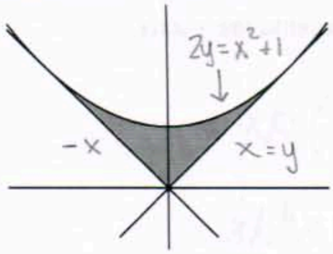
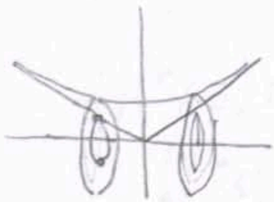
5. (12 total points) Let S be the region enclosed by the curves $y = x$, $y = -x$ and $2y = x^2 + 1$.



(a) (6 points) Find the volume of the solid obtained by rotating S about the x-axis.

① Draw Picture   Use <u>Washers</u>



② Find pts of intersection

$x=0 \quad x = \frac{x^2+1}{2}$

$x = 1$

and $-x = \frac{x^2+1}{2}$

$x=-1$

③

	$-1 \le x \le 0$	$0 \le x \le 1$
Outer	$x^2+1/2$	$x^2+1/2$
Inner	$-x$	x

④ Write integrals

$$\int_{-1}^{0} \pi\left[\left(\frac{x^2+1}{2}\right)^2 - (-x)^2\right] + \int_0^1 \pi\left[\left(\frac{x^2+1}{2}\right)^2 - (x)^2\right] dx$$

$$= 2\int_0^1 \pi\left[\frac{x^4+2x^2+1}{4} - x^2\right] dx$$

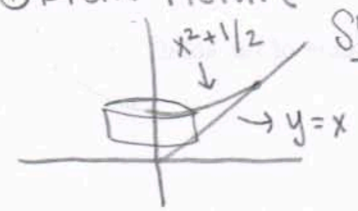
$$= 2\pi\left[\frac{1}{20}x^5 + \frac{1}{6}x^3 + \frac{1}{4}x - \frac{1}{3}x^3\right]\Big|_0^1$$

$$= 2\pi\left[\frac{1}{20} + \frac{1}{6} + \frac{1}{4} - 1/3\right]$$

$$\boxed{= 4\pi/15}$$

(b) (6 points) Find the volume of the solid obtained by rotating S about the y-axis.

① Draw Picture   <u>Shells</u>



② Points of intersection

$x=0$ and 1

③

	$0 \le x \le 1$
Height	$\frac{x^2+1}{2} - x$
Radius	x

↑ Rotating about y-axis

④ $$\int_0^1 2\pi x\left(\frac{x^2+1}{2} - x\right) = \int_0^1 2\pi\left(\frac{x^3}{2} + \frac{x}{2} - x^2\right) dx$$

$$= 2\pi\left(\frac{x^4}{8} + \frac{x^2}{4} - \frac{1}{3}x^3\right)\Big|_0^1$$

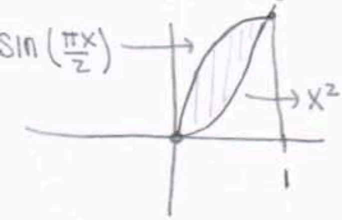
$$= 2\pi\left(\frac{1}{8} + \frac{1}{4} - \frac{1}{3}\right)$$

$$= \frac{2\pi}{24} = \boxed{\frac{\pi}{12}}$$

6. (12 total points) Let R be the region bounded by the curves $y = x^2$, $y = \sin(\pi x/2)$, $x = 0$, and $x = 1$.

   (a) (8 points) Find the volume of the solid obtained by rotating R around the y-axis.

① Draw Picture



use shells

② Pts of intersect.
$x = 0$ OR 1

③ Chart

	$0 \le x \le 1$
Height	$\sin(\frac{\pi x}{2}) - x^2$
Radius	x

rotating about y-axis

④ $\int_0^1 2\pi x \left(\sin(\frac{\pi x}{2}) - x^2\right) dx$

$= \int_0^1 2\pi x \sin(\frac{\pi x}{2}) - \int_0^1 2\pi x^3 dx$

$u = x \quad dv = \sin(\frac{\pi x}{2})$
$du = dx \quad v = \frac{-2}{\pi}\cos(\frac{\pi x}{2})$

$= 2\pi\left[\frac{-2}{\pi} x \cos(\frac{\pi x}{2})\Big|_0^1 - \int_0^1 \frac{-2}{\pi}\cos(\frac{\pi x}{2})\right] - 2\pi\left(\frac{1}{4}x^4\right)\Big|_0^1$

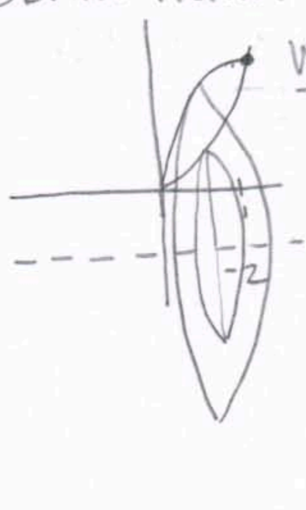
$= -4x\cos(\frac{\pi x}{2})\Big|_0^1 + \frac{8}{\pi}\sin(\frac{\pi x}{2})\Big|_0^1 - 2\pi\left(\frac{1}{4}\right)x^4\Big|_0^1$

$= 0 + 8/\pi - \frac{\pi}{2}$

$\boxed{= \frac{8}{\pi} - \frac{\pi}{2}}$

   (b) (4 points) Set up, BUT DO NOT EVALUATE, an integral to compute the volume of the solid obtained by rotating R about the horizontal line $y = -2$.

① Draw Picture



Washers

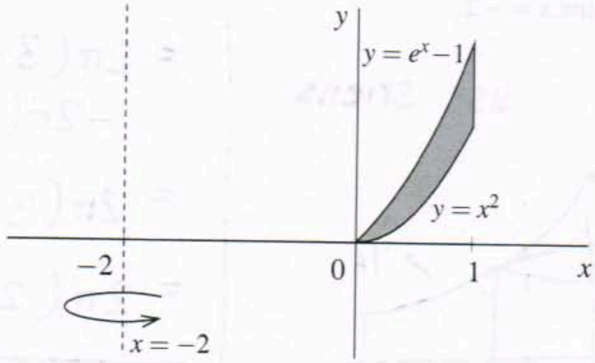
② Pts of inter
$x = 0, x = 1$

③

	$0 \le x \le 1$
Outer	$\sin(\frac{\pi x}{2}) + 2$
Inner	$x^2 + 2$

$$\boxed{④ \int_0^1 \pi\left[\left(\sin(\tfrac{\pi x}{2}) + 2\right)^2 - \left(x^2 + 2\right)^2\right] dx}$$

4. (8 total points) The region between $y = x^2$, $y = e^x - 1$, $x = 0$, and $x = 1$ is rotated about the vertical line $x = -2$ to form a solid.



(a) (4 points) Set up an integral for the volume of this solid using *CYLINDRICAL SHELLS*. *DO NOT EVALUATE THE INTEGRAL.*

Shells:

① Pts of inter

$x = 0$ and 1

② Chart

	$0 \le x \le 1$
Height	$e^x - 1 - x^2$
Radius	$x + 2$

③ Integral

$$\boxed{2\int_0^1 \pi\,(x+2)(e^x - 1 - x^2)\,dx}$$

(b) (4 points) Set up an integral (or integrals) for the volume of this solid using *WASHERS*. *DO NOT EVALUATE THE INTEGRAL(S).*

Washers: (Need to solve for x)

$\ln(y+1) = x$ and $\sqrt{y} = x$

① Points of intersection

$\underline{y=0}$ $x = 1 \Rightarrow \underline{y=1}$ and $y = \underline{e^1 - 1}$

② Chart

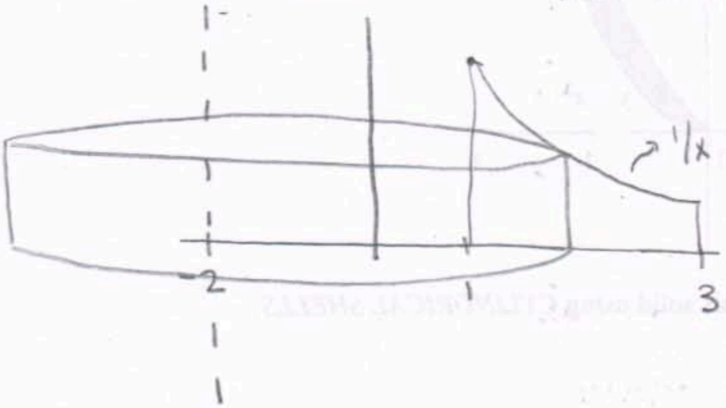
	$0 \le y \le 1$	$1 \le y \le e-1$
Outer	$\sqrt{y}+2$	$1+2$
Inner	$\ln(y+1)+2$	$\ln(y+1)+2$

③
$$\int_0^1 \pi\left([\sqrt{y}+2]^2 - [\ln(y+1)+2]^2\right)dy + \int_1^{e-1} \pi\left([1+2]^2 - [\ln(y+1)+2]^2\right)dy$$

7. (8 points) Let R be the region below the curve $y = \frac{1}{x}$, above the x-axis, and between the vertical lines $x = 1$ and $x = 3$. Set up and evaluate a definite integral for the volume of the solid obtained by rotating R about the vertical line $x = -2$.

① Draw Picture

use shells



② Points of intersection

$x=1,\ x=3$

③ Make chart

	$1 \le x \le 3$
Height	$1/x$
Radius	$x+2$

↑ for the -2

④ Integral

$$\int_1^3 2\pi(x+2)\left(\frac{1}{x}\right)dx$$

$$= \int_1^3 2\pi\left(1+\frac{2}{x}\right)dx$$

$$= 2\pi(x+2\ln|x|)\Big|_1^3$$

$$= 2\pi(3+2\ln(3)) - 2\pi(1+2\ln(1))$$

$$= 2\pi(3+2\ln(3)-1)$$

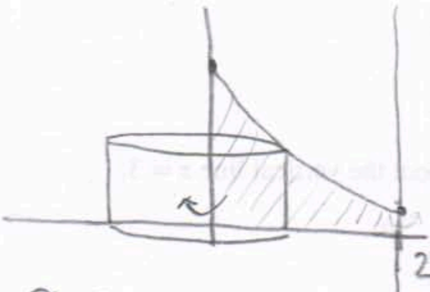
$$= 2\pi(2+2\ln(3))$$

$$\boxed{= 4\pi(1+\ln(3))}$$

7. (10 total points)

(a) (6 points) The region in the first quadrant bounded by the x-axis, the y-axis, the line $x = 2$, and the graph of $y = \dfrac{1}{1+x^2}$ is rotated around the y-axis to form a solid of revolution. Find the volume of this solid.

① Picture



Use Shells

② Points of intersection

$x = 0, x = 2$

③ Chart

	$0 \le x \le 2$
Height	$\frac{1}{1+x^2}$
Radius	x

↑ rotate about y-axis

④ Integral

$$2\pi \int_0^2 \left(\frac{1}{1+x^2}\right) x \, dx$$

$u = 1 + x^2 \quad du = 2x\,dx$

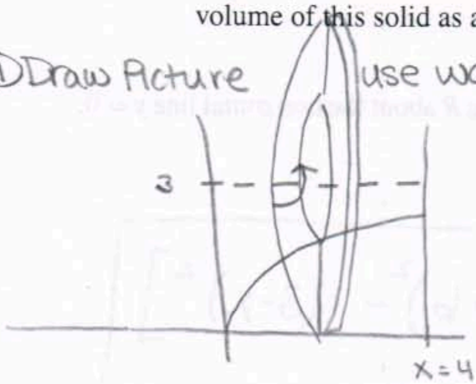
$$= \pi \int_1^5 \frac{1}{u} du$$

$$= \pi \ln|u| \Big|_1^5$$

$$\boxed{= \pi \ln(5)}$$

(b) (4 points) The region in the first quadrant bounded by the x-axis, the line $x = 4$, and the graph of $y = \sqrt{x}$ is rotated around the *horizontal* line $y = 3$ to form a solid of revolution. Express the volume of this solid as a definite integral, but DO NOT EVALUATE THIS INTEGRAL.

① Draw Picture



use washers

④ Integral

$$\boxed{\pi \int_0^4 \left(3^2 - [3 - \sqrt{x}]^2\right) dx}$$

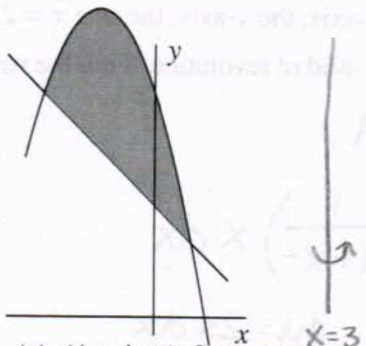
② Points of intersection

$x = 0, x = 4$

③ Make chart

	$0 \le x \le 4$
Outer	3
Inner	$3 - \sqrt{x}$

7. (8 total points) Let R be the region bounded by $y = -x^2 - 3x + 6$ and $x + y - 3 = 0$; see the picture.



(a) (4 points) Set up an integral for the volume obtained by rotating R about the vertical line $x = 3$. DO NOT EVALUATE THE INTEGRAL.

Use shells

Solve for y=...

$y = -x^2 - 3x + 6$

$x + y - 3 = 0$

$\Rightarrow y = 3 - x$

① Pts of intersection

$3 - x = -x^2 - 3x + 6$

$0 = -x^2 - 2x + 3$

$0 = (-x - 3)(x - 1)$

$x = -3$ or 1

② Make Chart

	$-3 \le x \le 1$
Height	$-x^2 - 3x + 6 - [3 - x]$
Radius	$3 - x$

③ Write Integral

$$\boxed{\int_{-3}^{1} 2\pi(3 - x)\left(-x^2 - 3x + 6 - [3 - x]\right)dx}$$

(b) (4 points) Set up an integral for the volume obtained by rotating R about the horizontal line $y = 0$. DO NOT EVALUATE THE INTEGRAL.

Use Washers

① Points of inters same as above $x = -3$ or 1

② Make chart

	$-3 \le x \le 1$
Outer	$-x^2 - 3x + 6$
Inner	$3 - x$

③ Write Integral

$$\boxed{\int_{-3}^{1} \pi\left[(-x^2 - 3x + 6)^2 - (3 - x)^2\right]}$$