Math 124 Final Examination Winter 2011

Print Your Name | Signature

Student ID Number | Quiz Section

Professor's Name | TA's Name

!!! READ...INSTRUCTIONS...READ !!!

1. Your exam contains 9 questions and 12 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 2 hours and 50 minutes for this final exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification.
4. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.
5. You are allowed one 8.5×11 sheet of handwritten notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed. Make sure your calculator is in radian mode.
6. Unless otherwise instructed, ALWAYS GIVE YOUR ANSWERS IN EXACT FORM. For example, 3π, $\sqrt{2}$, $\ln(2)$ are in exact form; the corresponding approximations 9.424778, 1.4142, 0.693147 are NOT in exact form.

Problem	Total Points	Score
1	16	
2	8	
3	6	
4	10	

Problem	Total Points	Score
5	10	
6	12	
7	12	
8	12	
9	14	
Total	100	

1. (16 points; 4pts each) Find the derivatives of the following functions. You do not have to simplify.

   (a) $y = (1 + \cos^3 x)^{2/3}$

   (b) $y = \arctan(e^{\arctan x})$

1. continued

(c) $y = (\cos x)^{\sin x}$

(d) $y = \dfrac{t}{(1+\sqrt{t})^{100}}$

2. (8 points)

   Calculate the following limits. Make sure to justify all your steps.

   (a) $\lim_{x \to 0} \frac{(\sin(x))^{12}}{x^{10}}$

   (b) $\lim_{x \to \infty} \left(\frac{x+1}{x-1} \right)^{x-1}$

3. (6 points) For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 6x, & \text{if } x < 5 \\ \\ x^3 - cx, & \text{if } x \geq 5 \end{cases}$$

4. (10 points) Consider the curve defined by the parametric equations

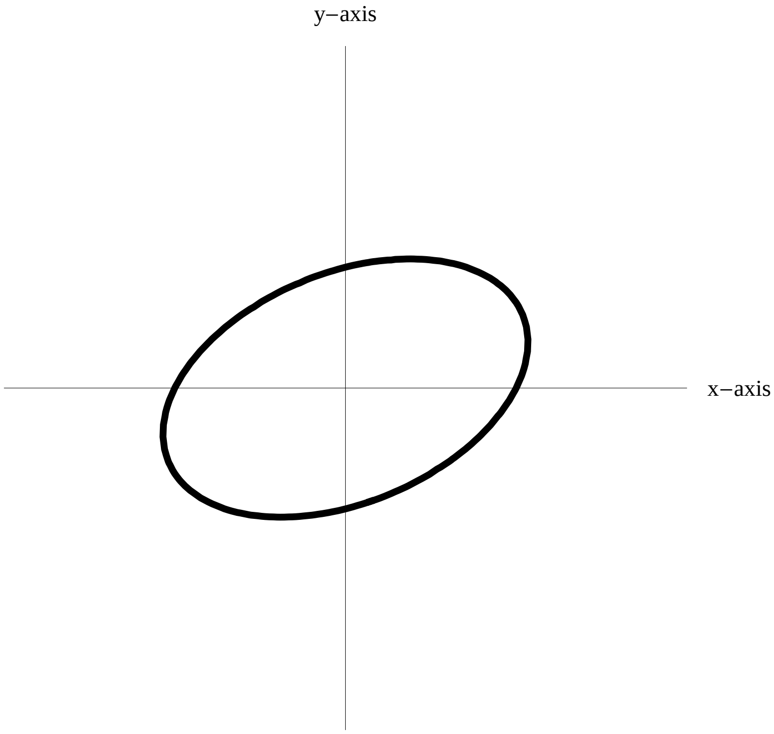
$$x = \frac{1}{3}t^3 - \ln t, \quad y = \frac{81}{2}t^2 + \frac{8}{t^2} + 3,$$

where $t > 0$.

(a) Find all the horizontal tangent lines to the curve.

(b) Find all the vertical tangent lines to the curve.

5. (10 points) The graph of the equation $x^2 - xy + 2y^2 = 4$ is a tilted ellipse, as pictured below.



(a) Find a formula for the implicit derivative $\frac{dy}{dx}$.

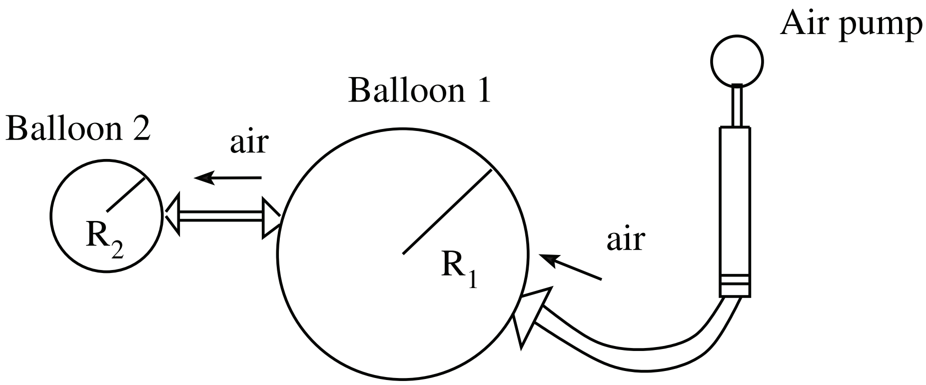
(b) Find the coordinates of a point on the ellipse where the tangent line is parallel to the line with equation $y = x + 4$. (Note: there are two correct answers; either will be accepted.) Give your answer in exact form.

6. (12 points) Balloon 1 is linked by a large tube to an air pump and by a smaller tube to Balloon 2 (see picture). The radius of Balloon 1 is R_1 and the radius of Balloon 2 is R_2.

   Air is being pumped in Balloon 1 at the constant rate of 101cm^3/minute and air is leaking out of Balloon 1 (and into Balloon 2) at a total rate equal to π times the rate of change of R_1, in cm^3/minute.

   At time t_0 measurements say that $R_1 = 5$ and $R_2 = 2$.

   Calculate the rate of change of R_2 at that time.



7. (12 points) A particle is traveling along a curve with parametric equations $x = x(t), y = y(t)$. The implicit equation of the curve is $y^2 = x^3 + 3x$. At time $t = 0$, the particle is located at the point $(1, -2)$ and its vertical velocity $\frac{dy}{dt}$ is 2 units/sec. Use the tangent line approximation to estimate the location of the particle at time $t = 0.1$.

8. (12 points) Nurl is designing a cylindrical container of volume 50π cubic centimeters. The top and bottom of the cylinder must be made of a material costing \$10 per square centimeter, while the rest of the container is made of a cheaper material that costs only \$3.20 per square centimeter. What is the surface area of the cheapest container Nurl can design?

9. (14 points) Let $f(x) = e^{\frac{1}{x-2}}$.

(a) Find the largest possible domain for the function.

(b) Find $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$.

(c) Find all asymptotes for f (either vertical or horizontal).

(d) Calculate the intervals where f is increasing or decreasing.

9. continued

(e) Find all local extrema (if any) for f.

(f) Calculate the intervals where f is concave up or concave down.

(g) Based on all of the above, sketch the graph of f, labeling all extrema and indicating any asymptotes.