Area Between Curves

Math 125

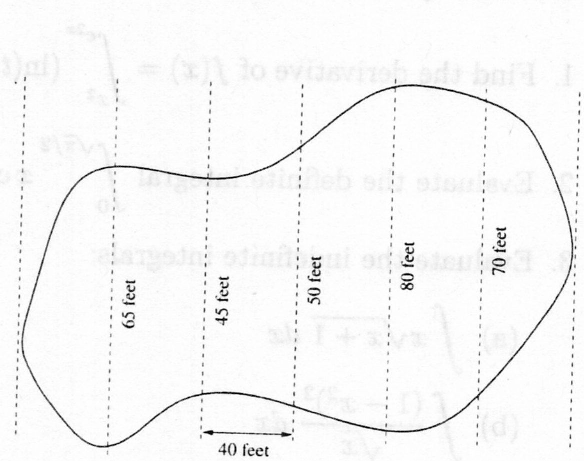
Name ______________________

Quiz Section ______________

In this work sheet we'll study the problem of finding the area of a region bounded by curves. We'll first estimate an area given numerical information. The we'll use calculus to find the area of a more complicated region.

The Lake

1 The widths, in feet, of a small lake were measured at 40 foot intervals. Estimate the area of the lake.



$L_6 = 40(0+65+45+50+80+70) = 12{,}400 \ (ft^2)$

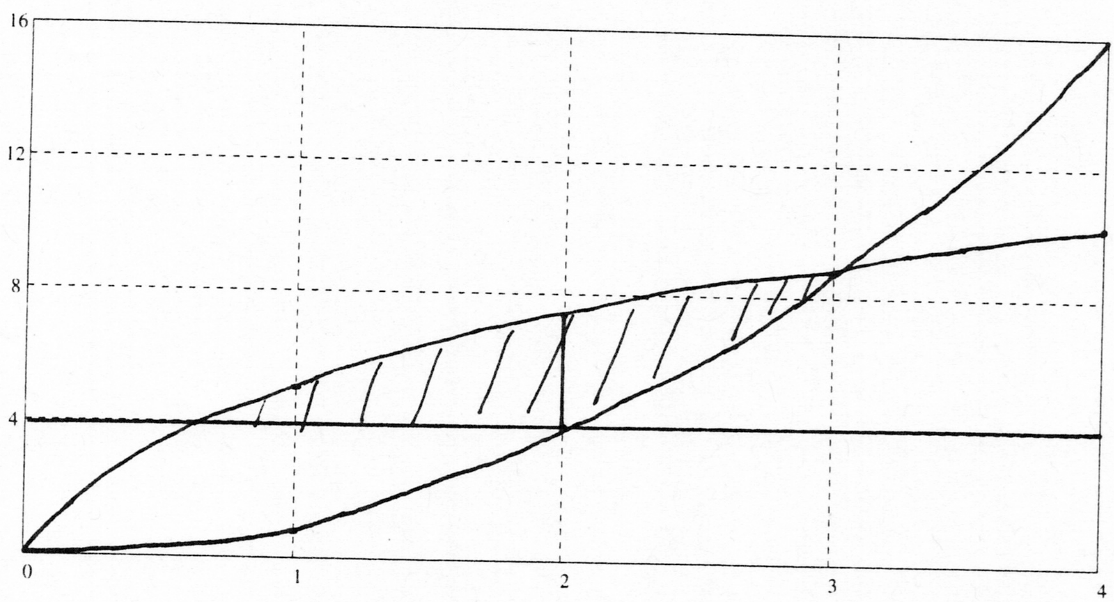
$R_6 = 40(65+45+50+80+70+0) = 12{,}400 \ (ft^2)$

[OR:

$M_3 = 80(65+50+70) = 14{,}800 \ ft^2$]

Area Bounded by Three Curves

2 On the grid below sketch the graphs of $y = 4$, $y = x^2$ and $y = \sqrt{27x}$. (The last one is just a piece of a sideways parabola).



3 Shade the "triangular" region bounded by the graphs of the three functions that lies above the horizontal line.

4 Compute the x-coordinate of the left endpoint of the region.

$$\sqrt{27x} = 4$$

$$27x = 16$$

$$x = \frac{16}{27} \approx 0.59259$$

5 Compute the x-coordinate of the right endpoint of the region.

$$x^2 = \sqrt{27x}$$

$$x^4 = 27x$$

$$x^3 = 27$$

$$x = 3$$

6 Note that the top of the region consists of a single curve, but the bottom of the region consists of two different curves. Find the x-coordinate where these two curves meet.

$$x^2 = 4$$

$$x = 2 \quad (\text{as } x > 0)$$

7 Sketch in a vertical line at the x-coordinate you found in the last problem. This divides the region into two smaller sub-regions.

8 Compute the area of the left sub-region.

$$\int_{16/27}^{2} \sqrt{27} \cdot x^{1/2} - 4 \, dx = 2\sqrt{3}\, x^{3/2} - 4x \Big|_{16/27}^{2}$$

$$= (4\sqrt{6} - 8) - \left(\frac{2^7}{3^4} - \frac{2^6}{3^3} \right)$$

$$= 4\sqrt{6} - 8 + \frac{64}{81}$$

$$= 4\sqrt{6} - \frac{584}{81}$$

9 Compute the area of the right sub-region. Add the two areas together to get the total area.

Right: $\int_2^3 \sqrt{27}\cdot x^{1/2} - x^2\, dx = 2\sqrt{3}\, x^{3/2} - \frac{x^3}{3} \Big|_2^3$

$= \left(18 - \frac{27}{3}\right) - \left(4\sqrt{6} - \frac{8}{3}\right) = \frac{35}{3} - 4\sqrt{6}$

Left + Right: $\left(4\sqrt{6} - \frac{584}{81}\right) + \left(\frac{35}{3} - 4\sqrt{6}\right) = \frac{361}{81}$

10 Recompute the area using the following trick. Solve for x as a function of y in the two non-constant functions. Find the area by integrating with respect to y. Is this easier?

on left: $x = y^2/27$ on right: $x = y^{1/2}$

$\int_4^9 y^{1/2} - y^2/27\, dy = \frac{2}{3} y^{3/2} - \frac{y^3}{81} \Big|_4^9$

$= (18 - 9) - \left(\frac{16}{3} - \frac{64}{81}\right)$

$= \frac{361}{81}$