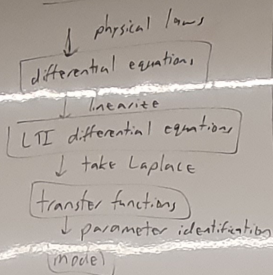


P is LTI
 $y(t) = (h * u)(t)$
 $e^{st} \xrightarrow{P} h(t) \xrightarrow{P} H(s) e^{st}$
 $U(s) \xrightarrow{H(s)} Y(s)$
 $Y(s) = H(s) U(s)$
 transfer function

System Modeling



Deriving Differential Equations

Mechanical systems (in translation)

- components:

masses



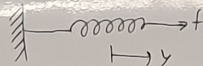
$$f(t) = M \frac{d^2 y}{dt^2}$$

Newton's 2nd Law

f is force

y is displacement

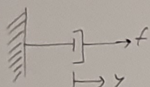
(linear) springs



$$f(t) = -K y(t)$$

$y=0 \Rightarrow$ spring is not stretched or contracted (at its natural length)

(linear) dashpots (damping elements)



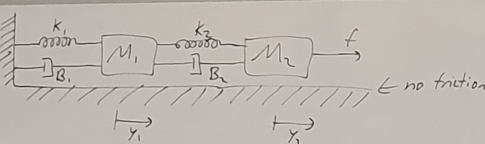
$$f(t) = -B \dot{y}(t)$$

$\frac{d}{dt}$

Procedure:

1. Draw free-body diagrams for each mass (FBDs)
2. Apply Newton's 2nd Law
- \rightarrow 3 system of differential equations

Ex.



$x_1 = x_2 = 0 \Rightarrow$ both springs are at their natural lengths

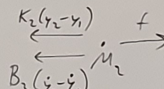
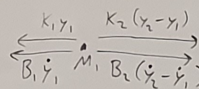
Newton's 2nd Law:

$$\begin{aligned}
 M_1 \ddot{x}_1 &= K_2(x_2 - x_1) + B_2(\dot{x}_2 - \dot{x}_1) - K_1 x_1 - B_1 \dot{x}_1 \\
 M_2 \ddot{x}_2 &= f - K_2(x_2 - x_1) - B_2(\dot{x}_2 - \dot{x}_1)
 \end{aligned}$$

system of differential equations

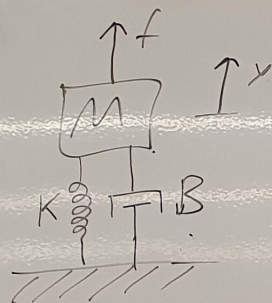
\rightarrow linear, LTI

FBDs:

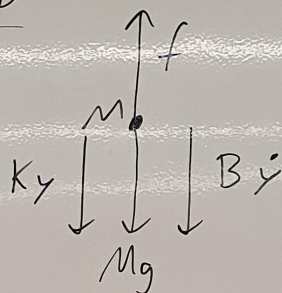


$$f(x_1, \dot{x}_1, \ddot{x}_1, x_2, \dot{x}_2, \ddot{x}_2, f) = 0$$

Ex.



FBD:



Newton's 2nd Law:

$$M \ddot{y} = f - K y - B \dot{y} - M g \Leftrightarrow M \ddot{y} + B \dot{y} + K y = f - M g \quad (1)$$

$$\Leftrightarrow 0 = f - K y - B \dot{y} - M \ddot{y} - \underbrace{M g}_{\Rightarrow \text{affine (not linear)}} \Rightarrow \text{not LTI}$$

Idea: change variables (coordinates) to cancel out t so that this system becomes LTI

\rightarrow define $\Delta y = y - y_0 \rightarrow$ unknown constant
 \Rightarrow translation or displacement relative to y_0

$$\Rightarrow \dot{\Delta y} = \dot{y}$$

$$\Rightarrow \ddot{\Delta y} = \ddot{y}$$

- Goal:
1. Choose y_0 so that Δy satisfies an LTI differential equation.
 2. Solve for Δy using LTI methods.
 3. Calculate $y = \Delta y + y_0$.

$$M \ddot{\Delta y} + B \dot{\Delta y} + K \underset{(y-y_0)}{\Delta y} = \underbrace{M \ddot{y} + B \dot{y} + K y - K y_0}_{f - Mg - Ky_0 \text{ (Eq. (1))}} = f - \cancel{Mg} + \cancel{Mg} = f$$

$$\text{Choose } y_0 \ni -Ky_0 = Mg \Leftrightarrow y_0 = \frac{-Mg}{K}$$

$$\Rightarrow M \ddot{\Delta y} + B \dot{\Delta y} + K \Delta y = f$$

$\Rightarrow \Delta y$ satisfies an LTI differential equation.