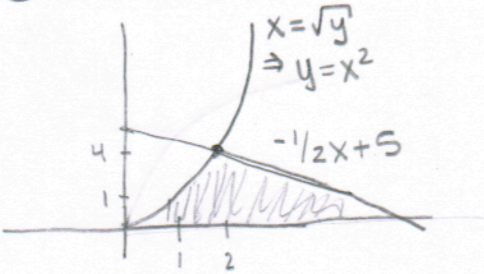
4. (10 total points) Let $\mathscr{R}$ be the region which is bounded on the left by the curve $x = \sqrt{y}$, bounded on the right by the line $y = -\frac{1}{2}x + 5$, and bounded below by the x-axis.

   (a) (5 points) Set up a definite integral (or integrals) *with respect to x* for the area of the region $\mathscr{R}$, and evaluate your integral(s). Give your answer in exact form.

① Draw Picture



② Points of intersection

$x=0$, $0 = -\frac{1}{2}x + 5 \Rightarrow x = 10$

and $x^2 = -\frac{1}{2}x + 5 \Rightarrow x = 2$

③ Make Chart

	$0 \le x \le 2$	$2 \le x \le 10$
Top	x^2	$-\frac{1}{2}x+5$
Bottom	0	0

④ Write integrals

$$\int_0^2 x^2\,dx + \int_2^{10} -\tfrac{1}{2}x + 5\,dx$$

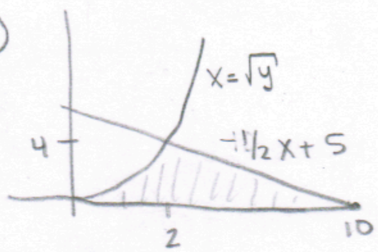
$$= \tfrac{1}{3}x^3 \Big|_0^2 + -\tfrac{1}{4}x^2 + 5x \Big|_2^{10}$$

$$= \frac{8}{3} - 25 + 50 + 1 - 10$$

$$\boxed{= 56/3}$$

   (b) (5 points) Set up a definite integral (or integrals) *with respect to y* for the area of the region $\mathscr{R}$, and evaluate your integral(s). Give your answer in exact form.

①



Rewrite

$-\frac{1}{2}x + 5 = y$

$\boxed{10 - 2y = x}$

② Points of intersection

$y = 0$ and $\sqrt{y} = 10 - 2y$

$y = 4$

③ Make Chart

	$0 \le y \le 4$
Right	$10-2y$
left	$\sqrt{y}$

✗ Note a and b should match b/c same region.

④ Write Integrals

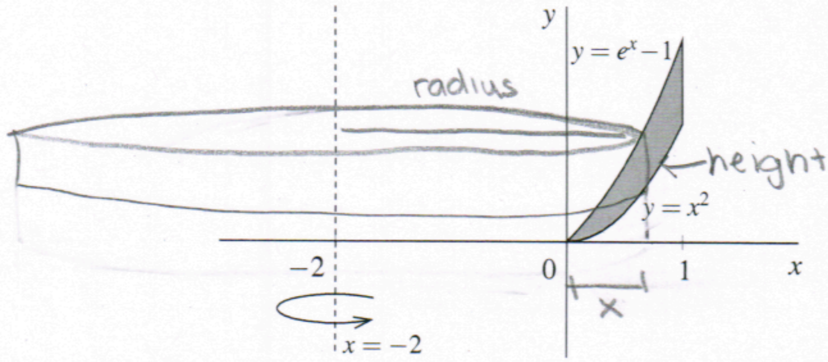
$$\int_0^4 10 - 2y - \sqrt{y}\,dy$$

$$= 10y - y^2 - \tfrac{2}{3}y^{3/2} \Big|_0^4$$

$$= 40 - 16 - \frac{16}{3}$$

$$\boxed{= 56/3}$$

4. (8 total points) The region between $y = x^2$, $y = e^x - 1$, $x = 0$, and $x = 1$ is rotated about the vertical line $x = -2$ to form a solid.



(a) (4 points) Set up an integral for the volume of this solid using *CYLINDRICAL SHELLS*.
DO NOT EVALUATE THE INTEGRAL.

① Draw Picture (Done)

② Look at Chart (See want $y = $ "x") Everything in x

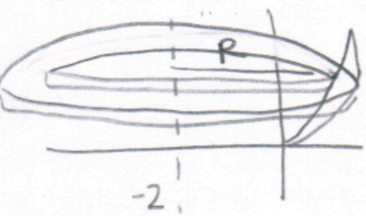
③ Find intersection pts From picture 0 and 1

④ Height $= (e^x - 1) - x^2$
Radius $= x + 2$

⑤
$$\boxed{\text{Volume} = 2\pi \int_0^1 (x+2)\left[(e^x - 1) - x^2\right] dx}$$

(b) (4 points) Set up an integral (or integrals) for the volume of this solid using *WASHERS*.
DO NOT EVALUATE THE INTEGRAL(S).

① Draw Picture



② Look at Chart (See $x = $ "y")
$y = x^2 \Rightarrow \sqrt{y} = x$
$y = e^x - 1 \Rightarrow \ln(y+1) = x$

③ Find intersections (in y)
$y = 0$ from picture
$\sqrt{y} = 1 \Rightarrow y = 1$ and $y = e^1 - 1 = e - 1$

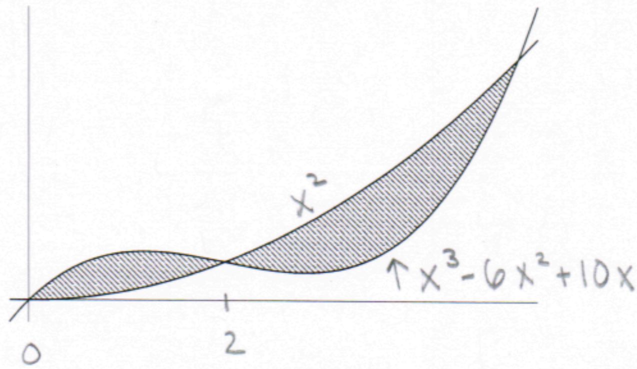
④ Chart (Find Radius)

	$0 \le y \le 1$	$1 \le y \le e-1$
Outer Radius	$\sqrt{y} + 2$	$1+2$
Inner Radius	$\ln(y+1) + 2$	$\ln(y+1) + 2$

⑤ Set up integral

$$\pi \int_0^1 (\sqrt{y} + 2)^2 - (\ln(y+1) + 2)^2 \, dy$$
$$+ \pi \int_1^{e-1} (3^2) - (\ln(y+1) + 2)^2 \, dy$$

4 (10 points) Compute the total area bounded by the curves $y = x^2$ and $y = x^3 - 6x^2 + 10x$.



① Draw Picture (Look at chart for y/x)

② Find intersection pts

$x^2 = x^3 - 6x^2 + 10x$

$0 = x^3 - 7x^2 + 10x$

$0 = x(x^2 - 7x + 10)$

$0 = x(x-5)(x-2)$

$x = 5,\ x = 0,\ x = 2$

③ Make chart

	$0 \le x \le 2$	$2 \le x \le 5$
Top	$x^3 - 6x^2 + 10x$	x^2
bottom	x^2	$x^3 - 6x^2 + 10x$

④ Write Integrals

$$\int_0^2 x^3 - 6x^2 + 10x - x^2\,dx + \int_2^5 x^2 - x^3 + 6x^2 - 10x\,dx$$

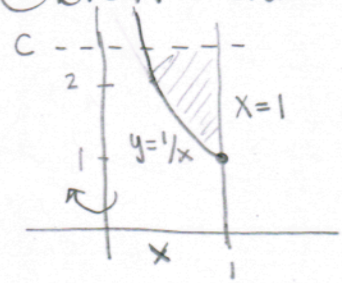
$$\frac{1}{4}x^4 - \frac{7}{3}x^3 + 5x^2 \Big|_0^2 + \frac{-1}{4}x^4 + 7/3x^3 - 5x^2 \Big|_2^5$$

$$= 16/3 + 125/12 + 16/3$$

$$= \boxed{\frac{253}{12}}$$

5. (8 points) Consider the region bounded by the curve $y = 1/x$, the line $x = 1$, and the line $y = c$ for some constant $c > 1$. Rotate this region about the y-axis. For what value of c is the volume of the resulting solid equal to 2π?

① Draw Picture



② Use chart: Since told you $y = 1/x$ and rotating about vert. line, use Shells. [Want x values]

③ Find intersections

$x = 1$ from picture

$1/x = c \Rightarrow x = 1/c$

④ Make Chart

	$1/c \le x \le 1$
Radius	x
Height	$c - 1/x$

⑤ Write integrals

$$2\pi \int_{1/c}^{1} x(c - 1/x)\,dx$$

Want

$$2\pi = 2\pi \int_{1/c}^{1} x(c - 1/x)\,dx$$

$$\Rightarrow 1 = \int_{1/c}^{1} cx - 1\,dx$$

$$\Rightarrow 1 = c/2\,x^2 - x \Big|_{1/c}^{1}$$

$$\Rightarrow 1 = c/2 - 1 - 1/2 - 1/c$$

$$\Rightarrow 5/2 = \frac{c}{2} - 1/c$$

$$\Rightarrow 5/2 = \frac{c^2 - 2}{2c} \Rightarrow 5c = c^2 - 2$$

$$\Rightarrow c^2 - 5c - 2 = 0$$

$$c = \frac{5 \pm \sqrt{25 + 4(2)(1)}}{2}$$

$$c = \frac{5 \pm \sqrt{33}}{2} \approx 5.3722$$

OR

$$-0.37228$$

$$\boxed{\text{Since } c > 1,\ c = \frac{5 + \sqrt{33}}{2}}$$