## General Modelling Procedure:

Apply physical laws.

→ Newton's 2nd law

→ Kirchoff's Laws (KCL, KVL)

System of differential equations.

↓ "linearize" about desired "operating point"

System of LTI systems.

↓ take Laplace transform + solve for relationship between I/O.

Transfer Functions.

↓ identify unknown parameters experimentally.

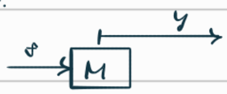
"model"

### Deriving Differential Equations.

Mechanical systems. (in translation)

- components.

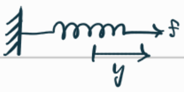
masses




$f(t) = M\dfrac{d^2y}{dt^2}$ Newton's 2nd law.

y = displacement

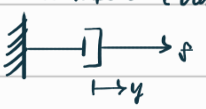
linear springs.




$f(t) = Ky(t)$

y = 0 → spring not stretched

linear dashpots (damping elements)

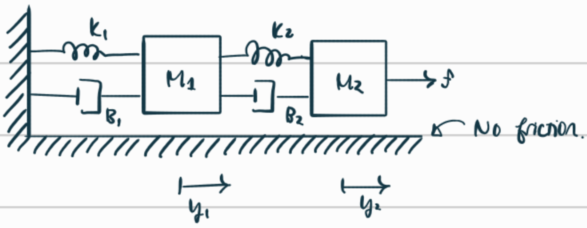



$f(t) = -B\dot{y}(t)$

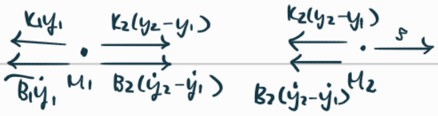
Procedure

1. FBD for each mass
2. Apply Newton's 2nd law.
3. System of DEs.

ex.




FBDs.



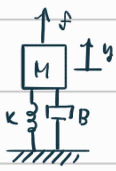

$y_1 = y_2 = 0$ both springs at natural lengths

$f(y_1, y_2, \dot{y}_1, \dot{y}_2, \ddot{y}_1, \ddot{y}_2, f) = 0$

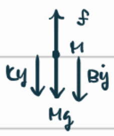
Newton's 2nd Law

$$\left.\begin{aligned} M_1\ddot{y}_1 &= K_2(y_2-y_1) + B_2(\dot{y}_2-\dot{y}_1) - K_1y_1 - B_1\dot{y}_1 \\ M_2\ddot{y}_2 &= f - K_2(y_2-y_1) - B_2(\dot{y}_2-\dot{y}_1) \end{aligned}\right\} \text{system of DEs, linear, LTI.}$$

ex.




FBD:




Newton's 2nd Law.

$M\ddot{y} = f - Ky - B\dot{y} - Mg \Leftrightarrow M\ddot{y} + B\dot{y} + Ky = f - Mg$

$\Leftrightarrow 0 = f - Ky - B\dot{y} - M\ddot{y} - Mg$ → affine (nonlinear) Not LTI.

Idea: change variables (coords) to cancel out constant term so that this system is LTI.

→ Define $\Delta y = y - y_0$ (unknown constant)

↳ translation/disp relative to $y_0$.

$\Delta\dot{y} = \dot{y}$

$\Delta\ddot{y} = \ddot{y}$

Goal:

1. Choose $y_0$ so that $\Delta y$ satisfies an LTI DE.
2. Solve for $\Delta y$ using LTI methods.
3. Calc. $y = \Delta y + y_0$

$$M\Delta\ddot{y} + B\Delta\dot{y} + K\underbrace{\Delta y}_{=y-y_0} = \underbrace{M\ddot{y} + B\dot{y} + Ky}_{f-Mg} - Ky_0$$

$$= f - Mg - Ky_0 \quad ① \quad = f - \cancel{Mg} + \cancel{Mg} = f$$

$\therefore$ LTI

Choose $y_0 \Rightarrow -Ky_0 = Mg \Leftrightarrow y_0 = \dfrac{-Mg}{K}$