3. (10 total points) A balloon is moving vertically up and down along a straight line above the ground, with the positive direction pointing up. The acceleration of the balloon at time t (in seconds) is given by $a(t) = -(t+5)$ ft/sec^2. The initial velocity of the balloon at time $t = 0$ is $v(0) = 12$ ft/sec.

   (a) (3 points) Find the velocity $v(t)$ of the balloon as a function of time t.

$a(t) = -(t+5)$

$v(t) = -(\frac{1}{2}t^2 + 5t) + C$

$v(0) = 12$ so

$12 = -(\frac{1}{2}0^2 + 5(0)) + C$

$\Rightarrow C = 12$

$\therefore$ $\boxed{v(t) = -(\frac{1}{2}t^2 + 5t) + 12}$

   (b) (4 points) Find the *total distance* traveled by the balloon from time $t = 0$ sec to time $t = 3$ sec.

Tot dist $= \int_0^3 |v(t)|dt$

① Find zero's

$0 = -\frac{1}{2}t^2 - 5t + 12$

$0 = t^2 + 10t - 24$

$0 = (t+12)(t-2)$

$t = 2$ or -12

Plug in $t=0$

$0^2 - 0 + 12 = 12$

Plug in $t=3$

$-\frac{1}{2}9 - 15 + 12$

$= -7.5$

② Chart

	$0 \le t \le 2$	$2 \le t \le 3$
$f(x)$	+	−

③ Make integral

$\int_0^2 -\frac{1}{2}t^2 - 5t + 12$

$+ \int_2^3 -(-\frac{1}{2}t^2 - 5t + 12)$

$= -\frac{1}{6}t^3 - \frac{5}{2}t^2 + 12t \Big|_0^2$

$+ \frac{1}{6}t^3 + \frac{5}{2}t^2 - 12t \Big|_2^3$

$= \frac{38}{3} - 9 + \frac{38}{3}$

$\boxed{= \frac{49}{3}}$

   (c) (3 points) The balloon hits the ground at time $t = 6$ sec. What was its initial height above the ground at time $t = 0$?

$v(t) = -\frac{1}{2}t^2 - 5t + 12$

$s(t) = -\frac{1}{6}t^3 - \frac{5}{2}t^2 + 12t + C$

$s(6) = 0$

$\Rightarrow 0 = -36 - 90 + 72 + C$

$54 = C$

$\boxed{\text{Hence, initial height was 54}}$

4. (8 points) A particle is moving along a straight line with acceleration $a(t) = 2t$. At time $t = 0$, its velocity is $v_0 = -4$. What is the *total distance* traveled by the particle from time $t = 0$ to time $t = 3$?

① $a(t) = 2t$

$v(t) = t^2 + C$

$v(0) = -4$

$\Rightarrow C = -4$

$v(t) = t^2 - 4$

~~$s(t) = \frac{1}{3}t^3 - 4t + C$~~

② Total Dist $= \int_0^3 |v(t)|$

a). Find zero's

$t^2 - 4 = 0 \quad t = 2$ or -2

b)

	$0 \le t \le 2$	$2 \le t \le 3$
$v(t)$	$-$	$+$

pick $t=0$: $0^2 - 4 = -4$

$t = 3$: $9 - 4 = 5$

c). Write Int.

$\int_0^2 -t^2 + 4 + \int_2^3 t^2 - 4$

$= \frac{-1}{3}t^3 + 4t \Big|_0^2 + \frac{1}{3}t^3 - 4t \Big|_2^3$

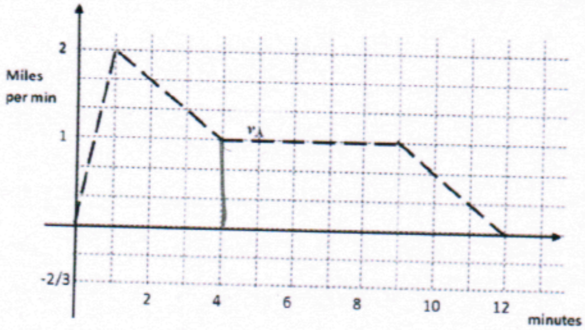
$= \frac{-8}{3} + 8 + 9 - 12 - \frac{8}{3} + 8$

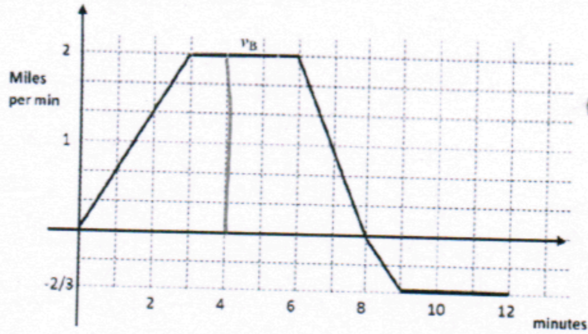
$\boxed{= \frac{23}{3}}$

5. (8 points) Approximate the integral $\frac{1}{\sqrt{2\pi}} \int_0^1 e^{-x^2/2}\,dx$ by using the trapezoidal rule with $n = 4$. Express your answer as a decimal.

5. (10 total points) Two cars, an Audi and a BMW, start driving in the same direction on a straight road at the same time and from the same place. The following two graphs show the velocities of the two cars (in miles per minute) along that road during the first 12 minutes of the trip. The dashed graph on the left labeled v_A represents the Audi's velocity, while the solid graph on the right labeled v_B shows the BMW's velocity. Use the graphs to answer the following questions.

Audi





BMW

(a) (2 points) What is the total distance traveled by the BMW during the 12 minutes?

Tot = |Area under curve|

$= \frac{(8+3)}{2} \cdot 2 + 3\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)\left(\frac{1}{2}\right)(1) = \boxed{40/3}$

(b) (2 points) How far apart are the cars from each other after 4 minutes, and which car is ahead at that time?

BMW Dist $= 2(3)(1/2) + 1(2) = 5$

Audi Dist $= 1(2)(1/2) + 3\left(\frac{2+1}{2}\right) = 5.5$

$\boxed{\text{Thus Audi ahead by } 1/2 \text{ mile}}$

(c) (2 points) What is the average velocity of the BMW during the first 4 minutes?

$\text{avg. vel} = \frac{d(4\text{ mins}) - d(0\text{ min})}{4-0} = \frac{5-0}{4} = \boxed{\frac{5}{4}}$

(d) (4 points) Define the function $f(x)$ to be $f(x) = \int_0^{2x} v_B(t)\,dt$. Compute $f'(2)$.

$f'(x) = v_B(2x) \cdot 2$ by FTC

$f'(2) = v_B(4) \cdot 2 = 2(2) = \boxed{4}$

3. (10 total points) An object is moving on the x-axis with acceleration $a(t) = 2t + 4$ for $0 \le t \le 4$. Its velocity at time $t = 0$ is $v(0) = -5$.

   (a) (5 points) What is the displacement of the object between times $t = 0$ and $t = 4$?

1

$a(t) = 2t + 4$

$v(t) = t^2 + 4t + C$

$v(0) = -5$

$-5 = 0^2 + 0 + C$

$C = -5$

$v(t) = t^2 + 4t - 5$

2

$\text{disp} = \int_0^4 t^2 + 4t - 5 \, dt$

$= \frac{1}{3}t^3 + 2t^2 - 5t \Big|_0^4$

$= 64/3 + 2(16) - 20$

$\boxed{= 100/3}$

   (b) (5 points) What is the total distance travelled by the object between times $t = 0$ and $t = 4$?

$\text{Total dist} = \int_0^4 |t^2 + 4t - 5| \, dt$

a Find zeros

$t^2 + 4t - 5 = 0$

$(t + 5)(t - 1) = 0$

$t = -5 \quad t = 1$

b Chart

	$0 \le t \le 1$	$1 \le t \le 4$
$v(t)$	$-$	$+$

$t = 0: \quad 0^2 + 0 - 5 = -5$

$t = 2: \quad 4 + 8 - 5 = 7$

c Integral

$\int_0^1 -t^2 - 4t + 5 + \int_1^4 t^2 + 4t - 5$

$= -\frac{1}{3}t^3 - 2t^2 + 5\Big|_0^1 + \frac{1}{3}t^3 + 2t^2 - 5t\Big|_1^4$

$= -\frac{1}{3} - 2 + 5 + \frac{64}{3} + 32 - 20 - \frac{1}{3} - 2 + 5$

$\boxed{= 116/3}$