

Problem 1: [8 points]

- i) Consider the following modulation scheme, where the input message is $m(t)$ and $k_d > 0$:
 $\varphi(t) = A \cos(\omega_c t + k_d \frac{d}{dt} m(t))$. If $m(t) = \sin^2 \omega_m t - \cos \omega_m t$, what is:

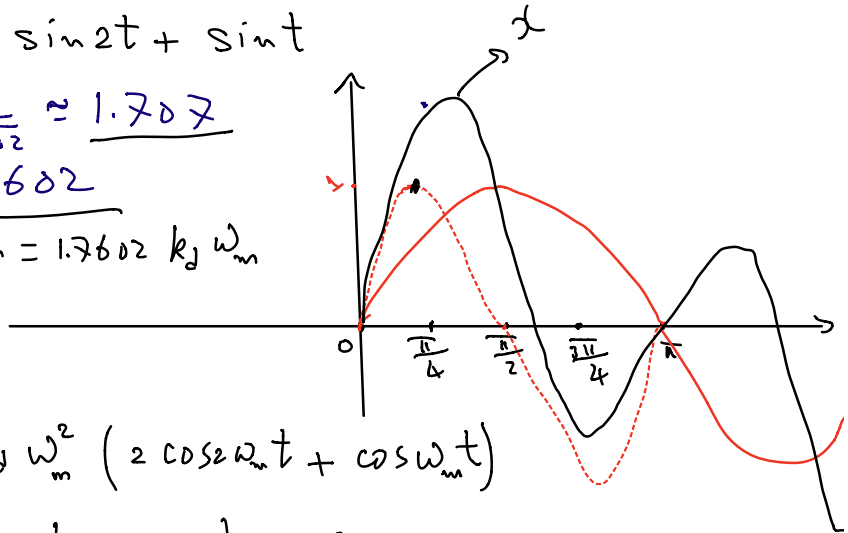
- a) [2 Points] The peak phase deviation of $\varphi(t)$?
 b) [2 Points] The peak frequency deviation of $\varphi(t)$

- ii) Consider a system with an input $\varphi(t)$ and a output $u(t) = [\varphi(t)]^2 - \frac{1}{2}$.

- a) [2 Points] If the input is $\varphi(t) = \cos(\omega_c t + k_p m(t))$, check whether $u(t)$ is AM, PM or FM modulated version of $m(t)$, or none of these? Justify your answer and find the carrier frequency and phase/frequency sensitivity constants.
 b) [1 Points] Repeat part a) but with $\varphi(t) = \cos(\omega_c t + k_f \int_0^t f(\tau) d\tau)$.
 c) [1 Points] Repeat part a) but with $\varphi(t) = [A + m(t)] \cos(\omega_c t)$.

$$\begin{aligned} \text{i.a)} \quad \frac{d m(t)}{dt} &= 2\omega_m \sin \omega_m t \cos \omega_m t + \omega_m \sin \omega_m t \\ &= \omega_m \sin 2\omega_m t + \omega_m \sin \omega_m t \\ \text{peak phase deviation} &= \omega_m k_d \max (\sin 2\omega_m t + \sin \omega_m t) \end{aligned}$$

Look at $x = \sin 2t + \sin t$
 $t = \frac{\pi}{4}, x = 1 + \frac{1}{\sqrt{2}} \approx 1.707$
 x_{\max} is 1.7602
 \Rightarrow Peak phase deviation = 1.7602 $k_d \omega_m$



$$\omega_i(t) = \omega_c + k_d \omega_m^2 (2 \cos 2\omega_m t + \cos \omega_m t)$$

$$\text{peak freq. deviation} = k_d \omega_m^2 3$$

$$t=0, 2\omega_m \cos \omega_m t + \cos \omega_m t = 3$$

ii-a) $\varphi(t) = A \cos(\omega_c t + k_p m(t))$, $A=1$

$$u(t) = \varphi^2(t) - \frac{1}{2}$$

$$= A^2 \left(\frac{1}{2} \cos(2\omega_c t + 2k_p m(t)) + \frac{1}{2} \right) - \frac{1}{2} = \frac{1}{2} \cos(2\omega_c t + 2k_p m(t))$$

is a PM : $2\omega_c$: freq.
 $2k_p$: phase dev. const.

ii.b) $\varphi(t) = \cos(\omega_c t + k_f \int_0^t m(\tau) d\tau)$

$$u(t) = \frac{1}{2} \cos(2\omega_c t + 2k_f \int_0^t m(\tau) d\tau)$$

FM : $2\omega_c$, $2k_f$

ii.c) $\varphi(t) = (A + m(t)) \cos \omega_c t$

$$u(t) = (A + m(t))^2 \left(\frac{1}{2} + \frac{1}{2} \cos 2\omega_c t \right) =$$

$$\frac{1}{2} (A^2 + m^2(t) + 2Am(t)) + \frac{1}{2} (A^2 + m^2(t) + 2Am(t)) \cos 2\omega_c t$$

neither FM, PM nor AM.

Look at $x(t) = \sin 2t + \sin t$
 $\max \{x(t)\}$. $x'(t) = 2 \cos 2t + \cos t = 0$

$$x'(t) = 2(2 \cos^2 t - 1) + \cos t = 4 \cos^2 t + \cos t - 2 = 0$$

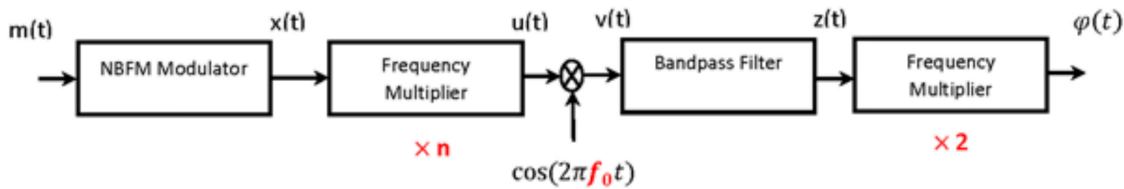
put $y = 4z^2 + z - 2$ ($z = \cos t$)

$$\Delta = 1 + 32 = 33, \quad z = \frac{-1 \pm \sqrt{33}}{8}$$

$$z = \frac{-1 + \sqrt{33}}{8} \Rightarrow \underline{x = 1.7602}$$

$$t = \arccos z + 2k\pi$$

Problem 2: [7 points] Consider the block diagram below.



In this block diagram, $m(t)$ is the input to a Narrowband Frequency Modulator (NBFM), that results in an output $x(t)$ with carrier frequency $\omega_c = 2\pi \times 10^5$ rad/s, and modulation index 0.1. Assume that $m(t)$ is such that $\max \left| \int_0^t m(\tau) d\tau \right| = 1$. The output $\phi(t)$ is to be a wideband FM signal with modulation index of 1.2, and center frequency of 1 MHz.

- a) [4 Points] Design the integer n , frequency f_0 and center frequency of the bandpass filter to meet these design constraints.
 b) [3 Points] If $m(t)$ is a 10 kHz cosine, what must the bandwidth of the bandpass filter be?

a) $x(t)$: $f_c = 100 \text{ kHz}$, $\beta = 0.1$
 $\phi(t)$: $f'_c = 1 \text{ MHz}$, $\beta' = 1.2$

$$2n = \frac{\beta'}{\beta} = \frac{1.2}{0.1} = 12 \Rightarrow n = 6$$

freq. of $u(t)$: $nf_c = 600 \text{ kHz}$

freq. of $v(t)$: $nf_c + f_0$, $nf_c - f_0$

choose f_0 , s.t. $2 |600 \text{ kHz} - f_0| = 1 \text{ MHz}$
 $\Rightarrow f_0 = 100 \text{ kHz}$ (other solution: $f_0 = 1.1 \text{ MHz}$)

centre freq. of BPF: $nf_c - f_0 = 500 \text{ kHz}$

b) $m(t)$ has bandwidth $B = 10 \text{ kHz}$

modulation index of $u(t)$: $\beta_n = n\beta = 0.6$

\Rightarrow bandwidth of $u(t)$ is: $2(1 + \beta_n)B = 32 \text{ kHz}$ (Carson)

\Rightarrow bandwidth of BPF = $B_u = 32 \text{ kHz}$

• using significant sidebands: $2mB$: m is largest integer: $J_m(0.6) > 0.01$
 $m = 2 \Rightarrow B_u = 40 \text{ kHz}$