3. (10 points) Find the derivative of the function

$$F(x) = \int_{\ln x}^{\cos(x)} \frac{e^t}{1-\tan(t)}\,dt.$$

① Rewrite integral so that constant on bottom and function on top.

$$\int_{\ln(x)}^{\cos(x)} \frac{e^t}{1-\tan t}\,dt = \int_{3}^{\cos(x)} \frac{e^t}{1-\tan t}\,dt - \int_{3}^{\ln(x)} \frac{e^t}{1-\tan(t)}$$

② Apply $g'(x) = f(h(x)) \cdot h'(x)$

$$\boxed{F'(x) = \frac{e^{\cos x}}{1-\tan(\cos x)} \cdot (-\sin x) - \frac{e^{\ln(x)}}{1-\tan(\ln(x))} \cdot \frac{1}{x}}$$

3. (8 points) Find a continuous function $f(x)$ and a number $a > 0$ such that $16 + \int_a^x t^2 f(t)\,dt = x^4$.
(*Hint: Differentiate both sides.*)

① Follow hint and differentiate both sides

$$0 + \frac{d}{dx}\int_a^x t^2 f(t)\,dt = 4x^3$$

a). Rewrite integral: Don't need to b/c already in form.

b). Use FTC part 1,

$$\frac{d}{dx}\int_a^x t^2 f(t)\,dt = x^2 f(x)$$

Hence $0 + x^2 f(x) = 4x^3 \Rightarrow \boxed{f(x) = 4x}$

② Plug back $f(x) = 4x$ into the integral

$$16 + \int_a^x t^2 f(t)\,dt = x^4 \Rightarrow 16 + \int_a^x t^2 \cdot 4t\,dt = x^4$$

③ Integrating yields

$$\Rightarrow 16 + t^4 \Big|_a^x = x^4$$

$$\Rightarrow 16 + x^4 - a^4 = x^4$$

$$\Rightarrow 16 = a^4$$

$$\Rightarrow \boxed{a = 2}$$

[2] (6 points) Compute the integral $\int_{-1}^{3} \left|4t - t^3\right| \, dt$.

DON'T DO YET

[3] (8 points) Let $f(x) = \int_{x^2}^{9} \cos\left(\pi\sqrt{t}\right) \, dt$. Compute the equation of the tangent line to $y = f(x)$ at the point where $x = 3$.

Recall, to find an eq. of line. Need slope and pt. You don't need just the derivative, you want the slope at $x=3$.

① Find slope

a). ReWRite integral

$$\int_{x^2}^{9} \cos(\pi\sqrt{t})\,dt = -\int_{9}^{x^2} \cos(\pi\sqrt{t})\,dt$$

b). Apply $g'(x) = f(h(x)) \cdot h'(x)$

$$f'(x) = -\cos(\pi\sqrt{x^2}) \cdot 2x$$

② Find slope at 3

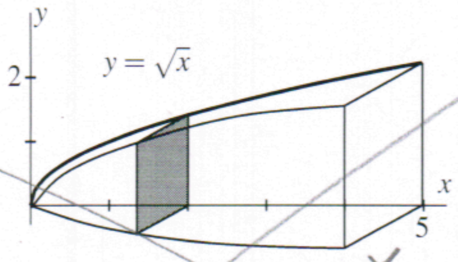
$$f'(3) = -\cos(3\pi) \cdot 2(3) = 6$$

③ Find pt. If $x=3$ then $f(3) = \int_9^9 \cos(\pi\sqrt{t})\,dt = 0$ since bounds are from 9 to 9

④ WRite eq. of line

$$y = f'(a)(x-a) + f(a) \Rightarrow \boxed{y = 6(x-3) + 0}$$

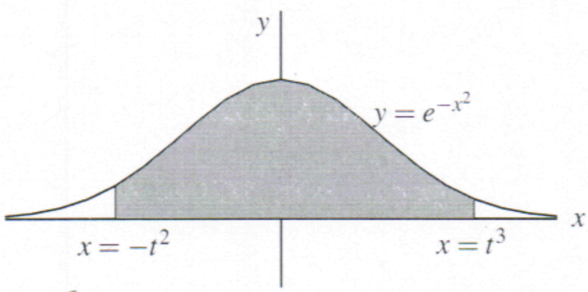
5. (4 points) Find the volume of the solid shown below. Each cross-section (slice) is a square.



DON'T DO

WE WILL DO THIS LATER

6. (8 points) At each time $t \geq 0$, $\mathscr{R}_t$ is the region above the x-axis, below the curve $y = e^{-x^2}$, with left side on the line $x = -t^2$ and right side on the line $x = t^3$ (see the figure). Let $A(t)$ be the area of $\mathscr{R}_t$. Find $\dfrac{dA}{dt}$ at time $t = 1$.



Note that $A(t) = \int_{-t^2}^{t^3} e^{-x^2}\,dx$ from the problem statement.

① Rewrite integral

$$\int_{-t^2}^{t^3} e^{-x^2}dx = \int_0^{t^3} e^{-x^2} - \int_0^{-t^2} e^{-x^2}$$

② Apply FTC part 1,

$$A'(t) = e^{-t^6} \cdot 3t^2 + e^{t^4} \cdot 2t$$

③ Find $A'(1)$ [Question is asking]

$$\boxed{A'(1) = e^{-1} \cdot 3 + e^{1} \cdot 2 = \frac{3}{e} + 2e}$$

3. (7 points) Find the interval (or intervals) on which the curve

$$y = \int_{2}^{x^2-x} (1+\sin^2(t))\,dt$$

is increasing.

Recall, to find where a function is increasing,
Find where $y' > 0$.

FIND DERIVATIVE

① Rewrite integral
(Don't Need to b/c already in form)

② Use FTC part 1,

$$y'(x) = [1+\sin^2(x^2-x)](2x-1)$$

FIND WHERE (The x values) the function is INCREASING

① Set $y'(x)=0$

$$0 = [1+\sin^2(x^2-x)](2x-1)$$

Since $1+\sin^2(x^2-x) \neq 0$, then $0 = 2x-1 \Rightarrow \boxed{x = 1/2}$

② Chose pts around 1/2 and see if $y' > 0$

	$-\infty < x < \frac{1}{2}$	$x = 1/2$	$1/2 < x < \infty$
y'	$-$	0	$+$

If $x=0$, $y'(0) = [1+\sin^2(0)][-1]$
$= 1(-1) = -1$

If $x=1$, $y'(1) = [1+\sin^2(0)][1]$
$= [1](1) = 1$

$\boxed{\text{Hence, } y' \text{ is increasing on the interval } [1/2, \infty)}$

5. (8 points) Find
$$\lim_{x\to 0} \frac{\int_0^x \cos(t^2)\,dt}{x}.$$

Recall, $\lim_{x\to a} \frac{f(x)-f(a)}{x-0} = f'(0)$

Hence $\lim_{x\to 0} \frac{\int_0^x \cos(t^2) - \int_0^0 \cos(t^2)}{x-0} = \lim_{x\to 0} \frac{\int_0^x \cos(t^2)dt}{x}$

= the derivative of $\int_0^x \cos(t^2)\,dt$ at $x=0$

① Rewrite integral: Already in good form

② Use FTC part 1,

Let $g(x) = \int_0^x \cos(t^2)\,dt$

$g'(x) = \cos(x^2)$

③ Find $g'(0)$

$\boxed{g'(0) = \cos(0^2) = 1}$