1. (16 total points) Evaluate the following integrals.

(a) (8 points) $\int x \arcsin x \, dx$ $\boxed{\text{IBP}}$

$u = \sin^{-1}(x)$ $dv = x\,dx$

$du = \frac{1}{\sqrt{1-x^2}}$ $v = \frac{1}{2}x^2$ $= \frac{1}{2}x^2\sin^{-1}(x) - \frac{1}{2}\underbrace{\int \frac{x^2}{\sqrt{1-x^2}}\,dx}_{①}$

① uses Trig

$x = \sin\theta$ $dx = \cos\theta\, d\theta$

Double angle

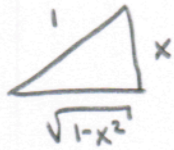
$$\int \frac{-\frac{1}{2}\sin^2\theta\cos\theta\, d\theta}{\sqrt{1-\sin^2\theta}} = \int -\frac{1}{2}\sin^2\theta\, d\theta = \frac{-1}{2}\int\left(\frac{1}{2}\right)\left[1+\cos(2\theta)\right]d\theta$$

$$= \frac{-1}{4}\theta - \frac{1}{8}\sin(2\theta) + C$$

$$= \frac{-1}{4}\theta - \frac{1}{8}\cdot 2\cdot\cos\theta\sin\theta + C$$

$$= \frac{-1}{4}\sin^{-1}(x) - \frac{1}{4}\sqrt{1-x^2}\,x + C$$

Triangle $x/1 = \sin\theta = o/h$



Hence

$$\boxed{\int x\sin^{-1}(x) = \frac{1}{2}x^2\sin^{-1}(x) - \frac{1}{4}\sin^{-1}(x) - \frac{1}{4}x\sqrt{1-x^2} + C}$$

(b) (8 points) $\int \sin^4(3t)\cos^3(3t)\,dt$ $\boxed{\text{TRIG}}$ Pull out even powers and get rid of odd

$$\int \sin^4(3t)\cos^3(3t)\,dt = \int \sin^2(3t)\sin^2(3t)\cos^2(3t)\cos(3t)$$

$u = \sin(3t)$

$du = \cos(3t)\cdot 3\,dt$

$$= \int u^2u^2\cos^2(3t)\left(\frac{1}{3}\right)du$$ $1-\sin^2(3t) = \cos^2(3t)$

$$= \int u^2u^2(1-u^2)\left(\frac{1}{3}\right)du = \frac{1}{3}\int u^4 - u^6\,du = \frac{1}{3}\left[\frac{1}{5}u^5 - \frac{1}{6}u^6\right] + C$$

$$\boxed{= \frac{1}{3}\left[\frac{1}{5}(\sin(3t))^5 - \frac{1}{6}(\sin(3t))^6\right] + C}$$

2. (8 points) Evaluate the integral

$$\int_{e^2}^{e^3} \frac{(\ln x)^3+1}{x\,((\ln x)^3-(\ln x)^2)}\,dx.$$

$u=\ln(x)$ $du = {}^1\!/\!_x\,dx$

$x\,du = dx$

$$= \int_2^3 \frac{u^3+1}{(u^3-u^2)}\,du.$$

Partial Fractions:

① Long division

$$\begin{array}{r} 1 \\ u^3-u^2 \overline{)\, u^3+1} \\ -\underline{u^3-u^2} \\ u^2+1 \end{array}$$

So, $\dfrac{u^3+1}{(u^3-u^2)} = 1 + \dfrac{u^2+1}{u^3-u^2} = 1 + \dfrac{u^2+1}{u^2(u-1)}$

② Form:

$$\frac{u^2+1}{u^2(u-1)} = \frac{A}{u} + \frac{B}{u^2} + \frac{C}{u-1} \Rightarrow u^2+1 = Au(u-1) + B(u-1) + Cu^2$$

$u=0$: $1=-B \Rightarrow B=-1$

$u=1$: $2=C$

$u=2$: $5 = A(2) - 1 + 8$ $-2 = A(2)$ $A=-1$

③ Integrate

$$\int 1 + \frac{u^2+1}{u^2(u-1)} = u + \int \frac{-1}{u} + \frac{-1}{u^2} + \frac{2}{u-1}$$

$$= u - \ln|u| + \frac{1}{u} + 2\ln|u-1| + C$$

$$\therefore \int_2^3 \frac{u^3+1}{(u^3-u^2)}\,du = u - \ln|u| + \frac{1}{u} + 2\ln|u-1| \Big|_2^3$$

$$= \frac{10}{3} - \ln|3| + 2\ln|2| - {}^5\!/\!_2 + \ln|2| - 0$$

$$\boxed{= \frac{5}{6} - \ln|3| + 3\ln|2|}$$

1. (12 total points) Evaluate the following integrals.

(a) (6 points) $\int \cos^4 x\, dx$

Use Double angle since left w/ $(\cos^2 x)^2$.

$\int \cos^4 x = \int \left[\frac{1}{2}(1+\cos(2x))\right]^2 = \frac{1}{4}\int 1 + 2\cos(2x) + \cos^2(2x)$

Do another double angle on $\cos^2(2x)$ to get

$\frac{1}{4}\int 1 + \frac{1}{4}\int 2\cos(2x) + \frac{1}{4}\int \frac{1}{2}(1+\cos(4x))\,dx$

$= \frac{1}{4}x + \frac{1}{4}\sin(2x) + \frac{1}{8}x + \frac{1}{32}\sin(4x) + C$

$\boxed{= \frac{3}{8}x + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C}$

(b) (6 points) $\int_0^{\pi/4} \frac{\sin t}{\cos^2 t}\, dt$

$u = \cos t$ $\qquad u = \cos(0) \Rightarrow u = 1$

$du = -\sin t$ $\qquad u = \cos(\pi/4) \Rightarrow u = \sqrt{2}/2$

$\int_1^{\sqrt{2}/2} \frac{-1}{u^2}\,du = \frac{1}{u}\Big|_1^{\sqrt{2}/2} = \frac{2}{\sqrt{2}} - 1 = \frac{2\sqrt{2}}{2} - 1 = \boxed{\sqrt{2} - 1}$

2. (12 total points) Evaluate the following integrals.

(a) (6 points) $\int x \ln(x+4)\,dx$

$u = x+4 \quad du = dx$

$\int (u-4)\ln(u)\,du$ IBP $w = \ln(u) \quad dv = u-4$

$dw = 1/u \quad v = \frac{1}{2}u^2 - 4u$

$= \left(\frac{1}{2}u^2 - 4u\right)\ln(u) - \int \frac{1}{2}u - 4\,du$

$= \left(\frac{1}{2}u^2 - 4u\right)\ln(u) - \frac{1}{4}u^2 + 4u + C$

$$\boxed{= \left(\frac{1}{2}(x+4)^2 - 4(x+4)\right)\ln(x+4) - \frac{1}{4}(x+4)^2 + 4(x+4) + C}$$

(b) (6 points) $\int_2^{\infty} \frac{1}{x\sqrt{x-2}}\,dx$

1. (14 total points) Evaluate the following integrals.

(a) (7 points) $\int_0^{\pi/4} t\cos^2(t)\,dt$ IBP $u=t$ $dv=\cos^2(t)\,dt$

$du=dt$ $v=\frac{1}{2}t+\frac{1}{4}\sin(2t)$

* Note double angle

$\int \cos^2(t) = \int \frac{1}{2}(1+\cos(2t))$

$= \frac{1}{2}t + \frac{1}{4}\sin(2t)$

$\therefore = t\left(\frac{1}{2}t+\frac{1}{4}\sin(2t)\right)\Big|_0^{\pi/4} - \int_0^{\pi/4} \frac{1}{2}t + \frac{1}{4}\sin(2t)\,dt$

$= \frac{1}{2}t^2 + \frac{1}{4}t\sin(2t)\Big|_0^{\pi/4} - \frac{1}{4}t^2 + \frac{1}{8}\cos(2t)\Big|_0^{\pi/4}$

$= \frac{\pi^2}{32} + \frac{\pi}{16} - \frac{\pi^2}{64} - \frac{1}{8}$

$\boxed{= \frac{\pi^2}{64} + \frac{\pi}{16} - \frac{1}{8}}$

(b) (7 points) $\int \frac{3x+5}{(x-1)^2(x+1)}\,dx$

Use partial Fractions

① No Long division

② Form. $\frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$

$3x+5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$

$x=1$: $8=2B$ $\boxed{B=4}$ $x=-1$: $2=4C$ $\boxed{C=1/2}$

$x=0$: $5=-A+4+1/2 \Rightarrow 1/2=-A$ $\boxed{A=-1/2}$

③ Integrate $\int \frac{-1/2}{x-1} + \int \frac{1/2}{x+1} + \int \frac{4}{(x-1)^2}$ → $u=x-1$ $du=dx$

→ $\int \frac{4}{u^2}$

$= \frac{-1}{2}\ln|x-1| + \frac{1}{2}\ln|x+1| - 4\left(\frac{1}{u}\right) + C$

$\boxed{= \frac{-1}{2}\ln|x-1| + \frac{1}{2}\ln|x+1| - \frac{4}{x-1} + C}$

2. (14 total points) Evaluate the following integrals.

(a) (7 points) $\int \frac{x^2}{\sqrt{2x-x^2}}\,dx$

① Complete the Square

$$2x-x^2 = -(x^2-2x) = -((x-1)^2-1)$$
$$= 1-(x-1)^2$$

② $\int \frac{x^2}{\sqrt{1-(x-1)^2}}\,dx \qquad u=x-1,\ du=dx$

$$= \int \frac{(u+1)^2}{\sqrt{1-u^2}}\,du \qquad u=\sin\theta,\ du=\cos\theta\,d\theta$$

$$= \int \frac{(\sin\theta+1)^2\cdot\cos\theta}{\sqrt{1-\sin^2\theta}}\,d\theta$$

$$= \int (\sin\theta+1)^2\,d\theta$$

$$= \int \sin^2\theta + 2\sin\theta + 1\,d\theta$$

$$= \int \tfrac{1}{2}(1-\cos(2\theta)) + \int 2\sin\theta + \int 1\,d\theta$$

$$= \tfrac{1}{2}\left(\theta - \tfrac{1}{2}\sin(2\theta)\right) - 2\cos\theta + \theta + C$$

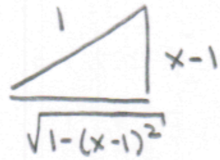
↓ identity

$$= \tfrac{1}{2}\theta - \tfrac{1}{4}2\cos\theta\sin\theta - 2\cos\theta + \theta + C$$

$$= \tfrac{3}{2}\theta - \tfrac{1}{2}\cos\theta\sin\theta - 2\cos\theta + C$$

③ Triangle

$u = \sin\theta = o/h = \frac{x-1}{1}$



$$\boxed{\therefore\ \tfrac{3}{2}(\sin^{-1}(x-1)) - \tfrac{1}{2}\left(\sqrt{1-(x-1)^2}\right)(x-1) - 2\sqrt{1-(x-1)^2} + C}$$

(b) (7 points) $\int w^5\sqrt{w^3-1}\,dw$

$u=w^3 \quad du=3w^2\,dw$

$$\int \tfrac{1}{3}w^3\sqrt{u-1}\,du = \int \tfrac{1}{3}u\sqrt{u-1}\,du \qquad v=u-1,\ dv=du \qquad \int \tfrac{1}{3}(v+1)\sqrt{v}\,dv$$

$$= \int \tfrac{1}{3}v^{3/2} + \tfrac{1}{3}v^{1/2}\,dv = \tfrac{2}{15}v^{5/2} + \tfrac{2}{9}v^{3/2} + C$$

$$\boxed{= \tfrac{2}{15}(w^3-1)^{5/2} + \tfrac{2}{9}(w^3-1)^{3/2} + C}$$

1. (14 total points) Evaluate the following integrals.

(a) (7 points) $\displaystyle\int \frac{dx}{x^3+x^2}$ Partial Fractions

① No long division

② Form

$$\frac{1}{x^3+x^2} = \frac{1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} \Rightarrow 1 = Ax(x+1) + B(x+1) + Cx^2$$

$x=0$: $1=B$ $\quad x=1$ $\quad 1 = 2A + 2 + 1$

$x=-1$: $1=C$ $\quad A=-1$

③ Integrate

$$\int \frac{1}{x^3+x^2}\,dx = \int \frac{-1}{x} + \int \frac{1}{x^2} + \int \frac{1}{x+1}$$

$$\boxed{= -\ln|x| - \frac{1}{x} + \ln|x+1| + C}$$

(b) (7 points) $\displaystyle\int \frac{(x+7)\,dx}{x^2+6x+13}$ * Note can't use partial fractions

① Complete the sq.

$x^2+6x+13 = (x+3)^2 - 9 + 13 = (x+3)^2 + 4$

② Integrate

$$\int \frac{(x+7)}{(x+3)^2+4}\,dx \quad \begin{array}{l} u = x+3 \\ du = dx \end{array} \quad \int \frac{u+4}{u^2+4} = \int \frac{u}{u^2+4} + \int \frac{4}{u^2+4} \leftarrow \text{arctan}$$

$v = u^2+4$

$dv = 2u\,du$

$$= \int \frac{1}{2}\left(\frac{1}{v}\right) + 4\left(\frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)\right) + C$$

$$= \frac{1}{2}\ln|u^2+4| + 2\tan^{-1}\left(\frac{u}{2}\right) + C$$

$$\boxed{= \frac{1}{2}\ln|(x+3)^2+4| + 2\tan^{-1}\left(\frac{x+3}{2}\right) + C}$$

2. (14 total points) Evaluate the following integrals. Leave your answers in exact form: do not use decimal expansions.

(a) (7 points) $\displaystyle\int_0^{\sqrt{5}} \frac{x^3\,dx}{\sqrt{9-x^2}}$ TRig Ignore the bounds until the end

$x = 3\sin\theta \quad dx = 3\cos\theta\, d\theta$

$\displaystyle\int \frac{27\sin^3\theta \cdot 3\cos\theta}{\sqrt{9-9\sin^2\theta}}\,d\theta = \int \frac{27\sin^3\theta\cdot 3\cos\theta}{3\cos\theta}\,d\theta = \int 27\sin^3\theta\,d\theta$

$= 27\int \sin^2\theta \sin\theta\,d\theta \quad u=\cos\theta \quad du = -\sin\theta\,d\theta$

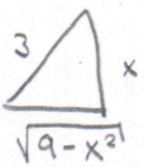
$\sin^2\theta = 1-\cos^2\theta$

$= -27\int \sin^2\theta\,du = -27\int (1-u^2) = -27(u - \frac{1}{3}u^3) + C$

$= -27(\cos\theta - \frac{1}{3}\cos^3\theta) + C$

TRiangle

$x/3 = \sin\theta = o/h$



$\therefore -27(\cos(\theta) - \frac{1}{3}\cos^3\theta) = -27\left(\frac{\sqrt{9-x^2}}{3}\right) + 9\left(\frac{\sqrt{9-x^2}}{3}\right)^3 \Bigg|_0^{\sqrt{5}}$

$= -27(2/3) + 9(8/27) + 27 - 9 = \boxed{8/3}$

(b) (7 points) $\displaystyle\int_1^4 \frac{\tan^{-1}(\sqrt{t})}{\sqrt{t}}\,dt$ (Recall that $\tan^{-1} = \arctan$.)

$u = \sqrt{t} \quad u^2 = t \quad 2u\,du = dt$

$\displaystyle\int_1^2 \frac{\tan^{-1}(u)\cdot 2u}{u} = 2\int_1^2 \tan^{-1}(u)\,du \qquad w = 2\tan^{-1}(u) \quad dv = du$

$dw = \dfrac{2}{1+u^2} \qquad v = u$

$= 2u\tan^{-1}(u)\Big|_1^2 - \displaystyle\int_1^2 \frac{2u}{1+u^2}\,du \qquad v = 1+u^2$

$dv = 2u\,du$

$= 2u\tan^{-1}(u)\Big|_1^2 - \displaystyle\int_2^5 \frac{1}{v}\,dv$

$= 2u\tan^{-1}(u)\Big|_1^2 - \ln|v|\Big|_2^5$

$\boxed{= 4\tan^{-1}(2) - 2(\pi/4) - \ln|5| + \ln|2|}$

3. (7 points) Evaluate the following integral.

$$\int \frac{x}{(x^2+2x-3)^{3/2}}\,dx$$

$\boxed{\text{TRIG}}$

① Complete the sq.

$$x^2+2x-3 = (x+1)^2-1-3 = (x+1)^2-4$$

② $\displaystyle\int \frac{x}{[(x+1)^2-4]^{3/2}}$ $\quad u=x+1,\ du=dx \quad$ $\displaystyle\int \frac{u-1}{(\sqrt{u^2-4})^3}\,du$ $\quad u=2\sec\theta,\ du=2\sec\theta\tan\theta\,d\theta$

$$=\int \frac{(2\sec\theta-1)\,2\sec\theta\tan\theta\,d\theta}{(\sqrt{4\sec^2\theta-4})^3} = \int \frac{(2\sec\theta-1)(\cancel{2}\sec\theta\cancel{\tan\theta})d\theta}{(\cancel{2}\cancel{\tan\theta})^3}$$

$$=\int \frac{(2\sec\theta-1)(\sec\theta)}{4\tan^2\theta}\,d\theta =$$

$$=\int \frac{2\sec^2\theta}{4\tan^2\theta}\,d\theta - \int \frac{\sec\theta}{4\tan^2\theta}\,d\theta$$

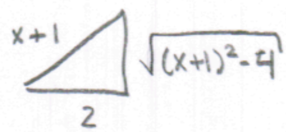
$$=\int \frac{2}{\cos^2\theta}\cdot\frac{\cos^2\theta}{4\sin^2\theta} - \frac{1}{4}\int \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}$$

$$=\int \frac{1}{2}\csc^2\theta - \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta} \qquad v=\sin\theta,\ dv=\cos\theta\,d\theta$$

$$=\frac{1}{2}\cot\theta - \frac{1}{4}\int \frac{1}{v^2} = \frac{1}{2}\csc\theta + \frac{1}{4\sin\theta} + C$$

③ Triangle $\quad \frac{u}{2}=\sec\theta=\frac{h}{a}$

$u=x+1$



$$\boxed{=\frac{1}{2}\left(\frac{2}{\sqrt{(x+1)^2-4}}\right)+\frac{1}{4}\left(\frac{x+1}{\sqrt{(x+1)^2-4}}\right)+C}$$

1. (12 total points) Evaluate the following integrals.

(a) (6 points) $\int x^3\cos(x^2)\,dx$ $\quad u=x^2 \quad du=2x\,dx$

$\int \frac{1}{2}x^2\cos(u)\,du = \frac{1}{2}\int u\cos(u)\,du$ IBP $\quad w=u \quad dv=\cos(u)$

$dw=du \quad v=\sin(u)$

$= \frac{1}{2}(u\sin(u)) - \frac{1}{2}\int \sin(u)\,du$

$= \frac{1}{2}u\sin(u) + \frac{1}{2}\cos(u) + C$

$$\boxed{= \frac{1}{2}x^2\sin(x^2) + \frac{1}{2}\cos(x^2) + C}$$

(b) (6 points) $\int \frac{(\sqrt{\ln(x)}+1)^3}{x\sqrt{\ln(x)}}\,dx$

$u=\sqrt{\ln(x)}+1$

$du = \frac{1}{2\sqrt{\ln(x)}}\cdot\frac{1}{x}\,dx$

$x\,2\sqrt{\ln(x)} = dx$

$= \int 2u^3\,du$

$= \frac{1}{2}u^4 + C = \boxed{\frac{1}{2}\left(\sqrt{\ln(x)}+1\right)^4 + C}$

2. (12 total points) Evaluate the following integrals.

(a) (6 points) $\displaystyle\int \frac{2x-1}{x^2-3x+2}\,dx$

$$\int \frac{2x-1}{(x-2)(x-1)}\,dx$$

Partial Fractions

① Long div. (No)

$$\boxed{3\ln|x-2| - \ln|x-1| + C}$$

② Form

$$\frac{2x-1}{(x-2)(x-1)} = \frac{A}{(x-2)} + \frac{B}{x-1}$$

$$\Rightarrow 2x-1 = A(x-1) + B(x-2)$$

$x=1$: $1 = B(-1) \Rightarrow B = -1$

$x=2$: $3 = A \quad \Rightarrow A = 3$

③ Integrate

$$\int \frac{3}{x-2} + \int \frac{-1}{x-1}$$

(b) (6 points) $\displaystyle\int \frac{\sqrt{3-2x-x^2}}{x+1}\,dx$ Trig sub.

① Complete sq.

$$-(x^2+2x-3) = -([x+1]^2-3-1) = 4-(x+1)^2$$

② $\displaystyle\int \frac{\sqrt{4-(x+1)^2}}{(x+1)}$ $u=x+1$ $du=dx$ $= \displaystyle\int \frac{\sqrt{4-u^2}}{u}\,du$ $\quad u = 2\sin\theta$, $du = 2\cos\theta\, d\theta$

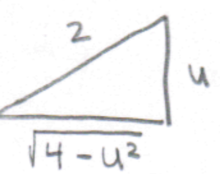
$$= \int \frac{\sqrt{4(1-\sin^2\theta)}}{2\sin\theta}\cdot 2\cos\theta\, d\theta = \int \frac{2\cos\theta\cdot 2\cos\theta}{2\sin\theta}\,d\theta = \int \frac{2\cos^2\theta}{\sin\theta}\,d\theta$$

$$= \int \frac{2(1-\sin^2\theta)}{\sin\theta} = \int \frac{2}{\sin\theta} - 2\sin\theta = 2\ln|\csc\theta - \cot\theta| + 2\cos(\theta) + C$$

③ Triangle

$$\frac{u}{2} = \sin\theta = \frac{o}{h}$$

$$\therefore \boxed{2\ln\left|\frac{2}{x+1} - \frac{\sqrt{4-(x+1)^2}}{x+1}\right| + 2\left(\frac{\sqrt{4-(x+1)^2}}{2}\right) + C}$$



$\underline{\underline{u = x+1}}$

1. (14 total points) Evaluate the following integrals.

(a) (7 points) $\int \sec^4(3x)\,dx$

Use Trig Integrals

* Pull out even powers and use u-sub to get rid of extra power.

$$\int \sec^2(3x)\cdot\sec^2(3x)\,dx \qquad u = \tan 3x \quad du = 3\sec^2(3x)\,dx$$

$$= \int \sec^2(3x)\left(\tfrac{1}{3}\right)du$$

$$= \int (1+\tan(3x)^2)\left(\tfrac{1}{3}\right)du$$

$$= \int \tfrac{1}{3}(1+u^2)\,du$$

$$= \tfrac{1}{3}\left(u + \tfrac{1}{3}u^3\right) + C$$ plug every-thing back in

$$\boxed{= \tfrac{1}{3}\left(\tan(3x) + \tfrac{1}{3}(\tan^3(3x))\right) + C}$$

(b) (7 points) $\int_3^{4.5} \sqrt{6x-x^2}\,dx$

① complete square

$$-(x^2-6x) = -([x-3]^2-9) = 9-(x-3)^2$$

$$\int_3^{4.5} \sqrt{(9-(x-3)^2)}\,dx \qquad u = x-3 \quad du = dx \qquad \int_0^{1.5} \sqrt{9-u^2}\,du \qquad u = 3\sin\theta \quad du = 3\cos\theta\,d\theta$$

$0 = 3\sin\theta$
$\theta = 0$
$\frac{3}{2} = 3\sin\theta$
$1/2 = \sin\theta$
$\theta = \pi/6$

$$= \int_0^{\pi/6} \sqrt{9-9\sin^2\theta}\cdot 3\cos\theta\,d\theta = \int_0 3\sqrt{1-\sin^2\theta}\;3\cos\theta\,d\theta$$

$$= \int_0^{\pi/6} 9\cos^2\theta\,d\theta \;\underset{\text{Double angle}}{\overset{\text{use}}{=}}\; \int_0^{\pi/6} \tfrac{9}{2}(1+\cos(2\theta))\,d\theta$$

$$= \tfrac{9}{2}\left(\theta + \tfrac{1}{2}\sin(2\theta)\right)\Big|_0^{\pi/6}$$

$$= \tfrac{9}{2}\left(\tfrac{\pi}{6} + \tfrac{1}{2}\sin(\pi/3)\right) - 0$$

$$\boxed{= \tfrac{9}{2}\left(\tfrac{\pi}{6} + \tfrac{\sqrt{3}}{4}\right)}$$

2. (14 total points) Evaluate the following integrals.

(a) (7 points) $\int \frac{\sin x \cos x}{\sin^2 x + 5 \sin x + 6} dx$ Let $u = \sin x$ $du = \cos x\, dx$

$\int \frac{u}{u^2+5u+6} du = \int \frac{u}{(u+2)(u+3)} du$

Use Partial Fractions:

① Long division (No)

② Form

$\frac{u}{(u+2)(u+3)} = \frac{A}{u+2} + \frac{B}{u+3} \Rightarrow$ $u = A(u+3) + B(u+2)$

$u=-3$: $-3 = B(-1)$ $B=3$

$u=-2$: $-2 = A(1)$ $A=-2$

③ $\int \frac{u}{(u+2)(u+3)} du = \int \frac{-2}{u+2} + \int \frac{3}{u+3} = -2\ln|u+2| + 3\ln|u+3| + C$

$\boxed{= -2\ln|\sin(x)+2| + 3\ln|\sin x + 3| + C}$

(b) (7 points) $\int_4^\infty \frac{\ln x}{x^{3/2}} dx$