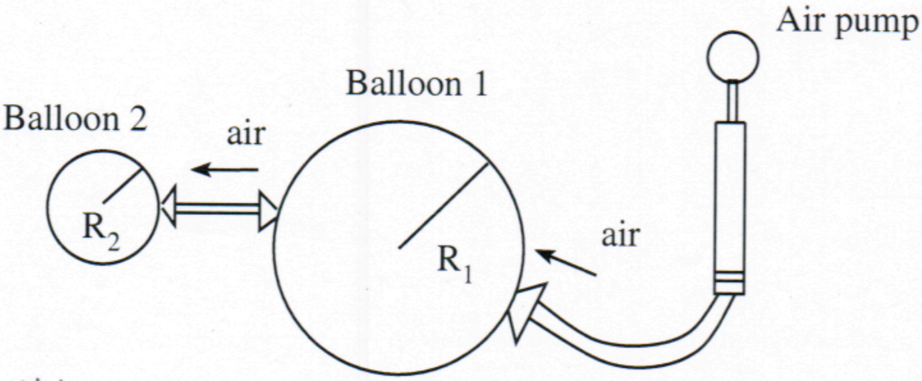
6. (12 points) Balloon 1 is linked by a large tube to an air pump and by a smaller tube to Balloon 2 (see picture). The radius of Balloon 1 is R_1 and the radius of Balloon 2 is R_2.

Air is being pumped in Balloon 1 at the constant rate of $101 \text{cm}^3/\text{minute}$ and air is leaking out of Balloon 1 (and into Balloon 2) at a total rate equal to π times the rate of change of R_1, in $\text{cm}^3/\text{minute}$.

At time t_0 measurements say that $R_1 = 5$ and $R_2 = 2$.

Calculate the rate of change of R_2 at that time.



Volume of Balloon 1 $= 4/3 \pi R_1^3 \Rightarrow \frac{dV_1}{dt} = 4\pi (R_1)^2 \cdot dR_1/dt$

$\frac{dV_1}{dt}$ = Rate in - Rate out $\Rightarrow 4\pi (R_1)^2 \frac{dR_1}{dt} = 101 - \pi \frac{dR_1}{dt}$

Volume of Ball. 2 $= 4/3 \pi R_2^3 \Rightarrow 4\pi R_2^2 \cdot \frac{dR_2}{dt} = \frac{dV_2}{dt} = \pi R_1/dt$

$\Rightarrow 4 R_2^2 \frac{dR_2}{dt} = \frac{dR_1}{dt}$

Plugging this in, we get that

$$4\pi (R_1)^2 \cdot 4R_2^2 \frac{dR_2}{dt} = 101 - \pi 4 R_2^2 \frac{dR_2}{dt}$$

$$\Rightarrow 4\pi (5^2) \cdot 4(2^2) \frac{dR_2}{dt} = 101 - \pi 4 (2^2) \frac{dR_2}{dt}$$

$$\boxed{\frac{dR_2}{dt} = \frac{101}{1616\pi}}$$