Problem 3.1

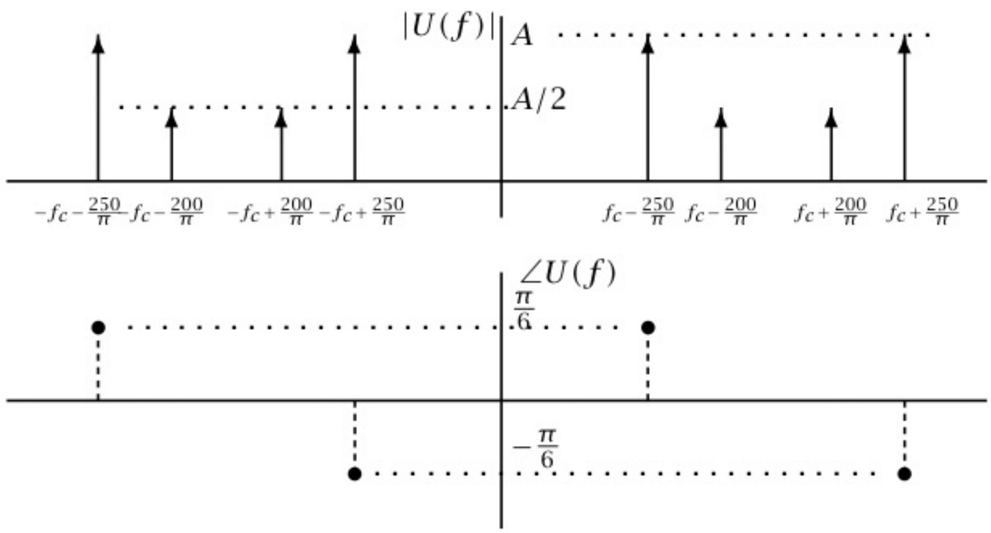
The modulated signal is

$$
\begin{aligned}
u(t) &= m(t)c(t) = Am(t)\cos(2\pi 4\times 10^3 t) \\
&= A\left[2\cos(2\pi\frac{200}{\pi}t) + 4\sin(2\pi\frac{250}{\pi}t + \frac{\pi}{3})\right]\cos(2\pi 4\times 10^3 t) \\
&= A\cos(2\pi(4\times 10^3 + \frac{200}{\pi})t) + A\cos(2\pi(4\times 10^3 - \frac{200}{\pi})t) \\
&\quad +2A\sin(2\pi(4\times 10^3 + \frac{250}{\pi})t + \frac{\pi}{3}) - 2A\sin(2\pi(4\times 10^3 - \frac{250}{\pi})t - \frac{\pi}{3})
\end{aligned}
$$

Taking the Fourier transform of the previous relation, we obtain

$$
\begin{aligned}
U(f) &= A\left[\delta(f-\frac{200}{\pi}) + \delta(f+\frac{200}{\pi}) + \frac{2}{j}e^{j\frac{\pi}{3}}\delta(f-\frac{250}{\pi}) - \frac{2}{j}e^{-j\frac{\pi}{3}}\delta(f+\frac{250}{\pi})\right] \\
&\quad \star \frac{1}{2}[\delta(f-4\times 10^3) + \delta(f+4\times 10^3)] \\
&= \frac{A}{2}\left[\delta(f-4\times 10^3 - \frac{200}{\pi}) + \delta(f-4\times 10^3 + \frac{200}{\pi})\right. \\
&\quad +2e^{-j\frac{\pi}{6}}\delta(f-4\times 10^3 - \frac{250}{\pi}) + 2e^{j\frac{\pi}{6}}\delta(f-4\times 10^3 + \frac{250}{\pi}) \\
&\quad +\delta(f+4\times 10^3 - \frac{200}{\pi}) + \delta(f+4\times 10^3 + \frac{200}{\pi}) \\
&\quad \left.+2e^{-j\frac{\pi}{6}}\delta(f+4\times 10^3 - \frac{250}{\pi}) + 2e^{j\frac{\pi}{6}}\delta(f+4\times 10^3 + \frac{250}{\pi})\right]
\end{aligned}
$$

The next figure depicts the magnitude and the phase of the spectrum $U(f)$.



To find the power content of the modulated signal we write $u^2(t)$ as

$$\begin{aligned} u^2(t) &= A^2\cos^2(2\pi(4\times 10^3+\frac{200}{\pi})t) + A^2\cos^2(2\pi(4\times 10^3-\frac{200}{\pi})t) \\ &\quad +4A^2\sin^2(2\pi(4\times 10^3+\frac{250}{\pi})t+\frac{\pi}{3}) + 4A^2\sin^2(2\pi(4\times 10^3-\frac{250}{\pi})t-\frac{\pi}{3}) \\ &\quad +\text{terms of cosine and sine functions in the first power} \end{aligned}$$

Hence,

$$P = \lim_{T\to\infty}\int_{-\frac{T}{2}}^{\frac{T}{2}} u^2(t)dt = \frac{A^2}{2}+\frac{A^2}{2}+\frac{4A^2}{2}+\frac{4A^2}{2} = 5A^2$$

Problem 3.2

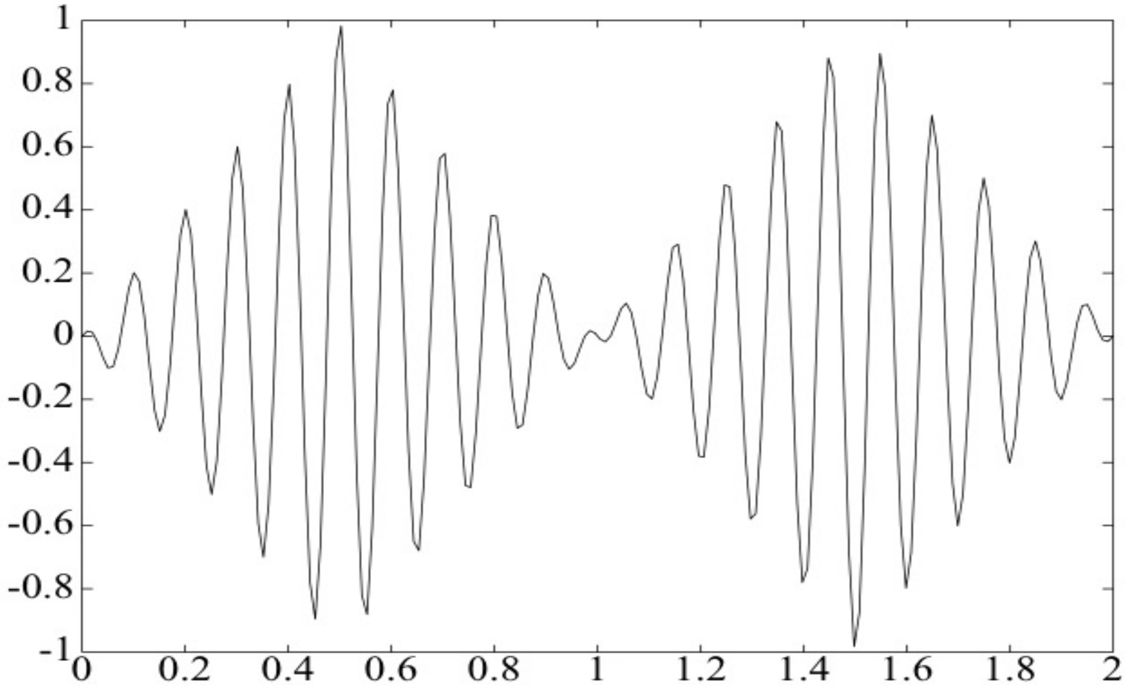
$$u(t) = m(t)c(t) = A(\text{sinc}(t)+\text{sinc}^2(t))\cos(2\pi f_c t)$$

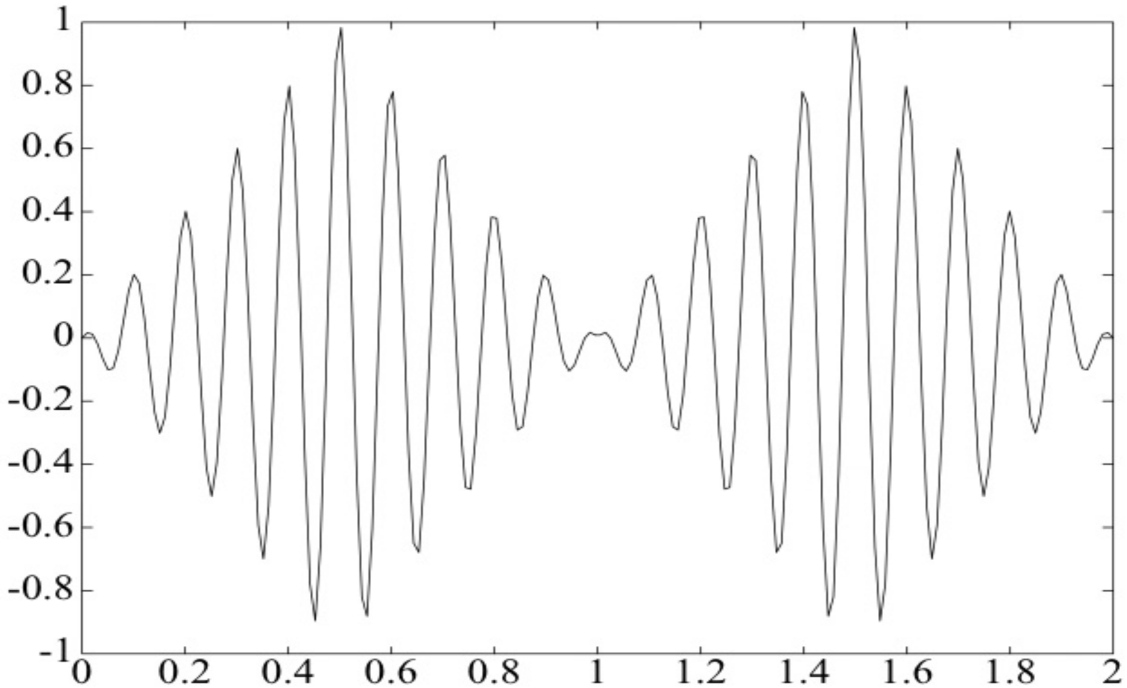
Taking the Fourier transform of both sides, we obtain

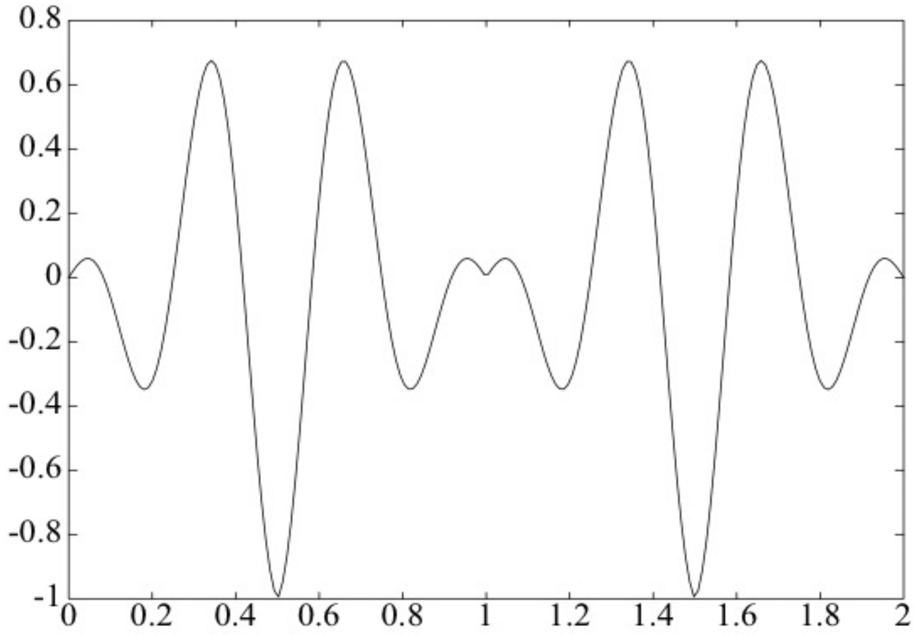
$$\begin{aligned} U(f) &= \frac{A}{2}[\Pi(f)+\Lambda(f)] \star (\delta(f-f_c)+\delta(f+f_c)) \\ &= \frac{A}{2}[\Pi(f-f_c)+\Lambda(f-f_c)+\Pi(f+f_c)+\Lambda(f+f_c)] \end{aligned}$$

$\Pi(f-f_c)\neq 0$ for $|f-f_c|<\frac{1}{2}$, whereas $\Lambda(f-f_c)\neq 0$ for $|f-f_c|<1$. Hence, the bandwidth of the bandpass filter is 2.

Problem 3.3
The following figure shows the modulated signals for $A=1$ and $f_0=10$. As it is observed both signals have the same envelope but there is a phase reversal at $t=1$ for the second signal $Am_2(t)\cos(2\pi f_0 t)$ (right plot). This discontinuity is shown clearly in the next figure where we plotted $Am_2(t)\cos(2\pi f_0 t)$ with $f_0=3$.


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2



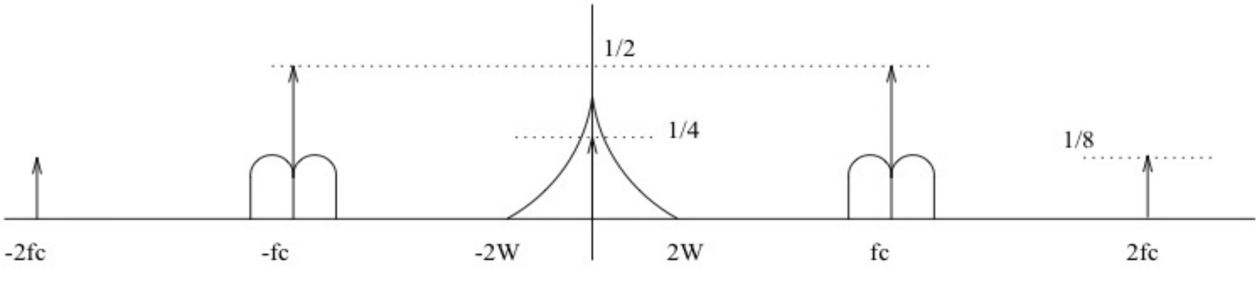
Problem 3.4

$$\begin{aligned}
y(t) &= x(t) + \frac{1}{2}x^2(t) \\
&= m(t) + \cos(2\pi f_c t) + \frac{1}{2}\left(m^2(t) + \cos^2(2\pi f_c t) + 2m(t)\cos(2\pi f_c t)\right) \\
&= m(t) + \cos(2\pi f_c t) + \frac{1}{2}m^2(t) + \frac{1}{4} + \frac{1}{4}\cos(2\pi 2 f_c t) + m(t)\cos(2\pi f_c t)
\end{aligned}$$

Taking the Fourier transform of the previous, we obtain

$$\begin{aligned}
Y(f) &= M(f) + \frac{1}{2}M(f) \star M(f) + \frac{1}{2}\left(M(f-f_c) + M(f+f_c)\right) \\
&\quad + \frac{1}{4}\delta(f) + \frac{1}{2}\left(\delta(f-f_c) + \delta(f+f_c)\right) + \frac{1}{8}\left(\delta(f-2f_c) + \delta(f+2f_c)\right)
\end{aligned}$$

The next figure depicts the spectrum $Y(f)$



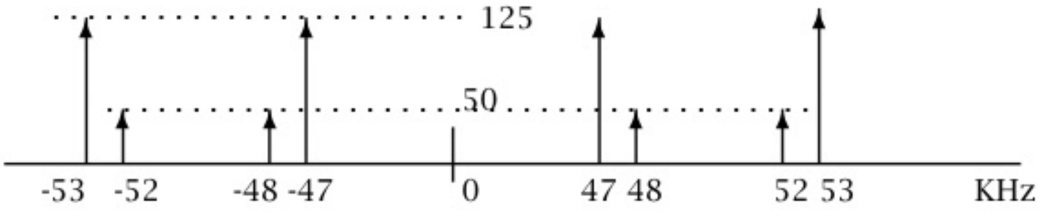
Problem 3.5

$$\begin{aligned} u(t) &= m(t) \cdot c(t) \\ &= 100(2\cos(2\pi 2000t) + 5\cos(2\pi 3000t))\cos(2\pi f_c t) \end{aligned}$$

Thus,

$$\begin{aligned} U(f) &= \frac{100}{2}\left[\delta(f-2000) + \delta(f+2000) + \frac{5}{2}(\delta(f-3000) + \delta(f+3000))\right] \\ &\quad \star [\delta(f-50000) + \delta(f+50000)] \\ &= 50\left[\delta(f-52000) + \delta(f-48000) + \frac{5}{2}\delta(f-53000) + \frac{5}{2}\delta(f-47000)\right. \\ &\quad \left. +\delta(f+52000) + \delta(f+48000) + \frac{5}{2}\delta(f+53000) + \frac{5}{2}\delta(f+47000)\right] \end{aligned}$$

A plot of the spectrum of the modulated signal is given in the next figure



Problem 3.6

The mixed signal $y(t)$ is given by

$$\begin{aligned} y(t) &= u(t) \cdot x_L(t) = Am(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \theta) \\ &= \frac{A}{2} m(t)\left[\cos(2\pi 2 f_c t + \theta) + \cos(\theta)\right] \end{aligned}$$

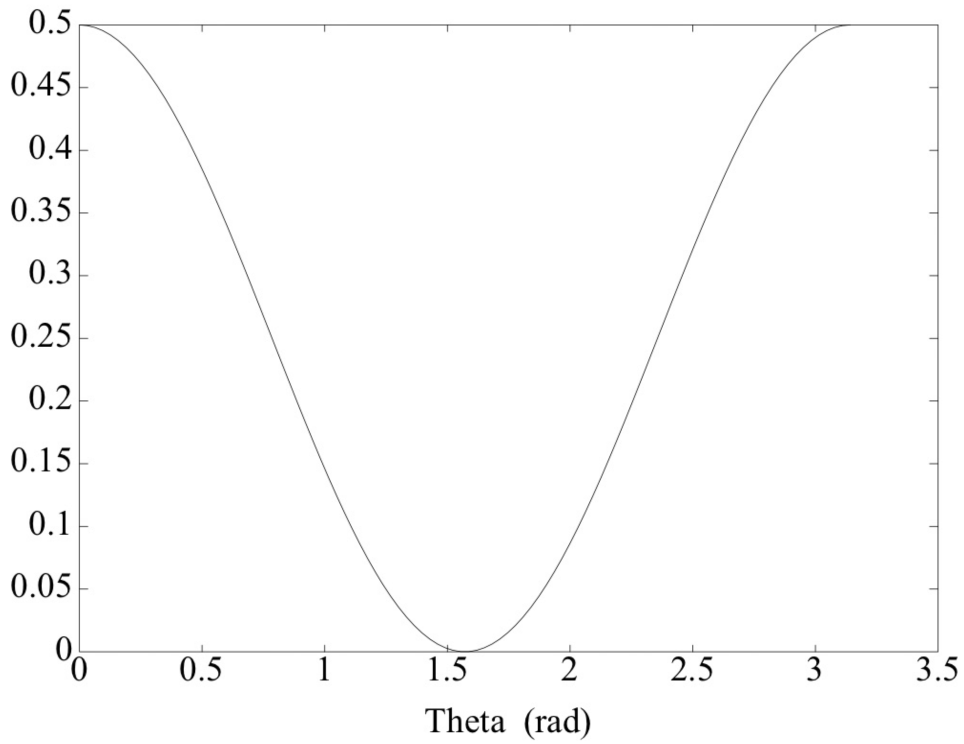
The lowpass filter will cut-off the frequencies above W, where W is the bandwidth of the message signal $m(t)$. Thus, the output of the lowpass filter is

$$z(t) = \frac{A}{2} m(t)\cos(\theta)$$

If the power of $m(t)$ is P_M, then the power of the output signal $z(t)$ is $P_{\text{out}} = P_M \frac{A^2}{4}\cos^2(\theta)$. The power of the modulated signal $u(t) = Am(t)\cos(2\pi f_c t)$ is $P_U = \frac{A^2}{2} P_M$. Hence,

$$\frac{P_{\text{out}}}{P_U} = \frac{1}{2}\cos^2(\theta)$$

A plot of $\frac{P_{\text{out}}}{P_U}$ for $0 \le \theta \le \pi$ is given in the next figure.

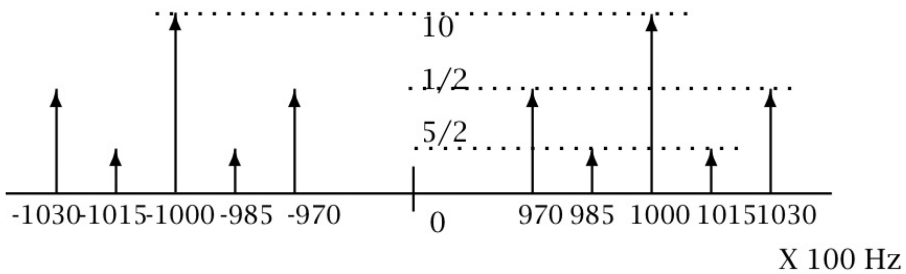


Problem 3.7

1) The spectrum of $u(t)$ is

$$\begin{aligned} U(f) &= \frac{20}{2}[\delta(f-f_c)+\delta(f+f_c)] \\ &\quad +\frac{2}{4}[\delta(f-f_c-1500)+\delta(f-f_c+1500) \\ &\quad +\delta(f+f_c-1500)+\delta(f+f_c+1500)] \\ &\quad +\frac{10}{4}[\delta(f-f_c-3000)+\delta(f-f_c+3000) \\ &\quad +\delta(f+f_c-3000)+\delta(f+f_c+3000)] \end{aligned}$$

The next figure depicts the spectrum of $u(t)$.



2) The square of the modulated signal is

$$\begin{aligned} u^2(t) &= 400\cos^2(2\pi f_c t)+\cos^2(2\pi(f_c-1500)t)+\cos^2(2\pi(f_c+1500)t) \\ &\quad +25\cos^2(2\pi(f_c-3000)t)+25\cos^2(2\pi(f_c+3000)t) \\ &\quad +\text{ terms that are multiples of cosines} \end{aligned}$$

If we integrate $u^2(t)$ from $-\frac{T}{2}$ to $\frac{T}{2}$, normalize the integral by $\frac{1}{T}$ and take the limit as $T \to \infty$, then all the terms involving cosines tend to zero, whereas the squares of the cosines give a value of $\frac{1}{2}$. Hence, the power content at the frequency $f_c = 10^5$ Hz is $P_{f_c} = \frac{400}{2} = 200$, the power content at the frequency P_{f_c+1500} is the same as the power content at the frequency P_{f_c-1500} and equal to $\frac{1}{2}$, whereas $P_{f_c+3000} = P_{f_c-3000} = \frac{25}{2}$.

3)

$$\begin{aligned} u(t) &= (20 + 2\cos(2\pi 1500t) + 10\cos(2\pi 3000t))\cos(2\pi f_c t) \\ &= 20(1 + \frac{1}{10}\cos(2\pi 1500t) + \frac{1}{2}\cos(2\pi 3000t))\cos(2\pi f_c t) \end{aligned}$$

This is the form of a conventional AM signal with message signal

$$\frac{\alpha}{|\min\{m(t)\}|}$$

$$\begin{aligned} m(t) &= \frac{1}{10}\cos(2\pi 1500t) + \frac{1}{2}\cos(2\pi 3000t) \\ &= \cos^2(2\pi 1500t) + \frac{1}{10}\cos(2\pi 1500t) - \frac{1}{2} \end{aligned}$$

The minimum of $g(z) = z^2 + \frac{1}{10}z - \frac{1}{2}$ is achieved for $z = -\frac{1}{20}$ and it is $\min(g(z)) = -\frac{201}{400}$. Since $z = -\frac{1}{20}$ is in the range of $\cos(2\pi 1500t)$, we conclude that the minimum value of $\alpha m(t)$ is $-\frac{201}{400}$.
$|\min\{m(t)\}|$
Hence, the modulation index is

$$\alpha = +\frac{201}{400}$$

4)

$$\begin{aligned} u(t) &= 20\cos(2\pi f_c t) + \cos(2\pi(f_c - 1500)t) + \cos(2\pi(f_c - 1500)t) \\ &= 5\cos(2\pi(f_c - 3000)t) + 5\cos(2\pi(f_c + 3000)t) \end{aligned}$$

The power in the sidebands is

$$P_{\text{sidebands}} = \frac{1}{2} + \frac{1}{2} + \frac{25}{2} + \frac{25}{2} = 26$$

The total power is $P_{\text{total}} = P_{\text{carrier}} + P_{\text{sidebands}} = 200 + 26 = 226$. The ratio of the sidebands power to the total power is

$$\frac{P_{\text{sidebands}}}{P_{\text{total}}} = \frac{26}{226}$$

A note on the modulation index calculation:
In textbook and lectures we adopted to normalize the message by the max of its absolute values: $m_n(t) = m(t)/\max|m(t)|$,

$$x_{AM}(t) = A_c\left(1 + \alpha\frac{m(t)}{\max|m(t)|}\right)\cos 2\pi f_c t$$

$\Rightarrow$ Identifying the terms from above gives:

$$\alpha = \frac{12}{20}$$

However, in some scenarios one can divide by the absolute value of the minimum of the message to obtain the normalized message $m_n(t) = \frac{m(t)}{|\min\{m(t)\}|}$ as is done in the solution to this exercise (the modulation index is always positive).
This is done sometimes since minimizing the message can be more tractable than maximizing its absolute value. Besides, it also guarantees that the envelope does not cross zero.
For most messages the two values are equal due to symmetry, that is: $\max|m(t)| = |\min\{m(t)\}|$.
In case they are not equal, the preferred way is to normalize by the max of the absolute value since the minimum can be zero.

In exam, use the max of the absolute value to obtain the normalized message

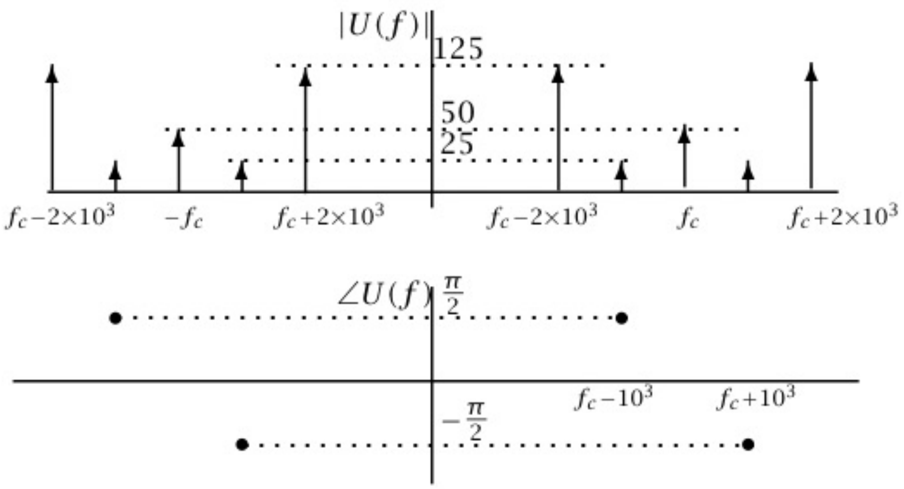
Problem 3.13

1) The modulated signal is

$$
\begin{aligned}
u(t) &= 100[1+m(t)]\cos(2\pi 8\times 10^5 t) \\
&= 100\cos(2\pi 8\times 10^5 t) + 100\sin(2\pi 10^3 t)\cos(2\pi 8\times 10^5 t) \\
&\quad +500\cos(2\pi 2\times 10^3 t)\cos(2\pi 8\times 10^5 t) \\
&= 100\cos(2\pi 8\times 10^5 t) + 50[\sin(2\pi(10^3+8\times 10^5)t) - \sin(2\pi(8\times 10^5-10^3)t)] \\
&\quad +250[\cos(2\pi(2\times 10^3+8\times 10^5)t) + \cos(2\pi(8\times 10^5-2\times 10^3)t)]
\end{aligned}
$$

Taking the Fourier transform of the previous expression, we obtain

$$
\begin{aligned}
U(f) &= 50[\delta(f-8\times 10^5)+\delta(f+8\times 10^5)] \\
&\quad +25\left[\frac{1}{j}\delta(f-8\times 10^5-10^3) - \frac{1}{j}\delta(f+8\times 10^5+10^3)\right] \\
&\quad -25\left[\frac{1}{j}\delta(f-8\times 10^5+10^3) - \frac{1}{j}\delta(f+8\times 10^5-10^3)\right] \\
&\quad +125\left[\delta(f-8\times 10^5-2\times 10^3)+\delta(f+8\times 10^5+2\times 10^3)\right] \\
&\quad +125\left[\delta(f-8\times 10^5-2\times 10^3)+\delta(f+8\times 10^5+2\times 10^3)\right] \\
&= 50[\delta(f-8\times 10^5)+\delta(f+8\times 10^5)] \\
&\quad +25\left[\delta(f-8\times 10^5-10^3)e^{-j\frac{\pi}{2}}+\delta(f+8\times 10^5+10^3)e^{j\frac{\pi}{2}}\right] \\
&\quad +25\left[\delta(f-8\times 10^5+10^3)e^{j\frac{\pi}{2}}+\delta(f+8\times 10^5-10^3)e^{-j\frac{\pi}{2}}\right] \\
&\quad +125\left[\delta(f-8\times 10^5-2\times 10^3)+\delta(f+8\times 10^5+2\times 10^3)\right] \\
&\quad +125\left[\delta(f-8\times 10^5-2\times 10^3)+\delta(f+8\times 10^5+2\times 10^3)\right]
\end{aligned}
$$



2) The average power in the carrier is

$$P_{\text{carrier}} = \frac{A_c^2}{2} = \frac{100^2}{2} = 5000$$

The power in the sidebands is

$$P_{\text{sidebands}} = \frac{50^2}{2} + \frac{50^2}{2} + \frac{250^2}{2} + \frac{250^2}{2} = 65000$$

3) The message signal can be written as

$$\frac{\alpha}{|\min m(t)|}$$

$$\begin{aligned} m(t) &= \sin(2\pi 10^3 t) + 5\cos(2\pi 2\times 10^3 t) \\ &= -10\sin^{2}(2\pi 10^3 t) + \sin(2\pi 10^3 t) + 5 \end{aligned}$$

As it is seen the minimum value ~~of $m(t)$~~ is -6 and is achieved for $\sin(2\pi 10^3 t) = -1$ or $t = \frac{3}{4\times 10^3} + \frac{1}{10^3}k$, with $k \in Z$. Hence, the modulation index is $\alpha = 6$.

4) The power delivered to the load is

$$P_{\text{load}} = \frac{|u(t)|^2}{50} = \frac{100^2(1+m(t))^2\cos^2(2\pi f_c t)}{50}$$

The maximum absolute value of $1 + m(t)$ is 6.025 and is achieved for $\sin(2\pi 10^3 t) = \frac{1}{20}$ or $t = \frac{\arcsin(\frac{1}{20})}{2\pi 10^3} + \frac{k}{10^3}$. Since $2\times 10^3 \ll f_c$ the peak power delivered to the load is approximately equal to

$$\max(P_{\text{load}}) = \frac{(100\times 6.025)^2}{50} = 72.6012$$

Problem 3.17

The input to the upper LPF is

$$\begin{aligned} u_u(t) &= \cos(2\pi f_m t)\cos(2\pi f_1 t) \\ &= \frac{1}{2}[\cos(2\pi(f_1 - f_m)t) + \cos(2\pi(f_1 + f_m)t)] \end{aligned}$$

whereas the input to the lower LPF is

$$\begin{aligned} u_l(t) &= \cos(2\pi f_m t)\sin(2\pi f_1 t) \\ &= \frac{1}{2}[\sin(2\pi(f_1 - f_m)t) + \sin(2\pi(f_1 + f_m)t)] \end{aligned}$$

If we select f_1 such that $|f_1 - f_m| < W$ and $f_1 + f_m > W$, then the two lowpass filters will cut-off the frequency components outside the interval $[-W, W]$, so that the output of the upper and lower LPF is

$$\begin{aligned} y_u(t) &= \cos(2\pi(f_1 - f_m)t) \\ y_l(t) &= \sin(2\pi(f_1 - f_m)t) \end{aligned}$$

The output of the Weaver's modulator is

$$u(t) = \cos(2\pi(f_1 - f_m)t)\cos(2\pi f_2 t) - \sin(2\pi(f_1 - f_m)t)\sin(2\pi f_2 t)$$

which has the form of a SSB signal since $\sin(2\pi(f_1 - f_m)t)$ is the Hilbert transform of $\cos(2\pi(f_1 - f_m)t)$. If we write $u(t)$ as

$$u(t) = \cos(2\pi(f_1 + f_2 - f_m)t)$$

then with $f_1 + f_2 - f_m = f_c + f_m$ we obtain an USSB signal centered at f_c, whereas with $f_1 + f_2 - f_m = f_c - f_m$ we obtain the LSSB signal. In both cases the choice of f_c and f_1 uniquely determine f_2.

Problem 3.18

The signal $x(t)$ is $m(t) + \cos(2\pi f_0 t)$. The spectrum of this signal is $X(f) = M(f) + \frac{1}{2}(\delta(f - f_0) + \delta(f + f_0))$ and its bandwidth equals to $W_x = f_0$. The signal $y_1(t)$ after the Square Law Device is

$$\begin{aligned}
y_1(t) &= x^2(t) = (m(t) + \cos(2\pi f_0 t))^2 \\
&= m^2(t) + \cos^2(2\pi f_0 t) + 2m(t)\cos(2\pi f_0 t) \\
&= m^2(t) + \frac{1}{2} + \frac{1}{2}\cos(2\pi 2 f_0 t) + 2m(t)\cos(2\pi f_0 t)
\end{aligned}$$

The spectrum of this signal is given by

$$Y_1(f) = M(f) \star M(f) + \frac{1}{2}\delta(f) + \frac{1}{4}(\delta(f - 2f_0) + \delta(f + 2f_0)) + M(f - f_0) + M(f + f_0)$$

and its bandwidth is $W_1 = 2f_0$. The bandpass filter will cut-off the low-frequency components $M(f) \star M(f) + \frac{1}{2}\delta(f)$ and the terms with the double frequency components $\frac{1}{4}(\delta(f - 2f_0) + \delta(f + 2f_0))$. Thus the spectrum $Y_2(f)$ is given by

$$Y_2(f) = M(f - f_0) + M(f + f_0)$$

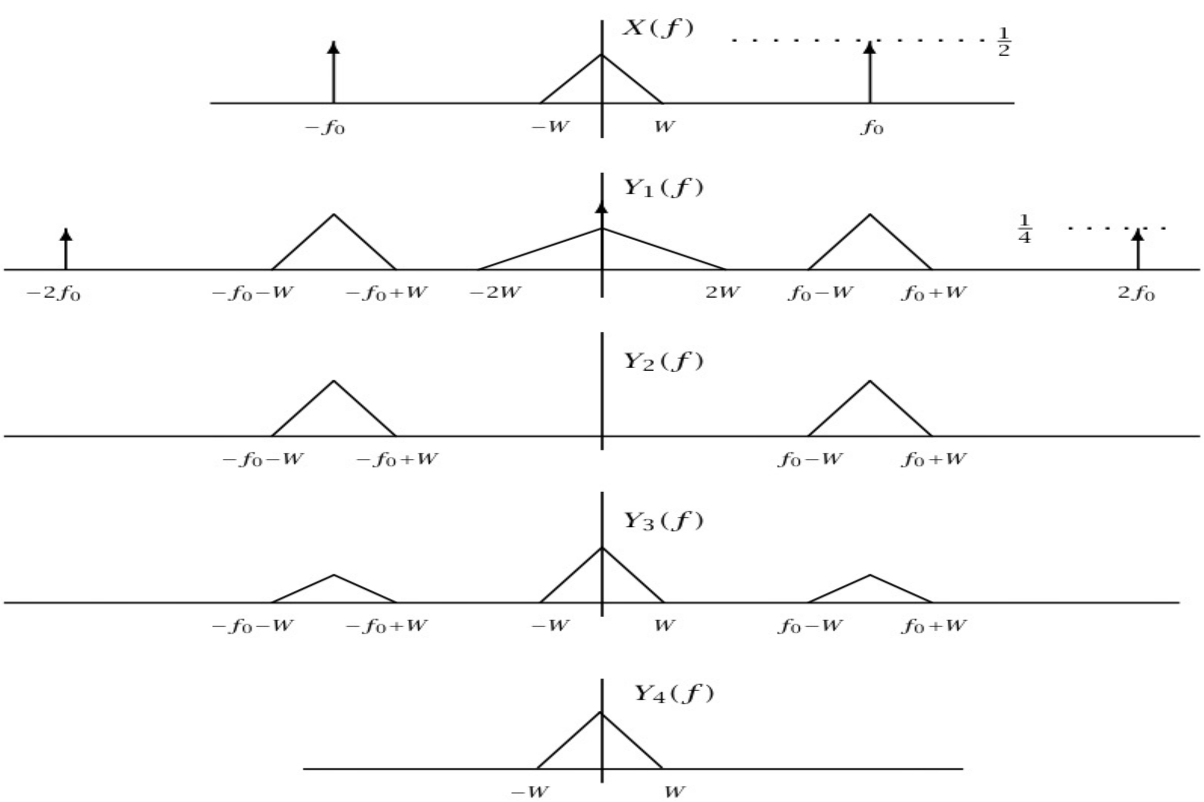
and the bandwidth of $y_2(t)$ is $W_2 = 2W$. The signal $y_3(t)$ is

$$y_3(t) = 2m(t)\cos^2(2\pi f_0 t) = m(t) + m(t)\cos(2\pi f_0 t)$$

with spectrum

$$Y_3(t) = M(f) + \frac{1}{2}(M(f - f_0) + M(f + f_0))$$

and bandwidth $W_3 = f_0 + W$. The lowpass filter will eliminate the spectral components $\frac{1}{2}(M(f - f_0) + M(f + f_0))$, so that $y_4(t) = m(t)$ with spectrum $Y_4 = M(f)$ and bandwidth $W_4 = W$. The next figure depicts the spectra of the signals $x(t)$, $y_1(t)$, $y_2(t)$, $y_3(t)$ and $y_4(t)$.



Problem 3.19

1)

$$\begin{aligned}
y(t) &= ax(t) + bx^2(t) \\
&= a(m(t) + \cos(2\pi f_0 t)) + b(m(t) + \cos(2\pi f_0 t))^2 \\
&= am(t) + bm^2(t) + a\cos(2\pi f_0 t) \\
&\quad + b\cos^2(2\pi f_0 t) + 2bm(t)\cos(2\pi f_0 t)
\end{aligned}$$

2) The filter should reject the low frequency components, the terms of double frequency and pass only the signal with spectrum centered at f_0. Thus the filter should be a BPF with center frequency f_0 and bandwidth W such that $f_0 - W_M > f_0 - \frac{W}{2} > 2W_M$ where W_M is the bandwidth of the message signal $m(t)$.

3) The AM output signal can be written as

$$u(t) = a(1 + \frac{2b}{a}m(t))\cos(2\pi f_0 t)$$

Since $A_m = \max[|m(t)|]$ we conclude that the modulation index is

$$\alpha = \frac{2bA_m}{a}$$

Problem 3.22

a) A DSB modulated signal is written as

$$\begin{aligned}
u(t) &= Am(t)\cos(2\pi f_0 t + \phi) \\
&= Am(t)\cos(\phi)\cos(2\pi f_0 t) - Am(t)\sin(\phi)\sin(2\pi f_0 t)
\end{aligned}$$

Hence,

$$\begin{aligned}
x_c(t) &= Am(t)\cos(\phi) \\
x_s(t) &= Am(t)\sin(\phi) \\
V(t) &= \sqrt{A^2 m^2(t)(\cos^2(\phi) + \sin^2(\phi))} = |Am(t)| \\
\Theta(t) &= \arctan\left(\frac{Am(t)\cos(\phi)}{Am(t)\sin(\phi)}\right) = \arctan(\tan(\phi)) = \phi
\end{aligned}$$

b) A SSB signal has the form

$$u_{\mathrm{SSB}}(t) = Am(t)\cos(2\pi f_0 t) \mp A\hat{m}(t)\sin(2\pi f_0 t)$$

Thus, for the USSB signal (minus sign)

$$\begin{aligned}
x_c(t) &= Am(t) \\
x_s(t) &= A\hat{m}(t) \\
V(t) &= \sqrt{A^2(m^2(t) + \hat{m}^2(t))} = A\sqrt{m^2(t) + \hat{m}^2(t)} \\
\Theta(t) &= \arctan\left(\frac{\hat{m}(t)}{m(t)}\right)
\end{aligned}$$

For the LSSB signal (plus sign)

$$\begin{aligned}
x_c(t) &= Am(t) \\
x_s(t) &= -A\hat{m}(t) \\
V(t) &= \sqrt{A^2(m^2(t)+\hat{m}^2(t))} = A\sqrt{m^2(t)+\hat{m}^2(t)} \\
\Theta(t) &= \arctan\left(-\frac{\hat{m}(t)}{m(t)}\right)
\end{aligned}$$

c) If conventional AM is employed, then

$$\begin{aligned}
u(t) &= A(1+m(t))\cos(2\pi f_0 t+\phi) \\
&= A(1+m(t))\cos(\phi)\cos(2\pi f_0 t) - A(1+m(t))\sin(\phi)\sin(2\pi f_0 t)
\end{aligned}$$

Hence,

$$\begin{aligned}
x_c(t) &= A(1+m(t))\cos(\phi) \\
x_s(t) &= A(1+m(t))\sin(\phi) \\
V(t) &= \sqrt{A^2(1+m(t))^2(\cos^2(\phi)+\sin^2(\phi))} = A|(1+m(t))| \\
\Theta(t) &= \arctan\left(\frac{A(1+m(t))\cos(\phi)}{A(1+m(t))\sin(\phi)}\right) = \arctan(\tan(\phi)) = \phi
\end{aligned}$$

Problem 3.23

1) If SSB is employed, the transmitted signal is

$$u(t) = Am(t)\cos(2\pi f_0 t) \mp A\hat{m}(t)\sin(2\pi f_0 t)$$

Provided that the spectrum of $m(t)$ does not contain any impulses at the origin $P_M = P_{\hat{M}} = \frac{1}{2}$ and

$$P_{\text{SSB}} = \frac{A^2 P_M}{2} + \frac{A^2 P_{\hat{M}}}{2} = A^2 P_M = 400\frac{1}{2} = 200$$

The bandwidth of the modulated signal $u(t)$ is the same with that of the message signal. Hence,

$$W_{\text{SSB}} = 10000 \text{ Hz}$$

2) In the case of DSB-SC modulation $u(t) = Am(t)\cos(2\pi f_0 t)$. The power content of the modulated signal is

$$P_{\text{DSB}} = \frac{A^2 P_M}{2} = 200\frac{1}{2} = 100$$

and the bandwidth $W_{\text{DSB}} = 2W = 20000$ Hz.

3) If conventional AM is employed with modulation index $\alpha = 0.6$, the transmitted signal is

$$u(t) = A[1+\alpha m(t)]\cos(2\pi f_0 t)$$

The power content is

$$P_{\text{AM}} = \frac{A^2}{2} + \frac{A^2\alpha^2 P_M}{2} = 200 + 200 \cdot 0.6^2 \cdot 0.5 = 236$$

The bandwidth of the signal is $W_{\text{AM}} = 2W = 20000$ Hz.