U-Substitution Problems 2

1. Midterm 1 (Perkins). Compute

$$\int_0^{\pi/4} \sec^2(\theta)\cos\left(\tan(\theta)\right)d\theta$$

$u=\tan\theta \quad du=\sec^2\theta\, d\theta$

$\Rightarrow \frac{du}{\sec^2\theta}=d\theta$

Change bounds

$u=\tan(\frac{\pi}{4})=1$

$u=\tan(0)=0$

$$\int_0^1 \cancel{\sec^2\theta}\cos(u)\frac{du}{\cancel{\sec^2\theta}} = \int_0^1 \cos(u)\,du$$

$$= \sin(u)\Big|_0^1 = \boxed{\sin(1)}$$

2. Midterm 1 (Palmieri). Compute

$$\int \sec^2(2x)\tan^5(2x)\,dx.$$

$u=2x \quad du=2dx$

$\frac{du}{2}=dx$

$$=\int \tfrac{1}{2}\sec^2(u)\tan^5(u)\,du$$

$v=\tan u$

$dv=\sec^2 u\,du$

$\frac{dv}{\sec^2 u}=du$

$$=\int \tfrac{1}{2}v^5\,dv$$

$$=\frac{1}{12}v^6+C$$

$$\boxed{=\frac{1}{12}(\tan(2x))^6+C}$$

3. Midterm 1 (Folland). Find the function $F(x)$ such that $F'(x) = x\sqrt{3x+1}$ and $F(0) = 0$.

Find the $\int x\sqrt{3x+1}\,dx$ $\quad u = 3x+1 \Rightarrow \frac{u-1}{3} = x$

$du = 3dx$

$\frac{du}{3} = dx$

$$\int \frac{u-1}{3}\sqrt{u}\,\frac{du}{3} = \frac{1}{9}\int u^{3/2} - u^{1/2}\,du$$

$$= \frac{2}{45}u^{5/2} - \frac{2}{27}u^{3/2} + C$$

$$F(x) = \frac{2}{45}(3x+1)^{5/2} - \frac{2}{27}(3x+1)^{3/2} + C$$

$$0 = \frac{2}{45} - \frac{2}{27} + C \Rightarrow C = \frac{4}{135} \Rightarrow \boxed{F(x) = \frac{2}{45}(3x+1)^{5/2} - \frac{2}{27}(3x+1)^{3/2} + 4/135}$$

4. Midterm 1 (Folland). Compute

$$\int_{\pi/6}^{\pi/3} \frac{\sin(3x)}{2+\cos(3x)}\,dx$$

$u = 3x \quad du = 3dx \Rightarrow \frac{du}{3} = dx$

$$\int_{\frac{\pi}{2}}^{\pi} \frac{\sin(u)}{2+\cos(u)} \cdot \frac{1}{3}du = \frac{1}{3}\int_0^{-1} \frac{-1}{2+v}\,dv$$

$v = \cos(u)$

$dv = -\sin(u)\,du$

$w = 2+v \quad dw = dv$

$$= \frac{1}{3}\int_2^1 \frac{-1}{w}\,dw = \frac{-1}{3}\ln|w|\Big|_2^1 = \frac{-1}{3}\ln|1| + \frac{1}{3}\ln|2|$$

$$= \boxed{\frac{1}{3}\ln|2|}$$

5. Midterm 1 (Burdyz). Compute the integral

$$\int_{-\pi}^{-\pi/2} [\cos x - (\cos x)^2]^2 \sin(x)\, dx.$$

$u = \cos x \quad du = -\sin x\, dx$

$$\int_{-1}^{0} -[u-u^2]^2\, du = -\int_{-1}^{0} u^4 - 2u^3 + u^2\, du$$

$$= -\frac{1}{5}u^5 + \frac{1}{2}u^4 - \frac{1}{3}u^3 \Big|_{-1}^{0}$$

$$= -\frac{1}{5} - \frac{1}{2} - \frac{1}{3} = \boxed{\frac{-31}{30}}$$

6. Midterm 1 (Perkins). Compute

$$\int_0^{\pi} \frac{\sin t}{1+\cos^2 t}\, dt$$

$u = \cos(t) \quad du = -\sin(t)\, dt$

$$= \int_{1}^{-1} \frac{-1}{1+u^2}\, du = -\arctan(u)\Big|_{1}^{-1}$$

$$= -\left(\frac{-\pi}{4}\right) - -\frac{\pi}{4} = \boxed{\frac{\pi}{2}}$$

7. Midterm 1 (H. Smith). Evaluate

$$\int \frac{\cos\left(1+\sqrt{x}\right)}{\sqrt{x}}\,dx$$

$u = 1+\sqrt{x}$ $\quad du = \frac{1}{2}x^{-1/2}\,dx$

$2\sqrt{x}\,du = dx$

$$\int \frac{\cos(u)}{\cancel{\sqrt{x}}} \cdot 2\cancel{\sqrt{x}}\,dx = 2\sin(u)+C$$

$$\boxed{= 2\sin(1+\sqrt{x})+C}$$

8. Midterm 1 (Palmieri). Compute

$$\int_1^2 x(2-x)^7\,dx.$$

$x = 2-u$

$u = 2-x$ $\quad du = -dx$

$$\int_1^0 (2-u)u^7 \,-du = -\int_1^0 2u^7 - u^8\,du$$

$$= \frac{-2}{8}u^8 + \frac{1}{9}u^9 \Big|_1^0$$

$$= \frac{2}{8} - \frac{1}{9}$$

$$\boxed{= \frac{5}{36}}$$