Lecture 4 (2.4 - 2.6)

Rui Li

September 28, 2018

Contents

Example:
Show that:

$$\neg(P \wedge (\neg Q \wedge R)) \equiv \neg(P \wedge Q) \vee R$$

Solution:

$$\begin{aligned}
\neg(P \wedge (\neg Q \wedge R)) &\equiv \neg P \vee \neg(\neg Q \wedge R) \ (By\ DML) \\
&\equiv \neg P \vee \neg\neg Q \vee \neg R \\
&\equiv \neg P \vee (Q \vee \neg R) \\
&\equiv \neg P \vee (\neg R \vee Q) \ (By\ Commutative\ Law) \\
&\equiv (\neg P \vee \neg R) \vee Q) \ (By\ Associative\ Law) \\
&\equiv \neg(P \wedge R) \vee Q \ (By\ DML)
\end{aligned}$$

2.4.1 Implication

Definition:

$$P \to Q$$
$$(P\ implies\ Q)$$
$$(If\ P\ then\ Q)$$

P	Q	$P \to Q$
True	True	True
True	False	False
False	True	True
False	False	True

$P \to Q$ is called a conditional statement, where P is the hypothesis and Q is the conclusion.

Examples:

P = "You Study", Q = "You pass", $P \to Q$ = "If you study then you pass"

For all real x, if $x > 2$ then $x^2 > 4$
For all real x, if $x \geq 2$ then $x^2 > 4$

Negating Implications

P	Q	$P \to Q$	$\neg P \vee Q$
True	True	True	True
True	False	False	False
False	True	True	True
False	False	True	True

$$P \to Q \equiv \neg P \vee Q$$

Thus,

$$\begin{aligned} \neg(P \to Q) &\equiv \neg(\neg P \vee Q) \\ &\equiv \neg\neg P \wedge \neg Q \\ &\equiv P \wedge \neg Q \end{aligned}$$

2.5 Converse and Contrapositive

Definition:
Converse of $P \to Q$ is $Q \to P$
Contrapositive of $P \to Q$ is $(\neg Q) \to (\neg P)$
However, $P \to Q \neq Q \to P$
Additionally $\neg Q \to \neg P \equiv P \to Q$

Example:
"If $a < b$ then $a \leq b$"
Converse: "If $a \leq b$ then $a < b$"
Contrapositive: "If $a > b$ then $a \geq b$"

2.6 If and Only If

Definition:
$P \leftrightarrow Q$ (P if and only if Q)

P	Q	$P \leftrightarrow Q$
True	True	True
True	False	False
False	True	False
False	False	False