1. (10 total points) Evaluate the following indefinite integrals.

(a) (5 points) $\int x^2 \ln x \, dx$ ← IBP

$u = \ln(x)$ $\quad dv = x^2 dx$

$du = 1/x \, dx$ $\quad v = \frac{1}{3}x^3$

$\frac{1}{3}x^3 \ln(x) - \int \frac{1}{3}x^2 dx = \boxed{\frac{1}{3}x^3 \ln(x) - \frac{1}{9}x^3 + C}$

(b) (5 points) $\int \tan^3(x) \sec(x) \, dx$

$\int \tan^2 x \tan x \sec x \, dx$ $\quad u = \sec x$ $\quad du = \sec x \tan x \, dx$

$= \int \tan^2 x \, du = \int \sec^2 x - 1 \, du = \int u^2 - 1 \, du$

$= \frac{1}{3}u^3 - u + C = \boxed{\frac{1}{3}(\sec^3 x) - \sec x + C}$

2. (10 total points) Evaluate the following definite integrals.

(a) (5 points) $\int_0^{\ln 2} \frac{7e^{2t}}{e^{2t}+3e^t+2}\,dt$ $\quad u = e^t \quad du = e^t\,dt$

$$= \int_1^2 \frac{7u}{u^2+3u+2}\,du \quad \text{partial fractions}$$

$$\frac{7u}{u^2+3u+2} = \frac{A}{(u+2)} + \frac{B}{u+1} \Rightarrow A(u+1) + B(u+2) = 7u$$

$u=-1 \quad B=-7$

$u=-2 \quad -A=-14 \Rightarrow A=14$ $\quad = \int_1^2 \frac{14}{u+2} + \frac{-7}{u+1}\,du$

$$= 14\ln|u+2|\Big|_1^2 - 7\ln|u+1|\Big|_1^2$$

$$= 14\ln(4) - 14\ln(3) - 7\ln(3) + 7\ln(2)$$

$$\boxed{= 14\ln(4) - 21\ln(3) + 7\ln(2)}$$

(b) (5 points) $\int_0^{1/2} \sqrt{2x-x^2}\,dx$ $\leftarrow$ Trig problem

Complete the square $\quad 2x - x^2 = -(x^2-2x) = -((x-1)^2-1))$

$= 4-(x-2)^2$

$\int_0^{1/2} \sqrt{1-(x-1)^2}\,dx \quad$ $x-1 = \sin\theta$, $dx = \cos\theta\,d\theta$ $\quad$ * Note $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$$\int_{-\pi/2}^{-\pi/6} \sqrt{1-\sin^2\theta}\,\cos\theta\,d\theta = \int_{-\pi/2}^{-\pi/6} \cos^2\theta\,d\theta = \int_{-\pi/2}^{-\pi/6} \tfrac{1}{2}(1+\cos(2\theta))\,d\theta$$

$$= \tfrac{1}{2}\theta + \tfrac{1}{4}\sin(2\theta)\Big|_{-\pi/2}^{-\pi/6} = \tfrac{1}{2}\left(\tfrac{-\pi}{6} + \tfrac{\pi}{2}\right) + \tfrac{1}{4}\sin\left(\tfrac{-\pi}{3}\right) - \tfrac{1}{4}\sin(-\pi)$$

$$\boxed{= \frac{\pi}{6} - \frac{\sqrt{3}}{8}}$$