Math 125 **Final Examination** **Autumn 2012**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.
- A scientific calculator is needed, but graphing and/or programmable calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text (p. 484 if you have the 6th edition of Stewart) without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	12	
2	12	
3	8	
4	8	
5	10	

Question	Points	Score
6	8	
7	10	
8	10	
9	10	
10	12	
Total	100	

1. (12 total points) Evaluate the following integrals.

   (a) (6 points) $\displaystyle\int \frac{2+3x}{\sqrt{4+x^2}}\,dx$

   (b) (6 points) $\displaystyle\int \frac{\ln x}{(1+x)^2}\,dx$

2. (12 total points) Evaluate the following integrals.

   (a) (6 points) $\displaystyle\int_0^{\pi} |x\cos x|\ dx$ Give your answer in exact form.

   (b) (6 points) $\displaystyle\int_2^{3} \frac{1}{x(\ln(3x))^4}\ dx$ Give your answer in exact form.

3. (8 points) Evaluate the improper integral

$$\int_0^\infty \frac{1}{\sqrt{x}(1+x)}\,dx.$$

Be sure to indicate the limit(s) you are taking to evaluate the integral because it is an improper integral.

4. (8 points) A manufacturing plant has a large vat holding water. The water volume is set to 20 cubic meters at noon every day. Some water is taken out of the vat and some recycled water flows into the vat. The water volume w (in cubic meters) is given by the following function of t for the first 4 hours after noon:

$$w(t) = 20 + \int_{t^2}^{e^t - 1} \sin(\sqrt{z})\, dz.$$

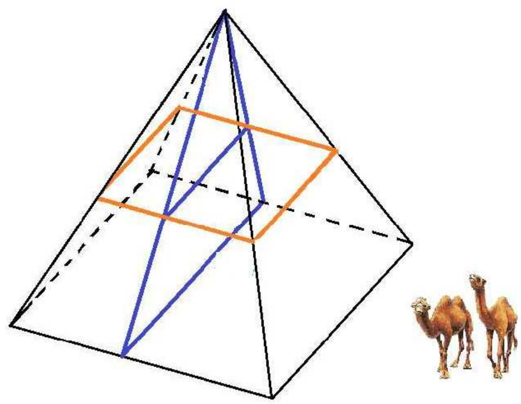
Time t is measured in hours and $t = 0$ represents noon.

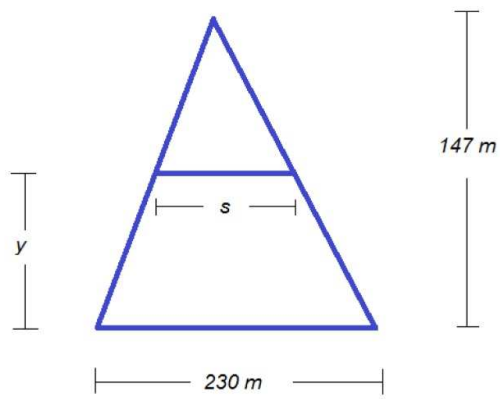
Determine the rate at which the water volume is changing at each of the times $t = 1$ and $t = 3$, and determine whether the water volume is increasing or decreasing at each of those times.

5. (10 points) Consider the region bounded between the parabola $y = x - x^2$ and the x-axis. Find the slope of the line through the origin that divides this region into two subregions with equal area.

6. (8 points) Consider the region between the lines $x = 0$ and $x = 1$, below the curve $y = 5x^3 + 2$, and above the x-axis. Find the volume of the solid obtained when this region is rotated around the vertical line $x = -1$.

7. (10 points) The Great Pyramid of Giza is believed to be built as a tomb for the Egyptian Pharoah Khufu. It is thought that when it was built, the side of the square base was 230 meters and it was 147 meters tall. Let $s(y)$ be the length of a side of its square horizontal cross-section at height y meters above the ground. A triangular vertical cross-section is pictured in detail on the right. Note that the diagrams are not to scale.





Set up and evaluate an integral to find the work done to build this pyramid. Assume that the rocks used to build the pyramid all started at ground level and had to be lifted into place as the pyramid was built, and the rocks fit together with no air between them (and no secret chambers!). Take the acceleration due to gravity to be 9.8 m/s^2 and the density of the rocks to be 2360 kg/m^3.

8. (10 total points)

   (a) (5 points) A termite is climbing up the outside of a termite mound. The termite moves along the graph of the function $y = 1.6\,x^{5/2}$ from $x = 0$ to $x = 1$. The units for both x and y are meters. Express the distance that the termite crawls as a definite integral.

   DO NOT EVALUATE THE INTEGRAL.

   (b) (5 points) Use the trapezoid rule with $n = 2$ subintervals to find an approximate value of the integral in part (a). Give your answer either in exact form, or in decimal form with at least three digits after the decimal point.

9. (10 points) Find the solution to the initial value problem

$$\frac{dy}{dx} = \frac{x\sqrt{x^2+1}}{e^y}, \qquad y(0) = 0.$$

Give your answer in the form $y = f(x)$.

10. (12 total points) A lake has a volume of 1000 cubic meters and contains pure water. A farm near the lake begins using a pesticide. Runoff from the farmland into the lake is 10 cubic meters of water per day, with a concentration of 50 grams of pesticide per cubic meter of water.

    The lake drains to the ocean at a rate of 10 cubic meters per day. You may assume that the pesticide mixes thoroughly with the water in the lake, and you should ignore other effects such as evaporation.

    (a) (6 points) Let $y(t)$ denote the total amount of pesticide (in grams) in the lake after t days. Set up a differential equation for $y(t)$.

    (b) (6 points) Fish can survive a maximum concentration of 1 gram of pesticide per cubic meter of water. Solve the differential equation you found in part (a) and determine whether the fish will be alive after 10 days.