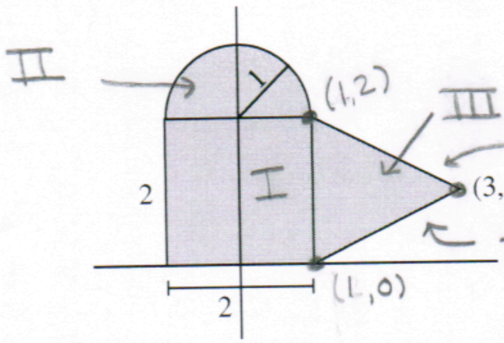
8. (10 points) Find the *x*-coordinate of the centroid of the shaded region below.



$\frac{2-1}{1-3} = \frac{-1}{2}$ $\quad y = \frac{-1}{2}(x-1)+2 = \frac{-1}{2}x + 5/2$

$\frac{1-0}{3-1} = \frac{1}{2}$ $\quad y = \frac{1}{2}(x-1) = \frac{1}{2}x - 1/2$

M_y of I = 0 by symmetry

M_y of II = 0 by symmetry

$$M_y \text{ of III} = \int_1^3 x\left[\frac{-1}{2}x + \frac{5}{2} - 1/2x + 1/2\right]$$

$$= \int_1^3 x[-x+3] = \int_1^3 -x^2 + 3x\,dx$$

$$= \frac{-1}{3}x^3 + \frac{3}{2}x^2\Big|_1^3 = \frac{9}{2} + \frac{1}{3} - \frac{3}{2} = \boxed{\frac{10}{3}}$$

Now

$$\bar{x} = \frac{M_y \text{ of I} + M_y \text{ of II} + M_y \text{ of III}}{\text{Area of I} + \text{Area of II} + \text{Area of III}}$$

Area of I $= 2 \cdot 2 = 4$

Area of II $= \frac{\pi(1)}{2} = \frac{\pi}{2}$

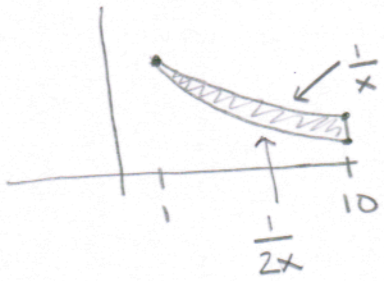
Area of III $= \frac{2(2)}{2} = 2$

$$\therefore \bar{x} = \frac{0 + 0 + 10/3}{4 + 2 + \pi/2} = \boxed{\frac{10}{18 + \frac{3}{2}\pi}}$$

9. (10 total points) Consider the region bounded by $x = 1$, $x = 10$, $y = \frac{1}{x}$, and $y = \frac{1}{2x}$.

(a) (8 points) Find the centroid of this region.

① Draw Picture



② Find Area

$$A = \int_1^{10} \frac{1}{x} - \frac{1}{2x} = \ln(x) - \frac{1}{2}\ln(x)\Big|_1^{10} = \ln(10) - \frac{1}{2}\ln(10)$$
$$= \frac{1}{2}\ln(10)$$

③ Find $\bar{x}$

$$\bar{x} = \frac{1}{A}\int_1^{10} x\left[\frac{1}{x} - \frac{1}{2x}\right] = \frac{1}{A}\int_1^{10} 1 - \frac{1}{2}\,dx = \frac{1}{A}\left(\frac{1}{2}x\right)\Big|_1^{10} = \frac{1}{A}(5 - \tfrac{1}{2})$$
$$= \frac{9}{2(1-\frac{1}{2})\ln(10)} = \boxed{\frac{9}{\ln(10)}}$$

④ Find $\bar{y}$

$$\bar{y} = \frac{1}{A}\cdot\frac{1}{2}\int_1^{10} \left(\frac{1}{x}\right)^2 - \left(\frac{1}{2x}\right)^2 dx = \frac{1}{2A}\left(\frac{-1}{x} + \frac{1}{4}\frac{1}{x}\right)\Big|_1^{10} = \frac{1}{A}\left(\frac{-3}{40} + \frac{3}{4}\right)\left(\frac{1}{2}\right)$$
$$= \frac{2}{\ln(10)}\left(\frac{27}{40}\right)\left(\frac{1}{2}\right)$$
$$\boxed{= \frac{27}{40\ln(10)}}$$

The centroid is $\left(\frac{9}{\ln(10)}, \frac{27}{40\ln(10)}\right)$

(b) (2 points) Determine whether the centroid lies inside the region.

To check whether inside region, plug in $\bar{x}$ into both the given eq.

$x = \frac{9}{\ln(10)}$: $\frac{1}{9/\ln(10)} = .2558427$

: $\frac{1}{2(9/\ln(10))} = .12792139$

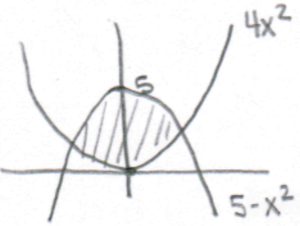
$\frac{27}{40\ln(10)} = .293148 \rightarrow$ This is not btwn those 2 values

Thus centroid lies outside of the region

9. (10 total points) Consider the region $\mathscr{R}$ bounded between the curves $y = 5 - x^2$ and $y = 4x^2$.

(a) (3 points) Find the area of $\mathscr{R}$.

① Draw Picture:



② Find Pts of Intersection

$5 - x^2 = 4x^2$

$5 = 5x^2 \quad x = \pm 1$

③

	$-1 \le x \le 1$
Top	$5-x^2$
Bottom	$4x^2$

④ Integrate

$A = \int_{-1}^{1} 5 - x^2 - 4x^2 \, dx$

$= \int_{-1}^{1} 5 - 5x^2 \, dx$

$= 5x - \frac{5}{3}x^3 \Big|_{-1}^{1}$

$= \frac{10}{3} + \frac{10}{3}$

$\boxed{= \frac{20}{3}}$

(b) (3 points) Find the x-coordinate $\bar{x}$ of the centroid (center of mass) of $\mathscr{R}$.

$\bar{x} = \frac{1}{A}\int_{-1}^{1} x(\text{Top} - \text{Bott})$

$= \frac{1}{A}\int_{-1}^{1} x(5 - x^2 - 4x^2)$

$= \frac{1}{A}\int_{-1}^{1} x(5 - 5x^2)$

$= \frac{1}{A}\int_{-1}^{1} 5x - 5x^3 \, dx$

$= \frac{1}{A}\left(\frac{5}{2}x^2 - \frac{5}{4}x^4\right)\Big|_{-1}^{1}$

$= \frac{1}{A}\left(\frac{5}{4} - \frac{5}{4}\right)$

$= 0$

$\boxed{\bar{x} = 0}$ ← by symmetry as well

(c) (4 points) Find the y-coordinate $\bar{y}$ of the centroid (center of mass) of $\mathscr{R}$.

$\bar{y} = \frac{1}{A}\left(\frac{1}{2}\right)\int_{-1}^{1} [5 - x^2]^2 - [4x^2]^2 \, dx$

$= \frac{1}{2A}\int_{-1}^{1} 25 - 10x^2 + x^4 - 16x^4 \, dx$

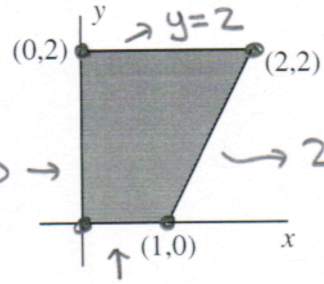
$= \frac{1}{2A}\int_{-1}^{1} 25 - 10x^2 - 15x^4 \, dx$

$= \frac{1}{2A}\left(25x - \frac{10}{3}x^3 - \frac{15}{5}x^5\right)\Big|_{-1}^{1}$

$= \frac{1}{2A}\left(\frac{56}{3} + \frac{56}{3}\right)$

$= \frac{1}{2(20/3)} \cdot \frac{112}{3} = \boxed{\frac{14}{5}}$

9. (8 points) Find the x-coordinate $\bar{x}$ of the center of mass of the region below.



$\rightarrow y=2$

$\rightarrow 2x-2$

$x=0 \rightarrow$

$\uparrow y=0$

* Find the equations of bounding functions

$\frac{2-0}{2-1} = 2 \leftarrow$ slope

$y = 2(x-1)+0$

$y = 2x-2$

FIND AREA

① Find pts of intersection:
From graph the pts are
$x=0,\ x=1,\ x=2$

② Make chart

	$0 \le x \le 1$	$1 \le x \le 2$
Top	2	2
Bott	0	$2x-2$

③ Integrate

$$A = \int_0^1 2\,dx + \int_1^2 2-(2x-2)\,dx$$

$$= 2(1) + \int_1^2 4-2x\,dx$$

$$= 2 + (4x - x^2)\Big|_1^2$$

$$= 2 + 8 - 4 - 4 + 1$$

$$= \boxed{3}$$

FIND $\bar{X}$

$$\bar{x} = \frac{1}{A}\int_0^1 x(2) + \frac{1}{A}\int_1^2 x(2-2x+2)\,dx$$

$$= \frac{1}{A}x^2\Big|_0^1 + \frac{1}{A}\int_1^2 4x - 2x^2\,dx$$

$$= \frac{1}{A} + \frac{1}{A}\left(2x^2 - \frac{2}{3}x^3\right)\Big|_1^2$$

$$= \frac{1}{A} + \frac{1}{A}\left(\frac{8}{3} - \frac{4}{3}\right)$$

$$= \frac{1}{A} + \frac{1}{A}\left(\frac{4}{3}\right)$$

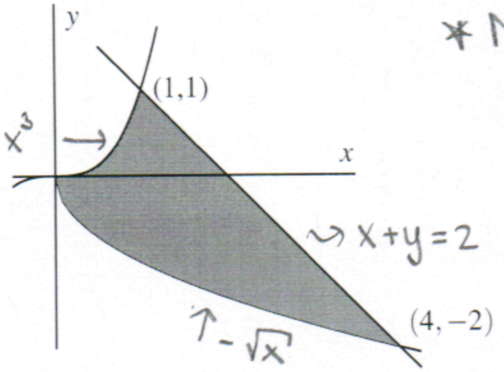
$$= \frac{1}{3} + \frac{1}{3}\left(\frac{4}{3}\right)$$

$$\boxed{= 7/9}$$

9. (8 points) Consider the region bounded by the curves

$$y = x^3, \quad x + y = 2, \quad y = -\sqrt{x}.$$

The area of this region is $49/12$. Find the x-coordinate of its center of mass. Leave your answer in exact form: do not use decimal expansions.



* Note Told you area of the region

① Find pts of intersection

$x=0,\ x=1,\ x=4$ (from graph)

② Make Chart

	$0 \le x \le 1$	$1 \le x \le 4$
Top	x^3	$2-x$
Bottom	$-\sqrt{x}$	$-\sqrt{x}$

Integrate

③

$$\bar{x} = \frac{1}{A}\left[\int_0^1 x(x^3+\sqrt{x})\,dx + \int_1^4 x(2-x+\sqrt{x})\,dx\right]$$

$$= \frac{1}{A}\left[\int_0^1 x^4 + x^{3/2}\,dx + \int_1^4 2x - x^2 + x^{3/2}\,dx\right]$$

$$= \frac{1}{A}\left[\left(\frac{1}{5}x^5 + \frac{2}{5}x^{5/2}\right)\Big|_0^1 + \left(x^2 - \frac{1}{3}x^3 + \frac{2}{5}x^{5/2}\right)\Big|_1^4\right]$$

$$= \frac{1}{A}\left[\left(\frac{3}{5}\right) + \left(\frac{112}{15}\right) - \left(\frac{16}{15}\right)\right]$$

$$= \frac{1}{(49/12)}\left[7\right] = \boxed{\frac{12}{7}}$$