4. (8 points) Let $f(x) = \cos(x^2)$. Find the average value of $\underline{f'(x)}$ on $[0, \sqrt{\pi}]$.

* Note it asks for $f'(x)$.

$$f'(x) = -\sin(x^2) \cdot 2x$$

The avg value $= \dfrac{1}{b-a} \displaystyle\int_a^b f(x)\,dx$

$$= \frac{1}{\sqrt{\pi} - 0} \int_0^{\sqrt{\pi}} -2x\sin(x^2)\,dx$$

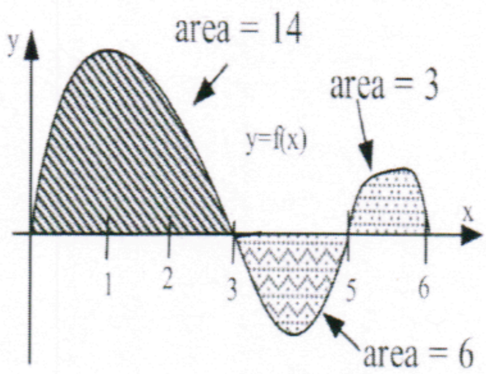
by def. of antiderivative

$$= \frac{1}{\sqrt{\pi}} \left(\cos(x^2)\right)\Big|_0^{\sqrt{\pi}}$$

$$= \frac{1}{\sqrt{\pi}}\left(\cos(\pi) - \cos(0)\right)$$

$$= \frac{1}{\sqrt{\pi}}(-2) = \boxed{\frac{-2}{\sqrt{\pi}}}$$

3. (8 total points) Use the area information given on this graph of $f(x)$ to evaluate the integrals below.



(a) (2 points) $\int_3^6 |f(x)|\,dx$

$= 6 + 3 = \boxed{9}$

↑
positive
b/c |·|

(b) (2 points) $\int_0^5 2 + f(x)\,dx = \int_0^5 2 + \int_0^5 f(x)$

$= 10 + [14 - 6]$

$= 10 + 8 = \boxed{18}$

(c) (2 points) $\int_6^5 2f(x)\,dx$

$= 2\int_6^5 f(x)\,dx = -2\int_5^6 f(x)\,dx = -2(3) = \boxed{-6}$

(d) (2 points) $\int_0^3 6x - f(x)\,dx$

$= \int_0^3 6x - \int_0^3 f(x)$

$= 3x^2 \Big|_0^3 - \int_0^3 f(x)$

$= 27 - 14$

$= \boxed{13}$

4. (8 total points) Determine if the following are **TRUE** or **FALSE**. You need not explain your answers. Each correct answer is +2 points, each wrong answer is −1 points, each blank answer is 0 points, but your total for this whole problem will not be less than 0 points. Put your **ANSWERS** in the **BOXES**.

(a) (2 points) The function $f(x) = \frac{e^x}{x}$ is a solution of the differential equation $x^2y' + xy = xe^x$.

Answer (T or F or leave blank):

$\boxed{T}$

$y = e^x/x \quad y' = \frac{xe^x - e^x}{x^2}$

so Left hand side

$= x^2\left(\frac{xe^x - e^x}{x^2}\right) + x\left(\frac{e^x}{x}\right)$

$= xe^x - e^x + e^x = xe^x =$ The Right hand side

(b) (2 points) $\frac{d}{dx}\int_2^{x^2+1} \ln(t)dt = \ln(x^2+1)$.

Answer (T or F or leave blank):

$\boxed{F}$

By FTC,

$\frac{d}{dx}\int_2^{x^2+1} \ln(t) = \ln(x^2+1)\cdot 2x$

(c) (2 points) The arc length of the curve $y = \tan x$ for $0 \le x \le \frac{\pi}{4}$ is $\int_0^{\pi/4} \sqrt{1+\sec^2 x}\,dx$.

Answer (T or F or leave blank):

$\boxed{F}$

arc length $= \int_0^{\pi/4} \sqrt{1+(dy/dx)^2}$

$\frac{d}{dx}(\tan x) = \sec^2 x$

arc length $= \int_0^{\pi/4} \sqrt{1+(\sec^2 x)^2}\,dx$

(d) (2 points) If f and f' are continuous on $[3,7]$, then $\underbrace{\int_3^7 f'(u)du = f(7) - f(3)}$.

Answer (T or F or leave blank):

$\boxed{T}$

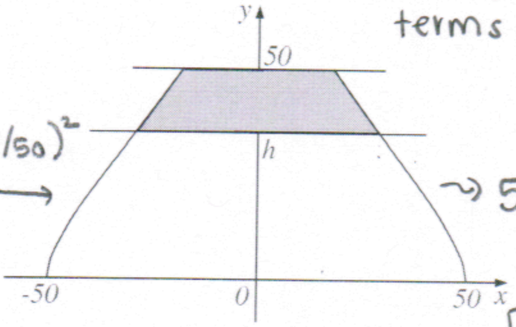
This is the 2nd part of the fund. thm of calculus.

4. (12 total points) A shape S is bounded by the x-axis, the line $y = 50$, the curve $x = 50e^{-(y/50)^2}$, and the curve $x = -50e^{-(y/50)^2}$. A barrier comes down and covers the shape S between height h and height 50.

(a) (3 points) Express the area not covered by the barrier (the unshaded area) in terms of an integral.

* Note do in terms of y

$x = -50e^{-(y/50)^2} \rightarrow$



$\rightsquigarrow 50e^{-(y/50)^2} = x$

① Point of inter in (y)
$y = h,\ y = 0$

② Chart:

	$0 \le y \le h$
Right	$50e^{-(y/50)^2}$
left	$-50e^{-(y/50)^2}$

$$\boxed{③\ A = \int_0^h 50e^{-(y/50)^2} - \left(-50e^{-(y/50)^2}\right) dy}$$

(b) (4 points) Suppose that the horizontal line $y = h$ at the bottom of the barrier starts at the top with zero velocity at time $t = 0$ and descends with acceleration $a(t) = -6t$. Find a formula for h in terms of t.

Find distance

$a(t) = -6t$

$v(t) = -3t^2 + C$

$s(t) = -t^3 + Ct + D$

$v(0) = 0 \Rightarrow C = 0$

$s(0) = 50 \Rightarrow D = 50$

Hence

$S(t) = -t^3 + 50$

Now if you think about this, $s(t)$ is representing "how high off the grnd "h" is. Thus

$$\boxed{h = -t^3 + 50}$$

(c) (5 points) If the barrier descends as in part b), find a formula in terms of t for the rate of change of the area not covered by the barrier.

* Rate of Change refers to the derivative of the Area.

Thus

$$\frac{d}{dt} A = \frac{d}{dt} \int_0^h 50e^{-(y/50)^2} + 50e^{-(y/50)^2} = \frac{d}{dt} \int_0^{-t^3+50} 2\left(50e^{-(y/50)^2}\right) dy$$

By FTC,

$$A'(t) = 2\left(50e^{-\left(\frac{-t^3+50}{50}\right)^2}\right) \cdot (-3t^2)$$