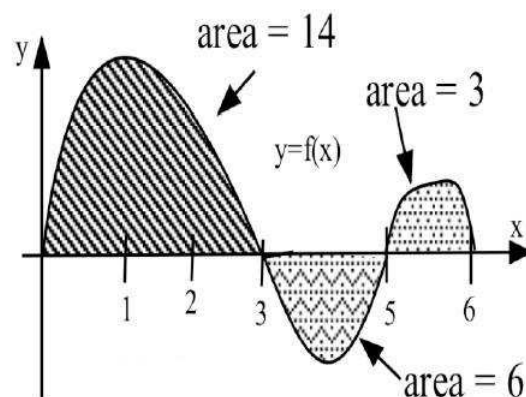


4. (8 points) Let  $f(x) = \cos(x^2)$ . Find the average value of  $f'(x)$  on  $[0, \sqrt{\pi}]$ .

3. (8 total points) Use the area information given on this graph of  $f(x)$  to evaluate the integrals below.



(a) (2 points)  $\int_3^6 |f(x)| dx$

(b) (2 points)  $\int_0^5 2 + f(x) dx$

(c) (2 points)  $\int_6^5 2f(x) dx$

(d) (2 points)  $\int_0^3 6x - f(x) dx$

4. (8 total points) Determine if the following are **TRUE** or **FALSE**. You need not explain your answers. Each correct answer is +2 points, each wrong answer is -1 points, each blank answer is 0 points, but your total for this whole problem will not be less than 0 points. Put your **ANSWERS** in the **BOXES**.

(a) (2 points) The function  $f(x) = \frac{e^x}{x}$  is a solution of the differential equation  $x^2y' + xy = xe^x$ .

Answer (T or F or leave blank):

☐

(b) (2 points)  $\frac{d}{dx} \int_2^{x^2+1} \ln(t) dt = \ln(x^2 + 1)$ .

Answer (T or F or leave blank):

☐

(c) (2 points) The arc length of the curve  $y = \tan x$  for  $0 \leq x \leq \frac{\pi}{4}$  is  $\int_0^{\pi/4} \sqrt{1 + \sec^2 x} dx$ .

Answer (T or F or leave blank):

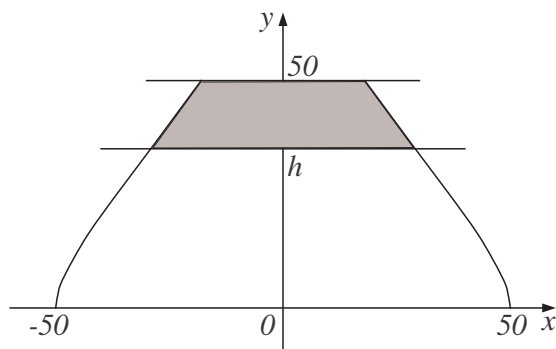
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(d) (2 points) If  $f$  and  $f'$  are continuous on  $[3, 7]$ , then  $\int_3^7 f'(u) du = f(7) - f(3)$ .

Answer (T or F or leave blank):

☐

4. (12 total points) A shape  $S$  is bounded by the  $x$ -axis, the line  $y = 50$ , the curve  $x = 50e^{-(y/50)^2}$ , and the curve  $x = -50e^{-(y/50)^2}$ . A barrier comes down and covers the shape  $S$  between height  $h$  and height 50.
- (a) (3 points) Express the area not covered by the barrier (the unshaded area) in terms of an integral.



- (b) (4 points) Suppose that the horizontal line  $y = h$  at the bottom of the barrier starts at the top with zero velocity at time  $t = 0$  and descends with acceleration  $a(t) = -6t$ . Find a formula for  $h$  in terms of  $t$ .
- (c) (5 points) If the barrier descends as in part b), find a formula in terms of  $t$  for the rate of change of the area not covered by the barrier.