Hanging Mobile

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1 The Problem

In this paper, we assume a hanging mobile is a set of point nodes and lines between nodes, as shown in Figure 1 (please ignore sticks...). The problem is to minimize the total length of lines between nodes and the number of lines connecting the the ceiling.

We want to minimize the total length of lines between nodes, because it saves the material and simplify the structure. Minimizing the number of lines

Figure 1: A hanging mobile art example

2 Method

Figure 2 is the force analysis of a node in a hanging mobile, where $F_1, F_2 \geq 0$ are forces along lines associate to the node, $x_1, x_2 \in \{0, 1\}$ indicates if the line exists (1) or not (0), mg is its gravity and $F_{yp}, F_{yn}, F_{xp}, F_{xn} \geq 0$ are decomposition of an external force $F_{ex} = (F_{xp} - F_{xn}, F_{yp} - F_{yn})$. Let $x_e \in \{0, 1\}$ be the variable indicating whether or not an external line connecting to the ceiling exists.

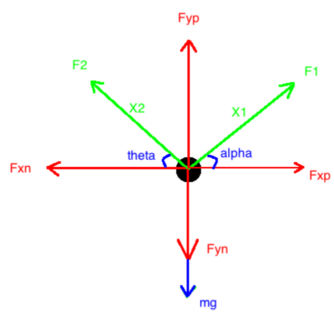

Figure 2: Force analysis of a node

2.1 Integer Linear Programming

We can formalize the problem as an integer linear programming problem.

2.1.1 Unknowns

The unknowns are:

$$X = [x_1, x_2, x_e, F_1, F_2, F_{yp}, F_{yn}, F_{xp}, F_{xn}] \tag{1}$$

2.1.2 Inequality Constraints

Inequality constraints are the domain of variables.

$$F_1, F_2, F_{yp}, F_{yn}, F_{xp}, F_{xn} \geq 0 \tag{2}$$

$$X_1, X_2, X_e \in 0, 1 \tag{3}$$

We can also add max force each line can resist by adding upper bound to each force.

2.1.3 Equality Constraints

Equality constraints defines the force equilibrium.

$$F_1 sin(\alpha) + F_{xp} - F_2 sin(\theta) - F_{xn} = 0 \tag{4}$$

$$F_1 cos(\alpha) + F_2 cos(\theta) + F_{yp} - mg - F_{yn} = 0 \tag{5}$$

2.1.4 Special Constraints

Special constraints defines the relationship between forces and lines.

1. if $F_1 = 0$, then $x_1 = 0$; else $x_1 = 1$
2. if $F_2 = 0$, then $x_2 = 0$; else $x_2 = 1$
3. if $F_{yp} + F_{yn} + F_{xp} + F_{xn} = 0$, then $x_e = 0$; else $x_e = 1$

which means if F_1 is not 0, the line x_1 must exist.

These if-else condition between variables can be written in a linear form:

$$\begin{cases} F_1 + M(1 - x_1) > 0 \\ F_1 - Mx_1 \leq 0 \end{cases} \tag{6}$$

$$\begin{cases} F_2 + M(1 - x_2) > 0 \\ F_2 - Mx_2 \leq 0 \end{cases} \tag{7}$$

$$\begin{cases} (F_{yp} + F_{yn} + F_{xp} + F_{xn}) + M(1 - x_e) > 0 \\ (F_{yp} + F_{yn} + F_{xp} + F_{xn}) - Mx_e \leq 0 \end{cases} \tag{8}$$

where $M \gg 0$ is a very large positive value.

2.1.5 Objective Function

Our objective is to minimize the total length of lines between nodes and the number of lines connecting the the ceiling.

Let the length of each line i be l_i. The objective function can be written as:

$$f(X) = l_1 x_1 + l_2 x_2 + L x_e \tag{9}$$

or

$$f(X) = \sum l_i x_i + \sum L x_{ei} \tag{10}$$

where $L > \sum_i l_i$ is a large positive number, and $\sum l_i x_i$ is the total length of lines between nodes.

With a lot of nodes. The solver must firstly minimize the sum of L, which is the number of nodes connected to the ceiling. Because, if $\sum L$ is not minimized, no matter how small $\sum_i l_i x_i$ is, it doesn't matter. When $\sum L$ is minimized, the solver will then minimize $suml_i x_i$.

References