Lecture 2 (1.2 - 1.4)

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1.2 - Sets

Definition: a set is a collection of objects, called it's elements.

Notation:
$a \in S$ (a is an element of S)
$a \notin S$ (a is not an element of S)

The Natural Numbers
$\mathbb{N}$ = {1, 2, 3..}
Thus $1 \in \mathbb{N}$, $2 \in \mathbb{N}$, etc. However $0 \notin \mathbb{N}$. (in MATH135)

The Integers
$\mathbb{Z}$ = {..-2, -1, 0, 1, 2..}
Thus $0 \in \mathbb{Z}$, $\pm 1 \in \mathbb{Z}$, $\frac{1}{2} \notin \mathbb{Z}$.

The Rational Numbers
$\mathbb{Q} = \{\frac{a}{b}$: a, b $\in \mathbb{Z}$, b$\neq 0\}$
Thus $1 = \frac{1}{1} \in \mathbb{Q}$; $\frac{1}{2} \in \mathbb{Q}$; $\sqrt{2} \notin \mathbb{Q}$ (Lecture 9).

The Real Numbers
$\mathbb{R} \to \sqrt{2} \in \mathbb{R}$; $\sqrt{-1} \notin \mathbb{R}$ (Lecture 9).

Examples:
Odd natural numbers less than 10 is {1, 3, 5, 7, 9}.
The set S = {1, 2, {3}} has three elements $1 \in S, 2 \in S, \{3\} \in S$ but $3 \notin S$.
{1, 2, 3} = {1, 3, 2} = {1, 2, 2, 3}

Order is not relevant with sets, neither is multiplicity (the second 2 above is redundant)

The Empty Set
$\varnothing = \{\}$ No elements.

Examples:
Set of integers both even and odd is $\varnothing$.
The set of elements common to both $\{3\}$, $\{\{3\}\}$ is $\varnothing$.
$\{\varnothing\}$ has one element, $\varnothing$ ($\varnothing \in \{\varnothing\}, \varnothing \notin \varnothing$).
$\{\varnothing, \{\varnothing\}\}$ has 2 elements.

1.3 - Statements and Negation

Definition: A statement is a sentence that is either true or false.
Examples of statements:
$1 + 1 = 2$
$1 + 1 = 3$
$\sqrt{2} \notin \mathbb{Q}$

Not statements:
$x^2 - x = 0$
$\frac{m-7}{2m+4} = 5$

Negation
Definition: Let p be a statement, the negation of p is denoted ¬P, and is the statement with the opposite truth value.

P	$\neg P$	$\neg(\neg P)$
True	False	True
False	True	False

Double negation has the same truth values as P, so P is logically equivalent to it's double negation.

$$\underline{P \equiv \neg(\neg P)}$$

Negation of 1+1=2 is $\neg(1 + 1 = 2)$, or $1 + 1 \neq 2$.
Negation of $\sqrt{2} \in \mathbb{Q}$ is $\sqrt{2} \notin \mathbb{Q}$. $\neg(\sqrt{2} \notin \mathbb{Q})$ is $\sqrt{2} \in \mathbb{Q}$ so $\neg(\neg(\sqrt{2} \in \mathbb{Q}))$ is $\sqrt{2} \in \mathbb{Q}$.

1.4 - Quantifiers

Definition:
A universally quantified statement is one of the form: $\forall x \in S, P(x)$
An existentially quantified statement: $\exists x \in S, P(x)$

Here:
$\forall$ "for all" - Universal
$\exists$ "there exists" - Existential

x is a variable
S is the domain of x (some set)
P(x) is an open sentence (one which becomes true or false when x is replaced by an element in S)

Examples:

$$\forall x \in \mathbb{R}, x^2 - x \geq 0 \tag{1}$$
$$\forall x \in \mathbb{Z}, x^2 - x \geq 0 \tag{2}$$
$$\exists x \in \mathbb{R}, x^2 - x < 0 \tag{3}$$
$$\exists x \in \mathbb{Z}, x^2 - x < 0 \tag{4}$$
$$\exists m \in \mathbb{Z}, \frac{m-7}{2m+4} = 5 \tag{5}$$
$$\exists m \in \mathbb{N}, \frac{m-7}{2m+4} = 5 \tag{6}$$
$$\forall m \in \mathbb{Z}, \frac{m-7}{2m+4} \neq 5 \tag{7}$$
$$\forall m \in \mathbb{N}, \frac{m-7}{2m+4} \neq 5 \tag{8}$$

(1) False
(2) True
(3) True
(4) False
(5) True, m = -3
(6) False
(7) False
(8) True

Practice:
a) 64 is a perfect square
b) $2^{2x-4} = 8$ has an integer solution
c) $y = x^3 - 2x + 1$ has x intercept
d) Every triangle is equivalent
e) $\sqrt{2} \notin \mathbb{Q}$