4.1 The message signal $m(t) = 10\,\text{sinc}(400t)$ frequency modulates the carrier $c(t) = 100 \cos 2\pi f_c t$. The modulation index is 6.

1. Write an expression for the modulated signal $u(t)$.
2. What is the maximum frequency deviation of the modulated signal?
3. What is the power content of the modulated signal?
4. Find the bandwidth of the modulated signal.

4.2 Signal $m(t)$ is shown in Figure P-4.2; this signal is used once to frequency modulate a carrier and once to phase modulate the same carrier.

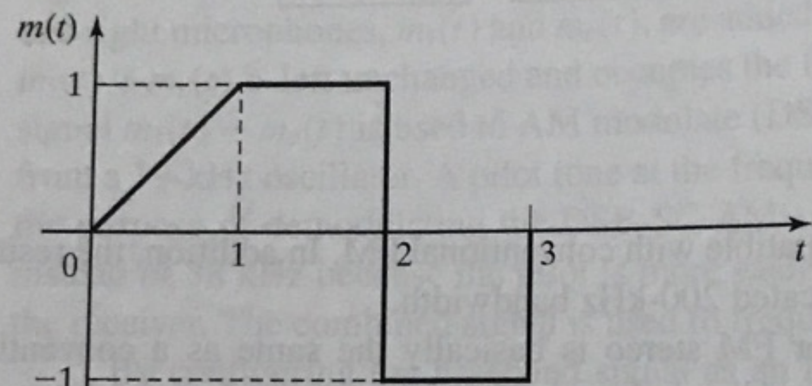

Figure P-4.2

1. Find a relation between k_p and k_f such that the maximum phase of the modulated signals in both cases are equal.
2. If $k_p = f_d = 1$, what is the maximum instantaneous frequency in each case?

4.3 Determine the in-phase and quadrature components as well as the envelope and the phase of FM- and PM-modulated signals.

4.4 An angle-modulated signal has the form

$$u(t) = 100 \cos\left[2\pi f_c t + 4 \sin 2000\pi t\right],$$

where $f_c = 10$ MHz.

1. Determine the average transmitted power.
2. Determine the peak-phase deviation.
3. Determine the peak-frequency deviation.
4. Is this an FM or a PM signal? Explain.

4.6 To generate wideband FM, we can first generate a narrowband FM signal, and then use frequency multiplication to spread the signal bandwidth. Figure P-4.6 illustrates such a scheme, which is called an Armstrong-type FM modulator. The narrowband FM signal has a maximum angular deviation of 0.10 radians to keep distortion under control.

1. If the message signal has a bandwidth of 15 kHz and the output frequency from the oscillator is 100 kHz, determine the frequency multiplication that is necessary to generate an FM signal at a carrier frequency of $f_c = 104$ MHz and a frequency deviation of $f = 75$ kHz.
2. If the carrier frequency for the wideband FM signal is to be within ± 2 Hz, determine the maximum allowable drift of the 100 kHz oscillator.

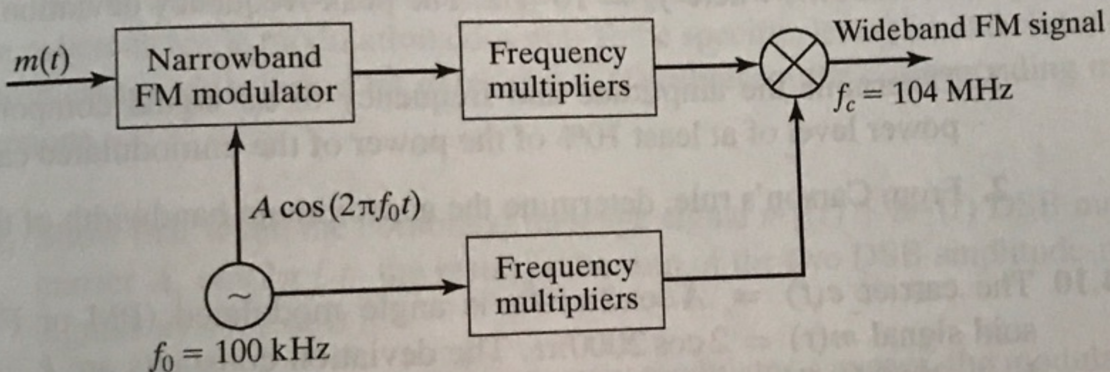

Figure P-4.6 Armstrong-type FM modulator.

4.8 An FM signal is given as

$$u(t) = 100\cos\left[2\pi f_c t + 100\int_{-\infty}^{t} m(\tau)\,d\tau\right],$$

where $m(t)$ is shown in Figure P-4.8.

1. Sketch the instantaneous frequency as a function of time.
2. Determine the peak-frequency deviation.

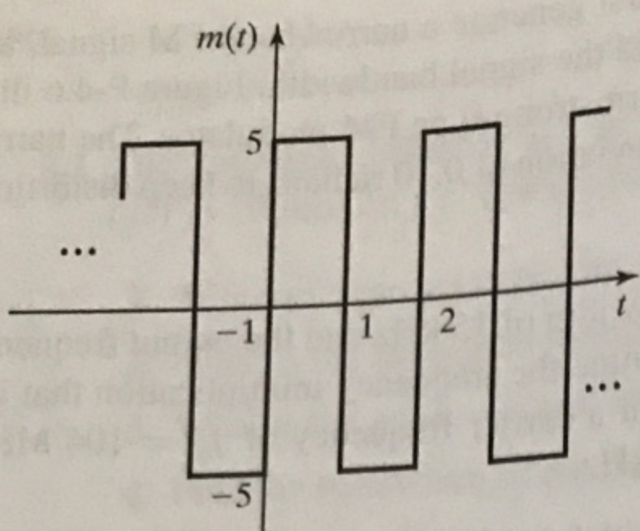

Figure P-4.8

4.9 The carrier $c(t) = 100\cos 2\pi f_c t$ is frequency modulated by the signal $m(t) = 5\cos 20{,}000\pi t$, where $f_c = 10^8$ Hz. The peak-frequency deviation is 20 kHz.

1. Determine the amplitude and frequency of all signal components that have a power level of at least 10% of the power of the unmodulated carrier component.
2. From Carson's rule, determine the approximate bandwidth of the FM signal.

4.11 The carrier $c(t) = 100\cos 2\pi f_c t$ is phase modulated by the signal $m(t) = 5\cos 2000\pi t$. The PM signal has a peak-phase deviation of $\pi/2$. The carrier frequency is $f_c = 10^8$ Hz.

1. Determine the magnitude spectrum of the sinusoidal components and sketch the results.
2. Using Carson's rule, determine the approximate bandwidth of the PM signal and compare the result with the analytical result in Part 1.

4.13 It is easy to demonstrate that amplitude modulation satisfies the superposition principle, whereas angle modulation does not. To be specific, let $m_1(t)$ and $m_2(t)$ represent two message signals and let $u_1(t)$ and $u_2(t)$ represent the corresponding modulated versions.

1. Show that when the combined message signal $m_1(t) + m_2(t)$ DSB modulates a carrier $A_c \cos 2\pi f_c t$, the result is the sum of the two DSB amplitude-modulated signals $u_1(t) + u_2(t)$.
2. Show that if $m_1(t) + m_2(t)$ frequency modulates a carrier, the modulated signal is not equal to $u_1(t) + u_2(t)$.

4.17 A message signal $m(t)$ has a bandwidth of 10 kHz and a peak magnitude $|m(t)|$ of 1 volt. Estimate the bandwidth of the signal $u(t)$ obtained when $m(t)$ frequency modulates a carrier with a peak-frequency deviation of (a) $f_d = 10$ Hz/V, (b) 100 Hz/V, and (c) 1000 Hz/V.