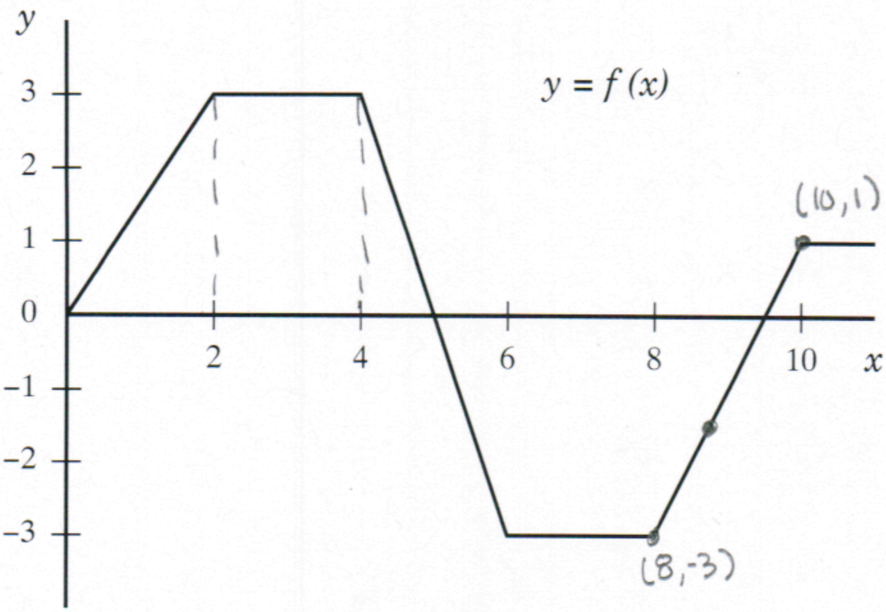
10. (8 total points) Let f be the function whose graph is given below, and let $g(x) = \int_4^x f(t)\,dt$.



Evaluate the following:

(a) (2 points) $g(0) =$

$g(0) = \int_4^0 f(t) = -\int_0^4 f(t) =$ - Area under curve from 4 to 0 $= -\left[\frac{1}{2}\cdot 2\cdot 3 + 2(3)\right]$

$\boxed{= -9}$

(b) (2 points) $g'(2) =$

By Fund Thm of Cal part 1,

$g'(x) = f(x) \Rightarrow g'(2) = f(2) = \boxed{3}$

(c) (2 points) $g''(9) =$

By FTC part 1,

$g'(x) = f(x) \Rightarrow g''(x) = f'(x) \Rightarrow g''(9) = f'(9)$

Recall, $f'(9)$ is the slope of tang. line at 9

~~(d) (2 points)~~ $\cancel{\int_0^2 t f(t^2)\,dt =}$

$\boxed{f'(9) = \frac{1+3}{10-8} = \frac{4}{2} = 2}$

Extra:

$u = t^2$

$du = 2t\,dt$

from part a

$\int_0^2 t f(t^2)\,dt = \frac{1}{2}\int_0^4 f(u)\,du = \frac{1}{2}(9) = \boxed{\frac{9}{2}}$

5. Pete is driving his car along a straight street. He starts at his work place and needs to deliver a packet to a customer. Not knowing the neighborhood too well he starts going in the wrong direction, but realizes his mistake soon. The velocity of his car is given by $v(t) = 90t^2 - 50t$ in mi/hour where t is measured in hours.

(a) (6pts) Pete reaches his destination after one hour. How far away does the customer live from Pete's work place?

They are asking for displacement.

Distance cust. lives away $= \int_0^1 90t^2 - 50t\,dt$

$= 30t^3 - 25t^2 \Big|_0^1$

$= (30 - 25) - [0 - 0]$

$\boxed{= 5 \text{ miles}}$

(b) (6pts) Pete's car is quite friendly to the environment, it can drive 35 miles per gallon fuel. How much fuel did Pete use up for this journey?

First, find how far Pete travelled. This is TOTAL distance

Total Dist $= \int_0^1 |90t^2 - 50t|\,dt$

① Find 0's

$90t^2 - 50t = 0 \Rightarrow t(90t - 50) = 0$

$t = 0$ OR $t = 5/9$

② Make chart. (Pick points)

$t<0$	0	$0<t<5/9$	$5/9$	$t>5/9$
+	0	−	0	+

$t = -1$: $v(-1) = 90(-1)^2 - 50(-1)$
$= 140$

$t = 1/2$ $v(\frac{1}{2}) = 90(\frac{1}{2})^2 - 50(\frac{1}{2})$
$= -2.5$

$t = 1$ $v(1) = 90(1^2) - 50(1)$
$= 40$

③ Break into pieces in the integral

$$\int_0^1 |90t^2-50t|\,dt = \int_0^{5/9} -(90t^2-50t)\,dt + \int_{5/9}^1 90t^2-50t$$

$$= -(30t^3-25t^2)\Big|_0^{5/9} + 30t^3-25t^2\Big|_{5/9}^1$$

$$= \frac{625}{243} + 30-25+\frac{625}{243}$$

$$\boxed{= \frac{2465}{243} \approx 10.144 \text{ miles}}$$

After finding distance, can calculate how many gallons

$$\text{Total gall} = \text{miles} \cdot \frac{\text{gall}}{\text{mile}}$$

$$= \frac{2465}{243} \cdot \frac{1}{35}$$

$$\boxed{= \frac{493}{1701} \approx .28983 \text{ gallons}}$$

3. (10 total points) A balloon is moving vertically up and down along a straight line above the ground, with the positive direction pointing up. The acceleration of the balloon at time t (in seconds) is given by $a(t) = -(t+5)$ ft/sec^2. The initial velocity of the balloon at time $t = 0$ is $v(0) = 12$ ft/sec.

   (a) (3 points) Find the velocity $v(t)$ of the balloon as a function of time t.

①Find equations $a(t), v(t)$

$a(t) = -(t+5)$

$v(t) = -\frac{1}{2}t^2 - 5t + C$

② initial conditions: $v(0) = 12$

$v(0) = -\frac{1}{2}(0^2) - 5(0) + C = 12$

$C = 12$

$\boxed{v(t) = -1/2\, t^2 - 5t + 12}$

   (b) (4 points) Find the *total distance* traveled by the balloon from time $t = 0$ sec to time $t = 3$ sec.

Total Distance $= \int_0^3 |-1/2\, t^2 - 5t + 12|$

①Find 0's

$0 = -1/2\, t^2 - 5t + 12$

$0 = -\frac{1}{2}(t-2)(t+12)$

$t = 2 \quad t = -12$

②Make Chart

$t<-12$	-12	$-12<t<2$	2	$2<t$
$-$	0	$+$	0	$-$

③ Rewrite integral

$\int_0^2 -\frac{1}{2}t^2 - 5t + 12 + \int_1^3 \frac{1}{2}t^2 + 5t - 12$

$= \left. \frac{-1}{6}t^3 - \frac{5}{2}t^2 + 12t \right|_0^2 + \frac{1}{6}t^3 + \frac{5}{2}t^2 - 12t \Big|$

$= \frac{38}{3} - 9 + \frac{38}{3}$

$\boxed{= 49/3 = 16.\overline{33}}$

   (c) (3 points) The balloon hits the ground at time $t = 6$ sec. What was its initial height above the ground at time $t = 0$?

1). Find $s(t)$:

$s(t) = \frac{-1}{6}t^3 - \frac{5}{2}t^2 + 12t + D$

2). Initial Conditions

$s(6) = 0$

$0 = \frac{-1}{6}(6^3) - \frac{5}{2}(6^2) + 12(6) + D$

$0 = -54 + D$

$\boxed{D = 54}$

* Note D is initial height.