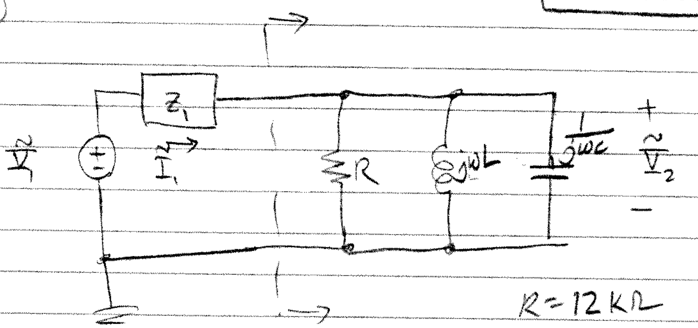
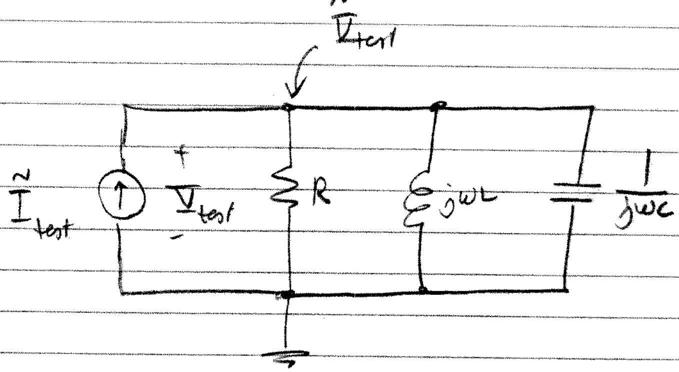
ECE2100
HW#11 SOLUTIONS
Nov. 2015
CLYMER

BC: 11.1



$R = 12\,k\Omega$
$L = 0.005\,H$
$C = 30\times10^{-9}\,F$

a.)



KCL at $\tilde{V}_{test}$:

$$0 = -\tilde{I}_{test} + \frac{\tilde{V}_{test} - 0}{R} + \frac{\tilde{V}_{test} - 0}{j\omega L} + \frac{\tilde{V}_{test} - 0}{1/(j\omega C)}$$

$$\tilde{I}_{test} = \tilde{V}_{test}\left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right)$$

②

$$\tilde{Z}_{EQ} = \frac{\tilde{V}_{test}}{\tilde{I}_{test}} = \frac{1}{\left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right)} \frac{(j\omega L)}{(j\omega L)}$$

$$\tilde{Z}_{EQ} = \frac{j\omega L}{1 + (j\omega C)(j\omega L) + j\frac{\omega L}{R}} = \frac{j\omega L}{1 - \omega^2 LC + j\frac{\omega L}{R}}$$

$$\tilde{Z}_{EQ} = \frac{j\omega(0.005)}{1 - \omega^2(0.005)(30\times10^{-9}) + j\omega\frac{0.005}{12\times10^3}}$$

$$\tilde{Z}_{EQ} = \frac{j\omega(0.005)}{1 - \omega^2(1.5\times10^{-10}) + j\omega(4.167\times10^{-7})}$$

TO EXPRESS AS $\tilde{Z}_{EQ} = R_{EQ} + jX_{EQ}$,
RATIONALIZE DENOMINATOR:

$$\tilde{Z}_{EQ} = \frac{(j\omega L)\left(1 - \omega^2 LC - j\frac{\omega L}{R}\right)}{(1 - \omega^2 LC)^2 + \left(\frac{\omega L}{R}\right)^2}$$

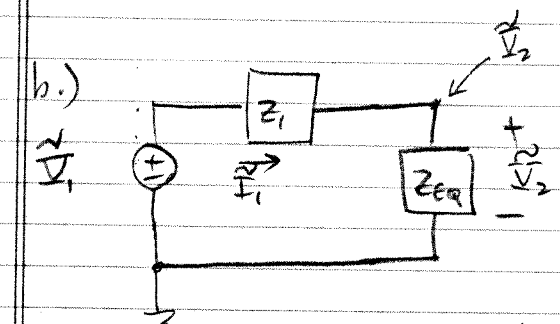
$$\tilde{Z}_{EQ} = \frac{\left(\frac{\omega L}{R}\right)^2 + j(\omega L)(1 - \omega^2 LC)}{(1 - \omega^2 LC)^2 + \left(\frac{\omega L}{R}\right)^2} = R_{EQ} + jX_{EQ}$$

③

$$R_{EQ} = \frac{\left(\frac{\omega L}{R}\right)^2}{(1-\omega^2 LC)^2 + \left(\frac{\omega L}{R}\right)^2} = \frac{\omega^2(1.73\times10^{-13})}{\left[1-\omega^2(1.5\times10^{-10})\right]^2 + \omega^2(1.73\times10^{-13})}$$

$$X_{EQ} = \frac{\omega L(1-\omega^2 LC)}{(1-\omega^2 LC)^2 + \left(\frac{\omega L}{R}\right)^2} = \frac{\omega(0.005 - 7.5\times10^{-13}\omega^2)}{\left[1-\omega^2(1.5\times10^{-10})\right]^2 + \omega^2(1.73\times10^{-13})}$$

b.)



KCL at $\tilde{V}_2$: $0 = -\tilde{I}_1 + \frac{\tilde{V}_2 - 0}{Z_{EQ}}$

$\tilde{I}_1 \cdot Z_{EQ} = \tilde{V}_2 \rightarrow \tilde{V}_2$ & $\tilde{I}_1$ HAVE SAME PHASE WHEN $\mathrm{Im}\{Z_{EQ}\} = 0$

$X_{EQ} \rightarrow 0 \rightarrow \boxed{\omega = 0 \text{ works} \rightarrow f = 0\,Hz}$

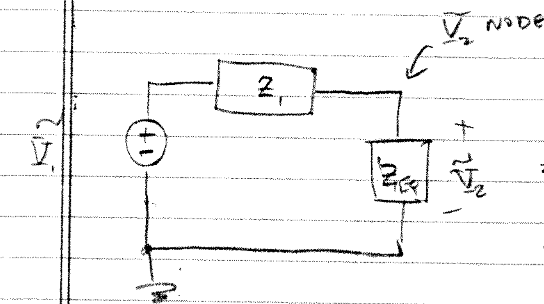
$0.005 = 7.5\times10^{-13}\omega^2 \rightarrow \omega^2 = \frac{0.005}{7.5\times10^{-13}} = 6.67\times10^{9}$ rad/s

$\omega = 81649.7$ rad/s $\rightarrow \boxed{f = \frac{\omega}{2\pi} = 12{,}995\,Hz}$

③

c.) $v_1(t) = 25\cos(2\pi\, 15000t)$ V

$\tilde{V}_1 = 25\angle 0°$ $(\omega = 2\pi\, 15000)$



$Z_1 = 0.02 + j0.015\,\omega$

$Z_1 = 0.02 + j(0.015)(2\pi\, 15000)$

$Z_1 = 0.02 + j\,1413.72$

$$Z_{EQ} = \frac{j\omega(0.005)}{1-\omega^2(0.005)(30\times10^{-9}) + j\omega\left(\frac{0.005}{12\times10^3}\right)}$$

$$Z_{EQ} = \frac{j(2\pi\, 15000)(}{1-(2\pi\, 15000)^2(0.005)(30\times10^{-9}) + j(2\pi\, 15000)\left(\frac{0.005}{12\times10^3}\right)}$$

$$Z_{EQ} = \frac{j\,94247.8}{-0.3324 + j\,0.0393} = \underline{165.2 - j\,1398.2}$$

(4)

KCL at $\tilde{V}_2$:

$$0 = \frac{\tilde{V}_2 - \tilde{V}_1}{Z_1} + \frac{\tilde{V}_2 - 0}{Z_{EQ}}$$

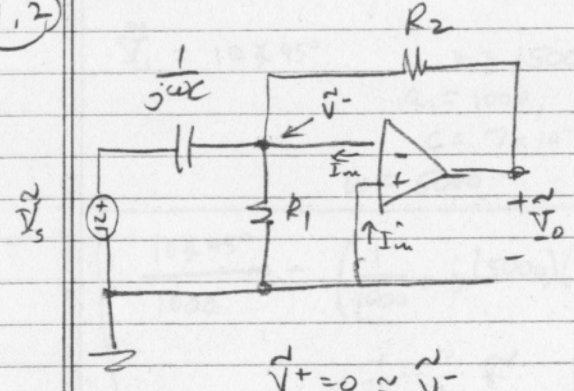
$$\frac{\tilde{V}_1}{Z_1} = \left(\frac{1}{Z_1} + \frac{1}{Z_{EQ}}\right)\tilde{V}_2$$

$$\tilde{V}_2 = \frac{\tilde{V}_1}{Z_1\left(\frac{1}{Z_1} + \frac{1}{Z_{EQ}}\right)} = \frac{\tilde{V}_1}{1 + \frac{Z_{11}}{Z_{EQ}}} = \frac{25\angle 0^\circ}{1 + \frac{0.02 + j1418}{165.2 - j1398}}$$

$$\boxed{\tilde{V}_2 = 5.06 - j212.1 = 212.12\angle -88.6^\circ}$$

$$\boxed{v_2(t) = 212.12\cos(2\pi 15000t - 88.6^\circ)}$$

(11.2)



$\omega = 2\pi \cdot 15\times10^3$ rad/s

$R_1 = 120\,\Omega$

$R_2 = 240\,\Omega$

$C = 400\times10^{-6}$ F

$$\tilde{V}^+ = 0 \cong \tilde{V}^-, \quad \tilde{I}_n \cong 0$$

$$\text{KCL at } \tilde{V}^-: \quad 0 = \frac{\cancel{\tilde{V}^-} - \tilde{V}_s}{1/(j\omega C)} + \frac{\cancel{\tilde{V}^-} - 0}{R_1} + \frac{\cancel{\tilde{V}^-} - \tilde{V}_o}{R_2} - \cancel{\tilde{I}_n}$$

$$\tilde{V}_S \cdot j\omega C = -\frac{\tilde{V}_o}{R_2}$$

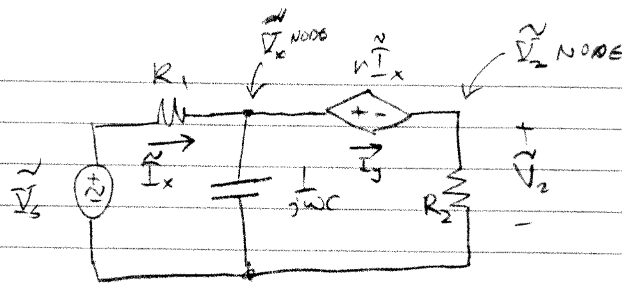
$$\tilde{V}_o = -\tilde{V}_s \cdot j\omega C R_2 \quad G_V = \frac{\tilde{V}_o}{\tilde{V}_s} = -j\omega C R_2$$

$$G_V = -j(2\pi 15\times10^3)\cdot(400\times10^{-6})(240)$$

$$= -j9048$$

$$\boxed{G_V = 9048\angle -90^\circ}$$

BC: 11.3



Ω'S LAW: $\frac{\tilde{V}_s - \tilde{V}_x}{R_1} = \tilde{I}_x$

KCL at $\tilde{V}_x$: $0 = \frac{\tilde{V}_x - \tilde{V}_s}{R_1} + \frac{\tilde{V}_x - 0}{1/(j\omega C)} + I_y$

$$\boxed{\frac{\tilde{V}_s}{R_1} = \left(\frac{1}{R_1} + j\omega C\right)\tilde{V}_x + \tilde{I}_y}$$

KCL at $\tilde{V}_2$: $0 = \frac{\tilde{V}_2 - 0}{R_2} - \tilde{I}_y$

$$\boxed{0 = \frac{\tilde{V}_2}{R_2} - \tilde{I}_y}$$

KVL FACT: $\tilde{V}_x - \tilde{V}_2 = r \cdot \tilde{I}_x = r\left[\frac{\tilde{V}_s - \tilde{V}_x}{R_1}\right]$

$$\boxed{\frac{r\tilde{V}_s}{R_1} = \tilde{V}_x\left(1 + \frac{r}{R_1}\right) - \tilde{V}_2}$$

$\tilde{V}_s = 10\angle 45°$

$r = 1500\,\Omega$

$R_1 = 1000$, $R_2 = 2000$

$C = 7\times10^{-6}\,F$

$\omega = 5000$

$$\frac{10\angle 45°}{1000} = \left(\frac{1}{1000} + j(5000)(7\times10^{-6})\right)\tilde{V}_x + \tilde{I}_y + 0 \cdot \tilde{V}$$

$$0 = 0\tilde{V}_x - \tilde{I}_y + \frac{1}{2000}\tilde{V}_2$$

$$\frac{1500 \cdot (10\angle 45°)}{1000} = \left(1 + \frac{1500}{1000}\right)\tilde{V}_x + 0 \cdot \tilde{I}_y - \tilde{V}_2$$

a.) SOLVING ON CALCULATOR:

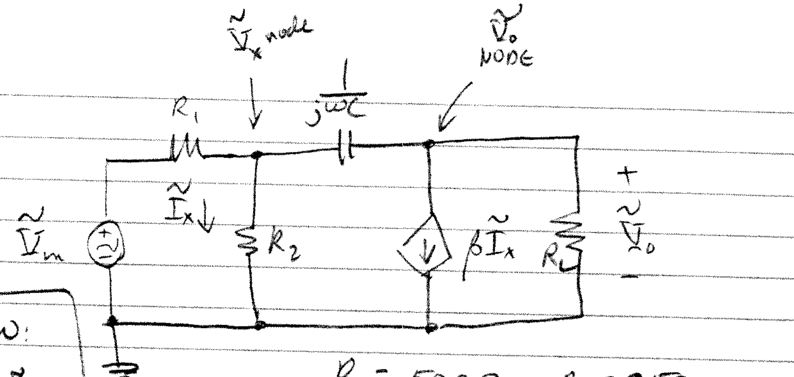
$\tilde{V}_x = 1.499\angle -41.3°$

$\tilde{I}_y = 0.0075\angle -130.2°$

$\tilde{V}_2 = 14.97\angle -130.2°$

b.) $v_2(t) = 14.97\cos(5000t - 130.2°)$

BC 11.4



Ω's LAW:

$$\boxed{\tilde{I}_x = \frac{\tilde{V}_x - 0}{R_2}}$$

$R_1 = 500\Omega \quad R_2 = 2500\Omega$

$R_L = 1000\Omega \quad C = 200\times10^{-12}F$

$\tilde{V}_m = 0.025\angle 0° \qquad \omega = 2\pi(20\times10^6)$ radians

KCL at $\tilde{V}_x$: $0 = \frac{\tilde{V}_x - \tilde{V}_m}{R_1} + \frac{\tilde{V}_x - 0}{R_2} + \frac{\tilde{V}_x - \tilde{V}_o}{1/(j\omega C)}$

$$\boxed{\frac{\tilde{V}_m}{R_1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + j\omega C\right)\tilde{V}_x - \tilde{V}_o(j\omega C)}$$

KCL at $\tilde{V}_o$ = $\quad \tilde{I}_x = \frac{\tilde{V}_x}{R_2}$

$$0 = \frac{\tilde{V}_o - \tilde{V}_x}{1/(j\omega C)} + \beta\tilde{I}_x + \frac{\tilde{V}_o - 0}{R_L}$$

$$\boxed{0 = \left(\frac{\beta}{R_2} - j\omega C\right)\tilde{V}_x + \left(\frac{1}{R_L} + j\omega C\right)\tilde{V}_o}$$

(2)

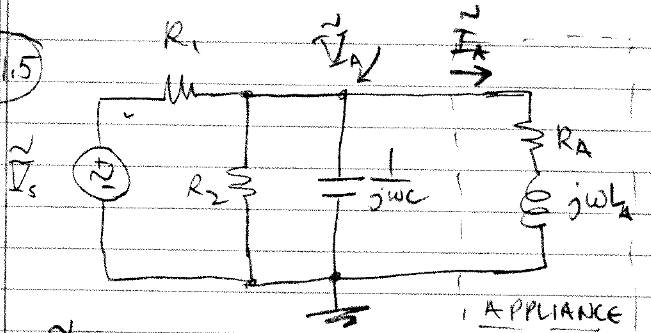
$$\frac{0.025\angle 0°}{500} = \left(\frac{1}{500} + \frac{1}{2500} + j(2\pi\, 2\times10^7)(2\times10^{-10})\right)\tilde{V}_x - j(2\pi\, 2\times10^7)(2\times10^{-10})$$

$$0 = \left(\frac{150}{2500} - j(2\pi 2\times10^7)(2\times10^{-10})\right)\tilde{V}_x + \left(\frac{1}{1000} + j(4\pi\times10^7)(2\times10^{-10})\right)\tilde{V}_o$$

$\tilde{V}_x = 7.9\times10^{-3}\angle -2.2°$

$$\boxed{\tilde{V}_o = 2.04\times10^{-3}\angle 67.4°}$$

BC: 11.5



$\tilde{V}_s = 120\angle 0°$ (rms)

$R_1 = 400\,\Omega$, $R_2 = 500\,\Omega$, $C = 40\times10^{-6}$ F

$\omega = 120\pi$ radians/s

KCL at $\tilde{V}_A$:

$$0 = \frac{\tilde{V}_A - \tilde{V}_s}{R_1} + \frac{\tilde{V}_A - 0}{R_2} + \frac{\tilde{V}_A - 0}{1/(j\omega C)} + \frac{\tilde{V}_A - 0}{R_A + j\omega L_A}$$

$$\frac{\tilde{V}_s}{R_1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + j\omega C + \frac{1}{R_A + j\omega L_A}\right)\tilde{V}_A$$

$$\tilde{V}_A = \frac{\tilde{V}_s}{R_1\left(\frac{1}{R_1} + \frac{1}{R_2} + j\omega C + \frac{1}{R_A + j\omega L_A}\right)} = \frac{\tilde{V}_s}{1 + \frac{R_1}{R_2} + j\omega C R_1 + \frac{R_1}{R_A + j\omega L_A}}$$

$$\tilde{V}_A = \frac{120\angle 0°}{\left(1 + \frac{400}{500} + j(120\pi)(40\times10^{-6})(400) + \frac{400}{50 + j(120\pi)(0.4)}\right)} \text{ (rms)}$$

$\tilde{V}_A = 26.8\angle -54.5°$ (Vrms)

$$\tilde{I}_A = \frac{\tilde{V}_A}{Z_A} = \frac{26.8\angle -54.5°}{50 + j(120\pi)(0.4)}$$

$\tilde{I}_A = 0.169\angle -126.2°$ (Arms)

$\tilde{I}_A^* = 0.169\angle +126.2°$ (Arms)

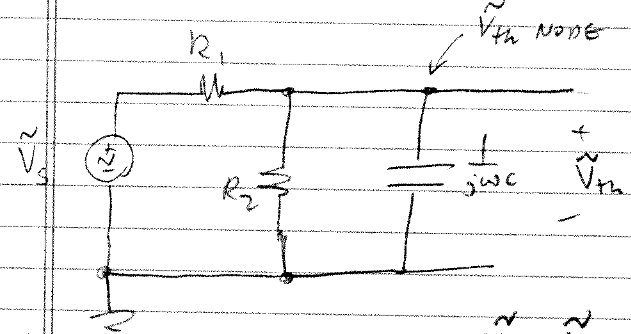
i.) $\vec{S}_A = \tilde{V}_A \cdot \tilde{I}_A^* = 4.53\angle 71.7°$ VA

$= 1.42 + j4.3$ VA

ii.) $P_A = 1.42$ W $= \mathrm{Re}\{\vec{S}\}$

iii.) $Q_A = 4.3$ VAR $= \mathrm{Im}\{\vec{S}\}$

iv.) p.f. $= \cos(71.7°) = 0.314$ LAGGING

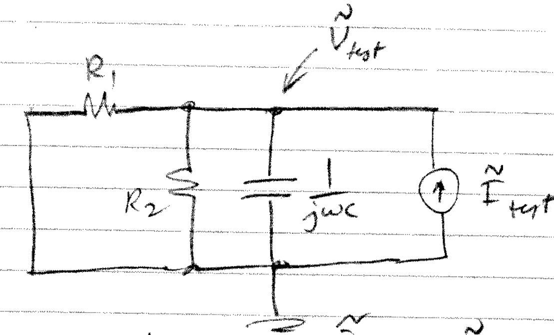


KCL at $\tilde{V}_{th}$: $0 = \dfrac{\tilde{V}_{th} - \tilde{V}_s}{R_1} + \dfrac{\tilde{V}_{th} - 0}{R_2} + \dfrac{\tilde{V}_{th} - 0}{1/(j\omega C)}$

$$\frac{\tilde{V}_s}{R_1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + j\omega C\right)\tilde{V}_{th}$$

$$\tilde{V}_{th} = \frac{\tilde{V}_s}{R_1\left(\frac{1}{R_1} + \frac{1}{R_2} + j\omega C\right)} = \frac{\tilde{V}_s}{1 + \frac{R_1}{R_2} + j\omega C R_1}$$

$$\boxed{\tilde{V}_{th} = \frac{120 \angle 0^\circ}{1 + \frac{400}{500} + j(120\pi)(40\times10^{-6})(400)} = 19.06 \angle -73.4^\circ}$$



KCL at $\tilde{V}_{test}$: $0 = \dfrac{\tilde{V}_{test}}{1/(j\omega C)} + \dfrac{\tilde{V}_{test}}{R_2} + \dfrac{\tilde{V}_{test}}{R_1} - \tilde{I}_{test}$

$$\tilde{V}_{test} = \frac{\tilde{I}_{test}}{\frac{1}{R_1} + \frac{1}{R_2} + j\omega C} = \frac{(1\angle 0^\circ)}{\left(\frac{1}{400} + \frac{1}{500} + j(120\pi)(40\times10^{-6})\right)}$$

$$\tilde{V}_{test} = 63.54 \angle -73.3^\circ$$

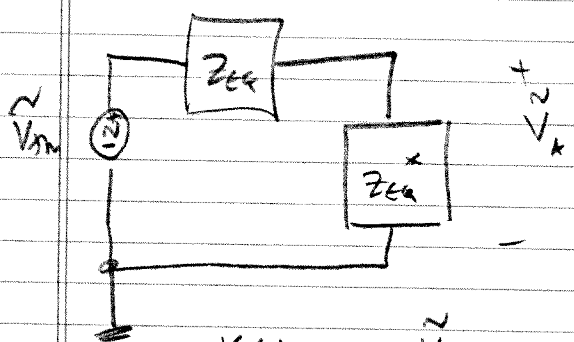
$$Z_{EQ} = \frac{\tilde{V}_{test}}{\tilde{I}_{test}} = \frac{63.54 \angle -73.3}{1 \angle 0} = 63.54 \angle -73.3$$

$$Z_{EQ} = 18.17 - j60.9$$

FOR MAXIMUM POWER TO LOAD,

WANT $Z_A = Z_{EQ}^* = 18.17 + j60.9$

$$\boxed{R_A = 18.17\Omega \qquad L_A = \frac{60.9}{120\pi} = 0.1615\ H}$$



KCL AT $\tilde{V}_A$:

$$0 = \frac{\tilde{V}_A - \tilde{V}_{th}}{Z_{EQ}} + \frac{\tilde{V}_A - 0}{Z_{EQ}^*}$$

$$\frac{\tilde{V}_{th}}{Z_{EQ}} = \tilde{V}_A\left(\frac{1}{Z_{EQ}} + \frac{1}{Z_{EQ}^*}\right)$$

$$\tilde{V}_A = \frac{\tilde{V}_{th}}{Z_{EQ}\left[\frac{1}{Z_{EQ}} + \frac{1}{Z_{EQ}^*}\right]} = \frac{19.06 \angle -73.4}{1 + \frac{18.17 - j60.9}{18.17 + j60.9}}$$

$$\tilde{V}_A = 33.3 \angle 0^\circ$$

$$\tilde{I}_A = \frac{\tilde{V}_A}{18.17 + j60.4} = 0.524 \angle -73.3 \rightarrow \tilde{I}_A^* = 0.524 \angle +73.3$$

$$P_A = Re\{\tilde{V}_A \cdot \tilde{I}_A^*\} = 33.0\,(0.15) = \underline{5.0\,W}$$