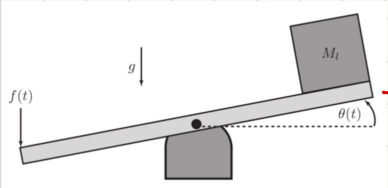
ASN #2

1) seesaw DEs



→ beam: length L

mass M_b

moment of inertia: $\frac{1}{12} M_b L^2$

torque: $F_t = rF_a \sin\theta$

applied force (Mg)

r → distance from pivot

$\Sigma\tau = I\alpha$

$I_b = 1/2 M_b L^2$

$I_M = MR^2$

$\tau = rF_a \sin\theta$

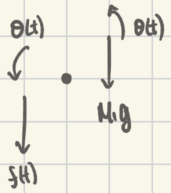
$\tau_{net} = m_1 g d_1 - m_2 g d_2$

$\Sigma\tau = I\alpha$

$\quad = r_L F_L \sin\theta - r_M F_M \sin\theta$

$\quad = L/2\,(f(t) - M_l g)\cos\theta(t)$

FBD:



$I_{tot} = I_b + I_M$

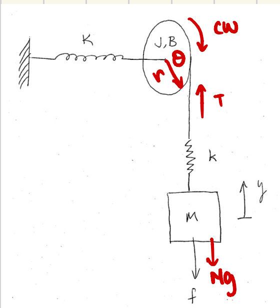
$\quad = 1/12 M_b L^2 + M_l (L/2)^2$

$\quad = 1/12 M_b L^2 + 1/4 M_l L^2$

$\Sigma\tau = I\alpha$ ⟶ Newton's 2nd Law for Rotational Motion

$$\left(1/12 M_b L^2 + 1/4 M_l L^2\right)\ddot{\theta}(t) = 1/2 L\,(f(t) - M_l g)\cos\theta(t)$$

2) DE for mechanical system



r, θ = rad, angle of pulley

K = torsion spring constant, $T_K = K\theta$

B = damping force, $T_B = B\dot{\theta}$

moment of inertia = J

spring constant = k

M = mass, F = force ($-y$ dir)

$y = -r\theta$ $\qquad T = ky$

$\dot{y} = -r\dot{\theta}$ $\qquad M\ddot{y} = f + Mg - ky$ ⟶ $f_{net} = ma$

$\ddot{y} = -r\ddot{\theta}$ $\qquad T = f + Mg - M\ddot{y}$

pulley torque: $\Sigma\tau = I\alpha = J\ddot{\theta}$

└ angular acceleration = $\ddot{\theta}$

$J = T_s + T_B + T_M$

$J\ddot{\theta} = rT - K\theta - B\dot{\theta}$

$$J\ddot{\theta} = r(f + Mg - M\ddot{y}) - K\theta - B\dot{\theta}$$

3) electromagnetic relay

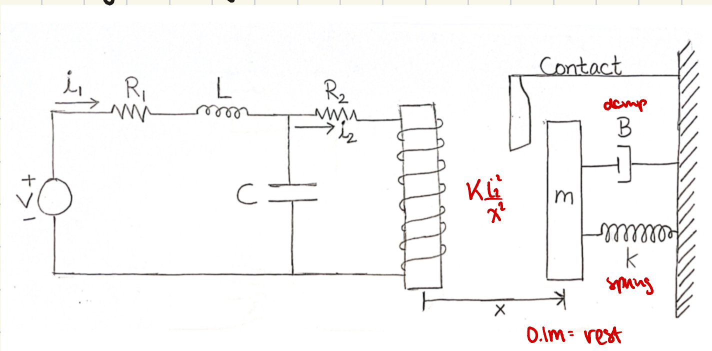


Figure 3: Electromagnetic Relay for Problem 3

mass:

$$F_m = \frac{Ki_2^2}{x^2}, \quad K > 0$$

$$F_m \longleftarrow \bullet \longrightarrow B\Delta\dot{x}, \quad \longrightarrow k\Delta x$$

$$\Delta x = x - x_e$$

$\quad x_e = 0.1\text{m}$ $\qquad F_s = k(x - 0.1)$

$$m\ddot{x} = F_m - B\dot{x} - k(x - 0.1)$$

$\quad \llcorner = Ki_2^2/x^2$

$$m\ddot{x} = \frac{Ki_2^2}{x^2} - B\dot{x} - k(x - 0.1)$$

KVL: $V = R_1 i_1 + L\dot{i}_1 + R_2 i_2$

$$0 = \frac{1}{C}\int (i_2 - i_1)\,dt + R_2 i_2$$

$\downarrow$ diff

$$0 = \frac{1}{C}(i_2 - i_1) + R_2\dot{i}_2 \longrightarrow i_1 = i_2 + CR_2\dot{i}_2$$

4) $M_1\ddot{y}_1 = B(\dot{y}_2 - \dot{y}_1) - K_1 y_1$ $\longrightarrow$ find transfer fn

$M_2\ddot{y}_2 = f - K_2 y_2 - B(\dot{y}_2 - \dot{y}_1)$

└ $Y_1(S) / F(S)$

Using Laplace Transforms: $\ddot{y} = s^2 Y(S)$ $\quad \dot{y} = sY(S)$ $\quad y = Y(S)$ $\quad f = F(S)$

$$M_1(S^2 Y_1(S)) = B(SY_2(S) - SY_1(S) - K_1 Y_1(S))$$
$$M_2(S^2 Y_2(S)) = F(S) - K_2 Y_2(S) - B(SY_2(S) - SY_1(S))$$

$$M_1 S^2 Y_1(S) = BSY_2(S) - BSY_1(S) - BK_1 Y_1(S)$$
$$BSY_2(S) = M_1 S^2 Y_1(S) + BSY_1(S) + BK_1 Y_1(S)$$
$$Y_2(S) = \frac{M_1 S^2 Y_1(S) + BSY_1(S) + BK_1 Y_1(S)}{BS} = \frac{Y_1(S)(M_1 S^2 + BS + K_1)}{BS}$$

$$F(S) = M_2 S^2 Y_2(S) + K_2 Y_2(S) + BSY_2(S) - BSY_1(S)$$
$$F(S) = Y_2(S)(M_2 S^2 + K_2 + BS) - BSY_1(S)$$

↳
$$F(S) = \left[\frac{Y_1(S)(M_1 S^2 + BS + K_1)}{BS}\right](M_2 S^2 + K_2 + BS) - BSY_1(S)$$
$$= Y_1(S)\left[\frac{(M_1 S^2 + BS + K_1)}{BS}(M_2 S^2 + K_2 + BS) - \frac{B^2 S^2}{BS}\right]$$

$$\frac{Y_1(S)}{F(S)} = \frac{BS}{(M_1 S^2 + BS + K_1)(M_2 S^2 + K_2 + BS) - B^2 S^2}$$

5) $M\ddot{y}_2 = -k_s(y_2 - y_1) - Mg - b_s(\dot{y}_2 - \dot{y}_1)$

$m\ddot{y}_1 = k_s(y_2 - y_1) + b_s(\dot{y}_2 - \dot{y}_1) - k_t(y_1 - y_0) - mg - b_t(\dot{y}_1 - \dot{y}_0) \longrightarrow$ find $\Delta Y_2(s)/Y_0(s)$

choose: $\Delta y_1 = y_1 - \hat{y}_1 \qquad \Delta y_2 = y_2 - \hat{y}_2$

└ const

$\Delta\dot{y}_1 = \dot{y}_1 \qquad \Delta\dot{y}_2 = \dot{y}_2$

$M\ddot{y}_2 = -k_s(y_2 - y_1) - Mg - b_s(\dot{y}_2 - \dot{y}_1)$ — set $\ddot{y}, \dot{y} = 0$

$0 = -k_s(y_2 - y_1) - Mg \qquad y_0 = 0$ (rest / equilibrium)

└ $y_2 = \Delta y_2 + \hat{y}_2$

$Mg = -k_s(\hat{y}_2 - \hat{y}_1)$

$m\ddot{y}_1 = k_s(y_2 - y_1) + b_s(\dot{y}_2 - \dot{y}_1) - k_t(y_1 - y_0) - mg - b_t(\dot{y}_1 - \dot{y}_0)$

$mg = k_s(\hat{y}_2 - \hat{y}_1) - k_t(\hat{y}_1)$

$mg = -Mg - k_t\hat{y}_1$

$$\hat{y}_1 = \frac{mg + Mg}{-k_t} = \frac{-(m+M)g}{k_t}$$

$Mg = -k_s(\hat{y}_2 - \hat{y}_1)$

$$\hat{y}_2 = \frac{-Mg}{k_s} + \hat{y}_1 = \frac{-Mg}{k_s} - \frac{(m+M)g}{k_t} \qquad \hat{y}_2 = \hat{y}_1 - Mg/k_s$$

time domain eqns: $M_2\Delta\ddot{y}_2 = -k_s(\Delta y_2 - \Delta y_1) - b_s(\Delta\dot{y}_2 - \Delta\dot{y}_1)$ LTI model

$m\Delta\ddot{y}_1 = k_s(\Delta y_2 - \Delta y_1) + b_s(\Delta\dot{y}_2 - \Delta\dot{y}_1) - k_t\Delta y_1 - b_t\Delta\dot{y}_1 + k_t y_0 + b_t\dot{y}_0$

transfer function: $A = k_s + b_s s \qquad B = k_t + B_t s$ — dynamic stiffness terms

$X_1 = \Delta Y_1 \qquad X_2 = \Delta Y_2 \quad Y_0 = Y_0$

$Ms^2 X_2 = -A(X_2 - X_1) \longrightarrow (Ms^2 + A)X_2 - AX_1 = 0$

$$X_1 = \frac{(Ms^2 + A)X_2}{A}$$

$ms^2 X_1 = A(X_2 - X_1) - BX_1 + BY_0$ sub

$(ms^2 + A + B)X_1 - AX_2 = BY_0$

$$(ms^2 + A + B)\frac{(Ms^2 + A)}{A}X_2 - AX_2 = BY_0$$

$(ms^2 + A + B)(Ms^2 + A)X_2 - A^2X_2 = ABY_0$

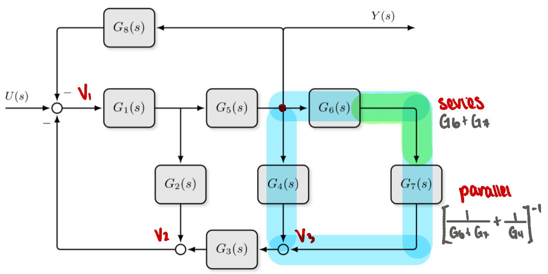
┌ $X_2 = \Delta Y_2$

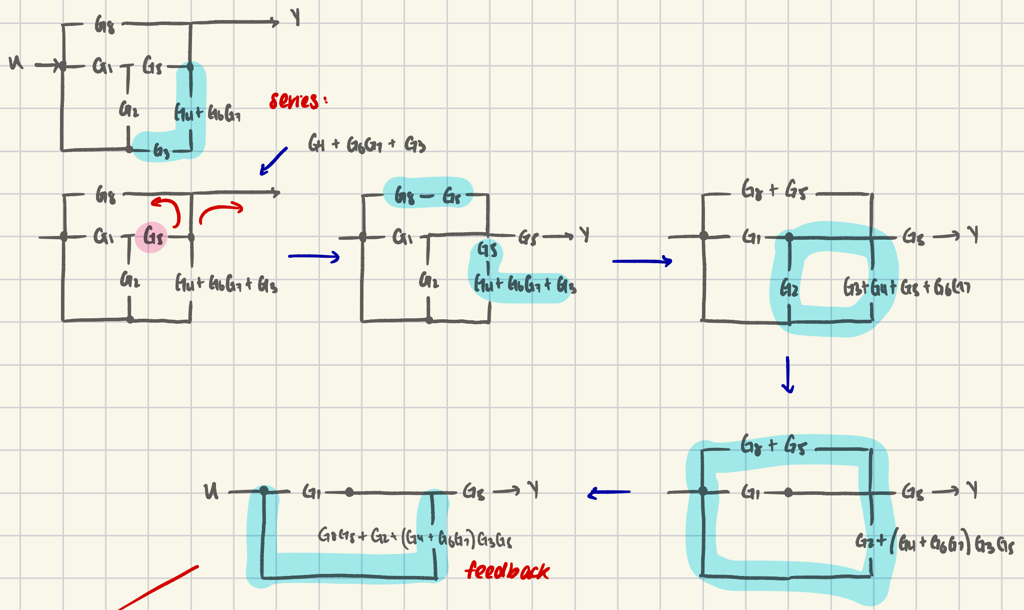
$$((ms^2 + A + B)(Ms^2 + A) - A^2)X_2 = ABY_0 \longrightarrow Y_0 = \frac{((ms^2 + A + B)(Ms^2 + A) - A^2)X_2}{AB}$$

$$\frac{\Delta Y_2(s)}{Y_0(s)} = \frac{AB}{((ms^2 + A + B)(Ms^2 + A) - A^2)} = \frac{(k_s + b_s s)(k_t + b_t s)}{(ms^2 + k_s + b_s s + k_t + b_t s)(Ms^2 + k_s + b_s s) - (k_s + b_s s)^2}$$

Problem 6

Reduce the block diagram in Figure 4 to a single block relating the output $Y(s)$ to the input $U(s)$. Do NOT use a system of algebraic equations to perform this reduction.



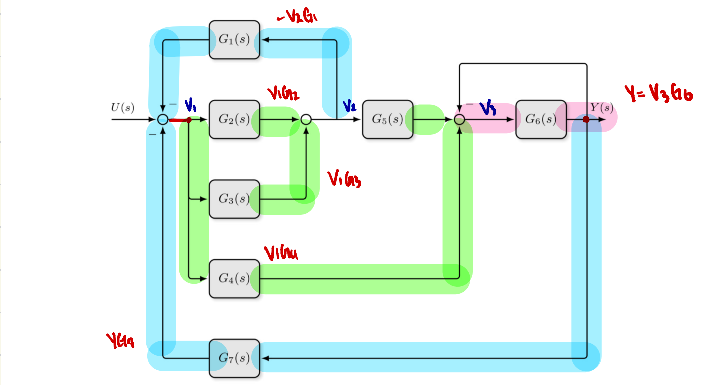


$$Z(s) = \frac{G_1}{1 - G_1[G_8G_5 + G_2 + (G_4 + G_6G_7)G_3G_5]}$$

$$U(s) \longrightarrow \frac{G_1G_5}{1 + G_1[G_8G_5 + G_2 + (G_4 + G_6G_7)G_3G_5]}$$

Problem 7

Reduce the block diagram in Figure 5 to a single block relating the output $Y(s)$ to the input $U(s)$.



$$V_1 = U - G_1V_2 - G_7Y = U - G_1V_1(G_2+G_3) - G_7Y$$

$$V_2 = V_1G_2 + V_1G_3 = V_1(G_2+G_3)$$

$$Y = V_3G_6 = (V_2G_5 + V_1G_4)G_6$$

$$= (V_1G_2G_5 + V_1G_3G_5 + V_1G_4)G_6$$

$$Y = V_1G_6(G_2G_5 + G_3G_5 + G_4)$$

$$V_1 = \frac{Y}{G_6(G_2G_5 + G_3G_5 + G_4)}$$

$$V_1 = U - G_1V_1(G_2+G_3) - G_7Y$$

$$V_1(1+G_1(G_2+G_3)) = U - G_7Y$$

$$V_1 = \frac{U - G_7Y}{(1+G_1G_2+G_2G_3)}$$

$$\frac{Y}{(G_2G_5 + G_3G_5 + G_4)G_6}(1+G_1G_2+G_2G_3) = U - G_7Y$$

$$Y \cdot \frac{(1+G_1G_2+G_2G_3)}{G_6(G_2G_5+G_3G_5+G_4)} + G_7Y = U$$

$$Y\left[\frac{(1+G_1G_2+G_2G_3)}{(G_2G_5+G_3G_5+G_4)G_6} + G_7\right] = U$$

$$Y = \left[\frac{(1+G_1G_2+G_2G_3) + G_7(G_2G_5+G_3G_5+G_4)}{(G_2G_5+G_3G_5+G_4)G_6}\right]U$$