Lecture 6 (3.2 - 3.3)

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September 28, 2018

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3.2 Proving existentially quantified statements

$\exists x \in S, P(x)$
Prove that there exists $m \in \mathbb{Z}$ such that,

$$\frac{m-7}{2m+4} = 5$$

Let $m = -3$ then $m \in \mathbb{Z}$, and,

$$\begin{aligned}\frac{m-7}{2m+4} &= \frac{-3-7}{2(-3)+4} \\ &= \frac{-10}{-2} \\ &= 5\end{aligned}$$

Essentially, for proving existentially quantified statements, give one example.

Disproving existentially quantified statements

For disproving existentially quantified statements, prove the universally quantified statement, $\forall x \in S, \neg P(x)$.

Disprove: There exists $x \in \mathbb{R}$ such that $cos(2x) + sin(2x) = 3$.
Proof: Let $x \in \mathbb{R}$ then,

$$-1 \leq cos(2x) \leq 1 \; and \; -1 \leq sin(2x) \leq 1$$

Hence,

$$cos(2x) + sin(2x) \leq 1 + 1 = 2 < 3$$

There exists no x such that the statement could be true.

3.3 Proving implications

Assume hypothesis is true, and try and get to conclusion.
DO NOT ASSUME CONCLUSION

Example: