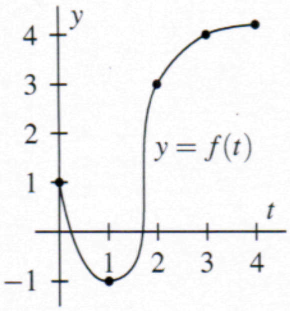
4. (8 points) Suppose that the graph of f is as shown:



Let $G(x) = \int_x^{x^2+x} t\,f(t)\,dt$. Find $G'(1)$.

① Rewrite in form w/ const on bott.

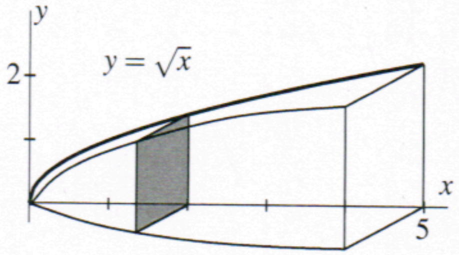
$$\int_0^{x^2+x} tf(t)dt + \int_x^0 tf(t)dt = \int_0^{x^2+x} tf(t) - \int_0^x tf(t)$$

② Use FTC

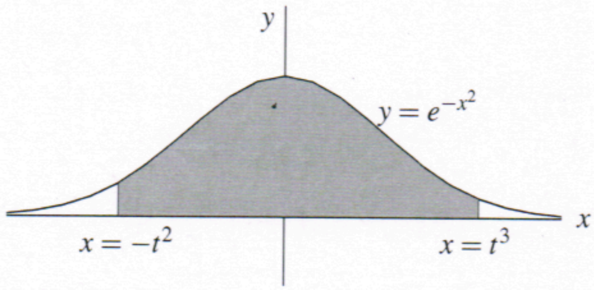
$$G'(x) = (x^2+x)f(x^2+x)\cdot(2x+1) - xf(x)\cdot 1$$

$$G'(1) = 2f(2)(3) - f(1)$$
$$= 6f(2) - f(1)$$
$$= 6(3) - (-1) \leftarrow \text{from graph}$$
$$= \boxed{19}$$

5. (4 points) Find the volume of the solid shown below. Each cross-section (slice) is a square.



6. (8 points) At each time $t \geq 0$, $\mathscr{R}_t$ is the region above the x-axis, below the curve $y = e^{-x^2}$, with left side on the line $x = -t^2$ and right side on the line $x = t^3$ (see the figure). Let $A(t)$ be the area of $\mathscr{R}_t$. Find $\frac{dA}{dt}$ at time $t = 1$.



$A(t) = \int_{-t^2}^{t^3} e^{-x^2}\,dx$

① Rewrite

$\int_0^{t^3} e^{-x^2} - \int_0^{-t^2} e^{-x^2}\,dx$

② Use FTC

$A'(t) = e^{-(t^3)^2} \cdot 3t^2 - e^{-(-t^2)^2} \cdot -2t$

$A'(1) = e^{-1} \cdot 3 - e^{1}(-2)$

$= \boxed{5e^{-1}}$

3. (6 points) Let

$$E(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt,$$

and let

$$G(x) = E(\sqrt{x}).$$

Compute $G'(x)$.

$$G(x) = E(\sqrt{x}) = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{x}} e^{-t^2}\, dx$$

① Rewrite (already done)

② Use FTC

$$\boxed{G'(x) = \frac{2}{\sqrt{\pi}} e^{-x} \cdot \frac{1}{2} x^{-1/2}}$$

5. (8 points) Find

$$\lim_{x \to 0} \frac{\int_0^x \cos(t^2)\,dt}{x}.$$

By definition, this is the derivative of $\int_0^x \cos(t^2)$ at $x=0$

① Rewrite: (Don't need to)

② FTC

$g(x) = \int_0^x \cos(t^2)$

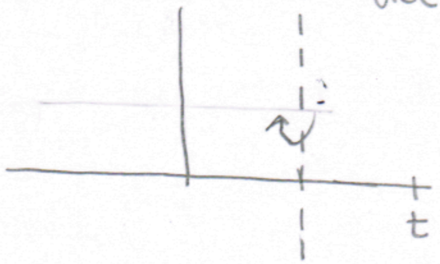
$g'(x) = \cos(x^2)$

$g'(0) = \cos(0) = \boxed{1}$

6. (12 total points) The region between the curve $y = e^{x^3}$ and the lines $y = 0$, $x = 1$, and $x = t$ is rotated about the vertical line $x = 1$. Here $t > 1$ is not further specified.

(a) (8 points) Write down an integral for the volume of the resulting solid. **Do not evaluate this integral.**

① Draw Picture Use shells



② Pt of intersections
$x=1$ and $x=t$

③ Chart

	$1 \le x \le t$
Height	e^{x^3}
Radius	$x-1$

④ Integral

$$\boxed{\text{Vol} = 2\pi \int_1^t (x-1)e^{x^3}\,dx}$$

(b) (4 points) The volume from part (a) is a function of t; call it $V(t)$. Find the derivative $V'(2)$.

$$V(t) = 2\pi \int_1^t (x-1)e^{x^3}\,dx$$

① Rewrite (Don't need to)

② Use FTC

$$V'(t) = 2\pi(t-1)e^{t^3}\cdot 1$$

$$V'(2) = 2\pi(2-1)e^{2^3}$$

$$\boxed{= 2\pi e^8}$$