

You are on a game show and given the choice of whatever is behind three doors. Behind one door is a fantastic prize (some examples use a car, others use cash) while behind the other two doors is a dud (some examples say a goat, others say it's just empty). You pick a door. Then the host opens one of the other two doors to reveal a dud. But here's the wrinkle: the host now gives you the opportunity to switch your door. What should you do?

Write up some notes on this problem, including how you think Bayes' Rule might apply. Drop a link to your notes below and discuss it with your mentor.

$$P(A | B) = P(B | A) * P(A) / P(B)$$

Intuitively, in this situation the contestant should stick with the door they chose. By revealing a dud behind the third door, the host has changed the probability question from $\frac{1}{3}$ to $\frac{1}{2}$ thus increasing the odds of winning the prize by about 27%, from 33.33% to 50%. At 50% odds changing my choice does not increase the probability of winning the prize.

However, as is often the case, the Bayes rule contradicts intuition. Since the host will not open the door with the prize behind it, whichever door the host does not choose automatically increases its odds of hiding the prize. I don't quite know how to show that with the Bayes' Rule formula....

$$P(\text{prize} | \text{dud}) = P(\text{dud} | \text{prize}) * (P(\text{prize})/P(\text{dud}))$$

For $P(\frac{1}{3})$:

$$P(\text{prize} | \text{dud}) = (\frac{1}{3} * \frac{2}{3}) * (\frac{1}{3} / \frac{2}{3}) = (2/9) * (\frac{1}{2}) = 2/18 = 1/9$$

For $P(\frac{1}{2})$:

$$P(\text{prize} | \text{dud}) = (1) * (\frac{1}{2} / \frac{1}{2}) = (1) * (\frac{1}{4}) = 1/4$$