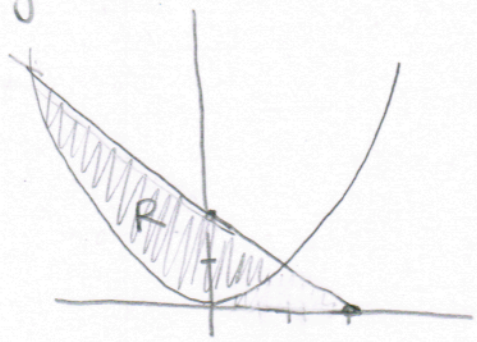
Problem #1:

Find the area of R

① graph the region:



② Find Pts of intersection

$$x^2 = 2 - x \Rightarrow x^2 + x - 2 = 0 \Rightarrow (x+2)(x-1) = 0$$

$$x = 1 \text{ and } -2$$

③ Set-up Chart

	$-2 \le x \le 1$
Top	$2 - x$
Bottom	x^2

④ Write Integral and solve

$$A = \int_{-2}^{1} \text{Top} - \text{bott} = \int_{-2}^{1} 2 - x - x^2 \, dx$$

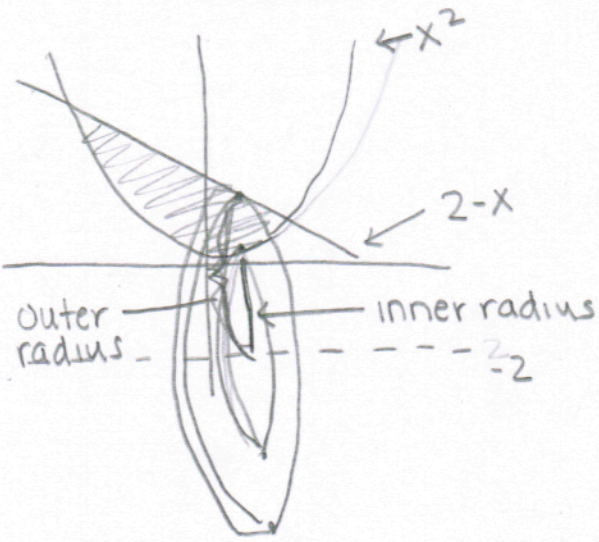
$$= 2x - \tfrac{1}{2}x^2 - \tfrac{1}{3}x^3 \Big|_{-2}^{1}$$

$$= \tfrac{7}{6} - (-10/3) = \boxed{9\tfrac{1}{2}}$$

Find volume rotating about $y = -2$

① graph

Note rotating horizontally and $y = x^2$; $y = 2 - x$ so use Washers



② Find pts of intersection

$x^2 = 2 - x \Rightarrow x = -2$ and 1

③ Make Chart

	$-2 \le x \le 1$
Outer	$2 - x + 2$
Inner	$x^2 + 2$

④ Make Integral

$$\pi \int R^2 - r^2 = \pi \int_{-2}^{1} (4-x)^2 - (x^2+2)^2 \, dx$$

$$= \pi \int_{-2}^{1} -x^4 - 3x^2 - 8x + 12 \, dx$$

$$= \pi \left(\tfrac{-1}{5}x^5 - x^3 - 4x^2 + 12x \right) \Big|_{-2}^{1}$$

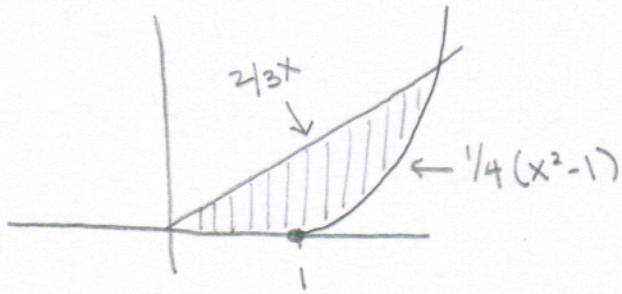
$$= [6.8 - (-25.6)]\,\pi$$

$$\boxed{= 162\pi/5}$$

Problem 2

a). Evaluate Area w/ respect to x

① Graph: Note everything is in x



② Find pts of intersection

- $\frac{2}{3}x = \frac{1}{4}(x^2-1)$

  $x = x = 3$

- $x = 0$ (from graph)
- $\frac{1}{4}(x^2-1) = 0 \Rightarrow x = 1$

③ Make Chart

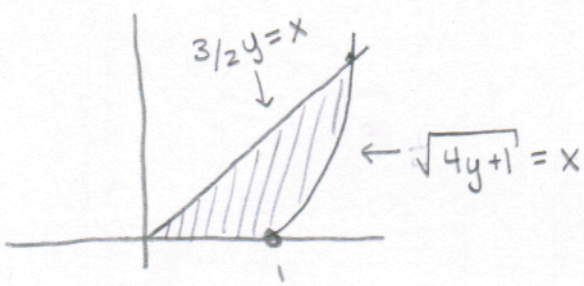
	$0 \le x \le 1$	$1 \le x \le 3$
Top	$\frac{2}{3}x$	$\frac{2}{3}x$
Bott.	0	$\frac{1}{4}(x^2-1)$

④ Make Integrals

$$\int_0^1 \frac{2}{3}x + \int_1^3 \frac{2}{3}x - \frac{1}{4}(x^2-1)\,dx$$

b). Evaluate Area w/ respect to y

① graph: Note everything in y



② Find pts of intersection

$$3/2y = \sqrt{4y+1} \Rightarrow y = 2$$

$y = 0$ from graph

③ Make chart

	$0 \le y \le 2$
Right	$\sqrt{4y+1}$
Left	$3/2\,y$

④ Write integrals

$$\int_0^2 \sqrt{4y+1} - 3/2\,y \, dy$$

c). Solve one of them

$$\underbrace{\int_0^2 \sqrt{4y+1}}_{①} - \underbrace{\int_0^2 \frac{3}{2}y}_{②}$$

① $u = 4y+1$

$\frac{du}{4} = dy \Rightarrow$ $\int_0^2 \sqrt{4y+1} = \int_1^9 \frac{1}{4}u^{1/2}\,du = \frac{2}{12}u^{3/2}\Big|_1^9 = \frac{9}{2} - \frac{1}{6}$

$= \boxed{\frac{13}{3}}$

② $-\int_0^2 \frac{3}{2}y\,dy = -\frac{3}{4}y^2\Big|_0^2 = \boxed{-3}$

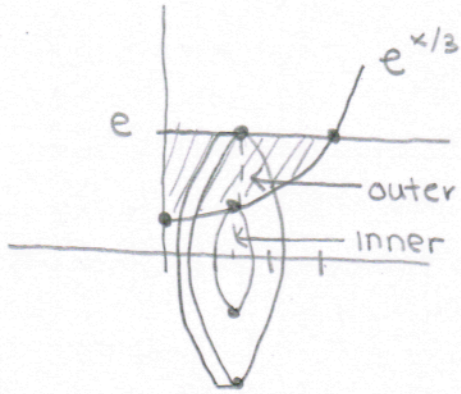
$\therefore \int_0^2 \sqrt{4y+1} - \frac{3}{2}y\,dy = \frac{13}{3} - 3 = \boxed{\frac{4}{3}}$

Problem 3:

a). Find the volume rotated about x-axis

① Graph: (Note rotating horizontal and $y=e^{x/3}$; $y=e$ use Washers) Draw Typical washer

* Everything should be in x



② Find pts of intersection

$e = e^{x/3}$ $\quad x = 3$

$x = 0$ from graph

③ Make chart

	$0 \le x \le 3$
outer	e
inner	$e^{x/3}$

④ Write integral

$$\pi \int_0^3 e^2 - e^{2x/3}\,dx = \pi \int_0^3 e^2\,dx - \pi \int_0^3 e^{2x/3}\,dx \qquad u = 2x/3,\ \frac{3du}{2} = dx$$

$$= \pi e^2 x \Big|_0^3 - \pi \int_0^2 \frac{3}{2} e^u\,du$$

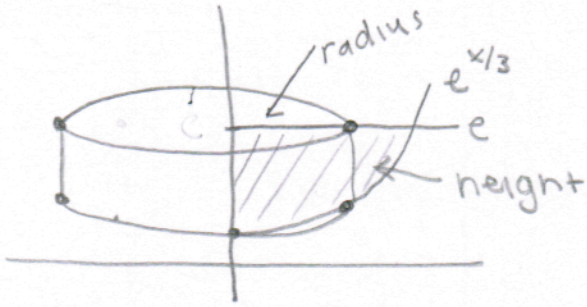
$$= \pi e^2 - \pi \frac{3}{2} e^u \Big|_0^2$$

$$= \pi e^2 - \frac{3\pi}{2} e^2 + \pi \frac{3}{2}$$

$$\boxed{= \frac{3\pi}{2} - \frac{\pi}{2} e^2}$$

b). Find the volume of solid rotated about y-axis.

① Graph: (Since rotating vertically and $y = e^{x/3}$ and e, then use shells). Note everything in x.



② Find pts of intersection

$x = 0$ (from graph)

$e = e^{x/3} \Rightarrow x = 3$

③ Make chart

	$0 \le x \le 3$
Height	$e - e^{x/3}$
radius	x

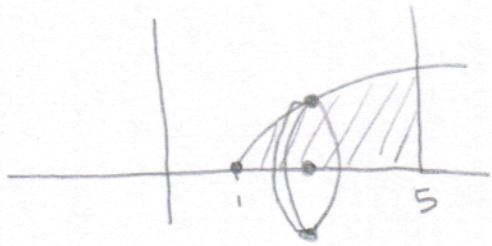
④ Write Integrals

$$2\pi \int_0^3 x(e - e^{x/3})\,dx$$

Problem 4

a). Find the volume rotated about x-axis.

① Graph (Note $y=\sqrt{x-1}$ and rotating horizontally, so use washers)
Everything in terms of x.



② Find pts of intersection: $x=1, x=5$ from chart

③ Make Chart:

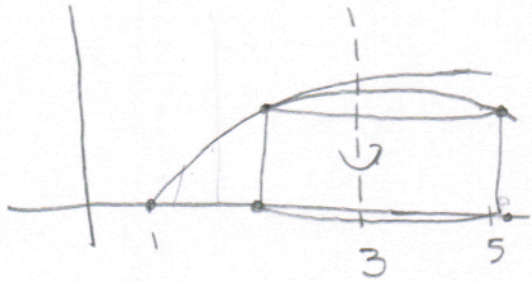
	$1 \le x \le 5$
Outer	$\sqrt{x-1}$
Inner	0

④ Write Integrals

$$\pi \int_1^5 (x-1)\,dx = \pi\left(\tfrac{1}{2}x^2 - x\right)\Big|_1^5 = \pi\left(\tfrac{15}{2} + \tfrac{1}{2}\right) = \boxed{\pi 8}$$

b). Find volume rotating about $x=3$

① graph (Note: $y=\sqrt{x-1}$ and rotating vertically, use shells)
Everything in terms of x.



② Find intersection pts: $x=1$ and $x=3$

③ Make chart

	$1 \le x \le 3$
Height	$\sqrt{x-1}$
radius	$3-x$

④ Write Integral

$$2\pi \int_1^3 (3-x)\sqrt{x-1}\,dx$$