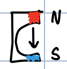
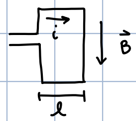
Electro-mechanical systems

"Armature-controlled" DC motor

★ field magnets create constant magnetic fields



★ 'armature' contains current loops



Lorentz force: $\vec{F} = i\vec{\ell} \times \vec{B}$

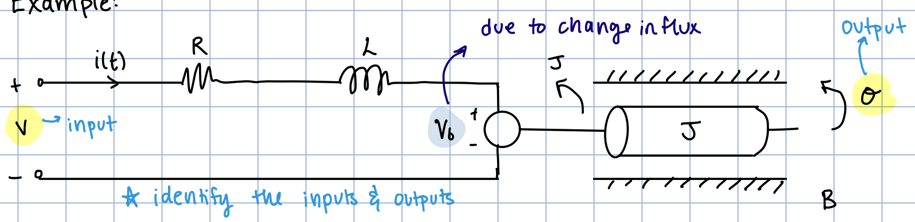
$|\vec{F}| = \ell B i$

greatest when loop is parallel to $\vec{B}$

★ magnetic force $\vec{F}$ leads to torque on the armature (rotor windings)

★ "commutator" reverses the current direction as the rotor turns ensuring continous torque in the same direction

Example:



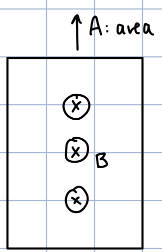
R, L: resistance / inductance

J: moment of inertia

B: damping coeffient (due to friction)

V: applied voltage

V_b: induced voltage (rotation



Lenz law: the direction of an induced current (due to a change in magnetic feild) will always oppose the change that produced it.

magnetic flux $= \phi_B = \vec{A} \times \vec{B}$

$= AB\sin\theta$

$\approx AB\theta$ (small angle approx.)

$\dot{\phi}_B \approx AB\dot{\theta} = V_b$ → didn't put (−) in the way we set up

$-\dfrac{d\phi_B}{dt} = V_{induced}$

Setting up equations:

KVL: $\dfrac{L \cdot i(t)}{dt} + Ri(t) = V(t) - V_b(t)$

Lorentz force law: $\tau(t) = K_m i(t)$

Newton's 2nd law (rotational systems): $\tau(t) - B\dot{\theta}(t) = J\ddot{\theta}(t)$

↳ damping due to friction

$\dot{\theta}$: angular velocity

$\ddot{\theta}$: angular acceleration

Faraday's law: $V_b(t) = K_b\,\dot{\theta}(t)$

How do we find transfer function from an LTI equations?

① take laplace transform with 0 initial conditions & solve for $Y(s)$ as a function of $U(s)$

② draw a block diagram to visualize inputs / transforms

⋮

more next class

Take laplace transforms (with 0 initial conditions)

1. $(L\cdot s + R)\, I(s) = V(s) - V_b(s)$ $\longrightarrow$ $I(s) = \frac{1}{sL+R}\left(V(s) - V_b(s)\right)$

2. $\tau(s) = K_m I(s)$

$T(s) - B\mathcal{L}\{\dot{\theta}(t)\} = J\cdot\mathcal{L}\{\ddot{\theta}(t)\}$

$T(s) = J\cdot\mathcal{L}\{\ddot{\theta}(t)\} + B\mathcal{L}\{\dot{\theta}(t)\}$

$\tau(s) = Js^2\theta(s) + Bs\theta(s)$

3. $T(s) = \theta(s)\,[Js^2 + Bs]$ $\longrightarrow$ $\theta(s) = \frac{1}{Js^2+Bs}\, T(s)$

$V_b(s) = K_b\mathcal{L}\{\dot{\theta}(t)\}$

4. $V_b(s) = K_b s\theta(s)$

Let $\Omega(s) = s\theta(s)$

Think of it as a block diagram