Limits on which specs can actually be achieved
- plant dynamics
- control effort by actuators

Calculating e_{ss}

Final value theorem (FVT)

If a signal $Y(s)$ has all of its poles in the OLHP, except possibly a single pole at $s=0$, then

$$y_{ss} = \lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s).$$

Proof. $L(\dot{y}(t)) = \int_0^\infty \dot{y}(t)e^{-st}dt = sY(s) - y(0)$

$$\Rightarrow \lim_{s\to 0}\int_0^\infty \dot{y}(t)e^{-st}dt = \lim_{s\to 0} sY(s) - \cancel{y(0)}$$

$$\int_0^\infty \dot{y}(t)\lim_{s\to 0} e^{-st}dt = \int_0^\infty \dot{y}(t)dt = [y(t)]_0^\infty = \lim_{t\to\infty} y(t) - y(0) = y_{ss} - \cancel{y(0)}$$

only well-defined if all poles of $Y(s)$ are in the OLHP, except possibly one at $s=0$

$$\Rightarrow y_{ss} = \lim_{s\to 0} sY(s).$$

$e(t) = r(t) - y(t)$

Corollary. If a system $H(s)$ is stable and its input is $u(t) = 1(t)$, then its steady-state output is: $y_{ss} = H(0)$. $H(0)$ is sometimes called the DC gain of the system.

Proof. $Y(s) = H(s)U(s) = H(s)\frac{1}{s}$ has all of its pole in the OLHP (since $H(s)$ is stable) except for exactly one pole at $s=0$

$\Rightarrow$ by FVT, $y_{ss} = \lim_{s\to 0} sY(s) = \lim_{s\to 0} \cancel{s}H(s)\frac{1}{\cancel{s}} = \lim_{s\to 0} H(s) = H(0)$

Ex. $H(s) = \frac{k}{sT+1} \Rightarrow y_{ss} = \frac{k}{0+1} = k$

Ex. $H(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \Rightarrow y_{ss} = 1$

Ex. $H(s) = 18\frac{(s+1)(s+2)}{(s+3)(s+4)} \Rightarrow y_{ss} = 18\frac{(1)(2)}{(3)(4)} = 3$

Ex. $H(s) = 5\frac{s+1}{s-2} \Rightarrow y_{ss}$ is undefined (or ∞)

unstable!

For a stable system $H(s)$, $e_{ss} = 1 - y_{ss} = 1 - H(0)$.

dynamic responses of LTI systems

no calculators
no cheat sheets
no notes