9. (10 points) Find the solution of the initial value problem

$$\frac{dy}{dt} = \frac{1}{ye^{t/2}}, \quad y(0) = -1.$$

Give your answer in the form $y = f(t)$.

1) Separate the variables

$$y\,dy = e^{-t/2}\,dt$$

2) Integrate both sides

$$\int y\,dy = \int e^{-t/2}\,dt$$

$$\frac{1}{2}y^2 = -2e^{-t/2} + C$$

3). Solve for y

$$y^2 = -4e^{-t/2} + C$$

$$y = \pm\sqrt{-4e^{-t/2} + C}$$

4). Solve for C

$$-1 = \pm\sqrt{-4 + C} \Rightarrow 1 = -4 + C \Rightarrow C = 5$$

$$\therefore \boxed{y = -\sqrt{-4e^{-t/2} + 5}}$$

9. (10 points) Find the solution of the initial value problem

$$\frac{dy}{dx} = \frac{(x+3)(y+2)}{x^2+9}, \quad y(0) = 10.$$

Give your answer in the form $y = f(x)$.

1). Separate the variables

$$\frac{1}{y+2}\,dy = \frac{x+3}{x^2+9}\,dx$$

2). Integrate both sides

$$\int \frac{1}{y+2}\,dy = \int \frac{x+3}{x^2+9}\,dx = \int \frac{x}{x^2+9}\,dx + \int \frac{3}{x^2+9}\,dx$$

$$\ln|y+2| \qquad u=x^2+9 \quad du=2x\,dx \qquad \arctan\left(\tfrac{x}{3}\right)$$

$$\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|x^2+9| + C$$

$$\Rightarrow \ln|y+2| = \frac{1}{2}\ln|x^2+9| + \arctan\left(\tfrac{x}{3}\right) + C$$

3). Solve for y

$$|y+2| = C\sqrt{x^2+9}\,e^{\arctan(x/3)}$$

$$y = C\sqrt{x^2+9}\,e^{\arctan(x/3)} - 2$$

4). Solve for coefficients

$$10 = C\sqrt{9}\,e^{\arctan(0/3)} - 2 = C\cdot 3e^0 - 2$$

$$4 = C$$

$$\Rightarrow \boxed{y = 4\left(\sqrt{x^2+9}\right)e^{\arctan(x/3)} - 2}$$

9. (10 points) Find the solution of the initial value problem

$$\frac{dy}{dt} = t\sin(t)\cos^2(y), \quad y(0) = \frac{\pi}{4}.$$

Give your answer in the form $y = f(t)$.

1). Separate the variables

$$\frac{1}{\cos^2 y}\,dy = t\sin(t)\,dt$$

2). Integrate both sides

$$\tan(y) = \int t\sin(t)\,dt \qquad u = t \quad dv = \sin(t)$$
$$du = dt \quad v = -\cos(t)$$

$$= -t\cos(t) + \int \cos(t)\,dt$$

$$= -t\cos(t) + \sin(t) + C$$

3). Solve for y

$$y = \arctan(-t\cos(t) + \sin(t) + C)$$

4). Solve for C

$$\frac{\pi}{4} = \arctan(C) \Rightarrow \tan\left(\frac{\pi}{4}\right) = C \Rightarrow C = \sqrt{2}/2$$

$$\boxed{\therefore\ y = \arctan\left(-t\cos(t) + \sin(t) + \sqrt{2}/2\right)}$$