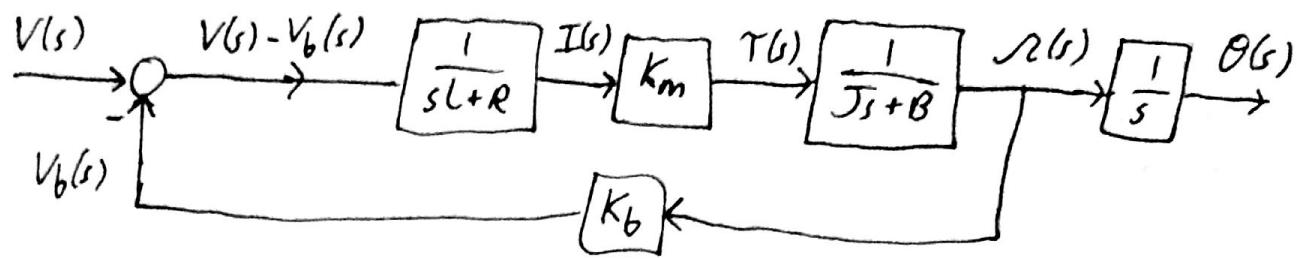


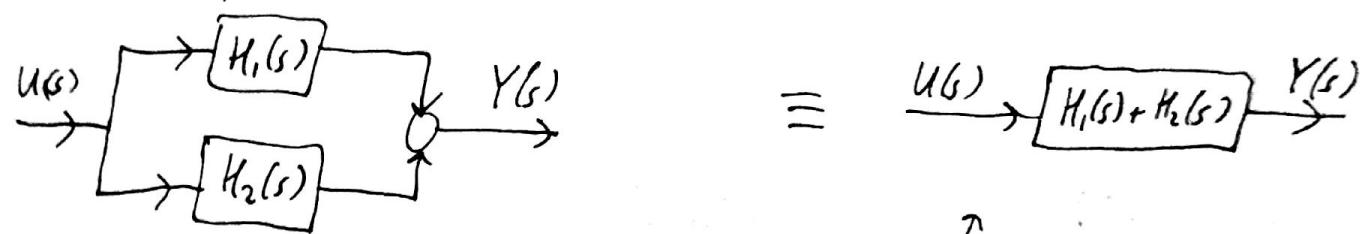
DC motor



Reduction of block diagrams:

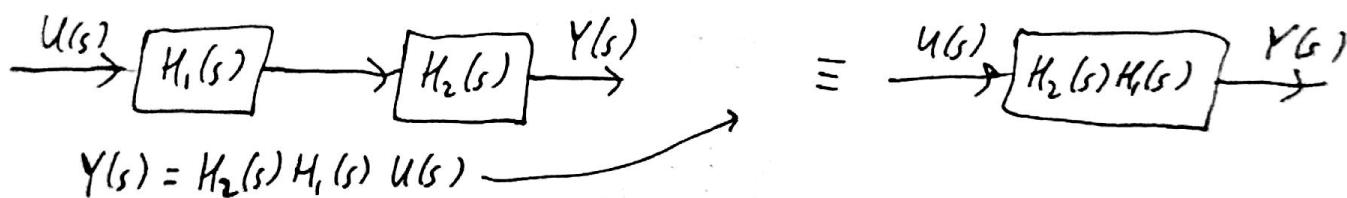
We can simplify block diagrams (or parts of them) in the following way:

- blocks in parallel:



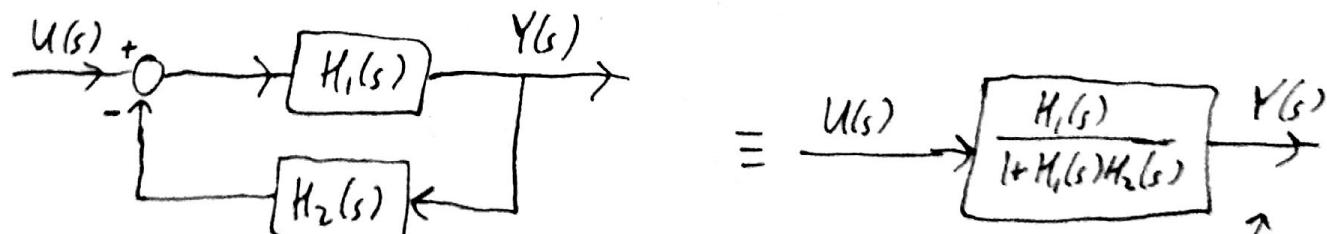
$$Y(s) = H_1(s)U(s) + H_2(s)U(s) = (H_1(s) + H_2(s))U(s)$$

- blocks in series:



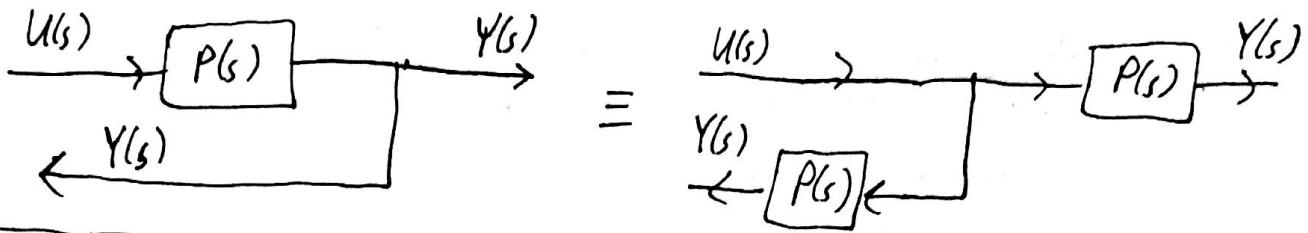
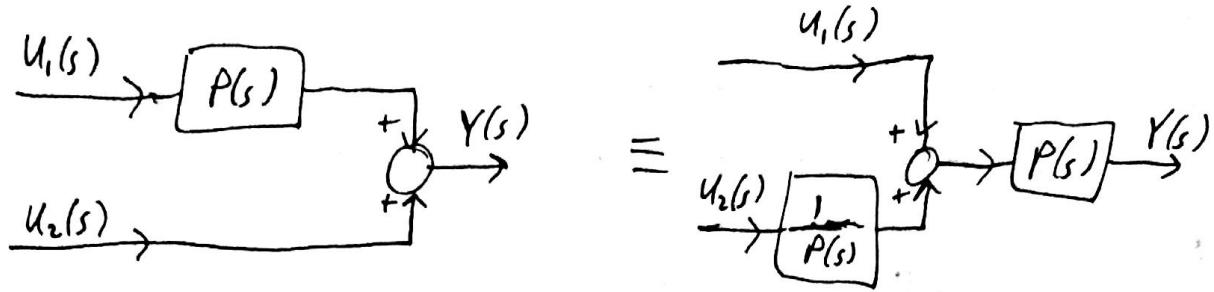
$$Y(s) = H_2(s)H_1(s)U(s)$$

- feedback loops:



$$\begin{aligned} Y(s) &= H_1(s)(U(s) - H_2(s)Y(s)) \Rightarrow (1 + H_1(s)H_2(s))Y(s) = H_1(s)U(s) \\ &\Rightarrow Y(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)}U(s) \end{aligned}$$

some other examples:



Reduction of motor block diagram:

Motor block diagram:

Input voltage $V(s)$ enters a summing junction. The output of this junction is fed into a block labeled $\frac{K_m}{(sL+R)(sJ+B)}$. The output of this block is $\sqrt{2}(s)$, which is then fed into a block labeled $\frac{1}{s}$. The final output is $\theta(s)$.

Feedback voltage $V_b(s)$ is fed into the negative input of the summing junction.

Reduction:

$$\frac{V(s)}{1 + \frac{K_m K_b}{(sL+R)(sJ+B)}} \times \frac{(sL+R)(sJ+B)}{(sL+R)(sJ+B)} = \frac{K_m}{(sL+R)(sJ+B) + K_m K_b}$$

Further reduction:

The reduced block diagram is:

$$\frac{V(s)}{\frac{K_m}{(sL+R)(sJ+B) + K_m K_b}} = T_{\theta V}(s)$$

The final reduced block diagram is:

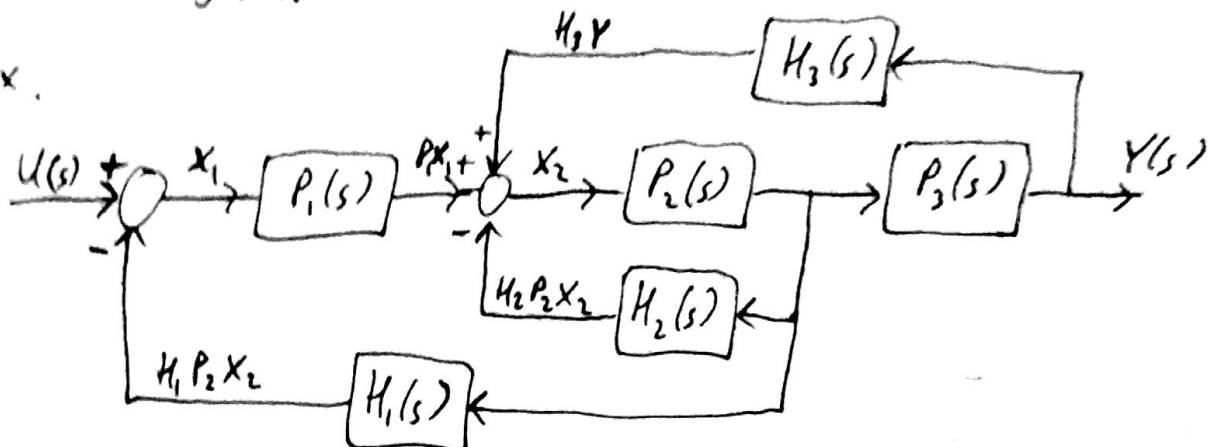
$$\frac{V(s)}{s[(sL+R)(sJ+B) + K_m K_b]} = T_{\theta V}(s)$$

$T_{\theta V}(s)$ is the transfer function from $U(s)$ to $Y(s)$
 $u(t)$ to $y(t)$

Alternative approach

Given a block diagram that is cumbersome to reduce step-by-step, an alternative is to convert it to a system of equations and solve using standard algebra:

Ex.



Procedure:

1. Add supplementary variables representing outputs of summers.
2. Write an expression for each summer input.
3. Write an equation for each summer relating its inputs to its outputs, and an equation for the system output.
4. Solve the system of equations and eliminate the supplementary variables.

$$1. X_1 = U - H_1 P_2 X_2$$

~~2. $X_2 = P_1 X_1 - H_2 P_2 X_2 + H_3 Y$~~

$$3. Y = P_3 P_2 X_2$$

$$(3) \text{ into } (2) \Rightarrow X_2 = P_1 X_1 - H_2 P_2 X_2 + H_3 P_3 P_2 X_2$$

$$\Rightarrow P_1 X_1 = (1 + P_2 H_2 - P_3 P_2 H_3) X_2 = (1 + P_2 (H_2 - P_3 H_3)) X_2$$

$$\Rightarrow X_1 = \frac{1 + P_2 (H_2 - P_3 H_3)}{P_1} X_2 \quad (4)$$

$$(4) \text{ into } (1) \Rightarrow \frac{1 + P_2 (H_2 - P_3 H_3)}{P_1} = U - H_1 P_2 X_2 \Rightarrow \frac{1 + P_2 (H_1 P_1 + H_2 - P_3 H_3)}{P_1} X_2 = U$$

$$\Rightarrow X_2 = \frac{P_1}{1 + P_2 (H_1 P_1 + H_2 - P_3 H_3)} U \quad (3) \Rightarrow Y = \frac{P_1 P_2 P_3}{1 + P_2 (H_1 P_1 + H_2 - P_3 H_3)} U \rightarrow = T_{uy}(s)$$