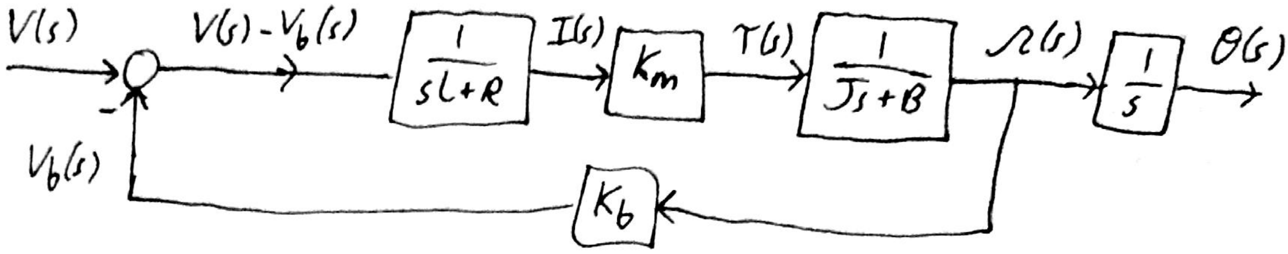
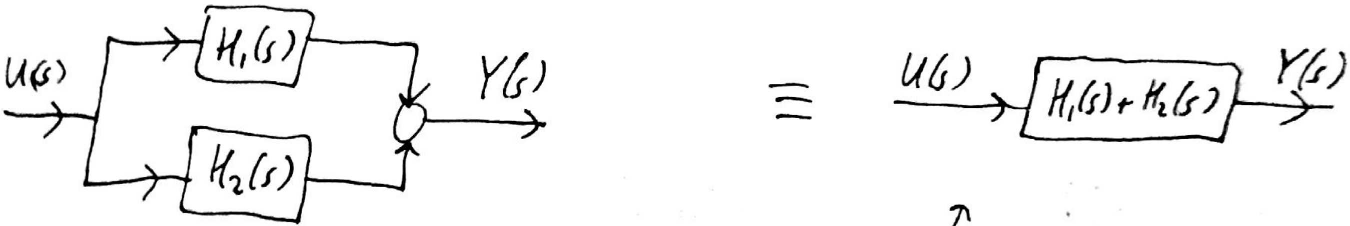
DC motor



Reduction of block diagrams:

We can simplify block diagrams (or parts of them) in the following ways:

- blocks in parallel:

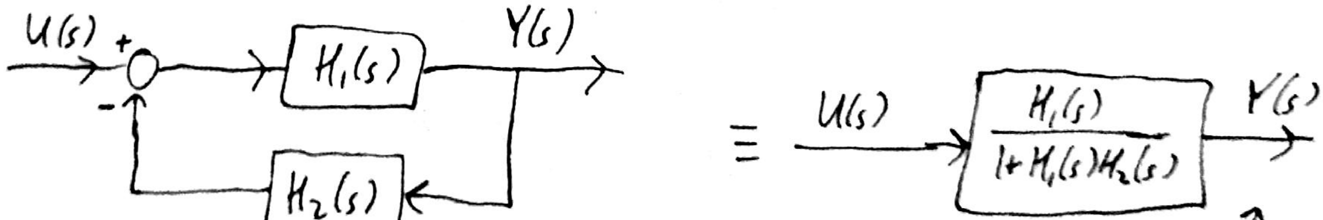


$$Y(s) = H_1(s)U(s) + H_2(s)U(s) = (H_1(s) + H_2(s))\,U(s)$$

- blocks in series:

$$Y(s) = H_2(s)\,H_1(s)\,U(s)$$

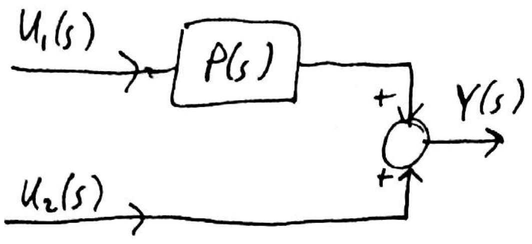
- feedback loops:



$$Y(s) = H_1(s)\left(U(s) - H_2(s)Y(s)\right) \Rightarrow \left(1 + H_1(s)H_2(s)\right)Y(s) = H_1(s)U(s)$$

$$\Rightarrow Y(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)}\,U(s)$$

some other examples:



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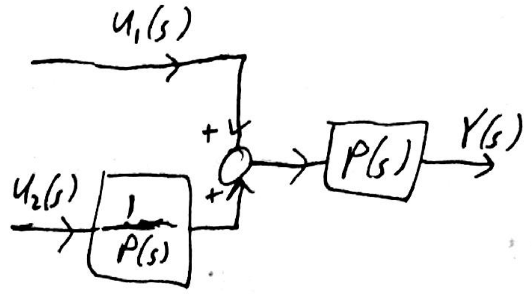

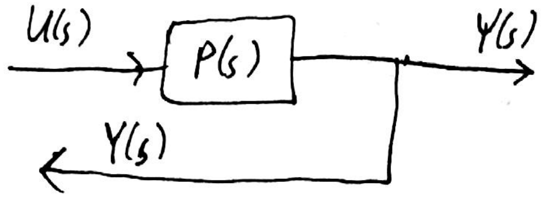

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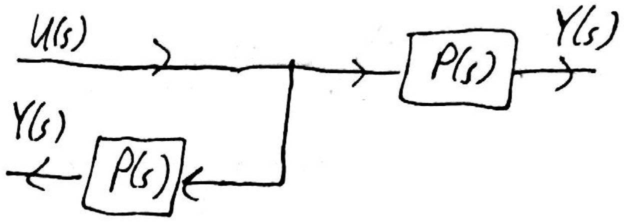
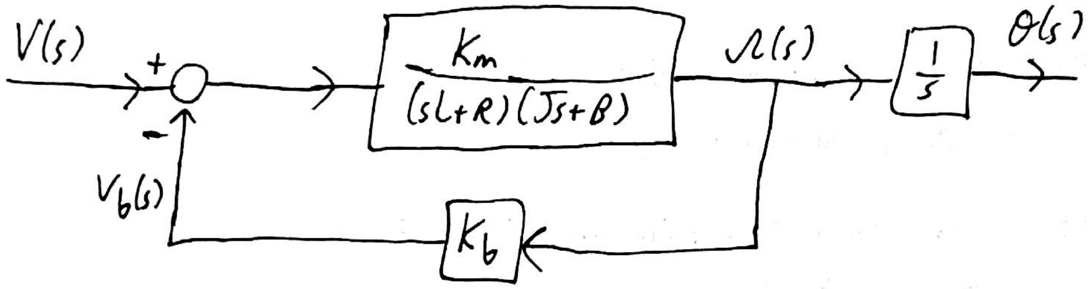


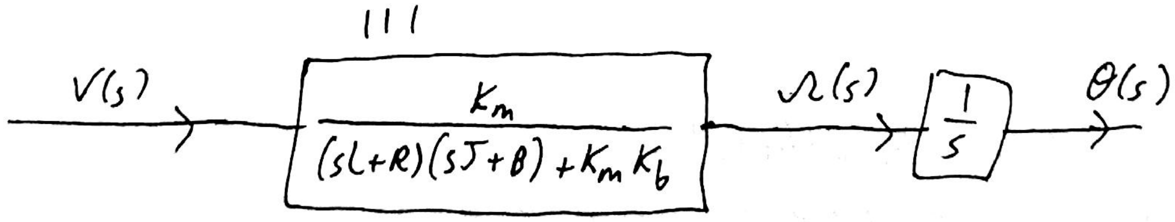
Reduction of motor block diagram:

$\parallel$

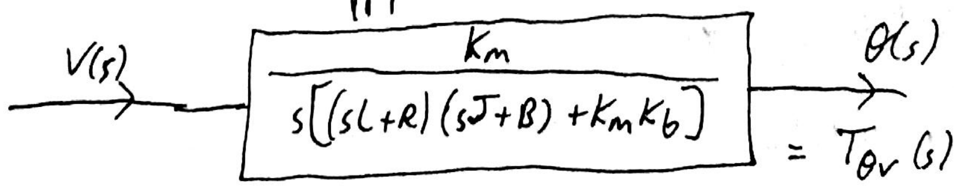


$$\frac{\dfrac{K_m}{(sL+R)(sJ+B)}}{1+\dfrac{K_mK_b}{(sL+R)(sJ+B)}} \times \frac{(sL+R)(sJ+B)}{(sL+R)(sJ+B)} = \frac{K_m}{(sL+R)(sJ+B)+K_mK_b}$$

$\parallel$



$\parallel$



$= T_{\theta v}(s)$

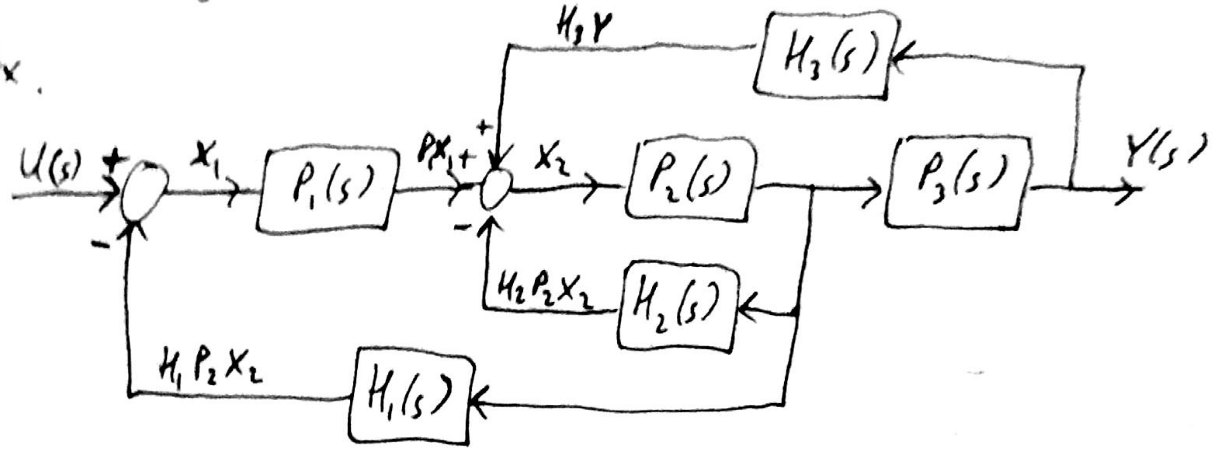
$T_{uy}(s)$ is the transfer function from $U(s)$ to $Y(s)$

$u(t)$ to $y(t)$

Alternative approach

Given a block diagram that is cumbersome to reduce step-by-step, an alternative is to convert it to a system of equations and solve using standard algebra:

Ex.



Procedure:

1. Add supplementary variables representing outputs of summers.
2. Write an expression for each summer input.
3. Write an equation for each summer relating its inputs to its outputs and an equation for the system output.
4. Solve the system of equations and eliminate the supplementary variables.

1. $X_1 = U - H_1 P_2 X_2$

2. $X_2 = P_1 X_1 - H_2 P_2 X_2 + H_3 Y$

3. $Y = P_3 P_2 X_2$

(3) into (2) $\Rightarrow X_2 = P_1 X_1 - H_2 P_2 X_2 + H_3 P_3 P_2 X_2$

$\Rightarrow P_1 X_1 = (1 + P_2 H_2 - P_2 P_3 H_3) X_2 = (1 + P_2 (H_2 - P_3 H_3)) X_2$

$\Rightarrow X_1 = \dfrac{1 + P_2 (H_2 - P_3 H_3)}{P_1} X_2 \quad (4)$

(4) into (1) $\Rightarrow \dfrac{1 + P_2 (H_2 - P_3 H_3)}{P_1} = U - H_1 P_2 X_2 \Rightarrow \dfrac{1 + P_2 (H_1 P_1 + H_2 - P_3 H_3)}{P_1} X_2 = U$

$\Rightarrow X_2 = \dfrac{P_1}{1 + P_2 (H_1 P_1 + H_2 - P_3 H_3)} U \underset{(3)}{\Rightarrow} Y = \dfrac{P_1 P_2 P_3}{1 + P_2 (H_1 P_1 + H_2 - P_3 H_3)} U \longrightarrow = T_{uy}(s)$