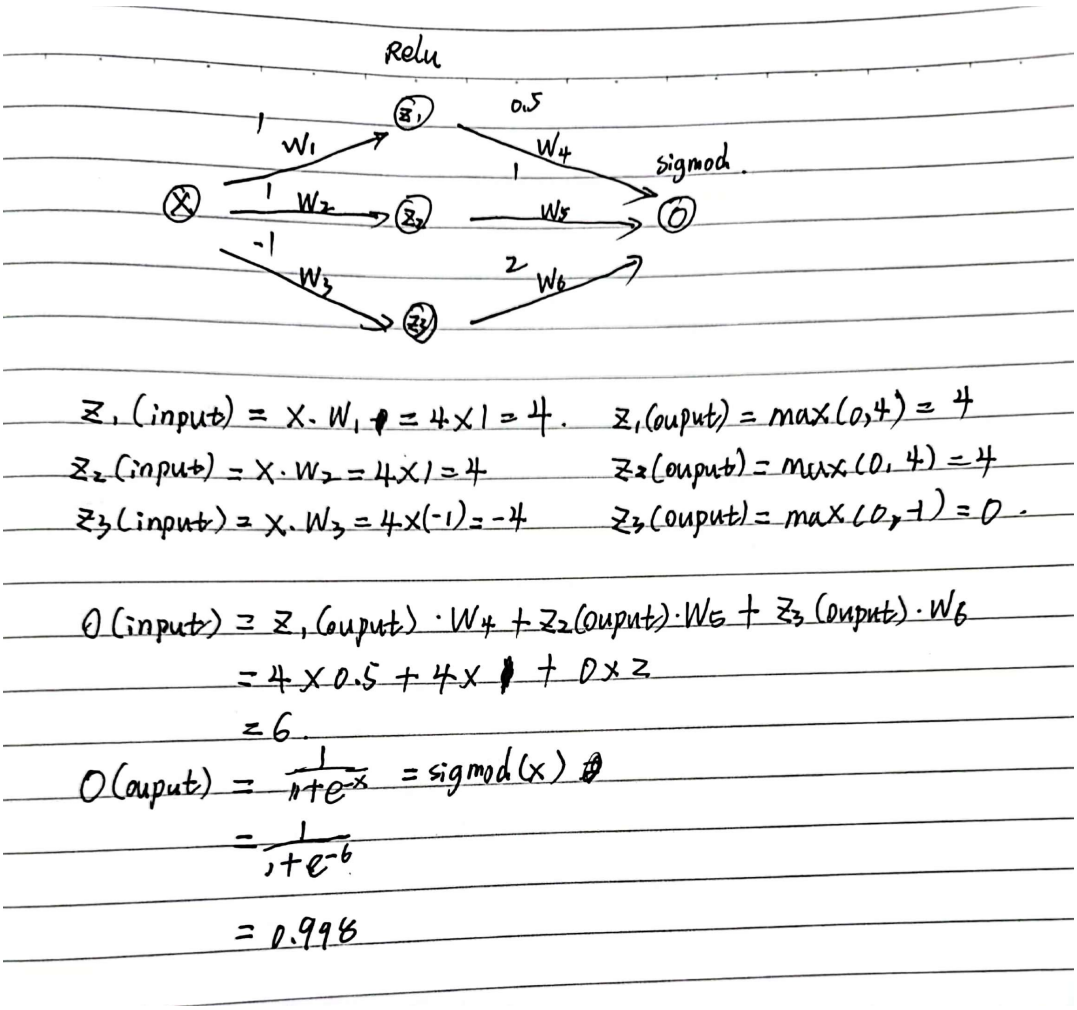
CS 422 Homework7

Xingli Li, Illinois Institute of Technology

1. Exercises

1.1-(a)

Answer:



$z_1(input) = X \cdot W_1 = 4 \times 1 = 4$. $z_1(ouput) = max(0, 4) = 4$

$z_2(input) = X \cdot W_2 = 4 \times 1 = 4$ $z_2(ouput) = max(0, 4) = 4$

$z_3(input) = X \cdot W_3 = 4 \times (-1) = -4$ $z_3(ouput) = max(0, -4) = 0$.

$O(input) = z_1(ouput) \cdot W_4 + z_2(ouput) \cdot W_5 + z_3(ouput) \cdot W_6$

$= 4 \times 0.5 + 4 \times 1 + 0 \times 2$

$= 6$.

$O(ouput) = \frac{1}{1+e^{-x}} = sigmod(x)$

$= \frac{1}{1+e^{-6}}$

$= 0.998$

1.1-(b)

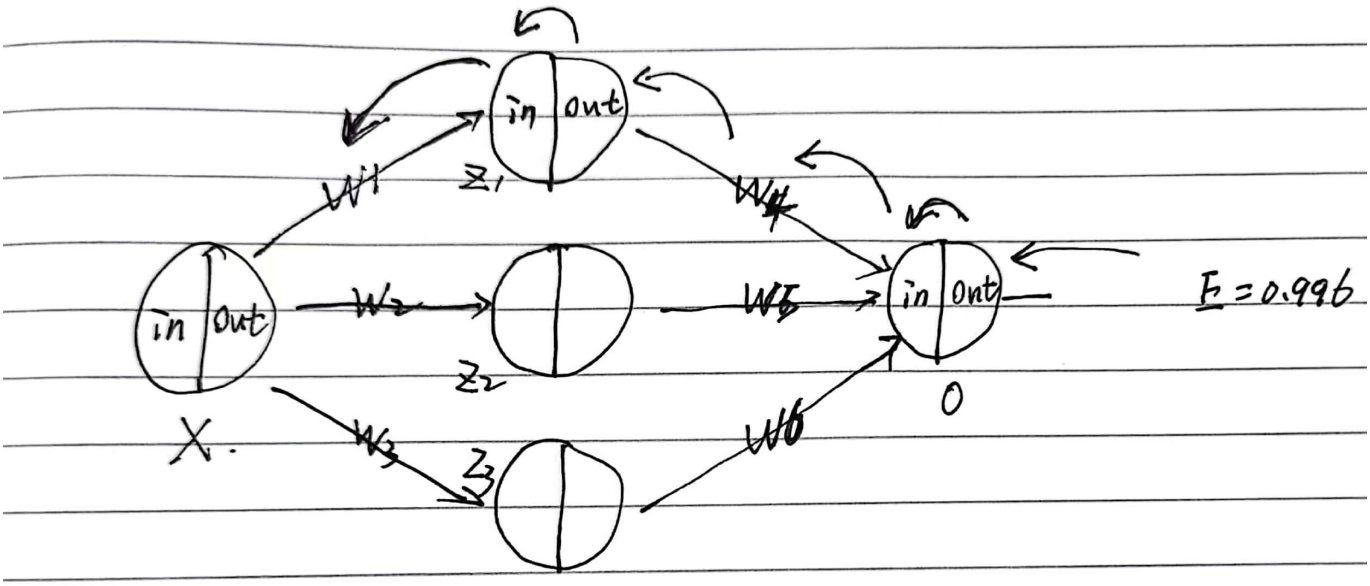
Answer:

$loss = (y - \hat{y})^2 = (0 - 0.998)^2 = 0.996$

1.1-(c)

Answer:

$\bigtriangledown E = [0.008, 0.016, 0.000, 0.016, 0.016, 0.000]^T$



$$\frac{\partial E}{\partial W_4} = \frac{\partial E}{\partial Out(O)} \times \frac{\partial Out(O)}{\partial in(O)} \times \frac{\partial in(O)}{\partial W_4}$$

$$\frac{\partial E}{\partial Out(O)} = -2(y - \hat{y}) = -1.996.$$

$$\frac{\partial Out(O)}{\partial in(O)} = sigmod' = out(O)(1 - out(O)) = 0.998(1 - 0.998)$$

$$\frac{\partial in(O)}{\partial W_4} = \frac{\partial (out(z_1) \cdot W_4)}{\partial W_4} = out(z_1) = Relu(z_1) = 4.$$

$$\frac{\partial E}{\partial W_4} = -1.996 \times 0.998(1 - 0.998) \times 4 = 0.016.$$

$$\frac{\partial E}{W1} = \frac{\partial E}{\partial out(z_1)} \cdot \frac{\partial out(z_1)}{\partial in((z_1)} \cdot \frac{\partial in(z)}{\partial W_1}$$

$$= \frac{\partial E}{\partial in(o_1)} \frac{\partial in(o_1)}{\partial out(z_1)} \cdot \frac{\partial out(z_1)}{\partial in(z_1)} \cdot \frac{\partial in(z_1)}{\partial W_1}$$

$$= \frac{\partial E}{\partial out(o_1)} \cdot \frac{\partial out(o_1)}{\partial in(o_1)} \cdot \frac{\partial in(o_1)}{\partial out(z_1)} \cdot \frac{\partial out(z_1)}{\partial out(z_1)} \cdot \frac{\partial in(z_1)}{\partial W_1}$$

$$= -2(0-0.998) \times 0.998(1-0.998) \times W_4 \times relu'(z_1) \cdot X$$

$$= -2(0-0.998) \times 0.998(1-0.998) \times 0.5 \times 1 \times 4$$

$$= 0.008$$

$$\frac{\partial E}{W_2} = \frac{\partial E}{\partial out(z_2)} \cdot \frac{\partial out(z_2)}{\partial in(z_2)} \cdot \frac{\partial in(z_2)}{\partial W_2}$$

$$= \frac{\partial E}{\partial in(o)} \cdot \frac{\partial in(o)}{\partial out(z_2)} \cdot \frac{\partial out(z_2)}{\partial in(z_2)} \cdot \frac{\partial in(z_2)}{\partial W_2}$$

$$= \frac{\partial E}{\partial out(o)} \cdot \frac{\partial out(o)}{\partial in(o)} \cdot [illegible] \cdot \frac{\partial in(o)}{\partial out(z_2)} \cdot \frac{\partial out(z_2)}{\partial in(z_2)} \cdot \frac{\partial in(z_2)}{\partial W_2}$$

$$= -2(0-0.998) \times 0.998(1-0.998) \times W_5 \times relu'(z_2) \cdot X$$

$$= 2 \times 0.998 \times 0.998(1-0.998) \times 1 \times 1 \times 4$$

$$= 0.016.$$

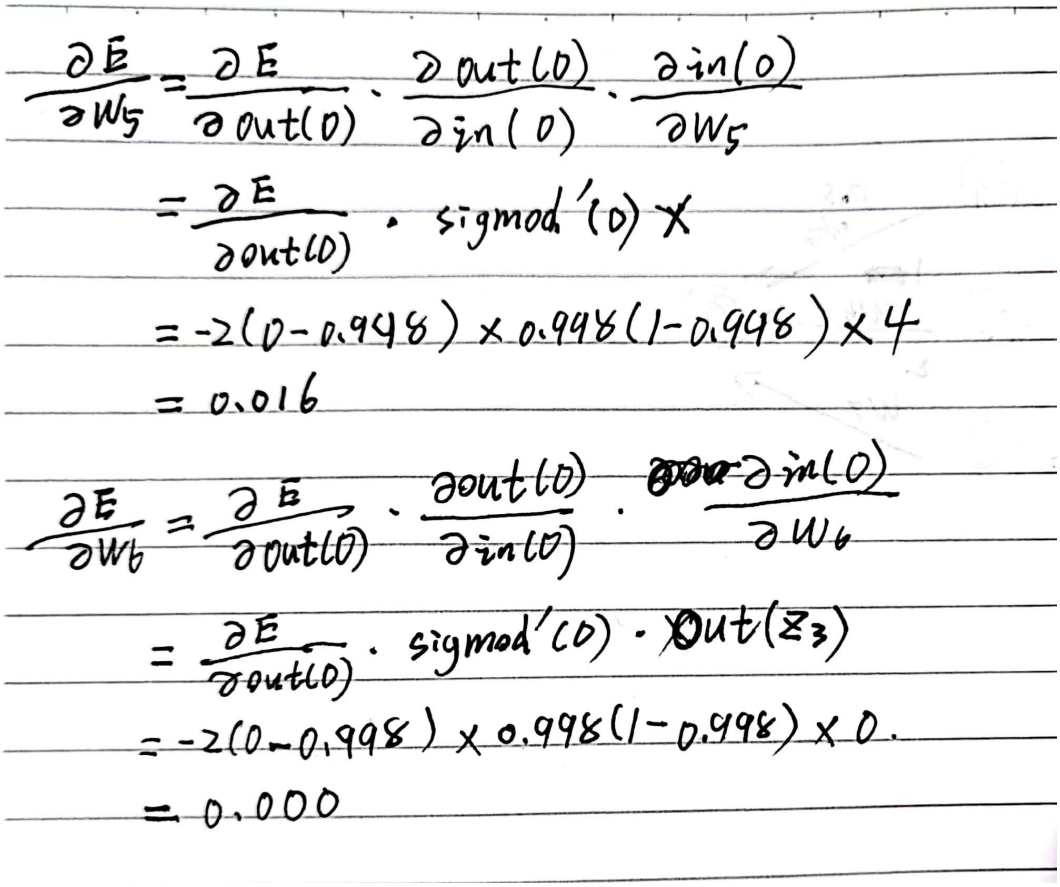
$$\frac{\partial E}{W_3} = \frac{\partial E}{\partial out(z_3)} \cdot \frac{\partial out(z_3)}{\partial in(z_3)} \cdot \frac{\partial in(z_3)}{\partial W_3}$$

$$= \frac{\partial E}{\partial out(o)} \cdot \frac{\partial out(o)}{\partial in(o)} \cdot \frac{\partial in(o)}{\partial out(z_3)} \cdot \frac{\partial out(z_3)}{\partial in(z_3)} \cdot \frac{\partial in(z_3)}{\partial W_3}$$

$$= 2 \times 0.998 \times 0.998(1-0.998) \times W_6 \times relu'(z_3)$$

$$= 2 \times 0.998 \times 0.998(1-0.998) \times 2 \times 0 \times 4$$

$$= 0.000$$



1.1-(d)

Answer:

w = w - $\eta \bigtriangledown E$

w1= 1-1*0.008 = 0.992

w2= 1-1*0.016 =0. 984

w3= -1-1*0.000 =-1

w4=0.5-1*0.016=0.484

w5=1-1*0.016=0.984

w6=2-1*0.000=2

O = Relu(z1)*0.484*+Relu(z2)*0.984*+Relu(z3)*2* =3.968*0.484+3.936*0.984+0=5.794

sigmod=$\frac{1}{1+e^{-5.794}}$ = 0.997

loss = (0-0.997)^2= 0.994

1.1-(e)

Answer:

```
In (b) and (d), the difference between the loss values represent  gradient
update.
Updated weight make the error reduction and accuracy improvement. The direction
of
the gradient is the direction of error reduction.
```