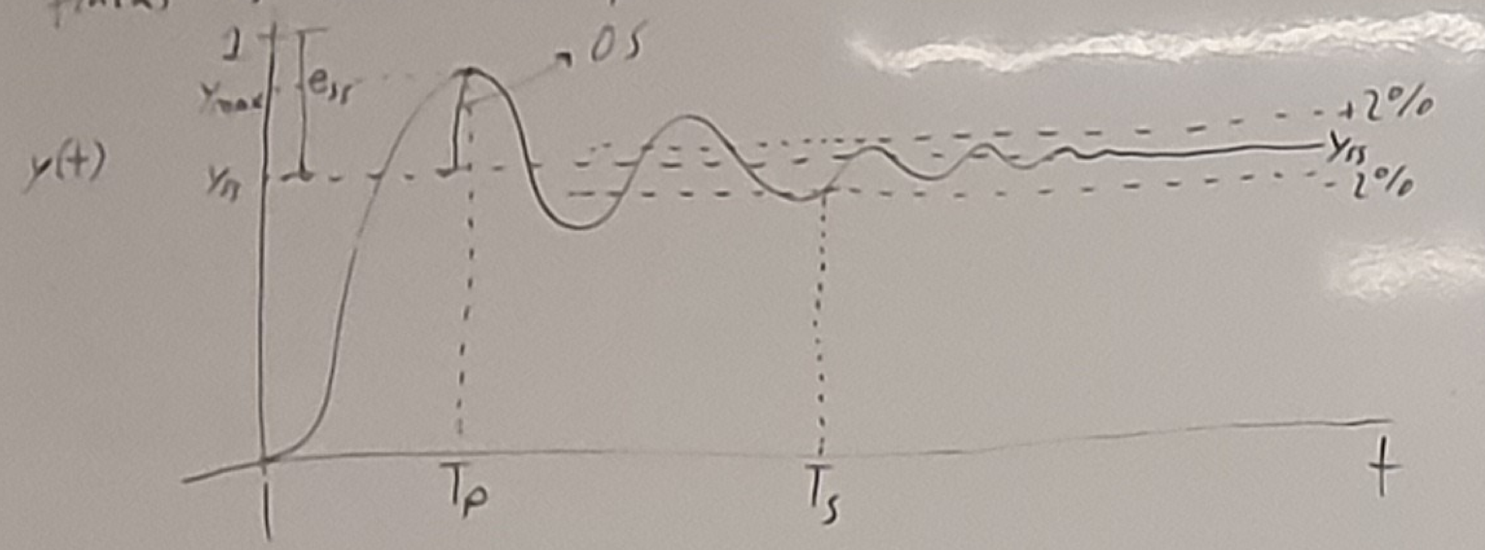
To quantify the effects of ζ and ω_n or, more broadly, $\mathrm{Re}(p)$ and $\mathrm{Im}(p)$ for any pole p, on the speed and oscillation of the transient response, [illegible] standard control engineering specifications (specs).

Specifically, let's look at the way the step response tracks its reference input (the unit step).



Our entire discussion only applies to stable systems!

Specs are typically how we mathematically represent desired performance.

$y_{ss} = \lim_{t\to\infty} y(t)$ is the steady-state value of $y(t)$

1. the steady-state error, denoted e_{ss}, $e_{ss} = \lim_{t\to\infty} e(t) = \lim_{t\to\infty} (r(t) - y(t)) = 1 - y_{ss}$

Ex. $r(t) = \mathbb{1}(t) \Rightarrow e_{ss} = 0$

$r(t) = 27.5\,\mathbb{1}(t) \Rightarrow e_{ss} = 0$

Ex. $r(t) = \mathbb{1}(t) \Rightarrow e_{ss} = \frac{1}{2}$

$r(t) = 4\,\mathbb{1}(t) \Rightarrow e_{ss} = 4 \cdot \frac{1}{2} = 2$

These specs are valuable for steps of all sizes, not just size 1

and for other reference signals too

- but the unit step makes it easy to quantify and compare different control systems (standardized signal for comparison)

2. the peak time, T_p, is defined as the time at which the step response reaches its maximum value

If $H(s)$ is a 2nd order system in standard form then

$$T_p = \frac{\pi}{\underbrace{\omega_n\sqrt{1-\zeta^2}}_{=\mathrm{Im}(p)}}$$

To derive this, set $\dot{y}(t) = 0$ and solve for the smallest t that satisfies this equation

- T_p gives a measure of the speed of the response

3. the settling time, T_s, is defined as the earliest time after which the step response stays within 2% of its steady-state value y_{ss} for all future time.

If $H(s)$ is a 2nd order system in standard form then

$$T_s \approx \frac{4}{\underbrace{\zeta\omega_n}_{\to \mathrm{Re}(p)}} \rightarrow \text{an approximation!}$$

- T_s gives another measure of the speed of the response

4. the overshoot, OS, is defined as the difference between the maximum and the steady-state values of the step response.

Often OS is expressed as a percentage of the steady-state value: $\%OS = \dfrac{\quad}{y_{ss}} \times$

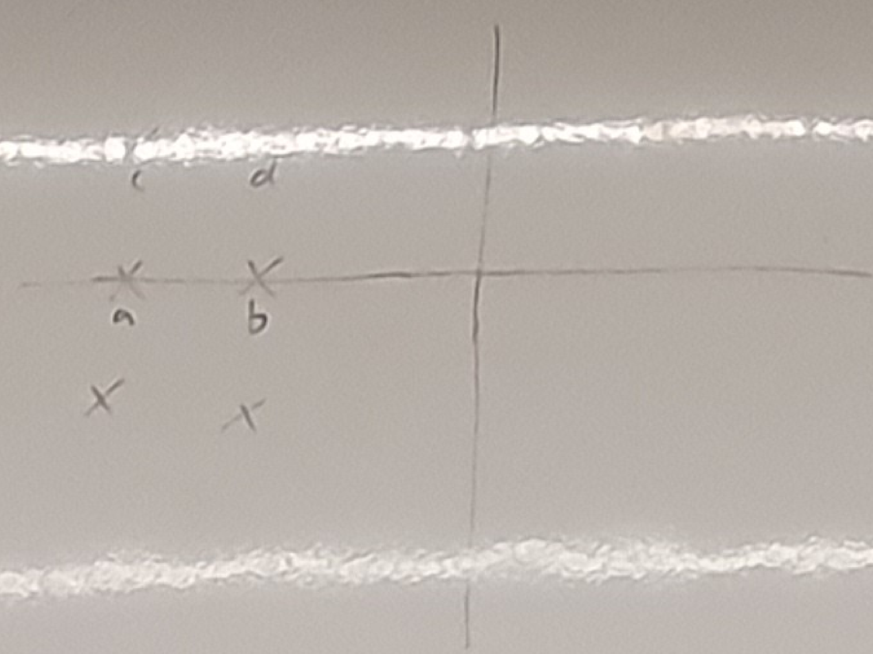
If $H(s)$ is a 2nd order system in standard form then

$$y_{max} = y(T_p) = y\left(\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}\right), \quad y_{ss} = 1$$

$$\Rightarrow \%OS = \frac{y(T_p)-1}{1} \times 100\% = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100\% \quad \rightarrow \frac{Re(p)}{Im(p)}$$

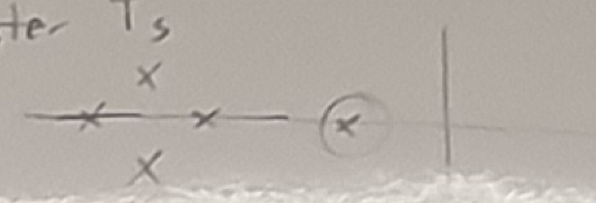
- %OS measures the extent to which the output exceeds its steady-state value
- often %OS is larger when the response is more oscillatory

How does the location of the poles affect the performance specs?



1. e_{ss}
   - pole locations have no effect

3. T_s
   - $Re(p)$ for each pole p determines the rate of decay
   - $\Rightarrow$ the further the poles are from the imaginary axis, typically the faster T_s
   - for higher order systems, typically the pole(s) closest to the imaginary axis (i.e., the slowest poles) approximately determine the settling time



   $\Rightarrow$ the slowest pole(s) are typically called the dominant poles

2, 4. T_p, %OS
   - $Im(p)$ determines the frequency of oscillations
   - $\Rightarrow$ the further the poles are from the real axis, typically the larger the %OS and smaller the T_p

Ex. (robot manipulator) $\rightarrow$ specs for control design

specs:
1. $e_{ss} = 0$
2. $T_s \le 1s$
3. $T_p \le 0.5s$
4. $\%OS \le 5\%$