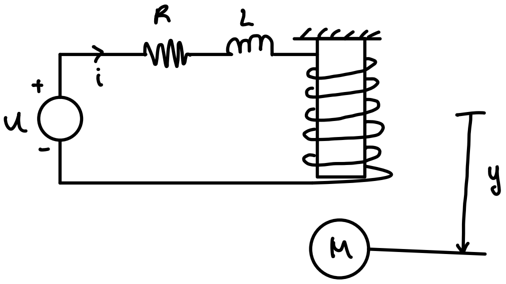
Tutorial 4
Modelling

Ex. 4.1 consider the following magnetic ball suspension system



u is the applied voltage (input)
y is the distance of the ball from the electromagnet
R is the resistance of the circuit
L is the inductance of the circuit
M is the mass of the ball
g is the acceleration due to gravity
K is a real constant - will explain later

a) derive the ODEs for the system
b) fix u = 5 and find all equilibrium points

a) KVL: $u = Ri + L\dot{i}$

$\underset{\uparrow \text{ input}}{u} = \underset{\uparrow \text{ resistor}}{Ri} + \underset{\uparrow \text{ inductor}}{L\dot{i}}$

Newton's 2nd law

$M\ddot{y} = \underset{\uparrow \text{ gravity}}{Mg} - F_m \leftarrow$ magnetic force

$$F_m = \frac{N^2 i^2 \mu_0 A}{2y^2} \quad K$$

N = # of turns
μ_0 = magnetic constant
A = area of core

$$= K\frac{i^2}{y^2}$$

$\Rightarrow$ ① $u = Ri + L\dot{i} \quad \Rightarrow \dot{i} = \frac{1}{L}(u - Ri) = f_1$

② $M\ddot{y} = Mg - K\frac{i^2}{y^2} \quad \Rightarrow \ddot{y} = g - \frac{K}{M}\frac{i^2}{y^2} = f_2$

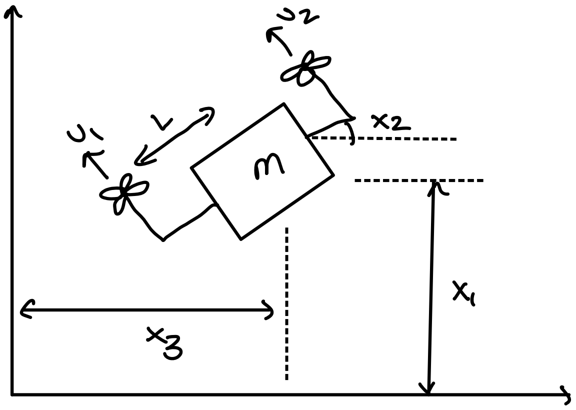
b) Let $\bar{u} = 5$ and suppose $u = \bar{u}$. Then to find the equilibrium points we want to find when system is at rest

① $i^* = \frac{1}{R}\bar{u} = \frac{5}{R}$

② $y^{*2} = \frac{Ki^{*2}}{Mg} = \frac{K5^2}{MgR^2} \Rightarrow y^* = \frac{5}{R}\sqrt{\frac{K}{Mg}}$

$$\boxed{i^* = \frac{5}{R} \qquad y^* = \frac{5}{R}\sqrt{\frac{K}{Mg}}}$$

Ex. 4.2 Consider the following simplified drone model



u_1 = left motor force
u_2 = right motor force
m = mass
x_1 = height
x_2 = tilt
x_3 = horizontal position
L = length of armature
I = moment of inertia

a) Find ODES
b) find an equilibrium point

a) Use Newton's 2nd law

① $m\ddot{x}_1 = (u_1+u_2)\cos x_2 - mg$ $\Rightarrow \ddot{x}_1 = \frac{1}{m}(u_1+u_2)\cos x_2 - g = f_1$

② $I\ddot{x}_2 = L(u_2-u_1)$ $\qquad \ddot{x}_2 = \frac{L}{I}(u_1+u_2) = f_2$

③ $m\ddot{x}_3 = -(u_1+u_2)\sin(x_2)$ $\qquad \ddot{x}_3 = -(u_1+u_2)\sin(x_2) = f_3$

b) To find equilibrium points, find system isn't moving.

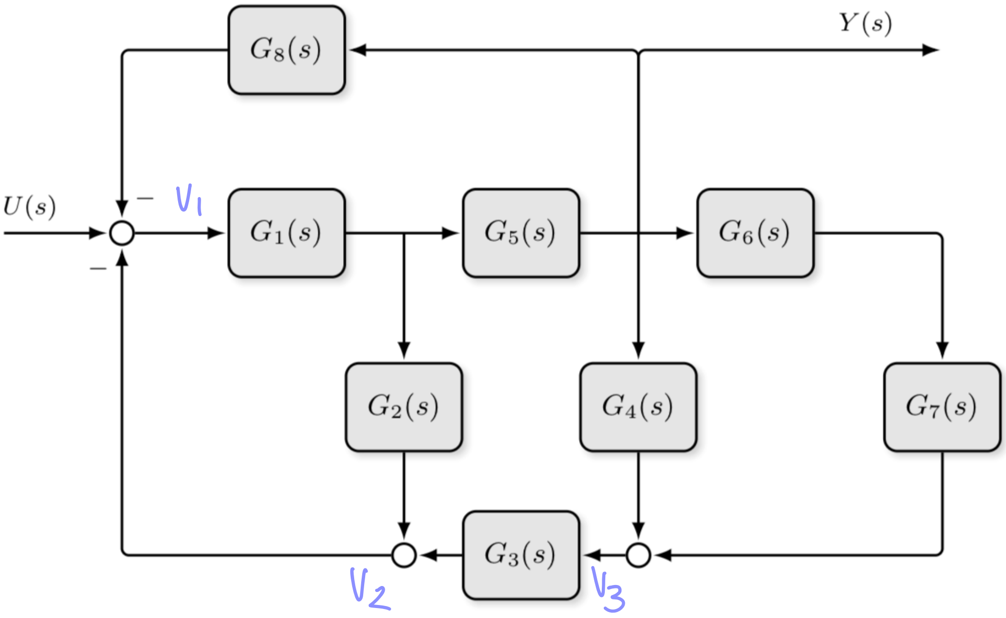
a solution to the above is

$u_1+u_2 = mg$ } $u_1^* = u_2^* = \frac{1}{2}mg$
$u_2^* = u_1^*$
$x_2^* = 0$
x_1^*, x_3^* can be anything pick $x_1 = x_3 = 0$ for now

Problem 8

Reduce each of the blocks below to a single block relating the output $Y(s)$ to the input $U(s)$.

(a)



$$V_3 = G_4 G_5 G_1 V_1 + G_7 G_6 G_5 G_1 V_1 = (G_4 + G_6 G_7) G_5 G_1 V_1$$

$$V_2 = G_2 G_1 V_1 + G_3 V_3 = G_2 G_1 V_1 + G_3 (G_4 + G_6 G_7) G_5 G_1 V_1$$

$$= (G_2 + G_3 G_4 G_5 + G_3 G_5 G_6 G_7) G_1 V_1$$

$$V_1 = u - G_8 G_5 G_1 V_1 - V_2 = u - G_8 G_5 G_1 V_1 - (G_2 + G_3 G_4 G_5 + G_3 G_5 G_6 G_7) G_1 V_1$$

$$\Rightarrow V_1 = \frac{1}{1 + G_1 (G_8 G_5 + G_2 + G_3 G_4 G_5 + G_3 G_5 G_6 G_7)} u$$

$$y = G_5 G_1 V_1$$

$$y = \frac{G_5 G_1}{1 + G_1 (G_8 G_5 + G_2 + G_3 G_4 G_5 + G_3 G_5 G_6 G_7)} u$$