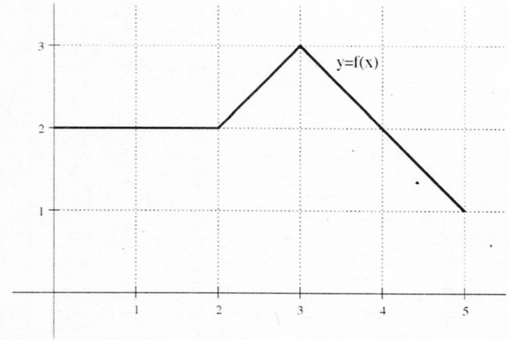
Name Solutions Quiz Section ______

In this worksheet, we explore the Fundamental Theorem of Calculus and applications of the Area Problem to problems involving distance and velocity. We also consider integrals involving net and total change.

FTC Practice

1 Let $f(x)$ be given by the graph to the right and define $A(x) = \int_0^x f(t)\,dt$. Compute the following.



$A(1) =$ 2 $A(2) =$ 4

$A(3) =$ $6+\frac{1}{2}=\frac{13}{2}$ $A(4) =$ 9

$A'(1) =$ 2 $A'(2) =$ 2

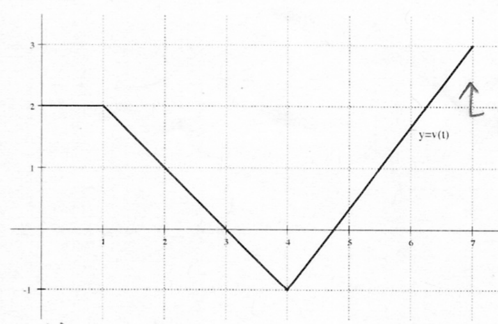
$A'(3) =$ 3 $A'(4) =$ 2

The maximum value of $A(x)$ on the interval $[0,5]$ is $10+\frac{1}{2}=\frac{21}{2}=A(5)$

The maximum value of $A'(x)$ on the interval $[0,5]$ is $3=A'(3)$

Velocity and Distance

2 A toy car is travelling on a straight track. Its velocity $v(t)$, in m/sec, be given by the graph to the right. Define $s(t)$ to be the position of the car in meters. Choose coordinates so that $s(0) = 0$. Compute the following.



$\frac{4}{3}$ slope

$s(2) =$ $3\frac{1}{2}$ $s(4) =$ $3\frac{1}{2}$ $s(6) =$ $\frac{25}{6}=4\frac{1}{6}$

$v(2) =$ 1 $v(4) =$ -1 $v(6) =$ $\frac{5}{3}=1\frac{2}{3}$

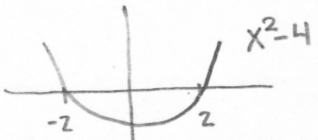
The maximum value of $s(t)$ on the interval $[0,7]$ is $s(7)=\frac{13}{2}=6\frac{1}{2}$

The minimum value of $s(t)$ on the interval $[0,7]$ is $s(0)=0$

The maximum value of $v(t)$ on the interval $[0,7]$ is $v(7)=3$

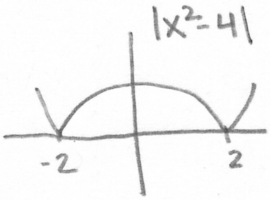
The minimum value of $v(t)$ on the interval $[0,7]$ is $v(4)=-1$

3 (a) Evaluate $\int_{-2}^{2} |x^2-4|\, dx$ and $\left|\int_{-2}^{2} (x^2-4)\, dx\right|$ and explain your answers.



① Find the zeros: $x^2-4=0 \Rightarrow x = \pm 2$

② Determine if + or − by plugging points



	$-\infty < x < -2$	$-2 < x < 2$	$2 < x < \infty$
x^2-4	$x=-3$ 5	$x=0$ −4	$x=3$ 5
$\lvert x^2-4\rvert$	$x=-3$ 5	$x=0$ 4	$x=3$ 5
multiple by ± for two above to agree	+	−	+

(**)

③ Break up integral using chart

$$\int_{-2}^{2} |x^2-4|\, dx = \int_{-2}^{2} -(x^2-4)\, dx = -\tfrac{1}{3}x^3 + 4x \Big|_{-2}^{2} = \frac{16}{3} + \frac{16}{3} = \frac{32}{3}$$

For

$$\left|\int_{-2}^{2} (x^2-4)\, dx\right| = \left| \tfrac{1}{3}x^3 - 4x \Big|_{-2}^{2} \right| = \left| -\frac{16}{3} - \frac{16}{3} \right| = \frac{32}{2}$$

These are the same b/c x^2-4 has the same sign for $-2 \le x \le 2$

(b) Now evaluate $\int_{-3}^{3} |x^2-4|\, dx$ and $\left|\int_{-3}^{3} (x^2-4)\, dx\right|$ and explain your answers.

① Find the zeros $x^2-4=0 \Rightarrow x = \pm 2$

② Determine if + or − From above,

$-\infty < x < -2$	$-2 < x < 2$	$2 < x < \infty$
+	−	+

③ Break up integral using Chart

$$\int_{-3}^{3} |x^2-4|\, dx = \int_{-3}^{-2} x^2-4\, dx + \int_{-2}^{2} -(x^2-4)\, dx + \int_{2}^{3} x^2-4\, dx$$

$$= \tfrac{1}{3}x^3 - 4x \Big|_{-3}^{-2} + \left(-\tfrac{1}{3}x^3 + 4x\right)\Big|_{-2}^{2} + \left(\tfrac{1}{3}x^3 - 4x\right)\Big|_{2}^{3}$$

$$= 46/3$$

For

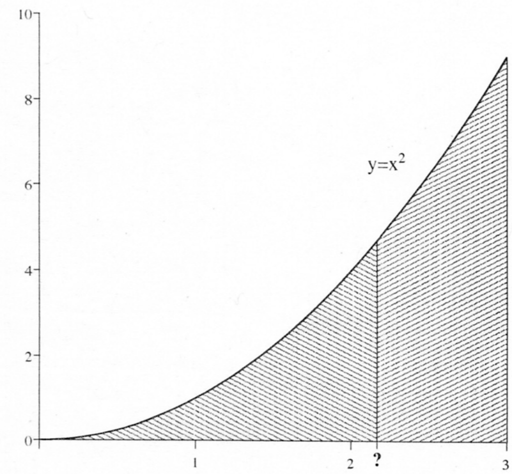
$$\left|\int_{-3}^{3} x^2-4\, dx\right| = \left| \tfrac{1}{3}x^3 - 4x \Big|_{-3}^{3} \right| = |9-12+9-12| = 6$$

Not the same b/c x^2-4 changes sign in the interval $-3 \le x \le 3$

4 An artist you know wants to make a figure consisting of the region between the curve $y = x^2$ and the x-axis for $0 \le x \le 3$

(i) Where should the artist divide the region with a vertical line so that each piece has the same area? (See the picture.)

(ii) Where should the artist divide the region with vertical lines to get 3 pieces with equal areas?



(i) ① Find the total area

Total Area = $\int_0^3 x^2\,dx = \frac{1}{3}x^3\Big|_0^3 = 9$

② Set up integral for 1/2 the area

$\frac{9}{2} = \int_0^a x^2\,dx = \frac{1}{3}x^3\Big|_0^a = \frac{1}{3}a^3 \Rightarrow a = \sqrt[3]{\frac{27}{2}} = \frac{3}{2^{1/3}} \approx 2.3811016$

(ii) ① Find the total area

Total Area = $\int_0^3 x^2\,dx = 9$

② Set up integral for 1/3 of the area

$3 = \int_0^a x^2 dx$ and $3 = \int_b^3 x^2\,dx = \frac{1}{3}x^3\Big|_b^3 = 9 - \frac{1}{3}b^3$

$3 = \frac{1}{3}x^3\Big|_0^a$ $\qquad 6 = \frac{1}{3}b^3$

$3 = \frac{1}{3}a^3 \Rightarrow 9^{1/3} = a \approx 2.0800838$ $\qquad b = 18^{1/3} \approx 2.62074$