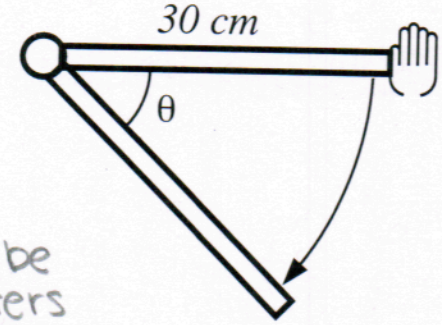
6. (8 points) A worker has to tighten a bolt using a wrench. She holds the wrench 30 cm away from the bolt. The more she tightens the bolt, the more force she has to use. The force she applies is equal to $3+\tan^2\theta$ newtons, where θ is the angle between the original position of the wrench and the current position, in radians.

   How much work does she do to turn the bolt $1/8$ of a full turn (that is, $\pi/4$ radians)?

   Give your answer in joules (1 joule = 1 newton-meter).

   (Hint: The distance traveled along a circle of radius r in moving an angle of $\Delta\theta$ radians is $r\Delta\theta$.)



① Force $= 3+\tan^2\theta$

② Distance $= r\,d\theta$
$= .30\,d\theta$

③ Bounds is $\theta = 0$ to $\pi/4$ ← Needs to be in meters

④ Work:

$$W = \int_0^{\pi/4} (3+\tan^2\theta)(.30)\,d\theta$$

$$= .30\int_0^{\pi/4} 3+[\sec^2\theta - 1]\,d\theta$$

$$= .30\,[3\theta + \tan\theta - \theta]\Big|_0^{\pi/4}$$

$$= .30\,[\pi/2 + \tan(\pi/4)]$$

$$= .30(\pi/2+1) = \boxed{.15\pi + .3}$$

6. (8 points) Hooke's law states that the force required to maintain a spring stretched x units beyond its natural length is proportional to x. The work required to stretch the spring from 2 feet beyond its natural length to 4 feet beyond its natural length is 18ft-lb. How far beyond its natural length can the spring be stretched with a force not exceeding 24 pounds?

Find k

① Force

kx ← distance in ft from natural length

② Distance = dx

↑ always

③ Bounds

2 to 4

$\therefore \quad 18 = \int_2^4 kx\,dx = \frac{k}{2}x^2\Big|_2^4 = k8 - 2k$

$18 = k(6) \quad \boxed{k = 3}$

Find how far spring stretched to not exceed 24 lbs ← This is a force

$F = kx \Rightarrow \quad 24 = 3x \Rightarrow \quad \boxed{8 = x}$

8. (10 total points) A spring has a natural length of 10 cm. The spring is now allowed to hang vertically, with the top end attached to a rigid support and the other end attached to a mass of 1 kg. This causes the spring to stretch 3 cm, to a length of 13 cm.

(a) (4 points) Find the spring constant k. The acceleration due to gravity is 9.8 m/sec^2.

Everything in meters

① Force = kx ← dist from natural length

ⓐ On the one hand
m = 1 kg a = 9.8
so F = 1 · 9.8 = 9.8

ⓑ On the other hand,
F = kx ⇒ k(.03)
∴ 9.8 = k(.03)
$k = \frac{980}{3}$

(b) (6 points) A small child pulls down on the mass, stretching the string to 15 cm. How much work does the child do?

① Force is
$F = kx = \frac{980}{3}x$

② dist = dx
↑always

③ Bounds (dist from natural length)
.03 to .15 − .1

④ Work
$$W = \int_{0.03}^{.05} \frac{980}{3} x \, dx = \frac{980}{3 \cdot 2} x^2 \Big|_{.03}^{.05} = \frac{980}{6}(.02)^2$$
$$= .065\overline{333} \text{ J}$$