ASN #1

PROBLEM 1

a) $z = -1 + j$

$|z| = (x^2+y^2)^{1/2} = ((-1)^2 + (1)^2)^{1/2} = \sqrt{2}$

$\angle z = \tan^{-1}(y/x) = \tan^{-1}(-1/1) = \tan^{-1}(-1) = -\pi/4 \Rightarrow \angle z = 3\pi/4$

$+\pi$ for quad 2 $\longrightarrow +\pi$

$\bar{z} = -1 - j$

b) $z = \cos(-4\pi/5) + j\sin(-4\pi/5)$

$|z| = 1$ // $z = re^{j\angle z}$

$\angle z = -4\pi/5$

$\bar{z} = \cos(-4\pi/5) - j\sin(-4\pi/5)$

c) $z = 1/(1+5j)^5 = (1+5j)^{-5}$

$|1+5j| = \sqrt{26}$

$\angle 1+5j = \tan^{-1}(5/1) = \tan^{-1}(5)$

$\frac{1}{(1+5j)} = \frac{1}{\sqrt{26}^5} e^{-j5\tan^{-1}(5)}$

$|z| = 1/\sqrt{26}^5 = 1/(26)^{5/2}$

$\angle z = -5\tan^{-1}(5)$

$\bar{z} = \frac{1}{\sqrt{26}^5} e^{+j5\tan^{-1}(5)}$

d) $z = e^{11\pi/4 j}$

$|z| = 1$

$\angle z = 11\pi/4 = 3\pi/4$

$\bar{z} = e^{-11\pi/4 j} = e^{j(-11\pi/4)} = e^{-j3\pi/4}$

d) $z = \frac{-1+j}{\cos(-4\pi/5) + j\sin(-4\pi/5)}$ $\quad -1+j = \sqrt{2}e^{j3\pi/4}$ $\quad \cos(-4\pi/5) + j\sin(-4\pi/5) = e^{-j(4\pi/5)} = e^{j6\pi/5}$

$z = \frac{\sqrt{2}e^{j3\pi/4}}{e^{j6\pi/5}} = \sqrt{2}e^{j(3\pi/4 - 6\pi/5)}$

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$|z| = \sqrt{2}$ $\quad \angle z = 3\pi/4 - 6\pi/5 = -9\pi/20 = 31\pi/20$ $\quad \bar{z} = \sqrt{2}e^{j9\pi/20}$

PROBLEM 2

no LT ☺

impulse response: $h(t) = e^{-4t} \Rightarrow$ find output $y(t)$

input: $u(t) = t$

↳ convolution: $y(t) = (g * u)(t) = \int_0^t g(t)u(t-\tau)d\tau$

$y(t) = \int_0^t e^{-4\tau}(t-\tau)d\tau = \int_0^t [e^{-4\tau} \cdot t - e^{-4\tau} \cdot \tau] d\tau$

$= t\int_0^t e^{-4\tau}d\tau - \int_0^t \tau e^{-4\tau}d\tau$

$a = \tau \quad db = e^{-4\tau}d\tau$

$d\tau \Rightarrow b = -1/4 e^{-4\tau}$

by parts!

$\int_0^t e^{-4\tau}d\tau = \frac{1-e^{-4t}}{4} \longrightarrow \int_0^t \tau e^{-4\tau}d\tau = -\frac{\tau}{4}e^{-4\tau} + \frac{1}{4}\int e^{-4\tau}d\tau = -\frac{\tau}{4}e^{-4\tau} - \frac{1}{16}e^{-4\tau} + C$

plug in $|_0^t$

$\left(-\frac{t}{4}e^{-4t} - \frac{1}{16}e^{-4t}\right) - \left(0 - \frac{1}{16}\right)$

$= \frac{1}{16} - e^{-4t}\left(\frac{t}{4} + \frac{1}{16}\right)$

$= t \cdot \frac{1-e^{-4t}}{4} - \left[\frac{1}{16} - e^{-4t}\left(\frac{t}{4} + \frac{1}{16}\right)\right] = \frac{t}{4} - \cancel{\frac{te^{-4t}}{4}} - \frac{1}{16} + \cancel{\frac{te^{-4t}}{4}} - \frac{e^{-4t}}{16} = \frac{t}{4} - \frac{1}{16} - \frac{e^{-4t}}{16}$

$y(t) = \left[\frac{t}{4} - \frac{1}{16} - \frac{e^{-4t}}{16}\right] 1(t)$

unit step, assume causal

PROBLEM 3

IR: $h(t) = 2e^{-3t} - 4e^{-2t} + 5\,1(t)$ (unit step) $\longrightarrow$ find $H(s)$ from $U(s) \to Y(s)$ (input, output)

$H(s) = \mathcal{L}\{h(t)\} = \mathcal{L}\{2e^{-3t}\} - \mathcal{L}\{4e^{-2t}\} + \mathcal{L}\{5\,1(t)\}$

$= 2\mathcal{L}\{e^{-3t}\} - 4\mathcal{L}\{e^{-2t}\} + 5\mathcal{L}\{1(t)\}$

LT table!

$= \frac{2}{s+3} - \frac{4}{s+2} + \frac{5}{s}$

PROBLEM 4

$h(t) = 2e^{-3t}$ ⇒ find $y(t)$ — output

$u(t) = e^{-17t}$

input

no LT of u

no convolution

LT of $h(t) = H(s) = \frac{2}{s+3}$ → when input conditions = 0 : $Y(s) = H(s)X(s)$

can't take this!

$a=1$, $b=3$, $c=2$

$\underbrace{ay'(t) + by(t) = cx(t)}_{\text{1st order system}} \Rightarrow \underbrace{a(sY(s))}_{\text{LT of sys}} + bY(s) = cX(s)$

$Y(s) = \frac{c}{as+b}X(s)$

$y'(t) + 3y(t) = 2\cancel{u(t)}$ e^{-17t} ⇒ $y' + 3y = 2e^{-17t}$ — integrating factor: $\frac{d}{dt}(e^{3t}y) = 2e^{-14t}$ ↳ $\int 2e^{-14t}dt = -\frac{1}{7}e^{-14t} + C$

$y(t) = -\frac{1}{7}e^{-14t} + Ce^{-3t}$ // using zero IC

$0 = -\frac{1}{7} + C$

$C = 1/7$

$y(t) = \left[-\frac{1}{7}e^{-14t} + \frac{1}{7}e^{-3t}\right]1(t)$

PROBLEM 5

a) $\frac{4s+11}{(s+2)(s+3)}$

$= \frac{3}{s+2} + \frac{1}{s+3}$

$4s+11 = \frac{A}{s+2} + \frac{B}{s+3} = As + 3A + Bs + 2B$

$4s = As + Bs \longrightarrow A+B = 4,\ A = 4-B$

$11 = 3A + 2B \longrightarrow 11 = 3(4-B) + 2B = 12 - 3B + 2B = 12 - B$

$B = 1,\ A = 3$

b) $\frac{3s^2 + 11s + 14}{(s+3)(s+1)^2}$

$= \frac{2}{s+3} - \frac{1}{s+1} + \frac{3}{(s+1)^2}$

$3s^2 + 11s + 14 = \frac{A}{s+3} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$

$= A(s+1)^2 + B(s+3)(s+1) + C(s+3)$

$= A(s^2+2s+1) + B(s^2+4s+3) + C(s+3)$

$= s^2\underbrace{(A+B)}_{3} + s\underbrace{(2A+4B+C)}_{11} + \underbrace{(A+3B+3C)}_{14}$

(a) $A+B = 3 \Rightarrow A = 3-B$

(b) $2(3-B) + 4B + C = 11,\ 6 - 2B + 4B + C = 11 \Rightarrow C = 5-2B$

(c) $(3-B) + 3B + 3(5-2B) = 14$

$-B + 3B - 6B = 14 - 3 - 15$

$-4B = -4 \Rightarrow B = 1$

$A = 3-1 = 2$

$C = 5 - 2(1) = 3$

PROBLEM 6

$h(t) = e^{-t}$ ⇒ find $y(t)$ → transfer fn property

LT!

$u(t) = te^{-2t}$

$Y(s) = H(s)X(s)$

↳ $U(s) = \mathcal{L}\{te^{-2t}\} = 1/(s+2)^2$

$H(s) = \mathcal{L}\{e^{-t}\} = 1/(s+1)$ // partial frac decomp!

$Y(s) = \frac{1}{(s+1)} \cdot \frac{1}{(s+2)^2} = \frac{1}{(s+1)(s+2)^2} \Rightarrow \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$

$1 = A(s+2)^2 + B(s+1)(s+2) + C(s+1)$

$= A(s^2+4s+4) + B(s^2+3s+2) + C(s+1)$

$= s^2(A+B) + s(4A+3B+C) + (4A+2B+C)$

↳ $A+B = 0,\ A = -B$

$-4B + 3B + C = 0,\ C = B$

$4(-B) + 2B + B = 1$

$-1B = 1 \Rightarrow B = -1$

$C = B = -1$

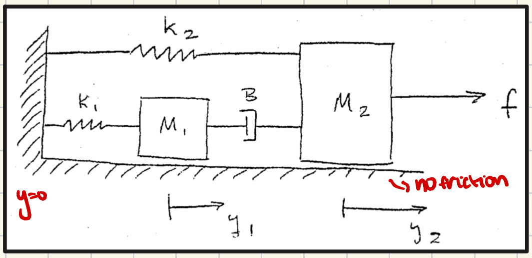
$A = -B = 1$

$Y(s) = \underbrace{\frac{1}{s+1} - \frac{1}{s+2} - \frac{1}{(s+2)^2}}_{\text{take } \mathcal{L}^{-1}}$

$y(t) = e^{-t} - e^{-2t} - te^{-2t}$

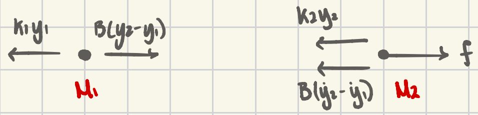
$y(t) = [e^{-t} - e^{-2t}(1+t)]1(t)$

PROBLEM 7



when $y_1 = y_2 = 0$, springs at rest

Step 1 : FBDs



Step 2 : Newton's 2nd Law // $f = ma$

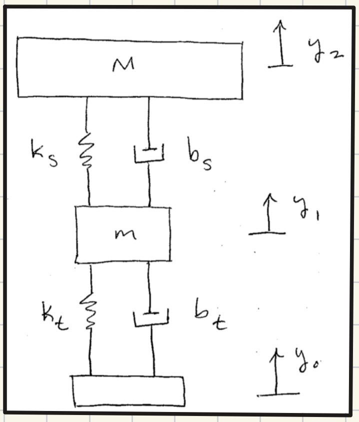
↳ $a = \ddot{d}$, since $d = y \Rightarrow f = M\ddot{y}$

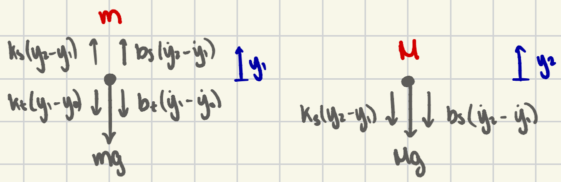
↳ net force on chosen mass from FBD

$$M_1\ddot{y}_1 = B(\dot{y}_2 - \dot{y}_1) - k_1 y_1$$
$$M_2\ddot{y}_2 = f - k_2 y_2 - B(\dot{y}_2 - \dot{y}_1)$$

Step 3 : System of DEs

PROBLEM 8





$$M\ddot{y}_2 + K_s(y_2 - y_1) + Mg + b_s(\dot{y}_2 - \dot{y}_1) = 0$$
$$M\ddot{y}_1 = K_s(y_2 - y_1) + b_s(\dot{y}_2 - \dot{y}_1) - K_t(y_1 - y_0) - b_t(\dot{y}_1 - \dot{y}_0) - mg$$