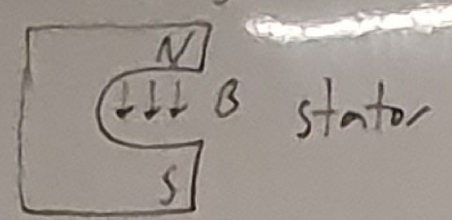
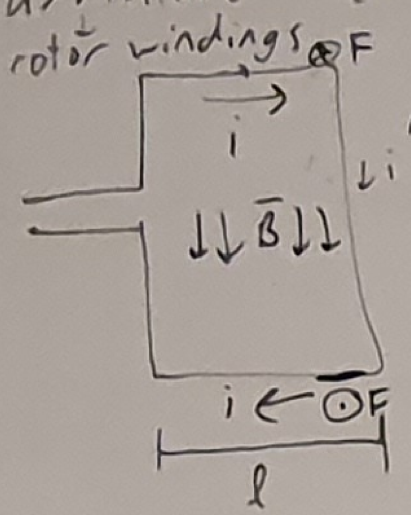
Electromechanical systems

"Armature-controlled" DC motor

- field magnets create constant magnetic field



- "armature" contains current loops
  rotor windings



Lorentz force law

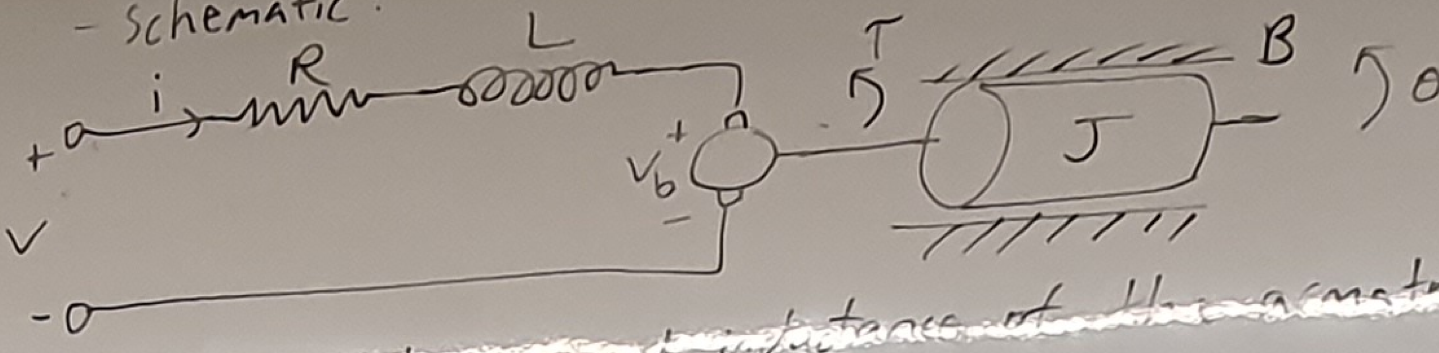
$\Rightarrow F = i\, l \times B$

$|F| = i l B$

- magnetic force F leads to a net torque on the armature (rotor windings) which is greatest when the loop is parallel to $\bar{B}$

- "commutator" reverses the current direction as the rotor turns, ensuring continuous torque (and, thus, rotation) in the same direction

- Schematic:



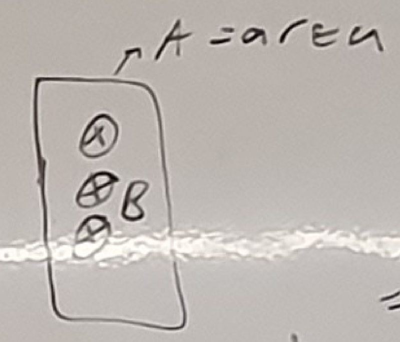
R, L = resistance and inductance of the armature

J, B - moment of inertia and damping coefficient (due to friction) of the armature

↓ NOT magnetic field

V = applied voltage

V_b = induced voltage (rotation causes changing magnetic flux through the loop → induces voltage so as to oppose this change)



magnetic flux $= \Phi_B = \bar{A} \times \bar{B} = AB\sin\theta \approx AB\theta$

$-\frac{d\Phi_B}{dt} = V_{induced}$ [small angle approx]

(Faraday's Law)

$\dot{\Phi}_B = AB\dot{\theta} = V_b$

Physical Laws:

KVL: $L\frac{di}{dt}(t) + Ri(t) = v(t) - v_b(t)$ ①

Lorentz force law: $T(t) = K_m i(t)$ ②

Newton's 2nd Law: $T(t) - B\dot{\theta}(t) = J\ddot{\theta}(t)$ ③ $\quad B\dot{\theta}(t) +$

Faraday's Law: $V_b(t) = K_b\dot{\theta}(t)$ ④

$w(t) = \dot{\theta}(t)$ ⑤

$\Rightarrow$ differential equations that represent the DC motor

→ LTI differential equations

$\delta(t) \xrightarrow{P} h(t)$

$y(t) = (h * u)(t)$

$U(s) \rightarrow \boxed{H(s)} \rightarrow Y(s)$

$Y(s) = H(s)U(s)$

$H(s)$ = transfer function

How do we find a transfer function from LTI differential equations?

physical laws

↓

diff eq.s

↓ linearize

LTI diff eqs

↓ Laplace

transfer functions

To find the transfer function, take Laplace transform with zero initial conditions and solve for $Y(s)$ as a function of $U(s)$

DC motor example:

$u(t) = v(t)$

$y(t) = \theta(t)$

Taking Laplace transform (with 0 initial conditions):

① $(sL+R)\,I(s) = V(s) - V_b(s) \longrightarrow I(s) = \frac{1}{sL+R}\left(V(s)-V_b(s)\right)$

② $T(s) = K_m\, I(s)$

③ $(Js^2 + Bs)\,\theta(s) = T(s) \longrightarrow \theta(s) = \frac{1}{Js^2+Bs}\, T(s)$

④ $V_b(s) = K_b\, s\,\theta(s) = K_b\,\Omega(s) \iff \Omega(s) = s\theta(s) = \frac{1}{Js+B}\, T(s)$

⑤ $\Omega(s) = s\theta(s) \rightarrow \theta(s) = \frac{1}{s}\,\Omega(s)$

We can represent these equations graphically as a block diagram: