① $\int e^{-\sqrt{x}}\,dx$ Let $u = -\sqrt{x}$ $du = -\frac{1}{2\sqrt{x}}\,dx \Rightarrow du = \frac{-1}{2u}\,dx$

$2u\,du = dx$

$= \int 2ue^{u}\,du$ Do IBP, $w = 2u$ $dv = e^{u}\,du$

$dw = 2du$ $v = e^{u}$

$= 2ue^{u} - \int 2e^{u}\,du$

$= 2ue^{u} - 2e^{u} + C$

$\boxed{= -2\sqrt{x}\,e^{-\sqrt{x}} - 2e^{-\sqrt{x}} + C}$

② $\int \cos^3 x\,dx = \int \cos^2 x \cos x\,dx$ $u = \sin x$

$du = \cos x\,dx$

$= \int \cos^2 x\,du$

$= \int 1 - \sin^2 x\,du = \int 1 - u^2\,du = u - \frac{1}{3}u^3 + C$

$\boxed{= \sin x - \frac{1}{3}(\sin^3 x) + C}$

③ $\int \frac{dt}{t\sqrt{4t^2-1}} = \int \frac{dt}{t\sqrt{4(t^2-\frac{1}{4})}} = \int \frac{dt}{2t\sqrt{t^2-\frac{1}{4}}}$ $t = \frac{1}{2}\sec\theta$

$dt = \frac{1}{2}\sec\theta\tan\theta\,d\theta$

$= \int \frac{\frac{1}{2}\sec\theta\tan\theta\,d\theta}{\sec\theta\sqrt{\frac{1}{4}\sec^2\theta - \frac{1}{4}}} = \int \frac{1}{2}\frac{\tan\theta\,d\theta}{\sqrt{\frac{1}{4}(\sec^2\theta-1)}} = \int \frac{\tan\theta\,d\theta}{\tan\theta} = \int d\theta$

$= \theta + C$

$\boxed{= \sec^{-1}(2t) + C}$

4

$$\int_{-2}^{2} \frac{1}{\sqrt{x^2+4x+8}}\,dx$$

Complete the Square

x^2+4x+8

$(x+2)^2+8-4$

$(x+2)^2+4$

$$= \int_{-2}^{2} \frac{1}{\sqrt{(x+2)^2+4}}\,dx$$

$u = x+2$

$du = dx$

$$= \int_{0}^{4} \frac{1}{\sqrt{u^2+4}}\,du$$

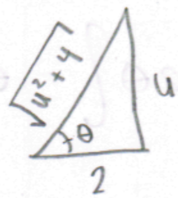
$u = 2\tan\theta$

$du = 2\sec^2\theta\, d\theta$

$$\Rightarrow \int \frac{2\sec^2\theta}{\sqrt{4\tan^2\theta+4}}\,d\theta = \int \frac{2\sec^2\theta}{2\sec\theta}\,d\theta = \int \sec\theta\, d\theta$$

$= \ln|\sec\theta + \tan\theta| + C$ From trig integrals

By Triangle, $u/2 = \tan\theta$



$$\ln|\sec\theta + \tan\theta| + C = \ln\left|\frac{\sqrt{u^2+4}}{2} + \frac{u}{2}\right| + C$$

$$\therefore \int_0^4 \frac{1}{\sqrt{u^2+4}}\,du = \ln\left|\frac{\sqrt{4^2+4}}{2} + 2\right| - \ln(0)$$

$$\boxed{= \ln\left|\frac{\sqrt{20}}{2} + 2\right|}$$

5 $\int 2x\ln(x+5)\,dx$ Let $u = x+5$ $du = dx$

$u-5 = x$

$= \int (2u-10)\ln(u)\,du$ By IBP, $w = \ln(u)$ $dv = 2u-10\,du$

$dw = \frac{1}{u}du$ $v = u^2-10u$

$= (u^2-10)\ln(u) - \int u - \frac{10}{u}\,du$

$= (u^2-10)\ln(u) - \frac{1}{2}u^2 + 10\ln|u| + C \Rightarrow \boxed{[(x+5)^2-10]\ln(x+5) - \frac{1}{2}(x+5)^2 + 10\ln|x+5| + C}$

⑥ $\int \sec(x)\tan^3 x\,dx = \int \sec(x)\tan^2(x)\tan(x)\,dx$

$u = \sec x$

$du = \sec x \tan x\,dx$

$= \int \tan^2 x\,du = \int \sec^2 x - 1\,du$

$= \int u^2 - 1\,du$

$= \frac{1}{3}u^3 - u + C$

$\boxed{= \frac{1}{3}(\sec^3 x) - \sec x + C}$

⑦ $\int x^3\sqrt{9+4x^2}$ $\quad u = 9 + 4x^2$

$du = 8x\,dx$

$= \frac{1}{8}\int x^2\sqrt{u}\,du$

$= \frac{1}{8}\int \frac{(u-9)}{4}\sqrt{u}\,du = \frac{1}{32}\int u^{3/2} - 9u^{1/2}\,du$

$= \frac{1}{32}\left(\frac{2}{5}u^{5/2} - 6u^{3/2}\right) + C$

$\boxed{= \frac{1}{32}\left(\frac{2}{5}(9+4x^2)^{5/2} - 6(9+4x^2)^{3/2}\right) + C}$

⑧ $\int_0^{\pi/8} \sin(2x)\cos(2x)\,dx \quad u = 2x \quad du = 2dx$

$= \int_0^{\pi/4} \frac{1}{2}\sin(u)\cos(u)\,du$ $\quad v = \sin(u)$

$dv = \cos(u)\,du$

$= \int_0^{\sqrt{2}/2} \frac{1}{2}v\,dv = \frac{1}{4}v^2\Big|_0^{\sqrt{2}/2} = \frac{1}{4}\left(\frac{2}{4}\right) = \boxed{\frac{1}{8}}$

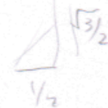
(9) Skip

(10) $\int \frac{t^3}{\sqrt{t^2+4}}\,dt$ $\quad u = t^2+4$, $du = 2t\,dt$ $\quad \int \frac{1}{2}\,\frac{t^2}{\sqrt{u}}\,du$

$$= \frac{1}{2}\int \frac{u-4}{\sqrt{u}}\,du = \frac{1}{2}\int u^{1/2} - 4u^{-1/2}\,du$$

$$= \frac{1}{2}\left(\frac{2}{3}u^{3/2} - 8u^{1/2}\right) + C$$

$$\boxed{= \frac{1}{2}\left(\frac{2}{3}(t^2+4)^{3/2} - 8(t^2+4)^{1/2}\right) + C}$$



(11) $\int_{\pi/4}^{\pi/3} \frac{\ln(\tan\theta)}{\sin\theta\cos\theta}\,d\theta$ $\quad u = \ln(\tan\theta)$ $\quad \ln(\tan(\pi/3)) = \ln(\sqrt{3})$

$$du = \frac{1}{\tan\theta}\cdot\frac{1}{\cos^2\theta}\,d\theta$$

$$= \frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\cos^2\theta}\,d\theta$$

$$= \frac{1}{\sin\theta\cos\theta}\,d\theta$$

$$= \int_0^{\sqrt{3}} u\,du = \frac{1}{2}u^2\Big|_0^{\sqrt{3}} = \boxed{3/2}$$

(12) $\int x^3\sin(x^2+1)\,dx$ $\quad u = x^2+1$, $du = 2x\,dx$

$$= \int \frac{1}{2}x^2\sin(u)\,du$$

$$= \int \frac{1}{2}(u-1)\sin(u)\,du$$

use IBP, $w = \frac{1}{2}(u-1)$ $\quad dv = \sin(u)\,du$

$dw = \frac{1}{2}\,du$ $\quad v = -\cos(u)$

$$= \frac{-1}{2}(u-1)\cos(u) + \int \frac{1}{2}\cos(u)\,du = \frac{-1}{2}(u-1)\cos(u) + \frac{1}{2}\sin(u) + C$$

$$\boxed{= \frac{-1}{2}(x^2)\cos(x^2+1) + \frac{1}{2}\sin(x^2+1) + C}$$

(13) $\int \ln(1+\sqrt{x})\,dx$ $\quad u = 1+\sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx \Rightarrow du = \frac{1}{2u}\,dx \Rightarrow 2u\,du = dx$

$= \int 2u\ln(u)\,du$ $\quad$ IBP $\quad w = \ln(u) \quad dv = 2u\,du$

$dw = \frac{1}{u}\,du \quad v = u^2$

$= u^2\ln(u) - \int u\,du$

$= u^2\ln(u) - \frac{1}{2}u^2 + C$

$$\boxed{= (1+\sqrt{x})^2\ln(1+\sqrt{x}) - \frac{1}{2}(1+\sqrt{x})^2 + C}$$

(14) $\int \frac{\sqrt{y-4}}{y}\,dy$ $\quad$ You don't know how to do this yet

(15) $\int_0^1 \frac{x^2}{\sqrt{4-x^2}}\,dx$ $\quad x = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$

$= \int_0^{\pi/6} \frac{4\sin^2\theta \cdot 2\cos\theta\,d\theta}{\sqrt{4-4\sin^2\theta}} = \int_0^{\pi/6} 4\sin^2\theta\,d\theta$

$= \int_0^{\pi/6} 4\left(\frac{1-\cos(2\theta)}{2}\right)d\theta$

$= 2\int_0^{\pi/6} 1-\cos(2\theta)\,d\theta$

$= 2\left(\theta - \tfrac{1}{2}\sin(2\theta)\right)\Big|_0^{\pi/6}$

$= 2\left(\frac{\pi}{6} - \frac{1}{2}\sin(\pi/3)\right)$

$= 2\left(\frac{\pi}{6} - \frac{1}{2}\,\sqrt{3}/2\right)$

$$\boxed{= \frac{\pi}{3} - \frac{1}{4}\sqrt{3}}$$

(16) $\displaystyle\int \frac{x}{\sqrt{6x-x^2}}\,dx$

Complete the Square

$-(x^2-6x)$

$= -([x-3]^2 - 9 + 0)$

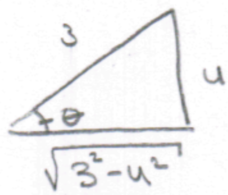
$= 9 - (x-3)^2$

$$= \int \frac{x}{\sqrt{9-(x-3)^2}}\,dx \qquad u = x-3 \quad du = dx$$

$$= \int \frac{u+3}{\sqrt{9-u^2}}\,du \qquad u = 3\sin\theta \quad du = 3\cos\theta\, d\theta$$

$$= \int \frac{3\sin\theta + 3}{\sqrt{9-9\sin^2\theta}} \cdot 3\cos\theta\, d\theta$$

Set up triangle $u/3 = \sin\theta = \frac{o}{h}$



$$= \int 3\sin\theta + 3$$

$$= -3\cos\theta + 3\theta + C$$

$$= -3\left(\frac{\sqrt{3^2-u^2}}{3}\right) + 3\sin^{-1}\left(\frac{u}{3}\right) + C$$

$$\boxed{= \sqrt{3^2-(x-3)^2} + 3\sin^{-1}\left(\frac{x-3}{3}\right) + C}$$