Useful Formulae

Trigonometric functions	**Fourier transform**
$\cos\alpha = \frac{e^{j\alpha}+e^{-j\alpha}}{2},\ \sin\alpha = \frac{e^{j\alpha}-e^{-j\alpha}}{2j}$	$X(f) \triangleq \mathcal{F}[x(t)] = \int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)dt$
$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$	$x(t) \triangleq \mathcal{F}^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)df$
$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$	$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt,\ \text{where}\ \omega = 2\pi f$
$\sin\alpha\sin\beta = -\frac{1}{2}[\cos(\alpha+\beta)-\cos(\alpha-\beta)]$	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$

$\mathrm{sinc}(\cdot)$, $\Pi(\cdot)$, and $\Lambda(\cdot)$ functions:

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x},\ \Pi(t) = \begin{cases} 1, & -\frac{1}{2} \le t \le \frac{1}{2} \\ 0 & \text{otherwise} \end{cases},\ \Lambda(t) = \Pi(t) * \Pi(t) = \begin{cases} 1+t, & -1 \le t \le 0 \\ 1-t & 0 < t \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Fourier transform pairs and properties:

Time Domain	Frequency Domain	Signal	Fourier Transform
$\delta(t)$	1	$\alpha x_1(t) + \beta x_2(t)$	$\alpha X_1(f) + \beta X_2(f)$
1	$\delta(f)$	$X(t)$	$x(-f)$
$\delta(t-t_0)$	$e^{-j2\pi f t_0}$	$x(t-t_0)$	$e^{-j2\pi f t_0}X(f)$
$e^{j2\pi f_c t}$	$\delta(f-f_c)$	$e^{j2\pi f_c t}x(t)$	$X(f-f_c)$
$\Pi(t)$	$\mathrm{sinc}(f)$	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)$
$\mathrm{sinc}(t)$	$\Pi(f)$	$x(t) * y(t)$	$X(f)Y(f)$
$\Lambda(t)$	$\mathrm{sinc}^2(f)$	$x(t)y(t)$	$X(f) * Y(f)$
$\mathrm{sinc}^2(t)$	$\Lambda(f)$	$\frac{d}{dt}x(t)$	$j2\pi f X(f)$
$e^{-\alpha t}u(t),\ \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
$e^{-\alpha\lvert t\rvert},\ \alpha > 0$	$\frac{2\alpha}{\alpha^2+(2\pi f)^2}$	$tx(t)$	$\left(\frac{j}{2\pi}\right)\frac{d}{df}X(f)$
$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$	$t^n x(t)$	$\left(\frac{j}{2\pi}\right)^n \frac{d^n}{df^n}X(f)$
$u(t)$	$\frac{1}{2}\left(\delta(f) + \frac{1}{j\pi f}\right)$	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{X(f)}{j2\pi f} + \frac{1}{2}X(0)\delta(f)$
$\sum_{n=-\infty}^{\infty}\delta(t-nT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty}\delta\left(f - n\frac{1}{T_0}\right)$	$R_x(\tau) = \int_{-\infty}^{\infty} x(t)x^*(t-\tau)dt$	$\lvert X(f)\rvert^2$

Parseval relation:

$$\int_{-\infty}^{\infty} x(t)y^*(t)dt = \int_{-\infty}^{\infty} X(f)Y^*(f)df$$