

Your Name

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Your Signature

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Student ID #

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Quiz Section

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Professor's Name

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TA's Name

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- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.
- A scientific calculator is needed, but no calculator with graphing, programming, symbolic manipulation, or calculus capabilities is allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text (p. 484 if you have the 6th edition of Stewart) without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet.**
- Place

a box around your answer

 to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Question	Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. (10 total points) Evaluate the following indefinite integrals.

(a) (5 points) $\int \frac{1}{x^2 \sqrt{25 - 9x^2}} dx$

(b) (5 points) $\int \frac{x+1}{x^3 + 4x^2 + 5x} dx$

2. (10 total points)

(a) (5 points) Evaluate the definite integral $\int_0^4 \frac{1}{(1 + \sqrt{x})^3} dx$. Give your answer in exact form.

(b) (5 points) Find the average value of the function $f(x) = \frac{\ln(2x)}{(2x)^{1/3}}$ on the interval $\left[\frac{1}{2}, 32\right]$.
Give your answer in exact form.

3. (10 points) Does the improper integral $\int_{-3}^{\infty} xe^{-x} dx$ converge or diverge? Justify your answer. If it converges, evaluate the integral and give your answer in exact form.

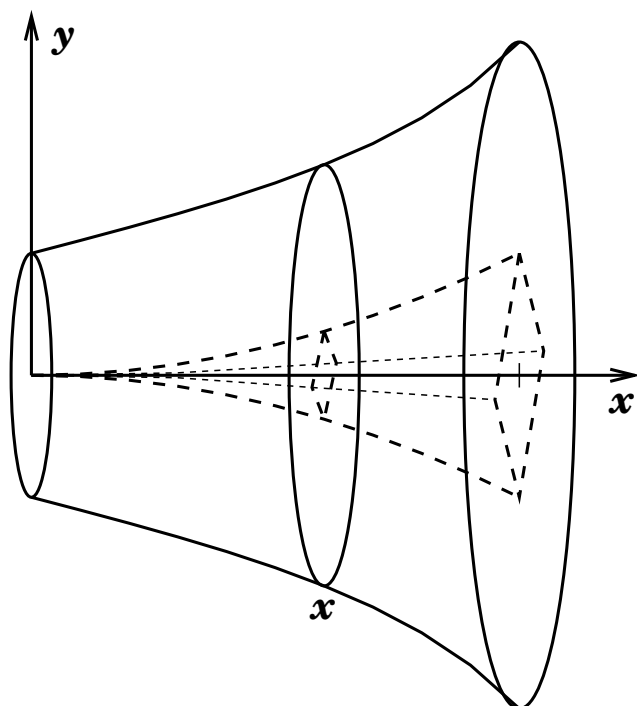
4. (10 total points) Starting at time $t = 0$, a car moves along the x -axis with velocity

$$v(t) = 20t^2 - 30t \text{ miles/hr,}$$

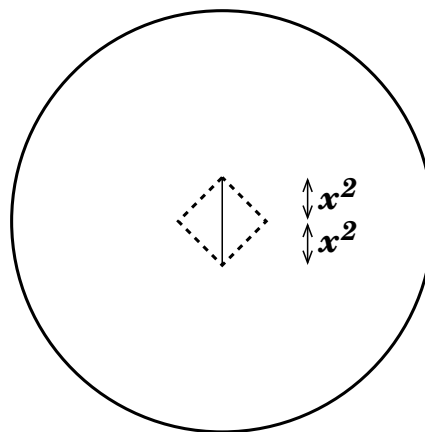
where t is time in hours. Answer the following questions. Include units in your answers.

- (a) (5 points) At time $t = 3$ hours, how far is the car from its starting location?
- (b) (5 points) Unfortunately, the car's gas mileage is only 6 miles per gallon. How many total gallons of gas are used from time $t = 0$ to time $t = 3$ hours?

5. (10 points) The region R is bounded by the curve $y = 1 + \tan\left(\frac{\pi}{3}x\right)$, the lines $x = 0$ and $x = 1$, and the x -axis. A solid is obtained by rotating R around the x -axis and then carving out a hole. For $0 \leq x \leq 1$ the vertical cross-section of the hole at x is a square whose diagonal joins the points (x, x^2) and $(x, -x^2)$. Find the volume of the solid. Give your answer in exact form.



cross-section at x



6. (10 total points) Two laborers dig a hole in the ground 10 feet deep, shoveling the dirt up to the top of the hole. The horizontal cross-section of the hole is a rectangle of length 8 feet and width 3 feet. The dirt weighs 100 pounds per cubic foot.
- (a) (5 points) What is the total work (in ft-lb) done by the two laborers?
(The dirt removed from the hole is cleared away by other laborers.)

- (b) (5 points) Suppose the first laborer digs the hole part of the way, and then the second laborer finishes digging the hole. How deep (in ft) should the first laborer dig in order to do half of the total work? Give your answer in decimal form with at least three digits after the decimal point.

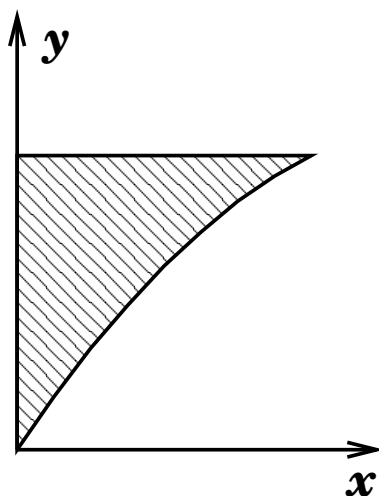
7. (10 total points) Consider the Lissajous curve given by the parametric equations

$$\begin{cases} x = \cos(5t), \\ y = \sin(2t). \end{cases}$$

(a) (5 points) Set up a definite integral for the arc length of the part of the curve for $0.10 \leq t \leq 0.25$.
DO NOT EVALUATE THE INTEGRAL.

(b) (5 points) Use the Trapezoid Rule with $n = 3$ subintervals to approximate the definite integral in part (a). Give your answer in decimal form with at least four digits after the decimal point.

8. (10 points) Consider a uniform flat plate in the first quadrant that is bounded by the y -axis, the line $y = 1/2$, and the curve $y = \sin x$. Find the y -coordinate \bar{y} of the center of mass of this plate. Give your answer in exact form.



9. (10 points) Find the solution of the initial value problem

$$y y' = e^{2x}(1 + y^2), \quad y(0) = -1.$$

Give your answer in the form $y = f(x)$.

10. (10 total points) A 167°F cup of coffee is brought into a 59°F room. After two minutes, the temperature of the coffee is 149°F . Assume the temperature of the room is constant, and that the rate of change of the temperature of the coffee is proportional to the difference between the temperature of the coffee and the temperature of the room.

(a) (3 points) Set up a differential equation for the temperature $T(t)$ of the coffee (in $^{\circ}\text{F}$) as a function of time t (in minutes) after it has been brought into the room.

(b) (5 points) Find $T(t)$ by solving the differential equation in part (a). Use separation of variables and solve for any unknown constant(s).

SHOW ALL OF YOUR STEPS. DO NOT QUOTE A FORMULA.

(c) (2 points) Find $T(4)$. Give your answer in exact form and simplify your answer.