Parity Violation in Gravity: New Constructions and Inflationary Signals

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Based on 2207.05094 w/ Stephon Alexander
and 2303.04815 with Stephon, Marc Kamionkowski, & Oliver Philcox

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Roadmap

I will…

I) Demonstrate two new constructions (i.e types of UV completions) of dynamical Chern-Simons (dCS) gravity that encompass a wide range of EFT scales

II) Show that dCS gravity yields a parity-violating primordial scalar trispectrum that can be made large.

III) Demonstrate that ratios of odd and even trispectra contain direct information about spin.

IV) Quantify how well upcoming and future experiments can constrain scalar trispectra.

V) Hint to how parity-violating trispectra can constrain baryogenesis

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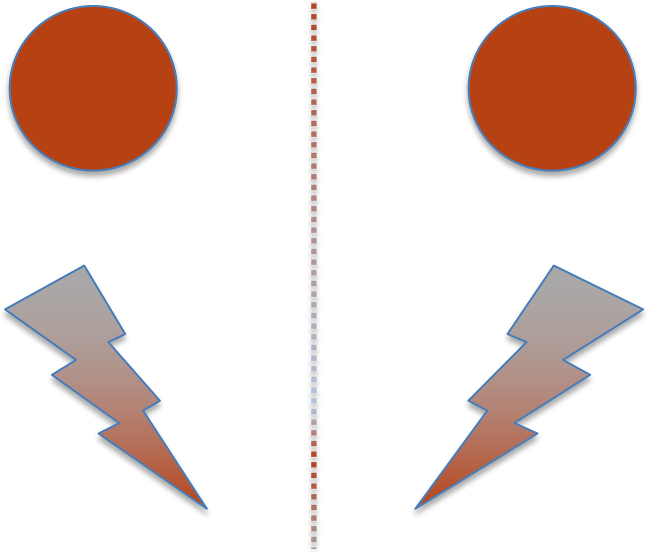
What is $\mathbb{P}$arity? How can Gravity violate it?

$$\mathbb{P}\mathbf{x} = -\mathbf{x}$$



$$A_L - A_R$$

Chern-Simons Gravity

What is [dynamical] Chern-Simons (dCS) Gravity?

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\left(\partial_\mu\phi\right)\left(\partial_\nu\phi\right) + \frac{M_{\rm pl}^2}{2}R - \frac{\phi}{4f}{}^*RR\right]$$

Spacetime Metric, Lorentzian Signature

dCS pseudo-scalar

Einstein-Hilbert Term

dCS Decay Constant

Pontryagin Density

$${}^*R\,R = {}^*R^\rho{}_\sigma{}^{\mu\nu}R^\sigma{}_{\rho\mu\nu}$$

$${}^*R^\rho{}_\sigma{}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}R^\rho{}_{\sigma\alpha\beta}$$

Effective Theory

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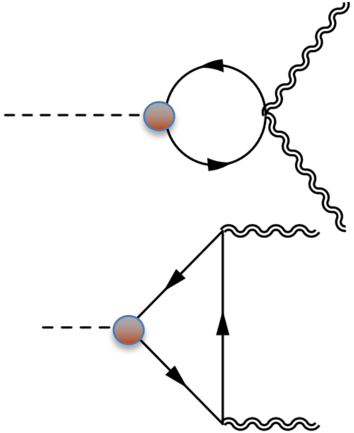
New Construction 1: Spontaneous Symmetry Breaking

$$\Phi = (1/\sqrt{2})(F+\sigma)\exp[ia(x)/F]$$

$$\mathcal{L}_\Phi = \sum_{j=L,R} i\bar{\Psi}_j\gamma^\mu\partial_\mu\Psi_j - \tilde{m}_\Psi\left(\bar{\Psi}_R\Psi_L + \bar{\Psi}_L\Psi_R\right)$$
$$+\,\partial_\mu\Phi\partial^\mu\Phi^* - V\left(|\Phi|^2\right) - \left(y_\Phi\Phi\bar{\Psi}_L\Psi_R + \text{h.c.}\right)$$

$$\mathcal{L}_\Phi = \bar{\Psi}\left[i\gamma^\mu\partial_\mu - \left(\tilde{m}_\Psi + \frac{y_\Phi F}{\sqrt{2}}\right)\right]\Psi$$
$$+\,\frac{1}{2}\partial_\mu a\partial^\mu a - \frac{y_\Phi}{\sqrt{2}}a\bar{\Psi}\gamma^5\Psi,$$



$$\mathcal{L}_g^{\text{eff}} = -\frac{g}{384\pi^2}\frac{a}{2m_\Psi}{}^*RR \longrightarrow f = 192\sqrt{2}\pi^2\frac{m_\Psi}{y_\Phi}$$

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New Construction 2: Dynamical Symmetry Breaking

$$\mathcal{L}_\Psi = \bar{\Psi}\left(i\gamma^\mu \partial_\mu - \tilde{m}_\Psi\right)\Psi - \underline{\lambda \bar{\Psi}\Psi\bar{\Psi}\Psi}$$

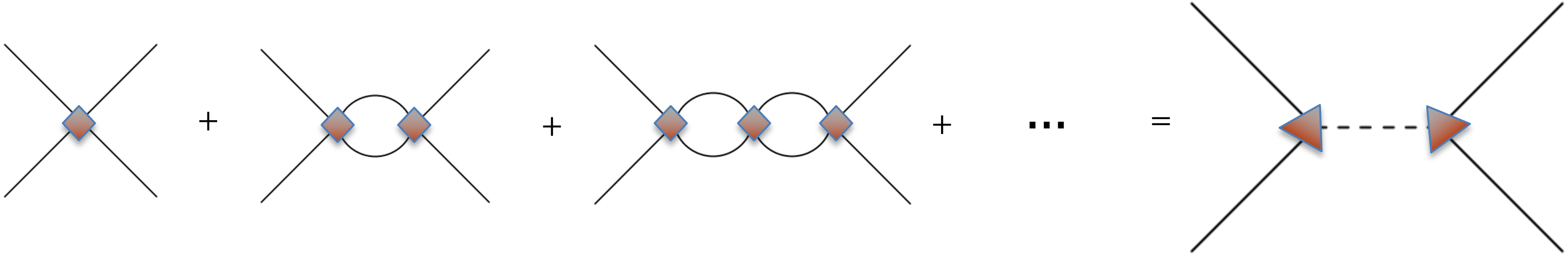
Cutoff Λ

Dirac Gamma Matrices

``bare'' mass

Dirac Fermion

(Attractive) Fermion Self-Coupling Constant



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New Construction 2: Dynamical Symmetry Breaking

$$Z_\alpha = \int \mathcal{D}\alpha \mathcal{D}\bar{\alpha} \exp\left(-\int d^4x\ \tilde{m}_\Phi^2 \bar{\alpha}\alpha\right)$$

$$+$$

$$\Psi = \Psi_\ell + \Psi_s$$

$$\Phi \ = \ \alpha - \tilde{m}_\Phi^{-2} \bar{\Psi}_s \Psi_s$$

$$+$$

$$\text{Integrate Out Short} - \text{Scale Modes}$$

$$\Longrightarrow$$

$$\tilde{\mathcal{L}}_\Psi = \bar{\Psi}(i\gamma^\mu \partial_\mu - \tilde{m}_\Psi)\Psi - \lambda \bar{\Psi}\Psi\bar{\Psi}\Psi$$
$$+ \left(\partial_\mu \Phi^*\right) \left(\partial^\mu \Phi\right) - \underline{y_\Phi \left(\Phi \bar{\Psi}\Psi + \mathrm{h.c.}\right)}$$
$$+ m_\Phi^2 \left|\Phi\right|^2 - \frac{\lambda_\Phi}{4} \left|\Phi\right|^4,$$

$$y_\Phi \sim \lambda \Lambda^2$$

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What are Observable Signatures of dCS?

1) Compact Binary Coalescence (CBC) Waveform-Modification
2) Birefringence of CBC Gravitational Waves
3) Modification of Orbits around Earth

$f \ll \mathrm{eV}$

Late-Time

4) Polarized Primordial Gravitational Waves
5) Primordial Imprints on Galaxy Clustering and the CMB

Early-Time

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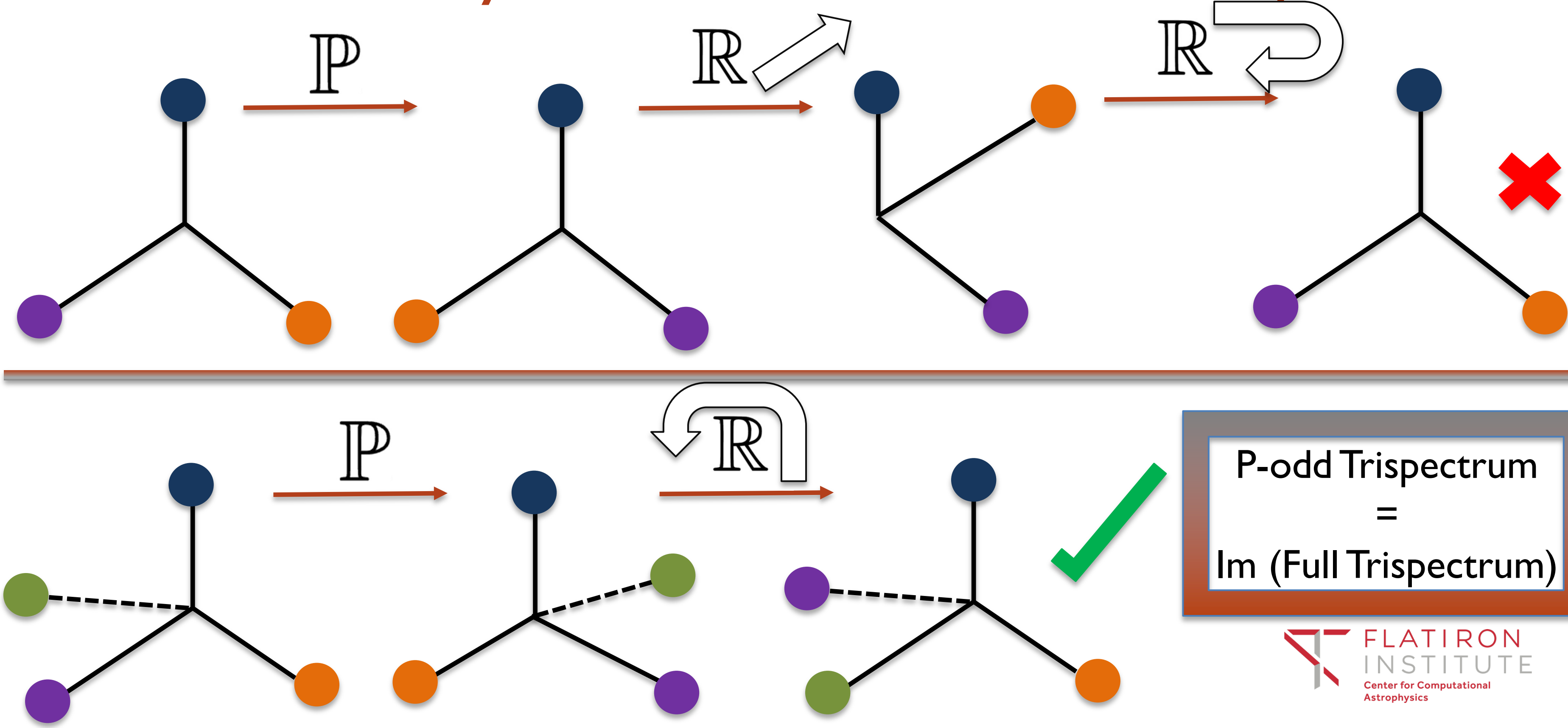
How is Galaxy Clustering Described?

Galaxy N-Point Functions!

$$\delta_g(\mathbf{x}) = \frac{n_g(\mathbf{x}) - \bar{n}_g}{\bar{n}_g}$$

Enter: The Galaxy Four Point Function / Trispectrum

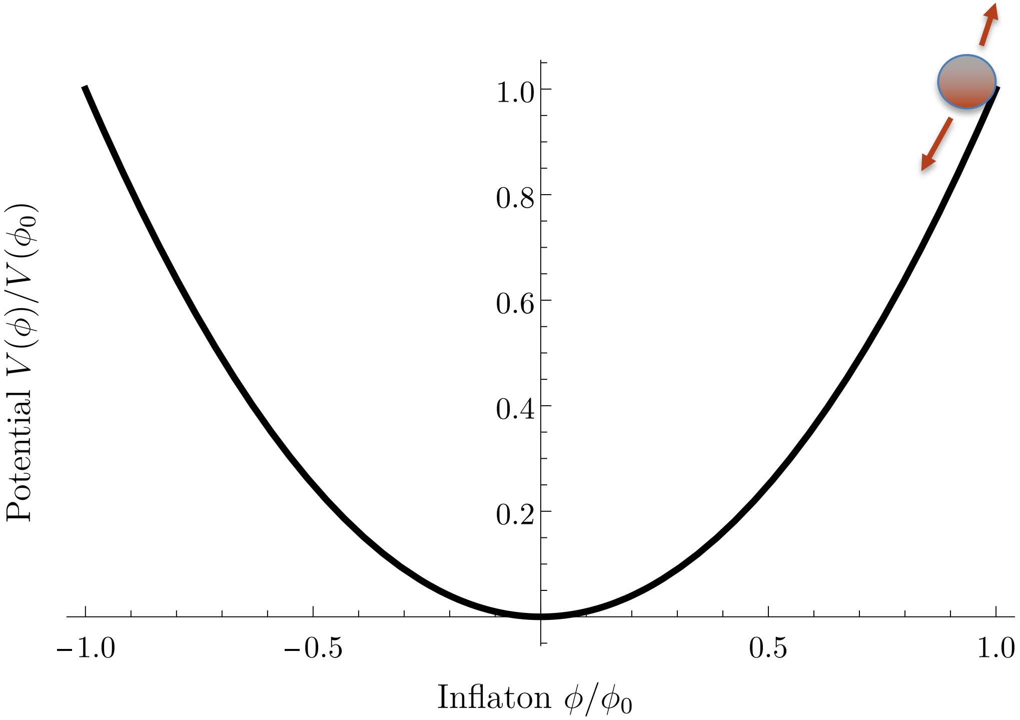


P-odd Trispectrum

=

Im (Full Trispectrum)

Inflation: A Lightning Review



Slow-Roll Parameter

$$\varepsilon = \frac{1}{2}\frac{\dot{\phi}^2}{M_{\rm pl}^2 H^2}$$

Solves:

1) The Horizon Problem
2) The Flatness Problem
3) Magnetic Monopole Problem

Quantum Inflaton Fluctuations Seed Density Perturbations

A.H. Guth (1980) The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems

dCS Inflation: Full Action

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \left(\partial_\mu \phi\right) \left(\partial_\nu \phi\right) + \frac{M_{\rm pl}^2}{2} R - \frac{\phi}{4f} {}^*RR \right]$$

Spacetime Metric, Lorentzian Signature

dCS pseudoscalar/Inflaton

Einstein-Hilbert Term

dCS Decay Constant

Pontryagin Density

$${}^*R\,R = {}^*R^\rho{}_\sigma{}^{\mu\nu} R^\sigma{}_{\rho\mu\nu}$$

$${}^*R^\rho{}_\sigma{}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} R^\rho{}_{\sigma\alpha\beta}$$

$$g_{\mu\nu} = a^2(\tau) \begin{pmatrix} -1 & 0 \\ 0 & \delta_{ij} + h_{ij} \end{pmatrix}$$

Inflaton Potential: $V \sim \Lambda^4$

$$\Lambda \sim \sqrt{H M_{\rm pl}} = 10^{16}\ \text{GeV}\ [H/(10^{14}\ \text{GeV})]^{1/2}$$

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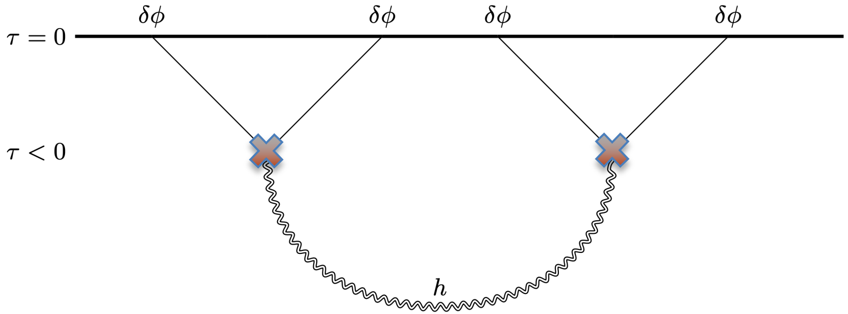
Primordial Scalar Trispectrum from dCS

$$\langle \delta\phi(\mathbf{k}_1)\delta\phi(\mathbf{k}_2)\delta\phi(\mathbf{k}_3)\delta\phi(\mathbf{k}_4)\rangle_c$$



$$\propto P_{\delta\phi} P_{\delta\phi} P_h$$

$$\zeta(\mathbf{k}) = -H[\delta\phi(\mathbf{k})/\dot{\phi}]$$

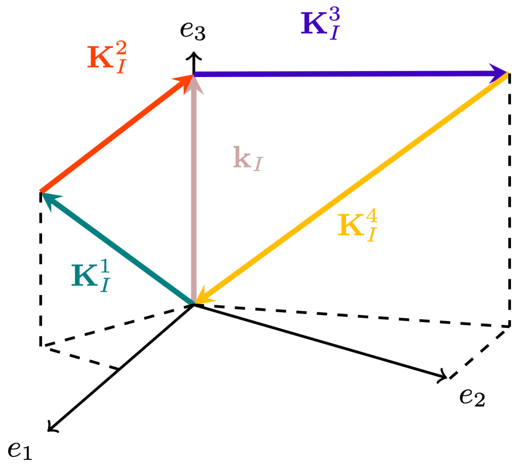
Perturbed Inflaton Kinetic Term

$$\frac{1}{2}\sqrt{-g}g^{\mu\nu}\partial_\mu\delta\phi\partial_\nu\delta\phi = \frac{1}{2}a^2\eta^{\mu\nu}\partial_\mu\delta\phi\partial_\nu\delta\phi + \boxed{\frac{1}{2}a^2 h^{ij}\partial_i\delta\phi\partial_j\delta\phi}$$

$$\mathbf{K}_s = \{\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4\} \qquad \mathbf{K}_t = \{\mathbf{k}_1, \mathbf{k}_3, \mathbf{k}_2, \mathbf{k}_4\}$$

$$\mathbf{K}_u = \{\mathbf{k}_1, \mathbf{k}_4, \mathbf{k}_3, \mathbf{k}_2\} \qquad \mathbf{K}_I^1 + \mathbf{K}_I^2 = -\mathbf{K}_I^3 - \mathbf{K}_I^4 = \mathbf{k}_I$$

$$I \in \{s, t, u\}$$



$$\begin{aligned} \hat{\mathbf{e}}_1 &= R_I\hat{\mathbf{x}} \\ \hat{\mathbf{e}}_2 &= R_I\hat{\mathbf{y}} \\ \hat{\mathbf{e}}_3 &= R_I\hat{\mathbf{z}} \end{aligned}$$

$$\hat{\mathbf{e}}_3 = \hat{\mathbf{k}}_I$$

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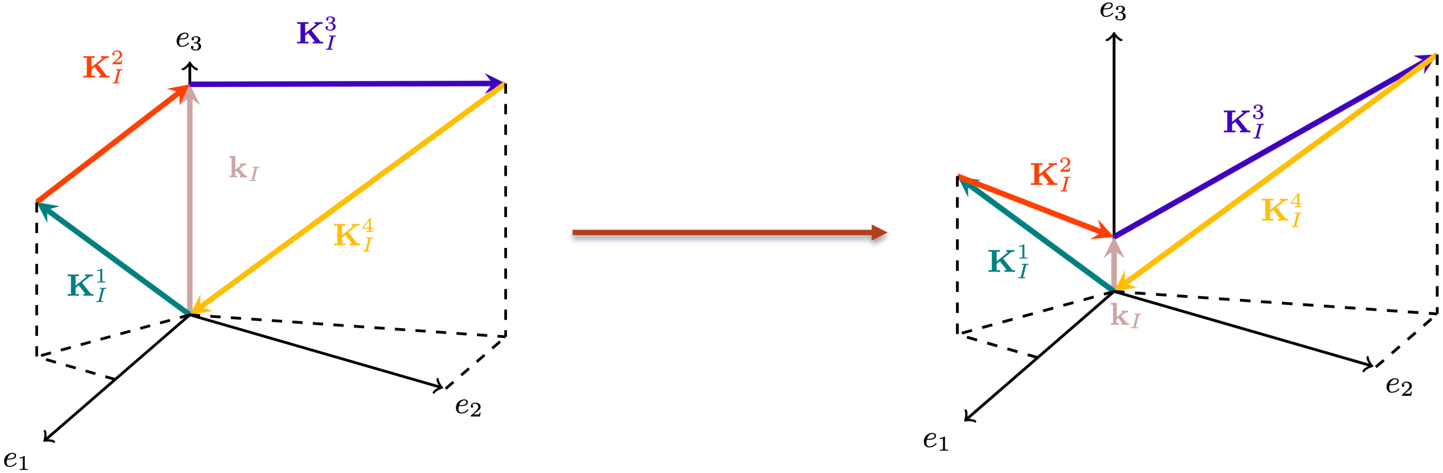
The Collapsed Limit

$$\mathbf{k}_i \gg \mathbf{k}_I \approx 0, \; i \in \{1,2,3,4\}$$

$$K_I^1 \approx K_I^2 \quad \Big| \quad K_I^3 \approx K_I^4$$



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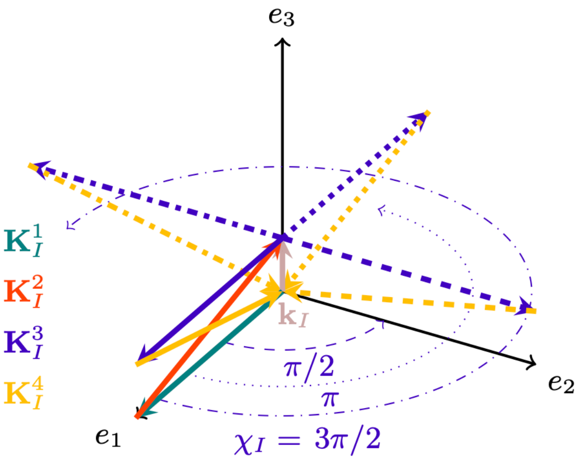
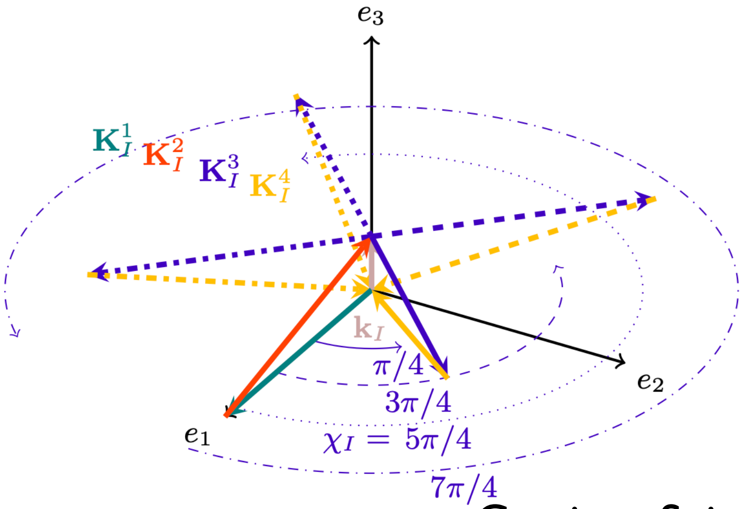
The Collapsed Limit Scalar Trispectra

$$\mathrm{Re}\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_3)\zeta(\mathbf{k}_4)\rangle_c^{\prime\mathrm{GE}}\Big|_{I-\mathrm{Coll.}}=\frac{9}{16}r\cos(2\chi_I)\sin^2(\theta_I^1)\sin^2(\theta_I^3)P_\zeta(k_I)P_\zeta(K_I^1)P_\zeta(K_I^3)\underline{\times F}$$

$$\mathrm{Im}\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_3)\zeta(\mathbf{k}_4)\rangle_c^{\prime\mathrm{GE}}\Big|_{I-\mathrm{Coll.}}=\frac{9}{16}\Pi_{\mathrm{circ}}r\sin(2\chi_I)\sin^2(\theta_I^1)\sin^2(\theta_I^3)P_\zeta(k_I)P_\zeta(K_I^1)P_\zeta(K_I^3)\underline{\times F}$$





Graviton Spin

$$\mathrm{Odd}/\mathrm{Even} = \Pi_{\mathrm{circ}}\cot(2\chi_I)$$

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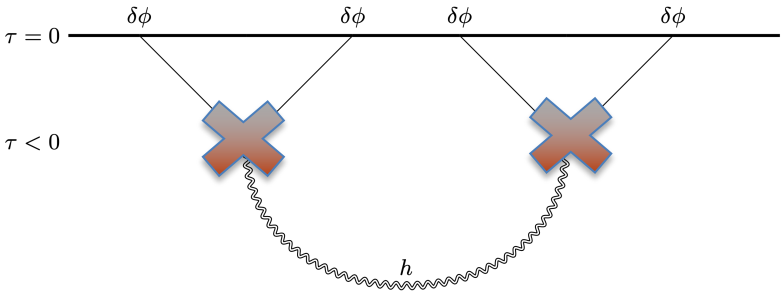
Two Examples Beyond Vanilla dCS Inflation

Superluminal Scalar Sound Speed

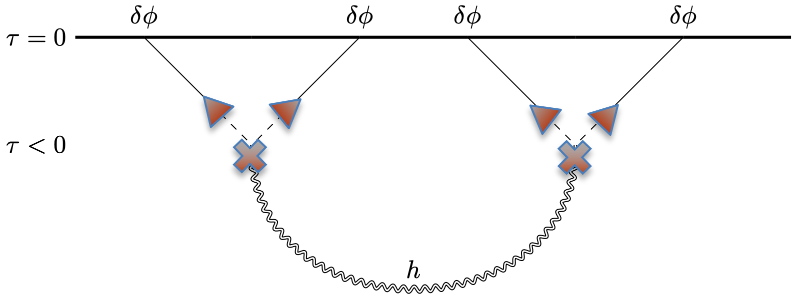
$$c_s^2 g^{ij} \left(\partial_i \phi\right)\left(\partial_j \phi\right)$$



$$F = 2 \times 10^6 \left(c_s/6\right)^8$$

Quasi-Single Field/Multi-Field

$$a^3 \lambda \delta\sigma \delta\dot{\phi} \qquad h^{ij}\partial_i(\delta\sigma)\partial_j(\delta\sigma)$$



$$F \sim 10^4 \left[w(\nu)/20\right]$$

$$\nu = \sqrt{9/4 - (m/H)^2} \lesssim 0.5$$

arXiv: 1504.05993

(Dimastrogiovanni, Fasiello, Kamionkowski)

$$F \sim 10^7 \left(N/60\right)^4$$

$$\nu \sim 3/2$$

arXiv: 0911.3380

(Xingang Chen, Yi Wang)

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The Trispectra can be Large

How Sensitive are we to Collapsed Trispectra?

$$\left[1+\delta^{P}_{\rm odd}\left(\Pi_{\rm circ}-1\right)\right] r \gtrsim 0.04\left(\frac{n}{3}\right)\left(\frac{8\times10^{5}}{F}\right)\left(\frac{10^{6}}{N_{\rm modes}}\right)^{1/2}\left(\frac{k_{\rm min}}{0.003\ h/{\rm Mpc}}\right)^{3/2}\left(\frac{0.3\ h/{\rm Mpc}}{k_{\rm max}}\right)^{3/2}$$

$$f \lesssim 4\times10^{9}\ {\rm GeV}\left(\frac{3}{n}\right)\left(\frac{F}{8\times10^{5}}\right)\left(\frac{\varepsilon}{10^{-2}}\right)^{3/2}\left(\frac{H}{10^{14}\ {\rm GeV}}\right)^{2}\left(\frac{N_{\rm modes}}{10^{6}}\right)^{1/2}\left(\frac{0.003\ h/{\rm Mpc}}{k_{\rm min}}\right)^{3/2}\left(\frac{k_{\rm max}}{0.3\ h/{\rm Mpc}}\right)^{3/2}$$

$$N_{\rm modes} = V\int_{k_{\rm min}}^{k_{\rm max}} \frac{d^3\mathbf{k}}{(2\pi)^3}\mathrm{W}\left(\mathbf{k}\right)\frac{\left[G^2(\mathbf{k},\bar{z})P^{gg}_{\rm L}(\mathbf{k},\bar{z})\right]^2}{\left[P^{gg}_{\rm NL}\left(\mathbf{k},\bar{z}\right)+\bar{n}^{-1}\right]^2}$$

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What about Experiments?

$$\left[1+\delta^P_{\rm odd}\left(\Pi_{\rm circ}-1\right)\right] r \gtrsim 0.04\left(\frac{n}{3}\right)\left(\frac{8\times10^5}{F}\right)\left(\frac{10^6}{N_{\rm modes}}\right)^{1/2}\left(\frac{k_{\rm min}}{0.003\ h/{\rm Mpc}}\right)^{3/2}\left(\frac{0.3\ h/{\rm Mpc}}{k_{\rm max}}\right)^{3/2}$$

Experiments								
Name	$[z_{\rm min}, z_{\rm max}]$	$f_{\rm sky}$	$[k_{\rm min}, k_{\rm max}]$ $[h/{\rm Mpc}]$	$\bar{n}$ $[(h/{\rm Mpc})^3]$	$\bar{b}$	V $[({\rm Gpc}/h)^3]$	$N_{\rm modes}$	F
BOSS[a] [112, 113]	$[0.43, 0.7]$	0.23	$[0.01, 0.24]$	3×10^{-4}	2.0	3.6	10^5	2×10^7
DESI[a] [115]	$[0.6, 1.7]$	0.34	$[0.003, 0.31]$	3.8×10^{-4}	1.4	45	8×10^5	8×10^5
Euclid[a] [114]	$[0.9, 1.8]$	0.36	$[0.003, 0.34]$	4.3×10^{-4}	1.7	44	10^6	6×10^5
MegaMapper[a] [116, 117]	$[2, 5]$	0.34	$[0.003, 0.64]$	2.5×10^{-4}	3.8	155	7×10^6	10^5
MSE[a] [120]	$[1.6, 4]$	0.24	$[0.003, 0.54]$	2.8×10^{-4}	3.7	91	6×10^6	10^5
SPHEREx[a] [118, 119]	$[0.1, 4.3]$	0.65	$[0.003, 0.46]$	2.0×10^{-3}	1.1	360	2×10^7	10^5
HIRAX[b] [121]	$[0.8, 2.5]$	0.36	$[(0.01, 0.1), 0.38]$	10^{-3}	1.9	88	3×10^6	$(2\times10^6, 7\times10^7)$
PUMA-32K[b] [122]	$[2, 6]$	0.5	$[(0.01, 0.1), 0.71]$	7.6×10^{-3}	6.3	290	7×10^7	$(2\times10^5, 5\times10^6)$

[a]Spectroscopic Galaxy Survey
[b]21-cm Experiment

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dCS Inflation: Lepto/Baryogenesis

$$a^4 \frac{\phi(\tau)}{4f}\, {}^*RR$$

$$\frac{\dot{\phi}}{4f} = \frac{1}{2}\frac{M_{\rm pl}}{H}\left[\sqrt{\frac{\varepsilon}{2}}\,\frac{H^2}{M_{\rm pl} f}\right] = \frac{1}{2}\frac{M_{\rm pl}^2}{H}\mu$$

Canonical Normalization

$$\partial_\mu J_5^\mu = {}^*RR/(384\pi^2) \qquad J_5^0 = n_L - n_R$$

$$\sim \underbrace{H\mu(n_L - n_R)}_{\text{Leptogenesis}}$$

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Conclusions/Future Extensions

1) Two new constructions of dCS, yielding a large range of decay constants
2) First example of massless spin-exchange yielding a parity-violating scalar trispectrum
3) Scalar trispectra can probe dCS decay constants at very high values
4) Odd to even ratios, in collapsed limit, (i.e. the phase of the trispectrum) give direct information about spin
5) Potential new probes of baryogenesis

dCS Extensions

1) Removal of the dCS Ghost
2) Parity-Odd Signal for all configurations
3) Where do the trispectra peak?
4) CMB signal
5) Intermediary dCS constants? (Gravi-axion DM/DE)?

Baryogenesis Extensions

1) What type of spin-1 baryogenesis can be probed? (What vertex interactions, shapes, sensitivities, UV completions etc)

2) Other CMB/LSS probes?

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Extra Content

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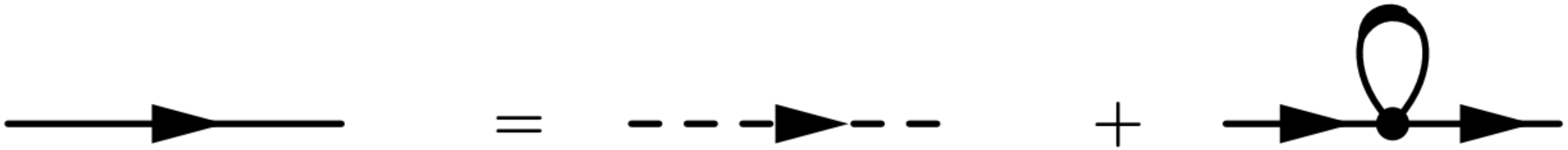
The Gap Equation



$$\frac{\Delta m_\Psi}{m_\Psi}\left(\frac{2\pi^2}{\lambda\Lambda^2}\right) = 1 - \frac{m_\Psi^2}{\Lambda^2}\log\left(1+\frac{\Lambda^2}{m_\Psi^2}\right) \quad , \quad \Delta m_\Psi = y_\Phi F/\sqrt{2}$$

$\Lambda \gg m_\Psi$	$\Lambda \ll m_\Psi$
$F \approx 0.9 m_\Psi$	$F \approx 0.45\Lambda^2/m_\Psi$

Remember! $y_\Phi \sim \lambda\Lambda^2$

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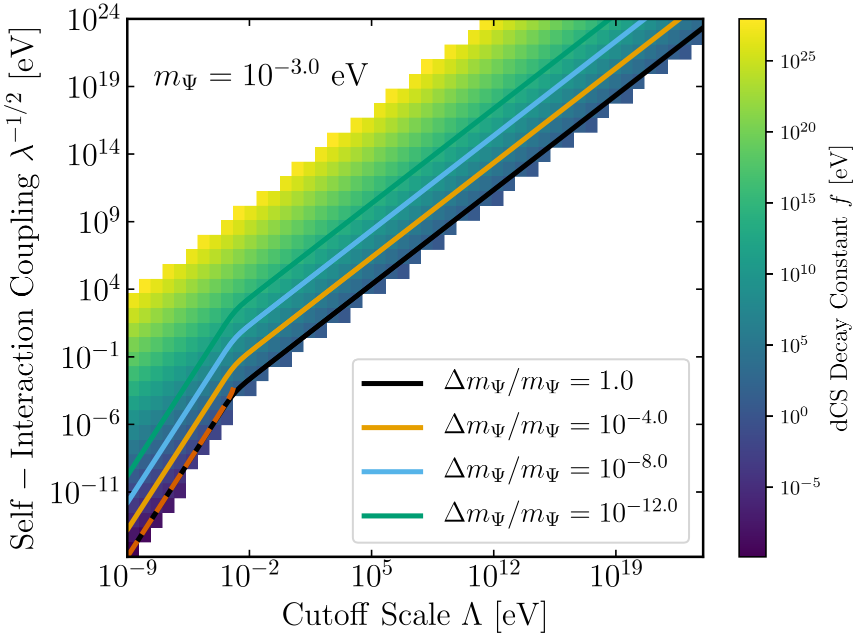
General Parameter Space



$$\Lambda \gg m_\Psi$$

$$f = 1.7 \text{ eV} \left(\frac{\Delta m_\Psi}{m_\Psi}\right)^{-1} \left(\frac{m_\Psi}{10^{-3} \text{ eV}}\right)$$

$$\Lambda \ll m_\Psi$$

$$f = 0.85 \text{ eV} \left(\frac{\Delta m_\Psi}{m_\Psi}\right)^{-1} \left(\frac{\Lambda}{m_\Psi}\right) \left(\frac{\Lambda}{10^{-3} \text{ eV}}\right)$$

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UV Completions for Fermion SI

(Scalar) Yukawa

$$\mathcal{L} \supset g_\chi \chi \bar{\Psi}\Psi$$

$$\Lambda \sim m_\chi$$

$$\lambda = (g_\chi/m_\chi)^2$$

$$f = 34 \text{ eV } g_\chi^{-2} \left(\frac{m_\Psi}{10^{-3} \text{ eV}}\right)$$

Gravitational Torsion

$$\mathcal{L} \supset (2/\gamma) R^{\mu\nu} \wedge e_\mu \wedge e_\nu$$

$$\Lambda \sim M_{\text{pl}}$$

$$\lambda = 3\pi/\Lambda_T^2$$

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dCS Inflation: Quadratic Action

$$S^{(2)} = \int d^4x \left\{ \underbrace{-\frac{1}{2}a^2 \left(\delta\phi'\right)^2 - \frac{1}{2}a^2 \left(\partial_i \delta\phi\right)\left(\partial^i \delta\phi\right)}_{\text{Inflaton Kinetic Term}} + \frac{M_{\text{pl}}^2}{8} a^2 \left[\underbrace{\left(h^i_j\right)' \left(h^j_i\right)' + \left(\partial_k h^i_j\right)\left(\partial^k h^j_i\right)}_{\text{Einstein-Hilbert Term}} \right] - a^4 \frac{\phi(\tau)}{4f} {}^*RR \right\}$$

Pontryagin Density

Background Inflaton

Canonical Normalization Required

Collaborators: Oliver Philcox, Stephon Alexander, Marc Kamionkowski

dCS Inflation: Dynamics and Primordial Power Spectra

$$u''(\tau,\mathbf{k}) - \frac{2}{\tau}u'(\tau,\mathbf{k}) + k^2 u(\tau,\mathbf{k}) = 0,$$

$$u''_\pm(\tau,\mathbf{k}) - \frac{2}{\tau}u'_\pm(\tau,\mathbf{k}) + \left(k^2 \mp \frac{2k\mu}{\tau}\right) u_\pm(\tau,\mathbf{k}) = 0,$$

$$u(\tau,\mathbf{k}) = \frac{H}{\sqrt{2k^3}}(1+ik\tau)e^{-ik\tau}$$

$$u_\pm(\tau,\mathbf{k}) = \frac{H}{M_{\rm pl}\sqrt{k^3}}(1+ik\tau)e^{-ik\tau}\left[\frac{U\left(2\mp i\mu, 4, 2ik\tau\right)}{U\left(2,4,2ik\tau\right)}\right]\exp\left(\pm\frac{\pi}{2}\mu\right)$$

$$\Pi_{\rm circ} = 0.9\left(\frac{\varepsilon}{10^{-2}}\right)^{1/2}\left(\frac{H}{10^{14}\ \mathrm{GeV}}\right)^2\left(\frac{10^9\ \mathrm{GeV}}{f}\right)$$

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Collaborators: Oliver Philcox, Stephon Alexander, Marc Kamionkowski

dCS Inflation: Dynamics and Primordial Power Spectra

$$P_\zeta(k) = \frac{2\pi^2}{k^3}A, \quad A = 2.1 \times 10^{-9}$$

$$P_t(k) = \frac{2\pi^2}{k^3}rA, \quad r = \Theta_{\text{even}}\bar{r}$$

$$\Theta_{\text{even}} = 1 + \frac{2\pi^2 - 3}{3}\mu^2, \quad \bar{r} = 16\varepsilon$$

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Full Graviton-Exchange Trispectrum Shape

$$-\frac{\bar{A}_t}{k_I^3}\frac{H^4}{\left[\prod_{i=1}^4 2k_i^3\right]}\left\{\frac{K_I^1+K_I^2}{\left[a_{34}^I\right]^2}\left[\frac{1}{2}\left(a_{34}^I+k_I\right)\left(\left[a_{34}^I\right]^2-2b_{34}^I\right)+k_I^2(K_I^3+K_I^4)\right]+[(1,2)\leftrightarrow(3,4)]\right.$$

$$+\frac{K_I^1K_I^2}{k_t}\left[\frac{b_{34}^I}{a_{34}^I}-k_I+\frac{k_I}{a_{12}^I}\left(K_I^3K_I^4-k_I\frac{b_{34}^I}{a_{34}^I}\right)\left(\frac{1}{k_t}+\frac{1}{a_{12}^I}\right)\right]+[(1,2)\leftrightarrow(3,4)]$$

$$\left.-\frac{k_I}{a_{12}^Ia_{34}^Ik_t}\left[b_{12}^Ib_{34}^I+2k_I^2\left(\prod_{i=1}^4 k_i\right)\left(\frac{1}{k_t^2}+\frac{1}{a_{12}^Ia_{34}^I}+\frac{k_I}{k_ta_{12}^Ia_{34}^I}\right)\right]\right\}$$

$$a_{ij}^I=K_I^i+K_I^j+k_I$$

$$b_{ij}^I=(K_I^i+K_I^j)k_I+K_I^iK_I^j \qquad k_t=\sum_{i=1}^4 k_i$$

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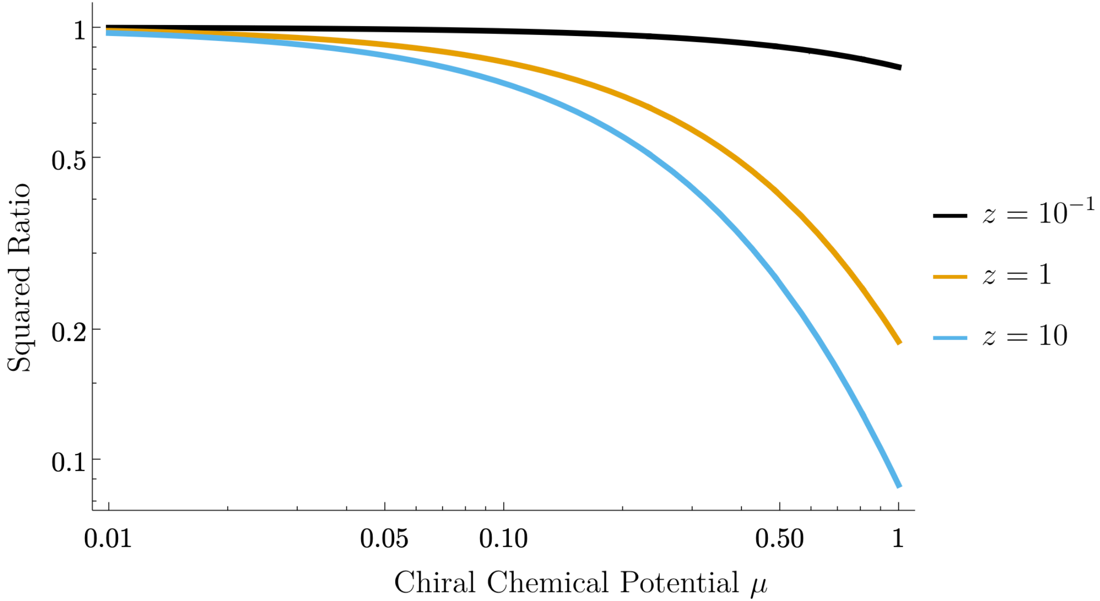
Amplitude-Approximation Validity Plot



Non-Linearities, Foregrounds, and Noise, Oh My

$$N_{\mathrm{modes}} = V \int_{k_{\min}}^{k_{\max}} \frac{d^3\mathbf{k}}{(2\pi)^3} \mathrm{W}\left(\mathbf{k}\right) \frac{\left[G^2(\mathbf{k},\bar{z}) P_{\mathrm{L}}^{gg}(\mathbf{k},\bar{z})\right]^2}{\left[P_{\mathrm{NL}}^{gg}\left(\mathbf{k},\bar{z}\right) + \bar{n}^{-1}\right]^2}$$

$$\boxed{P_a^{gg}\left(\mathbf{k},z\right) = \left[b(z) + f(z)\mu^2\right]^2 D^2(z) P_a^m\left(k\right)}$$

$$G\left(\mathbf{k},z\right) \simeq \exp\left[-\frac{1}{2}\left(k_\perp^2 + k_{||}^2\left[1+f(z)\right]^2\right)\Sigma^2(z)\right] \quad \Sigma(z) = \int_0^\infty \frac{dk}{6\pi^2} P_{\mathrm{L}}^m(k,z)$$

$$\mathrm{W}\left(\mathbf{k}\right) = \Theta\left\{k_{||} - \max\left[k_\perp \frac{\chi(z)H(z)}{(1+z)} \sin\left[\theta_w(z)\right], k_{||\min}\right]\right\}, \quad \theta_w(z) = N_w \frac{1.22}{2\sqrt{0.7}} \frac{\lambda_{\mathrm{obs}}^{21}(z)}{D_{\mathrm{phys}}}$$

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Angular Integrals!

$$\left(\frac{S}{N}\right)^2_{rF} = \frac{(rF)^2}{24} \int_{k_{\min}}^{k_{\max}} \left[\prod_{i=1}^{4} \frac{d^3\mathbf{k}_i}{(2\pi)^3}\right] \frac{[S^{\text{even}}_{\text{GE}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)]^2}{P(k_1)P(k_2)P(k_3)P(k_4)}$$
$$= \frac{(rF)^2}{24} N_{\text{modes}} \mathcal{N}_{\text{GE}},$$

$$\mathcal{N}^{I-\text{Coll.}}_{\text{GE}} = \frac{81}{256} \frac{1}{N_{\text{modes}}} \int \frac{d\Omega_{\mathbf{k}_I}}{(2\pi)^3} k^3_{\min} P^2_\zeta(k_{\min}) \int_{k_{\min}}^{k_{\max}} \left[\frac{d^3\mathbf{K}^1_I d^3\mathbf{K}^3_I}{(2\pi)^6}\right] \sin^4\left(\theta^1_I\right) \sin^4\left(\theta^3_I\right) \cos^2\left[2(\varphi^1_I - \varphi^3_I)\right]$$
$$= \frac{3}{200} \frac{k^3_{\max}}{k^3_{\min}} A^2_s,$$

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