U-Substitution Problems

1. Midterm 1 (Perkins). Compute the integral:

$$\int \frac{3x^2}{\sqrt{2-x}}\,dx$$

Solution: Let $u=2-x \Rightarrow du=-dx$ and $2-u=x$. Plugging back in
$-du=dx$

$$\int \frac{-3(2-u)^2}{\sqrt{u}}\,du = \int \frac{-3(u^2-4u+4)}{\sqrt{u}}\,du = -3\int u^{3/2}-4u^{1/2}+4u^{-1/2}\,du$$

$$= -3\left(\tfrac{2}{5}u^{5/2}-\tfrac{8}{3}u^{3/2}+8u^{1/2}\right)+C$$

must sub in what u is $\rightarrow$

$$= \boxed{-3\left[\tfrac{2}{5}(2-x)^{5/2}-\tfrac{8}{3}(2-x)^{3/2}+8(2-x)^{1/2}\right]+C}$$

2. Midterm 1 (Chen). Evaluate

$$\int_1^2 x\sqrt{2x-1}\,dx.$$

Let $u=2x-1$ $du=2dx$ so $du/2=dx$. For bounds, $u=2(2)-1=3$
$u=2(1)-1=1$

$$\int_1^2 x\sqrt{2x-1}\,dx = \int_1^3 \frac{x\sqrt{u}}{2}\,du$$

but $\frac{u+1}{2}=x$

$$= \int_1^3 \tfrac{1}{4}(u+1)\sqrt{u}\,du$$

$$= \tfrac{1}{4}\int_1^3 u^{3/2}+u^{1/2}\,du$$

$$= \tfrac{1}{4}\left[\tfrac{2}{5}u^{5/2}+\tfrac{2}{3}u^{3/2}\right]\Big|_1^3$$

3. Midterm 1 (Burdyz). Compute the integral

$$\int \frac{e^x}{e^{2x} + 2e^x + 1}\, dx.$$

Solution. Let $u = e^x$ $du = e^x dx$. Hence

$$\int \frac{e^x}{e^{2x} + 2e^x + 1}\, dx = \int \frac{1}{u^2 + 2u + 1}\, du = \int \frac{1}{(u+1)^2}\, du$$

$v = u + 1$ $dv = du$

$$= \int \frac{1}{v^2}\, dv = \frac{-1}{v} + C \underset{\uparrow}{=} \boxed{\frac{-1}{e^x + 1} + C}$$

sub back
$v = u + 1$ and
$u = e^x$

4. Midterm 1 (Burdyz). Compute

$$\int_0^{\sqrt{\pi}} \frac{x \sin(x^2)}{1 + \left(\cos(x^2)\right)^2}\, dx.$$

Let $u = x^2$ $du = 2x\, dx$
Change bounds $u = (\sqrt{\pi})^2$ $u = 0^2$

$$\int_0^{\sqrt{\pi}} \frac{x \sin(x^2)}{1 + (\cos(x^2))^2}\, dx = \int_0^{\pi} \frac{1}{2} \left(\frac{\sin(u)}{1 + (\cos(u))^2} \right) du$$

$v = \cos(v)$
$dv = -\sin(v)\, dv$
Change bounds
$v = \cos(\pi) = -1$
$v = \cos(0) = 1$

$$= \int_1^{-1} -\frac{1}{2} \left(\frac{1}{1 + v^2} \right) dv$$

$$= \frac{-1}{2} \arctan(v) \Big|_1^{-1}$$

$$= \frac{-1}{2}\left(\frac{-\pi}{4}\right) - \frac{-1}{2}\left(\frac{\pi}{4}\right) = \boxed{\frac{\pi}{4}}$$