Name________________________________

Math 125 **Second Midterm** **10:00 a.m., Feb. 26, 2015**

(80 minutes — 100 points)

Please show all your work clearly, and cross out any erroneous work that you do not want considered. If you need more space, you can use the reverse side. A sheet of notes is permitted, but no calculator or other electronic device. Except for the standard anti-derivative formulas on p. 495 of the textbook, you must show your work in deriving any other integrals.

1. (20 points) Using an inverse trig substitution and a triangle, change the following definite integral to a new definite integral involving a power of one or more trig functions. Be sure to include the limits of integration. Do **not** evaluate the integral.

$$\int_3^4 (4x - x^2)^{9/2} dx.$$

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2. (20 points) Evaluate the indefinite integral

$$\int \frac{3x^2 + 12x + 11}{(x+1)(x+2)(x+3)} dx.$$

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3. (20 points) Let R be the region between $\sin^2(\pi x)$ and the x-axis for $0 \leq x \leq \frac{1}{4}$. Find the volume of the solid obtained by rotating R around the y-axis.

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4. (20 points) (a) Evaluate

$$\int_1^b \frac{\ln x}{x^4}\,dx.$$

Your answer should involve b.

(b) Using part (a), find

$$\int_1^\infty \frac{\ln x}{x^4}\,dx.$$

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5. (20 points) Use Simpson's rule with $n = 4$ subdivisions to estimate $\int_0^2 x^3 dx$. Please show all your work and do the arithmetic carefully. Then find the **exact** value of this integral and see how close the estimate is.

ANSWERS

1. Write $4x - x^2 = -(x^2 - 4x) = -((x-2)^2 - 4) = 2^2 - (x-2)^2$ and set up a triangle with hypotenuse 2, opposite side $x - 2$, and adjacent side $\sqrt{4x - x^2}$. The limits of integration are from $\theta = \pi/6$ to $\theta = \pi/2$, so you get $\int_{\pi/6}^{\pi/2} (2\cos(\theta))^9 (2\cos(\theta) d\theta) = 2^{10} \int_{\pi/6}^{\pi/2} \cos^{10}(\theta) d\theta$.

2. Writing $\frac{3x^2+12x+11}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$ and clearing denominators, you solve for A, B, and C by setting $x = -1$, $x = -2$, and $x = -3$. You find that all three constants are 1, so the integral is $\ln|x+1| + \ln|x+2| + \ln|x+3| + C$, or equivalently $\ln|(x+1)(x+2)(x+3)| + C$. (NOTE: This is not a coincidence, since the numerator $3x^2 + 12x + 11$ was chosen to be the derivative of the denominator; if you knew this in advance you could evaluate the integral in one step using a u-substitution rather than partial fractions.)

3. $2\pi \int_0^{1/4} x \sin^2(\pi x) dx = \pi \int_0^{1/4} x(1 - \cos(2\pi x)) dx$ (after canceling 2's). The first part gives $\pi \frac{1}{2} x^2 \Big|_0^{1/4} = \pi/32$, and the part after the minus is $\pi \int_0^{1/4} x \cos(2\pi x) dx = \pi \Big(\frac{1}{2\pi} x \sin(2\pi x) \Big|_0^{\frac{1}{4}}$ $- \frac{1}{2\pi} \int_0^{1/4} \sin(2\pi x) dx \Big) = \pi \Big(\frac{1}{8\pi} + \frac{1}{(2\pi)^2} \cos(2\pi x) \Big|_0^{\frac{1}{4}} \Big) = \pi(\frac{1}{8\pi} + \frac{1}{(2\pi)^2}(0 - 1) = \frac{1}{8} - \frac{1}{4\pi}$. So the answer is $\frac{\pi}{32} - \frac{1}{8} + \frac{1}{4\pi}$.

4. (a) $\int_1^b \frac{\ln x}{x^4} dx = -\frac{1}{3} \frac{\ln x}{x^3} \Big|_1^b + \frac{1}{3} \int_1^b \frac{dx}{x^4} = -\frac{1}{3} \frac{\ln b}{b^3} + \frac{1}{9}(1 - \frac{1}{b^3})$. (b) As $b \longrightarrow \infty$, the integral in part (a) has limit 1/9.

5. $\frac{1/2}{3}(1 \cdot 0 + 4 \cdot \frac{1}{2^3} + 2 \cdot 1 + 4 \cdot \frac{3^3}{2^3} + 1 \cdot 2^3) = \frac{1}{6}(\frac{1}{2} + 2 + \frac{27}{2} + 8) = 24/6 = 4$, and the exact value is $(x^4/4) \Big|_0^2 = 4$ also. It can be shown that the Simpson approximation gives the exact value for the integral of any polynomial of degree at most 3.