1. (10 total points) Evaluate the following indefinite integrals.

(a) (5 points) $\displaystyle\int \frac{x^3+3x^2+4x+4}{x^2+2x-3}\,dx$

① Long division

$$\begin{array}{r} x+1 \\ x^2+2x-3 \overline{)\, x^3+3x^2+4x+4} \\ x^3+2x^2-3x \\ x^2+7x+4 \\ -\; x^2+2x-3 \\ 5x+7 \end{array}$$

$$\int x+1+\frac{5x+7}{x^2+2x-3}\,dx$$

$$=\int x+1+\frac{5x+7}{(x+3)(x-1)}\,dx$$

$$\frac{5x+7}{(x+3)(x-1)}=\frac{A}{x+3}+\frac{B}{x-1}$$

$$5x+7=A(x-1)+B(x+3)$$

$x=1$: $12=4B$ $B=3$

$x=-3$ $-8=-4A$ $A=2$

$$\int x+1+\frac{2}{x+3}+\frac{3}{x-1}\,dx$$

$$=\boxed{\tfrac{1}{2}x^2+x+2\ln|x+3|+3\ln|x-1|+C}$$

(b) (5 points) $\displaystyle\int \frac{dt}{\sqrt{t^2-6t+13}}$ ← Trig

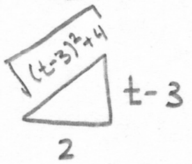
Complete the square

$$t^2-6t+13=(t-3)^2-9+13=(t-3)^2+4$$

$$\int \frac{dt}{\sqrt{(t-3)^2+4}} \qquad t-3=2\tan\theta \qquad dt=2\sec^2\theta\,d\theta$$

$$\int \frac{2\sec^2\theta\,d\theta}{\sqrt{4\tan^2\theta+4}}=\int\frac{\cancel{2}\sec^2\theta}{\cancel{2}\sec\theta}\,d\theta=\int\sec\theta\,d\theta$$

$$\frac{t-3}{2}=\tan\theta=\frac{o}{a}$$



$$=\ln|\sec\theta+\tan\theta|+C$$

↑ on pg 495 of text

$$\boxed{=\ln\left|\frac{\sqrt{(t-3)^2+4}}{2}+\frac{t-3}{2}\right|+C}$$

2. (10 total points) Evaluate the following definite integrals.

(a) (5 points) $\int_0^{\pi/6} \tan^3(2x)\sec(2x)\,dx$

$\int_0^{\pi/6} \tan^2(2x)\tan(2x)\sec(2x)\,dx$ $u = \sec(2x)$ $du = 2\sec(2x)\tan(2x)$

$= \frac{1}{2}\int_1^2 u^2 - 1\,du = \frac{1}{6}u^3 - \frac{1}{2}u \Big|_1^2 = \frac{8}{6} - 1 - \frac{1}{6} + \frac{1}{2}$

$= \boxed{\frac{2}{3}}$

(b) (5 points) $\int_0^{\pi} e^{\cos t}\sin 2t\,dt$ $\sin(2t) = 2\sin t\cos t$

$2\int_0^{\pi} e^{\cos t}\sin(t)\cos(t)\,dt$ $u = \cos t$ $du = -\sin t$

$2\int_1^{-1} -ue^u\,du$ $w = -2u$ $dv = e^u\,du$ $dw = -2du$ $v = e^u$

$-2ue^u\Big|_1^{-1} + 2\int_1^{-1} e^u\,du = \frac{2}{e} + 2e + 2e^u\Big|_1^{-1}$

$= \frac{2}{e} + 2e + \frac{2}{e} - 2e = \boxed{\frac{4}{e}}$

1. (10 total points) Evaluate the following indefinite integrals.

(a) (5 points) $\int \frac{x}{\sqrt{x+2}}\,dx$

$u=\sqrt{x+2}$ $\quad u^2=x+2$

$2u\,du=dx$

$\int \frac{(u^2-2)2\cancel{u}}{\cancel{u}}\,du = 2\left[\frac{1}{3}u^3-2u\right]+C$

$= \boxed{\frac{2}{3}\left(\sqrt{x+2}\right)^3-4\left(\sqrt{x+2}\right)+C}$

(b) (5 points) $\int e^{2x}\sec(e^{2x})\tan^3(e^{2x})\,dx$

$u=e^{2x}$ $\quad du=2e^{2x}dx$

$\frac{1}{2}\int \sec(u)\tan^3(u)\,du = \frac{1}{2}\int \sec(u)\tan(u)\tan^2(u)\,du$

$v=\sec(u)$ $\quad dv=\sec u\tan u\,du$

$=\frac{1}{2}\int \tan^2(u)\,dv = \frac{1}{2}\int v^2-1\,dv = \frac{1}{2}\left(\frac{1}{3}v^3-v\right)+C$

$\boxed{=\frac{1}{6}\left(\sec^3(e^{2x})\right)-\frac{1}{2}\left(\sec(e^{2x})\right)+C}$

2. (10 total points) Evaluate the following definite integrals.

(a) (5 points) $\int_1^2 \frac{\ln x}{x^3}\,dx$ IBP $u = \ln(x)$ $dv = x^{-3}dx$

$du = \frac{1}{x}dx$ $v = \frac{-1}{2}x^{-2}$

$= \frac{-\ln(x)}{2x^2}\Big|_1^2 + \frac{1}{2}\int_1^2 x^{-3}\,dx$

$= \frac{-\ln(x)}{2x^2}\Big|_1^2 - \frac{1}{4}x^{-2}\Big|_1^2 = \frac{-\ln(2)}{8} - \frac{1}{16} + \frac{1}{4}$

$= \boxed{\frac{-\ln(2)}{8} + \frac{3}{16}}$

(b) (5 points) $\int_2^3 \sqrt{4x - x^2}\,dx$

Complete the square $4x - x^2 = -(x^2 - 4x) = -[(x-2)^2 - 4]$

$\int_2^3 \sqrt{4 - (x-2)^2}\,dx$ $x - 2 = 2\sin\theta$ $-\pi/2 \le \theta \le \pi/2$

$dx = 2\cos\theta\,d\theta$

$\int_0^{\pi/6} \sqrt{4 - 4\sin^2\theta} \cdot 2\cos\theta\,d\theta$

$= \int_0^{\pi/6} 4\cos^2\theta\,d\theta = \int_0^{\pi/6} 2(1 + \cos(2\theta))\,d\theta$

$= 2\left[\theta + \frac{1}{2}\sin(2\theta)\right]\Big|_0^{\pi/6}$

$= 2\left[\frac{\pi}{6} + \frac{1}{2}\sin\left(\frac{\pi}{3}\right)\right] = \boxed{\frac{\pi}{3} + \frac{\sqrt{3}}{2}}$