5. A ball is dropped from a window located 64 ft above the ground. Assume that the gravitational acceleration of the ball is $-32\,\text{ft/sec}^2$.

(a) (4 points) Suppose that the ball is released with no initial vertical velocity. Find the time of the impact and the vertical velocity of the ball at the time of impact.

① Find $a(t)$, $v(t)$, $s(t)$ by computing antiderivatives

$a(t) = -32$

$v(t) = -32t + C$

$s(t) = -16t^2 + Ct + D$

② Find C and D by plugging in given conditions

Observe ⓐ Ball dropped from window located at 64ft

$\Rightarrow s(0) = 64$ at time 0 ball located at height of 64

ⓑ No initial velocity $\Rightarrow$ at time 0 ball has no velocity

$v(0) = 0$

$0 = -32(0) + C \Rightarrow C = 0$

$64 = -16(0)^2 + 0 \cdot 0 + D \Rightarrow D = 64$

$\therefore a(t) = -32,\ v(t) = -32t,\ s(t) = -16t^2 + 64$

③ Answer questions

Impact $\Rightarrow$ height $= 0$

so $0 = s(t) = -16t^2 + 64$

$t = \pm 2$

(b) (6 points) What initial vertical velocity should be given to the ball so that the ball hits the ground in half the time computed in the first part of the problem?

For velocity,

$v(2) = -32(2) = -64$ ft/sec

① Write $a(t)$, $v(t)$, $s(t)$

$a(t) = -32$

$v(t) = -32t + C$

$s(t) = -16t^2 + Ct + D$

② Plug in given initial conditions to find C and D

→ know $s(0) = 64$ (don't know inital vel.)

$\Rightarrow s(0) = -16(0)^2 + C(0) + D = 64$

$\Rightarrow D = 64$.

Also know that $s(1) = 0$ (ball hits ground in ½ time)

$\Rightarrow 0 = -16(1^2) + C(1) + 64 = 0 \Rightarrow C = -48$

$\therefore a(t) = -32,\ v(t) = -32t - 48,\ s(t) = -16t^2 - 48t + 64$

For initial velocity want $v(0) \Rightarrow v(0) = -32(0) - 48 = -48$ ft/sec