

Application #3 Description

In Project 3, you implemented two methods for clustering sets of data. In this Application, we will analyze the performance of these two methods on various subsets of our county-level cancer risk data set. In particular, we will compare these two clustering methods in three areas:

- **Efficiency** - Which method computes clusterings more efficiently?
- **Automation** - Which method requires less human supervision to generate reasonable clusterings?
- **Quality** - Which method generates clusterings with less error?

Efficiency

The next four questions will consider the efficiency of hierarchical clustering and k-means clustering. Note that successfully computing the 3108 county images for Questions 2 and 3 in desktop Python may require some fine tuning of your code for one or both methods.

Question 1 (2 pts)

Write a function `gen_random_clusters(num_clusters)` that creates a list of clusters where each cluster in this list corresponds to one randomly generated point in the square with corners $(\pm 1, \pm 1)$. Use this function and your favorite Python timing code to compute the running times of the functions `slow_closest_pair` and `fast_closest_pair` for lists of clusters of size 2 to 200.

Once you have computed the running times for both functions, plot the result as two curves combined in a single plot. (Use a line plot for each curve.) The horizontal axis for your plot should be the the number of initial clusters while the vertical axis should be the running time of the function in seconds. Please include a legend in your plot that distinguishes the two curves.

Once you are satisfied with your plot, upload your plot into the peer assessment. Your plot will be assessed based on the answers to the following questions:

- Does the plot follow the formatting guidelines for plots? Does the plot include a legend?
- Do the two curves in the plot have the correct shapes?

Question 2 (1 pt)

Use `alg_project3_viz` to create an image of the 15 clusters generated by applying hierarchical clustering to the 3108 county cancer risk data set. You may submit an image with the 3108 counties colored by clusters or an enhanced visualization with the original counties colored by cluster and linked to the center of their corresponding clusters by lines. You can generate such an enhanced plot using our `alg_clusters_matplotlib` code by modifying the last parameter of `plot_clusters` to be `True`. Note that plotting only the resulting cluster centers is not acceptable.

Once you are satisfied with your image, upload your image in the peer assessment. Your submitted image will be assessed based on whether it matches our solution image. You do not need to include axes, axis labels, or a title for this image.

Question 3 (1 pt)

Use `alg_project3_viz` to create an image of the 15 clusters generated by applying 5 iterations of k-means clustering to the 3108 county cancer risk data set. You may submit an image with the 3108 counties colored by clusters or an enhanced visualization with the original counties colored by cluster and linked to the center of their corresponding clusters by lines. As in Project 3, the initial clusters should correspond to the 15 counties with the largest populations.

Once you are satisfied with your image, upload your image in the peer assessment. Your submitted image will be assessed based on whether it matches our solution image. You do not need to include axes, axis labels, or a title for this image.

Question 4 (1 pt)

Which clustering method is faster when the number of output clusters is either a small fixed number or a small fraction of the number of input clusters? Provide a short explanation in terms of the asymptotic running times of both methods. You should assume that `hierarchical_clustering` uses `fast_closest_pair` and that k-means clustering always uses a small fixed number of iterations.

Automation

In the next five questions, we will compare the level of human supervision required for each method.

Question 5 (1 pt)

Use `alg_project3_viz` to create an image of the 9 clusters generated by applying hierarchical clustering to the 111 county cancer risk data set. You may submit an image with the 111 counties colored by clusters or an enhanced visualization with the original counties colored by cluster and linked to the center of their corresponding clusters by lines.

Once you are satisfied with your image, upload your image in the peer assessment. Your submitted image will be assessed based on whether it matches our solution image. You do not need to include axes, axes labels, or a title for this image.

Question 6 (1 pt)

Use `alg_project3_viz` to create an image of the 9 clusters generated by applying 5 iterations of k-means clustering to the 111 county cancer risk data set. You may submit an image with the 111 counties colored by clusters or an enhanced visualization with the original counties colored by cluster and linked to the center of their corresponding clusters by lines. As in Project 3, the initial clusters should correspond to the 9 counties with the largest populations.

Once you are satisfied with your image, upload your image in the peer assessment. Your submitted image will be assessed based on whether it matches our solution image. You do not need to include axes, axes labels, or a title for this image.

Question 7 (1 pt)

The clusterings that you computed in Questions 5 and 6 illustrate that not all clusterings are equal. In particular, some clusterings are better than others. One way to make this concept more precise is to formulate a mathematical measure of the error associated with a cluster. Given a cluster C , its *error* is the sum of the squares of the distances from each county in the cluster to the cluster's center, weighted by each county's population. If p_i is the position of the county and w_i is its population, the cluster's error is:

$$\text{error}(C) = \sum_{p_i \in C} w_i (d_{p_i c})^2$$

where c is the center of the cluster C . The `Cluster` class includes a method `cluster_error(data_table)` that takes a `Cluster` object and the original data table associated with the counties in the cluster and computes the error associated with a given cluster.

Given a list of clusters L , the *distortion* of the clustering is the sum of the errors associated with its clusters.

$$\text{distortion}(L) = \sum_{C \in L} \text{error}(C).$$

Write a function `compute_distortion(cluster_list)` that takes a list of clusters and uses `cluster_error` to compute its distortion. Now, use `compute_distortion` to compute the distortions of the two clusterings in questions 5 and 6. Enter the values for the distortions (with at least four significant digits) for these two clusterings in the box below. Clearly indicate the clusterings to which each value corresponds.

As a check on the correctness of your code, the distortions associated with the 16 output clusters produced by hierarchical clustering and k-means clustering (with 5 iterations) on the 290 county data set are approximately 2.575×10^{11} and

2.323×10^{11} , respectively.

Question 8 (1 pt)

Examine the clusterings generated in Questions 5 and 6. In particular, focus your attention on the number and shape of the clusters located on the west coast of the USA.

Describe the difference between the shapes of the clusters produced by these two methods on the west coast of the USA. What caused one method to produce a clustering with a much higher distortion? To help you answer this question, you should consider how k-means clustering generates its initial clustering in this case.

In explaining your answer, you may need to review the geography of the west coast of the USA.

Question 9 (1 pt)

Based on your answer to Question 8, which method (hierarchical clustering or k-means clustering) requires less human supervision to produce clusterings with relatively low distortion?

Quality

In the last two questions, you will analyze the quality of the clusterings produced by each method as measured by their distortion.

Question 10 (4 pts)

Compute the distortion of the list of clusters produced by hierarchical clustering and k-means clustering (using 5 iterations) on the 111, 290, and 896 county data sets, respectively, where the number of output clusters ranges from 6 to 20 (inclusive). **Important note:** To compute the distortion for all 15 output clusterings produced by **hierarchical_clustering**, you should remember that you can use the hierarchical cluster of size 20 to compute the hierarchical clustering of size 19 and so on. Otherwise, you will introduce an unnecessary factor of 15 into the computation of the 15 hierarchical clusterings.

Once you have computed these distortions for both clustering methods, create three separate plots (one for each data set) that compare the distortion of the clusterings produced by both methods. Each plot should include two curves drawn as line plots. The horizontal axis for each plot should indicate the number of output clusters while the vertical axis should indicate the distortion associated with each output clustering. For each plot, include a title that indicates the data set used in creating the plots and a legend that distinguishes the two curves.

Once you are satisfied with your plots, upload these plots (separately) into the peer assessment. Your plots will be assessed based on the answers to the following questions:

- Do the plots follow the formatting guidelines for plots? Does the title of each plot indicate which data was used to create the plot? Do the plots include a legend?
- Do the two curves in each plot have the correct shapes?

Question 11 (1 pt)

For each data set (111, 290, and 896 counties), does one clustering method consistently produce lower distortion clusterings when the number of output clusters is in the range 6 to 20? If so, indicate on which data set(s) one method is superior to the other.

Overall evaluation (optional, no credit)

Which clustering method would you prefer when analyzing these data sets? Provide a summary of each method's strengths and weaknesses on these data sets in the three areas considered in this application. Your summary should be at least a paragraph in length (4 sentences minimum).

✓ Terminer

