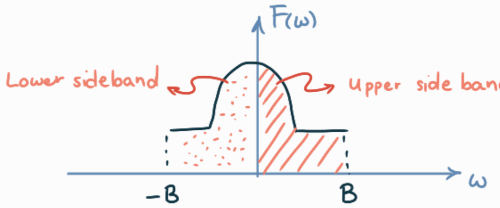
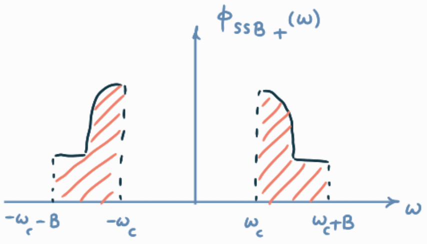
ECE 318 - TUTORIAL 4

* SINGLE SIDEBAND (SSB) MODULATION

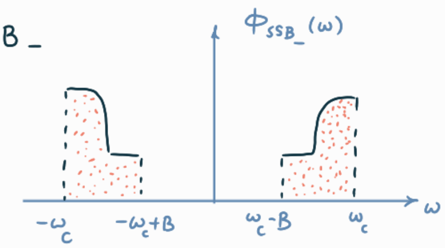


- Consider a message signal $f(t)$ that has the spectrum on the right:
- In SSB, we would like to transmit only one of the two sidebands:

a) SSB_+



b) SSB_-



SSB_+ : only upper sideband is transmitted

SSB_- : only lower sideband is transmitted

There are different methods to generate an SSB signal:

(a) Use Hilbert transform "theoretical"

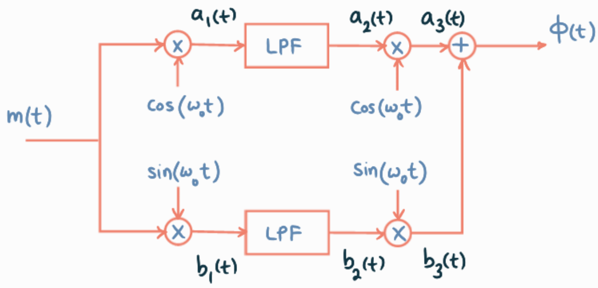
The Hilbert transform of a signal $f(t)$ is : $\hat{f}(t) \triangleq f(t) * \frac{1}{\pi t} = \frac{1}{\pi} \int_{\tau=-\infty}^{\infty} \frac{f(\tau)}{t-\tau} \, d\tau$

Then the upper sideband SSB (SSB_+) is given by $\phi_+(t) = f(t) \cdot \cos(\omega_c t) - \hat{f}(t) \cdot \sin(\omega_c t)$

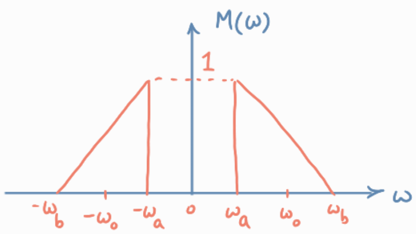
The lower sideband SSB (SSB_-) is given by: $\phi_-(t) = f(t) \cdot \cos(\omega_c t) + \hat{f}(t) \cdot \sin(\omega_c t)$

(b) The Weaver's method "practical" As discussed in example 1

Ex. Consider the block diagram on the right, used to generate an SSB modulated signal.



The message signal $m(t)$ has the spectrum shown on the right side, where $\omega_0 = \frac{\omega_a + \omega_b}{2}$



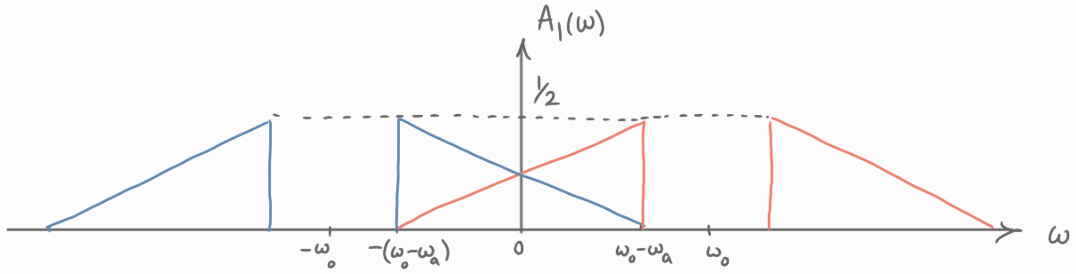
Assume that the LPF is ideal with cutoff frequency $\omega_{LPF} = \omega_0 - \omega_a = \omega_b - \omega_0$, and also that $\omega_c \gg \omega_0$.

(a) Sketch the spectrum of the signals, $a_i(t)$ and $b_i(t)$, $i = 1, 2, 3$.

(b) This block diagram generates SSB_+. How do you generate SSB_- with this block diagram?

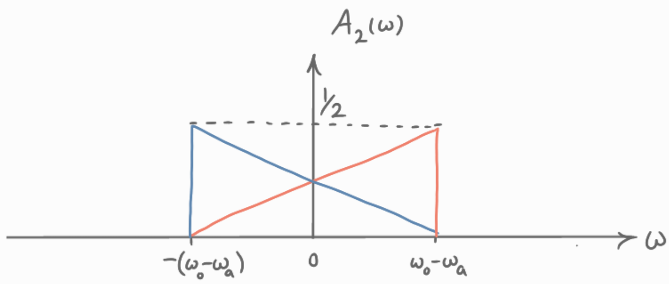
SOL : First consider the upper branch.

(1) Since $a_1(t) = m(t) \cdot \cos(\omega_o t) \Rightarrow A_1(\omega) = \frac{1}{2}\{M(\omega-\omega_o) + M(\omega+\omega_o)\}$.

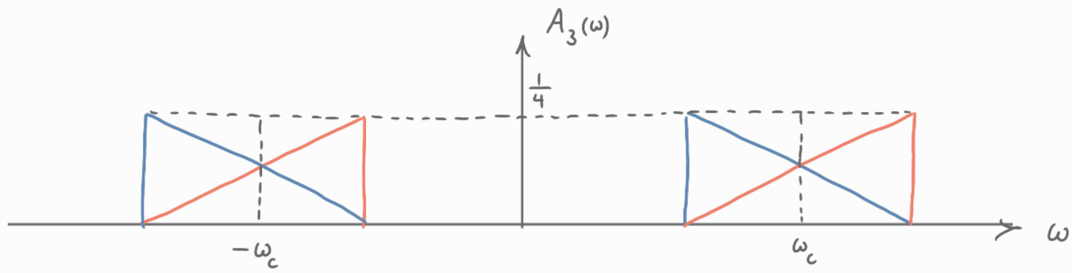


Note that since $\omega_o < \omega_b$, the lower sideband of the right shift and the upper sideband of the left shift overlap completely.

(2) Next, $a_1(t)$ passes through an ideal LPF and produces $a_2(t)$. The LPF filters out all frequency components above $\omega_o - \omega_a$. Thus, $A_2(\omega) = \mathcal{F}[a_2(t)]$ is given by:



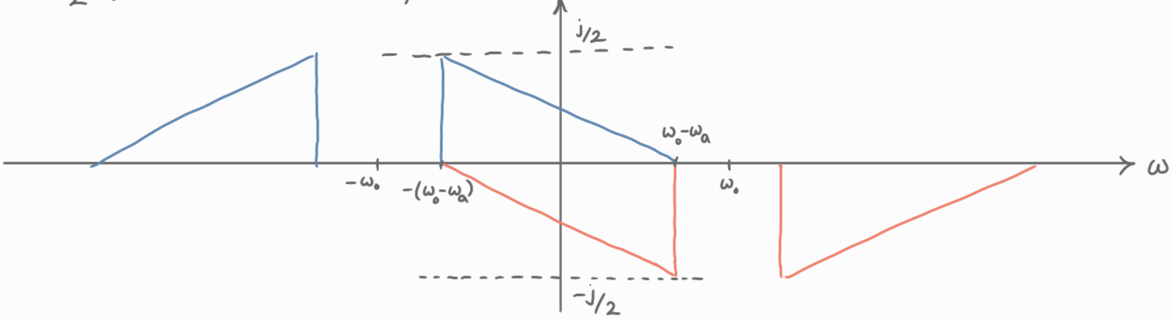
(3) Finally, $a_3(t) = a_2(t) \cdot \cos(\omega_c t) \Rightarrow A_3(\omega) = \frac{1}{2}\{A_2(\omega-\omega_c) + A_2(\omega+\omega_c)\}$



Next, consider the lower branch.

(4) Since $b_1(t) = m(t) \cdot \sin(\omega_o t) \Rightarrow B_1(\omega) = \frac{1}{2j}\{M(\omega-\omega_o) - M(\omega+\omega_o)\}$.

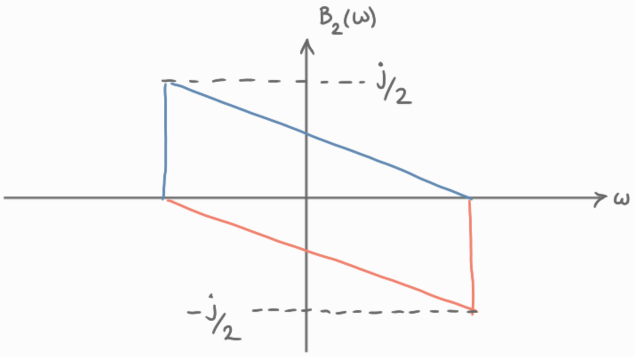
Thus, $B_1(\omega) = j/2 \{M(\omega+\omega_o) - M(\omega-\omega_o)\}$.



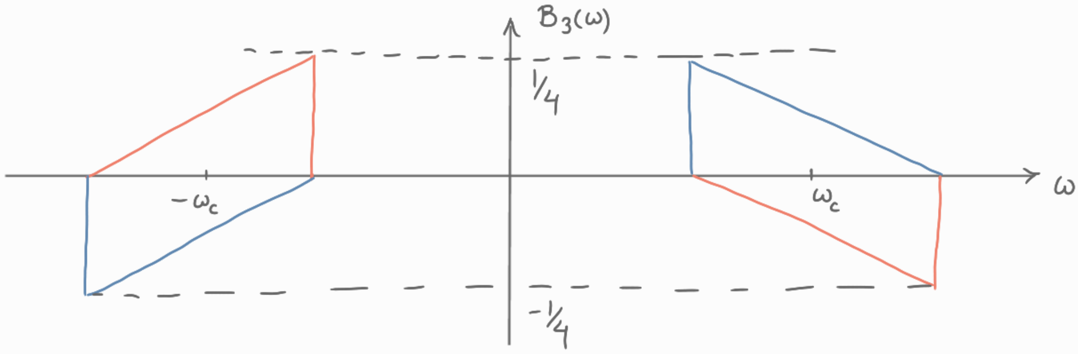
NOTE : ⊛ When $a_1(t)$ is multiplied by $\cos(\omega_o t)$, the resulting spectrum only has real parts, which is why $A_1(\omega)$ has only real part and no imaginary part.

⊛ When $b_1(t)$ is multiplied by $\sin(\omega_o t)$, the resulting spectrum has only imaginary parts. Thus, $B_1(\omega)$ has only imaginary part, and varies between $+j/2$ and $-j/2$.

(5) Next, $b_1(t)$ is passed through an ideal LPF, and it produces $b_2(t)$. Therefore, $B_2(\omega)$ is presented as follows, since the cutoff frequency of the LPF is $\omega_o - \omega_a$.



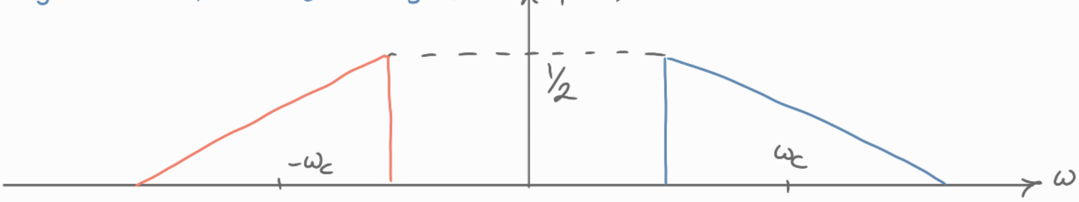
(6) Finally, $b_3(t) = b_2(t) \cdot \sin(\omega_c t) \Rightarrow B_3(\omega) = \frac{j}{2}\left\{B_2(\omega+\omega_c) - B_2(\omega-\omega_c)\right\}$



NOTE : The spectra $B_1(\omega)$ and $B_2(\omega)$ have only imaginary part. However, when $B_2(\omega)$ is multiplied by $\sin(\omega_c t)$, the resulting spectrum is derived as follows: $B_2(\omega)$ is shifted to right and multiplied by $(-j/2)$, and $B_2(\omega)$ is also shifted to the left and multiplied by $(+j/2)$.

The right side of the spectrum has magnitude $(-j^2/4) = 1/4$ and the left side of the spectrum has magnitude $(j^2/4) = -1/4$ thus the spectrum is inverted about the ω-axis.

$\Rightarrow \phi(t) = a_3(t) + b_3(t) \Rightarrow \Phi(\omega) = A_3(\omega) + B_3(\omega)$



NOTE (a) $B_3(\omega)$ has spectrum that has only real parts, and $A_3(\omega)$ too has only real parts. Thus, they could be easily added to get $\phi(\omega) = A_3(\omega) + B_3(\omega)$. Also note that the blue-ink triangles add up constructively, while the red-ink triangles cancel each other.

(b) Note that the current block diagram produces an upper sideband SSB signal (SSB_+).

In order to generate a lower sideband SSB (SSB_-) signal, we multiply the lower branch by -1 (e.g. multiply by $-\cos(\omega_c t)$ instead of $\cos(\omega_c t)$). This way, the upper sidebands are cancelled, and only the lower sideband remains, resulting in the spectrum $\phi(\omega) = A_3(\omega) - B_3(\omega)$.

END OF TUTORIAL #