Lecture 2 (1.2 - 1.4)

Rui Li

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Contents

1.2 - Sets

Definition: a set is a collection of objects, called it's elements.

Notation:
$a \in S$ (a is an element of S)
$a \notin S$ (a is not an element of S)

The Natural Numbers
$\mathbb{N} = \{1, 2, 3..\}$
Thus $1 \in \mathbb{N}$, $2 \in \mathbb{N}$, etc. However $0 \notin \mathbb{N}$. (in MATH135)

The Integers
$\mathbb{Z} = \{..\text{-}2, \text{-}1, 0, 1, 2..\}$
Thus $0 \in \mathbb{Z}$, $\pm 1 \in \mathbb{Z}$, $\frac{1}{2} \notin \mathbb{Z}$.

The Rational Numbers
$\mathbb{Q} = \{\frac{a}{b}$: a, b $\in \mathbb{Z}$, b$\neq 0\}$
Thus $1 = \frac{1}{1} \in \mathbb{Q}; \frac{1}{2} \in \mathbb{Q}; \sqrt{2} \notin \mathbb{Q}$ (Lecture 9).

The Real Numbers
$\mathbb{R} \to \sqrt{2} \in \mathbb{R}; \sqrt{-1} \notin \mathbb{R}$ (Lecture 9).

Examples:
Odd natural numbers less than 10 is $\{1, 3, 5, 7, 9\}$.
The set S = $\{1, 2, \{3\}\}$ has three elements $1 \in S, 2 \in S, \{3\} \in S$ but $3 \notin S$.
$\{1, 2, 3\} = \{1, 3, 2\} = \{1, 2, 2, 3\}$

Order is not relevant with sets, neither is multiplicity (the second 2 above is redundant)

The Empty Set
$\varnothing = \{\}$ No elements.

Examples:
Set of integers both even and odd is $\varnothing$.
The set of elements common to both $\{3\}$, $\{\{3\}\}$ is $\varnothing$.
$\{\varnothing\}$ has one element, $\varnothing$ ($\varnothing \in \{\varnothing\}, \varnothing \notin \varnothing$).
$\{\varnothing, \{\varnothing\}\}$ has 2 elements.

1.3 - Statements and Negation

Definition: A statement is a sentence that is either true or false.
Examples of statements:
$1 + 1 = 2$
$1 + 1 = 3$
$\sqrt{2} \notin \mathbb{Q}$

Not statements:
$x^2 - x = 0$
$\frac{m-7}{2m+4} = 5$

Negation
Definition: Let p be a statement, the negation of p is denoted ¬P, and is the statement with the opposite truth value.

P	$\neg P$	$\neg(\neg P)$
True	False	True
False	True	False

Double negation has the same truth values as P, so P is logically equivalent to it's double negation.

$$P \equiv \neg(\neg P)$$

Negation of 1+1=2 is $\neg(1 + 1 = 2)$, or $1 + 1 \neq 2$.
Negation of $\sqrt{2} \in \mathbb{Q}$ is $\sqrt{2} \notin \mathbb{Q}$. $\neg(\sqrt{2} \notin \mathbb{Q})$ is $\sqrt{2} \in \mathbb{Q}$ so $\neg(\neg(\sqrt{2} \in \mathbb{Q}))$ is $\sqrt{2} \in \mathbb{Q}$.

1.4 - Quantifiers