Math 124 Final Examination Autumn 2010

Print Your Name | Signature

Student ID Number | Quiz Section

Professor's Name | TA's Name

!!! READ...INSTRUCTIONS...READ !!!

1. Your exam contains 8 questions and 9 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 2 hours and 50 minutes for this final exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification.
4. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.
5. You are allowed one 8.5×11 sheet of handwritten notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed. Make sure your calculator is in radian mode.
6. Unless otherwise instructed, ALWAYS GIVE YOUR ANSWERS IN EXACT FORM. For example, 3π, $\sqrt{2}$, $\ln(2)$ are in exact form; the corresponding approximations 9.424778, 1.4142, 0.693147 are NOT in exact form.

Problem	Total Points	Score
1	12	
2	13	
3	15	
4	12	

Problem	Total Points	Score
5	12	
6	12	
7	12	
8	12	
Total	100	

1. (12 points; 4pts each) Find the derivatives of the following functions. You do not have to simplify.

   (a) $y = x^e + 5\,e^{x^2}$

   (b) $y = \sin\left(\sqrt{x\ \cos x}\right)$

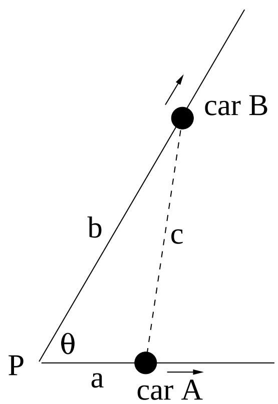
   (c) $y = x^{(2^x)}$

2. (13 points) *In this problem you should use the law of cosines:*

$$a^2 + b^2 - 2ab\cos\theta = c^2,$$

where θ is an angle in the triangle, c is the opposite side, and a and b are the adjacent sides.

Two straight roads intersect at point P at a $60°$ angle. Car A is traveling away from P on one road, and car B on the other road. You're in car A, which has a device that can measure distance from car B and also the rate at which that distance is increasing. At a certain moment you have traveled 3 km away from P and are moving at 80 km/hr. At that time the device shows that your distance from car B is 7 km, and this distance is increasing at 100 km/hr.

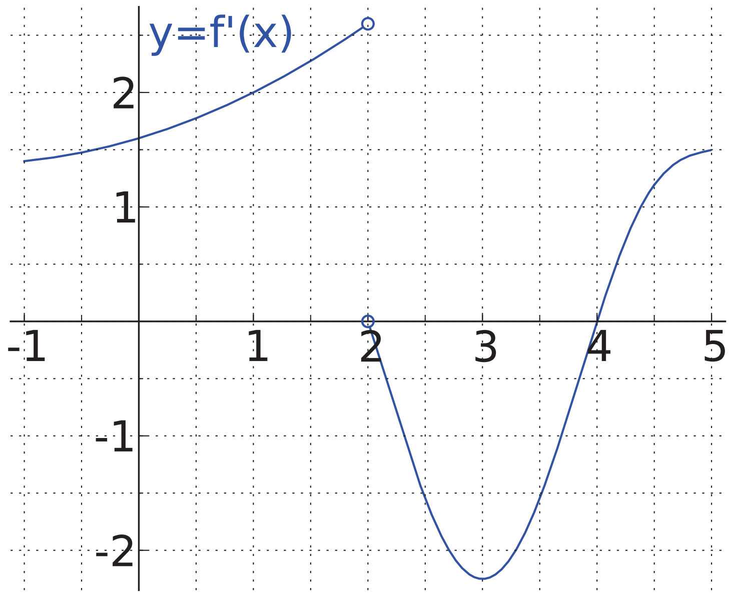


At that instant find

(a) (5 pts) the distance car B has traveled from P;

(b) (8 pts) the speed at which car B is moving.

3. (15 points) Below is the graph of the DERIVATIVE of a continuous function $f(x)$.

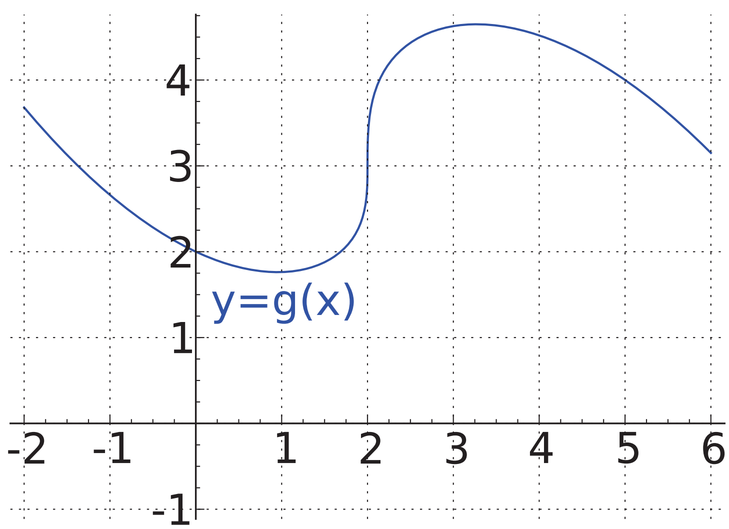


a. On what interval(s) is f increasing?

b. On what interval(s) is f concave down?

c. Find all the value(s) of c such that $f(x)$ has a local maximum at c.

Below is the graph of a continuous function $g(x)$.



d. Find all the value(s) of c such that $g(x)$ has a local minimum at c.

e. Circle the x values that are among the critical numbers for $f(g(x))$ (where $f(x)$ is the function whose derivative is shown at the top of the page).

$x = -1$ $\qquad$ $x = 0$ $\qquad$ $x = 4$ $\qquad$ $x = 5$

4. (12 points)

Consider the curve defined by the equation

$$y^2 = 3x + 4\cos(xy).$$

(a) (4pts) Implicitly differentiate to find $\dfrac{dy}{dx}$.

(b) (4pts) Find the tangent line equation(s) at the y-intercept(s).

(c) (4pts) Find the tangent line equation(s) at the x-intercept(s).

5. (12 points)

Let $f(t)$ be defined by the formula

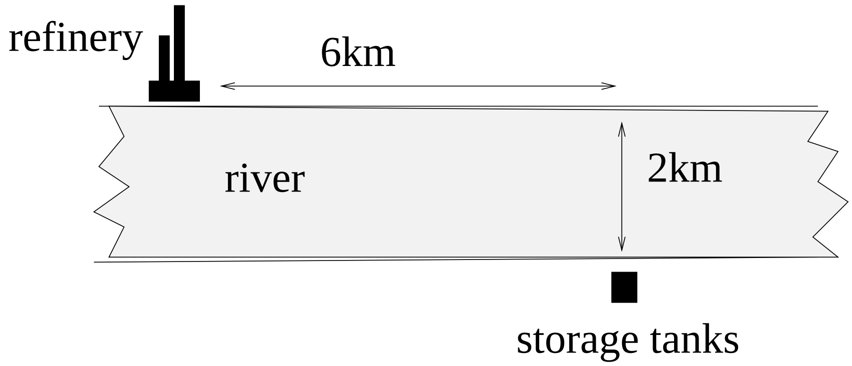
$$f(t) = \frac{t^2 e^{t-4}}{1+t}.$$

(a) Find the derivative of $f(t)$.

(b) Find the tangent line to the graph of $f(t)$ at the point $(4, f(4))$.

(c) Use the tangent line approximation to find a number t for which $f(t) \approx 3$. Give your answer in decimal form to four digits after the decimal point.

6. (12 points) An oil refinery is located on the north bank of a straight river that is 2 km wide. A pipeline is to be constructed from the refinery to storage tanks located on the south bank of the river 6 km east of the refinery. The cost of laying pipe is \$300,000/km over land to a point P on the north bank and \$500,000/km under the river to the tanks.



(a) To minimize the cost of the pipeline, where should P be located? Be sure to justify you have found the minimum.

(b) What is the resultant minimum cost?

7. (12 points)

(a) (6pts) Compute the limit. If it is correct to say that the limit is ∞ or $-\infty$, then say so. If the limit does not exist, explain why.

$$\lim_{x \to 0} (e^x + x)^{1/x}$$

(b) (6pts) Find a value of c so that the following function is continuous everywhere.

$$f(x) = \begin{cases} \frac{4x - 2\sin(2x)}{10x^3} & \text{if } x \neq 0 \\ c & \text{if } x = 0 \end{cases}$$

8. (12 points) A curve has parametric equations:

$$\begin{aligned} x &= 3t^2 + 2 \\ y &= 4t^3 + 2, \end{aligned}$$

with $t > 0$. Find the equation of the tangent line to the curve that passes through the point $(6, 2)$.