Week 2: Questions Taken From Past Midterms

(1) If $\int_2^4 (f(x)+g(x))\,dx = 11$, $\int_{-2}^3 f(x^2)x\,dx = 13$, and $\int_2^4 (3g(x)+2)\,dx = 7$,

$$\text{Find} \quad \int_2^9 f(x)\,dx.$$

(2) Compute the following integrals
   (a) $\int \frac{arctanx}{x^2+1}\,dx$
   (b) $\int x\sqrt[3]{x+2}\,dx$
   (c) $\int_0^{\pi/2} \sin(2x)\,dx$
   (d) $\int_{\pi/6}^{\pi/3} \frac{\sin(3x)}{2+cos(3x)}\,dx$
   (e) $\int \frac{e^{2\sqrt{x}}}{\sqrt{x}}\,dx$
   (f) $\int (\sin x)^3\,dx$

(3) Let $f(x) = \int_0^{\sqrt{2x+7}} \frac{dt}{t^4+9}$. Compute $f'(x)$.

(4) Pete is driving his car along a straight street. He starts at his work place and needs to deliver a packet to a customer. Not knowing the neighborhood too well he starts going in the wrong direction, but realizes his mistake soon. The velocity of his car is given by $v(t) = 90t^2 - 50t$ in mi/hour where t is measured in hours.
   (a) Pete reaches his destination after one hour. How fair away does the customer live from Pete's work place?
   (b) Pete's car is quite friendly to the environment, it can drive 35 miles per gallon fuel. How much fuel did Pete use up for this journey?

(5) For $F(x) = \int_{\sqrt{x}}^1 \tan^{-1}(u)\,du$, find $F(1)$ and $F'(1)$.

Additional Practice Questions From Another Calculus Book

(1) Evaluate the integral:
   (a) $\int_0^5 |x^2 - 4x + 3|\,dx$
   (b) $\int_0^{\pi} |\cos(x)|\,dx$

(2) Calculate the Derivative
   (a) $\frac{d}{dx}\int_{-u}^{3u} \sqrt{x^2+1}\,dx$
   (b) $\frac{d}{dx}\int_{x^2}^{x^4} \sqrt{t}\,dt$

(3) Let $f(x) = x^2 - 5x - 6$ and $F(x) = \int_0^x f(t)\,dt$
   (a) Find the critical points of $F(x)$ and determine whether they are local minima or local maxima.
   (b) Find the points of inflection of $F(x)$ and determine whether the concavity changes from up to down or down to up.

(4) Calculate the sums:
   (a) $\sum_{j=3}^4 \sin\left(j\frac{\pi}{2}\right)$
   (b) $\sum_{j=0}^2 3^{j-1}$
   (c) $\sum_{k=3}^5 \frac{1}{k-1}$

(5) Describe the area represented by the limits
   (a) $\lim_{N\to\infty} \frac{5}{N}\sum_{j=1}^N e^{-2+\frac{5j}{N}}$
   (b) $\lim_{N\to\infty} \frac{3}{N}\sum_{j=1}^N \left(2+\frac{3j}{N}\right)^4$

(6) Calculate the limit for the given function and interval.

(a) $f(x) = 9x$, $[0, 2]$

(b) $f(x) = \frac{1}{2}x + 2$, $[1, 4]$