Name________________________________

Math 125 **First Midterm** **10:00 Jan. 29, 2015**

(7 problems, 80 minutes, 100 points)

1. (12 points) Evaluate the indefinite integral

$$\int x\sqrt{1-x^2}\; e^{\left((1-x^2)^{3/2}\right)}\; dx.$$

2. (12 points) By making the substitution $u = \sec(\pi x)$, convert the following definite integral to a new, simpler-looking definite integral, but do **not** evaluate it:

$$\int_0^{1/3} \sqrt{1+\sec^3(\pi x)}\,\sec^2(\pi x)\tan(\pi x)dx.$$

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3. (15 points) In this problem use the **midpoint** rule with $n = 4$ subintervals. Suppose an object starts from v_0 and accelerates. You measure the acceleration $a_0 = a(0)$, $a_1 = a(1)$, $a_2 = a(2)$,..., $a_8 = a(8)$ at 1-second intervals for 8 sec. Find an expression in terms of v_0 and the a_i for the velocity after 8 sec.

4. (15 points) Let R be the region in the first quadrant that's bounded by the line $y = 9x$ and curve $y = x^3$.

(a) Find the x- and y-coordinates of the intersection points of the line and the curve.

(b) **Using the washer method**, find an integral for the volume of the solid of revolution obtained by revolving R **around the $y-$axis**. Do **not** evaluate the integral.

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5. (15 points) (a) Write

$$\lim_{n\to\infty} \frac{\pi}{n} \sum_{i=1}^{n} 2\pi\left(5 + i\frac{\pi}{n}\right) \sin\left(i\frac{\pi}{n}\right)$$

as a definite integral.

(b) Explain what volume is given by the integral in part (a). Use a clear word description and/or a clearly labeled diagram to explain what volume it is.

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6. (15 points) Your purpose in this problem is to find the gravitational constant g on an airless moon. You see a ball thrown up from ground level at initial velocity v_0. (You do not know the value of v_0.) At $t = 2$ sec the ball is at height 40 m, and at $t = 6$ sec it is at height 90 m.

(a) Using the information at $t = 2$ and $t = 6$, set up two equations.

(b) Solve for g.

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7. (16 points) At time $t = 0$ an object starts at $x_0 = 6$ traveling **to the left** at 2 units/sec. It is acted on by a force that causes it to accelerate with $a(t) = \sin(0.5t)$. Find equations for

(a) the object's velocity $v(t) = \dot{x}(t)$, and

(b) the object's displacement $x(t)$. Please show all your work clearly.

Midterm Answers, Jan. 29, 2015

1. Substituting $u = (1-x^2)^{3/2}$, we get $du = \frac{3}{2}\sqrt{1-x^2}(-2x)dx = -3x\sqrt{1-x^2}dx$, and so

$$\int x\sqrt{1-x^2}e^{\left((1-x^2)^{3/2}\right)}dx = -\frac{1}{3}\int e^u du = -\frac{1}{3}e^{\left((1-x^2)^{3/2}\right)} + C$$

.

2. $\frac{1}{\pi}\int_1^2 u\sqrt{1+u^3}du$

3. $v_0 + 2(a_1 + a_3 + a_5 + a_7)$.

4. (a) $(0,0)$ and $(3,27)$. (b) $\pi\int_0^{27}(y^{2/3} - \frac{1}{81}y^2)dy$.

5.

$$\lim_{n\to\infty}\frac{\pi}{n}\sum_{i=1}^{n}2\pi\left(5+i\frac{\pi}{n}\right)\sin\left(i\frac{\pi}{n}\right) = 2\pi\int_0^{\pi}(5+x)\sin(x)dx,$$

which is the volume of one hump of the sine curve (between $x = 0$ and $x = \pi$) rotated around the line $x = -5$.

6. (a) The two equations are $-\frac{1}{2}\cdot 2^2 g + 2v_0 = 40$ and $-\frac{1}{2}\cdot 6^2 g + 6v_0 = 90$. (b) Subtracting the second equation from three times the first one gives $12g = 30$, and so $g = 2.5$ m/sec^2.

7. (a) $v(t) = -2\cos(0.5t) + C$, and the fact that $v(0) = -2$ means that $C = 0$; (b) $x(t) = -4\sin(0.5t) + C'$, and the fact that $x(0) = 6$ means that $C' = 6$, so that $x(t) = -4\sin(0.5t) + 6$.