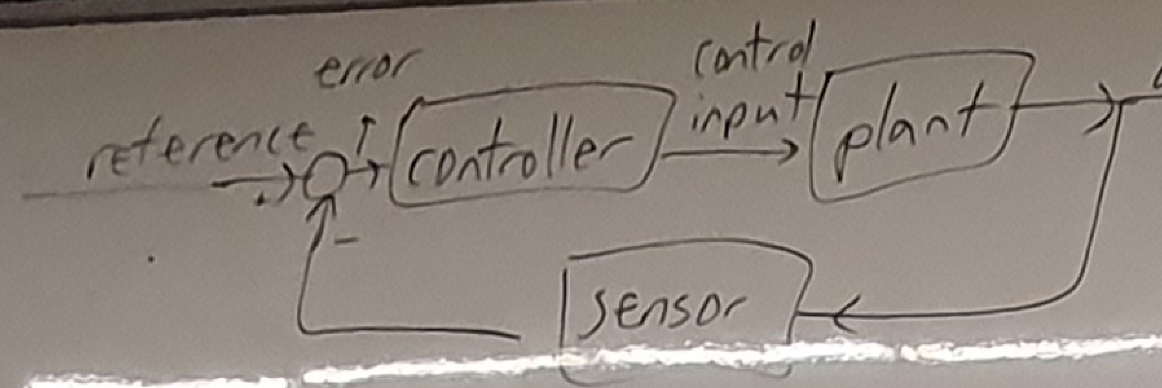


feedback

Ex.s

- $M\ddot{y}(t) = u(t)$, $u(t) = 0 \ \forall t \leq t_0$, $y(t_0) = \dot{y}(t_0) = 0$

  $\Rightarrow$ linear

- $y(t) = u(t) + 1$) — P

  $\Rightarrow$ nonlinear

$u_1(t) = 0 \ \forall t$

$u_2(t) = 1 \ \forall t \geq t_0$

$u_1(t) \xrightarrow{P} y_1(t) = 1$

$u_2(t) \xrightarrow{P} y_2(t) = 2$

$u(t) = u_1(t) + u_2(t) = 1 \xrightarrow{P} y(t) = 2$

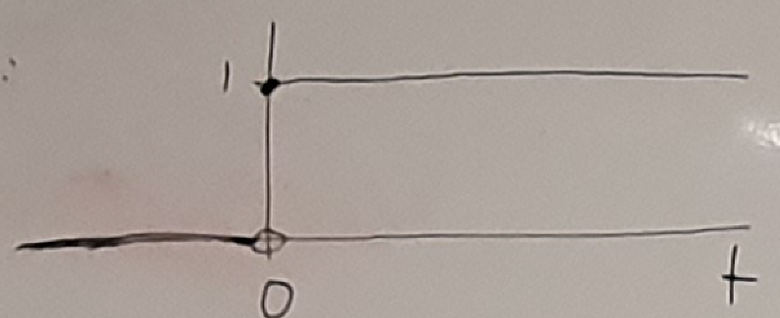
$y_1(t) + y_2(t) = 3$

$\Rightarrow y(t) \neq y_1(t) + y_2(t)$

Signals

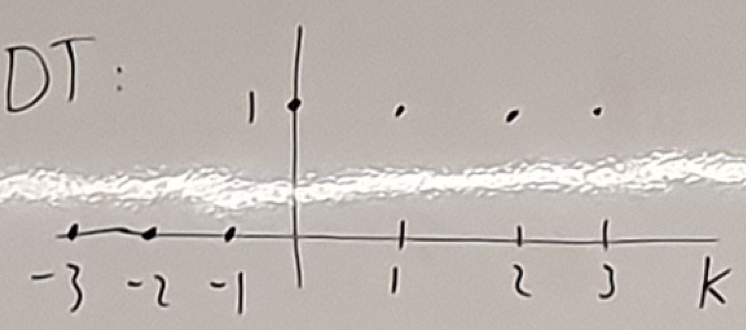
A simple signal that is often used as a reference in control systems is a unit step:

CT:



$\mathbb{1}(t)$

DT:



$\mathbb{1}[k]$

Ex. $\dot{y} = u$

$\Rightarrow y(t) = \int_0^t u(\tau)\,d\tau$

Systems:

- informally, a device or process whereby certain "input" signals determine certain "output" signals
- mathematically, a mapping (function) from a set U of input signals to a set Y of output signals

Notation: $U \xrightarrow{P} Y$

$u(t) \xrightarrow{P} y(t)$ where $y(t) = (Pu)(t)$

$\rightarrow y = Pu$

Output $y(t) = (Pu)(t)$ is called the system's <u>response</u> to the input $u(t)$

Properties of Systems:

1. CT, DT, or hybrid

- if the input and output signals are of CT signals then the system is CT
  - e.g., most models of physical systems
- if the input and output signals are of DT signals then the system is DT
  - e.g., digital hardware
- in a hybrid system, the input and output signals are of different kinds
  - e.g., analog/digital (A/D) converters
    D/A converters

Ex. A differential equation may represent a CT system
- assuming that there exists a unique solution

Ex. DT systems are often represented by difference equations

e.g., $y[k] + a_1 y[k-1] + a_2 y[k-2] + \dots + a_n y[k-n]$
$= b_0 u[k] + b_1 u[k-1] + \dots + b_m u[k-m]$

initial conditions give the values of the output
- in CT: and its derivatives at some time t_0
- in DT: at multiple different times k

2. Memoryless vs. dynamic

- in a memoryless system, the instantaneous output value $y(t)$ depends only on the current input value $u(t)$
  - e.g., ideal amplifier:
    $\underbrace{V_{out}(t)}_{y(t)} = K \underbrace{V_{in}(t)}_{u(t)}$
- a system that is not memoryless is dynamic
  - e.g., mechanical system
    $M\ddot{y}(t) = u(t)$, $\quad u(t) = 0 \ \forall t \le t_0$
    $y(t_0) = \dot{y}(t_0) = 0$

$\Rightarrow \quad y(t) = \frac{1}{M}\int_{-\infty}^{t}\left(\int_{-\infty}^{\tau} u(\rho)\,d\rho\right)d\tau$

$\Rightarrow$ $y(t)$ depends on the past history of $u(t)$ (not just its current value)

$\Rightarrow$ this system is dynamic

Most interesting control problems involve dynamic plants

3. Causality

A system P is causal if $y(t) = (Pu)(t)$ depends only on $\{u(\tau) : \tau \leq t\}$

- i.e., only on prior and present values of the input

In other words, if $u_1(\tau) = u_2(\tau)$ $\forall\, \tau \leq t$, and if $y_1 = Pu_1$ and $y_2 = Pu_2$

then $y_1(\tau) = y_2(\tau)$ $\forall\, \tau \leq t$

Ex.s:

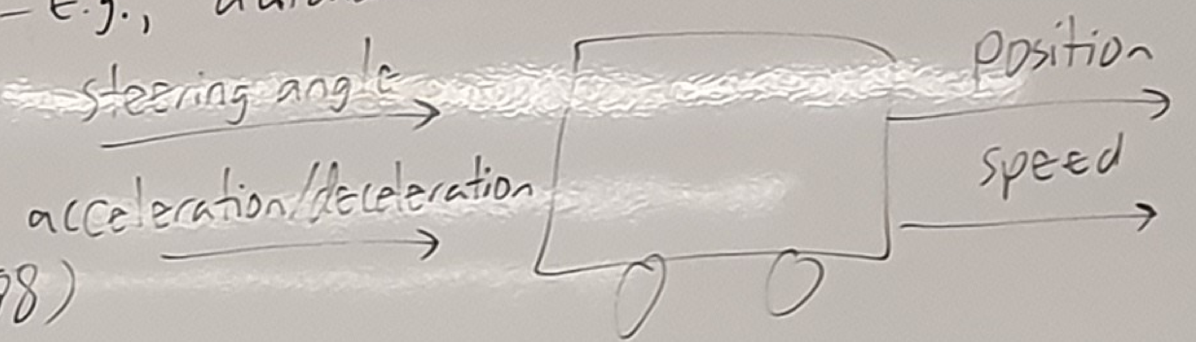
- memoryless system $\rightarrow$ causal
- $y[k] = u[k-1] + 2u[k] \rightarrow$ causal
- $y[k] = u[k+2] \rightarrow$ not causal

Real-time plants and controllers are causal, but (offline) signal processing involves noncausal systems

4. Multivariable vs. scalar

- multivariable - systems with multiple inputs and outputs
- scalar - single input, single output (SISO)

- e.g., autonomous vehicle



Multivariable systems pose special problems for control (ECE 488)

5. Linearity

linear combinations: $c_1 f_1(t) + c_2 f_2(t)$ where $c_1, c_2 \in \mathbb{C}$ are called weights

or $\sum_{i=1}^{n} c_i f_i(t)$

or $\sum_{i=1}^{\infty} c_i f_i(t)$

or $\int_a^b c(\tau) f(t, \tau)\, d\tau$

- a system is linear if when the input is a linear combination of signals then the output is a linear combination with the same weights of the outputs of those individual signals

$u_1 \xrightarrow{P} y_1$

$u_2 \xrightarrow{P} y_2$

$c_1 u_1 + c_2 u_2 \xrightarrow{P} c_1 y_1 + c_2 y_2$

$u_i \xrightarrow{P} y_i \ \forall\, i$

$\Rightarrow \sum_{i=1}^{\infty} c_i u_i \xrightarrow{P} \sum_{i=1}^{\infty} c_i y_i$

$u(t, \tau) \xrightarrow{P} y(t, \tau) \ \forall\, \tau$

$\Rightarrow \int_a^b c(\tau) u(t, \tau)\, d\tau \xrightarrow{P} \int_a^b c(\tau) y(t, \tau)\, d\tau$