Linear Algebra Review

1D Arrays: v/c vectors = Python list

2D Array / Matrix: list of lists (rows) *

↳ consider M_{mxn}, $m=2$, $n=3$

$M = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 3 & 4 \end{bmatrix}$ → python: [[1,2,3], [-2,3,7]]

↳ basic matrix operations

assume $v = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Note: VM is NOT valid *

$Mv = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1\cdot1 + 2\cdot0 + 3\cdot-1 \\ -2\cdot1 + 3\cdot0 + 4\cdot-1 \end{bmatrix} = \begin{bmatrix} -2 \\ -9 \end{bmatrix}$

$w = [1\ 5\ -3\ 2]^T$, $x = [8\ 2\ 4\ 7]^T$ → $w^T x = [1\ 5\ -3\ 2]\begin{bmatrix} 8 \\ 2 \\ 4 \\ 7 \end{bmatrix}$ // scalar answer

Differential Calc Review

Let f be a function that takes a vector $v = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix}^T$ as input and returns the scalar value $1 \cdot v_1 + 2 \cdot v_2 + 3 \cdot v_3 + \cdots + n \cdot v_n$.

$\frac{\partial f(v)}{\partial v_2}$ → derivative of f wrt v_2 only! // result: scalar // answer: 3

↳ recall gradient notation $\nabla_v f(v)$ → column vector

$\frac{\partial f(v)}{\partial v} = \begin{bmatrix} \frac{\partial f}{\partial v_1} & \cdots & \frac{\partial f}{\partial v_n} \end{bmatrix}^T = [1\ 2\ 3\ \ldots\ n]^T$

↳ conceptual gradient: roll ball down hill with weight f ... where does it go?

ANS: since ∇f is largest at v_n, it will roll "down" away from v_n → e.g. $-v_n$ dir

important: arrays = list of lists in Python

↳ code using matrix mult concept

some more matrix mash: $w = [1\ 5\ 0]^T$ $x = [8\ 2]^T$

$wx^T = \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix}[8\ 2] = \begin{bmatrix} 8 & 2 \\ 40 & 10 \\ 0 & 0 \end{bmatrix}$ // 3x1 // 1x2 = M_{3x2}

$M_{py} = [[8,2], [40,10], [0,0]]$

given $(AB)^T$ → B^TA^T is equivalent → transposing reverses order of a product!

↳ rule *: only square matrices w nonzero determinants have inver se

(Q15) We have the following relationship: $Wx = y$. Assuming W is full rank and square and the dimensions of W, x, and y are appropriate, which of the following expressions gives us a solution for x?

1. $x = W^T y$
2. $x = y/W$
3. $x = (W^T W)^{-1} y$
4. $x = W^{-1} y$
5. $x = (W^T W)^{-1} W y$
6. None of these

$W^{-1}Wx = W^{-1}y$

$x = W^{-1}y$

↳ matrix inversion rules

// rule: review MATH 115 for more

Python Review

adding 2 lists: def add_two_lists(a,b):

```
def add_two_lists(a,b):
    return [a[i] + b[i] for i in range(len(a))]
```

dot product: returns scalar prod of 2 vectors

```
def dot(v1, v2)
    return sum(v1[i] * v2[i] for i in range(len(v1)))
```

functions as objects:

```
def add_n(n):
    def new_fun(v):
        return [elt + n for elt in v]
    return new_fun
```