7. (8 total points)

(a) (4 points) Set up but DO NOT EVALUATE an integral to compute the arc length of the curve $y = \sin^2(\pi x)$, for $0 \leq x \leq 1$.

arc length $= \int_a^b \sqrt{1+(dy/dx)^2}$

① Find dy/dx

$$\frac{dy}{dx} = 2\sin(\pi x)\cos(\pi x)\cdot\pi$$

② Set up integral

$$\boxed{\text{arc length} = \int_0^1 \sqrt{1+(2\pi\sin(\pi x)\cos(\pi x))^2}\,dx}$$

(b) (4 points) Approximate the length of the above curve via Simpson's rule with $n = 4$. SIMPLIFY THE SUM, but LEAVE YOUR ANSWER IN EXACT FORM.

① Find plug in pts

$\Delta x = \frac{1-0}{4} = \frac{1}{4}$

0   1/4   1/2   3/4   1

$$\therefore \frac{1/4}{3}\left[f(0)+4f(1/4)+2f(1/2)+4f(3/4)+f(1)\right]$$

$$= \frac{1}{12}\left[1+4\sqrt{1+\pi^2}+2(1)+4\sqrt{1+\pi^2}+1\right]$$

$$= \boxed{\frac{1+4\sqrt{1+\pi^2}}{3}}$$

8. (8 points) The odometer on your car is broken. However, you occasionally checked the speedometer during an 8 hour trip and obtained the data below. Use the trapezoidal rule to estimate the distance traveled.

time (in hours)	0	2	4	6	8
speed (in mph)	50	58	66	62	61

Trapezoid rule

$$= \frac{2}{2}\left[f(0) + 2f(2) + 2f(4) + 2f(6) + f(8)\right]$$

$$= \left[50 + 2(58) + 2(66) + 2(62) + 61\right]$$

$$= \boxed{483 \text{ miles}}$$

8. (8 total points)

   (a) (4 points) Set up but DO NOT EVALUATE an integral to compute the arc length of the curve $y = x^3$, for $0 \le x \le 1$.

$$\text{arc length} = \int_a^b \sqrt{1 + (dy/dx)^2}\, dx$$

① $dy/dx = 3x^2$

$$\boxed{\therefore \text{ arc length} = \int_0^1 \sqrt{1 + (3x^2)^2}\, dx}$$

   (b) (4 points) Approximate the length of the above curve using the trapezoidal rule with $n = 5$. Do not simplify the sum: leave your answer in exact form.

① $\Delta x = \frac{b-a}{5} = \frac{1-0}{5} = \frac{1}{5}$

0 1/5 2/5 3/5 4/5 1

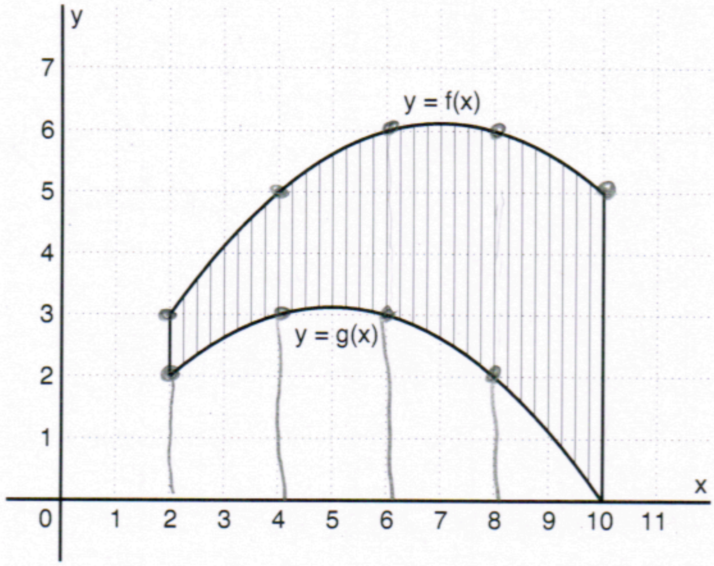
Trap. Rule

$$= \frac{1/5}{2}\left[f(0) + 2f(\tfrac{1}{5}) + 2f(2/5) + 2f(3/5) + 2f(4/5) + f(1)\right]$$

$$= \frac{1}{10}\left[1 + 2\sqrt{1+9(\tfrac{1}{625})} + 2\sqrt{1+9(16/625)} + 2\sqrt{1+9(81/625)} + 2\sqrt{1+9(256/625)} + \sqrt{10}\right]$$

6. (10 points) Set up an integral (in terms of $f(x)$ and $g(x)$) for the volume of the solid of revolution that is obtained by rotating the region shown below around the x-axis. Then use Simpson's rule with $n = 4$ subintervals to approximate the volume. Show your work, and give your final answer as a decimal.

Use Washers



① The Volume is

$$\int_2^{10} \pi\left(f(x)^2 - g(x)^2\right)dx$$

② Simpson's Rule

$$\Delta x = \frac{10-2}{4} = 2$$

$$= \frac{\Delta x}{3}\left[f(2) + 4f(4) + 2f(6) + 4f(8) + f(10)\right]$$

$$= \frac{2}{3}\pi\left[(3^2-2^2) + 4[5^2-3^2] + 2[6^2-3^2] + 4[6^2-2^2] + [5^2-0^2]\right]$$

$$= \frac{2\pi}{3}\left[5 + 64 + 54 + 128 + 25\right]$$

$$\boxed{= 184\pi}$$