QFT Neutrino Lecture

Outline:

I) Review of Representation Theory in Particle Physics (Continuous Symmetries)

II) Minor Aside: Discrete Symmetries

III) Spin-$\frac{1}{2}$ Particles (Weyl, Majorana, Dirac)

IV) Neutrinos

I) Goal: A formulation of a relativistic theory of particles.
(1) (2)

(1) Symmetries of Minkowski: Lorentz Group (3 rotations & 3 boosts) $O(1,3)$
+ Group of Translations (4 directions) $\mathbb{R}^{1,3}$

$\Rightarrow$ Poincaré Group (10 dimensional noncompact) $\mathbb{R}^{1,3} \rtimes O(1,3)$
$\uparrow$ semidirect product

(2) Projective Hilbert Space $P\mathcal{H} = \bigoplus_n P\mathcal{H}_n$
(i.e Fock Space)

$\rightarrow$ Classify states in $P\mathcal{H}$ based on transformation rules of $\mathbb{R}^{1,3} \rtimes O(1,3)$? i.e find irreducible representations (irreps) of $\mathbb{R}^{1,3} \rtimes O(1,3)$? ~~Wrong!~~

$P\mathcal{H}$ is projective so that representations need to be faithful up to a phase.

How to find projective irreps? Won't prove, but

i) projective irreps of a group G $\longleftrightarrow$ Irreps of Universal Cover of G $= \tilde{G}$

ii). Irreps of the Lie Algebra of $\tilde{G}$ ~~are~~ easier to find than irreps of $\tilde{G}$.

iii) irreps of Lie Algebra of $\tilde{G}$ $\longleftrightarrow$ irreps of Lie Algebra of G.

$\Rightarrow$ We find projective irreps of the Poincare Group by looking @ irreps of the Poincare Algebra.

Assume gone through calculations of Poincare Algebra but see (Srednicki Ch.2, Schwartz 8.1 & 10.1)

Poincare Algebra iso(1,3) in 4-vector notation

so(1,3):

$[J_i, J_j] = i\hbar\,\varepsilon_{ijk} J_k$ (Angular Momentum)

$[J_i, K_j] = i\hbar\,\varepsilon_{ijk} K_k$ (k transforms as 3vector)

$[K_i, K_j] = -i\hbar\,\varepsilon_{ijk} J_k$ (boosts = rotations)

$\mathbb{R}^{1,3}$

$[P_i, P_j] = 0$

$[P_i, H] = 0$

$[J_i, H] = 0$

$[J_i, P_j] = i\hbar\,\varepsilon_{ijk} P_k$ (3 vector)

$[K_i, H] = i\hbar\, P_i$

$[K_i, P_j] = i\hbar\,\delta_{ij} H$

boosts mix time & space

Irreps of $\mathbb{R}^{1,3}$ (algebra) [Wigner Classification] Look images of stabilizer subgroups ~~[illegible]~~

1) $P^\mu = 0$ $\Rightarrow$ Vacuum. $\to$ Trivial

2) $P^0 > 0$, $m=0$, stabilizer of $P^\mu = (k, 0, 0, -k)$ $\to$ double cover of $SE(2)$ also a "continuous spin"

3) $m > 0$, stabilizer of $P^\mu = (m, 0, 0, 0)$ $\to$ double cover of $SO(3)$ $\updownarrow$ swap (wraps)

4) $m < 0$ (ignore)

For $m=0$ $P^0 > 0$

labeled by helicity $P^\mu J_{\mu\nu}$

$p^i J_i / |P|$ (Pauli-Lubanski Pseudovector...)

$$SO(3) = SU(2)$$

$$J_i^+ \equiv \tfrac{1}{2}(J_i + i k_i) \qquad J_i^- \equiv \tfrac{1}{2}(J_i - i k_i)$$

$$[J_i^+, J_j^+] = i \varepsilon_{ijk} J_k^+$$

$$[J_i^-, J_j^-] = i \varepsilon_{ijk} J_k^-$$

$$[J_i^+, J_j^-] = 0$$

Two copies of $SO(3)$ $\Rightarrow$ representations labeled by (A,B) with $(2A+1)(2B+1)$ degrees of freedom.

with A, B half-integers

$SU(2) \oplus SU(2)$	$(0,0)$	$(\frac{1}{2},0)$	$(0,\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$ ...
$SO(3)$	0	$\frac{1}{2}$	$\frac{1}{2}$	$0 \oplus 1$

II) The symmetries considered so far have been continuous, i.e we have been looking @ Lie Groups

$SO^+(1,3) \to$ orthochronous, proper subgroup of $O(1,3)$

~~Even within O(1,~~ Rather How do we get to the other parts of $O(1,3)$ (non-orthochronous, non proper?)

Parity Operator $P = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

Time-Reversal $T = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

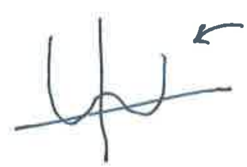
As a result, $O(1,3)$ is 4 disconnected copies of $SO^+(1,3)$

~~for~~ $SO^+(1,3) \cup P[SO^+(1,3)] \cup T[SO^+(1,3)] \cup (PT)[SO^+(1,3)]$

why do we look for reps of $so(1,3)$ & not P or T? Nature....

There are other discrete symmetries found often in QFT

C maps ~~$(n,n') \to (n',n)$~~ particles to antiparticles [Charge Conjugation]



$\leftarrow \mathbb{Z}_2$

More exotic ... $\mathbb{Z}_n$

D_3

Classification of ~~$\mathbb{Z}$~~ finite discrete symmetries is done..

often seen in more complicated condensed matter systems.

III. We now move to describe spin $\overset{L}{(\tfrac{1}{2},0)}$ & $\overset{R}{(0,\tfrac{1}{2})}$ representations. Such irreps have $J=\tfrac{1}{2}$ s.t. $2J+1=2$ degrees of freedom. Therefore we need to find 2×2 matrices that satisfy

$$[J_i^+, J_j^+] = i\varepsilon_{ijk} J_k^+$$

$$[J_i^-, J_j^-] = i\varepsilon_{ijk} J_k^-$$

$$[J_i^+, J_j^-] = 0$$

These are the Pauli matrices!

$$\left[\frac{\sigma_i}{2}, \frac{\sigma_j}{2}\right] = i\varepsilon_{ijk}\frac{\sigma_k}{2}$$

For the $(\tfrac{1}{2},0)$ we set $J_i^- = \frac{\sigma_i}{2}$ & $J_i^+ = 0$

& $(0,\tfrac{1}{2})$ $J_i^+ = \frac{\sigma_i}{2}$ & $J_i^- = 0$

Remember for real Lorentz transformations

$$J_i^\delta = J_i^+ + J_i^- \quad \& \quad K_i = i(J_i^- - J_-^+)$$

$$\text{so} \quad J_i = \frac{\sigma_i}{2} \quad \& \quad K_i = \tfrac{1}{2}\sigma_i \;(\tfrac{1}{2},0)$$

$$= -\tfrac{1}{2}\sigma_i \;(0,\tfrac{1}{2})$$

Given the full characterization of spin $\tfrac{1}{2}$ particles we now move ~~from~~ to ~~describe~~ describe the dynamics of the states (i.e finding Lorentz invariant Lagrangians)

The simplest case is one $(\frac{1}{2},0)$ spinor $\psi_L = \psi$ $\quad \sigma^\mu \equiv (I, \sigma_i)$, $\bar\sigma^\mu \equiv (I, -\sigma_i)$

$\partial^\mu \psi_L \partial_\mu \psi_L + \partial^\mu \psi_L^\dagger \partial_\mu \psi_L^\dagger$ is unbounded below ... so we need a kinetic term with ψ_L & $\psi_L^\dagger$. Let's look at $i\psi^\dagger \bar\sigma^\mu \partial_\mu \psi$

$$(i\psi^\dagger \bar\sigma^\mu \partial_\mu \psi)^\dagger = (i\psi^\dagger_{\dot a} \bar\sigma^{\mu \dot a c} \partial_\mu \psi_c)^\dagger$$

$$= -i \partial_\mu \psi^\dagger_{\dot c} (\bar\sigma^{\mu \dot a c})^* \psi_a \quad \cancel{\bar\sigma^\mu \text{ hermitian}}$$

$$(\bar\sigma^\mu \text{hermitian}) = -i\partial_\mu \psi^\dagger_{\dot c} \bar\sigma^{\mu \dot c a} \psi_a$$

$$(\text{chain rule}) = i\psi^\dagger_{\dot c} \bar\sigma^{\mu \dot c a} \partial_\mu \psi_a - i\partial_\mu (\psi^\dagger_{\dot c} \bar\sigma^{\mu \dot c a} \psi_a)$$

$$(\text{supress indices}) = i\psi_L^\dagger \bar\sigma^\mu \partial_\mu \psi_L - i\partial_\mu (\psi_L^\dagger \bar\sigma^\mu \psi_L).$$

so $i\psi^\dagger \bar\sigma^\mu \partial_\mu \psi$ is Hermitian!

$$\Rightarrow \mathcal{L} = i\psi_L^\dagger \bar\sigma^\mu \partial_\mu \psi_L - \tfrac{1}{2} m \psi_L \psi_L - \tfrac{1}{2} m^* \psi_L^\dagger \psi_L^\dagger$$

$$\mathcal{L} = \tfrac{i}{2} \Psi^T C \gamma^\mu \partial_\mu \Psi - \tfrac{1}{2} m \Psi^T C \Psi \qquad \Psi = \begin{pmatrix} \psi_c \\ \psi^{\dagger \dot c} \end{pmatrix}$$

Majorana Lagrangian

Theory of a single spinor!

$$C = \begin{pmatrix} -\varepsilon^{ac} & 0 \\ 0 & -\varepsilon_{\dot a \dot c} \end{pmatrix}$$

$$= \begin{pmatrix} \varepsilon_{ac} & 0 \\ 0 & \varepsilon^{\dot a \dot c} \end{pmatrix}$$

$$\gamma^\mu \equiv \begin{pmatrix} 0 & \sigma^\mu_{a \dot c} \\ \bar\sigma^{\mu \dot a c} & 0 \end{pmatrix}$$

[Weyl Representation for the Gamma Matrices] what about two spinors?

$$\mathcal{L} = i\psi_i^\dagger \bar{\sigma}^\mu \partial_\mu \psi_i - \tfrac{1}{2} m \psi_i \psi_i - \tfrac{1}{2} m \psi_i^\dagger \psi_i^\dagger$$

This has an $SO(2)$ internal symmetry

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

we can rewrite so that this Lagrangian instead has a $U(1)$ symmetry

$$\chi = \frac{1}{\sqrt{2}}(\psi_1 + i\psi_2)$$

$$\xi = \frac{1}{\sqrt{2}}(\psi_1 - i\psi_2)$$

$$\mathcal{L} = i\chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi + i\xi^\dagger \bar{\sigma}^\mu \partial_\mu \xi - m\chi\xi - m\xi^\dagger \chi^\dagger$$

$$\Psi \equiv \begin{pmatrix} \chi_c \\ \xi^{\dagger \dot{c}} \end{pmatrix} \qquad \bar{\Psi} \equiv \Psi^\dagger \beta \qquad \beta \equiv \begin{pmatrix} 0 & \delta^{\dot{a}}{}_{\dot{c}} \\ \delta_a{}^c & 0 \end{pmatrix}$$

($\beta = \gamma^0$ numerically, but spinor index structure diff)

$$\rightarrow \mathcal{L} = i\bar{\Psi}\gamma^\mu \partial_\mu \Psi - m\bar{\Psi}\Psi$$

has $\underline{U(1)}$ symmetry! Theory of $\underline{two}$ spinors.

$$C^{-1}\chi_a C = \xi_a$$

$$C^{-1}\xi_a(x) C = \chi_a$$

Dirac is the same as Majorana with replacement $\bar{\Psi} \rightarrow \Psi^T C$ as Ψ

IV) Neutrinos... what are they?

Overview of them - not extremely technical

~~SM has 3~~ Experiments indicate the existence of 3 left-handed $[(\frac{1}{2},0)]$ spinors in the SM that interact via weak interactions and are electrically neutral.

Processes like β-decay

$$n \rightarrow p + e^- + \bar{\nu}_e$$

or ~~$\pi^+ \rightarrow$~~ pions

or $\pi^\pm \rightarrow \mu^\pm + \bar{\nu}_\mu$

meson decay
$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$
$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

Our Knowledge of neutrinos mainly comes from these processes (i.e. when neutrinos interact with other particles and we measure the other particles)

Corresponds to measurements when we diagonalize the interaction terms in ν $\mathcal{L}$

but if ν's have mass, if you diagonalize mass states (energy eigenstates) int. not necessarily diagonalize

This is what puzzled physicists for a while (Solar Neutrino Problem)

We now know ν's have masses, but oscillations only give info. about mass differences! (Normal vs. Inverted)

Moreover, we do not know if this mass term is Majorana or Dirac. Search for ν-less double beta decay can put bounds

(March 4th this year 2203.02139)

KamLAND performed first search for this decay

$$2n \to 2p + 2e^-$$

put upper limit on this mass as 36-156 meV

How do we go from mass to interacting basis?

$$\underbrace{\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}}_{\text{interactions}} = \underbrace{U}_{\text{Unitary}} \underbrace{\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}}_{\text{mass}}$$

How do we parametrize?

1) $N \times N$ unitary matrix has N^2 real parameters.

2) $2N-1$ not significant as a phase can be absorbed into each field (remember projective hilbert space)

$$\Rightarrow N^2 - (2N-1) = (N-1)^2$$

$\frac{N(N-1)}{2}$ are ~~rotation~~ mixing angles

$\frac{(N-1)(N-2)}{2}$ are complex phases (CP violation)

$$\text{PMNS} \Rightarrow U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

MSW effect! (neutrinos propagate in matter, interact, ~~mass~~ changing propagation speed of diff. states).