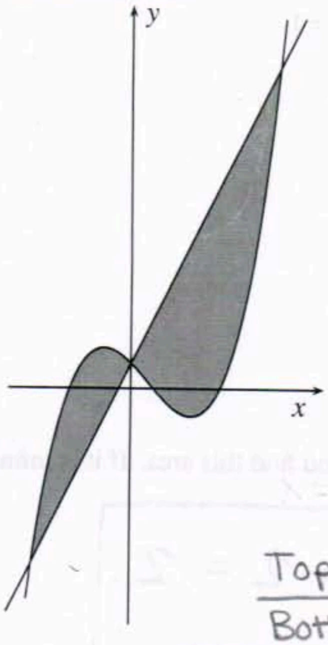
5. (8 points) Find the area of the shaded region between the curves $y = x^3 - x^2 - 2x + 1$ and $y = 4x + 1$, as shown below.



① Find pts of interrection

$4x + 1 = x^3 - x^2 - 2x + 1$

$0 = x^3 - x^2 - 6x$

$0 = x(x^2 - x - 6)$

$0 = x(x-3)(x+2)$

$x = 0, 3, -2$

② Create Chart

	$-2 \le x \le 0$	$0 = x$	$0 \le x \le 3$
Top	$x^3 - x^2 - 2x + 1$	0	$4x + 1$
Bott	$4x + 1$	0	$x^3 - x^2 - 2x + 1$

③ Create Int ÷ Solve

$$\int_{-2}^{0} x^3 - x^2 - 2x + 1 - 4x - 1 + \int_{0}^{3} 4x + 1 - x^3 + x^2 + 2x - 1$$

$$= \int_{-2}^{0} x^3 - x^2 - 6x \, dx + \int_{0}^{3} -x^3 + x^2 + 6x \, dx$$

$$= \tfrac{1}{4}x^4 - \tfrac{1}{3}x^3 - 3x^2 \Big|_{-2}^{0} - \tfrac{1}{4}x^4 + \tfrac{1}{3}x^3 + 3x^2 \Big|_{0}^{3}$$

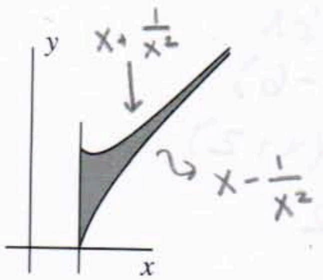
$$= 0 - [4 + 8/3 - 12] - \frac{81}{4} + 9 + 27$$

$$= \boxed{\frac{253}{12} \approx 21.08\overline{33}}$$

6. (8 points) Consider the *unbounded* region S contained within the curves

$$y = x + \frac{1}{x^2}, \qquad y = x - \frac{1}{x^2} \qquad \text{and} \qquad x = 1$$

as shown in the picture below.



Is the area of S finite or infinite? If it is finite, justify your conclusion and find this area. If it is infinite, carefully explain why.

① Find pts of intersection

$x = 1$ (from graph)

② Make Chart

	$x \geq 1$
Top	$x + \frac{1}{x^2}$
Bott	$x - 1/x^2$

③ Construct Integral

$$\int_1^\infty x + \frac{1}{x^2} - x + 1/x^2 \, dx$$

$$= \int_1^\infty 2/x^2 \, dx$$

$$= \lim_{b \to \infty} \int_1^b \frac{2}{x^2} \, dx$$

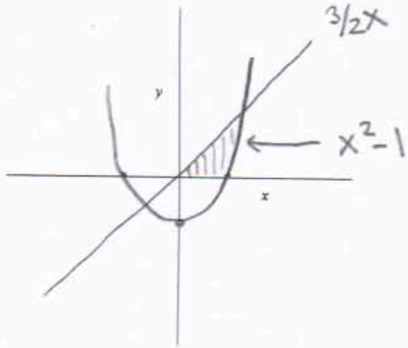
$$= \lim_{b \to \infty} \frac{-2}{x} \Big|_1^b$$

$= \lim$ b–

$$\boxed{= \lim_{b \to \infty} \frac{-2}{b} + 2 = 2}$$

Therefore converges

4. (8 points) Consider the region bounded by the line $y = \frac{3}{2}x$, the parabola $y = x^2 - 1$, and lying above the x-axis. Sketch this region and find its area.



① Find pt of intersection

$\underline{x=0}$

$x^2 - 1 = 3/2\,x \Rightarrow \underline{x = 2}$ or $-1/2$

and $\underline{x=1}$

② Make Chart

	$0 \le x \le 1$	$1 \le x \le 2$
Top	$3/2\,x$	$3/2\,x$
Bott	0	$x^2 - 1$

③ Make Integral

$$\int_0^1 \frac{3}{2}x\,dx + \int_1^2 3/2\,x - x^2 + 1\,dx = \frac{3}{4}x^2\Big|_0^1 + \frac{3}{4}x^2 - \frac{1}{3}x^3 + x\Big|_1^2$$

$$= \frac{3}{4} + 3 - \frac{8}{3} + 2 - \frac{3}{4} + \frac{1}{3} - 1$$

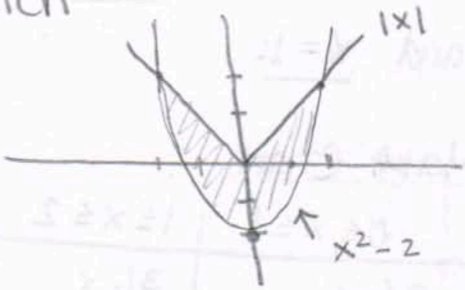
$$\boxed{= 5/3}$$

6. (7 points) Let R be the region enclosed by the curves

$$y = |x|, \quad y = x^2 - 2.$$

Sketch R and find its area.

① Sketch



② Find pts of intersection

$x=0,\ x=-2,\ x=2$

③ Make Chart

	$-2 \le x \le 0$	$0 \le x \le 2$
Top	$-x$	x
Bott	x^2-2	x^2-2

④ Integrate

$$\int_{-2}^{0} -x - x^2 + 2\,dx + \int_{0}^{2} x - x^2 + 2\,dx$$

$$= -\tfrac{1}{2}x^2 - \tfrac{1}{3}x^3 + 2x \Big|_{-2}^{0} + \tfrac{1}{2}x^2 - \tfrac{1}{3}x^3 + 2x \Big|_{0}^{2}$$

$$= 0 - [-2 + 8/3 - 4] + 2 - 8/3 + 4 - 0$$

$$\boxed{= \frac{20}{3}}$$