3. (10 points) Find the derivative of the function

$$F(x) = \int_{\ln x}^{\cos(x)} \frac{e^t}{1 - \tan(t)}\, dt.$$

3. (7 points) Find the interval (or intervals) on which the curve

$$y = \int_2^{x^2-x} (1+\sin^2(t))\,dt$$

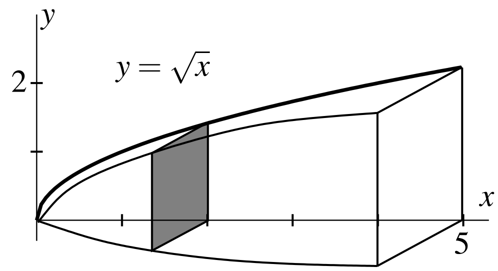
is increasing.

$\boxed{2}$ (6 points) Compute the integral $\displaystyle\int_{-1}^{3} \left|4t - t^3\right| \, dt$.

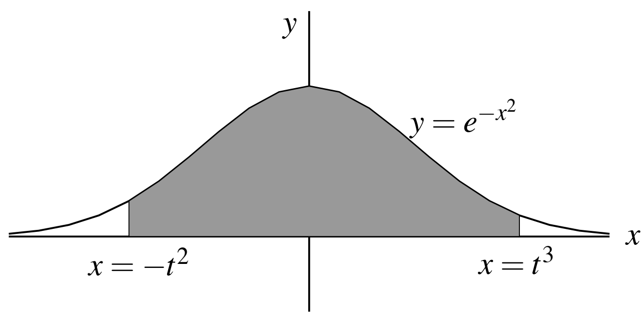
$\boxed{3}$ (8 points) Let $f(x) = \displaystyle\int_{x^2}^{9} \cos\left(\pi\sqrt{t}\right) \, dt$. Compute the equation of the tangent line to $y = f(x)$ at the point where $x = 3$.

3. (8 points) Find a continuous function $f(x)$ and a number $a > 0$ such that $16 + \int_a^x t^2 f(t)\,dt = x^4$.
(*Hint: Differentiate both sides*.)

5. (4 points) Find the volume of the solid shown below. Each cross-section (slice) is a square.



6. (8 points) At each time $t \geq 0$, $\mathscr{R}_t$ is the region above the x-axis, below the curve $y = e^{-x^2}$, with left side on the line $x = -t^2$ and right side on the line $x = t^3$ (see the figure). Let $A(t)$ be the area of $\mathscr{R}_t$. Find $\dfrac{dA}{dt}$ at time $t = 1$.



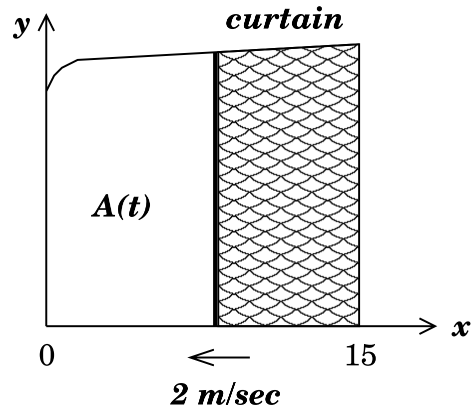
5. (8 points) Find

$$\lim_{x\to 0} \frac{\int_0^x \cos(t^2)\,dt}{x}.$$

8. (8 total points) A stage opening is bounded by the x-axis, the y-axis, the line $x = 15$, and the curve

$$y = \sqrt{10 + x^{1/3}}.$$

The units on the x and y axes are meters. Initially, the stage curtain is completely open. At time $t = 0$, a vertical pole pulling the curtain starts on the right side of the stage opening ($x = 15$) and moves to the left at a constant speed of 2 m/sec. Let $A(t)$ be the area that is not yet covered by the curtain at time t seconds (the enclosed white area in the figure below).



(a) (4 points) Express $A(t)$ as a definite integral.

(b) (4 points) Find $\dfrac{dA}{dt}$ when $t = 3.5$ sec. Give your answer in exact form and include correct units.

4. (8 points) A manufacturing plant has a large vat holding water. The water volume is set to 20 cubic meters at noon every day. Some water is taken out of the vat and some recycled water flows into the vat. The water volume w (in cubic meters) is given by the following function of t for the first 4 hours after noon:

$$w(t) = 20 + \int_{t^2}^{e^t - 1} \sin(\sqrt{z})\, dz.$$

Time t is measured in hours and $t = 0$ represents noon.

Determine the rate at which the water volume is changing at each of the times $t = 1$ and $t = 3$, and determine whether the water volume is increasing or decreasing at each of those times.