Math 124 Final Examination Spring 2011

Print Your Name | Signature

Student ID Number | Quiz Section

Professor's Name | TA's Name

!!! READ...INSTRUCTIONS...READ !!!

1. Your exam contains 9 questions and 11 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 2 hours and 50 minutes for this final exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification.
4. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.
5. You are allowed one 8.5×11 sheet of handwritten notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed. Make sure your calculator is in radian mode.
6. Unless otherwise instructed, ALWAYS GIVE YOUR ANSWERS IN EXACT FORM. For example, $3\pi, \sqrt{2}, \ln(2)$ are in exact form; the corresponding approximations $9.424778, 1.4142, 0.693147$ are NOT in exact form.

Problem	Total Points	Score
1	12	
2	12	
3	10	
4	12	

Problem	Total Points	Score
5	12	
6	10	
7	12	
8	8	
9	12	
Total	100	

1. (12 points; 4pts each) Find the derivatives of the following functions. You do not have to simplify.

   (a) $f(x) = \sqrt{3+x} \cdot \sqrt[3]{5x^2 - 6}$

   (b) $f(x) = \left(e^x - \dfrac{2}{4x^3}\right)^3$

   (c) $f(x) = (\tan x)^{\ln x}$

2. (12 points) Given the curve

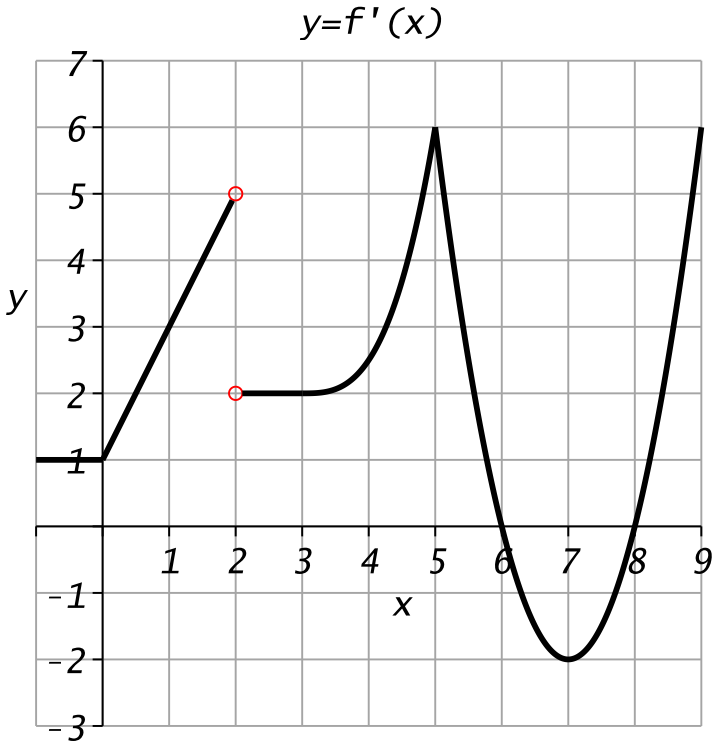
$$x^{2/3} + y^{2/3} = 5$$

answer the following.

(a) Verify that the point $(8, 1)$ is on the curve and find the equation of the tangent line to the curve at this point.

(b) Is the graph concave up or concave down at that point?

3. (10 points) The following is a graph of $y = f'(x)$, the *derivative* of $f(x)$. Given that $f(0) = 0$, answer the following questions. You do not need to explain your answers. Each part is worth 1 point with no partial credit.



(a) $\lim_{h\to 0} \frac{f(5+h)-f(5)}{h} =$

(b) $\lim_{x\to 0} f(x) =$

(c) Is the graph of $y = f(x)$ concave up or concave down at $x = 1$?

(d) $\lim_{h\to 0^+} \frac{f(2+h)-f(2)}{h} =$

(e) $\lim_{x\to 5} f'(x) =$

(f) $f''(1) =$

(g) $\lim_{x\to 0} \frac{f(x)}{x} =$

(h) What are the critical points of f in the domain $(-1, 9)$?

(i) Is f increasing or decreasing at $x = 7$?

(j) Is f positive or negative at $x = -\frac{1}{2}$?

4. (12 points) The length of a rectangle increases by 3 feet per minute while the width decreases by 2 feet per minute. When the length is 15 feet and the width is 8 feet, what is the rate at which the following changes. Make sure to state whether the rate is increasing or decreasing and include units.

(a) The area.

(b) The perimeter.

(c) The length of the diagonal.

5. (12 points) A particle is moving in the plane and traces out a curve with parametric equations:

$$x(t) = \sqrt{3}\cos(\pi t) \qquad y(t) = \sin(\pi t),$$

with $0 \leq t \leq 3$ seconds.

(a) Find the points on the curve where the slope is 1.

(b) On the given time interval, find the **last** time when the slope is 1.

6. (10 points) Use linear approximation to approximate the value of

$$9^{1/3} - 2.$$

Show all of your work. You will receive **no credit** for simply evaluating this using your calculator.

7. (12 points) Compute the limit. If it is correct to say that the limit is ∞ or $-\infty$, then say so. If the limit does not exist, explain why.

(a) $\lim_{x\to 0} \dfrac{\sin(3x)\,\sin(2x)}{x\sin(5x)}$

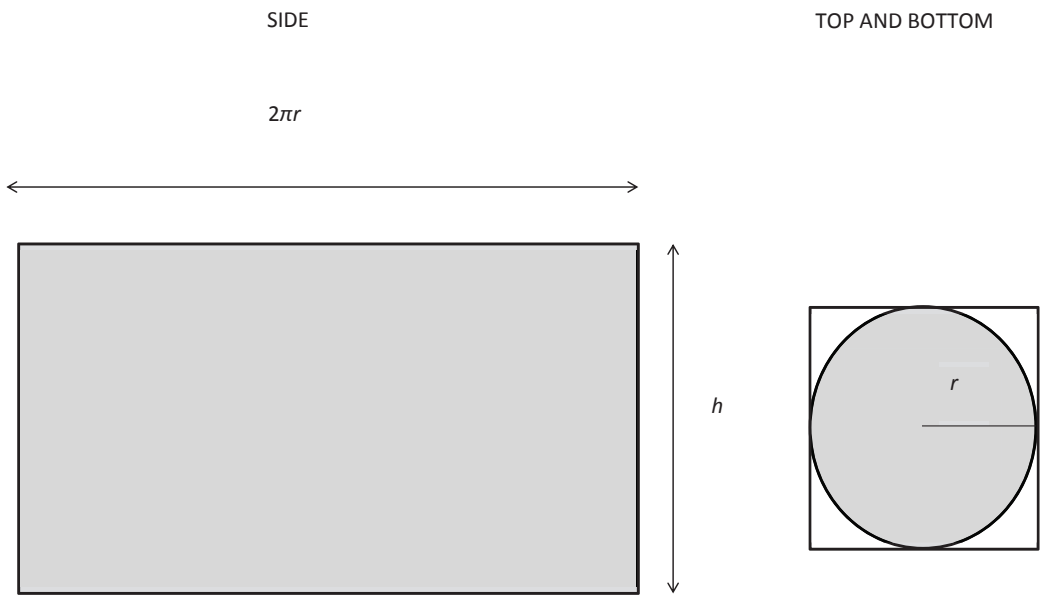
(b) $\lim_{x\to 2} \dfrac{e^{x^2}-e^4}{x-2}$

(c) $\lim_{t\to 0} \left(\dfrac{1}{t\sqrt{1+t}} - \dfrac{1}{t}\right)$

8. (8 points) For what values of a and b is the line $4x + y = b$ tangent to the parabola $y = ax^2$ when $x = 5$?

9. (12 points) A cylindrical can of volume 250 cubic centimeters is to be made from aluminum. The side is made from a thin sheet which costs 0.2 cents per square centimeter. The top and bottom of the can is made from a thicker sheet which costs 0.4 cents per square centimeter. Moreover, since the top and the bottom are circles, they have to be cut from square pieces. The wasted area between the circle and the square can be sold back to the aluminum supplier at a price of 0.1 cent per square centimeter to be recycled. What are the radius r and height h of the minimal cost can?

   Give your answer in exact form and as a decimal approximation. Note: Given a cylinder of radius r and height h, the volume is $V = \pi r^2 h$.



Use this page if you need more space on #9.