ECE 318 : TUTORIAL 1

A review of :

- SIGNALS : standard signals, periodic signals, energy vs. power signals, odd vs. even
- FOURIER TRANSFORM (FT)
- FOURIER SERIES (FS)

* Standard Signals :

1/ Step signal "function"

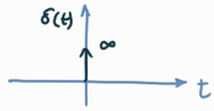
$x(t) = A \cdot u(t)$ where $u(t)$ is the unit step function.

$$x(t) = \begin{cases} 0 & , t<0 \\ A & , t \geq 0 \end{cases}$$

2/ Delta signal "function"

$$\int_{-\infty}^{t} \delta(t)\, dt = 1$$

$$\delta(t) = \begin{cases} \infty & t=0 \\ 0 & t \neq 0 \end{cases}$$



3/ Sinusoidal signals :

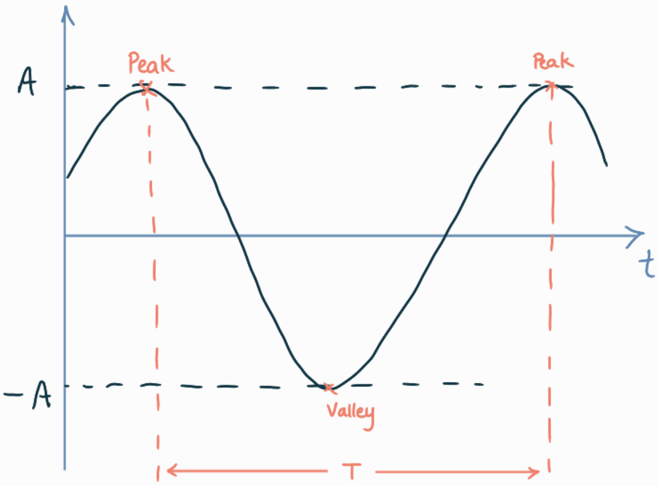
$x(t) = A \cdot \cos(\omega t + \theta)$

OR

$x(t) = A \cdot \sin(\omega t + \theta)$

A: amplitude or peak
θ: phase shift
t: time
ω: angular frequency
f: frequency
T: fundamental period

$$f = \frac{\omega}{2\pi} \qquad T = \frac{1}{f} = \frac{2\pi}{\omega}$$



* Complex Exponential :

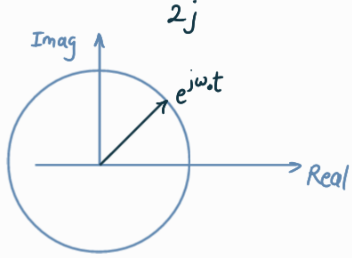
$x(t) = e^{j\omega_0 t}$, constant magnitude $|e^{j\omega_0 t}| = 1$, time-varying phase $\omega_0 t$

Using "Euler identity" :

$x(t) = e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t$ OR $\sin \omega_0 t = \dfrac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$ and $\cos \omega_0 t = \dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

$e^{j0} = e^{j2\pi} = 1$, $e^{j\pi} = -1$

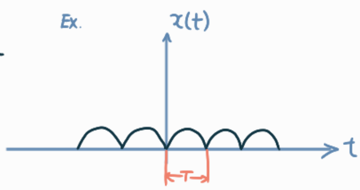
$e^{j\pi/2} = j$, $e^{j3\pi/2} = -j$



* Periodic signals or non-peridic / "aperiodic"

A signal is said to be periodic if for some constant $T > 0$: $x(t) = x(t+T) \quad \forall t, \; T \in \mathbb{R}$

Ex.



If $x(t)$ is periodic then $x(t) = x(t+T) = x(t+2T)$ and T is the smallest period and called "fundamental period"

Remark The sum of periodic signals is not necessarily a periodic signal

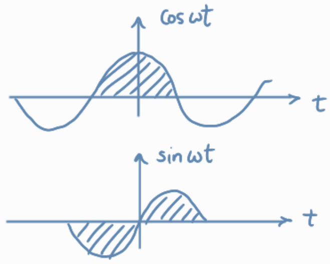
Remark The sum of "continuous" periodic signals is periodic if the least common multiplier of their fundamental period exists.

Ex. $x_1(t)$ is periodic with period T_1

$x_2(t)$ is periodic with period T_2

then $x_3(t) = x_1(t) + x_2(t)$ is periodic if $\frac{T_1}{T_2} = \frac{\text{integer}}{\text{integer}}$ = rational number

*** Odd & even signals:**



If $x(t)$ is an even signal, then $x(t) = x(-t)$ Ex. $\cos \omega t = \cos -\omega t$

If $x(t)$ is an odd signal, then $x(t) = -x(-t)$ Ex. $\sin \omega t = -\sin -\omega t$

For even signals: $x(t)$ is even, then: $\int_{-a}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt$

For odd signals: $x(t)$ is odd, then: $\int_{-a}^{a} x(t)\,dt = 0$

*** Energy vs. power signals:**

- A signal is said to be an energy signal if it has finite energy ($E < \infty$), e.g. time-limited signals or decaying signals.

$$E \triangleq \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

- A signal is said to be a power signal if its power is finite & $\neq 0$

$$\text{Average power} = P \triangleq \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt$$

Remark The average power for periodic signals can be obtained from one period, i.e.:

$$P = \frac{1}{T}\int_{0}^{T} |x(t)|^2\,dt \rightsquigarrow \text{for periodic signals only}$$

Remark Any periodic signal is a power signal but the reverse is not always true.

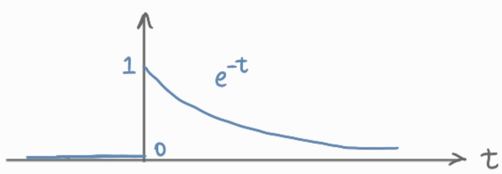
Ex. Consider the signal: $f(t) = e^{-t}u(t)$. Calculate its average power, energy and its type.

<u>SoL</u>:

$$E \triangleq \int_{-\infty}^{\infty} |e^{-t}u(t)|^2\,dt$$

$$= \int_{0}^{\infty} e^{-2t}\,dt$$

$$= \frac{e^{-2t}}{-2}\Bigg|_{t=0}^{\infty} = \frac{1}{2}$$



$u(t) = 0,\ t<0$ & $|e^{-t}| = e^{-t},\ t \geq 0$

Since $E = 1/2 < \infty$ then this signal is an energy signal and hereby, its average power is zero, $P_{av} = 0$.

NOTE If the signal is an energy signal, then its average power is zero. On the other hand, if the signal is a power signal then its energy is infinity.

Ex. Find the average power of $f(t)$: $f(t) = \sum_{n=-\infty}^{\infty} f_o(t - nT_o)$, $T_o = 4$, $f_o(t) = \begin{cases} t^3, & |t| \leq 2 \\ 0, & \text{otherwise} \end{cases}$

Sol: In this example, it is very useful to check if the given signal is periodic or not since if it is a periodic signal, you will be able to use the formula of the average power of the periodic signals and this will save much time.

Check for periodicity: The signal is periodic if $f(t) = f(t+T)$ for any $T>0$.

$f(t+T) = \sum_{n=-\infty}^{\infty} f_o(t+T-nT_o)$ is there any $T>0$ that makes $f(t+T) = f(t)$?

Let us use $T = T_o$, then $f(t+T) = \sum_{n=-\infty}^{\infty} f_o(t-(n-1)T_o)$. Using a change of variable $z = n-1$, then:
boundary of sum $-\infty, \infty$

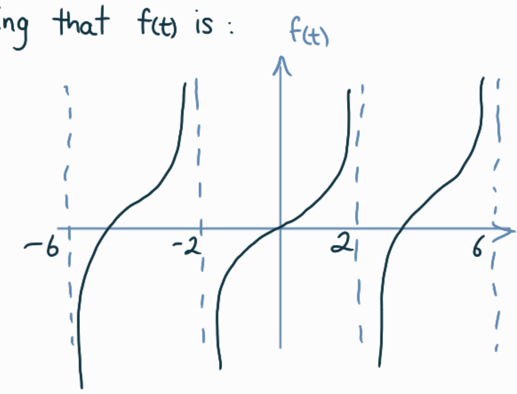
$f(t+T) = \sum_{z=-\infty}^{\infty} f_o(t - zT_o)$

As a result, $f(t+T) = f(t)$ which means that the signal is periodic with period $T_o = 4$

Since the signal is periodic, $P_{av} = \frac{1}{T_o}\int_{-T_o/2}^{T_o/2} |f(t)|^2 dt$

$$P_{av} = \frac{1}{4}\int_{-2}^{2} (t^3)^2 dt = \frac{1}{4}\frac{t^7}{7}\Big|_{t=-2}^{2} = \frac{64}{7} \text{ watts}$$

Note: If you did not check the periodicity of the signal in the beginning, you would evaluate P_{av} using this definition: $P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left|\sum_{n=-\infty}^{\infty} f_o(t-nT_o)\right|^2 dt$ noting that $f(t)$ is:



Ex. Find the average power of the signal $f(t) = \sin(t) + \sin(\pi t)$

Sol: we should do a periodicity check first:

→ $\sin(t)$ is periodic with $T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{1} = 2\pi$

→ $\sin(\pi t)$ is periodic with $T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{\pi} = 2$

However, $f(t)$ is a sum of periodic signals and may not be periodic. Check for ratio of T_1/T_2:

$\frac{T_1}{T_2} = \frac{2\pi}{2} = \pi$ irrational number. As a result $f(t)$ is not periodic :(

In this case we have to calculate its average power using the general formula:

$$\left.\begin{aligned} &P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |f(t)|^2 dt \\ &|f(t)|^2 = |\sin(t) + \sin(\pi t)|^2 \rightarrow \text{is an even and real function} \end{aligned}\right\} \Rightarrow P_{av} = \lim_{T\to\infty} \frac{2}{T}\int_{0}^{T/2} f(t)^2 dt$$

Now

$$f^2(t) = [\sin(t) + \sin(\pi t)]^2$$

$$= \sin^2(t) + \sin^2(\pi t) + 2\sin(t)\sin(\pi t)$$

$$= \frac{1}{2}(1 - \cos(2t)) + \frac{1}{2}(1 - \cos(2\pi t)) + \cos((\pi-1)t) - \cos((\pi+1)t)$$

$$= 1 + \cos((\pi-1)t) - \cos((\pi+1)t) - \frac{\cos(2t)}{2} - \frac{\cos(2\pi t)}{2}$$

$$P = \lim_{T\to\infty} \left\{ \frac{2}{T}\int_0^{T/2} 1\,dt + \frac{2}{T}\int_0^{T/2} \cos(\pi-1)t\,dt - \frac{2}{T}\int_0^{T/2} \cos(\pi+1)t\,dt - \frac{2}{T}\int_0^{T/2} \frac{\cos 2t}{2}\,dt - \frac{2}{T}\int_0^{T/2} \frac{\cos 2\pi t}{2}\,dt \right\}$$

The integration of the cosine terms are bounded and when divided by T within the limit $T\to\infty$, the result will approach zero. The only remaining term will be 1 and thus:

$$P = \lim_{T\to\infty} \frac{2}{T}\int_{t=0}^{T/2} 1\cdot dt = \lim_{T\to\infty} \frac{2}{T}\,\frac{T}{2} = 1$$

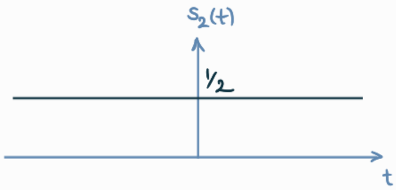
$\Rightarrow$ Power of $f(t)$ is $P_{av} = 1$. The signal is a power signal.

Ex. $X(t) = \cos^2(2t - \pi/3)$ is it periodic?

SoL: Using $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

$$X(t) = \frac{1}{2}\left[1 + \cos\left(4t - \frac{2\pi}{3}\right)\right]$$

$$X(t) = \underbrace{\frac{1}{2}}_{s_2} + \underbrace{\frac{1}{2}\cos(4t - 2\pi/3)}_{s_1}$$



$s_1 = \frac{1}{2}\cos(4(t - \pi/6))$ is periodic with $T_1 = \frac{2\pi}{4} = \frac{\pi}{2}$

Is s_2 periodic or not? Let us draw it first:

s_2 is a constant signal which means it has a flexible $T_2 > 0$

Since T_2 is the smallest period, we say $T_2 = \varepsilon$ where ε is a very small number > 0

However we really care about the ratio $\frac{T_1}{T_2}$ to be $\frac{\text{integer}}{\text{integer}}$, hence: $\frac{T_1}{T_2} = \frac{\pi/2}{\varepsilon} = \frac{n_1}{n_2}$, n_1, n_2 are integers

$T = n_2 \cdot \pi/2 = n_1 \cdot \varepsilon$

If we choose $n_2 = 1$, we still have the flexibility to set n_1 & ε such that their product is $\frac{\pi}{2}$

As a result, $T = \pi/2$ and $X(t)$ is periodic.

Ex. $X(t) = \cos(t) + \sin(2t)$ is $X(t)$ periodic? What is the period?

SoL: $\cos(t)$ is periodic with $T_1 = \frac{2\pi}{1} = 2\pi$, since $\omega = 1$

$\sin(2t)$ is periodic with $T_2 = \frac{2\pi}{2} = \pi$, since $\omega = 2$

Then $X(t)$ is periodic if $\frac{T_1}{T_2} = \frac{n_1}{n_2} = \frac{\text{integer}}{\text{integer}}$: $\frac{T_1}{T_2} = \frac{2\pi}{\pi} = \frac{2}{1}$ and $T = 1\cdot T_1 = 2\cdot T_2 = 2\pi$

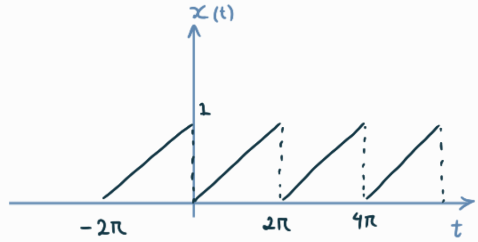
$\Rightarrow$ $X(t)$ is periodic with period 2π.

Ex. Calculate the average power of $e^{-j\omega_0 t}$

SoL: $$P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |e^{-j\omega_0 t}|^2\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} 1^2\,dt = \lim_{T\to\infty} \frac{1}{T}\, t\Big|_{-T/2}^{T/2} = \lim_{T\to\infty} \frac{1}{T}\times T = \lim_{T\to\infty} 1 = 1$$

Ex. Find the exponential Fourier series and sketch the corresponding spectrum for the following signal.



SOL: $x(t)$ is periodic with a fundamental period $T = 2\pi$.

As a result $\omega_0 = \frac{2\pi}{T} = 1$

The Fourier series coefficients are: $X_n = \frac{1}{T}\int x(t)\, e^{-jn\omega_0 t}\, dt$

You can choose the period of $x(t)$ over which the signal $x(t)$ is nicely expressed mathematically. In the given case, $x(t)$ is nicely expressed between $[0, 2\pi]$

* $x(t)$ is called sawtooth signal and it is used alot in communication and control systems. For example, sweeping signal in screens

$$X_n = \frac{1}{2\pi}\int_0^{2\pi} \frac{t}{2\pi} e^{-jn\omega_0 t}\, dt = \frac{1}{(2\pi)^2}\int_0^{2\pi} t\, e^{-jn\omega_0 t}\, dt$$

Usually it is better to isolate X_0

$$X_0 = \frac{1}{(2\pi)^2}\int_0^{2\pi} t\, dt = \frac{1}{(2\pi)^2}\cdot \frac{t^2}{2}\Big|_0^{2\pi} = \frac{1}{2}$$

For $n \neq 0$:

$$X_n = \frac{1}{(2\pi)^2}\int_0^{2\pi} t\, e^{-jn\omega_0 t}\, dt \quad , \quad n \neq 0$$

Using integration by parts: $\int_a^b f\cdot g' = f\cdot g\Big|_a^b - \int_a^b f'\cdot g$

In the given case $f = t$, $g' = e^{-jn\omega_0 t} \Rightarrow g = \int e^{-jn\omega_0 t}\, dt = \frac{e^{-jn\omega_0 t}}{-jn\omega_0}$

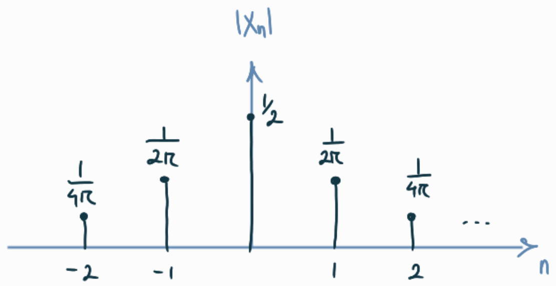
$$X_n = \frac{1}{(2\pi)^2}\left[t\cdot \frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_0^{2\pi} - \int_0^{2\pi} \frac{e^{-jn\omega_0 t}}{-jn\omega_0}\, dt \right] \quad , \quad n \neq 0$$

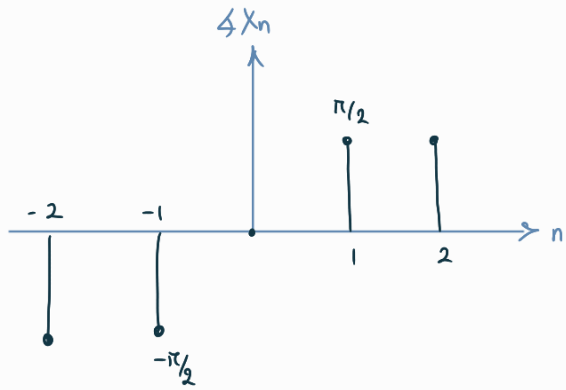
$$X_n = \frac{1}{(2\pi)^2}\left[\frac{2\pi e^{-jn\omega_0 2\pi}}{-jn\omega_0} + \frac{1}{jn\omega_0}\left(\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right)\Big|_0^{2\pi} \right] \quad , \quad n \neq 0 \qquad \omega_0 = \frac{2\pi}{2\pi} = 1$$

$$X_n = \frac{1}{(2\pi)^2}\left[\frac{2\pi e^{-j2\pi n}}{-jn} + \frac{1}{jn}\left(\frac{e^{-jnt}}{-jn}\right)\Big|_0^{2\pi} \right]$$

$$X_n = \frac{1}{(2\pi)^2}\cdot\frac{2\pi}{-jn}\left[\underbrace{e^{-j2\pi n}}_{1} + \frac{1}{2\pi jn}\left(\underbrace{e^{-j2\pi n}}_{1} - 1\right) \right] = \frac{1}{-j2\pi n} = \frac{j}{2\pi n} \quad , \quad n \neq 0$$

$$|X_n| = \left|\frac{j}{2\pi n}\right| = \frac{1}{2\pi n} \quad , \quad \measuredangle X_n = \measuredangle j - \measuredangle 2\pi n = \frac{\pi}{2} - \begin{cases} 0 & , n > 0 \\ \pi & , n < 0 \end{cases} = \begin{cases} \pi/2 & , n > 0 \\ -\pi/2 & , n < 0 \end{cases}$$





Ex. Determine the average power in the signal using Fourier series. $x(t) = \sin^2(2t)\cos(t)$

SOL: For trigonometric functions, one can get Fourier series expansion using Euler formula:

$$\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j} , \quad \cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2} \Rightarrow \sin 2t = \frac{e^{j2t}-e^{-j2t}}{2j} , \quad \cos t = \frac{e^{jt}+e^{-jt}}{2}$$

$$x(t) = \left[\frac{e^{j2t}-e^{-j2t}}{2j}\right]^2\left[\frac{e^{jt}+e^{-jt}}{2}\right] = -\frac{1}{8}\left[e^{j4t}+e^{-j4t}-2\right]\left[e^{jt}+e^{-jt}\right]$$

$$= -\frac{1}{8}\left[e^{j5t}+e^{-j3t}-2e^{jt}+e^{j3t}+e^{-j5t}-2e^{-jt}\right]$$

$$= \frac{1}{4}e^{jt}+\frac{1}{4}e^{-jt}-\frac{1}{8}e^{j3t}-\frac{1}{8}e^{-j3t}-\frac{1}{8}e^{j5t}-\frac{1}{8}e^{-j5t}$$

$x(t)$ can be written in the form of Fourier series expansion as $x(t) = \sum_{n=-5}^{5} X_n e^{jn\omega_0 t}$

$X_0 = 0$, $X_1 = X_{-1} = \frac{1}{4}$, $X_2 = X_{-2} = 0$, $X_3 = X_{-3} = X_5 = X_{-5} = -\frac{1}{8}$

Using Parseval theorem: $P_{av} = \sum_{n=-\infty}^{\infty} |X_n|^2 = \sum_{n=-5}^{5} |X_n|^2 = \frac{1}{64}(4+4+1+1+1+1) = \frac{12}{64}$

Ex. Determine the Fourier transform of the following signals:

a) $u(t)$ b) $\mathrm{sgn}(t) \triangleq \begin{cases} 1 & , t>0 \\ 0 & , t=0 \\ -1 & , t<0 \end{cases}$ c) $\mathrm{rect}\left(\frac{t}{\tau}\right)$ d) $\frac{1}{t}$ e) $e^{j\omega_0 t}u(t)$

SOL:

a) $u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$ (I) $\rightarrow$ One definition of $u(t)$, using Fourier transform properties:

Integration property $\int_{-\infty}^{t} x(\tau)\, d\tau \xrightarrow{\mathcal{F}} \frac{X(\omega)}{j\omega} + \pi X(0)\,\delta(\omega)$

$x(t)$		$X(\omega)$
$\delta(t)$	$\xrightarrow{\mathcal{F}}$	1
$\int_{-\infty}^{t} \delta(\tau)d\tau$	$\xrightarrow{\mathcal{F}}$	$\frac{X(\omega)}{j\omega} + \pi X(0)\,\delta(\omega)$, $X(\omega)=1$, $X(0)=1$
$u(t)$	$\xrightarrow{\mathcal{F}}$	$\frac{1}{j\omega} + \pi\,\delta(\omega)$

$$F\{u(t)\} = \frac{1}{j\omega} + \pi\,\delta(\omega)$$

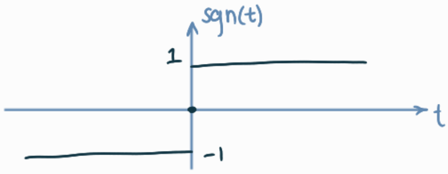
b) $\operatorname{sgn}(t) = u(t) - u(-t) \Rightarrow \mathcal{F}\{\operatorname{sgn}(t)\} = \mathcal{F}\{u(t)\} - \mathcal{F}\{u(-t)\}$ ← using linearity property of Fourier transform

From part (a) $\mathcal{F}\{u(t)\} = \frac{1}{j\omega} + \pi\delta(\omega)$ $\quad \mathcal{F}\{u(-t)\} = ?$

$u(-t)$ is related to $u(t)$ by scaling the time in $u(t)$ by -1, so we can utilize the scaling property.

$\mathcal{F}\{x(a\cdot t)\} = \frac{1}{|a|} X\left(\frac{\omega}{a}\right) \Rightarrow \mathcal{F}\{u(-t)\} = \frac{1}{|-1|}\left[\frac{1}{j\frac{\omega}{-1}} + \pi\delta\left(\frac{\omega}{-1}\right)\right] = \frac{-1}{j\omega} + \pi\delta(\omega)$ since $\delta(t)$ is an even function

$\mathcal{F}\{\operatorname{sgn}(t)\} = \frac{1}{j\omega} + \cancel{\pi\delta(\omega)} + \frac{1}{j\omega} - \cancel{\pi\delta(\omega)} \Rightarrow \mathcal{F}\{\operatorname{sgn}(t)\} = \frac{2}{j\omega}$



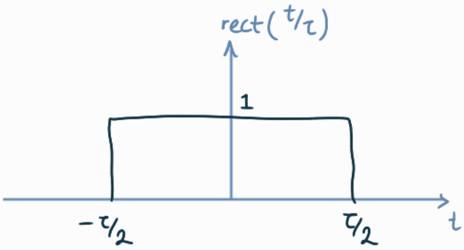
c) $\operatorname{rect}\left(\frac{t}{\tau}\right) = u\left(t+\frac{\tau}{2}\right) - u\left(t-\frac{\tau}{2}\right)$

$\mathcal{F}\left\{\operatorname{rect}\left(\frac{t}{\tau}\right)\right\} = \mathcal{F}\left\{u\left(t+\frac{\tau}{2}\right)\right\} - \mathcal{F}\left\{u\left(t-\frac{\tau}{2}\right)\right\}, \quad \mathcal{F}\{u(t)\} = \frac{1}{j\omega} + \pi\delta(\omega)$

$\mathcal{F}\left\{u\left(t-\frac{\tau}{2}\right)\right\} = e^{-j\omega\frac{\tau}{2}}\left[\frac{1}{j\omega} + \pi\delta(\omega)\right]$ using time-shift property

$\mathcal{F}\left\{u\left(t+\frac{\tau}{2}\right)\right\} = e^{j\omega\frac{\tau}{2}}\left[\frac{1}{j\omega} + \pi\delta(\omega)\right]$

$\Rightarrow \mathcal{F}\left\{\operatorname{rect}\left(\frac{t}{\tau}\right)\right\} = \frac{e^{j\omega\frac{\tau}{2}} - e^{-j\omega\tau/2}}{j\omega} + \left[e^{j\omega\frac{\tau}{2}} - e^{-j\omega\frac{\tau}{2}}\right]\pi\delta(\omega)$

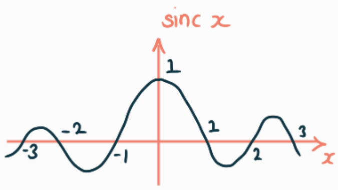


Recall $x(t)\cdot\delta(t-t_0) = x(t_0)$ $\Rightarrow e^{j\omega\tau/2}\delta(\omega) = e^{j0\tau/2} = 1, \quad e^{-j\omega\tau/2}\delta(\omega) = 1 \Rightarrow \left[e^{j\omega\tau/2} - e^{-j\omega\frac{\tau}{2}}\right]\delta(\omega) = 0$

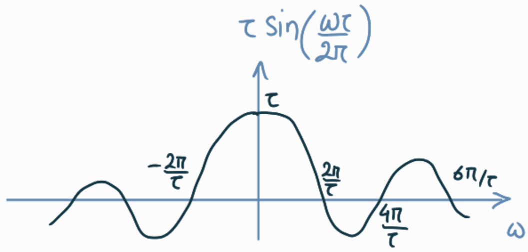
$\mathcal{F}\{\operatorname{rect}(t/\tau)\} = \frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{j}\cdot\frac{1}{\omega} = \frac{2\sin(\omega\tau/2)}{\omega}$ using Euler theorem

$\mathcal{F}\{\operatorname{rect}(t/\tau)\} = \frac{2\sin(\omega\tau/2)}{\omega} = \frac{2\sin\left(\pi/\pi\ \omega\tau/2\right)\cdot\tau/2}{\pi/\pi\cdot\omega\cdot\tau/2}$

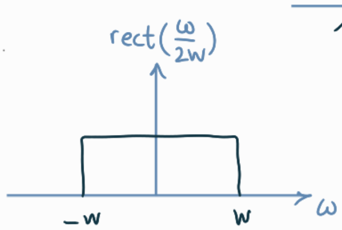
$\operatorname{Sinc}\square = \frac{\sin(\pi\square)}{\pi\square}$



$= 2\frac{\tau}{2}\frac{\sin\left(\pi\frac{\omega\tau}{2\pi}\right)}{\pi\frac{\omega\tau}{2\pi}} \Rightarrow \mathcal{F}\{\operatorname{rect}(t/\tau)\} = \tau\operatorname{sinc}\left(\frac{\omega\tau}{2\pi}\right)$



* Ex. Find the Fourier inverse of $\operatorname{rect}\left(\frac{\omega}{2W}\right)$.



$\mathcal{F}^{-1}\left\{\operatorname{rect}\left(\frac{\omega}{2W}\right)\right\} = ?$

$\mathcal{F}\{\operatorname{rect}(t/\tau)\} = \tau\operatorname{sinc}\left(\frac{\omega\tau}{2\pi}\right)$

duality property

$\mathcal{F}\left\{\tau\operatorname{sinc}\left(\frac{t\tau}{2\pi}\right)\right\} = 2\pi\operatorname{rect}\left(\frac{-\omega}{\tau}\right)$, rect is an even function

$\mathcal{F}\left\{\operatorname{sinc}\left(\frac{t\tau}{2\pi}\right)\right\} = \frac{2\pi}{\tau}\operatorname{rect}\left(\frac{\omega}{\tau}\right) \Rightarrow \frac{\tau}{2\pi}\mathcal{F}\left\{\operatorname{sinc}\left(\frac{t\tau}{2\pi}\right)\right\} = \operatorname{rect}\left(\frac{\omega}{\tau}\right)$, Let $\tau = 2W$:

$\mathcal{F}\left\{\frac{W}{\pi}\operatorname{sinc}\left(\frac{Wt}{\pi}\right)\right\} = \operatorname{rect}\left(\frac{\omega}{2W}\right)$

d) $1/t$

We know from (b) that $\mathcal{F}\{\operatorname{sgn}(t)\} = \frac{2}{j\omega}$. Using duality:

$\mathcal{F}\left\{\frac{2}{jt}\right\} = 2\pi \operatorname{sgn}(-\omega)$, sgn is an odd function

$\mathcal{F}\left\{\frac{1}{t}\right\} = \frac{-j}{2} \cdot 2\pi \operatorname{sgn}(\omega) \Rightarrow \mathcal{F}\left\{\frac{1}{t}\right\} = -\pi j \operatorname{sgn}(\omega)$

e) $e^{j\omega_0 t} u(t)$

From (a) $\mathcal{F}\{u(t)\} = \frac{1}{j\omega} + \pi\delta(\omega)$. Using frequency shift property:

$\mathcal{F}\{e^{j\omega_0 t} u(t)\} = \frac{1}{j(\omega-\omega_0)} + \pi\delta(\omega-\omega_0)$

Ex. Find the Fourier transform of : a) $e^{-t}u(t)$ b) $\frac{1}{1-jt}$

SOL :

a) $\mathcal{F}\{e^{-t}u(t)\} = \int_{-\infty}^{\infty} e^{-t}u(t)e^{-j\omega t}dt = \int_{0}^{\infty} e^{-t}e^{-j\omega t}dt = \int_{0}^{\infty} e^{-(j\omega+1)t}dt = \frac{-e^{-(j\omega+1)t}}{j\omega+1}\Big|_0^{\infty} = \frac{1}{j\omega+1}$

b) From (a) $\mathcal{F}\{e^{-t}u(t)\} = \frac{1}{j\omega+1}$, we can use the time scaling property to make $\frac{1}{1-jt}$ from $\frac{1}{1+j\omega}$

$\mathcal{F}\{\underbrace{e^{t}u(-t)}_{X(t)}\} = \frac{1}{1-j\omega}$, Then using duality : $\mathcal{F}\left\{\frac{1}{1-jt}\right\} = 2\pi X(-\omega) = 2\pi e^{-\omega}u(\omega)$

END OF TUTORIAL 1 #