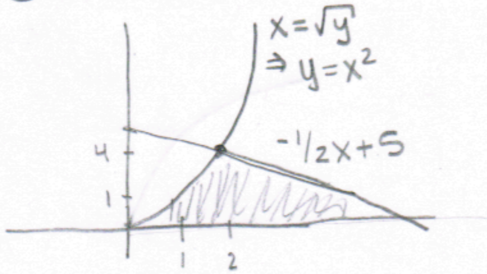
4. (10 total points) Let $\mathscr{R}$ be the region which is bounded on the left by the curve $x = \sqrt{y}$, bounded on the right by the line $y = -\frac{1}{2}x + 5$, and bounded below by the x-axis.

   (a) (5 points) Set up a definite integral (or integrals) *with respect to x* for the area of the region $\mathscr{R}$, and evaluate your integral(s). Give your answer in exact form.

① Draw Picture



② Points of intersection

$x=0,\ 0=-\frac{1}{2}x+5 \Rightarrow x=10$

and $x^2 = -\frac{1}{2}x+5 \Rightarrow x=2$

③ Make Chart

	$0 \le x \le 2$	$2 \le x \le 10$
Top	x^2	$-\frac{1}{2}x+5$
Bottom	0	0

④ Write integrals

$$\int_0^2 x^2\,dx + \int_2^{10} -\tfrac{1}{2}x+5\,dx$$

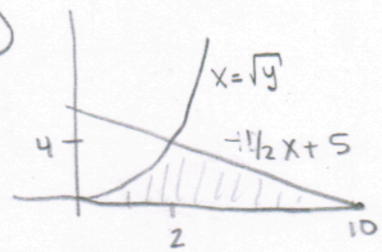
$$= \tfrac{1}{3}x^3 \Big|_0^2 + -\tfrac{1}{4}x^2 + 5x \Big|_2^{10}$$

$$= \tfrac{8}{3} - 25 + 50 + 1 - 10$$

$$\boxed{= 56/3}$$

   (b) (5 points) Set up a definite integral (or integrals) *with respect to y* for the area of the region $\mathscr{R}$, and evaluate your integral(s). Give your answer in exact form.

①



Rewrite

$-\frac{1}{2}x+5=y$

$\boxed{10-2y=x}$

② Points of intersection

$y=0$ and $\sqrt{y}=10-2y$

$y=4$

③ Make Chart

	$0 \le y \le 4$
Right	$10-2y$
left	$\sqrt{y}$

✱ Note a and b should match b/c same region.

④ Write Integrals

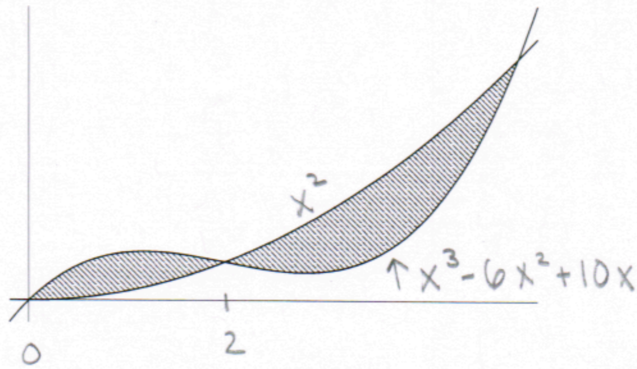
$$\int_0^4 10-2y-\sqrt{y}\,dy$$

$$= 10y - y^2 - \tfrac{2}{3}y^{3/2} \Big|_0^4$$

$$= 40 - 16 - \tfrac{16}{3}$$

$$\boxed{= 56/3}$$

$\boxed{4}$ (10 points) Compute the total area bounded by the curves $y = x^2$ and $y = x^3 - 6x^2 + 10x$.



(1) Draw Picture (Look at chart for y/x)

(2) Find intersection pts

$x^2 = x^3 - 6x^2 + 10x$

$0 = x^3 - 7x^2 + 10x$

$0 = x(x^2 - 7x + 10)$

$0 = x(x-5)(x-2)$

$x = 5,\ x = 0,\ x = 2$

(3) Make chart

	$0 \le x \le 2$	$2 \le x \le 5$
Top	$x^3 - 6x^2 + 10x$	x^2
bottom	x^2	$x^3 - 6x^2 + 10x$

(4) Write Integrals

$$\int_0^2 x^3 - 6x^2 + 10x - x^2\,dx + \int_2^5 x^2 - x^3 + 6x^2 - 10x\,dx$$

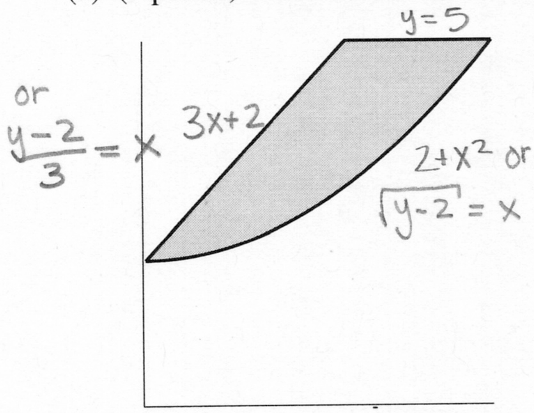
$$\frac{1}{4}x^4 - \frac{7}{3}x^3 + 5x^2 \Big|_0^2 + \frac{-1}{4}x^4 + 7/3x^3 - 5x^2 \Big|_2^5$$

$$= 16/3 + 125/12 + 16/3$$

$$= \boxed{\frac{253}{12}}$$

5. (10 total points) Let $\mathscr{R}$ be the region in the first quadrant bounded by the curves $y = 2 + x^2$ on the right, $y = 5$ on top, and $y = 3x + 2$ on the left.

(a) (5 points) Find the area of the region $\mathscr{R}$.



① Find the pts of intersection

$3x+2 = 2+x^2$

$\Rightarrow 0 = x^2 - 3x = x(x-3)$

$x = 0$ or $x = 3$

[observe when $x=3$, $2+(3^2) = 11 > 5$]

The pts in terms of y are $y=2$ and $y=5$

② Make Chart -

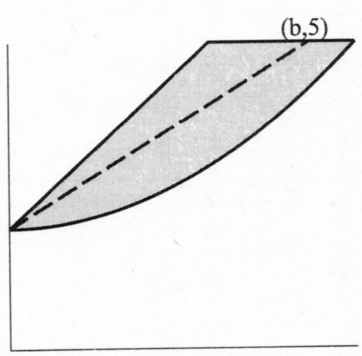
	$2 \le y \le 5$
Right	$\sqrt{y-2}$
Left	$y-2/3$

③ Integrate

$$\int_2^5 \sqrt{y-2} - \frac{1}{3}(y-2)\,dy \qquad u = y-2 \quad du = dy$$

$$\int_0^3 \sqrt{u} - \frac{1}{3}u\,du = \frac{2}{3}u^{3/2} - \frac{1}{6}u^2 \Big|_0^3 = \frac{2}{3}(3^{3/2}) - \frac{1}{6}(3^2) = \boxed{2\sqrt{3} - \frac{3}{2}}$$

(b) (5 points) The line through $(0,2)$ and $(b,5)$ divides $\mathscr{R}$ into two regions of equal area. Find b.



Find the equation of the line

$\frac{5-2}{b-0} = \frac{3}{b} = m \qquad y = \frac{3}{b}(x-0)+2$

$\Rightarrow \frac{b}{3}(y-2) = x$

Total area

$$\int_2^5 \frac{b}{3}(y-2) - \frac{y-2}{3}\,dy = \frac{1}{2}\left(2\sqrt{3} - \frac{3}{2}\right)$$

$$\Rightarrow \frac{b}{3}\left(\frac{1}{2}y^2 - 2y\right) - \frac{1}{6}y^2 + \frac{2}{3}y \Big|_2^5 = \sqrt{3} - \frac{3}{4}$$

$$\frac{5b}{6} - \frac{25}{6} + \frac{10}{3} + \frac{2b}{3} + \frac{4}{6} - \frac{4}{3}$$

$$\frac{9}{6}b - \frac{3}{2} = \sqrt{3} - \frac{3}{4} \Rightarrow \frac{3}{2}b = \sqrt{3} - 3/4$$

$$\boxed{b = \frac{2\sqrt{3}}{3} + \frac{1}{2}}$$