Techniques of Integration

Math 125

Name ____________________

Quiz Section ______

The following integrals are more challenging than the basic ones we've seen in the textbook so far. You will probably have to use more than one technique to solve them. Don't hesitate to ask for hints if you get stuck.

1. $\displaystyle\int \frac{\sin(t)\cos(t)}{\sin^2(t)+6\sin(t)+8}\,dt = \int \frac{x}{x^2+6x+8}\,dx = \int \frac{x}{(x+2)(x+4)}\,dx$

$x = \sin t$

$dx = \cos t \, dt$

Partial fractions

$$\frac{x}{(x+2)(x+4)} = \frac{A}{x+2} + \frac{B}{x+4}$$

$$x = A(x+4) + B(x+2)$$

eval. at $x=-2$: $-2 = 2A \rightarrow A = -1$

eval. at $x=-4$: $-4 = -2B \rightarrow B = 2$

$$= \int \frac{2}{x+4} - \frac{1}{x+2}\,dx = 2\ln|x+4| - \ln|x+2| + C$$

$$= 2\ln(\sin t + 4) - \ln(\sin t + 2) + C$$

(don't need $|\cdot|$ any more, as >0)

2. $\displaystyle\int \left(\sin^{-1}(x)\right)^2\,dx = x(\sin^{-1}x)^2 - \int \frac{2x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

$u = (\sin^{-1}x)^2$ $\quad dv = dx$

$du = \dfrac{2\sin^{-1}x}{\sqrt{1-x^2}}$ $\quad v = x$

$u = \sin^{-1}x$ $\quad dv = \dfrac{2x}{\sqrt{1-x^2}}\,dx$

$du = \dfrac{1}{\sqrt{1-x^2}}$ $\quad v = -2\sqrt{1-x^2}$

$$= x(\sin^{-1}x)^2 - \left[-2\sqrt{1-x^2}\cdot\sin^{-1}x - \int \frac{-2\sqrt{1-x^2}}{\sqrt{1-x^2}}\,dx\right]$$

$$= x(\sin^{-1}x)^2 + 2\sin^{-1}x\sqrt{1-x^2} - 2x + C$$

<u>OR:</u> $x = \sin\theta$

$dx = \cos\theta\,d\theta$

$\displaystyle\int = \int \theta^2\cos\theta\,d\theta = \underset{\text{int by parts}}{\cdots} = \theta^2\sin\theta + 2\theta\cos\theta - 2\sin\theta + C$

$$= x(\sin^{-1}x)^2 + 2\sin^{-1}x\sqrt{1-x^2} - 2x + C$$

3. $\displaystyle\int \frac{y^2}{(1-y^2)^{7/2}}\,dy = \int \frac{\sin^2\theta\cos\theta}{\cos^7\theta}\,d\theta = \int \tan^2\theta\sec^4\theta\,d\theta$

$y = \sin\theta$

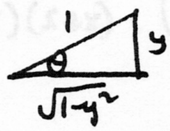
$dy = \cos\theta\,d\theta$

$$= \int \tan^2\theta\,(1+\tan^2\theta)\sec^2\theta\,d\theta$$

$u = \tan\theta$

$du = \sec^2\theta\,d\theta$

$\theta = \sin^{-1} y$



$$= \int u^2 + u^4\,du$$

$$= \frac{1}{3}u^3 + \frac{1}{5}u^5 + C$$

$\cos\theta = \sqrt{1-y^2}$

$\tan\theta = \dfrac{y}{\sqrt{1-y^2}}$

$$= \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta + C$$

$$= \frac{1}{3}\left(\frac{y}{\sqrt{1-y^2}}\right)^3 + \frac{1}{5}\left(\frac{y}{\sqrt{1-y^2}}\right)^5 + C$$

4. $\displaystyle\int \frac{1}{x+2\sqrt{x}+1}\,dx = \int \frac{2u\,du}{u^2+2u+1} = \int \frac{2(v-1)}{v^2}\,dv$

rationalizing substitution

$u = \sqrt{x}$

$x = u^2$

$dx = 2u\,du$

$v = u+1$

$dv = du$

$$= \int \frac{2}{v} - \frac{2}{v^2}\,dv$$

$$= 2\ln|v| + \frac{2}{v} + C$$

$$= 2\ln(1+\sqrt{x}) + \frac{2}{1+\sqrt{x}} + C$$