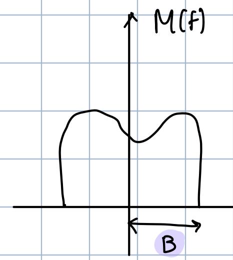
Modulation: the process of transforming m(t) [message] to make it suitable for transmission over the channel



$c(t) = A_c \cos(w_c t + \theta_c) \longrightarrow$ carrier signal ($f_c >> B$, $w_c = 2\pi f_c$)

① varying amplitude of c(t) according to message ⟶ amplitude modulation

② Varying angle of c(t) according to message ⟶ angle modulation
- ↳ phase modulation
- ↳ frequency modulation

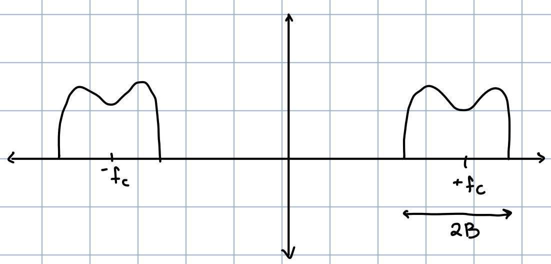
↳ angle of c(t) is $\theta(t) = w_c t + \theta_c$

Types of amplitude modulation

① Double side band - supressed carrier (DSC-SC)

modulated signal: $A_c m(t) \cos(w_c t) = \varphi(t)$ $(\theta_c = 0)$

$\phi(f) = \frac{A_c}{2}[M(f-f_c) + M(f+f_c)]$



frequency multiplexing

↳ allocating different frequiences for different stations so that they can send messages at same time

↳ this is why we need carrier modulation

demodulation: $\varphi(t) \cdot \cos w_c t = A_c m(t) \cos^2 w_c t = \frac{A_c}{2}(m(t) + \xrightarrow{LPF} \cancel{m(t)\cos 2w_c t})$

modulated signal

$= \frac{A_c}{2} m(t)$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

Shannon's formula

$C = B \log(1 + SNR)$ — signal to noise ratio

channel width

(or) $SNR = 2^{C/B} - 1$ ⟶ as $B\uparrow \rightarrow SNR\downarrow$

$-A_c m(t) \le A_c m(t) \cos w_c t \le A_c m(t)$

$\varphi(t) = A_c m(t) \cos w_c t$

$E(t) = |A_c m(t)|$

↳ envelope of $\varphi(t)$

add unmodulate large carrier to DSB-SC signal $\varphi(t)$

s.t the envelope contains the message

$\varphi_{AM}(t) = A_c(\underbrace{1 + m(t)}_{\geq 0})\cos(\omega_c t)$

DSB-LC

(conventional AM)

first one proposed ~1910s

DC term that you can get rid of by a transformer

we are assuming $-1 \leq m(t) \leq 1$ such that +1 would work

if not, we would need to normalize the message

normalized message

$\varphi_{AM}(t) = A_c(1 + \alpha m_n(t))\cos\omega_c t$

$m_n = \dfrac{m(t)}{\max|m(t)|}$ → using the max can raise the envelope more than nessesary

$0 \leq \alpha \leq 1$

modulation index

$\varphi_{AM}(t) = A_c\cos(\omega_c t) + Am(t)\cos(\omega_c t)$

for $1 + m(t) \geq 0$, we need $\min(m(t)) \geq -1$

$\therefore \; m_n(t) = \dfrac{m(t)}{|\min(m(t))|}$

not an issue in practice since:

$\min(m(t)) = -\max(m(t))$

for signals of interest

Power

average

$$\overline{\varphi_{AM}(t)^2} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \varphi_{AM}(t)^2\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left(A_c^2\cos^2\omega_c t + A_c^2\alpha^2 m_n(t)^2\cos^2\omega_c t + 2A_c\alpha m_n(t)\cos\omega_c t\right)dt$$

$$= \overline{A_c^2\cos^2\omega_c t} + \overline{A_c^2\alpha^2 m_n(t)^2\cos^2\omega_c t} + \overline{2A_c\cancel{\alpha m_n(t)}\cos\omega_c t}$$

$$= \frac{A_c^2}{2} + \underbrace{\frac{A_c^2\alpha^2 P_{m_n}}{2}}_{\text{useful power}}$$

since f_c is big, and $m_n(t)$ is slowly varying compared to $\cos^2\omega_c t$,

as $T\to\infty$, we can treat $m_n(t)$ to be constant

AM power effiency $= \mu = \dfrac{\text{useful power}}{\text{total power}} = \dfrac{\frac{\alpha^2 A_c^2}{2} P_{mn}}{\frac{A_c^2}{2} + \frac{A_c^2\alpha^2}{2} P_{mn}}$

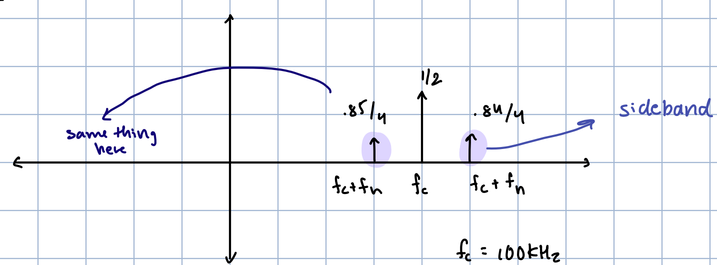
$m(t) = 3\cos(300\pi t)$, $c(t) = \cos\omega_c t$ $\quad \omega_c = 2\pi 10^5$

$m_n(t) = \cos(300\pi t)$

$\varphi_{AM} = (1 + .85\cos 300\pi t)\cos\omega_c t$

$\overline{\varphi_{AM}(t)^2} = \frac{1}{2} + \frac{.85^2}{2}\cdot\frac{1}{2}$

$\mu = \dfrac{.85^2 \cdot 1/4}{\frac{1}{2} + \frac{.85^2}{4}} \approx \frac{1}{3}$

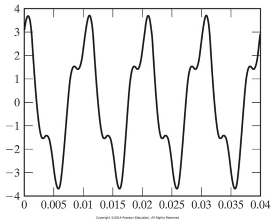


$f_c = 100\text{kHz}$

$f_n = 150\text{Hz}$

Example:

Example: The signal $m(t) = 3\cos 200\pi t + \sin 600\pi t$ is used to modulate the carrier $c(t) = \cos(2\pi 10^5 t)$. The modulation index is $\alpha = 0.85$. Determine the power in the carrier component and the sideband components of the AM modulated signal.



Normalizing message function

$$\varphi_{AM} = A_c (1 + \alpha m_n(t)) \cos w_c(t)$$

$$m(t) = 3\cos 200\pi t + \sin 600\pi t$$

$$m_n(t) = \frac{m(t)}{\max(m(t))}$$

$$\max(m(t)) \Rightarrow m'(t) = 0$$

$$-\cancel{3 \cdot 200}\pi \sin 200\pi t + \cancel{600\pi} \cos 600\pi t = 0$$

$$\sin 200\pi t = \cos 600\pi t$$

$$\cos\left(200\pi t - \frac{\pi}{2}\right) = \cos(600\pi t)$$

$$\pm\left(200\pi t - \frac{\pi}{2}\right) + 2\pi k = 600\pi t$$

$$t_0 = \frac{1}{1600}$$

$$m(t_0) = 3.7 = \max(m(t))$$