Problem 5

Sin Approximation : $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

$\sin\left(\frac{1}{n}\right) - \frac{1}{n} = \left(\frac{1}{n} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots\right)$

$n = n, 3, 5, 7, 9$.

$\therefore O(n^{-n}) \Rightarrow O(n^{-3})$

Cosine Approximation :

$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \dots$

$\cos\left(\frac{1}{n}\right) - 1 + 1\ (2n^2) = \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \right.$

$\hookrightarrow + \frac{1}{2n^2}$

$O(n^{-n})$ decreases

$n = 2, 4, 6$