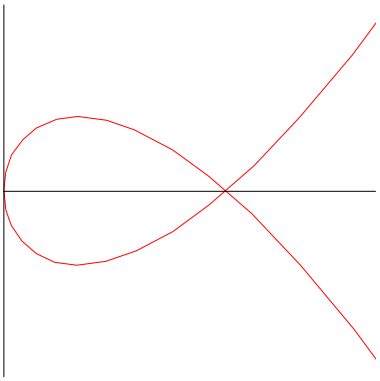
5. (12 points) The location of a particle moving in the plane at time t seconds is given by these parametric equations:

$$x = (t-2)^2 \qquad\qquad y = (t-2)^3 - 3(t-2).$$

The path is graphed below for $0 \leq t \leq 4$.



(a) Find all of the times when the particle crosses the x axis.

(b) Find the equation of the tangent line to the path the first time the particle crosses the x axis.

(c) Find the equation of the tangent line to the path the last time the particle crosses the x axis.

6. (12 points) Consider the curve in the plane defined by the equation

$$y^3 - 2y^2 - x^2 + 3xy = 0.$$

(a) How many points on the curve have x-coordinate 0? Show that $(0, 2)$ is one of them.

(b) Find $\dfrac{dy}{dx}$ in terms of x and y.

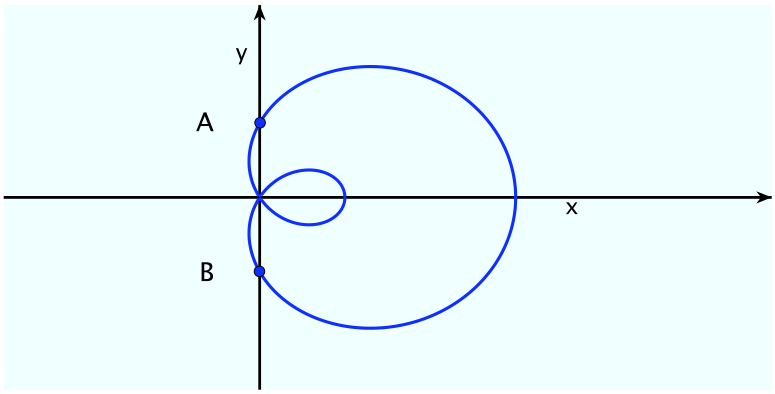
(c) Find the equation of the tangent line to the curve at $(0, 2)$.

5. (12 points)

The curve defined implicitly by

$$(x^2 - 2x + y^2)^2 = x^2 + y^2$$

is called a *limaçon trisectrix*. This curve is pictured below, along with the y-intercepts, labeled A and B.



Find the coordinates of the point where the tangent lines at A and B intersect.

8. (12 points) A particle moves through the plane along the curve C defined by the parametric equations $x(t) = 3t^2 - 4t$, $y(t) = t^2 + 4t + 4$, where $t \geq 0$. Let $P(t) = (x(t), y(t))$ be the location of the particle at time t.

   (a) [6pts] Find the equation of the tangent line to the curve C at time $t = 1$.

   (b) [6pts] Find the time(s) when a tangent line to the curve at $P(t)$ passes through the point $(2, 0)$.

4. (10 points) Consider the curve defined by the parametric equations

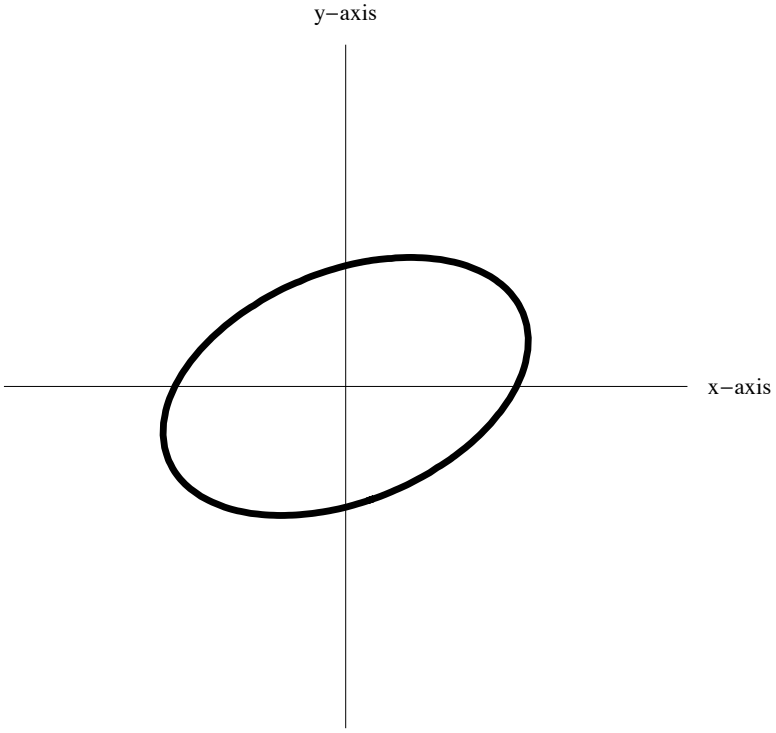
$$x = \frac{1}{3}t^3 - \ln t, \quad y = \frac{81}{2}t^2 + \frac{8}{t^2} + 3,$$

where $t > 0$.

(a) Find all the horizontal tangent lines to the curve.

(b) Find all the vertical tangent lines to the curve.

5. (10 points) The graph of the equation $x^2 - xy + 2y^2 = 4$ is a tilted ellipse, as pictured below.



(a) Find a formula for the implicit derivative $\frac{dy}{dx}$.

(b) Find the coordinates of a point on the ellipse where the tangent line is parallel to the line with equation $y = x + 4$. (Note: there are two correct answers; either will be accepted.) Give your answer in exact form.