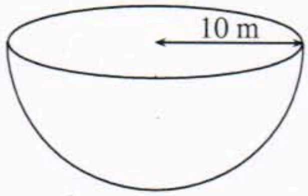
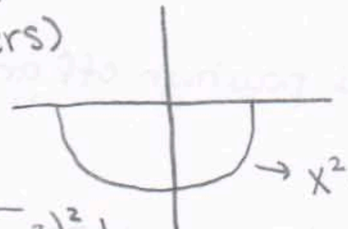
6. (10 points) A tank has the shape of an open-top hemisphere with radius 10 m that is full of water with density 1000 kg/m^3. Set up an integral which computes the work required to empty the tank by pumping all of the water to the top of the tank. DO NOT EVALUATE THIS INTEGRAL.



* This is Rotating Volume

1) Vol. of Slice
(use washers)



$\to x^2+y^2=100$

$\therefore \text{Vol} = \pi\left(\sqrt{100-y^2}\right)^2 dy$ $x=\sqrt{100-y^2}$

(2) Find mass

$m = 1000\pi(100-y^2)dy = \text{den}\cdot\text{vol}$

(3) Find Force

$F = m\cdot a = (9.8)(1000\pi)(100-y^2)dy$

(4) Find distance

At y height, have to move slice up y...

$\text{dist} = -y$

(5) Bounds; water moving

$y = -10$ to 0

(6) Set up int. and solve

$$\boxed{\int_{-10}^{0} (9.8)(1000\pi)(100-y^2)(-y)\,dy}$$

$$= \pi(9.8)(1000)\int_{-10}^{0} -100y + y^3\,dy$$

$$= \pi(9.8)(1000)\left[-50y^2 + \tfrac{1}{4}y^4\right]\Big|_{-10}^{0} =$$

$$= \pi(9.8)(1000)\left[50(100) - 2500\right]$$

$$\boxed{= \pi(24{,}500{,}000)}$$

7. (10 points) An 8 foot chain weighs 120 pounds. A large robot is holding one end of the chain 3 feet above the ground, so that 5 feet of the chain are on the ground. How much work must the robot do to lift this end of the chain from a height of 3 feet to a height of 10 feet?

① Forces at Work

- Rope

② Find mass as how high off grnd (y)

None

③ Find Force as how high off ground (y)

$$\frac{120 \text{ lbs}}{8 \text{ ft}} = 15 \text{ lbs/ft}$$

Then Force of Rope = $15y$

④ Bounds $y = 3$ to 10

⑤ dist = dy

⑥ Work Integral

$$W = \int_3^{10} 15y\,dy = \frac{15}{2}y^2 \Big|_3^{10} = \frac{15}{2}(100) - \frac{15}{2}(9) = \boxed{682.5}$$

7. (8 total points)

The electric force (in Newtons) acting on a charged particle A as a result of the presence of a second charged particle B is given by Coulomb's Law

$$F = \frac{k q_A q_B}{r^2},$$

where r is the distance (in meters) between the particles, q_A and q_B are the charges of A and B in Coulombs, and $k = 9 \times 10^9$ is a constant.

Assume that two particles A and B have opposite charges, with $q_A = 1$ Coulomb and $q_B = -1$ Coulomb. (The force F is negative, indicating that the particles are attracting each other.) Assume that particle A is kept fixed, and that the initial distance between the two particles is 1 meter.

(a) (4 points) Find the work done to move particle B from its initial position to a position 2 meters away from particle A.

- Force $= \frac{kq_Aq_B}{r^2} = \frac{k}{r^2}$ as $q_A=1, q_B=-1$
- Distance $= dr$
- Bounds (1 to 2)
- $W = \int_1^2 \frac{k}{r^2}dr = \left.\frac{-k}{r}\right|_1^2 = -k\left(\frac{1}{2}-1\right) = 9\times10^9\left(\frac{1}{2}\right) =$

$\boxed{=4.5\times10^9}$

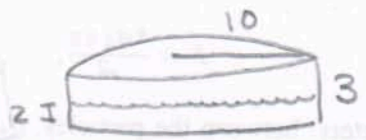
(b) (4 points) Find the work done to move particle B from its initial position to an infinite distance away from particle A.

$W = \int_1^\infty \frac{k}{r^2} = \lim_{b\to\infty}\int_1^b \frac{k}{r^2} = \lim_{b\to\infty} \left.\frac{-k}{r}\right|_1^b$

$= \lim_{b\to\infty} \frac{-k}{b} - \frac{-k}{1} = \lim_{b\to\infty} 9\times10^9\left(\frac{-1}{b}+1\right)$

$\boxed{= 9\times10^9}$

8. (8 points) A small circular pool has a radius of 10 ft, the sides are 3 ft high, and the depth of the water is 2 ft. How much work (in ft-lb) is required to pump all of the water out over the side of the pool? (Water weighs 62.5 lb/ft^3.)



① Volume of a slice
(Not rotating)
$\text{Vol.} = \pi(10^2)\,dy$

② Mass (Not in this case)

③ Force = m×a

$\text{Force} = 62.5(\pi)(100)\,dy$

④ distance (at height y how far to move water)

$\text{dist} = 3 - y$

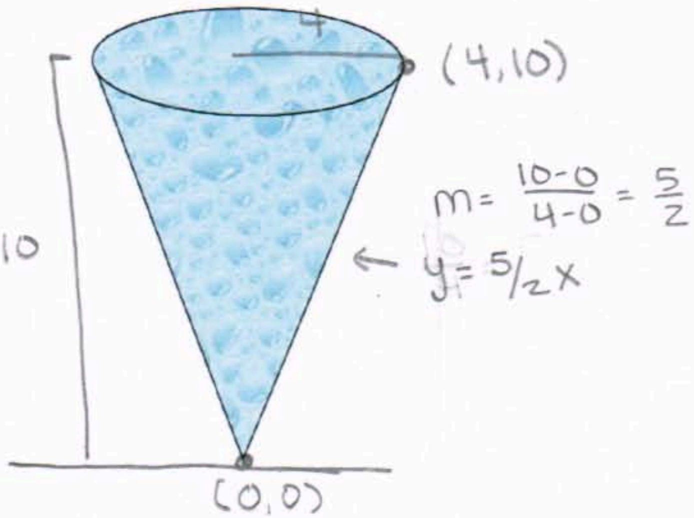
⑤ Bounds (water moving)

$y = 0$, to 2

⑥ Work

$$\text{Work} = \int_0^2 62.5\pi(100)(3-y)\,dy$$
$$= 62.5\pi(100)\left[3y - \tfrac{1}{2}y^2\right]\Big|_0^2$$
$$= 62.5\pi(100)[6-2]$$
$$\boxed{= 25{,}000\pi}$$

6. (10 points) A container has the shape of an inverted circular cone with height 10 feet and top radius 4 feet. It is filled with a liquid weighing 60 lb/ft^3. Find the work required to pump the top 5 feet of the liquid to the top of the container, and give your answer in decimal form.
Please label your origin and coordinate axis on the fi gure.



① Vol. of slice (Rotating)

$V = \pi (2/5 y)^2 dy$

② Find mass (Not in this case)

③ Find Force

Force = vol $\cdot$ 60 = $60\pi (4/25) y^2 dy$

④ Distance (at height y)

$(10-y)$ = distance

⑤ Bounds (water moving)

$y = 5$ to 10

↑ moving top 5 ft

⑥ Integral

$$\int_5^{10} \frac{240\pi}{25} y^2 (10-y)\, dy$$

$$\int_5^{10} \frac{240\pi}{25} (10y^2 - y^3)\, dy$$

$$= \frac{240\pi}{25} \left(\frac{10}{3}y^3 - \frac{1}{4}y^4\right)\Big|_5^{10}$$

$$= \frac{240\pi}{25} \left(\frac{10}{3}(1000) - \frac{1}{4}(10{,}000)\right)$$

$$- \frac{240\pi}{25} \left(\frac{10}{3}(125) - \frac{1}{4}(625)\right)$$

$$= \frac{240\pi}{25} \left(\frac{6875}{12}\right)$$

$$\boxed{= 5500\pi}$$