University of Waterloo

ECE 380: Analog Control Systems

Problem Set 2

January 30, 2026

Problem 1

Figure 1 is a model of a seesaw. The beam has length l with evenly distributed mass M_b. A mass M_l sits on one end. A vertically downward force $f(t)$ is applied at the other end. The moment of inertia of the beam rotating about its center of mass is $\frac{1}{12}M_b l^2$.

Find a differential equation that models this seesaw.

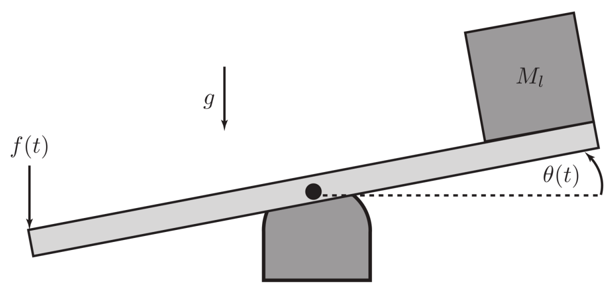

Figure 1: Seesaw for Problem 1

Problem 2

Figure 2 is a model of a mechanical system. Let r and θ denote the radius and angle of the pulley, respectively, where θ is measured in the clockwise (negative radian) direction. The torsion spring constant is K, which exerts a torque of magnitude $K\theta$ on the pully, and the rotational damping constant is B, which exerts a torque of magnitude $B\dot{\theta}$ on the pulley. The moment of inertia of the pulley is J, the spring constant is k, and the mass M experiences an applied force f downwards.

Find differential equations that model this mechanical system.

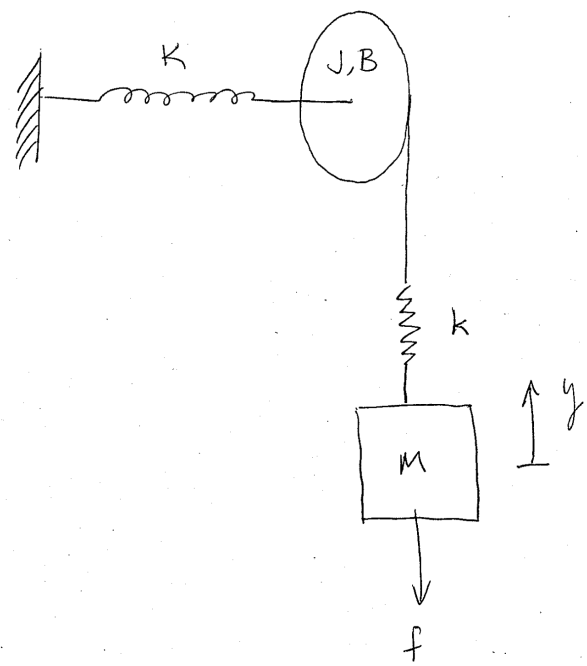

Figure 2: Mechanical System for Problem 2

Problem 3

Figure 3 is a model of an electromagnetic relay. The electromagnet exerts a magnetic force on the conducting mass m of magnitude $K\frac{i_2^2}{x^2}$, where $K > 0$ is a constant. When the current i_2 is positive, this force pulls the mass towards the electromagnet. The spring is at its natural length when the displacement $x = 0.1m$. The circuit has an applied voltage v, resistances R_1 and R_2, inductance L, and capacitance C. The spring and damping coefficients for the mass are k and B, respectively.

Find differential equations that model this relay.

Note: when the mass touches the contact, it closes another circuit that is regulated by the relay. However, you do not need to include the contact in your model.

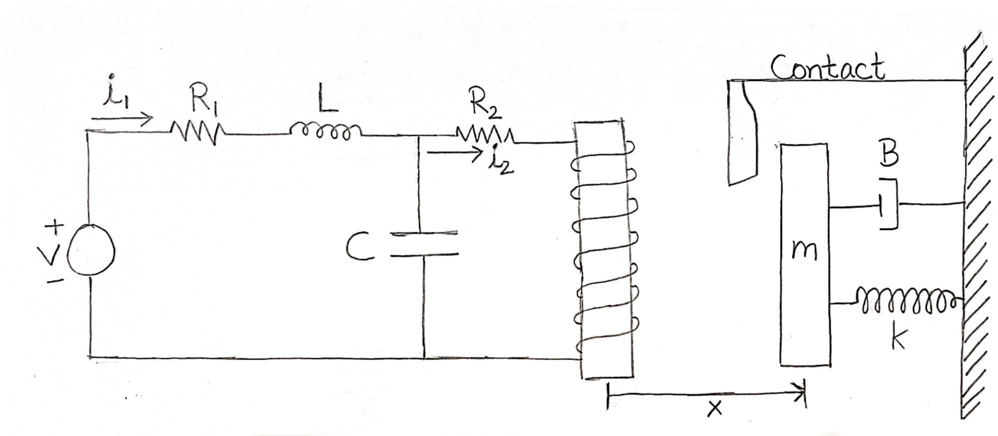

Figure 3: Electromagnetic Relay for Problem 3

Problem 4

Consider the following differential equations, which model the mechanical system of Problem 7 on Assignment 1:

$$
\begin{aligned}
M_1\ddot{y}_1 &= B(\dot{y}_2 - \dot{y}_1) - K_1 y_1 \\
M_2\ddot{y}_2 &= f - K_2 y_2 - B(\dot{y}_2 - \dot{y}_1).
\end{aligned}
$$

Find the transfer function from f to y_1.

Problem 5

Consider the following differential equations, which model the car suspension system of Problem 8 on Assignment 1:

$$
\begin{aligned}
M\ddot{y}_2 &= -k_s(y_2 - y_1) - Mg - b_s(\dot{y}_2 - \dot{y}_1) \\
m\ddot{y}_1 &= k_s(y_2 - y_1) + b_s(\dot{y}_2 - \dot{y}_1) - k_t(y_1 - y_0) - mg - b_t(\dot{y}_1 - \dot{y}_0).
\end{aligned}
$$

Find an equivalent model for this system that is LTI.
Then, find the transfer function from y_0 to Δy_2.

Hint: define new variables $\Delta y_1 = y_1 - \hat{y}_1$ and $\Delta y_2 = y_2 - \hat{y}_2$, where $\hat{y}_1$ and $\hat{y}_2$ are constants.

Problem 6

Reduce the block diagram in Figure 4 to a single block relating the output $Y(s)$ to the input $U(s)$. Do NOT use a system of algebraic equations to perform this reduction.

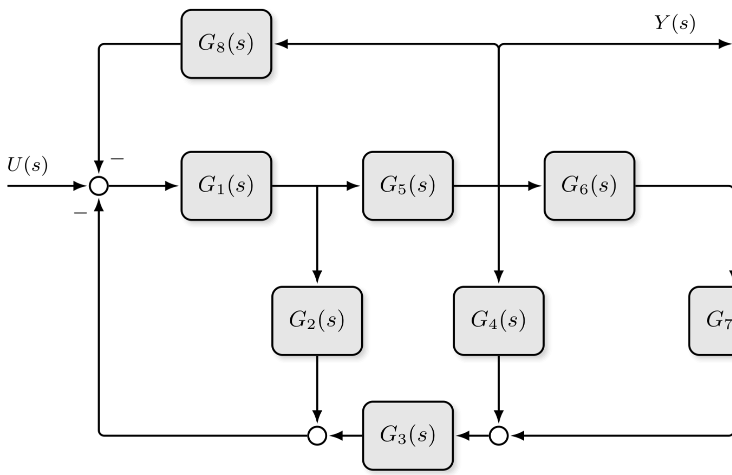

Figure 4: Block Diagram for Problem 6

Problem 7

Reduce the block diagram in Figure 5 to a single block relating the output $Y(s)$ to the input $U(s)$.

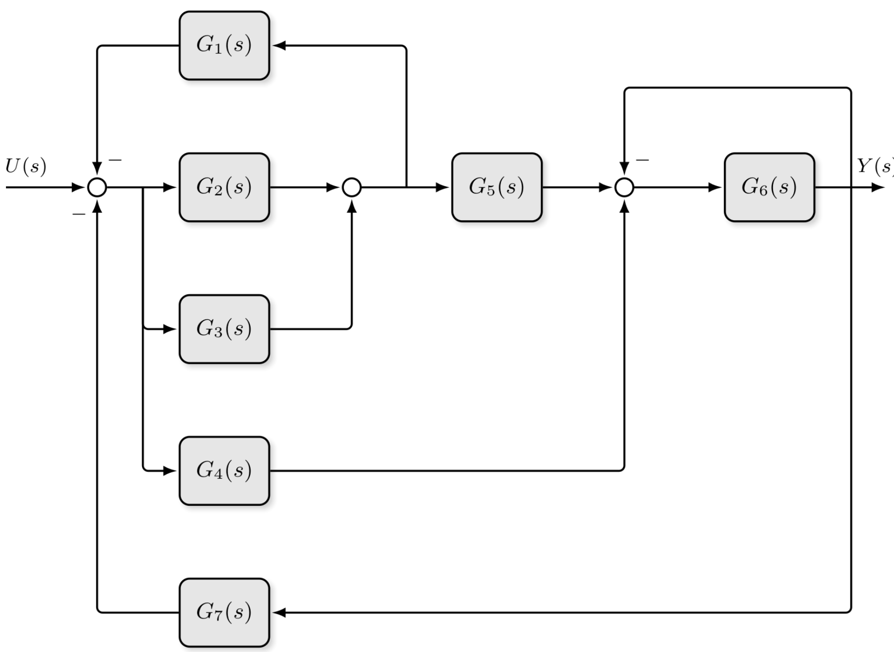

Figure 5: Block Diagram for Problem 7