**Math 124 Final Examination Autumn 2011**

Print Your Name | Signature

Student ID Number | Quiz Section

Professor's Name | TA's Name

## !!! READ...INSTRUCTIONS...READ !!!

1. Your exam contains 8 questions and 11 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 2 hours and 50 minutes for this final exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification.
4. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.
5. You are allowed one $8.5 \times 11$ sheet of handwritten notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed. Make sure your calculator is in radian mode.
6. Unless otherwise instructed, ALWAYS GIVE YOUR ANSWERS IN EXACT FORM. For example, $3\pi, \sqrt{2}, \ln(2)$ are in exact form; the corresponding approximations $9.424778, 1.4142, 0.693147$ are NOT in exact form.

| Problem | Total Points | Score |
|---|---|---|
| 1 | 12 | |
| 2 | 12 | |
| 3 | 12 | |
| 4 | 14 | |

| Problem | Total Points | Score |
|---|---|---|
| 5 | 12 | |
| 6 | 12 | |
| 7 | 12 | |
| 8 | 14 | |
| Total | 100 | |

1. (12 points; 4 pts each) Find the derivatives of the following functions. You do not have to simplify.

   (a) $g(x) = \ln(1 + \cos(3x - 1))$

   (b) $h(x) = \dfrac{5 + \tan^{-1} x}{e^x + 1}$

   (c) $u(x) = (\ln x)^{(3^x)}$

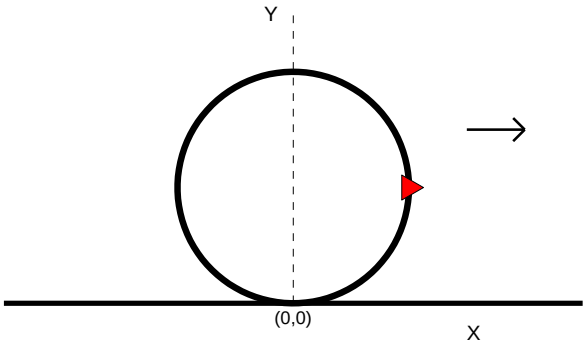
2. (12 points) An isosceles triangle has a base of length $b(t)$, which varies with time, and two equal sides of constant length $5$ cm.

   (a) (6 pts) Let $h(t)$ be the height of the triangle (the distance from the base to the vertex between the equal sides). Find a formula for $\dfrac{dh}{dt}$ in terms of $b$ and $\dfrac{db}{dt}$.

   (b) (6 pts) Let $\theta(t)$ be the angle between the two equal sides. If $\dfrac{db}{dt} = 7$ cm/sec at the time when $\theta = \dfrac{\pi}{3}$, find $\dfrac{d\theta}{dt}$.

3. (12 points) A triangle is painted on the rim of a rolling bicycle wheel. The position of the triangle at time $t$ is given by the parametric equations:

$$\begin{aligned} x(t) &= t + \cos t \\ y(t) &= 1 - \sin t \end{aligned}$$

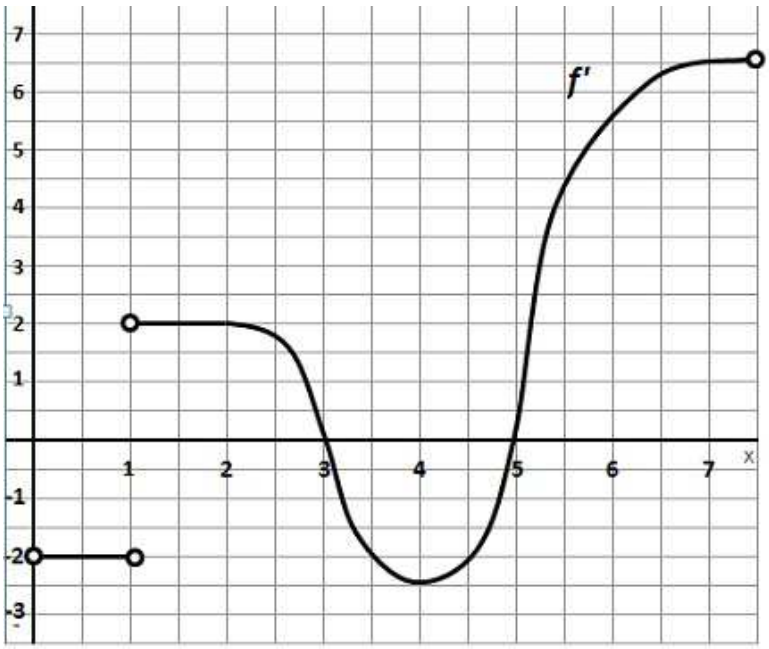



(a) (5 pts) Find the time $t$ in the interval $[0, 2\pi]$ when the horizontal velocity of the triangle is a minimum. What is the minimum horizontal velocity? What are the coordinates $x(t)$ and $y(t)$ of the triangle at this time?

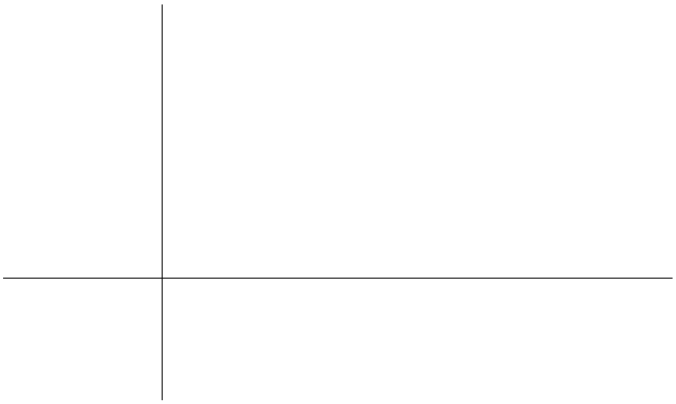
(b) (5 pts) Find the time $t$ in the interval $[0, 2\pi]$ when the horizontal velocity of the triangle is a maximum. What is the maximum horizontal velocity? What are the coordinates $x(t)$ and $y(t)$ of the triangle at this time?

(c) (2 pts) Describe in words, the position of the triangle with respect to the wheel when its horizontal velocity is a maximum and when it is a minimum.

4. (14 points; 2 pts each part) The following is the graph of the **DERIVATIVE f'** of a function f (f is not shown). Answer the following questions; you do not need to explain your answers.




(a) Suppose $f(1) = 0$ and $f$ is continuous for $x \geq 0$. Use the derivative graph above to sketch the graph of $f$ over the interval $0 \leq x \leq 2$. Label the $y$-intercept.



(b) List the longest open interval over which $f$ is decreasing (recall that the given graph is not $f$, but its derivative).

4. continued

(c) List the longest interval over which $f$ is concave up.

(d) List $x$-coordinates of the local minimal points in the interval $2 < x < 7$.

(e) Compute the following. If the answer is "does not exist", say so.

$$\lim_{h \to 0} \frac{f(0.5+h) - f(0.5)}{h} =$$

(f) Compute the following. If the answer is "does not exist", say so.

$$\lim_{x \to 1} f'(x) =$$

(g) Compute the following. If the answer is "does not exist", say so.

$$f''(4) =$$

5. (12 points) A boat leaves a dock at 2:00 PM and travels due east for an hour. At time $t$ (in hours) after it leaves the dock, the boat is $10t^{3/2}$ km east of the dock. Another boat, coming from north of the dock, is heading due south at a speed of 10 km/h and reaches the dock at 3:00 PM. At what time were the two boats closest together? Give your answer to the closest minute. Justify your answer.

6. (12 points; 4 pts each part) Compute the limit. If it is correct to say that the limit is $\infty$ or $-\infty$, then say so. If the limit does not exist, explain why. Justify your answers.

(a) $\lim_{x \to 1^-} \dfrac{x+7}{x^2-1}$

(b) $\lim_{x \to 0} \dfrac{(e^{3x}-1)}{\sin(x/4)}$

(c) $\lim_{x \to \infty} \dfrac{\sqrt{1+5x^4}+\cos(x)}{3x^2+1}$

7. (12 points) A right circular cone of height $h$ and base radius $r$ has total surface area consisting of its base area $\pi r^2$ plus its side area $\pi r\sqrt{r^2+h^2}$. Suppose you start out with a cone of height 8 cm and base radius 6 cm, and you want to change the dimensions in such a way that the total surface area remains the same. If you change the height to 8.04 cm, what is your new value for the base radius? Use implicit differentiation and the tangent line approximation. Please show your work clearly.

8. (14 points) Let $y = f(x) = \ln(x) - \sqrt{x}$ for $x > 0$.

(a) Compute the following limit. If it is correct to say that the limit is $\infty$ or $-\infty$, then say so. If the limit does not exist, explain why. Justify your answers.

$\lim_{x \to 0^+}$ f(x).

(b) Compute the following limit. If it is correct to say that the limit is $\infty$ or $-\infty$, then say so. If the limit does not exist, explain why. Justify your answers.

$\lim_{x \to +\infty}$ f(x).

(c) Find any vertical or horizontal asymptote(s).

(d) Compute $f'(x)$ and $f''(x)$. Label and box each derivative.

8. continued

(e) Find the interval(s) where f(x) is increasing, the interval(s) where f(x) is decreasing, and any local maximum or local minimum point(s) (both coordinates).

(f) Find the interval(s) where f(x) is concave upward, the interval(s) where f(x) is concave downward, and any point(s) of inflection (both coordinates).

(g) Sketch the graph of $y = f(x)$, showing clearly all points you found in parts (e) and (f).