Multimedia Security

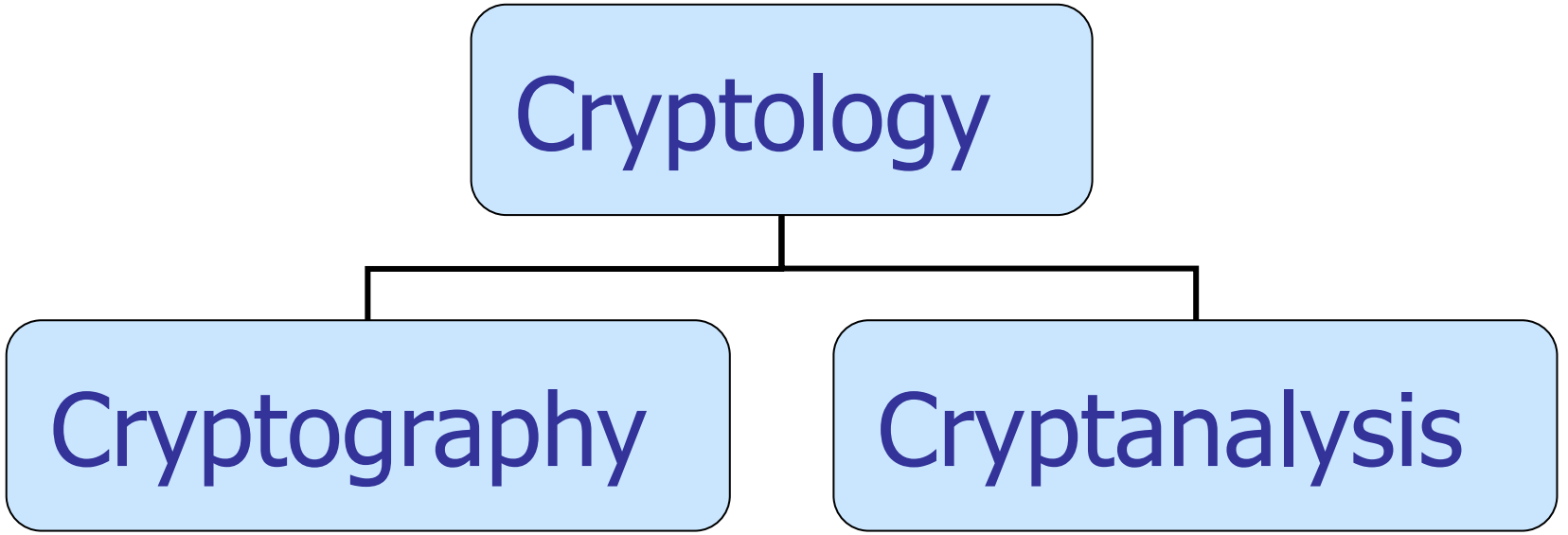
Foundations of Cryptology

Mauro Barni
University of Siena

Cryptography

- Cryptography is the art or science of keeping messages secret;
  - the word cryptography is derived from Greek and literally means **secret** (crypto-) **writing** (-graphy)
- It allows to protect the content against disclosure or modification both during data transmission or storage.
- It deals with all aspects of secure messaging, authentication, digital signatures, electronic money ...

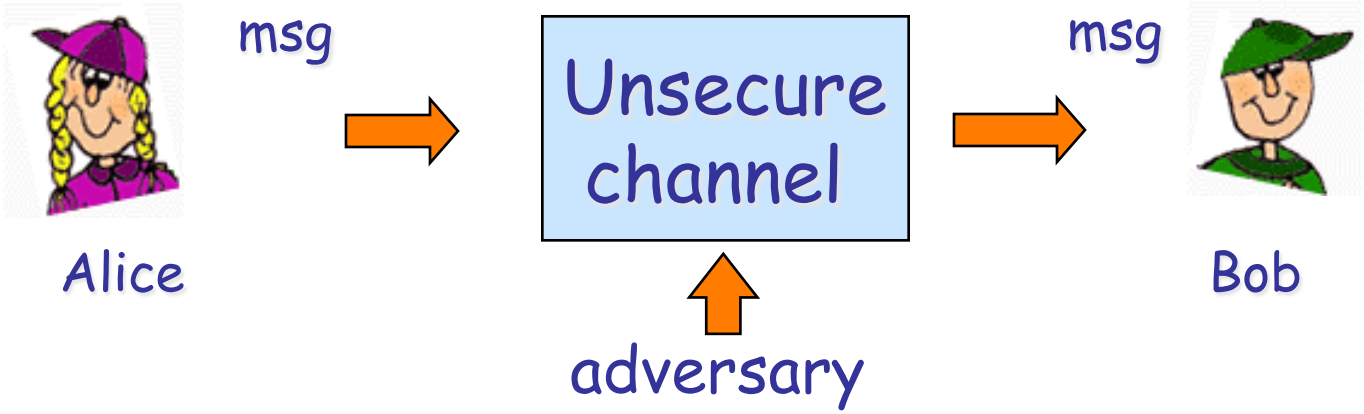
What is Cryptology?



Cryptography vs cryptanalysis

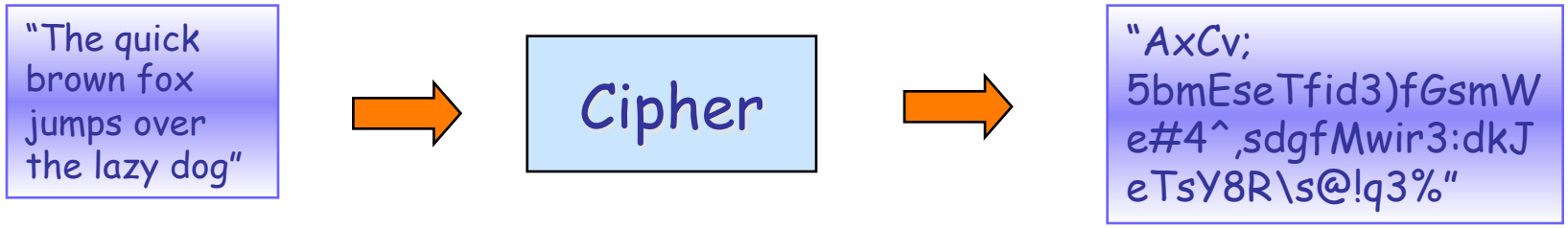
- Cryptology
  - it is the study of cryptography and cryptanalysis.
- Cryptography
  - it is the study of mathematical techniques to solve issues of information security, such as confidentiality, data integrity, entity authentication, and data origin authentication.
- Cryptanalysis
  - it is the study of mathematical techniques for attempting to defeat information security services.

Framework



- A *sender* is an entity in a two-party communication which is the legitimate transmitter of information.
- A *receiver* is an entity which is the intended recipient of information.
- An *adversary* is an entity which is neither the sender nor receiver, and which tries to break the security (often the confidentiality) of the transmission.

Basic Terminology

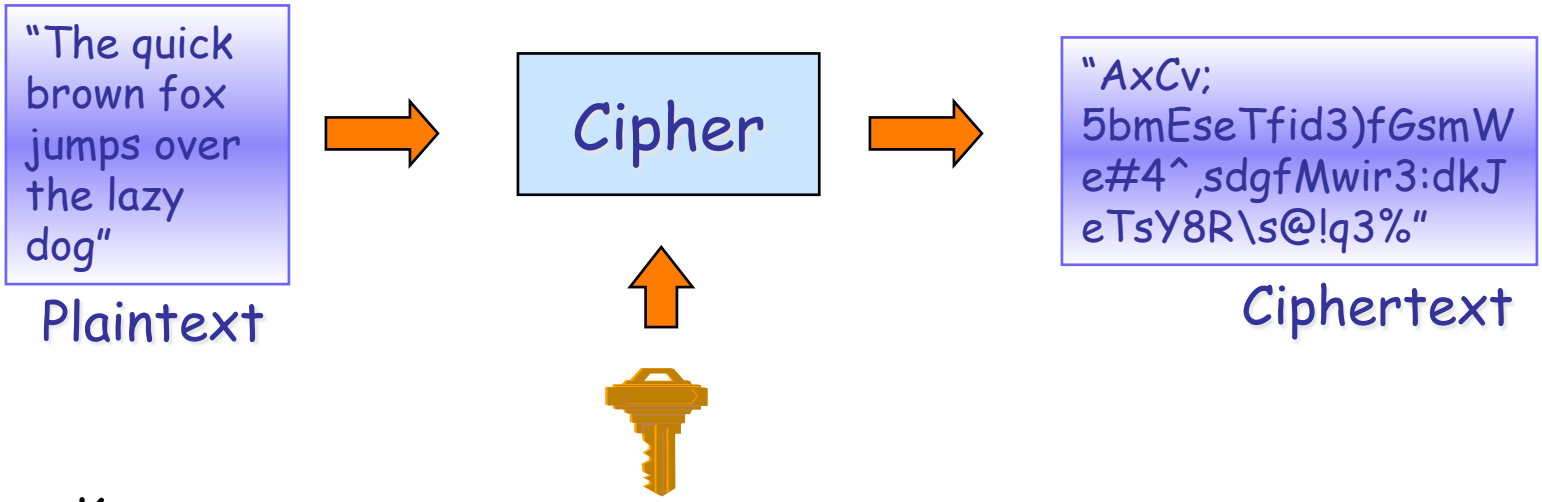


Plaintext
the original message, i.e. the stuff to secure.

Ciphertext
the coded message, apparently random sequence of data and thus unreadable

Cipher
algorithm for transforming plaintext to ciphertext and viceversa, composed by encipher and decipher

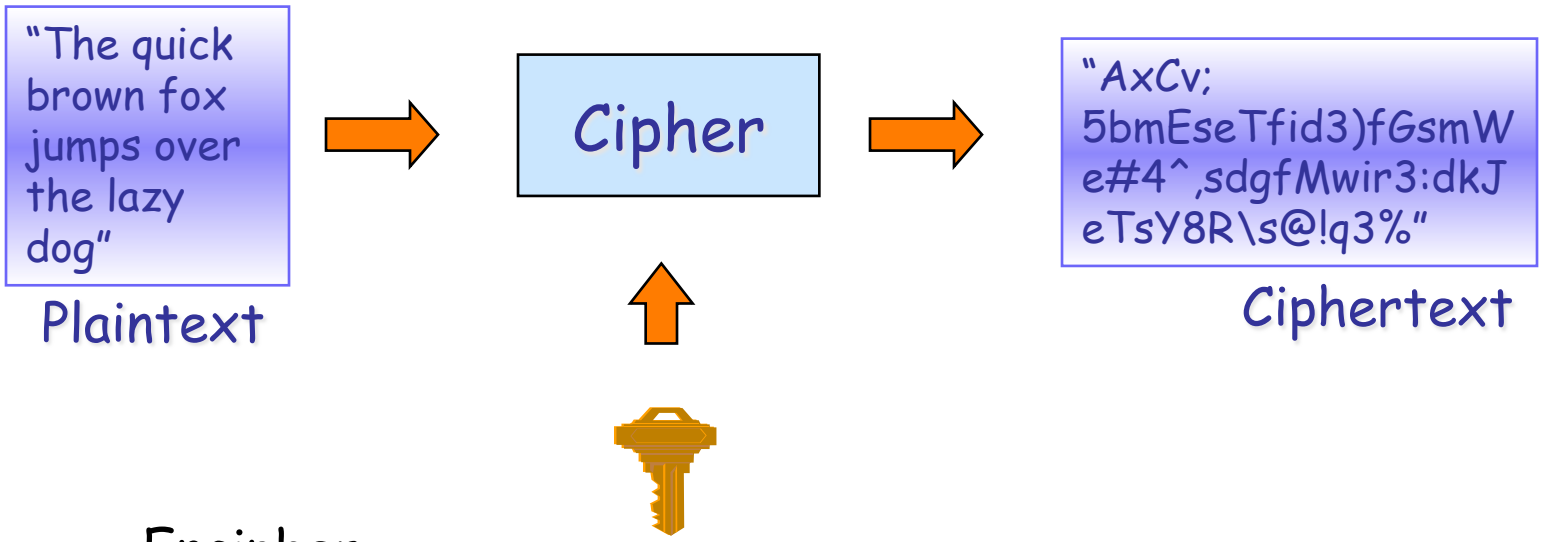
Basic Terminology



Key

some critical information used by the cipher, **independent** from plaintext. The cipher, given an input, will produce different outputs for different keys.

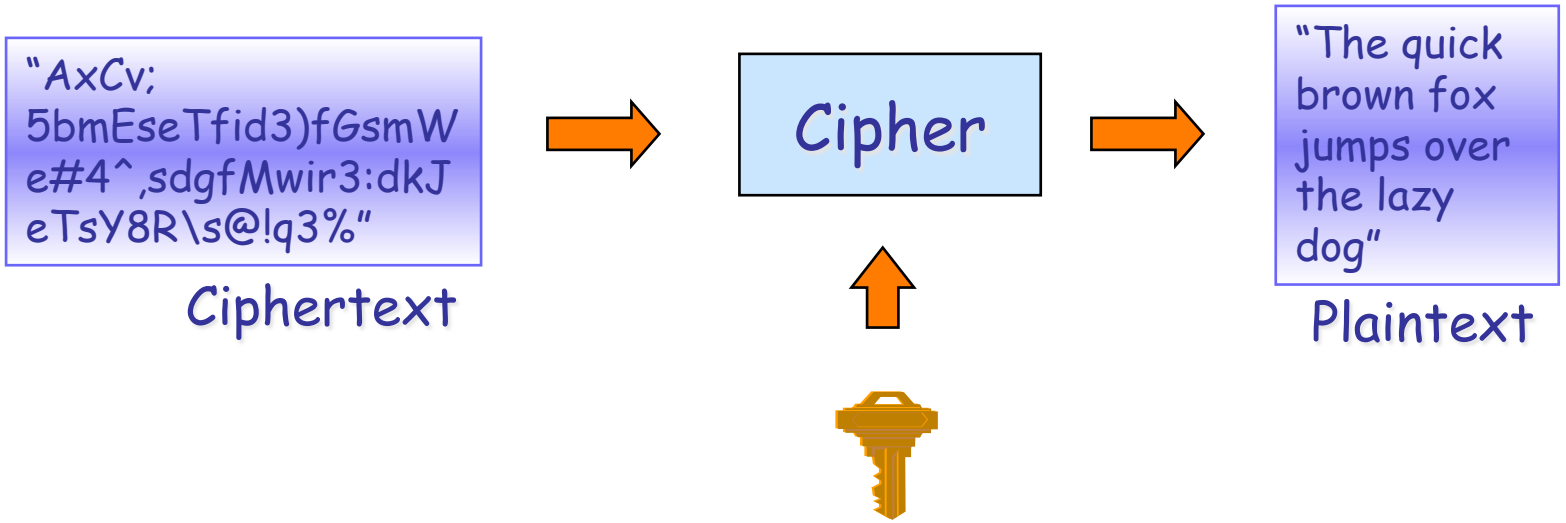
Basic Terminology



Encipher

(encryption) the process of converting plaintext to ciphertext using a cipher and a key

Basic Terminology



Decipher

(decryption) the process of converting ciphertext back into plaintext using a cipher and a key

Basic Terminology

- Plaintext $\mathbf{x} = [x_1, x_2, \ldots, x_M]$ *where* $\mathbf{x} \in \boldsymbol{M}$, $x_i \in \boldsymbol{A}$

- ***A*** denotes a finite set called the *alphabet of definition*
  - English letters, bits, digits
- ***M*** denotes a set called the *message space*.
- ***M*** consists of strings of symbols from ***A***
- For example, ***M*** may consist of binary strings, English text, computer code, etc.
  - In cipher DES **x** = string of 64 bits
  - In cipher RSA **x** = integer number

Basic Terminology

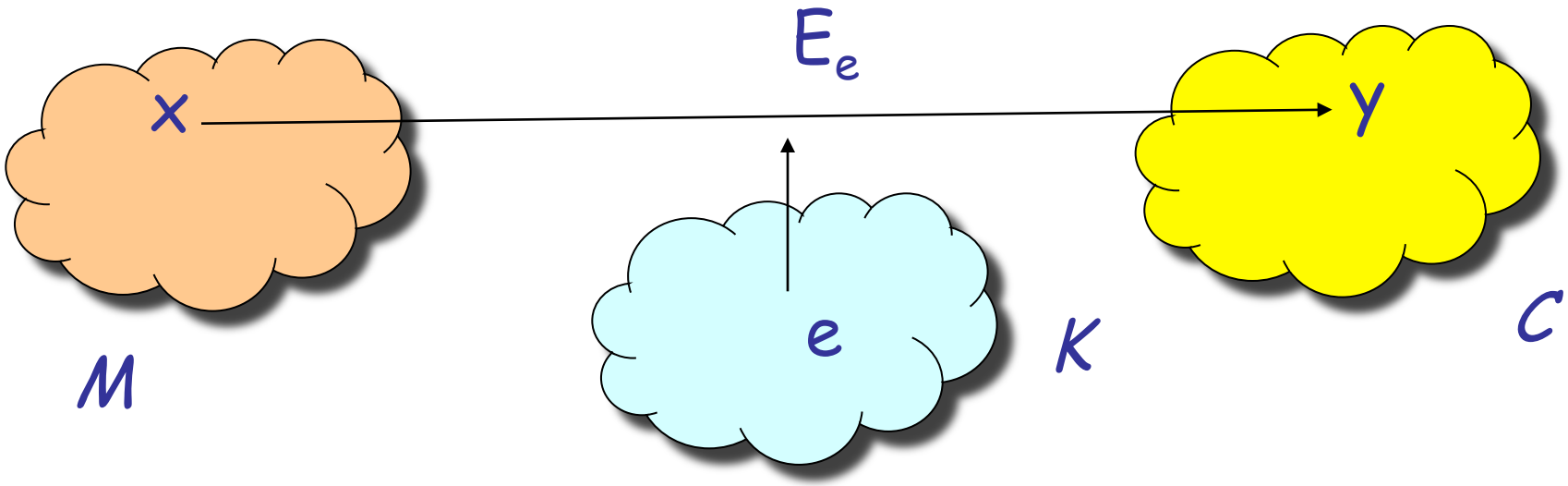
- Ciphertext $\mathbf{y} = [y_1, y_2, \ldots, y_M]$ *where* $\mathbf{y} \in \boldsymbol{C}$

- ***C*** denotes a set called the *ciphertext space*.
- ***C*** consists of strings of symbols from an alphabet of definition, which may differ from the alphabet of definition for ***M***.

Basic Terminology

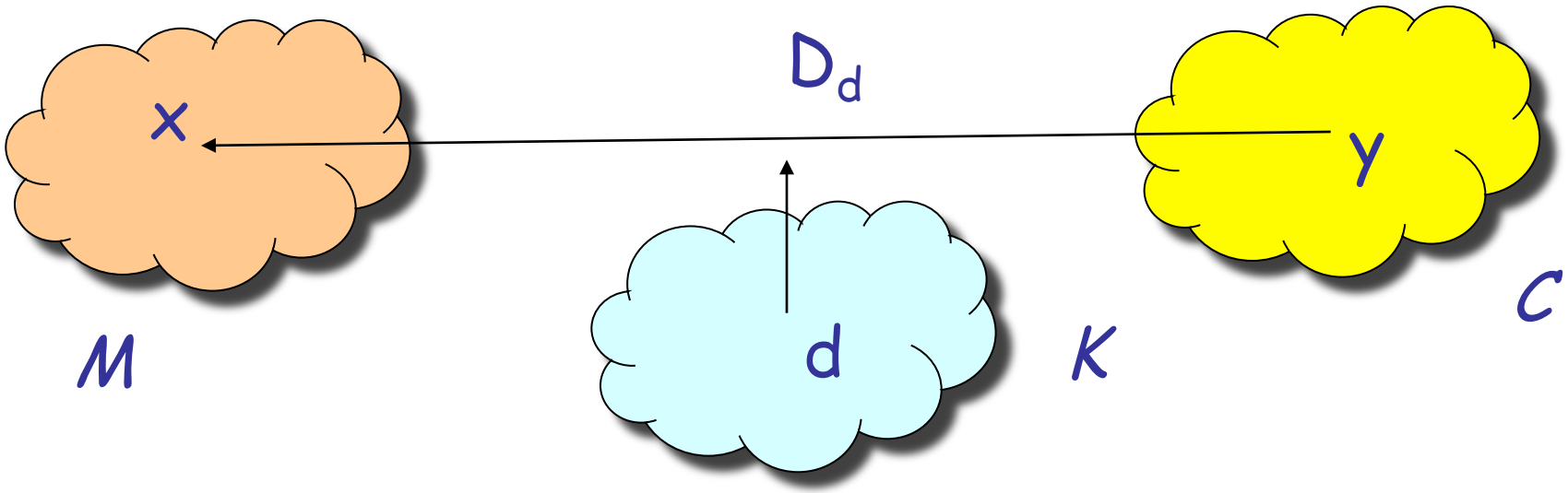
- Key **k** = $[k_1, k_2, \ldots, k_L]$, *where* **k** $\in$ ***K***
- ***k*** denotes a set called the *key space*
- ***k*** consists of strings of symbols from an alphabet of definition, which may differ from the alphabet of definition of***M***

Basic Terminology



- Each element **e** $\in$ ***K*** uniquely determines a bijection from ***M*** to ***C***, denoted by E_e
- E_e is called an *encryption function* or *transformation*.
  - E_e is a bijection if the process can be reversed
  - *a function f from a set X to a set Y is said to be bijective if for every y in Y there is exactly one x in X such that f(x) = y.*

Basic Terminology



- Each element **d** $\in$ ***K*** uniquely determines a bijection from ***C*** to ***M***, denoted by D_d
- D_d is called a *decryption function* or a *decryption transformation*.
- NOTE: this may not hold for probabilstic encryption

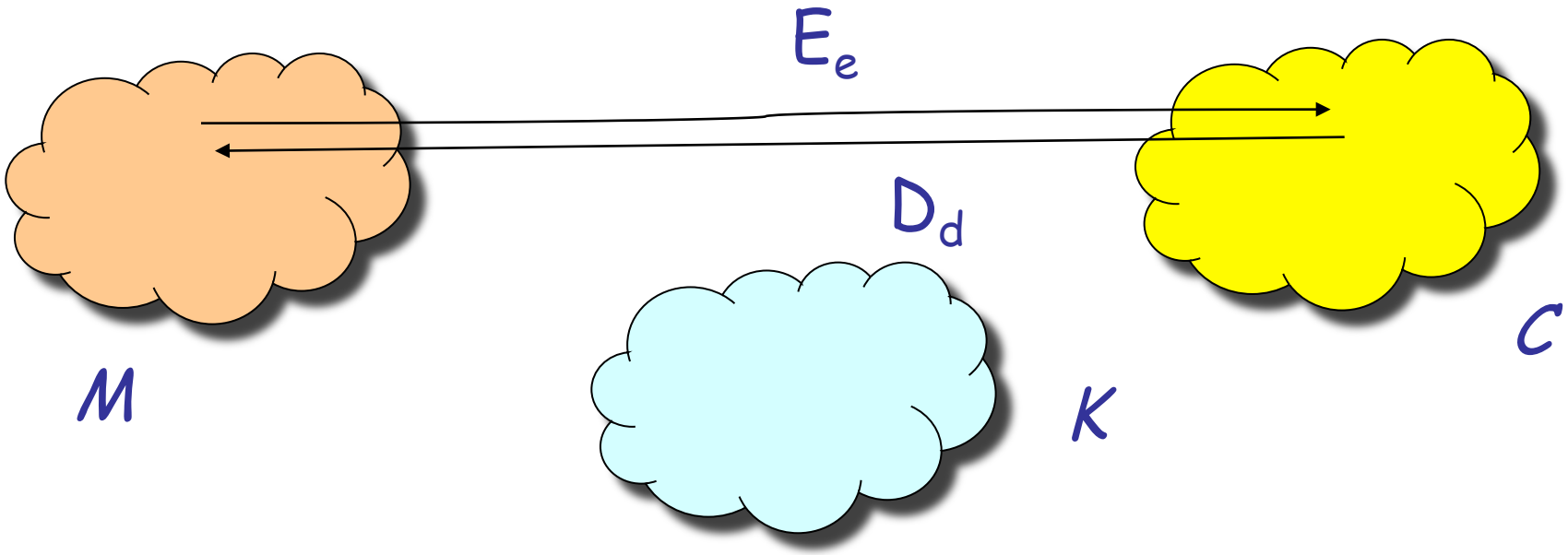
Basic Terminology

- A cipher or encryption scheme or cryptosystem consists of:
  - a set $\{E_e: \mathbf{e} \in \boldsymbol{K}\}$ of encryption transformations
  - a set $\{D_d: \mathbf{d} \in \boldsymbol{K}\}$ of decryption transformations
  - with the property that for each $\mathbf{e} \in \boldsymbol{K}$ there is a unique key $\mathbf{d} \in \boldsymbol{K}$ such that $D_d(E_e(\mathbf{m})) = \mathbf{m}$, for all $\mathbf{m} \in \boldsymbol{M}$.

Basic Terminology

- Keys **e** and **d** are referred to as a *key pair* and sometimes denoted by (**e**,**d**).
- Note that **e** and **d** could be the same!
- Why are keys necessary? Why not just choose one encryption and decryption function?
  - If an encryption/decryption transformation is revealed then one does not have to redesign the entire scheme but simply change the key
  - As a physical analogue, consider an ordinary resettable combination lock. The structure of the lock is available to anyone who wishes to purchase one but the combination is chosen and set by the owner

Basic Terminology



- To *construct* a cryptosystem it is necessary to select a message space ***M*** , a ciphertext space ***C***, a key space ***K*** , a set of encryption transformations $\{E_e: \mathbf{e} \in \boldsymbol{K}\}$, and a corresponding set of decryption transformations $\{D_d: \mathbf{d} \in \boldsymbol{K}\}$.

Basic Terminology

Example (*encryption scheme*) Let $\mathcal{M} = \{m_1, m_2, m_3\}$ and $\mathcal{C} = \{c_1, c_2, c_3\}$. There are precisely $3! = 6$ bijections from $\mathcal{M}$ to $\mathcal{C}$. The key space $\mathcal{K} = \{1, 2, 3, 4, 5, 6\}$ has six elements in it, each specifying one of the transformations. Figure 1.5 illustrates the six encryption functions which are denoted by E_i, $1 \leq i \leq 6$. Alice and Bob agree on a trans-

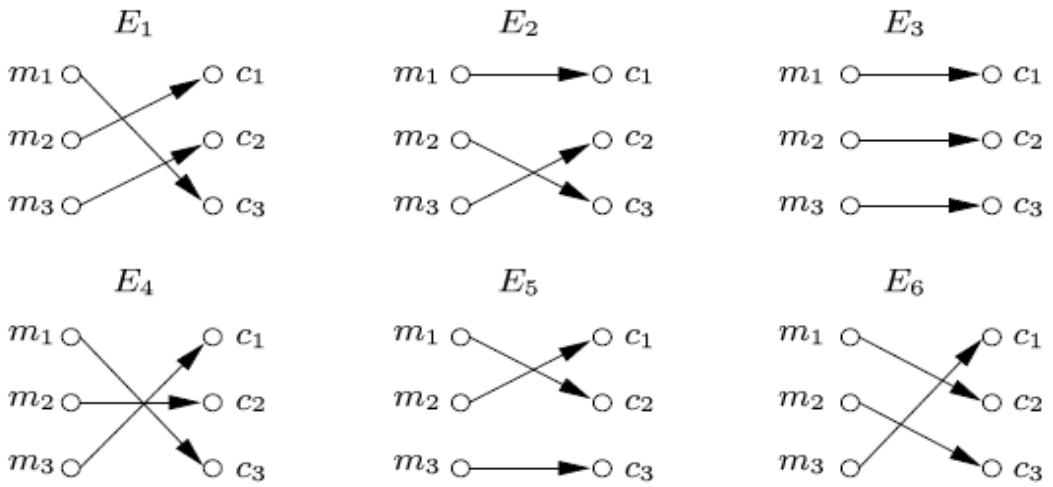


Figure 1.5: *Schematic of a simple encryption scheme.*

formation, say E_1. To encrypt the message m_1, Alice computes $E_1(m_1) = c_3$ and sends c_3 to Bob. Bob decrypts c_3 by reversing the arrows on the diagram for E_1 and observing that c_3 points to m_1.

Basic Terminology

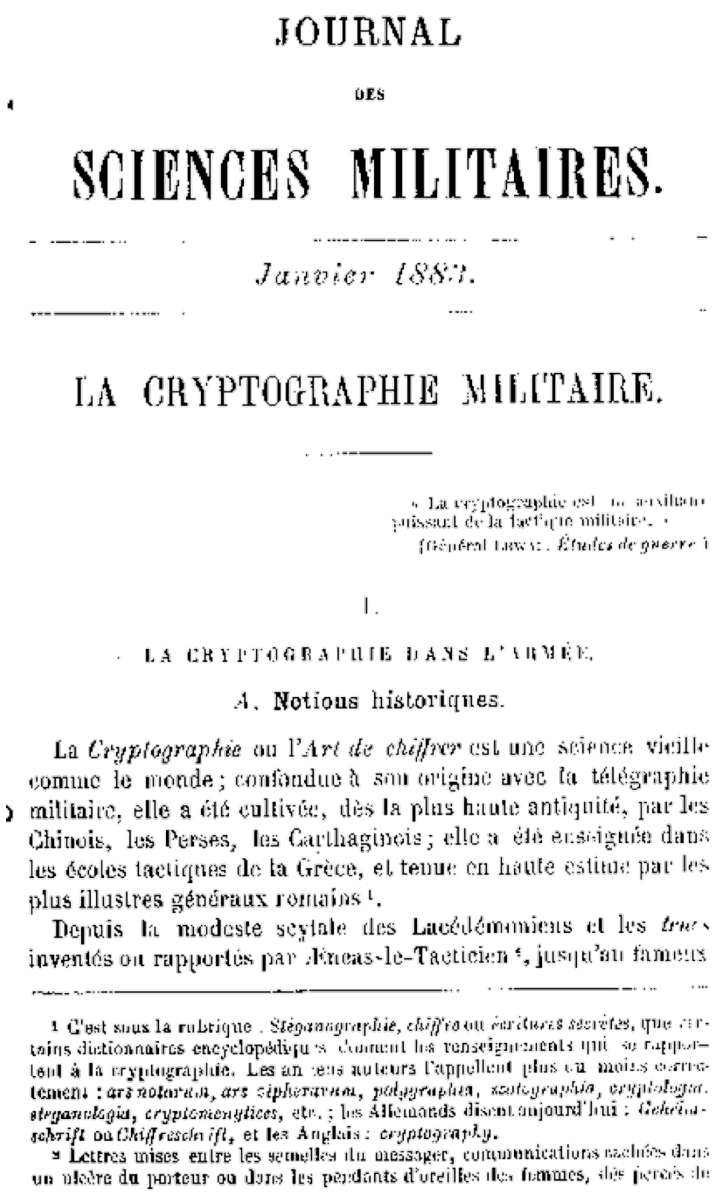
- Fundamental premise: sets ***M, C, K,*** $\{E_e: \mathbf{e} \in \boldsymbol{K}\}$, $\{D_d: \mathbf{d} \in \boldsymbol{K}\}$ are public knowledge !
  - When two parties wish to communicate securely, the only thing that they keep secret is the particular key pair (**e**,**d**) they are using.
  - One can keep the class of encryption/decryption transformations secret but one should not base the security on this approach.

Basic Terminology

- **Kerckhoffs' principle**,stated by Auguste Kerckhoffs in the 19th century
- *“a cryptosystem should be secure even if everything about the system, except the key, is public knowledge”*

Basic Terminology

- In contrast to security through obscurity
- History has shown that maintaining the secrecy of the transformations is very difficult indeed.
  - Auguste Kerckhoffs, *La cryptographie militaire*, Journal des sciences militaires, vol. IX, pp. 5–83, Jan. 1883.



JOURNAL

DES

SCIENCES MILITAIRES.

Janvier 1883.

LA CRYPTOGRAPHIE MILITAIRE.

« La cryptographie est [illegible] auxiliaire puissant de la tactique militaire. »
Général Lewal. *Études de guerre*.

I.

LA CRYPTOGRAPHIE DANS L'ARMÉE.

A. Notions historiques.

La *Cryptographie* ou l'*Art de chiffrer* est une science vieille comme le monde ; confondue à son origine avec la télégraphie militaire, elle a été cultivée, dès la plus haute antiquité, par les Chinois, les Perses, les Carthaginois ; elle a été enseignée dans les écoles tactiques de la Grèce, et tenue en haute estime par les plus illustres généraux romains [1].

Depuis la modeste scytale des Lacédémoniens et les *tru*[illegible] inventés ou rapportés par Énéas-le-Tacticien [2], jusqu'au fameux

[1] C'est sous la rubrique *Stéganographie, chiffre* ou *écritures secrètes*, que certains dictionnaires encyclopédiques donnent les renseignements qui se rapportent à la cryptographie. Les anciens auteurs l'appellent plus ou moins correctement : *ars notoria, ars ciphcratica, polygraphia, scatégraphie, cryptologia, steganologia, cryptomenytices*, etc. ; les Allemands disent aujourd'hui : *Geheimschrift* ou *Chiffrirkunst*, et les Anglais : *cryptography*.

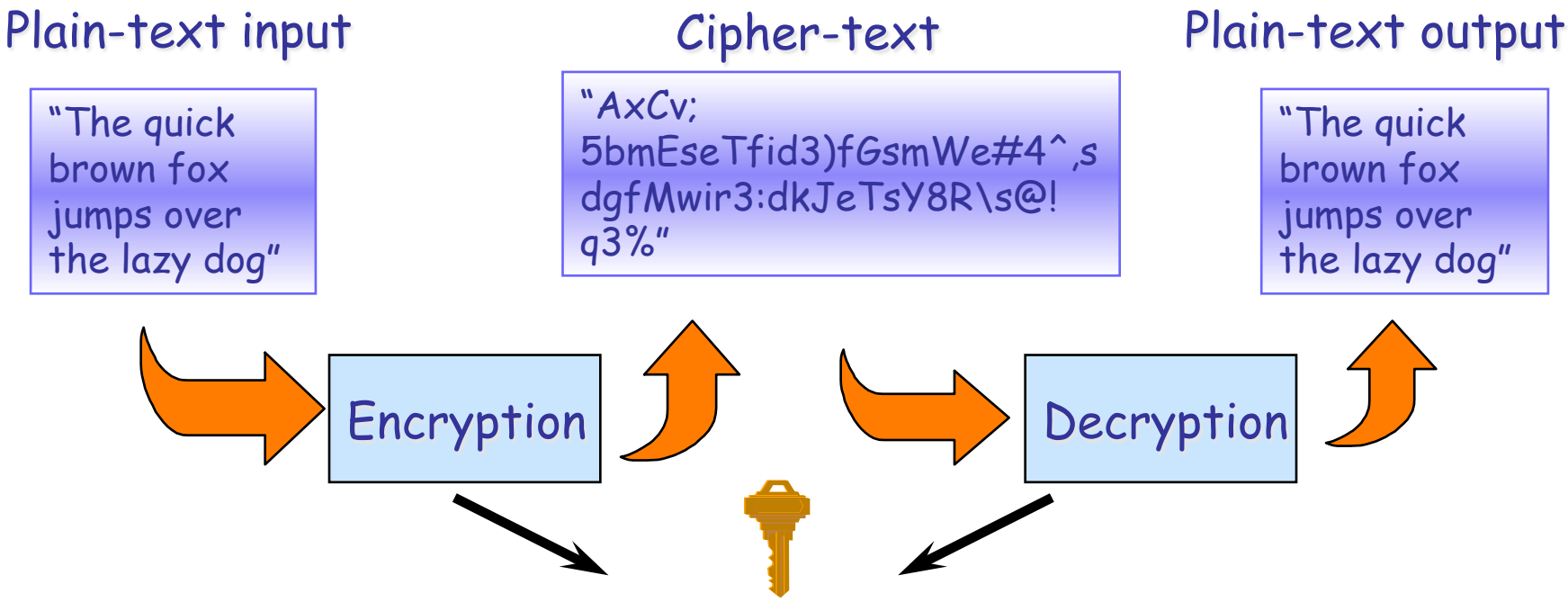
[2] Lettres mises entre les semelles du messager, communications cachées dans un ulcère du porteur ou dans les pendants d'oreilles des femmes, dès jorcés de

Basic Terminology

We can characterize algorithms by:

- Number of keys used
  - single-key or secret or symmetric
  - two-key or public or asymmetric
- Encryption operations used to transform **x** in **y**
  - Substitution
  - Transposition
  - Product
- Way in which plaintext is processed
  - lock
  - stream

Symmetric Cryptography



- all classical encryption algorithms are private-key
- it was only type prior to invention of public-key in 1970's
- it implies a secure channel to distribute the key

Symmetric Cryptography

- Two classes of symmetric-key schemes
  - *block ciphers*
  - *stream ciphers*.
- A *block cipher* is an encryption scheme which breaks up the plaintext messages to be transmitted into *blocks* of fixed length *t*, and encrypts one block at a time.
- A *stream cipher* is a very simple block cipher having block length equal to one.

Example of Symmetric Cryptography

- *Alphabet of definition* ***A*** = {A,B,C, …., X,Y,Z}
- ***M*** and ***C*** be the set of all strings of length 5 over ***A***.
- ***K:*** set of all the possible permutations on ***A*** , that is 26!
- The key **e** is a permutation on ***A .***
- To encrypt:
  - a message is broken up into groups of 5 letters (with padding if the length of the message is not a multiple of five)
  - a permutation **e** is applied to each letter one at a time.
- To decrypt, the inverse permutation $\mathbf{d} = \mathbf{e}^{-1}$, is applied to each letter of the ciphertext.

Symmetric Cryptography

- For instance, the key **e** is chosen to be the permutation which maps each letter to the one which is 3 positions to its right, as shown below:

- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
- D E F G H I J K L M N O P Q R S T U V W X Y Z A B C

- The key **d** is the inverse permutation that maps each letter to the one which is 3 positions on the left.

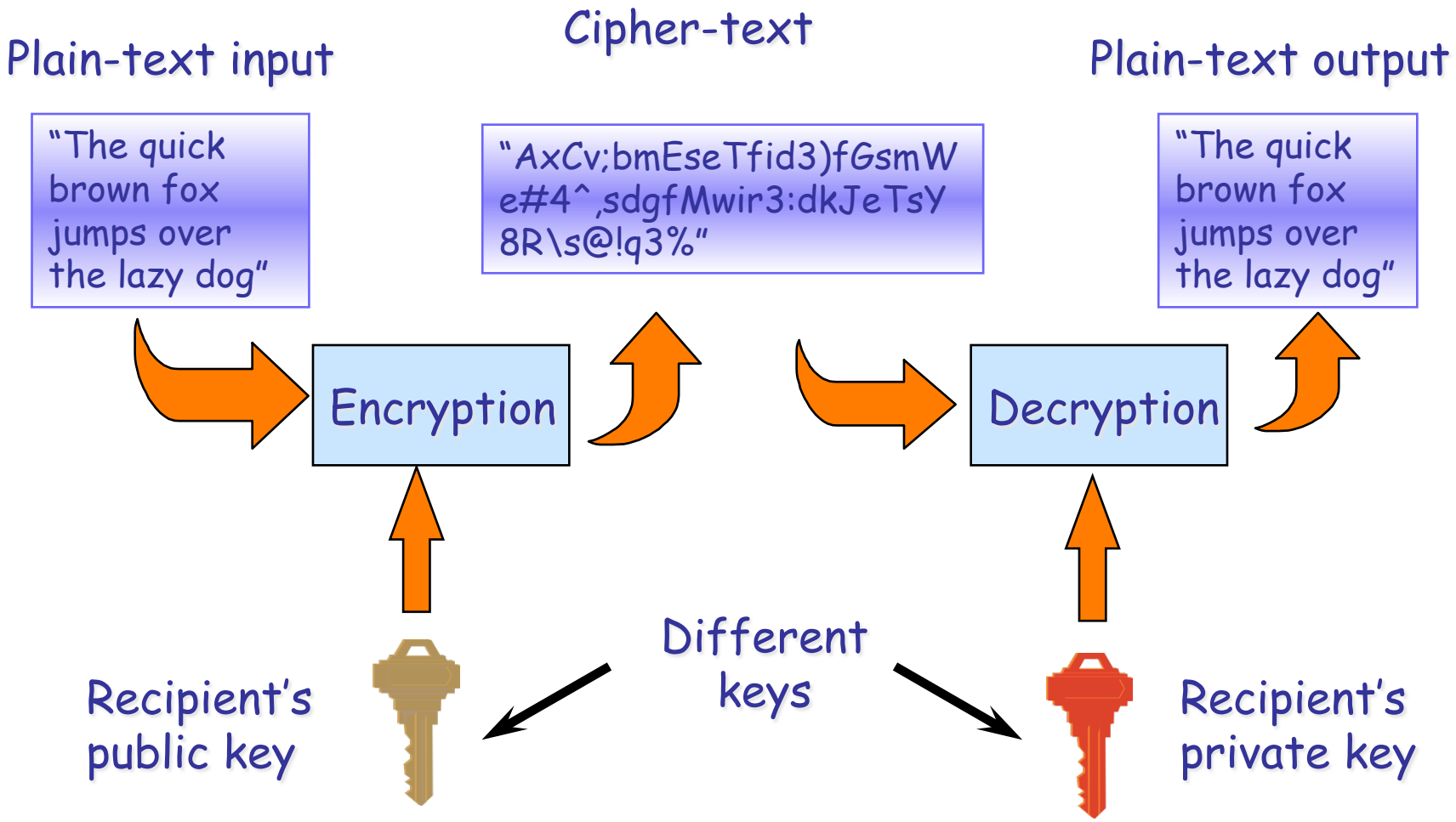
Symmetric Cryptography

- Message
  THIS CIPHER IS CERTAINLY NOT SECURE

- m = THISC IPHER ISCER TAINL YNOTS ECURE
- $E_e(m)$= WKLVF LSKHU LVFHU WDLQO BQRWV HFXUH

- Encrypted message:
  WKLVFLSKHULVFHUWDLQOBQRWVHFXUH

Asymmetric Cryptography



Asymmetric Cryptography

- Probably most significant advance in the 3000 year history of cryptography
- Uses **two** keys – a public & a private key
- **Asymmetric** since parties are **not** equal
- Based on number theory

Advantages of symmetric cryptography

- Very high data throughput (in HW achieves encrypt rates of ~ 100 MB/s, in SW MB/s).
- Relatively short keys
- Employed as primitive to construct various cryptographic mechanisms (e.g. PNRGs, hash functions, digital signature schemes)
- Can be composed to produce stronger ciphers.
- Sometimes symmetric-key encryption is perceived to be more secure because of its long history

Disadvantages of symmetric crypto

- Most disadvantages are linked to key management
  - In a two-party communication, the key must remain secret at both ends.
  - In a two-party communication between A and B, sound cryptographic practice dictates that the key be changed frequently, and perhaps for each communication session

Advantages of public-key cryptography

- Only the private key must be kept secret
  - authenticity of public keys must, however, be guaranteed
- A private key/public key pair may remain unchanged for long periods, e.g., many sessions
- Many public-key schemes yield relatively efficient digital signature mechanisms.
- In a large network, the number of keys may be considerably smaller than in the symmetric-key.

Symmetric vs. Asymmetric

- Number of keys needed for n users:
  - in public key: *2n* keys (2 for each user);
  - in private key: *n*(n-1)/2* keys (1 for each link between 2 users);

users	Keys in public-key system	Keys in private-key system
10	20	45
100	200	4.950
1.000	2.000	499.500
10.000	20.000	49.995.000
100.000	200.000	4.999.950.000
1.000.000	2.000.000	499.999.500.000

Disadvantages of public-key crypto

- Throughput is several orders of magnitude smaller than that of the best known symmetric-key schemes
- Key size is typically much larger than that required for symmetric-key encryption.
- No public-key scheme has ever been proven to be secure
  - The security of most schemes is based on the presumed difficulty of a small set of number-theoretic problems
- Public-key cryptography does not have an extensive history as symmetric-key (it was discovered in 1970's)

Symmetric vs. Asymmetric

- *Key size*
  - keys in public-key systems must be larger (e.g., 1024, 2048, 4096 bits) than in symmetric-key systems (e.g., 64, 128, 256)
  - for equivalent security, symmetric keys have bit lengths considerably smaller than that of private keys in public-key systems, e.g. by a factor >= 10.

Symmetric vs. Asymmetric

- Complementary advantages
- Strengths of each are exploited: Public-key encryption used to transmit a key for a symmetric-key system
  - A and B can take advantage of the long term nature of the keys of the public-key scheme and the performance of the symmetric-key scheme

Cryptanalysis

- To attack a cryptographic algorithm, two kinds of attacks are possible:
  - Brute force search
  - Cryptanalysis

Brute Force Search

- (or Exhaustive key search), is the basic technique of trying every possible key in turn until the correct key is identified.
  - most basic attack, proportional to key size: we assume it needs to test one half of all keys
  - assume either *know / recognise* plaintext

Brute Force Search

- Consider the following example
  - I know that ***M*** and ***C*** are the set of all strings of length 5 over ***A*** = english alphabet
  - I know that the key **e** is chosen to be a shift on ***A*** (it maps each letter to the one which is *n* positions to its right).
  - I know the ciphertext c= WKLVF LSKHU
- I wish to find the plaintext (in this case THISC IPHER), and the value of the key **e** (that is a mapping of each letter to the one which is 3 positions to its right)

Brute Force Search

- We can find the key $d=e^{-1}$ by trying all possible shifts of each letter to the one which is *n* positions to its left, where *n* = 1,2,3,…25.
  - ciphertext c = WKLVF LSKHU
  - For n=1: $D_d(c)$= VJKUE KRJGT
  - For n=2: $D_d(c)$= UIJTD JQIFS
  - For n=3: $D_d(c)$= THISC IPHER
- Only for n=3 we obtain a meaningful message !!!
- The key is e=3

Brute Force Search

- A necessary, but usually not sufficient, condition for an encryption scheme to be secure is that the key space be large enough to preclude exhaustive search

	Key Size (bits)	Number of Alternative Keys	Time required at 1 encryption/μs	Time required at 10^6 encryptions/μs
	32	$2^{32} = 4.3 \times 10^9$	$2^{31}\ \mu s = 35.8$ minutes	2.15 milliseconds
DES	56	$2^{56} = 7.2 \times 10^{16}$	$2^{55}\ \mu s = 1142$ years	10.01 hours
AES	128	$2^{128} = 3.4 \times 10^{38}$	$2^{127}\ \mu s = 5.4 \times 10^{24}$ years	5.4×10^{18} years
TripleDES	168	$2^{168} = 3.7 \times 10^{50}$	$2^{167}\ \mu s = 5.9 \times 10^{36}$ years	5.9×10^{30} years
	26 characters (permutation)	$26! = 4 \times 10^{26}$	$2 \times 10^{26}\ \mu s = 6.4 \times 10^{12}$ years	6.4×10^{6} years

Brute Force Search

Reference	Magnitude
Seconds in a year	$\approx 3 \times 10^7$
Age of our solar system (years)	$\approx 6 \times 10^9$
Seconds since creation of solar system	$\approx 2 \times 10^{17}$
Clock cycles per year, 50 MHz computer	$\approx 1.6 \times 10^{15}$
Binary strings of length 64	$2^{64} \approx 1.8 \times 10^{19}$
Binary strings of length 128	$2^{128} \approx 3.4 \times 10^{38}$
Binary strings of length 256	$2^{256} \approx 1.2 \times 10^{77}$
Number of 75-digit prime numbers	$\approx 5.2 \times 10^{72}$
Electrons in the universe	$\approx 8.37 \times 10^{77}$

Reference numbers comparing relative magnitudes.

An important question then is "How large is large?".

In order to gain some perspective on the magnitude of numbers, the Table lists various items along with an associated magnitude.

Cryptanalysis

- It is the art (science) of cracking codes.
  - It exploits the characteristics of the cryptosystem, and, depending on the type of attack, the knowledge of the plaintext/ciphertext characteristics
  - The aim is to find the key and, in some cases, the plaintext
  - In order to design a robust encryption algorithm or cryptographic protocol, one should use cryptanalysis to find and correct any weaknesses

Cryptanalytic Attacks

- Attack categories are usually listed ordering them according to the quality of information available to the cryptanalyst
- Equivalently, in decreasing order with respect to the level of difficulty for the cryptanalyst.
- The ideal for a cryptanalyst is to extract the key K, so that any $E_K[X]=Y$ can be decrypted.

Types of Cryptanalytic Attacks

We assume that the algorithm and the ciphertext to be decrypted are known. Then we have:

- ciphertext only
- known plaintext
- chosen plaintext
- chosen ciphertext

Ciphertext only

- **Only ciphertext to be decrypted is known**
  - Easiest attack to cope with
  - The attacker has only access to some enciphered messages and tries to derive the key
  - Attacker does not try all the possible keys
    - probably they are too many…

Ciphertext only

- Without any knowledge about the plaintext, only statistical attacks can be used:
  - some hypothesis about the statistical distribution of the letters in the alphabet
  - some hypothesis about the header format (e.g. email begins with “dear ..”, or it is a .ps file)

Known plaintext

- **One or more pairs of plaintext & ciphertext known**
- The attacker knows (or strongly suspects) some plaintext-ciphertext pairs (blocks)
  - Suppose he/she knows that "bcmjhvb" corresponds to the encryption of "caligula"
  - this knowledge can be used to decrypt the other parts of the cyphertext ...
  - ... or to discover the key used for encryption

Chosen plaintext

- **The opponent has obtained temporary access to the encryption machinery**
- Hence, he is able to **select a plaintext and to obtain the corresponding ciphertext**
  - the aim is to obtain the key used for encryption
  - the attacker uses knowledge of algorithm structure in attack to choose properly formatted plaintexts to be encrypted
  - This type of attack is generally most applicable to public-key cryptosystems

Chosen ciphertext

- **The opponent has obtained temporary access to the decryption machinery**
- **Then, he is able to select a ciphertext and obtain the corresponding plaintext** (with the exclusion of the one he wants to decrypt)
  - allows further knowledge of algorithm structure to be used for the attack
  - This type of attack is generally most applicable to public-key cryptosystems

Security definition

- One of the most important properties of a cryptographic system is a proof of security.
- However, every design involves a trade-off between the strength of the security and further important qualities of a cryptosystem, such as efficiency and practicality.
- The most popular security models currently used in cryptographic research include **computational security** and **unconditional security**.

Computational security

- It is based on the amount of computation required to break a system by the best currently known cryptanalytic method
- A proposed scheme is *computationally secure if the level of computation required to defeat it (using the best attack known) exceeds, by a comfortable margin, the computational resources of the hypothesized adversary*

Computational security

- Most of the currently used public-key cryptosystems (RSA, Diffie-Hellman) as well as private-key systems (DES, IDEA, RC5) fall into this category.
- In principle, all of them can be broken by trying the possible keys in sequence.
- But in practice, such attacks are considered as unfeasible because they would take from months to millions of years on the fastest of today's computers.

Computational security

- Sometimes computational security can be proven
- A cryptographic method is provably secure if the difficulty of defeating it can be shown to be essentially as difficult as solving a well-known and supposedly difficult (typically number-theoretic) problem.
- Factoring large numbers is a typical example.
- It is believed that there is essentially no better way for solving these problems than trying all possible solutions in sequence, but nobody has found a proof for this yet.

Unconditional security

- It is based on information theory and imposes no limits on the adversary's computational power
- Unconditionally secure systems can not be broken even if all possible keys could be tried within short time
- The first definition of information-theoretic secrecy was given by Shannon
  - C.E Shannon, “Communication Theory of Secrecy Systems”, *Bell System Technical Journal*, vol. 28(4), pp. 656–715, 1949.

Unconditional security

- Cryptosystems are unconditionally secure against a ciphertext-only attack if plaintext X and ciphertext Y are statistically independent
  - The attacker has the same probability of obtaining X whether he knows Y or not
- This is equivalent to saying that the cryptanalyst can do no better than guessing the plaintext without knowledge of the encrypted data, no matter how much time and computing power is used

Unconditional security

- If we denote the *a priori* probability that plaintext *x* occurs by $p_{\boldsymbol{M}}(x)$,
- A cryptosystem has perfect secrecy if $p_{\boldsymbol{M}}(x|y) = p_{\boldsymbol{M}}(x)$ *for all* $x \in \boldsymbol{M}$ and $y \in \boldsymbol{C}$
- This is equivalent to requiring that $I(X;Y) = 0$
- Shannon proved that unconditional security can be achieved only when the key-length is at least equal to the message entropy, which precludes the applications of these schemes in practice

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