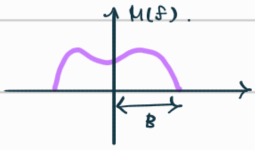
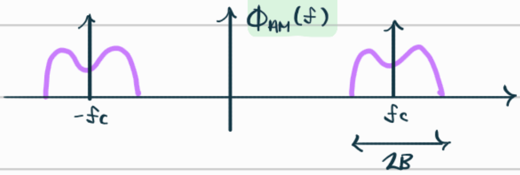
Jan 22 2026



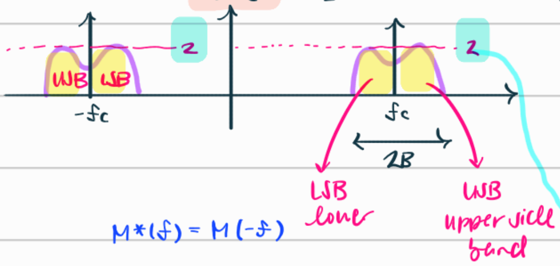
m(t) baseband usage.

$\varphi_{AM}(t) = A_c(1 + \alpha m_n(t))\cos 2\pi f_c t$ carrier freq.

$\varphi_{DSB\text{-}SC}(t) = A_c m(t) \cos 2\pi f_c t$



$\Phi_{DSB\text{-}LC}(f) = \frac{A_c}{2}(M(f-f_c) + M(f+f_c))$



$M^*(f) = M(-f)$

Pays a price to be power efficient

But still bandwidth inefficient → Hermitian symmetry

we only need to send one side to recover full signal

(either upper/lower is used depending on freq. required?)

Draw spectrum of upper side band. (SSB+) (single side band +)

$\Phi_{SSB+}(f) = \Phi_{USB}(f)$

→ Now the question is

how do we recover time domain signal?

Find SSB (single side band):

$\Phi_{SSB+}(t) = ?$

Start w/ DSB-LC → $\varphi_{DSB\text{-}LC}(t)$

$\Phi_{SSB+}(f) = \frac{\cancel{A_c}}{\cancel{2}} u(f-f_c) M(f-f_c) + \frac{\cancel{A_c}}{\cancel{2}} u(-f-f_c) M(f+f_c) \times$ 2 to cancel out $\frac{1}{2}$ Assume $A_c = 1$

$\mathcal{F}^{-1}\{\Phi_{SSB+}(f)\} = ?$ to time domain.

Let $X(f) = u(f) M(f) \rightarrow x(t) = \mathcal{F}^{-1}\{u(f)\} * m(t)$

$Y(f) = u(-f) M(f) \rightarrow y(t) = \mathcal{F}^{-1}\{u(-f)\} * m(t)$.

Hilbert transform.

$m(t) \rightarrow \boxed{h(t) = \frac{1}{\pi t}} \rightarrow m(t) * \frac{1}{\pi t} = \hat{m}(t)$

$\Phi_{SSB+}(t) = e^{j2\pi f_c t} x(t) + e^{-j2\pi f_c t} y(t)$

$\mathcal{F}\{u(t)\} = \frac{\delta(f)}{2} + \frac{1}{j2\pi f}$

$\mathcal{F}\{\frac{\delta(t)}{2} + \frac{j}{2\pi t}\} = u(f)$.

$x(t) = \frac{1}{2}\left(\delta(t) + \frac{j}{\pi t}\right) * m(t) = \left(m(t) + \frac{j}{\pi t} * m(t)\right)\frac{1}{2}$
$= (m(t) + j\hat{m}(t))\frac{1}{2}$

$y(t) = \frac{1}{2}\left(\delta(t) - \frac{j}{\pi t}\right) * m(t) = \left(m(t) - \frac{j}{\pi t} * m(t)\right)\frac{1}{2}$
$= (m(t) - j\hat{m}(t))\frac{1}{2}$

$\Phi_{SSB+}(t) = \left(\frac{e^{2\pi f_c t} + e^{-j2\pi f_c t}}{2}\right) m(t) + \left(\frac{je^{j2\pi f_c t} - je^{2\pi f_c t}}{2}\right)\hat{m}(t)$

$\Phi_{SSB+}(t) = m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t$ (message on cos - hilbert on sine)

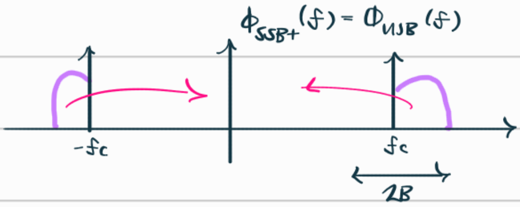
$\Phi_{SSB-}(t) = m(t)\sin 2\pi f_c t + \hat{m}\cos 2\pi f_c t$.

For a given modulation f_c, we have 2 channels at our disposal

$\cos 2\pi f_c t$ In-phase channel
$\sin 2\pi f_c t$ Quadrature channel

} orthogonal
no cross-talk between channels
$\langle \cos 2\pi f_c t, \sin 2\pi f_c t \rangle = \frac{1}{T}\int \cos 2\pi f_c t \sin 2\pi f_c t \, dt = 0.$

Given USB → Recover $m(t)$



shift back to recover frequencies.

Multiply by cos, cancel sin channel
(correlate w/ cos)

Recover $\hat{m}(t)$ → correlate w/ sin, LPF to cancel cos

filter baseband → shift
shift → filter sideband.

Demodulation is similar to DSB-SC demod.

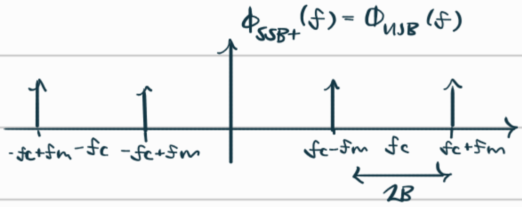
$\varphi_{SSB+}(t) \rightarrow \otimes \rightarrow \boxed{\text{LPF } B}$
↑
$\cos 2\pi f_c t$.

ex. upper side band modulation.

$m(t) \rightarrow \cos 2\pi f_c t$.

$(f_m \ll f_c)$.

$m(t) \rightarrow \boxed{\text{DSB-SC}} \rightarrow 2\cos 2\pi f_m t \cos 2\pi f_c t$

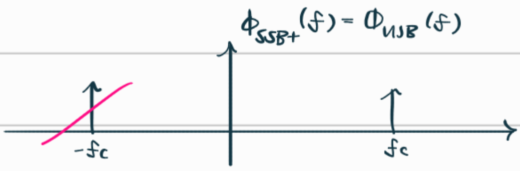


$$= \underbrace{\cos 2\pi(f_c+f_m)t}_{USB} + \underbrace{\cos 2\pi(\cancel{f_c-f_m})t}_{LSB} \rightarrow \text{filter USB}$$

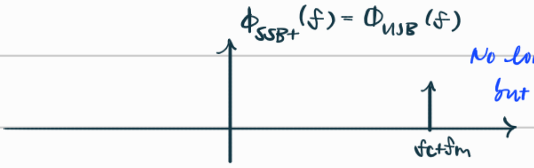
After BPF → $\varphi_{SSB+}(t) = \cos 2\pi(f_c+f_m)t$. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

$$= \underbrace{\cos 2\pi f_m t}_{m(t)} \cos 2\pi f_c t - \underbrace{\sin 2\pi f_m t}_{\hat{m}(t)} \sin 2\pi f_c t.$$

send message on inphase-channel
and quadrature.



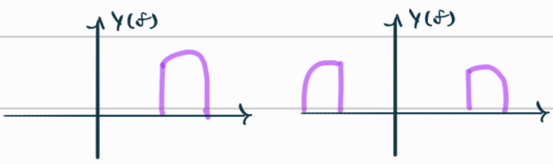
→



No longer hermitian
but complex signal

$m(t) \rightarrow \boxed{\text{baseband filter}} \rightarrow e^{j2\pi f_m t} \xrightarrow{\times e^{j2\pi f_c t}} \mathrm{Re}\{\underbrace{e^{j2\pi f_m t} \cdot e^{j2\pi f_c t}}\} = \cos 2\pi(f_c+f_m)t$

shift by complex exp.
reflect about y.
take real,



$$s(t) = \mathrm{Re}\{y(t)\} = \frac{y(t)+y^*(t)}{2}$$

For a given m(t) → define z(t) (analytic signal associated w/ m(t))

$$Z(f) = 2M(f)u(f)$$ Filter in base band ⇒ $z(t) = m(t) + j\hat{m}(t)$.

$$\varphi_{SSB+}(t) = Re\{z(t)\underline{e^{j2\pi f_c t}}\}$$ → take real pt.

shift by complex exp.

$$= m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t.$$

2 ways to get → search