Integral Game Solutions

Problem 1

$$\int \frac{x^4-2x^7}{x^3} - 4e^x - \sin x \, dx = \int x - 2x^4 - 4e^x - \sin x \, dx$$

$$\boxed{= \frac{1}{2}x^2 - \frac{2}{5}x^5 - 4e^x + \cos(x) + C}$$

Problem 2

$$\int \frac{e^x}{e^{2x}+2e^x+1} dx$$ Let $u = e^x$ $du = e^x dx$

$du/e^x = dx$

$$\therefore \int \frac{1}{u^2+2u+1} du = \int \frac{1}{(u+1)^2} du$$ Let $v = u+1$ $dv = du$

$$= \int \frac{1}{v^2} dv = \frac{-1}{v} + C$$

$$= \frac{-1}{u+1} + C$$

$$\boxed{= \frac{-1}{e^x+1} + C}$$

Problem 3

$$\int_{-\pi}^{-\pi/2} (\cos(x) - \cos^2 x)^2 \sin(x) dx$$ Let $u = \cos x$

$du = -\sin x \, dx$

$-du/\sin x = dx$

$$= \int_{-1}^{0} -(u - u^2) du = \int_{-1}^{0} u^2 - u \, du$$

$$= \frac{1}{3}u^3 - \frac{1}{2}u^2 \Big|_{-1}^{0}$$

$$= \frac{1}{3} + \frac{1}{2} = \boxed{\frac{5}{6}}$$

Problem 4

$$\int_0^{\sqrt{\pi}} \frac{x\sin(x^2)}{1+\cos^2(x^2)}\,dx \qquad \text{Let } u = x^2$$

$$du = 2x\,dx \Rightarrow du/2x = dx$$

$$= \int_0^{\pi} \frac{1}{2}\frac{\sin(u)}{1+\cos^2 u}\,du \qquad \text{Let } v = \cos(u)$$

$$dv = -\sin(u)\,du$$

$$= \int_1^{-1} -\frac{1}{2}\frac{1}{1+v^2}\,dv = \frac{-1}{2}\arctan(v)\Big|_1^{-1}$$

$$= \frac{\pi}{8} + \frac{\pi}{8} = \boxed{\frac{\pi}{4}}$$

Problem 5

$$\int 3x^4 - 1/x + 5\cos x\,dx = \boxed{\frac{3}{5}x^5 - \ln|x| + 5\sin(x) + C}$$

Problem 6

$$\int \sec^2(2x)\tan^5(2x) \qquad \text{Let } u = 2x$$

$$du = 2dx$$

$$= \int \frac{1}{2}\sec^2(u)\tan^5(u)\,du \qquad \text{Let } v = \tan(u)$$

$$dv = \sec^2(u)\,du$$

$$dv/\sec^2(u) = du$$

$$= \int \frac{1}{2} v^5\,dv = \frac{1}{12} v^6 + C$$

$$\boxed{= \frac{1}{12}(\tan(2x))^6 + C}$$

Problem 7

$$\int_1^2 x(2-x)^7\,dx \qquad \text{Let } u = 2-x$$

$$du = -dx$$

$$= -\int_1^0 (2-u)u^7\,du = \int_0^1 2u^7 - u^8\,du$$

$$= \frac{1}{4}u^8 - \frac{1}{9}u^9$$

$$\boxed{= 5/36}$$

Problem 8

$$\int_1^{\sqrt{3}} \frac{5}{1+y^2} = 5\tan^{-1}(y)\Big|_1^{\sqrt{3}} = 5\left(\pi/3 - \frac{\pi}{4}\right) = \boxed{\frac{5\pi}{12}}$$

Problem 9

$$\int_0^{\pi/4} \sec^2\theta \cos(\tan(\theta))\,d\theta \quad \text{let } u = \tan\theta$$
$$du = \sec^2\theta\, d\theta$$

$$= \int_0^1 \cos(u)\,du = \sin(u)\Big|_0^1 = \boxed{\sin(1)}$$

Problem 10

$$\int \frac{3x^2}{\sqrt{2-x}}\,dx \quad u = 2-x \quad du = -dx$$
$$-du = dx$$

$$= \int \frac{3(2-u)^2}{\sqrt{u}}\,du = \int \frac{3u^2 - 12u + 6}{\sqrt{u}}\,du$$

$$= \int 3u^{3/2} - 12u^{1/2} + 6u^{-1/2}\,du$$

$$= \frac{6}{5}u^{5/2} - 8u^{3/2} + 12u^{1/2} + C$$

$$\boxed{= \frac{6}{5}(2-x)^{5/2} - 8(2-x)^{3/2} + 12(2-x)^{1/2} + C}$$

Problem 11

$4t - t^3 = 0 \Rightarrow t(4-t^2) = 0 \quad t = 0, \pm 2,$

$t<-2$	-2	$-2<t<0$	0	$0<t<2$	2	$t>2$
+	0	−	0	+	0	−

$$\int_{-1}^{3} |4t - t^3|\,dt = \int_{-1}^{0} t^3 - 4t\,dt + \int_0^2 4t - t^3 + \int_2^3 t^3 - 4t\,dt$$

$$= \frac{1}{4}t^4 - 2t^2\Big|_{-1}^{0} + 2t^2 - \frac{1}{4}t^4\Big|_0^2 + \frac{1}{4}t^4 - 2t^2\Big|_2^3$$

$$= -1/4 + 2 + 8 - 4 + 81/4 - 18 - 4 + 8 = \boxed{12}$$

Problem 12

$$\int_1^8 \frac{2x+5}{x^{3/2}} = \int_1^8 2x^{-1/2} + 5x^{-3/2}$$

$$= 4x^{1/2} - 10x^{-1/2} \Big|_1^8$$

$$= 4\sqrt{8} - 10/\sqrt{8} - 4 + 10$$

$$\boxed{= \frac{11 + 6\sqrt{2}}{\sqrt{2}}}$$

Problem 13

$$\int_0^{\pi} \frac{\sin(t)}{1+\cos^2 t}\, dt \qquad u = \cos(t)$$
$$du = -\sin(t)\, dt$$

$$= \int_1^{-1} \frac{-1}{1+u^2}\, du = -\tan^{-1}(u) \Big|_1^{-1} = \frac{\pi}{4} + \frac{\pi}{4} = \boxed{\frac{\pi}{2}}$$

Problem 14

$$\int y^3 \sqrt{y^2-7} \qquad u = y^2 - 7 \qquad du/2y = dy$$
$$du = 2y\, dy$$

$$= \int \frac{1}{2} y^2 \sqrt{u}\, du$$

$$= \int \frac{1}{2}(u+7)\sqrt{u}\, du = \int \frac{1}{2}\left(u^{3/2} + 7u^{1/2}\right) du$$

$$= \frac{1}{2}\left(\frac{2}{5}u^{5/2} + \frac{14}{3}u^{3/2}\right) + C$$

$$\boxed{= \frac{1}{5}(y^2-7)^{5/2} + \frac{14}{6}(y^2-7)^{3/2} + C}$$

Problem 15

$\int xe^{x^2}\sec^2(e^{x^2})dx$ Let $u = e^{x^2}$

$du = e^{x^2} \cdot 2x\,dx$

$= \frac{1}{2}\int \sec^2(u)\,du$

$= \frac{1}{2}\tan(u) + C = \boxed{\frac{1}{2}\tan(e^{x^2}) + C}$

Problem 16

$\int_1^{\sqrt{e}} \frac{\cos(\pi \ln(x))}{x}\,dx$ $u = \pi \ln(x)$ $\frac{x\,du}{\pi} = dx$

$du = \pi/x\,dx$

$= \int_0^{\pi/2} \frac{1}{\pi}\cos(u)\,du = \frac{1}{\pi}\sin(u)\Big|_0^{\pi/2}$

$\boxed{= \frac{1}{\pi}}$

Problem 17

$\int \frac{5xe^{x^2}}{e^{x^2}-5}$ Let $u = e^{x^2}$ $du/e^{x^2} \cdot 2x = dx$

$du = e^{x^2} \cdot 2x\,dx$

$= \int \frac{5}{2}\left(\frac{1}{u-5}\right)du$ Let $v = u-5$

$dv = du$

$= \int \frac{5}{2}\,\frac{1}{v}\,dv = \frac{5}{2}\ln|v| + C = \boxed{\frac{5}{2}\ln|e^{x^2}-5| + C}$

Problem 18

$\int \tan(x)\ln(\cos(x))dx$ $u = \cos(x)$

$du = -\sin(x)dx$

$= \int \frac{\sin(x)}{\cos(x)}\ln(\cos(x))\,dx = \int \frac{-1}{u}\ln(u)\,du$ Let $v = \ln(u)$

$dv = 1/u\,du$

$u\,dv = du$

$= \int -v\,dv$

$= -1/2\,v^2 + C = -\frac{1}{2}(\ln(u))^2 + C = \boxed{-\frac{1}{2}\left[\ln(\cos(x))\right]^2 + C}$

Problem 19

$\int_{-1}^{1} (2-x)^6 dx$ $u = 2-x$, $du = -dx$ $= \int_{3}^{1} -(u)^6 = \frac{-1}{7} u^7 \Big|_3^1$

$= \frac{-1}{7} + \frac{2187}{7} = \boxed{\frac{2186}{7}}$

Problem 20

$\int_1^2 \frac{1+x^2}{x^3} dx = \int_1^2 x^{-3} + x^{-1} dx = \frac{-1}{2} x^{-2} + \ln|x| \Big|_1^2$

$= \frac{-1}{8} + \ln|2| + \frac{1}{2} = \boxed{\frac{3}{8} + \ln|2|}$

Problem 21

$\int \sin^3(x) dx = \int \sin(x)(1-\cos^2 x) dx$ $u = \cos x$ $du = -\sin x\, dx$

$= \int -(1-u^2) du = \int u^2 - 1\, du = \frac{1}{3}u^3 - u + C$

$\boxed{= \frac{1}{3}(\cos x)^3 - \cos x + C}$

Problem 22

$\int_0^1 x f(x^2) dx$ $u = x^2$ $du = 2x\, dx$

$= \frac{1}{2}\int_0^1 f(u) du = \boxed{\frac{7}{2}}$ b/c $\int_0^1 f(x) dx = 7.$

Problem 23

$$\int_2^e \frac{1}{x\sqrt{\ln(x)}}\,dx \quad \text{let } u=\ln(x) \quad du = 1/x\,dx$$
$$x\,du = dx$$

$$= \int_{\ln(2)}^{1} u^{-1/2}\,du = 2u^{1/2}\Big|_{\ln(2)}^{1} = \boxed{2 - 2\sqrt{\ln(2)}}$$

Problem 24

$$\int_0^{\pi/3} \frac{\sin(x)}{\cos^4(x)}\,dx \qquad u=\cos x$$
$$du = -\sin x\,dx$$

$$= \int_1^{1/2} \frac{-1}{u^4}\,du = \frac{1}{3}u^{-3}\Big|_1^{1/2} =$$
$$\frac{8}{3} - \frac{1}{3} = \boxed{\frac{7}{3}}$$

Problem 25

$$\int \frac{(x+1)^2}{\sqrt{x-1}} \qquad u = x-1 \qquad u+1 = x$$
$$du = dx$$

$$\int \frac{(u+2)^2}{\sqrt{u}}\,du = \int \frac{u^2+4u+4}{\sqrt{u}} = \int u^{3/2} + 4u^{1/2} + 4u^{-1/2}$$
$$= \frac{2}{5}u^{5/2} + \frac{8}{3}u^{3/2} + 8u^{1/2} + C$$
$$\boxed{= \frac{2}{5}(x-1)^{5/2} + 8/3\,(x-1)^{3/2} + 8(x-1) + C}$$

Problem 26

$$\int \frac{x}{2x+11}\,dx \qquad u = 2x+11 \qquad = \frac{1}{2}\int \frac{\frac{u-11}{2}}{u}$$
$$du = 2dx$$

$$= \frac{1}{4}\int \frac{u-11}{u} = \frac{1}{4}\int 1 - \frac{11}{u} = \frac{1}{4}(u - 11\ln|u|) + C$$

$$\boxed{= \frac{1}{4}\left[(2x+11) - 11\cdot\ln|2x+11|\right] + C}$$