"We are now coming to realise that human behaviour is determined as much by social context as by rational thinking"

–Alex Pentland

The Stubborn-Effect in Opinion Dynamics - an attempt to forecast “Brexit”

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Agenda

1. Introduce background
2. Introduce the Galam sequential model
3. Model results
4. Application to “Brexit” forecast

What is agent-based modelling?

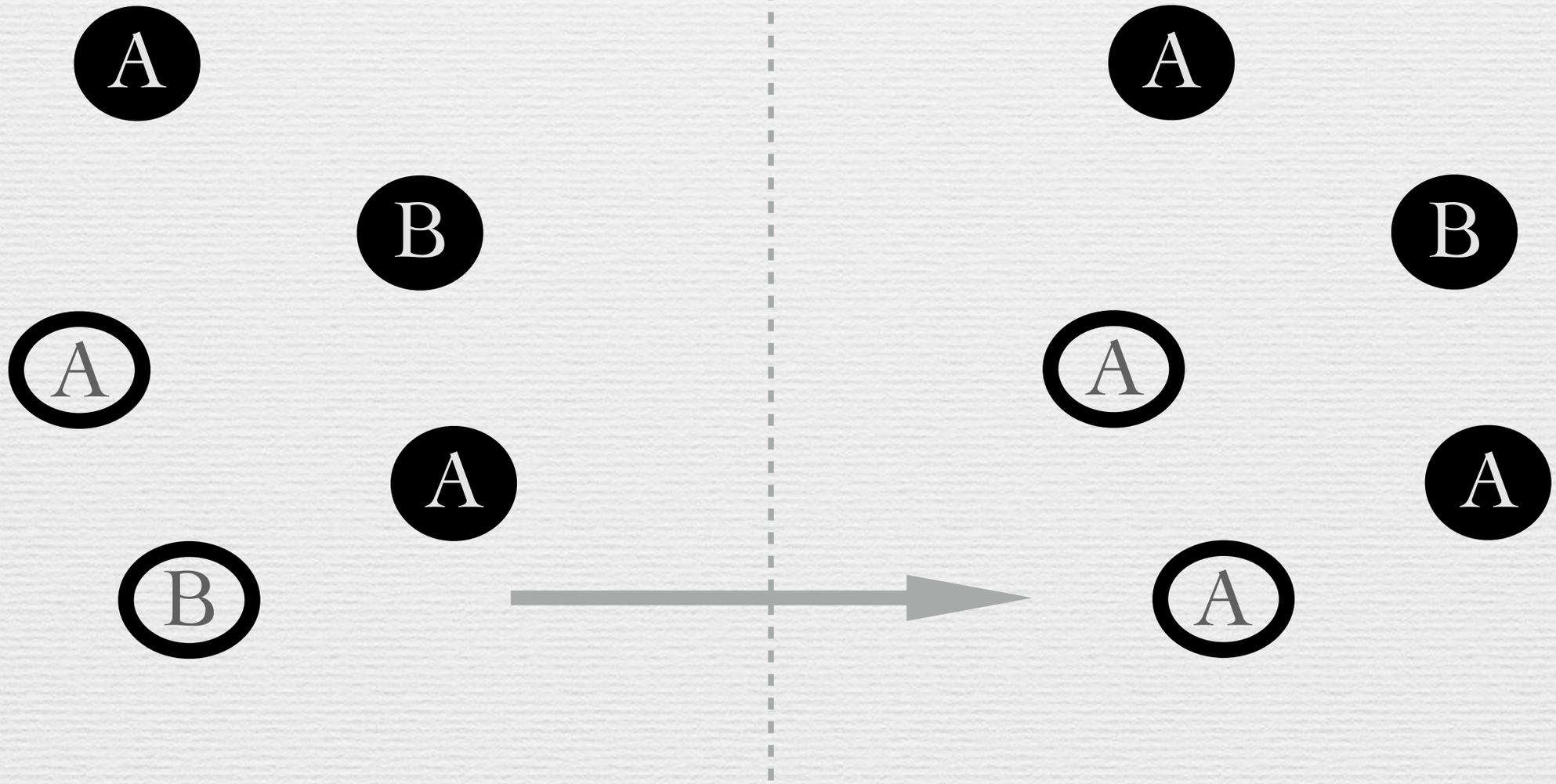
- "An **agent-based model** (**ABM**) is a class of computational models for simulating the actions and interactions of autonomous agents with a view to assessing their effects on the system as a whole."

Galam Sequential Model

- Assumption: there are individuals in the society who are "louder" and more influential than others.
- Some definitions:
  - A- B opinion: A = Brexit
  - Agent nature: stubborn (a, b) or open-minded
  - P_t= support for A at time t
- "Social rules" : distribute, update, shuffle

Interaction example

Solid = Stubborn,
Hollow=open-minded
A is local majority



Studying theoretical model

- Q: studying how does the initial make-up of stubborn agents alter the dynamics of the system
- Study for r=3 case, polynomial of degree 3.

Key Equations

~ Binomial(r , p_t) :

two outcomes, group size r, one outcome with probability p_t

Simple Model:

$$p_{t+1} = \sum_{m=\frac{r+1}{2}}^{r} \binom{r}{m} p_t^m (1-p_t)^{r-m}$$

Mixed Model:

$$p_{t+1} = \sum_{m=\frac{r+1}{2}}^{r} \binom{r}{m} p_t^m (1-p_t)^{r-m} - \Sigma_1 + \Sigma_2$$

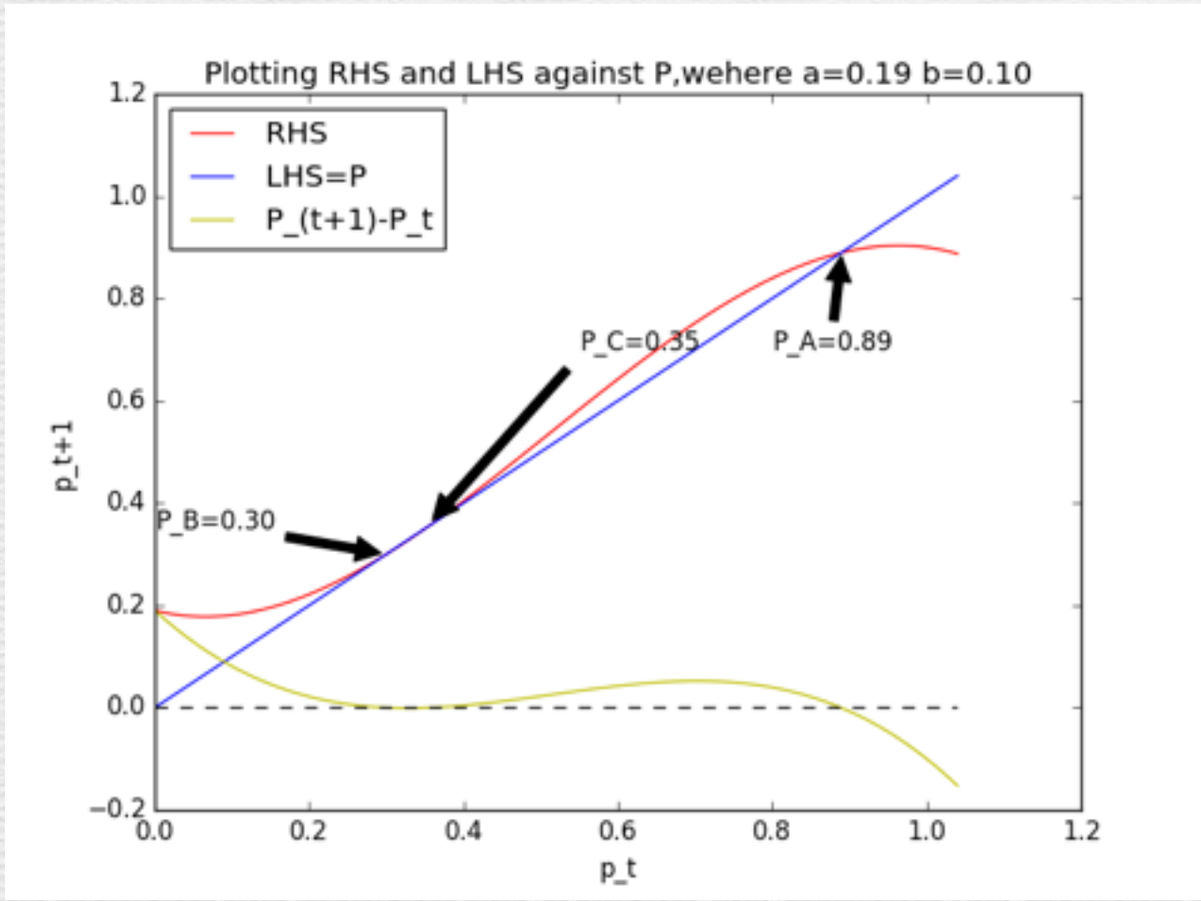
where

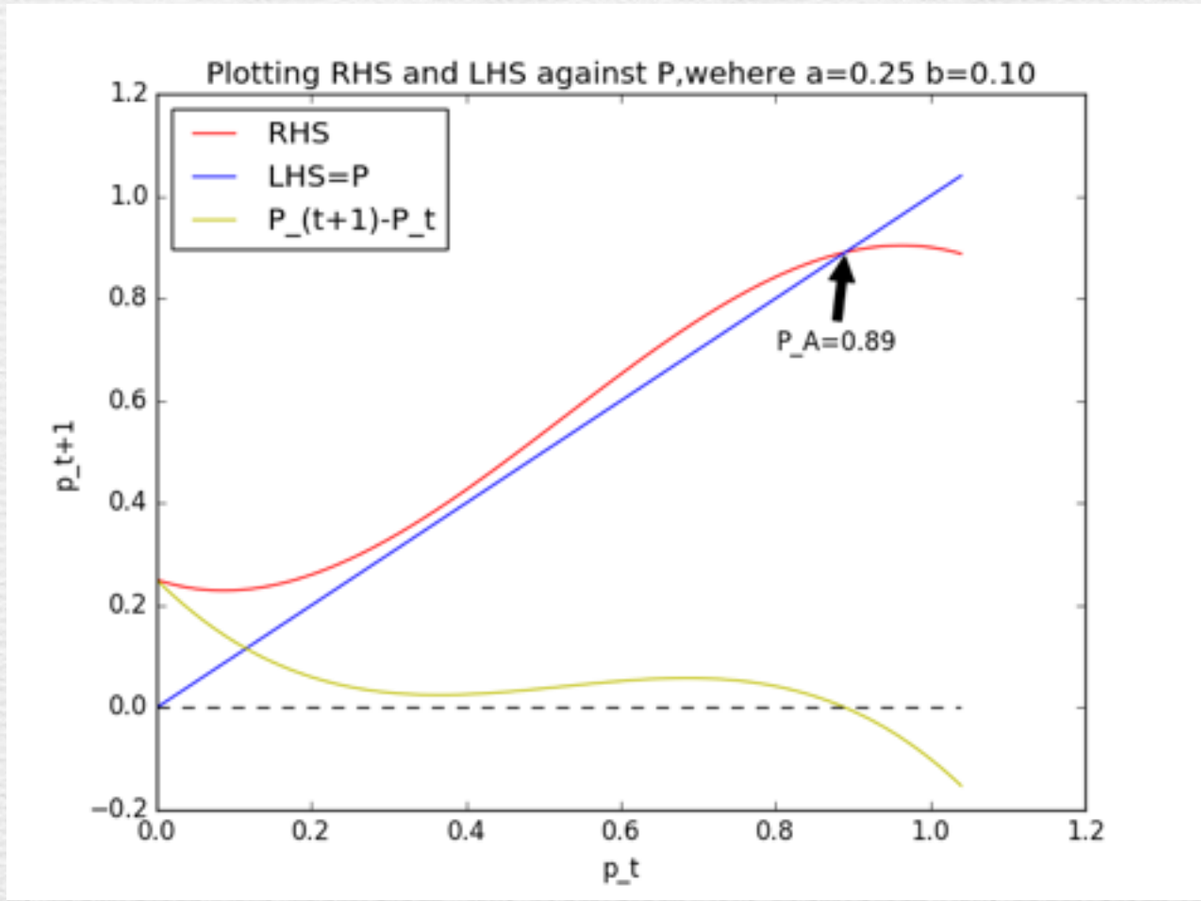
$$\Sigma_1 = \sum_{m=\frac{r+1}{2}}^{r} \sum_{k=0}^{r-m} \binom{r}{m,k,r-m-k} p_t^m b^k (1-p_t-b)^{r-m-k} \frac{k}{r}$$

$$\Sigma_2 = \sum_{m=\frac{r+1}{2}}^{r} \sum_{k=0}^{r-m} \binom{r}{m,k,r-m-k} (1-p_t)^m a^k (p_t-a)^{r-m-k} \frac{k}{r}$$

Sample results for mixed Model, r=3:

$$p_{t+1} = p_t^3 + 3p_t^2(1-p_t) - p_t^2 b + (1-p_t)^2 a$$

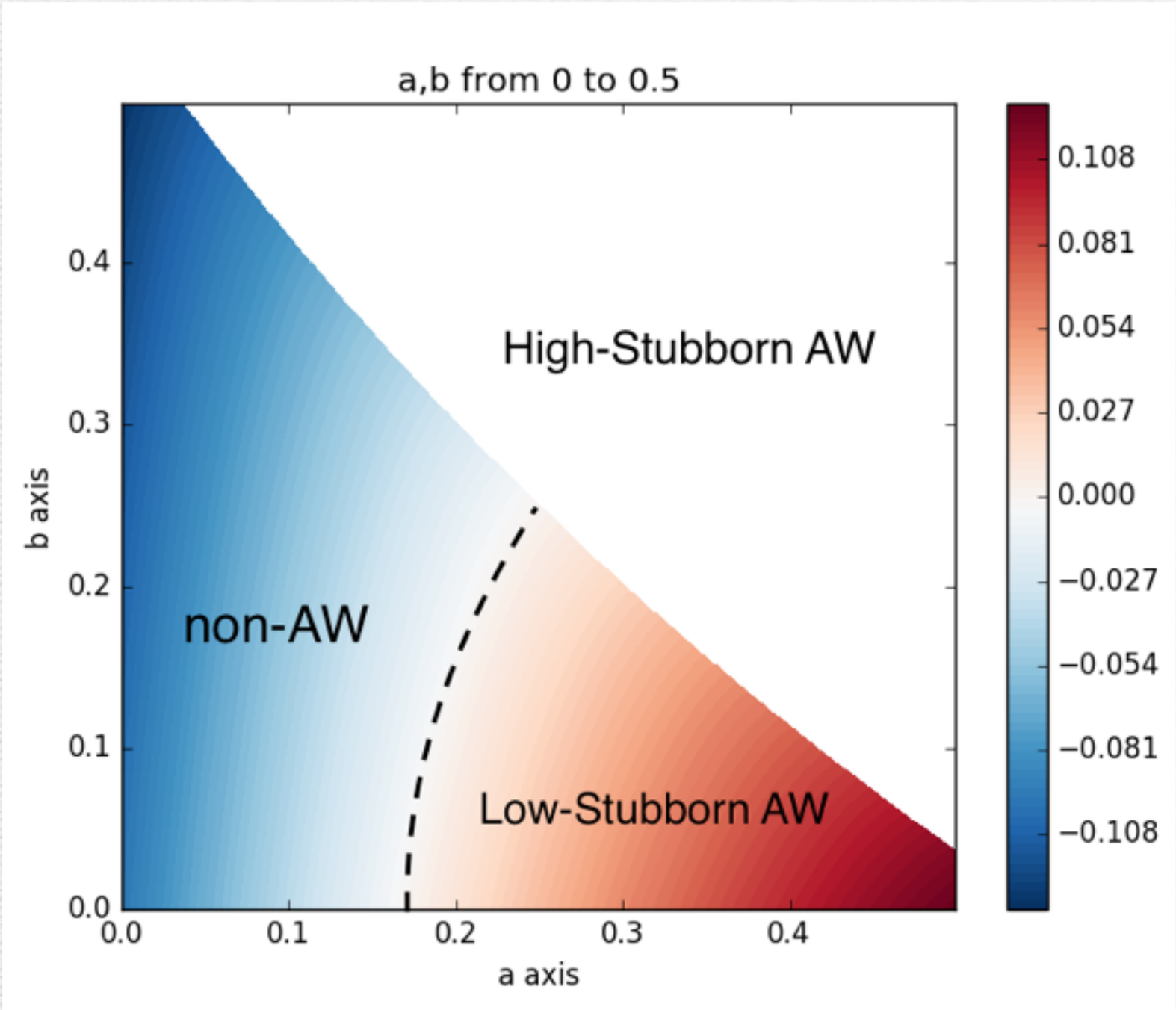




P_C =0.35, we need 35% "Brexit" votes in a poll to have the "Brexit". When stubborn-A is higher than stubborn-B, P_C skewed to the left.

P_C does not exist, P_A is the only fixed point: at this level of stubborn-A and stubborn-B, initial support doesn't matter, the outcome is determined. We call this AW (absolute winning) situation.

Theoretical model conclusion:



- AW: outcome is determined by the level of stubborn agents.
- non-AW: outcome depends on the initial support.
- Colour intensity: how "certain" we are about our forecast.

Changing r size

1. Computer simulation

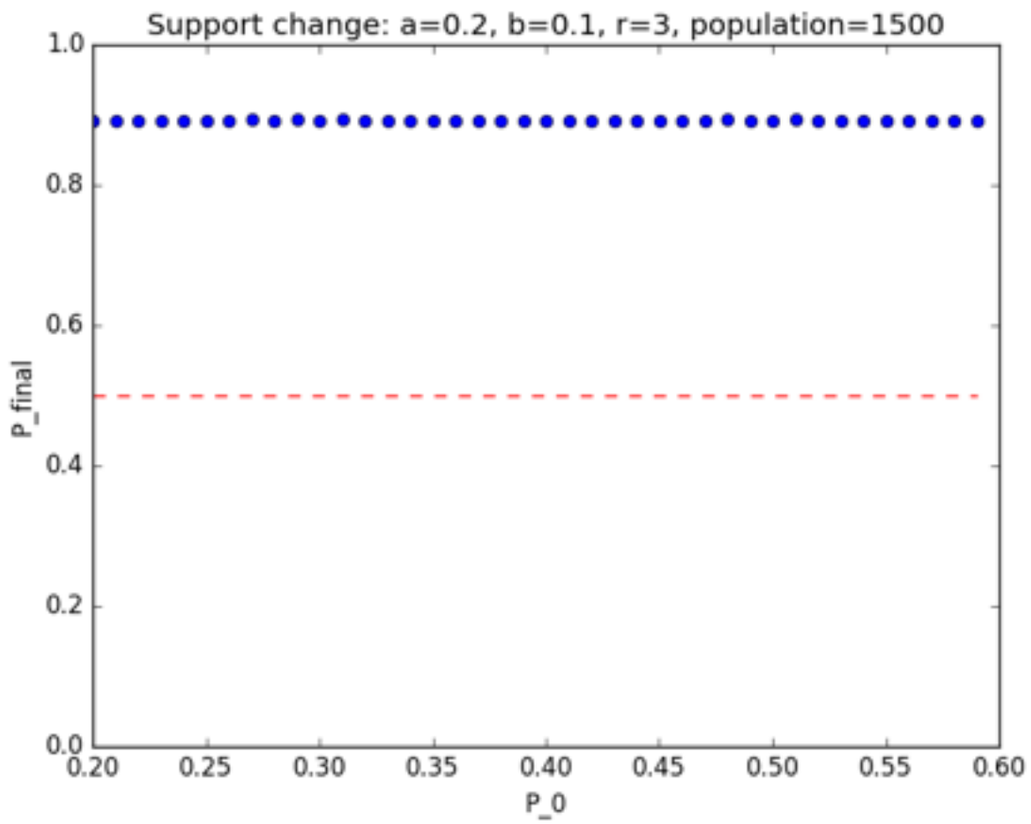
2. Sufficient population size: 1500

3. Measure P_final: after infinite iterations (shuffles)

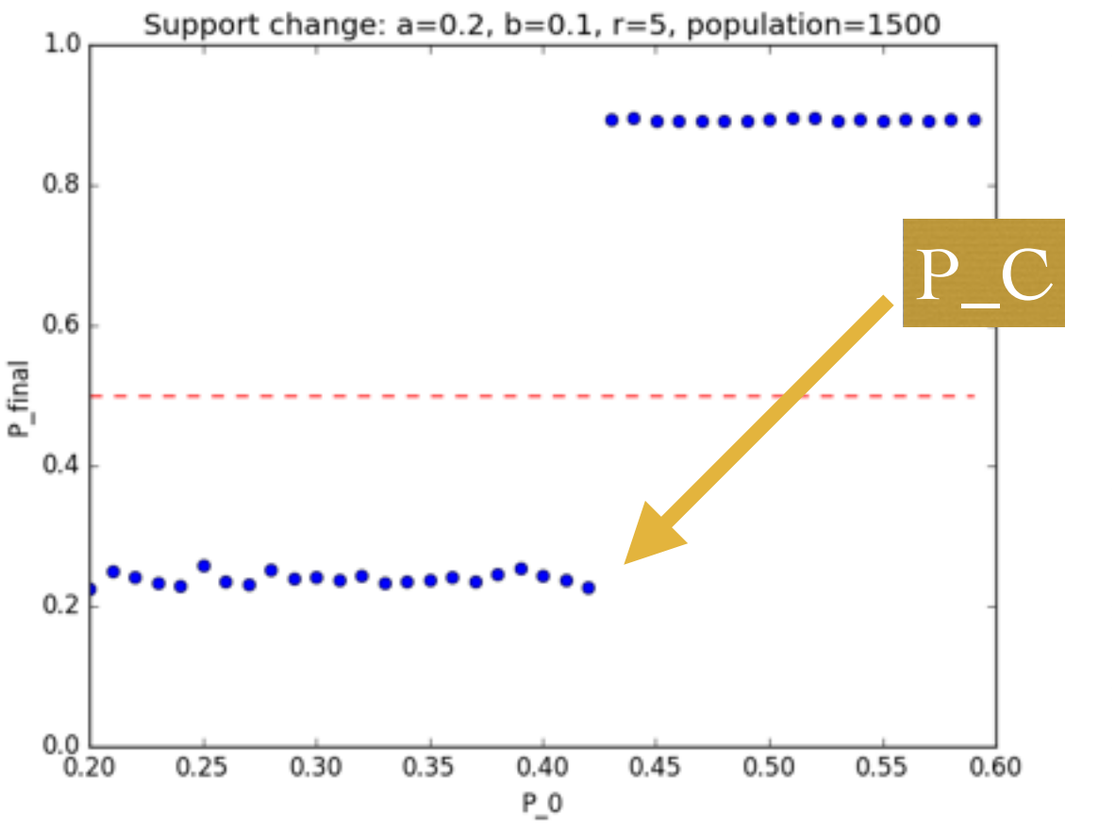
4. “Jump” means non-AW, and vice versa.

Sample results

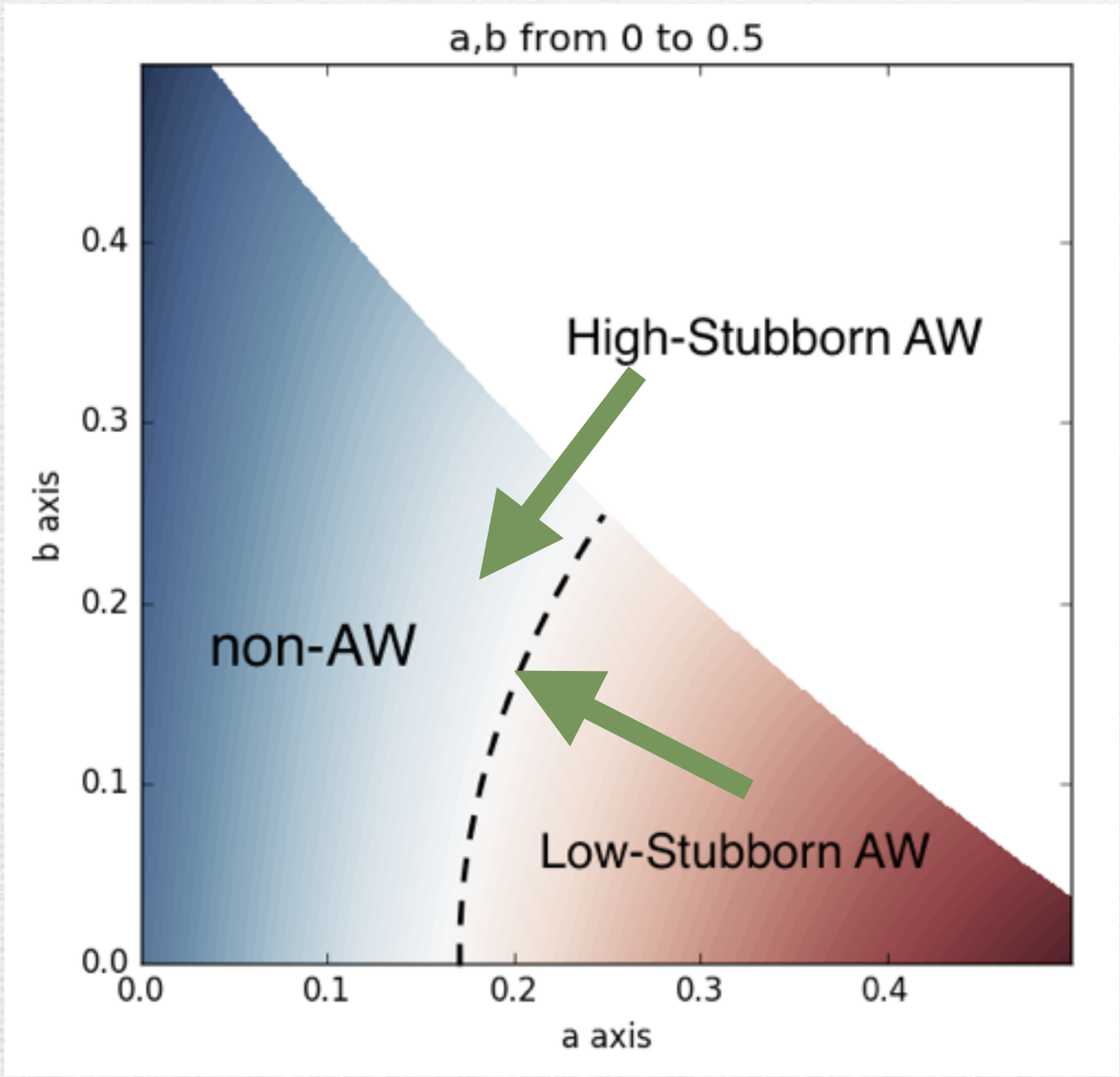
r=3: AW situation



r=5: non-AW situation



Computer simulation results:



- Theoretical calculations are verified
- Smaller r lead to enhanced stubborn-effect

Model conclusions:

1. The level of stubborn agents separate the outcome into 2 situations:
   - AW: outcome is determined by stubborn agents
   - non-AW: outcome depends on the initial support.
2. Depending on the group size, as r decrease, stubborn-effect is more dramatic.

Voting Context: “Brexit”

Three remarkable points:

1. Increasing level of connectivity: internet reduce r

2. High level of open minded: younger generation

3. Emotional campaigns -> angry & stubborn voter

Forecast strategy:
using poll data to estimate a, b, P_0

ICM survey

	Total	Remain a member of EU	Leave the EU
Unweighted base	2030	830	796
Weighted base	2030	802	822
10 - Absolutely certain to vote	1269 62%	534 67%	621 76%
9	208 10%	87 11%	83 10%
8	155 8%	64 8%	63 8%
7	88 4%	47 6%	25 3%
6	48 2%	18 2%	10 1%
5	72 4%	34 4%	13 2%
4	11 1%	4 *	1 *
3	15 1%	5 1%	2 *
2	23 1%	11 1%	3 *
1 - Certain not to vote	46 2%	- -	- -
Don't know	97 5%	- -	- -

YouGov / The Times Survey Results

Sample Size: 2001 GB Adults
Fieldwork: 5th - 6th June 2016

		EU Referendum			
	Total	Remain	Leave	Would not vote	Don't know
Weighted Sample	2001	867	838	74	223
Unweighted Sample	2001	885	843	56	217
	%	%	%	%	%

	30-31 May	5-6 June				
If there was a referendum on Britain's membership of the European Union and this was the question, how would you vote: Should the United Kingdom remain a member of the European Union or leave the European Union?						
Remain a member of the European Union	*41*	43	100	0	0	0
Leave the European Union	*41*	42	0	100	0	0
Would not vote	*4*	4	0	0	100	0
Don't know	*13*	11	0	0	0	100
And even if you aren't sure yet, which way do you think you will end up voting on the 23rd June? *[Asked to respondents who said they 'Don't know' how they would vote in an EU referendum; n=217]*						
Will probably end up voting to remain a member of the European Union	*21*	21	*0*	*0*	*0*	21
Will probably end up voting to leave the European Union	*17*	20	*0*	*0*	*0*	20
Will probably end up not voting	*11*	11	*0*	*0*	*0*	11
Don't know	*51*	48	*0*	*0*	*0*	48
And on a scale of 0 (certain NOT to vote) to 10 (absolutely certain to vote), how likely are you to vote in the EU referendum?						
0 - Certain NOT to vote	*6*	5	1	1	75	13
1	*1*	1	0	0	5	1
2	*1*	1	1	1	9	2
3	*2*	1	0	1	0	3
4	*1*	1	1	1	5	4
5	*5*	4	4	2	4	15
6	*2*	3	2	2	0	8
7	*4*	3	3	3	1	6
8	*7*	6	6	5	0	5
9	*6*	8	9	8	0	6
10 - Absolutely certain to vote	*66*	67	72	77	1	36

Source: https://www.icmunlimited.com/wp-content/uploads/2016/06/13-Jun.pdf

Raw data & estimation methodology

ICM		stubborn leave	stubborn remain	total leave votes	total remain votes	Total voters
	08/04/2016	621	534	796	866	1945
	20/05/2016	651	548	783	885	1946
YouGov						
	25/04/2016	482	512	688	711	1650
	05/06/2016	649	637	843	885	1945
	17/06/2016	608	574	742	746	1641
Survation						
	15/06/2016	333	291	416		815

total voters = vote leave + vote remain + uncertain
a= absolutely certain leave / total voters
b= absolutely certain remain / total voters
P_0= leave / (leave+ remain)

Poll Data

		a	b	P_0
ICM Survey - ONLINE & Telephone				
	08/04/2016	32%	27%	48%
	20/05/2016	33%	28%	47%
YouGov Survey - ONLINE				
	25/04/2016	29%	31%	49%
	05/06/2016	33%	33%	49%
	17/06/2016	37%	35%	50%
Survation - Telephone				
	15/06/2016	41%	36%	51%

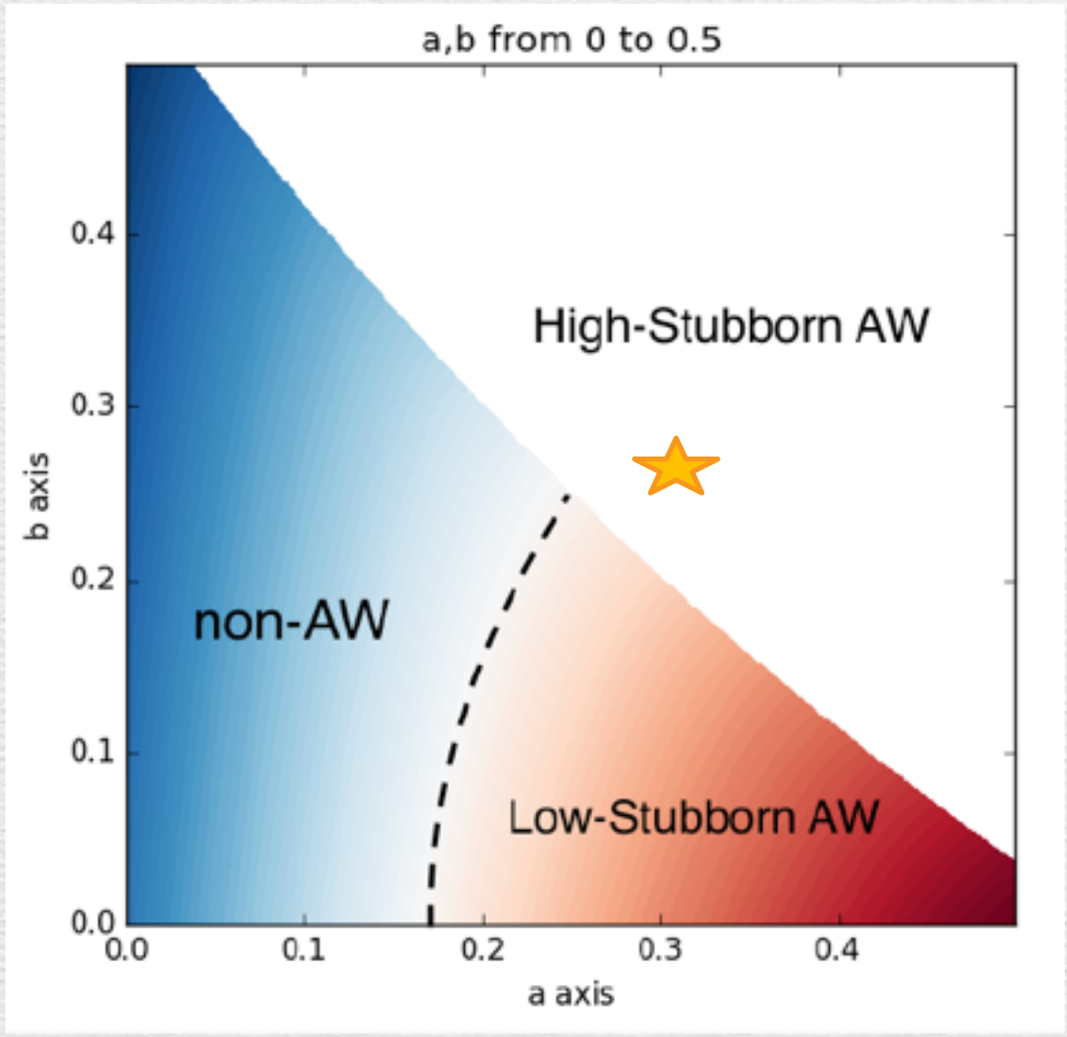
		Model Prediction	Poll Prediction
ICM Survey			
	08/04/2016	Brexit	Brexit
	20/05/2016	Brexit	N/A
YouGov Survey			
	25/04/2016	Remain	Brexit
	05/06/2016	Remain - uncertain	Remain
	17/06/2016	Brexit	Remain
Survation			
	15/06/2016	Brexit	Brexit

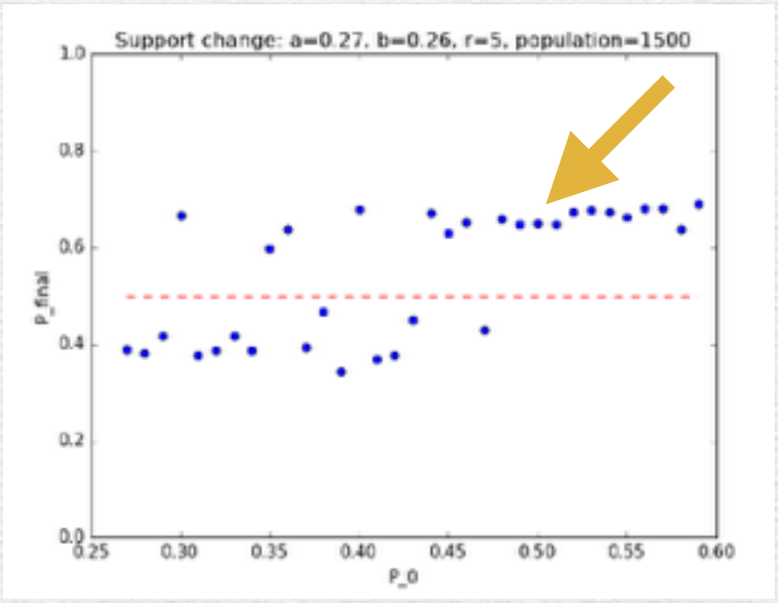
- Model prediction and poll prediction differ. In general, model prediction outperforms poll prediction.
- For some poll results: e.g. ICM, performance is persistent, and with high conviction from an early date.

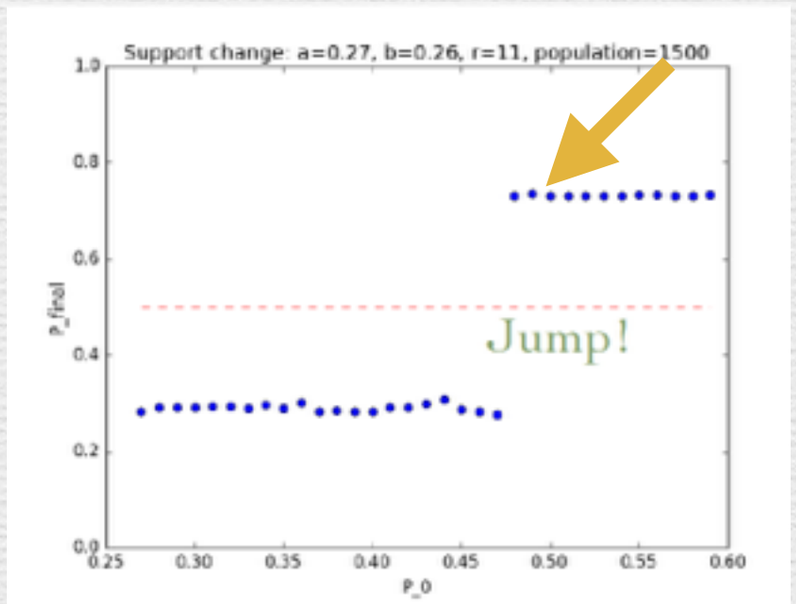
In some cases, assumption for r size doesn't matter…

ICM Survey April 8, 2016		
a	b	P_0
32%	27%	48%

If we test for r=5,7, result is the same that Brexit happens. Because initial support P_0 surpasses critical point.







Further Research:

- Open to ideas & suggestions :)
- Network Structure
- Drop the assumption of infinite shuffle
- People become stubborn after a while

Thank you

Special thanks to: Andrew & Rob H

References

- Backstrom, L., Boldi, P., Rosa, M., Ugander, J. & Vigna, S. (2011) Four Degrees of Separation.
- Galam, S. (2011) Collective beliefs versus individual inflexibility: The unavoidable biases of a public debate. *Physica A: Statistical Mechanics and its Applications.* 390 (17), 3036-3054.
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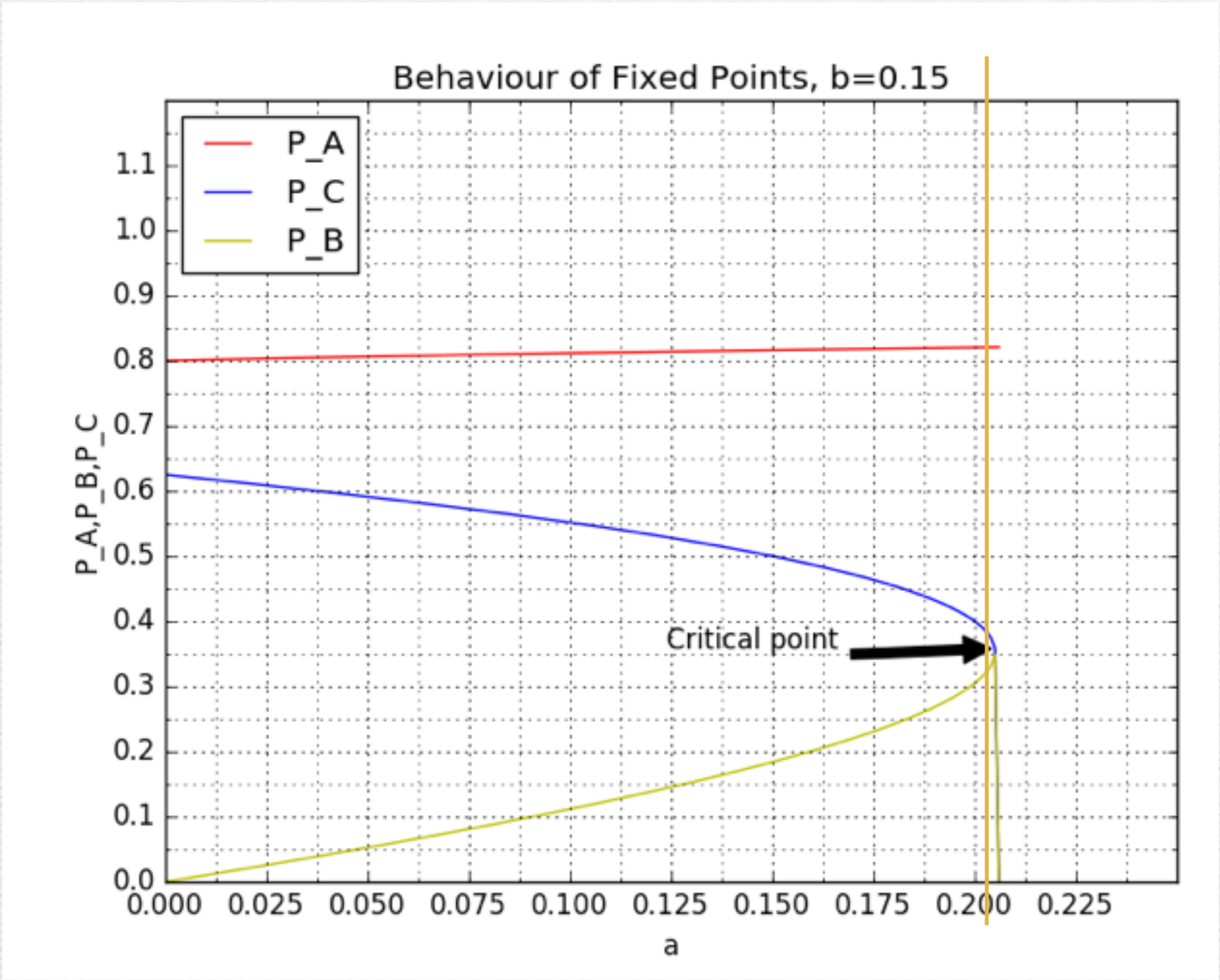
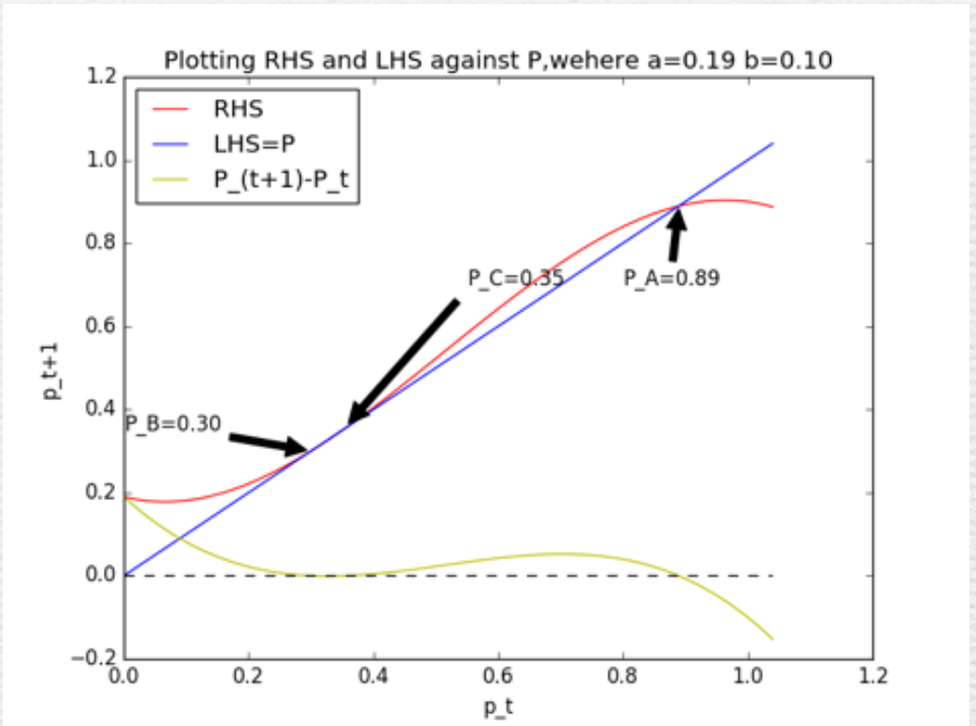
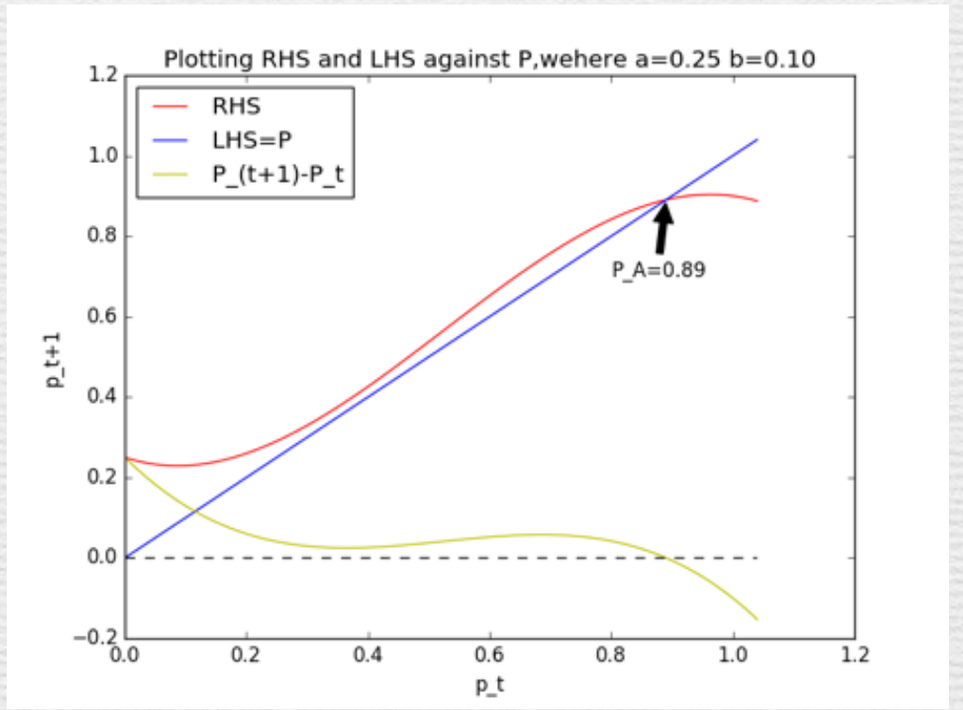
- Pentland, Alex,,. (2014) *Social physics : how good ideas spread-the lessons from a new science.*
- White, M. (July 2) **The Brexit vote aftermath, explained: a wild week in UK politics**. *The Guardian.*

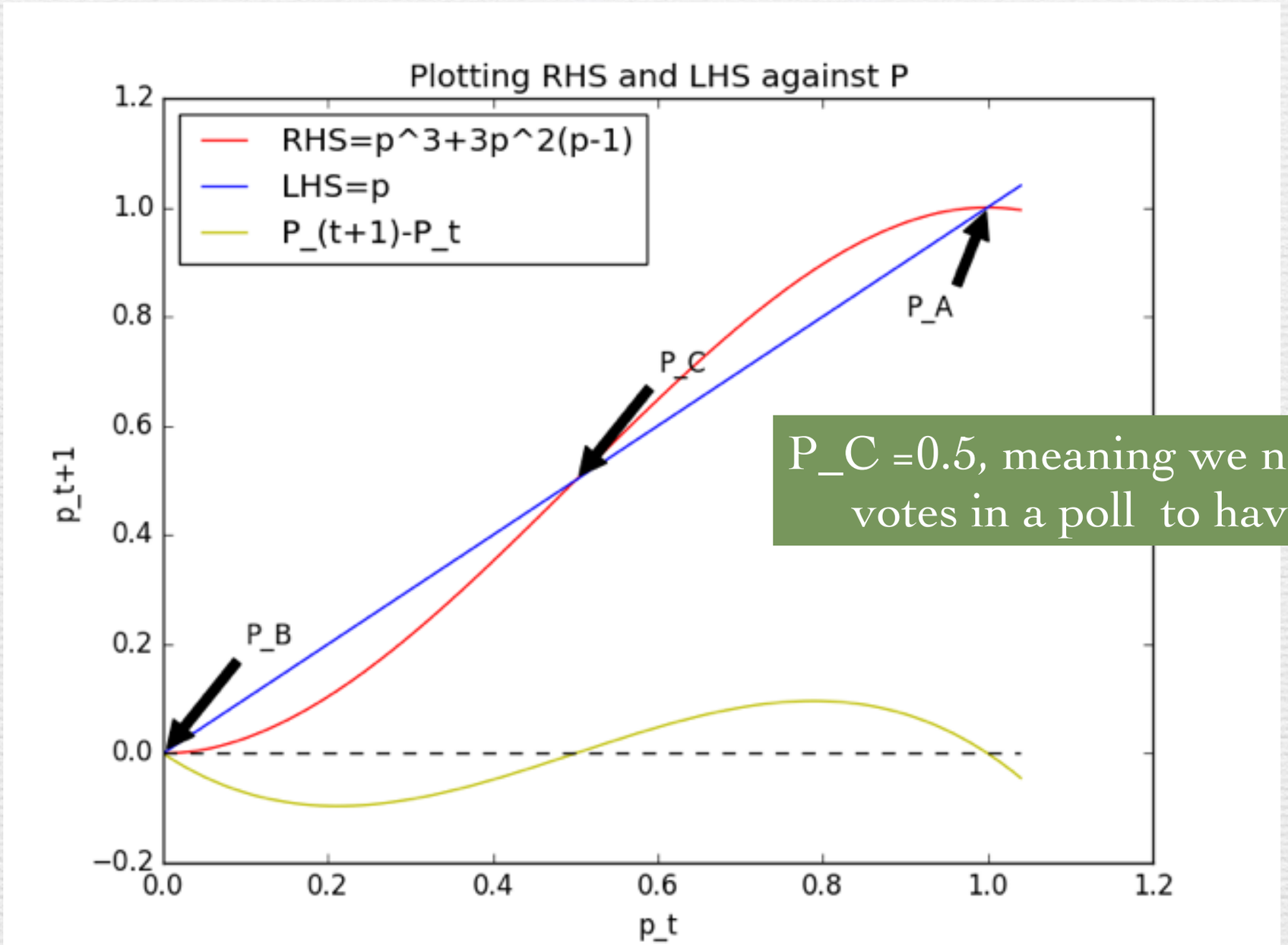
One-parameter





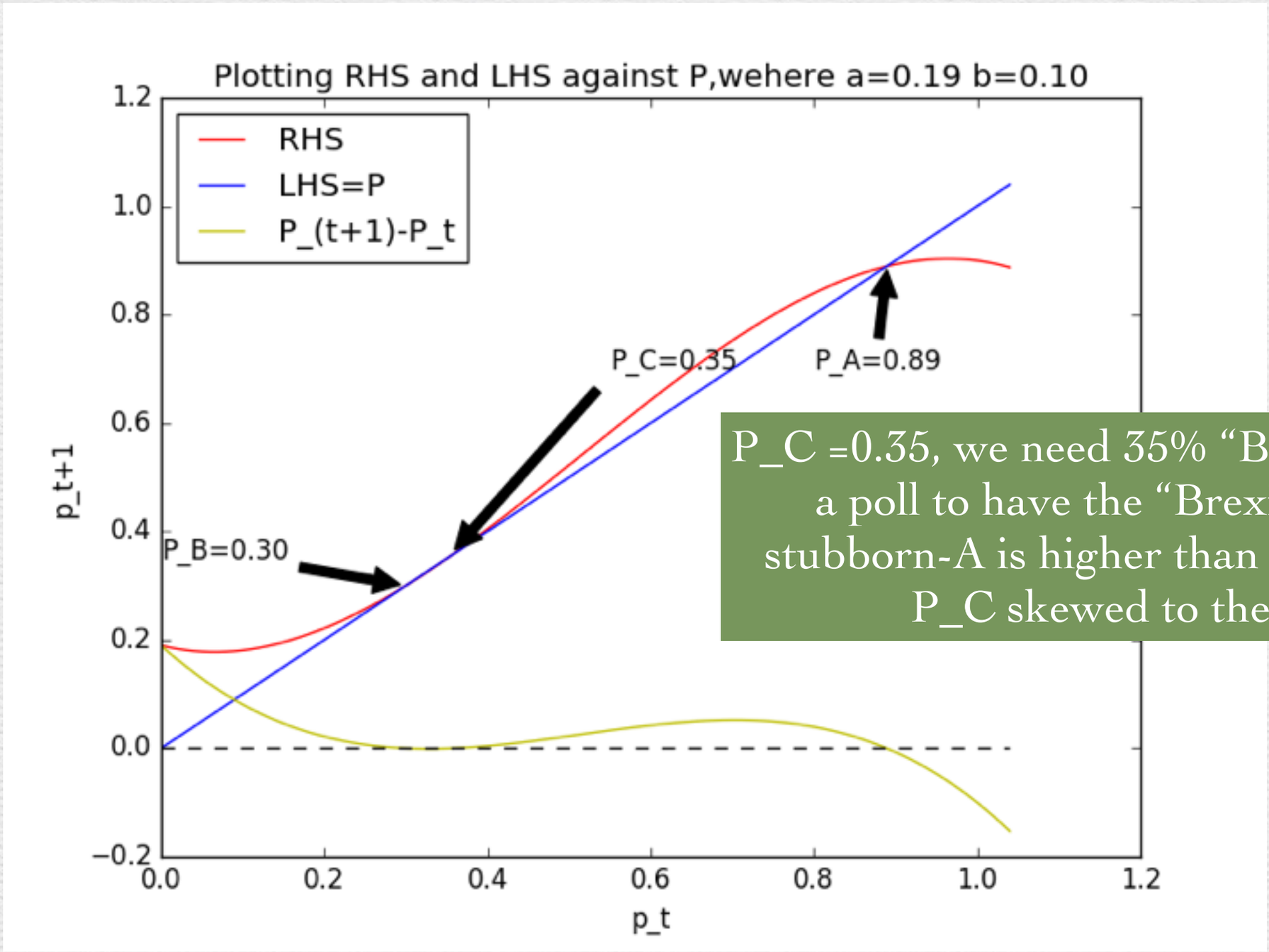


Take r=3, a=b=0 : no stubborn agents



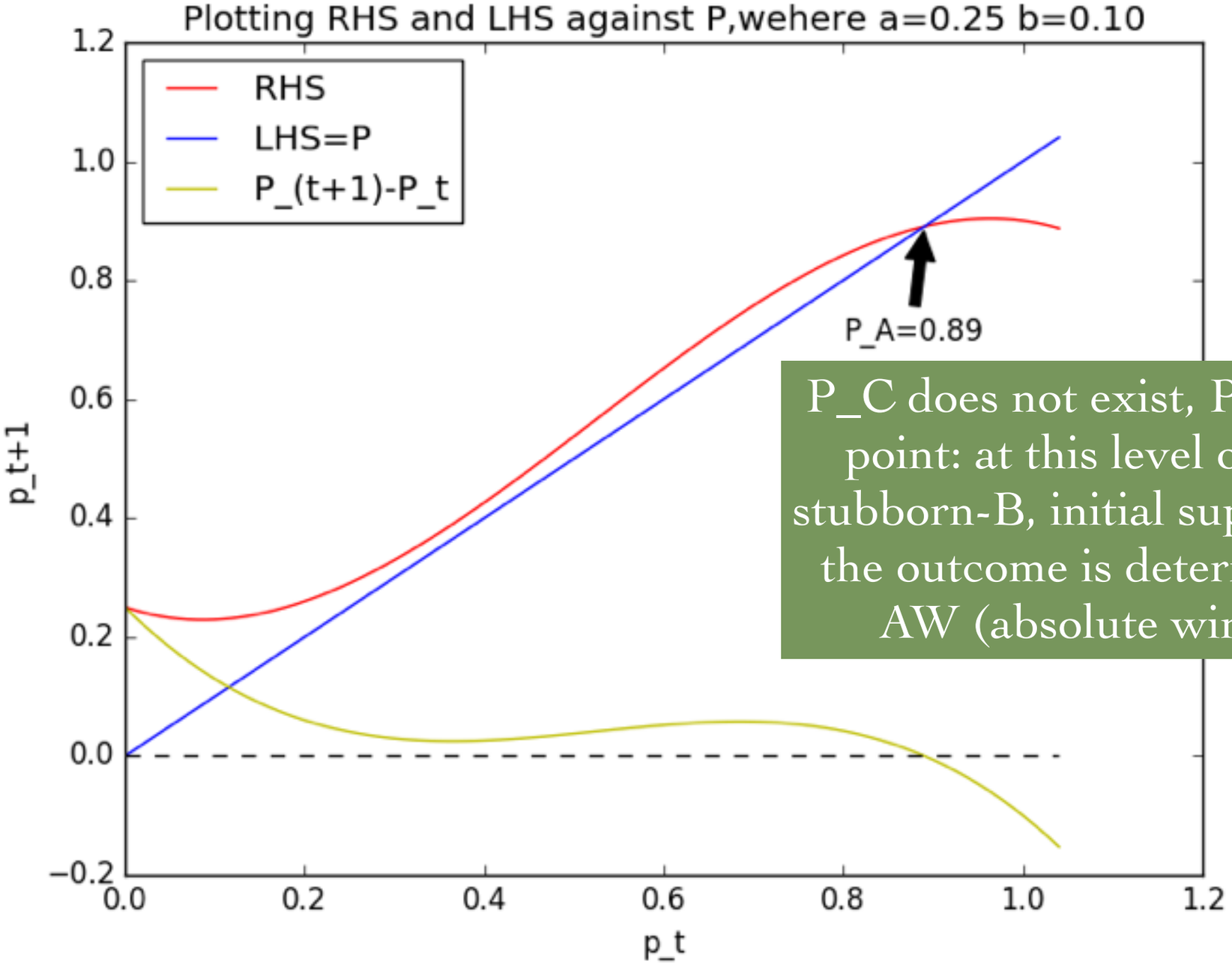
P_C =0.5, meaning we need 50% “Brexit” votes in a poll to have the “Brexit”.

Flow 2: a=0.19, b=0.10
19% stubborn-Brexiters, 10% stubborn-Remainers



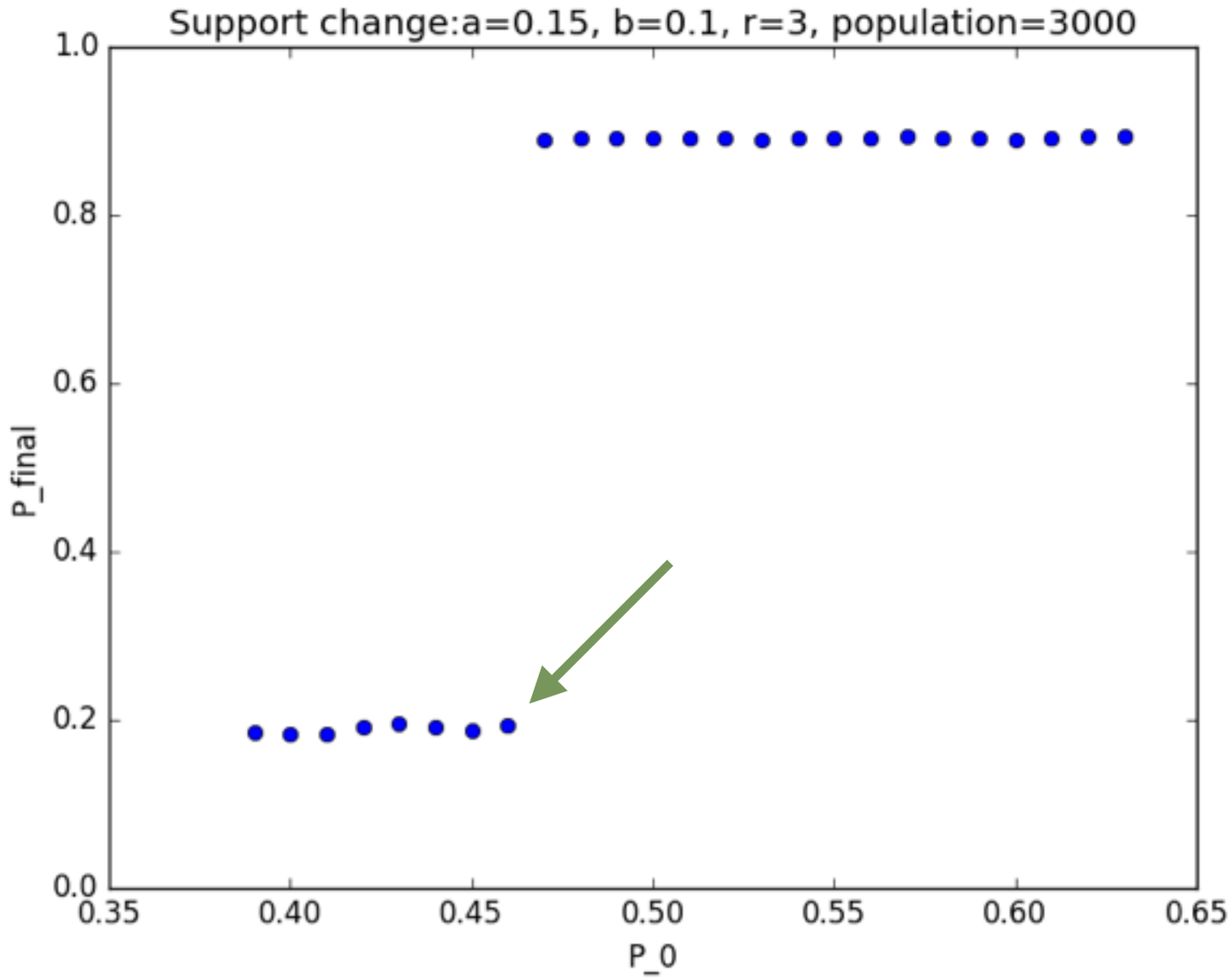
P_C =0.35, we need 35% “Brexit” votes in a poll to have the “Brexit”. When stubborn-A is higher than stubborn-B, P_C skewed to the left.

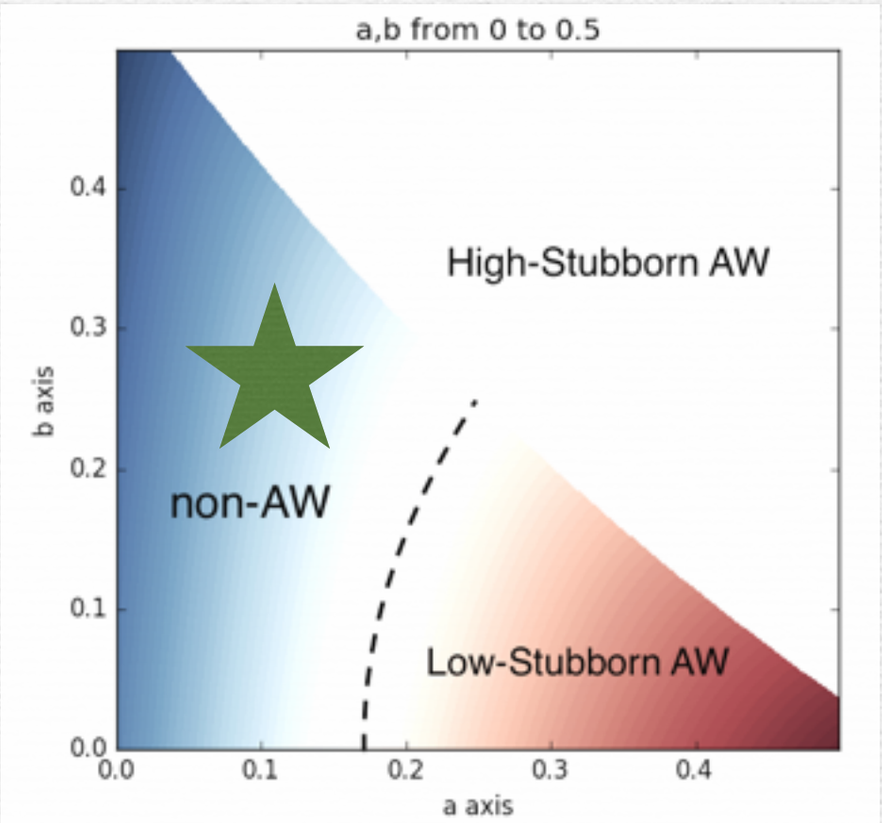
Flow 3: a=0.25, b=0.10
25% stubborn-Brexiters, 10% stubborn-Remainers



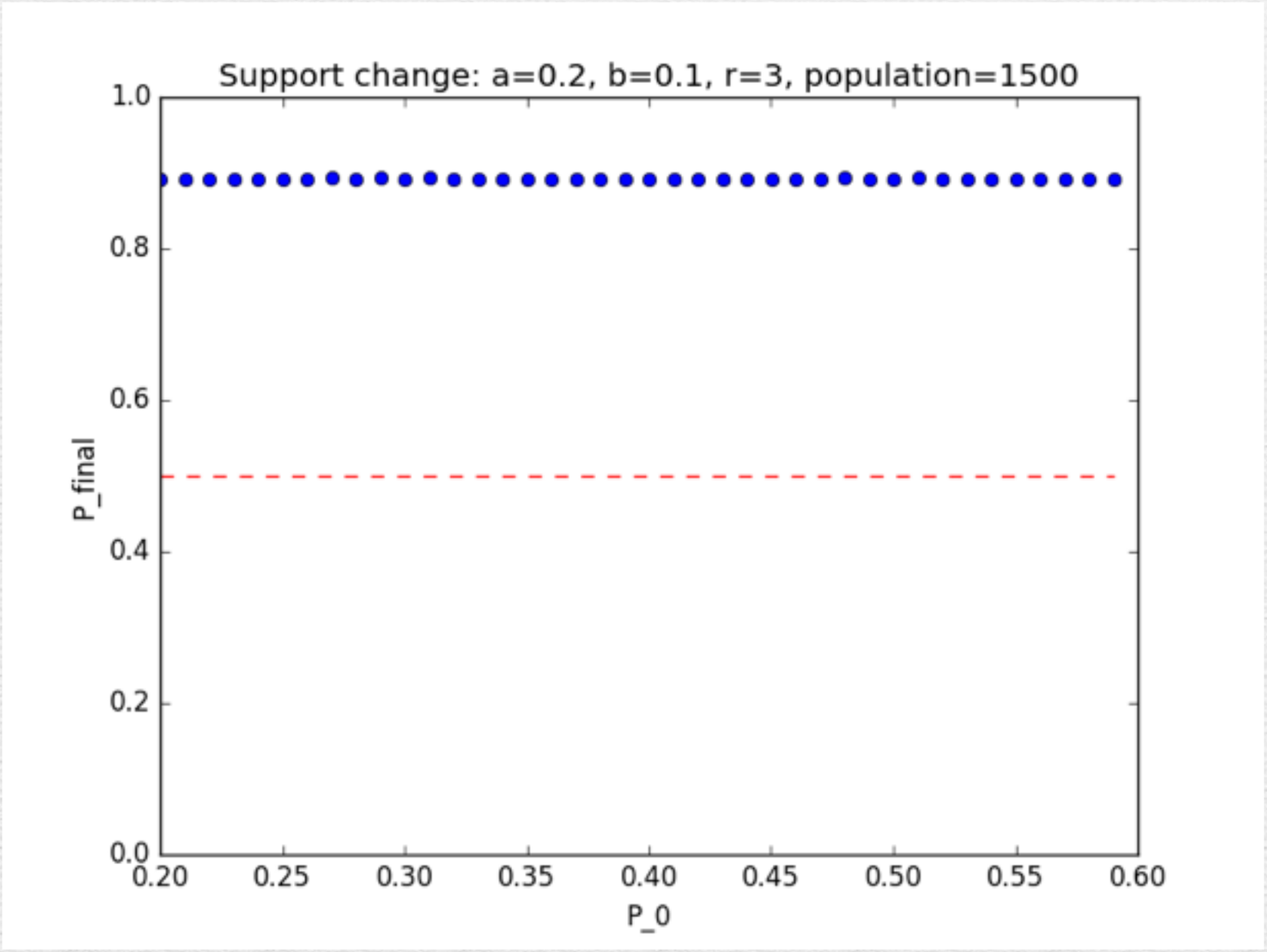
P_C does not exist, P_A is the only fixed point: at this level of stubborn-A and stubborn-B, initial support doesn't matter, the outcome is determined. We call this AW (absolute winning) situation.

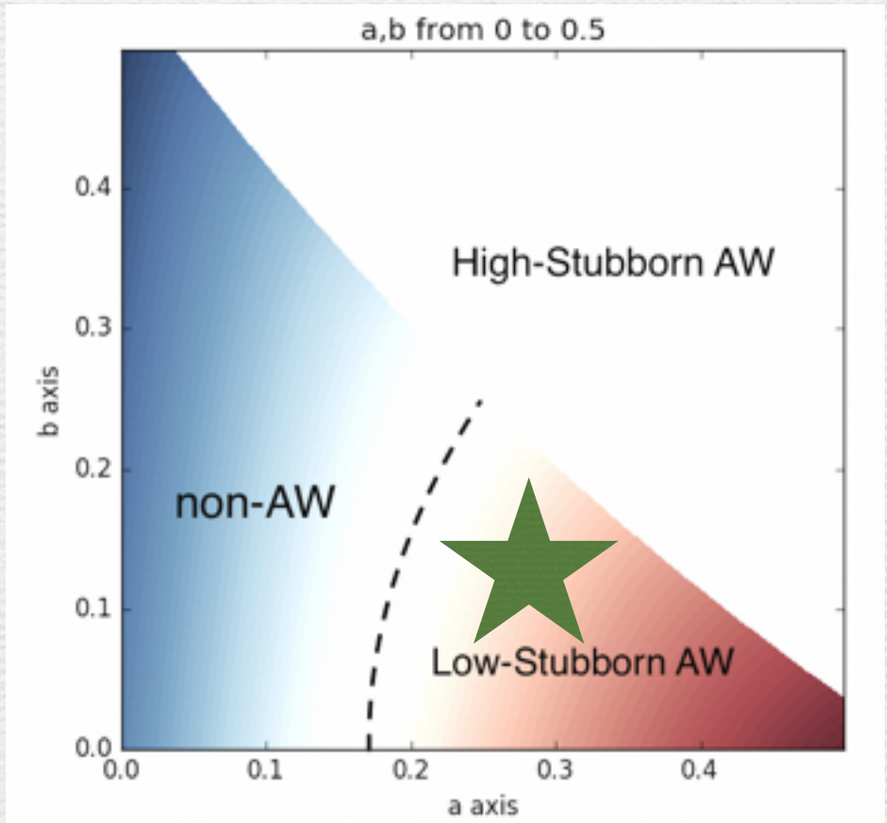
a=0.15, b=0.1, r=3: non-AW, P_0=0.47



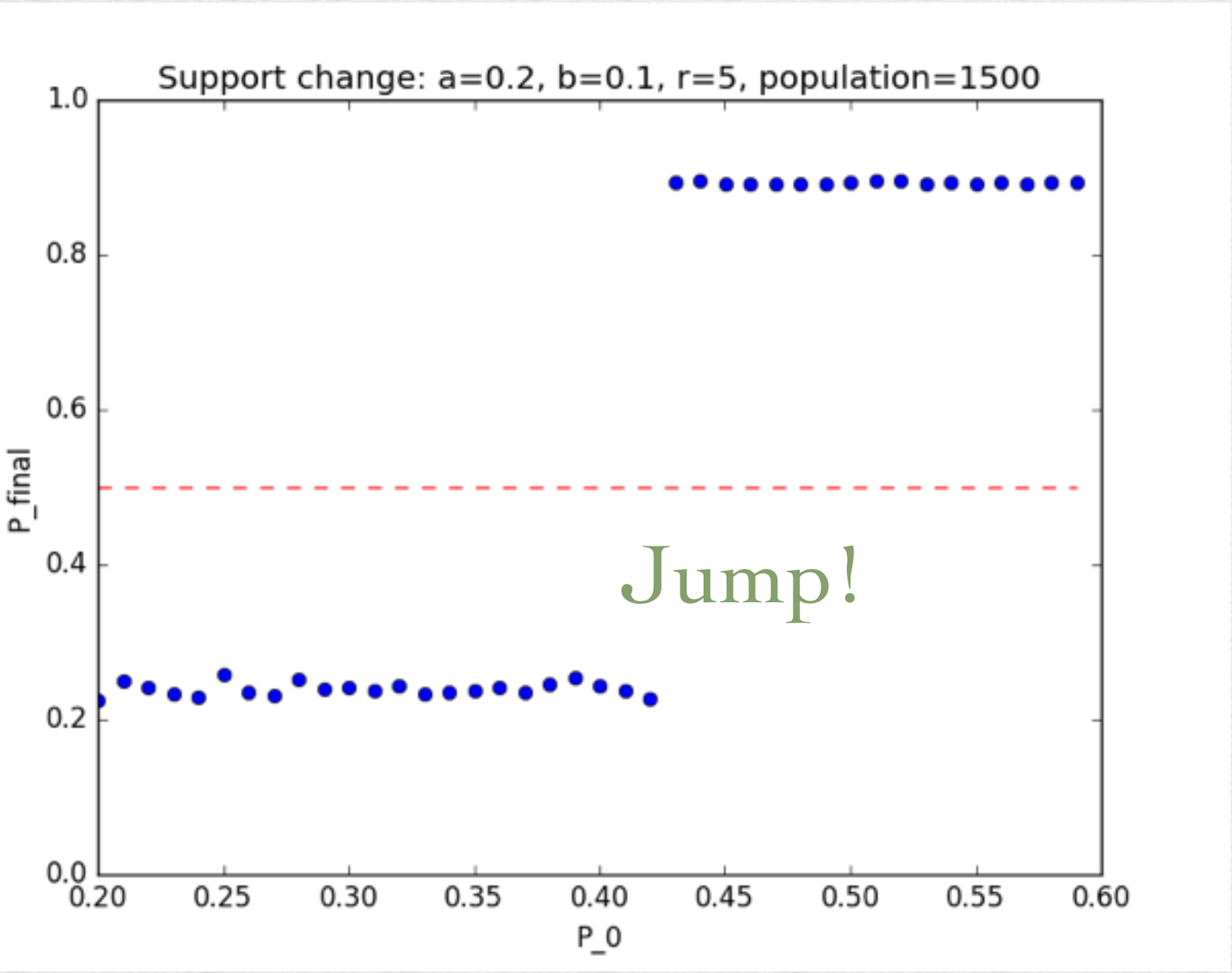


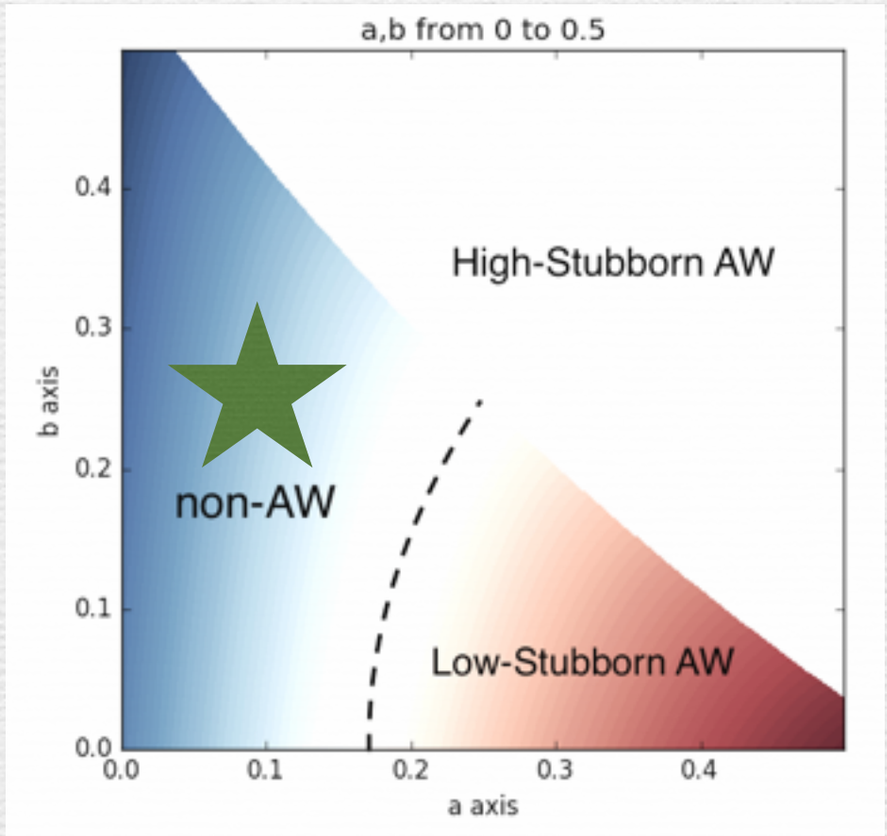
a=0.2, b=0.1, r=3: AW for A



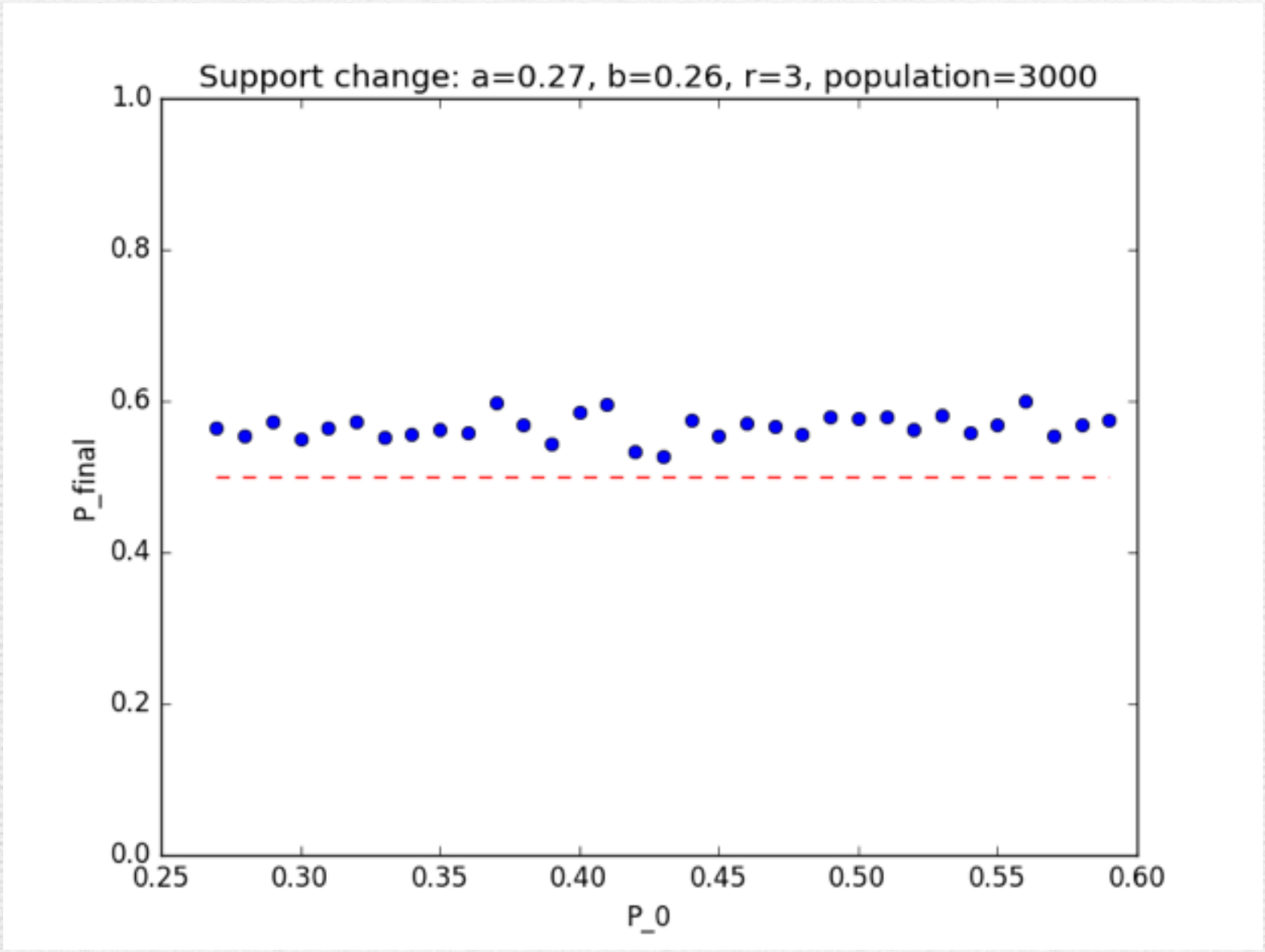


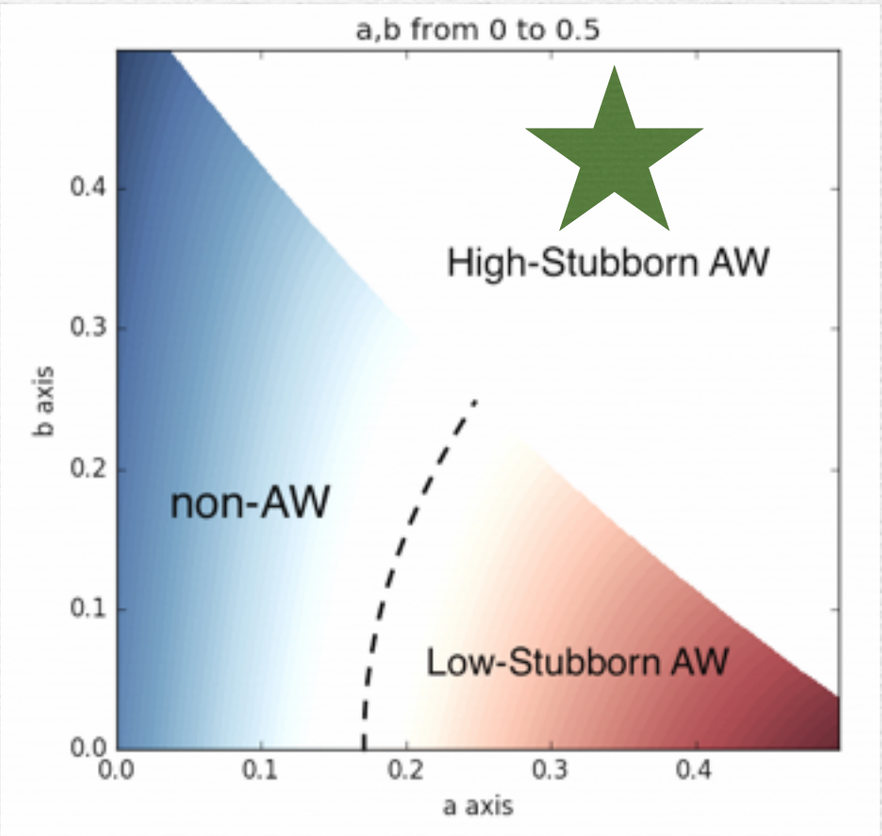
r=5: non-AW, P_0=0.43



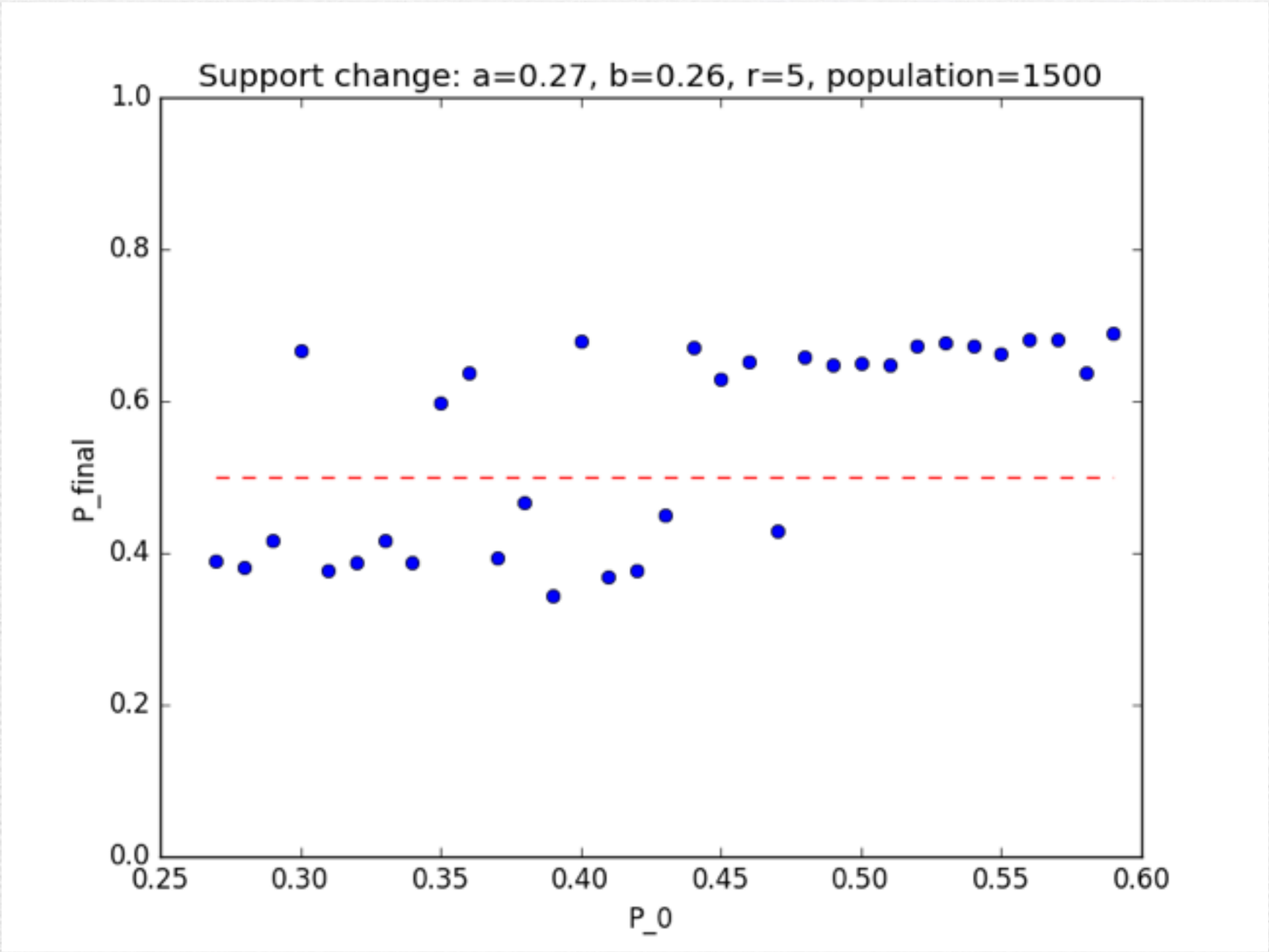


a=0.27, b=0.26, r=3: AW for A





Group size r=5



Confused….

Group size r=7: non-AW