Math 125 **Final Examination** **Winter 2012**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.
- A scientific calculator is allowed, but graphing calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text (p. 484 if you have the 6th edition of Stewart) without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	12	
2	12	
3	8	
4	10	
5	10	

Question	Points	Score
6	10	
7	10	
8	8	
9	10	
10	10	
Total	100	

1. (12 total points) Evaluate the following integrals.

   (a) (6 points) $\displaystyle\int \frac{1-x}{\sqrt{1-x^2}}\,dx$

   (b) (6 points) $\displaystyle\int \frac{x^2-x+8}{x^3+4x}\,dx$

2. (12 total points) Evaluate the following integrals.

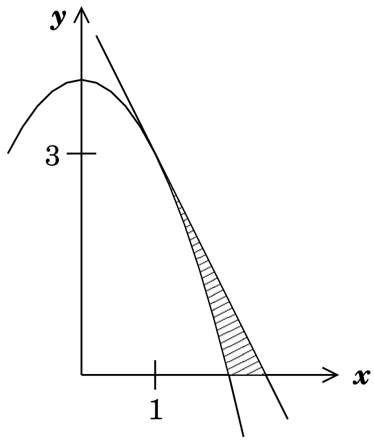
   (a) (6 points) $\int x\ (e^x + \ln(x))\ dx$

   (b) (6 points) $\int_0^2 \frac{1}{\sqrt{x^2+2x+4}}\, dx$ Give your answer in exact form.

3. (8 points) Consider the improper integral $\displaystyle\int_1^2 \frac{x}{\sqrt{x-1}}\,dx$.

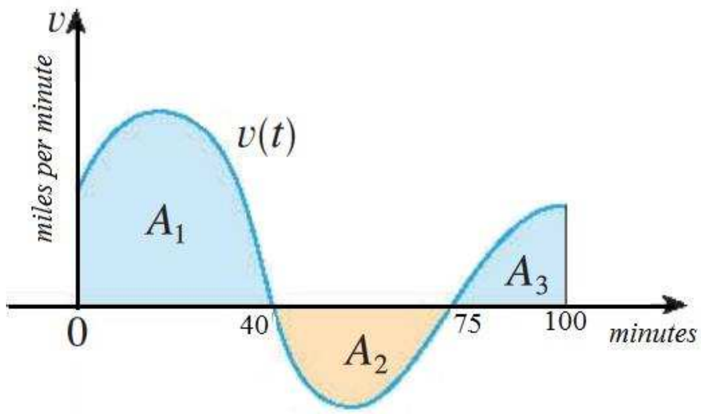
   Evaluate the integral (if it converges) or explain why it does not converge.

4. (10 points) Let R be the shaded region in the figure below, bounded by the x-axis, by the curve $y = 4 - x^2$, and by the line tangent to $y = 4 - x^2$ at the point $(1, 3)$. Find the area of the region R.



5. (10 total points) Bob the bicyclist is riding back and forth on a straight road. His velocity $v(t)$ (in miles per minute) is depicted in the graph below. Suppose the values of the areas labeled on the graph are:

$$A_1 = 16, \; A_2 = 6, \text{ and } A_3 = 4.$$



Answer the following questions. Include units in your answers when appropriate.

(a) (2 points) What was the total distance traveled by Bob during the time interval $0 \le t \le 100$ minutes?

(b) (2 points) How far from his initial position is Bob at the end of the 100 minutes?

(c) (2 points) At what time, between $t = 0$ and $t = 100$ minutes, is Bob the farthest away from his starting position?

(d) (2 points) How many times during the time interval $0 \le t \le 100$ minutes is Bob exactly 11 miles away from his starting point?

(e) (2 points) Compute the value of the definite integral $\displaystyle\int_{40}^{100} (20v(t) + 3)\; dt$.

6. (10 total points) Let $\mathscr{R}$ be the region between the lines $x = 2$ and $x = 4$ that is bounded on the top by the curve $y = t(x)$ and bounded on the bottom by the curve $y = b(x)$, where $t(x)$ and $b(x)$ are continuous functions satisfying $t(x) > b(x)$.

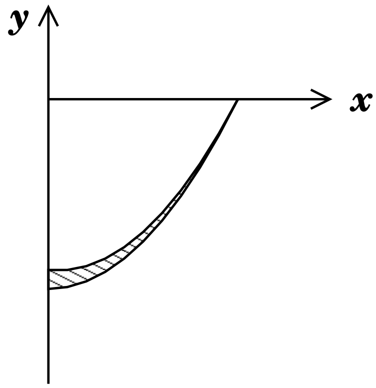
   (a) (6 points) Set up a definite integral *with respect to* x for the volume of the solid obtained by rotating the region $\mathscr{R}$ about the vertical line $x = -3$. Your answer should be a definite integral that involves $t(x)$ and $b(x)$.

   (b) (4 points) Suppose we have the following table of values for $t(x)$ and $b(x)$.

x	$b(x)$	$t(x)$
0	0.0	12.0
0.5	6.4	12.1
1	8.3	12.5
1.5	9.6	13.1
2	10.5	14.0
2.5	11.2	15.1
3	11.8	16.5
3.5	12.3	18.1
4	12.7	20.0
4.5	13.1	22.1
5	13.4	24.5

Use the relevant data from this table and the Trapezoid Rule with $n = 4$ subintervals to approximate the value of the volume of the solid in part (a). Give your answer in decimal form, correct to at least the first digit after the decimal point.

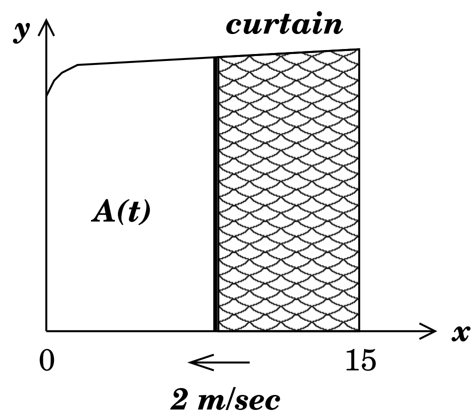
7. (10 points) Find the center of mass of the region in the fourth quadrant bounded above by the curve $y = \dfrac{9}{10}(x^2 - 1)$, bounded below by the curve $y = x^2 - 1$, and bounded on the left by the y-axis.



8. (8 total points) A stage opening is bounded by the x-axis, the y-axis, the line $x = 15$, and the curve

$$y = \sqrt{10 + x^{1/3}}.$$

The units on the x and y axes are meters. Initially, the stage curtain is completely open. At time $t = 0$, a vertical pole pulling the curtain starts on the right side of the stage opening ($x = 15$) and moves to the left at a constant speed of 2 m/sec. Let $A(t)$ be the area that is not yet covered by the curtain at time t seconds (the enclosed white area in the figure below).



(a) (4 points) Express $A(t)$ as a definite integral.

(b) (4 points) Find $\dfrac{dA}{dt}$ when $t = 3.5$ sec. Give your answer in exact form and include correct units.

9. (10 points) Find the solution of the differential equation

$$\frac{dy}{dt} = \sqrt{5t}\ (1+y^2)$$

that satisfies the initial condition $y(0) = 1$. Solve for y, giving your answer in the form $y = f(t)$.

10. (10 points) Let P denote the total weight (in kilograms) of water lilies on the surface of a lake. The rate of increase of P depends on the amount of sunlight and is given by the equation

$$\frac{dP}{dt} = s(t)\,P.$$

Here, t denotes the time (in hours) after midnight, and

$$s(t) = 0.03\cos^2\left(\frac{\pi}{12}t\right) \quad \text{(for } 6 \le t \le 18\text{)}$$

is a function that depends on the amount of sunlight at time t.

One day, at 6:00am, there were 100 kilograms of lilies on the lake. How many kilograms of lilies were on the lake at 6:00pm that day? Give your answer in decimal form, correct to at least the second digit after the decimal point.