Chern-Simons Gravity and Neutrino Self-Interactions

Cyril Creque-Sarbinowski

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Center for Computational Astrophysics

What is (dynamical) Chern-Simons Gravity [dCS]?

$$S = \int d^4x\sqrt{-g}\left[\kappa R + \frac{a}{4f}\,{}^*R\,R - \frac{1}{2}\left(\nabla_\mu a\right)\left(\nabla^\mu a\right)\right]$$

Spacetime Metric, Lorentzian Signature

Einstein-Hilbert Term, $\kappa = (16\pi G)^{-1}$

dCS Decay Constant

Pontryagin Density

$${}^*R\,R = {}^*R^\rho{}_\sigma{}^{\mu\nu} R^\sigma{}_{\rho\mu\nu}$$

$${}^*R^\rho{}_\sigma{}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} R^\rho{}_{\sigma\alpha\beta}$$

dCS (pseudo)scalar

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What are Fermion Self-Interactions?

$$\mathcal{L}_\Psi = \bar{\Psi}\left(i\gamma^\mu \partial_\mu - \tilde{m}_\Psi\right)\Psi - \lambda \bar{\Psi}\Psi\bar{\Psi}\Psi$$

Dirac Gamma Matrices

``bare'' mass

Dirac Fermion

(Attractive) Fermion Self-Coupling Constant

★ Cutoff Λ

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How will we obtain dCS from Fermion-SI?

1. Identify and realize a scalar bound state in our theory of Fermion Self-Interactions (SI)

2. Demonstrate that the angular part of the (complex) scalar becomes the dCS scalar through fermion-loops with gravity after spontaneous symmetry breaking.

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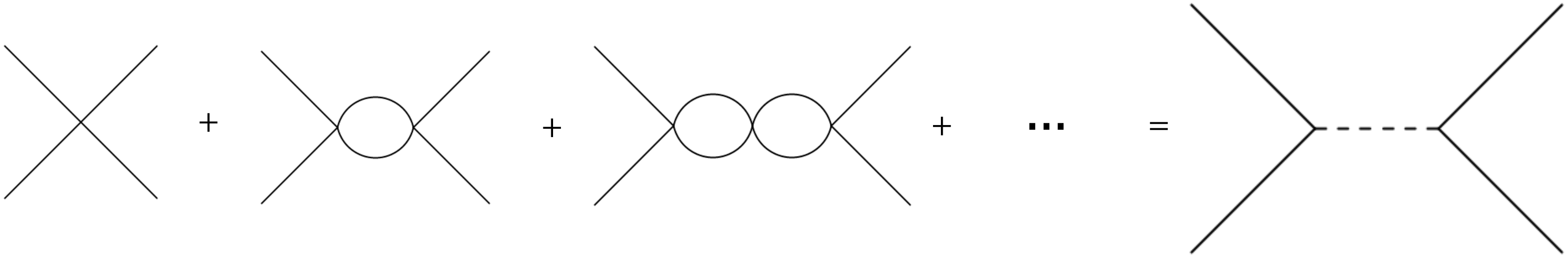
How is there a bound state?

Geometric Summation to identify pole



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How to realize the bound state?

$$Z_\alpha = \int \mathcal{D}\alpha \mathcal{D}\bar{\alpha} \exp\left(-\int d^4x \; \tilde{m}_\Phi^2 \bar{\alpha}\alpha\right)$$

$$+$$

$$\Psi = \Psi_\ell + \Psi_s$$

$$\Phi = \alpha - \tilde{m}_\Phi^{-2} \bar{\Psi}_s \Psi_s$$

$$+$$

$$\text{Integrate Out Short} - \text{Scale Modes}$$

$$\Longrightarrow$$

$$\begin{aligned}\tilde{\mathcal{L}}_\Psi = &\bar{\Psi}(i\gamma^\mu \partial_\mu - \tilde{m}_\Psi)\Psi - \lambda \bar{\Psi}\Psi\bar{\Psi}\Psi \\ &+ (\partial_\mu \Phi^*)\,(\partial^\mu \Phi) - y_\Phi\left(\Phi\bar{\Psi}\Psi + \text{h.c.}\right) \\ &+ m_\Phi^2 |\Phi|^2 - \frac{\lambda_\Phi}{4}|\Phi|^4,\end{aligned}$$

$$y_\Phi \sim \lambda \Lambda^2$$

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Spontaneous Symmetry Breaking

$$\Phi = (1/\sqrt{2})(F+\sigma)\exp[ia(x)/F].$$

$$\mathcal{L}_\Phi = \sum_{j=L,R} i\bar{\Psi}_j\gamma^\mu\partial_\mu\Psi_j - \tilde{m}_\Psi\left(\bar{\Psi}_R\Psi_L + \bar{\Psi}_L\Psi_R\right) + \partial_\mu\Phi\partial^\mu\Phi^* - V\left(|\Phi|^2\right) - \left(y_\Phi\Phi\bar{\Psi}_L\Psi_R + \text{h.c.}\right)$$

?

$$\mathcal{L}_\Phi = \bar{\Psi}\left[i\gamma^\mu\partial_\mu - \left(\tilde{m}_\Psi + \frac{y_\Phi F}{\sqrt{2}}\right)\right]\Psi + \frac{1}{2}\partial_\mu a\partial^\mu a - \frac{y_\Phi}{\sqrt{2}}a\bar{\Psi}\gamma^5\Psi,$$

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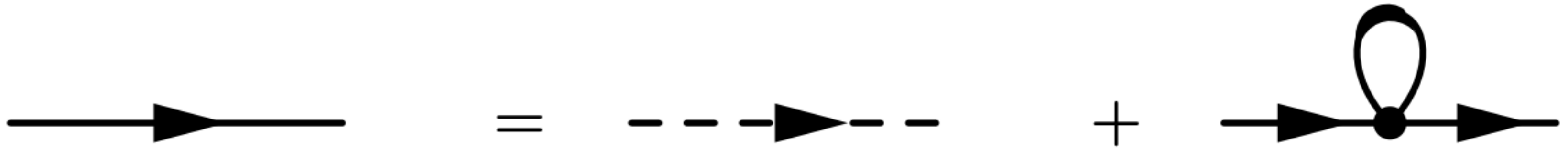
The Gap Equation



$$\frac{\Delta m_\Psi}{m_\Psi}\left(\frac{2\pi^2}{\lambda\Lambda^2}\right) = 1 - \frac{m_\Psi^2}{\Lambda^2}\log\left(1+\frac{\Lambda^2}{m_\Psi^2}\right) \,, \quad \Delta m_\Psi = y_\Phi F/\sqrt{2}$$

$\Lambda \gg m_\Psi$	$\Lambda \ll m_\Psi$
$F \approx 0.9 m_\Psi$	$F \approx 0.45\Lambda^2/m_\Psi$

Remember! ★ $y_\Phi \sim \lambda\Lambda^2$

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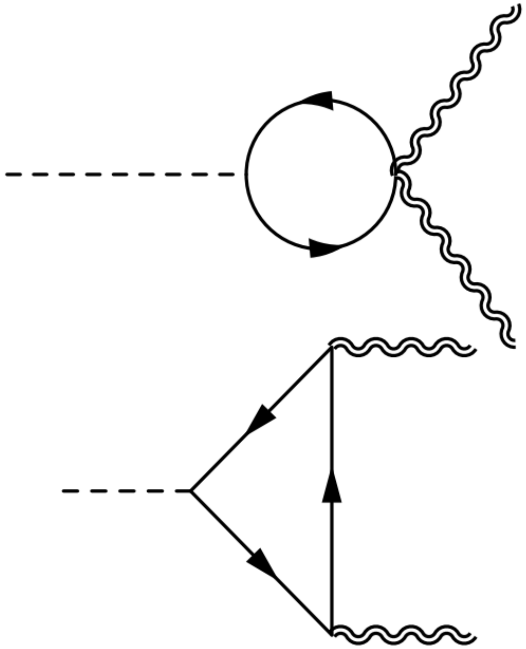
arXiv: 2207.05094, Collaborator: Stephon Alexander

Loop Generation of dCS



$$\mathcal{L}_g^{\text{eff}} = -\frac{g}{384\pi^2}\frac{a}{2m_\Psi}{}^*RR$$

$$f = 192\sqrt{2}\pi^2\frac{m_\Psi}{y_\Phi}$$

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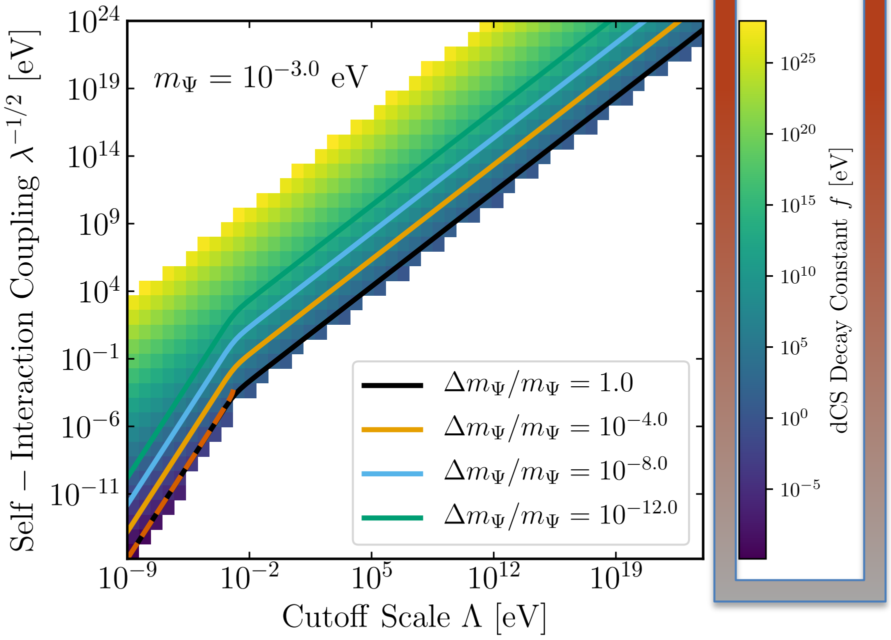
General Parameter Space



$$\Lambda \gg m_\Psi$$

$$f = 1.7 \text{ eV} \left(\frac{\Delta m_\Psi}{m_\Psi}\right)^{-1} \left(\frac{m_\Psi}{10^{-3} \text{ eV}}\right)$$

$$\Lambda \ll m_\Psi$$

$$f = 0.85 \text{ eV} \left(\frac{\Delta m_\Psi}{m_\Psi}\right)^{-1} \left(\frac{\Lambda}{m_\Psi}\right) \left(\frac{\Lambda}{10^{-3} \text{ eV}}\right)$$

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UV Completions for Fermion SI

(Scalar) Yukawa

$$\mathcal{L} \supset g_\chi \chi \bar{\Psi}\Psi$$

$$\Lambda \sim m_\chi$$

$$\lambda = (g_\chi / m_\chi)^2$$

$$f = 34 \text{ eV } g_\chi^{-2} \left(\frac{m_\Psi}{10^{-3} \text{ eV}}\right)$$

Gravitational Torsion

$$\mathcal{L} \supset (2/\gamma) R^{\mu\nu} \wedge e_\mu \wedge e_\nu$$

$$\Lambda \sim M_{\text{pl}}$$

$$\lambda = 3\pi / \Lambda_T^2$$

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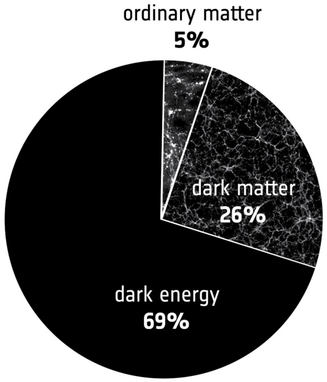
Fermion Candidates

Source: https://stringfixer.com/files/26342399.jpg



Source:
https://cdn.sci.esa.int/documents/33220/35293/Cosmic_energy_energy_budget.jpg/840136ea-a65b-125d-e421-82d2ae889c06?version=1.0&t=1572356777315

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Summary

1. If fermions self-interact, a complex scalar bound state forms
2. The bound state undergoes spontaneous symmetry breaking, giving a Yukawa term
3. The Yukawa term generates dCS through loops interactions with gravity
4. Light fermions, such as neutrinos (and possible DM), are ideal candidates
5. The complex scalar can be a fundamental scalar!

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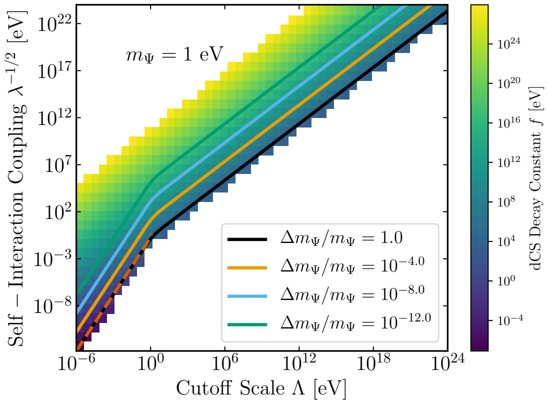
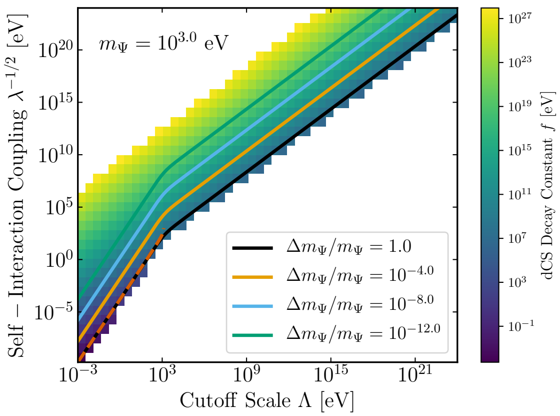
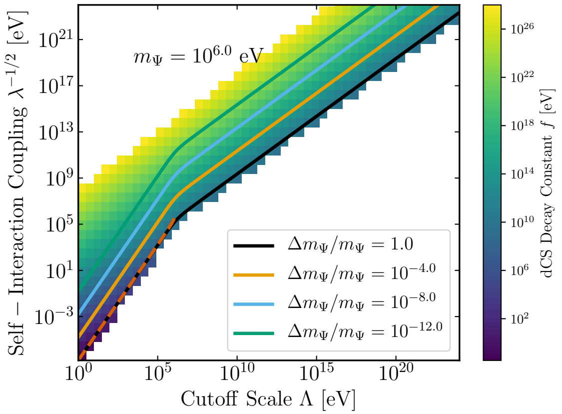
More Parameter Space Plots







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