Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.
- A scientific calculator is needed, but no calculator with graphing, programming, symbolic manipulation, or calculus capabilities is allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text (p. 484 if you have the 6th edition of Stewart) without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	8	
4	12	
5	10	

Question	Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. (10 total points) Evaluate the following indefinite integrals.

   (a) (5 points) $\displaystyle\int \frac{x^3+3x^2+4x+4}{x^2+2x-3}\,dx$

   (b) (5 points) $\displaystyle\int \frac{dt}{\sqrt{t^2-6t+13}}$

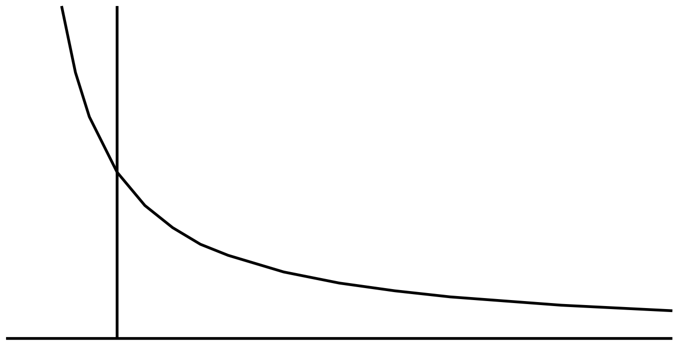
2. (10 total points) Evaluate the following definite integrals.

(a) (5 points) $\displaystyle\int_0^{\pi/6} \tan^3(2x)\ \sec(2x)\,dx$

(b) (5 points) $\displaystyle\int_0^{\pi} e^{\cos t} \sin 2t\, dt$

3. (8 points) Let R be the region *in the first quadrant* which is *above* the hyperbola $y = \dfrac{12}{x+2}$ and *below* the line $y = \dfrac{15}{2} - x$. Sketch the region R and find its area.

The graph of the hyperbola is given below to get you started.



4. (12 total points) Consider the region R bounded between the parabola $y = x^2$ and the line $y = 3x$.

   (a) (2 points) Sketch the region.

   (b) (4 points) Find the volume of the solid obtained by rotating the region R about the y-axis. (Set up a definite integral AND evaluate it.)

   (c) (6 points) Set up two separate definite integrals for the volume obtained by rotating the region R about the *horizontal* line $y = 10$. In the first integral use the method of *shells*, and in the second integral use the method of *washers*. DO NOT EVALUATE EITHER INTEGRAL.

   Shells:

   Washers:

5. (10 points) A rectangular oil tank has height 10 feet, length 5 feet, and width 6 feet. The top of the tank lies 8 feet below ground level. It is currently filled with oil whose weight is 70 pounds per cubic foot. Find the work required to empty the oil tank by pumping all of the oil out of the tank and into an oil truck 5 feet above the ground.

6. (10 total points) Consider the *improper* integral

$$\int_0^1 x^k \ln x \, dx,$$

where k is a constant.

(a) (4 points) Does this improper integral converge when $k = -1$? Justify your answer.

(b) (6 points) Determine the values of $k \neq -1$ for which the improper integral above converges. Justify your answer.

7. (10 total points) You notice a bug flying around the room. The distance $s(t)$ (in feet) that the bug has traveled after t seconds is given by

$$s(t) = \int_0^{3t^2} \sqrt{2x^2 + \cos^2(\pi x)}\, dx.$$

Give units for your answer to both parts below.

(a) (5 points) Find the speed $\frac{ds}{dt}$ of the bug at time $t = \frac{1}{2}$ seconds. Don't forget units.

(b) (5 points) After $t = 1$ second, the bug has traveled a distance

$$s(1) = \int_0^{3} \sqrt{2x^2 + \cos^2(\pi x)}\, dx.$$

Use Simpson's rule with $n = 4$ subintervals to approximate the value of this integral.
You can either give your answer in exact form, or in decimal form with at least three digits after the decimal point. Don't forget units.

8. (10 total points) Consider the region $\mathscr{R}$ bounded between the curves $y = 5 - x^2$ and $y = 4x^2$.

(a) (3 points) Find the area of $\mathscr{R}$.

(b) (3 points) Find the x-coordinate $\bar{x}$ of the centroid (center of mass) of $\mathscr{R}$.

(c) (4 points) Find the y-coordinate $\bar{y}$ of the centroid (center of mass) of $\mathscr{R}$.

9. (10 points) Find the solution of the differential equation

$$\frac{dy}{dx} = \frac{x\sqrt{4-y^2}}{\sqrt{x^2-1}}$$

that satisfies $y(\sqrt{2}) = 1$. Give your answer in the form $y = f(x)$.

10. (10 total points) You would like to be a multimillonaire in 30 years. You might win the lottery, or you can start investing. This problem is about investing.
Let $A(t)$ be the amount of money (in dollars) you have in your investment account at time t (in *years*). Let M be the amount (in dollars) that you deposit every *month*, so $12M$ is the amount (in dollars) that you deposit every *year*. The rate of change of the amount A in your account has two parts: the interest and your deposits. The part of the rate of change coming from the interest is proportional to the amount in your account, and the annual interest rate is 10%. Interest is compounded continuously, and assume that your deposits are also applied continuously.

(a) (4 points) Set up a differential equation for the rate of change of A with respect to time, taking into account both the interest you earn and the deposits you make. Be careful with units.

(b) (4 points) You start with $A = 0$ at time $t = 0$. Solve the differential equation above. Your solution will involve M.

(c) (2 points) Determine the value of M, the amount that you have to deposit every month, if you want to have 5 million dollars in your account after 30 years.