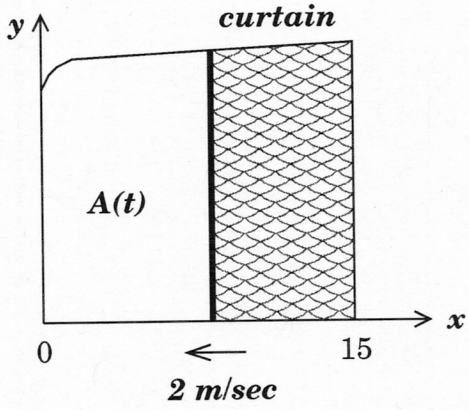
8. (8 total points) A stage opening is bounded by the x-axis, the y-axis, the line $x = 15$, and the curve

$$y = \sqrt{10 + x^{1/3}}.$$

The units on the x and y axes are meters. Initially, the stage curtain is completely open. At time $t = 0$, a vertical pole pulling the curtain starts on the right side of the stage opening ($x = 15$) and moves to the left at a constant speed of 2 m/sec. Let $A(t)$ be the area that is not yet covered by the curtain at time t seconds (the enclosed white area in the figure below).



(a) (4 points) Express $A(t)$ as a definite integral.

$$A(t) = \int_0^{15-2t} \sqrt{10 + x^{1/3}}\, dx$$

Note since the curtain is moving, the upper bound on the integral is changing. As the speed is constant, then the upper bound is changing by $15-2t$

(b) (4 points) Find $\frac{dA}{dt}$ when $t = 3.5$ sec. Give your answer in exact form and include correct units.

By FTC,

$$\frac{dA}{dt} = \sqrt{10 + (15-2t)^{1/3}} \cdot -2$$

↑ this is the derivative of $(15-2t)$

Hence

$$A'(3.5) = \sqrt{10 + (15-2(3.5))^{1/3}} \cdot -2 = -4\sqrt{3}\ \frac{m^2}{s}$$

Since x and y are in meters, then $A(t)$ has units m^2. As time is in seconds and differentiating w/ respect to t, then $A'(t)$ has units $\frac{m^2}{s}$.