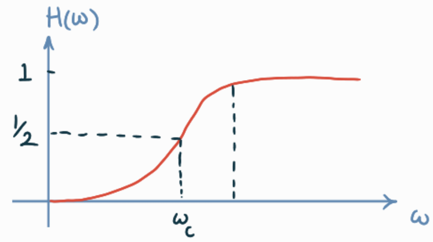
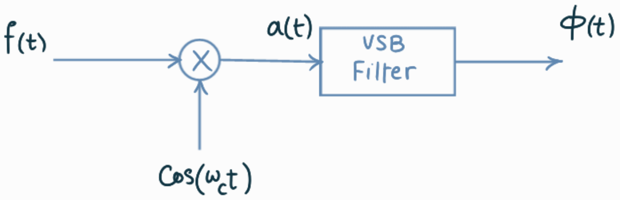
ECE 318 TUTORIAL 5 Vestigial Sideband (VSB) Modulation

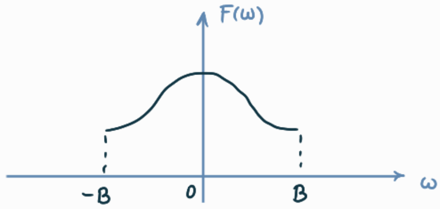
* An SSB signal is generated by passing a DSB-SC signal through an ideal BPF that lets only one sideband through.

* BPFs are not ideal. What if a non-ideal BPF (with spectrum on right) is used to generate the SSB?





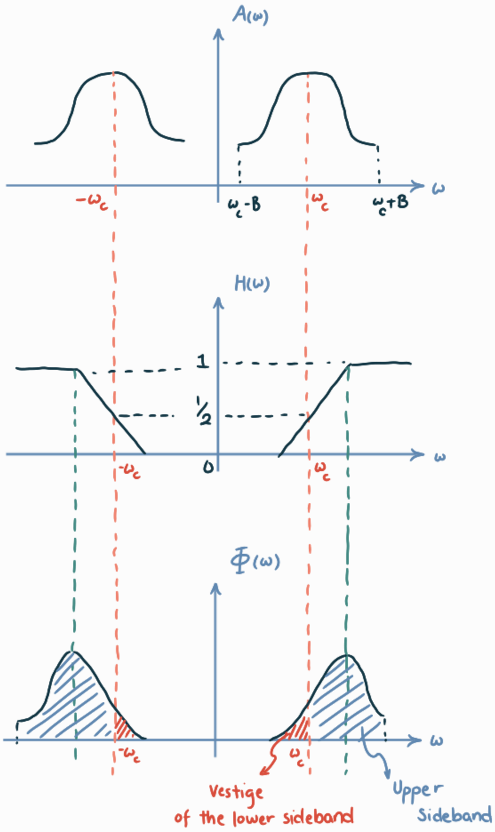
Let $F(\omega)$ be the Spectrum of $f(t)$:



Since $a(t) = f(t)\cdot \cos(\omega_c t) \Rightarrow A(\omega) = \frac{1}{2}\left[F(\omega-\omega_c) + F(\omega+\omega_c)\right]$

The VSB filter's Spectrum:

The resulting signal $\phi(t)$ has the spectrum $\Phi(\omega)$

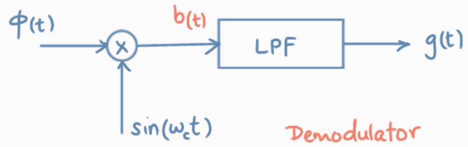


Note: The VSB filter is now a non-ideal BPF. But is there any other criteria the filter needs to satisfy so that the message $f(t)$ can be recovered properly?

For example, we may need to design the roll-off region of $H(\omega)$, so that (a) $f(t)$ is recovered and (b) some additional criteria are satisfied (say a certain percentage of the lower sideband is to be transmitted). We use a couple of examples to demonstrate these.

Under what condition is $f(t)$ recovered?

Ex. Consider the VSB modulator in the previous page where $f(t)$ is the message signal with $BW = B$ rad/sec. We use the demodulator on the right to recover $f(t)$, but use $\sin(\omega_c t)$ instead of $\cos(\omega_c t)$. The LPF is ideal with $BW = B$ rad/sec. Under what condition on $H(\omega)$ will we have $g(t) = k \cdot f(t)$, for some $k \neq 0$?



Demodulator

SoL: Let's find the spectrum of $a(t)$ and $\phi(t)$ since $a(t) = f(t) \cdot \cos(\omega_c t) \Rightarrow A(\omega) = \frac{1}{2}\{F(\omega-\omega_c)+F(\omega+\omega_c)\}$

$$\therefore \quad \phi(\omega) = A(\omega) \cdot H(\omega) = \frac{H(\omega)}{2}\{F(\omega-\omega_c) + F(\omega+\omega_c)\}$$

In the demodulation stage, $b(t) = \phi(t) \sin(\omega_c t)$

$$\therefore \ B(\omega) = \frac{j}{2}\{\phi(\omega+\omega_c) - \phi(\omega-\omega_c)\} = \frac{j}{2}\left\{\frac{H(\omega+\omega_c)}{2}\left[F(\omega)+F(\omega+2\omega_c)\right] - \frac{H(\omega-\omega_c)}{2}\left[F(\omega-2\omega_c)+F(\omega)\right]\right\}$$

Since LPF only passes signals that are in $-B \leq \omega \leq B$ rad/sec we have:

$$G(\omega) = \frac{j}{4} F(\omega)\{H(\omega+\omega_c) - H(\omega-\omega_c)\} \qquad |\omega| \leq B \qquad (*1)$$

Since we want $g(t) = k \cdot f(t)$ for some $k \neq 0$, we have: $\quad G(\omega) = k \cdot F(\omega) \qquad (*2)$

Combining $(*1)$ and $(*2)$, the condition on $H(\omega)$ is:

$$j\{H(\omega+\omega_c) - H(\omega-\omega_c)\} = 4k, \quad \text{for some } k \neq 0 \quad \text{for } |\omega| \leq B$$

Or, $H(\omega)$ and specifically its rolloff region should be such that they add up to a constant value (here $\frac{4k}{j}$) within $|\omega| \leq B$

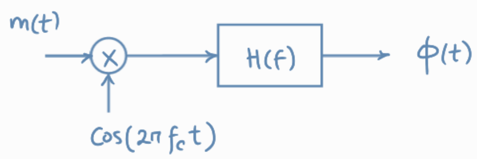
Note: If we were to use $\cos(\omega_c t)$ in the demodulation stage, the condition would be:

$$H(\omega-\omega_c) + H(\omega+\omega_c) = k_o, \quad \text{for some } k_o \neq 0, \ |\omega| \leq B.$$

In fact, it is common to assume that $H(\omega)$ satifies $H(\omega-\omega_c) + H(\omega+\omega_c) = 1$ and $H(\omega_c) = 1/2$ for $|\omega| \leq B$.

How to deal with additional criteria on $H(\omega)$?

Ex. A message $m(t)$ with a BW of 10 KHz modulates a 100 KHz cosine carrier. The signal is then filtered to produce the VSB signal. Design a high-pass filter $H(f)$ such that 20% of the lower sideband is transmitted. Assume that the maximum magnitude of $H(f)$ is 1.



SOL : The VSB filter must satisfy the following properties:

(i) $H(f-f_c) + H(f+f_c) = 1$, $|f| \leq 10$ KHz, (Here, $f_c = 100\,000$)

(ii) $H(f_c) = 1/2$,

(iii) $H(f)$ should pass only 20% of the lower sideband.

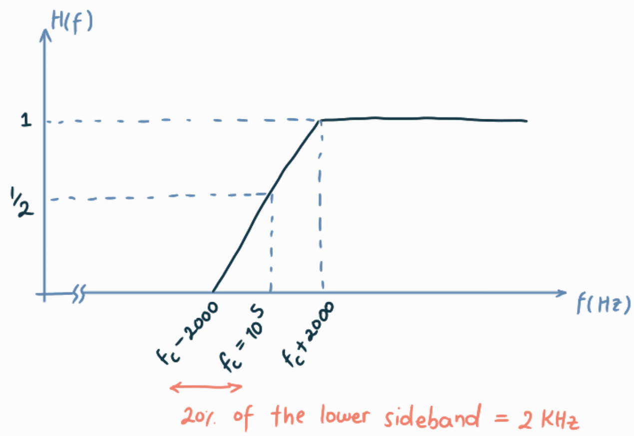
Designing a VSB filter usually amounts to designing the roll-off region, since beyond the roll-off region its response will be 1 (after all VSB filter is a high-pass filter).

Recall that at the midpoint of the roll-off region, we have $H(f_c) = 1/2$, hence the roll-off curve passes through $(f_c, 1/2)$.

The roll-off region can be modeled as a straight line or a raised-cosine function, but obviously straight-line is the simplest one (which we use here). Note that this straight line should pass through $(f_c, 1/2)$.

Also note that 20% of the lower sideband (20% of 10 KHz = 2 KHz) should be passed, thus:

$H(f) = 0$ at $f = f_c - 2$ KHz. See the following figure.



Thus the roll-off region straight line passes through the two points $(100\,000, 1/2)$ and $(98000, 0)$.

Hence the filter response in function form is:

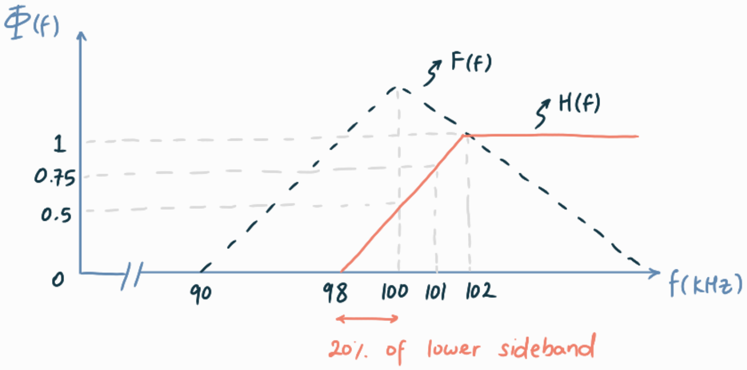
$$H(f) = \begin{cases} 0, & |f| \leq 98 \text{ KHz} \\ a(|f| - b), & 98\text{K} < |f| \leq 102 \text{ KHz} \\ 1, & |f| > 102 \text{ KHz} \end{cases}$$

One can easily find a and b from (ii) $H(100\,000) = 1/2 = a \times 2000 \Rightarrow a = 1/4000$

(i) $H(98000) = 0 = a(98000 - b) \Rightarrow b = 98000$

Thus, $$H(f) = \begin{cases} 0, & 0 \leq |f| \leq 98\text{K} \\ \dfrac{|f| - 98000}{4000}, & 98\text{K} \leq |f| \leq 102\text{K} \\ 1, & |f| > 102\text{K} \end{cases}$$

How does the filter look like?



What if the design criteria was: 40% of the lower sideband should be transmitted and $H(f = 101\ kHz) = 0.625$.

We basically go through the same procedure, and we use two known data points $(100K, 0.5)$ and $(96K, 0)$ to find the slope of the roll-off region straight line. We find the following response:

$$H(\omega) = \begin{cases} 0, & 0 \leq f \leq 96K & \text{stopband} \\ \dfrac{|f| - 96000}{8000}, & 96K < |f| \leq 104K & \text{roll-off region} \\ 1, & |f| > 104K & \text{passband} \end{cases}$$

END OF TUTORIAL