# Python Notes

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# 1 Syntax & Fundamentals

## 1.1 Methods

Dunder method  $\_getitem\_$  is used for example is used with Double UNDerscore before and aftER. List of available methods:

```
1.1.1 __getitem__
```

## 1.1.2 \_\_repr\_\_

With repr this only represents object with '¡Vector object at 0x10e100070;'

```
1.1.3 __abs__
```

- 1.1.4 \_\_add\_\_
- 1.1.5 \_\_mul\_\_
- 1.1.6 \_\_and\_\_

Used to represent a&b which is the intersection of two sets.

#### 1.1.7 \_\_len\_\_

- ♦ Collections
- ♦ Attribute access
- $\diamond$  Iteration
- ♦ Async (Asynchronous Iteration)
- ♦ Operator overloading
- $\diamond$  Function Invocation
- ♦ String representation and Formatting
- Asynchronous programming (await)
- $\diamond$  Object creation and destruction
- ♦ Managed contexts (async)

## 1.1.8 Emulating numeric types

```
class MyInt:
    def __init__(self, value):
        self.value = value

    def __add__(self, other):
        return MyInt(self.value + other.value)

# Usage
a = MyInt(5)
b = MyInt(3)
c = a + b # This will call the __add__ method
```

#### 1.1.9 String representation of objects

```
class MyClass:
    def __init__(self, name):
        self.name = name

def __str__(self):
        return f'Hello, {self.name}!'

def __repr__(self):
        return f'MyClass({self.name})'

# Usage
obj = MyClass('Alice')
print(obj) # This will call the __str__ method
print(repr(obj)) # This will call the __repr__ method
```

#### 1.1.10 Boolean value of an object

```
class MyList:
    def __init__(self, items):
        self.items = items

    def __bool__(self):
        return bool(self.items)

# Usage
obj = MyList([])
if obj:
    print('The object is truthy')
else:
    print('The object is falsy') # This will be printed
```

#### 1.1.11 Implementing collections

A collection is a container that holds other objects (know as elements/items), python has different available collection types such as list/tuple/set/dictionary

```
from collections.abc import MutableSequence

class MyList(MutableSequence):
    def __init__(self, items):
        self._items = items

def __getitem__(self, index):
        return self._items[index]

def __setitem__(self, index, value):
        self._items[index] = value

def __delitem__(self, index):
        del self._items[index]

def __len__(self):
        return len(self._items)

def insert(self, index, value):
```

```
self._items.insert(index, value)

# Usage
obj = MyList([1, 2, 3])
obj.insert(1, 'a')
print(obj) # This will print [1, 'a', 2, 3]
```

# 1.2 Formatting

## 1.2.1 f"

```
name = "John"
print(f'Hi my name is {name}')
```

## 1.2.2 .format()

```
>>> a = 3
>>> print("Hi, do you know the number {}".format(a))
Hi, do you know the number 3
>>>
```

## 1.3 Functions

## 1.3.1 Declaration structure

```
def f(x, y, z):
    """
    Purpose of the function description

    :param x: description of the paramterer x
    :param y: description of the paramterer y
    :param z: description of the paramterer z
    :return: what the function returns
    """
```

## 1.3.2 lambda method

```
f = lambda x, y: x + y
```

f is the function's name, x, y are the input variables and : x + y is the result to return.

#### 1.3.3 Decorators

## 1.4 Classes

## 1.4.1 \_\_init\_\_

```
class State:
    def __init__(self):
        self.counter = 0
    def __call__(self, x):
        self.counter += x
        print(self.counter)

a = State()
a(2) => 2
a(2) => 4
```

## 1.4.2 \_\_call\_\_

Integer division

```
a // b # 2//3 = 0 (rounded division)
```

Slicing list

```
var_name[i:j] # returns |j-i| elements
```

Create a class

```
class Person:
    def __init__(self, age, name):
        self.name = name
    def greet(self):
        print("Hi: ", self.name)
```

# 2 Major Modules

#### 2.1 Itertools

## 2.1.1 Import

```
from itertools import ...
```

#### 2.1.2 combinations

## 2.1.3 permutations

## 2.2 OS

## 2.2.1 Import

```
import os
```

## 2.2.2 Execute command in prompt

```
os.system("command to execute")
```

## 2.2.3 Get current path

```
os.path.abspath(os.path.filename(__file__))
```

## 2.2.4 Join paths

```
os.path.join(path_1, path_2)
```

# 2.3 Requests

# **2.3.1** Import

```
import requests
```

## 2.3.2 Get request to url

```
response = requests.get(url)
```

## 2.4 **JSON**

## 2.4.1 Save json file

```
with open(path, "w") as file:
    json.dump(data, file, indent=4)
```

## 2.4.2 Load json file

```
with open(path, "r") as file:
   data = json.load(file)
```

# 2.5 Pickle

## 2.5.1 Dump

```
from pickle import dump
dump(model, open(filename, 'wb'))
```

#### 2.5.2 Load

```
from pickle import load
loaded_model = load(open(filename, 'rb'))
```

## 2.6 Matplotlib

#### 2.6.1 Import

```
import matplotlib as plt
import matplotlib.pyplot as plt
```

## 2.6.2 Save figure

```
plt.savefig(<file_path>, format="png", dpi=300)
```

## 2.6.3 Multiple Graphs

#### First method

```
# Create a new figure
fig = plt.figure(figsize=(10, 6))
# Add subplots
ax1 = fig.add_subplot(221) # 2 rows, 2 columns, index 1
ax2 = fig.add_subplot(222) # 2 rows, 2 columns, index 2
ax3 = fig.add_subplot(223) # 2 rows, 2 columns, index 3
ax4 = fig.add_subplot(224) # 2 rows, 2 columns, index 4
# Plot on each subplot
ax1.plot([1, 2, 3], [1, 2, 3])
ax1.set_title('Plot 1')
ax2.plot([1, 2, 3], [3, 2, 1])
ax2.set_title('Plot 2')
ax3.plot([1, 2, 3], [2, 1, 2])
ax3.set_title('Plot 3')
ax4.plot([1, 2, 3], [1, 3, 2])
ax4.set_title('Plot 4')
plt.tight_layout()
plt.show()
```

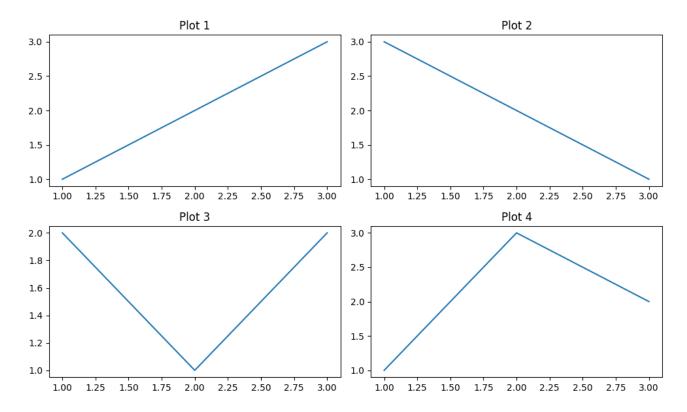


Figure 1: Different graphs stacked on matplotlib

#### Second method

```
# Create a figure and subplots
fig, (ax1, ax2) = plt.subplots(2, 1, figsize=(10, 8))

# Plot prices on the top subplot
ax1.plot(A, color='blue')
ax1.set_title('Dataset A')
ax1.grid(True)

# Plot RSI values on the bottom subplot
ax2.plot(B, color='green')
ax2.set_title('Dataset B')
ax2.grid(True)
```

## 2.6.4 Plot dataframe column in graph

```
goog_data['Close'].plot(ax=ax1, color='r', lw=2.)
```

ax1 is the subplot on which to plot the data, color is the hyperparameter for color and lw is the hyperparameter for the line width.

#### 2.6.5 Buy and sell markers

```
# Fetch Tesla stock data
tesla = yf.Ticker("TSLA")
data = tesla.history(period="1y")
```

```
# Calculate 50-day simple moving average
data['SMA_50'] = data['Close'].rolling(window=50).mean()
\mbox{\tt\#} Generate buy and sell signals based on SMA strategy
data['Signal'] = 0
data.loc[data['Close'] > data['SMA_50'], 'Signal'] = 1 # Buy signal
data.loc[data['Close'] < data['SMA_50'], 'Signal'] = -1 # Sell signal</pre>
# Filter buy and sell signals
buy_signal = data[data['Signal'] == 1]
sell_signal = data[data['Signal'] == -1]
plt.plot(data.index, data['Close'], label='Tesla Stock Price')
plt.plot(data.index, data['SMA_50'], label='50-Day SMA', linestyle='--', color='
   orange')
plt.scatter(buy_signal.index, buy_signal['Close'], marker='^', color='green', label=
   'Buy Signal', s=100)
plt.scatter(sell_signal.index, sell_signal['Close'], marker='v', color='red', label=
   'Sell Signal', s=100)
```

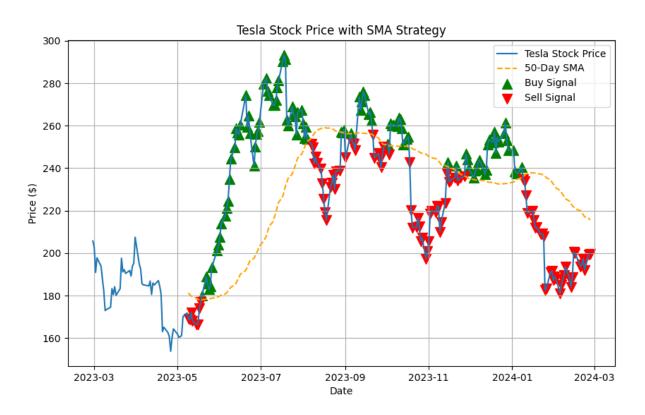


Figure 2: Buying and selling points on graph

## 2.6.6 Modern graph

```
import seaborn as sns
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np

# Generate random data
np.random.seed(0)
n_samples = 100
```

```
n_features = 5
n_{clusters} = 3
X = np.random.randn(n_samples, n_features)
Y_train = np.random.randint(0, n_clusters, n_samples)
# Create DataFrame from random data
dfsvd = pd.DataFrame(X, columns=['c{}'.format(c) for c in range(n_features)])
# Select columns for plotting
plotdims = 2
dfsvdplot = dfsvd.iloc[:, :plotdims]
dfsvdplot['Close_pred'] = Y_train
# Create pairplot using seaborn
ax = sns.pairplot(dfsvdplot, hue='Close_pred', height=2.5)
# Ensure tight layout for better visualization
plt.tight_layout()
# Display the plot
plt.show()
```

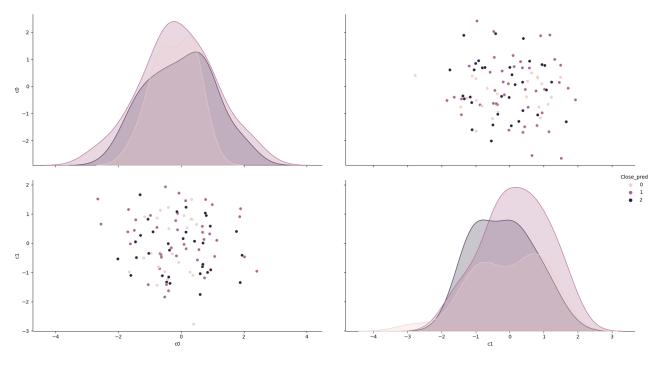


Figure 3: Modern scatter and line graph

## 2.6.7 3D Scatter Plot

```
np.random.seed(42)

def randrange(n, vmin, vmax):
    """
    Helper function to make an array of random numbers having shape (n, )
    with each number distributed Uniform(vmin, vmax).
    """
    return (vmax - vmin)*np.random.rand(n) + vmin

fig = plt.figure(figsize=(16, 10))
ax = fig.add_subplot(projection='3d')
```

```
m = 100

# For each set of style and range settings, plot n random points in the box
# defined by x in [23, 32], y in [0, 100], z in [zlow, zhigh].
for m, zlow, zhigh in [('o', -50, -25), ('v', -30, -5), ('^', -30, -5)]:
    xs = randrange(n, 23, 32)
    ys = randrange(n, 0, 100)
    zs = randrange(n, zlow, zhigh)
    ax.scatter(xs, ys, zs, marker=m)

ax.set_xlabel('X Label')
ax.set_ylabel('Y Label')
ax.set_zlabel('Z Label')
plt.show()
```

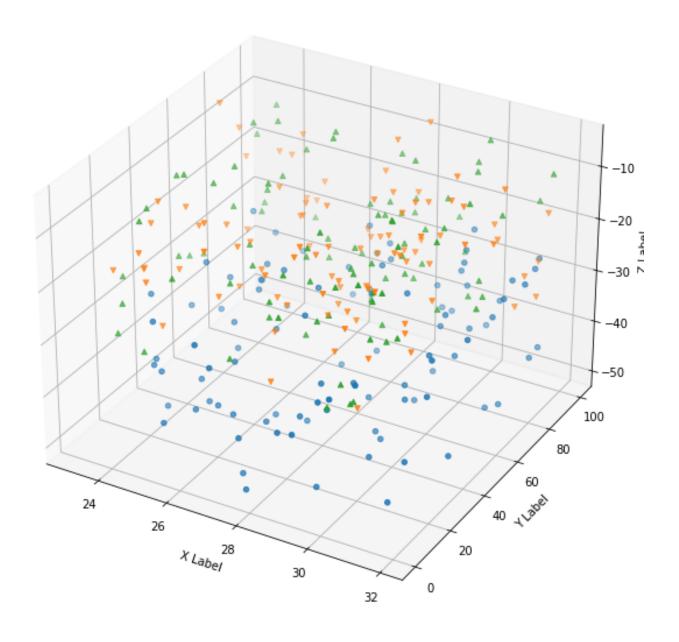


Figure 4: 3D Scatter graph

## 2.6.8 Graph settings

```
plt.xlabel("text for the x label") # label for the x axis
plt.ylabel("text for the y label")
plt.show() # display graph
```

## 2.6.9 Line plot

```
plt.plot(x, y)
```

## 2.6.10 Histogram

```
plt.hist(data, number_bins, color="blue")
```

## 2.6.11 Scatter plot

```
plt.scatter(x, y, alpha=0.1, s=50)
```

Where x and y are the coordinates of the points, alpha is the transparency of it, s is the size of the point. To plot based on classes, make use of:

```
test_points = np.array([[1, 2], [2, 1], [3, 4], [4, 5]])
classes = np.array([0, 0, 1, 1])
plt.scatter(test_points[:, 0], test_points[:, 1], c=classes, cmap='cool')
```

Which results in the following:

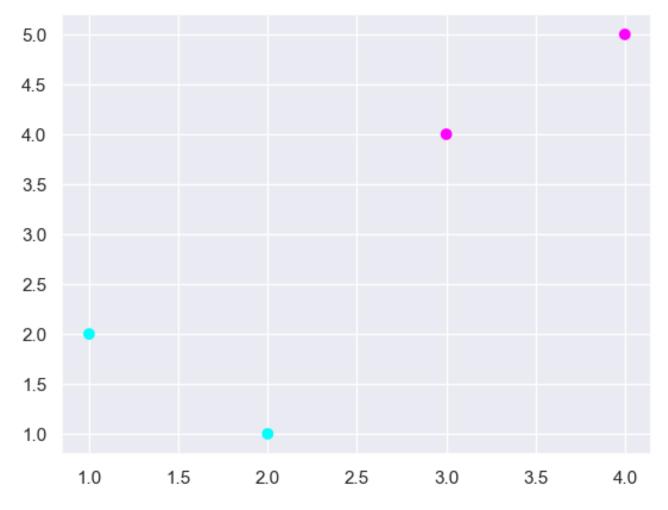


Figure 5: Scatter plot with color based on classes

## 2.6.12 Heatmap

```
plt.imshow(dataset, cmap="gray_r")
```

# 2.7 Seaborn

## 2.7.1 Pair plot

For higher than 3 dimensions impossible to plot all the data on a single graph.

```
import seaborn as sns
sns.pairplot(df)
```

With df the dataframe containing all the data.

An example showcasing it would be with the iris dataset:

```
import pandas as pd
from sklearn.datasets import load_iris
import seaborn as sns

iris = load_iris()
df = pd.DataFrame(iris.data, columns=iris.feature_names)
```

```
df['class'] = iris.target
df['class name'] = iris.target_names[iris['target']]
df.head()
sns.pairplot(df)
```

Noticeable that ignores textual classes (strings) because they can't be plotted. Result should look like this:

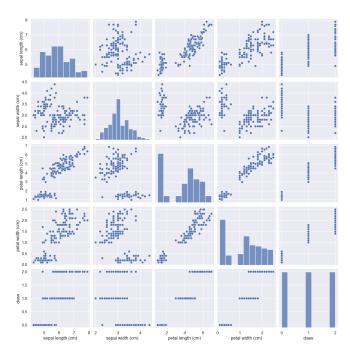


Figure 6: Pair plot of the iris setosa dataset

pairplot seaborn iris setosa

- 2.8 SciPy
- 2.9 Numpy
- 2.9.1 Import

```
import numpy as np
```

## 2.9.2 Operators

```
arr = np.array(-1, 1)
arr > 0 # => array([False, True])
```

## 2.9.3 Vertical slicing

```
a = np.array([[1, 2], [1, 2], [3, 4]])
# [[1, 2],
# [1, 2],
# [3, 4]
# ]
```

```
first_column = a[:, 0] # => [1, 1, 3]
```

## 2.9.4 np.astype(type)

```
arr = np.array([-1, 0.2, 2.43])
arr.astype(int) # => array([-1, 0, 2])
```

## 2.9.5 np.trunc()

```
np.trunc(8.99) # => 8.0
np.trunc([8.3, 3.7]) # => [8.0, 3.0]
```

#### 2.9.6 np.choice(sample, size, replace)

```
urn = ["b", "b", "b", "w", "w"]
chosen = np.random.choice(urn, size=2, replace=False)
```

Size represents the number of elements in the list returned, replace set to false means that once the marble is selected it is not returned to the urn (one by one, not in bulk, consider a size=4 in an urn of 3 elements, won't throw an error if replace is set to True but will if it is set to False), with urn being the list from which to pick selected items. Returns an array.

## 2.9.7 np.mean()

```
valeurs = [True, False, True]
np.mean(valeurs) # => 0.66 because True counts as a one and False as a zero
```

#### 2.9.8 np.unique()

Setting  $return_counts$  to false returns a list containing all the elements that appear at least once.

## 2.9.9 np.where()

Useful to iterate through pandas dataframe and create new column of data

```
df['signal'] = np.where(condition, a, b)
```

Where the 'signal' column is created and a is put in the row if the condition is True otherwise writes b in the dataframe. *condition* might be dataset['SMA\_short'] ¿ dataset['SMA\_long']

## 2.9.10 np.eye()

Create an identity matrix.

```
eye = np.eye(4)
```

#### 2.9.11 np.reshape(x, y)

```
array.reshape(x, y)
```

Where x and y are desired dimensions (row x column).

#### 2.9.12 np.reshape(-1, 1)

Reshape as column vector.

## 2.9.13 np.reshape(1, -1)

Reshape as horizontal array.

#### 2.9.14 .\_c

Turn two arrays into columns of bigger array:

#### 2.9.15 np.linspace(start, end, steps)

```
np.linspace(0, 10, 100)
array([ 0.
             , 0.1010101 , 0.2020202 , 0.3030303 , 0.4040404 ,
        0.50505051, 0.60606061, 0.70707071, 0.80808081, 0.90909091,
        1.01010101, 1.111111111, 1.21212121, 1.31313131, 1.41414141,
        1.51515152, 1.61616162, 1.71717172, 1.81818182, 1.91919192,
        2.02020202, 2.12121212, 2.22222222, 2.32323232, 2.42424242,
        2.52525253, 2.62626263, 2.72727273, 2.82828283, 2.92929293,
        3.03030303, 3.13131313, 3.23232323, 3.33333333, 3.43434343,
        3.53535354, 3.63636364, 3.73737374, 3.83838384, 3.93939394,
        4.04040404, 4.14141414, 4.24242424, 4.34343434, 4.44444444,
        4.54545455, 4.64646465, 4.74747475, 4.84848485, 4.94949495,
        5.05050505\,,\quad 5.15151515\,,\quad 5.25252525\,,\quad 5.35353535\,,\quad 5.45454545\,,
        5.55555556\,,\quad 5.65656566\,,\quad 5.75757576\,,\quad 5.85858586\,,\quad 5.95959596\,,
         6.06060606 \,, \quad 6.16161616 \,, \quad 6.26262626 \,, \quad 6.36363636 \,, \quad 6.46464646 \,, \\
        6.56565657,
                    6.66666667, 6.76767677, 6.86868687, 6.96969697,
                                              7.37373737,
        7.07070707,
                    7.17171717,
                                 7.27272727,
                                                           7.47474747,
        7.57575758,
                    7.67676768,
                                 7.7777778,
                                              7.87878788,
                                                           7.97979798,
                    8.18181818, 8.28282828,
        8.08080808,
                                              8.38383838,
                                                           8.48484848,
                    8.68686869, 8.78787879,
                                              8.8888889,
        8.58585859,
                                                           8.98989899,
                    9.19191919, 9.29292929,
                                              9.39393939,
        9.09090909,
                                                           9.49494949,
        9.5959596 , 9.6969697 , 9.7979798 ,
                                              9.8989899 , 10.
                                                                     ])
```

# 2.9.16 np.cov(x, y)

Covariance matrix can be displayed as such:

$$\begin{bmatrix} cov(x,x) & cov(x,y) \\ cov(y,x) & cov(y,y) \end{bmatrix}$$

## 2.9.17 np.var(x, y, ddof=k)

Where k is 0 by default and represents the divider N-k.

## 2.9.18 np.cumsum()

```
np.cumsum(data_array)
```

## 2.9.19 np.random.randn(n)

Generate n random numbers, thus comparable to noise generation.

```
noise = np.random.randn(100)
```

#### 2.9.20 np.random.standard\_normal((rows, columns))

Generate random matrix.

```
np.random.standard_normal((5, 4))
```

# 2.9.21 np.eye()

Meshgrid to be used for 3D graphing.

```
x = np.array([1, 2, 3])
y = np.array([4, 5, 6])
X, Y = np.meshgrid(x, y)
##################################
X = [[1, 2, 3],
        [1, 2, 3],
        [1, 2, 3]]

Y = [[4, 4, 4],
        [5, 5, 5],
        [6, 6, 6]]
```

Import module

```
import numpy as np
```

Create identity matrice (I)

```
eye = np.eye(4)
```

Create array

```
x = np.array([[1,2,3][4,5,6]])
```

## 2.9.22 np.repeat(array, iterations)

## 2.9.23 np.convolve()

mathematical appendix explain how convolution works?

# 2.9.24 np.ones()

#### 2.9.25 np.linalg.inv(A)

Calculate inverse of matrix.

```
np.linalg.inv()
```

## 2.9.26 np.linalg.pinv(A)

Moore-Penrose inverse calculation.

```
np.linalg.pinv()
```

## 2.10 Pandas

## 2.10.1 Import

```
import pandas as pd
```

## 2.10.2 Axis representation

Axis = 0 represents a row, axis = 1 represents a colum.

#### 2.10.3 Load dataframe

Load dataframe from file path.

```
df = pd.read_csv(<file_path>, header=0)
```

header = 0 to use first column as column titles

#### 2.10.4 Create dataframe

```
data = {"column_name":[row1, row2, row3, ...]}
df = pd.DataFrame(data)
```

## 2.10.5 Create dataframe with dict

```
df = pd.DataFrame({'A': [1, 2, 3], 'B': [4, 5, 6]})
```

## 2.10.6 Get dataframe shape

```
df.shape()
```

## 2.10.7 Iterate through rows

```
for index, row in df.iterrows():
    print(f'Index: {index}, A: {row["A"]}, B: {row["B"]}')
```

#### 2.10.8 Change column datatype

```
df['Close'] = df['Close'].astype(float)
```

#### 2.10.9 Get rows specific index

Get index of rows with specific Contents

```
tesla = yf.Ticker("TSLA")
data = tesla.history(period="1y")
data['SMA_50'] = data['Close'].rolling(window=50).mean()
data['Signal'] = 0
data.loc[data['Close'] > data['SMA_50'], 'Signal'] = 1  # Buy signal
data.loc[data['Close'] < data['SMA_50'], 'Signal'] = -1  # Sell signal

buy_signal = data[data['Signal'] == 1]
sell_signal = data[data['Signal'] == -1]</pre>
```

Where the content of the buy\_signal and sell\_signal dataframes are the following:

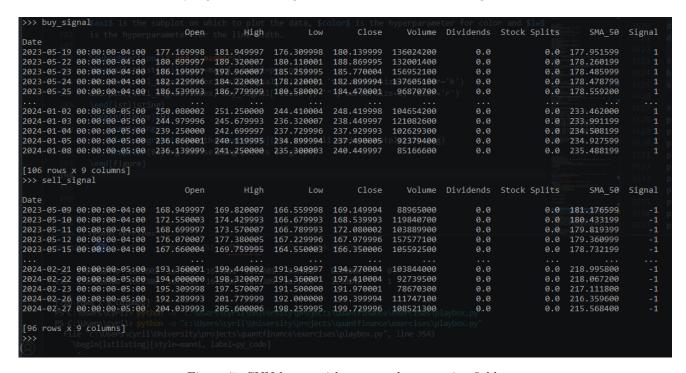


Figure 7: CNN layers with rectangular receptive fields

#### 2.10.10 Shuffle dataframe

```
data = {
     'A': [1, 2, 3, 4, 5],
     'B': ['a', 'b', 'c', 'd', 'e']
}
df = pd.DataFrame(data)
shuffled_df = df.sample(frac=1, random_state=42)
```

#### 2.10.11 Confusion matrix

```
df_cm = pd.DataFrame(confusion_matrix(y_test_actual, y_test_pred), \
columns=np.unique(y_test_actual), index = np.unique(y_test_actual))
df_cm.index.name = 'Actual'
df_cm.columns.name = 'Predicted'
sns.heatmap(df_cm, cmap="Blues", annot=True, annot_kws={"size": 16})
```

#### 2.10.12 Load from the web

```
url = "https://raw.githubusercontent.com/jeffprosise/Applied-Machine-Learning/main/
    Chapter%201/Data/customers.csv"
customers = pd.read_csv(url)
print(customers.head())
```

## 2.10.13 .loc(row, column)

```
df.loc(row, column)
```

row can bet set to  $x_i: x_f$  where  $x_i$  is the initial row and  $x_f$  is the final one (e.g.  $df.loc(x_i: x_f, column)$ ), the number of elements collected can be calculated by  $x_f - x_i$ , using: will select the whole column (indeed all the rows). Multiple columns can be sliced by listing them in a list (e.g.  $df.loc(row, [column_1, column_2, ...]))$ . To get a whole column  $df[column_name]$  also works.

## 2.10.14 .iloc()

```
df.iloc[:, 1:5]
```

Returns column 1 to 4.

#### 2.10.15 .values

```
df.iloc[:, 1:5].values
```

```
array([[
           1,
                19,
                       15,
                             39],
                21,
                       15,
                             81],
           1,
           0,
                20,
                       16,
                              6],
                23,
                      16,
           0,
                             77],
           0,
                31,
                       17,
                             40],
                22,
           0,
                       17,
                             76],
           0,
                35,
                       18,
                              6],
                23,
                       18,
                             94],
           1,
                64,
                       19,
                              3],
                       19,
                30,
                             72],
           0,
                67,
           1,
                       19,
                             14],
                35,
                       19,
                             99],
           0,
                58,
                       20,
           0,
                             15],
                24,
                       20,
                             77],
           1,
                37,
                       20,
                             13],
                       20,
           1,
                22,
                             79],
                35,
                       21,
                             35],
           0,
                20,
                       21,
                            66],
                52,
                       23,
                             29],
                35,
                       23,
                            98],
```

(b) After OHE

 Gender
 Age
 Annual Income (k\$)
 Spending Score (1-100)

 0
 1
 19
 15
 39

 1
 1
 21
 15
 81

 2
 0
 20
 16
 6

 3
 0
 23
 16
 77

 4
 0
 31
 17
 40

 ...
 ...
 ...
 ...
 ...

Figure 8: Dataframe values extraction

## 2.10.16 .merge(df1, df2, left\_on, right\_on, how)

```
result = pd.merge(df1, df2, left_on='df1col', right_on='df2col', how='inner')
```

how set to inner will only match if existing matching keys;  $left\_on$  is the column from the df1 to merge with the  $right\_on$  columns from df2.

## 2.10.17 .drop([column1, column2, ...])

```
df_bis = df.drop([column1, column2, ...], axis=1)
```

Axis set to 1 to drop columns.

## 2.10.18 .info()

```
df.info()
```

Gives important information about size and data type.

# 2.10.19 .describe()

```
df.describe()
```

Time	V1	V2	V3	V4	V5	V6
284807.000000	2.848070e+05	2.848070e+05	2.848070e+05	2.848070e+05	2.848070e+05	2.848070e+05
94813.859575	1.168375e-15	3.416908e-16	-1.379537e-15	2.074095e-15	9.604066e-16	1.487313e-15
47488.145955	1.958696e+00	1.651309e+00	1.516255e+00	1.415869e+00	1.380247e+00	1.332271e+00
0.000000	-5.640751e+01	-7.271573e+01	-4.832559e+01	-5.683171e+00	-1.137433e+02	-2.616051e+01
54201.500000	-9.203734e-01	-5.985499e-01	-8.903648e-01	-8.486401e-01	-6.915971e-01	-7.682956e-01
84692.000000	1.810880e-02	6.548556e-02	1.798463e-01	-1.984653e-02	-5.433583e-02	-2.741871e-01
139320.500000	1.315642e+00	8.037239e-01	1.027196e+00	7.433413e-01	6.119264e-01	3.985649e-01
172792.000000	2.454930e+00	2.205773e+01	9.382558e+00	1.687534e+01	3.480167e+01	7.330163e+01
	284807.000000 94813.859575 47488.145955 0.0000000 54201.500000 84692.000000 139320.500000	284807.000000 2.848070e+05 94813.859575 1.168375e-15 47488.145955 1.958696e+00 0.000000 -5.640751e+01 54201.500000 -9.203734e-01 84692.000000 1.810880e-02 139320.500000 1.315642e+00	284807.000000 2.848070e+05 2.848070e+05 94813.859575 1.168375e-15 3.416908e-16 47488.145955 1.958696e+00 1.651309e+00 0.000000 -5.640751e+01 -7.271573e+01 54201.500000 -9.203734e-01 -5.985499e-01 84692.000000 1.810880e-02 6.548556e-02 139320.500000 1.315642e+00 8.037239e-01	284807.000000 2.848070e+05 2.848070e+05 2.848070e+05 94813.859575 1.168375e-15 3.416908e-16 -1.379537e-15 47488.145955 1.958696e+00 1.651309e+00 1.516255e+00 0.000000 -5.640751e+01 -7.271573e+01 -4.832559e+01 54201.500000 -9.203734e-01 -5.985499e-01 -8.903648e-01 84692.000000 1.810880e-02 6.548556e-02 1.798463e-01 139320.500000 1.315642e+00 8.037239e-01 1.027196e+00	284807.000000	284807.000000 2.848070e+05 2.84

Figure 9: Dataset description

# 2.10.20 .diff()

Difference between two rows

```
df["c"] = df["c"].diff()
```

#### 2.10.21 .sort\_values

```
sorted_dataframe = df.sort_values(by='<column_name>', ascending=False)
```

# 2.10.22 shuffle(df, random\_state)

```
df = shuffle(df, random_state=42)
```

# $2.10.23 \quad .concat([df1,\,df2])$

Used to concatenate datasets.

```
df_conc = pd.concat([df1, df2])
```

# 2.10.24 .sample(frac)

```
sample_df = df.sample(frac=0.05)
```

## 2.10.25 .shift()

```
s = pd.Series([10, 20, 30, 40, 50])
shifted_s = s.shift(-1)
Original Series:
   10
   20
1
2
   30
3
   40
   50
4
dtype: int64
Shifted Series:
   20.0
   30.0
   40.0
   50.0
3
   NaN
4
dtype: float64
```

# 2.10.26 .count()

Count classified instances.

```
class_names = {0:'Not Fraud', 1:'Fraud'}
print(dataset.Class.value_counts().rename(index = class_names)) #//# count class
    columns to see how many are fraud and how many are not
```

Bar chart of instances count.

```
fig = plt.figure()
plot = dataset.groupby(['parsed_signals']).size().plot(kind='barh', color='red')
plt.show()
```

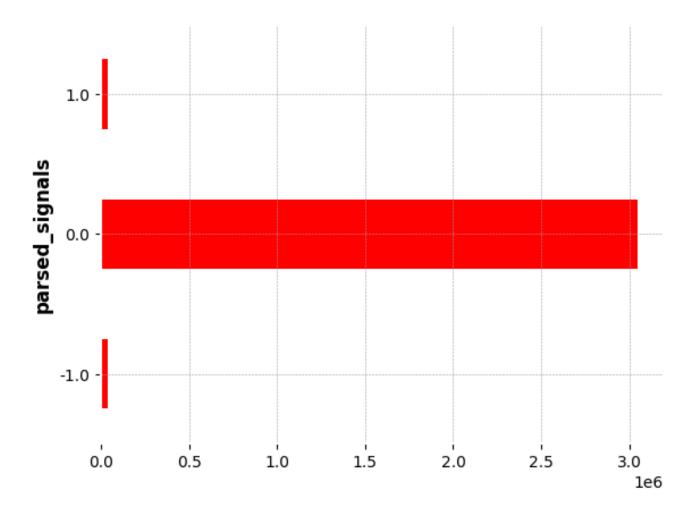


Figure 10: Signal count

# 2.10.27 .head()

Prints out the head of the dataset

df.head()

# 2.10.28 .ffill()

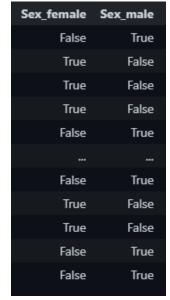
Forward fill (fill missing future value with previous value).

df.ffill()

# $\mathbf{2.10.29} \quad .\mathbf{get\_dummies}()$







(b) After dummies split

Figure 11: Dummies split resulting effect

### 2.10.30 .set\_index()

Define index column based on existing column.

```
df.set_index('Time', inplace=True)
```

### 2.10.31 .reset\_index()

```
df.reset_index(drop=True, inplace=True)
A B C
0 1 4 7
1 2 5 8
2 3 6 9
```

# 2.10.32 .pct\_change()

Change from previous row

```
a = pd.DataFrame([2, 4, 2, 1])
a.pct_change()
0 NaN
1 1.0
2 -0.5
3 -0.5
```

# 2.10.33 .corr()

Correlation matrix of various cryptocurrency assets.

```
correlation = dataset.corr()
plt.figure(figsize=(20, 20))
plt.title('Correlation Matrix')
sns.heatmap(correlation, vmax=1, square=True, annot=True, cmap='cubehelix')
```

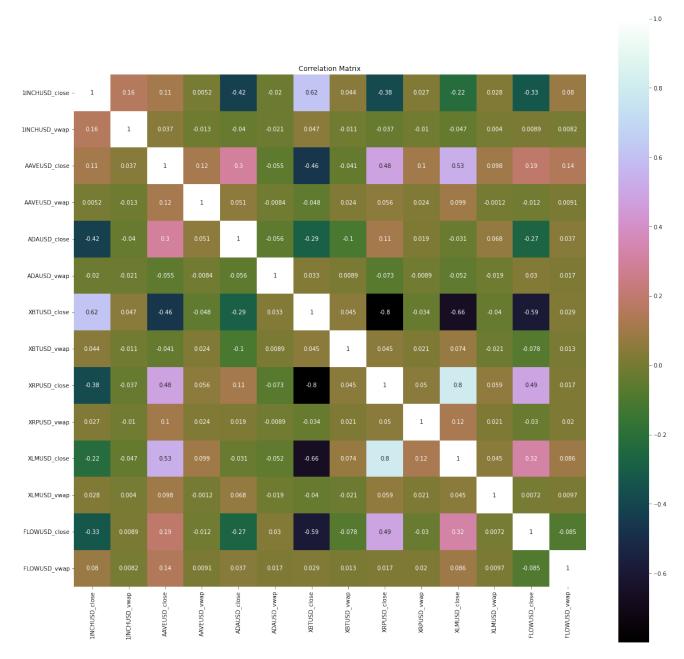


Figure 12: Correlation of various cryptocurrencies

### 2.10.34 .rolling()

Used for moving average as follows:

```
data["sma_size"] = data['Close'].rolling(window=window_size).mean()
```

# 2.10.35 .isin()

Keep only rows containing specific data

```
0 1 foo
2 3 foo
4 5 foo
```

### 2.10.36 .at()

Get element at [row, column]

### 2.10.37 .merge()

Outer merge might be the most useful.

```
# Creating two sample DataFrames
df1 = pd.DataFrame({'key': ['A', 'B', 'C', 'D'],
                    'value': [1, 2, 3, 4]})
df2 = pd.DataFrame({'key': ['B', 'D', 'E', 'F'],
                    'value': [5, 6, 7, 8]})
# Performing an inner merge based on the 'key' column
inner_merge = pd.merge(df1, df2, on='key', how='inner')
print("Inner Merge:")
print(inner_merge)
# Performing a left merge based on the 'key' column
left_merge = pd.merge(df1, df2, on='key', how='left')
print("\nLeft Merge:")
print(left_merge)
# Performing a right merge based on the 'key' column
right_merge = pd.merge(df1, df2, on='key', how='right')
print("\nRight Merge:")
print(right_merge)
# Performing an outer merge based on the 'key' column
outer_merge = pd.merge(df1, df2, on='key', how='outer')
print("\nOuter Merge:")
print(outer_merge)
**************************************
Inner Merge:
 key value_x value_y
           2
  В
                     5
1 D
                     6
Left Merge:
 key value_x value_y
O A 1 NaN
```

```
В 2
1
              5.0
2 C
      3
                NaN
3 D
         4
                6.0
Right Merge:
key value_x value_y
             5
0 B
        2.0
1
   D
        4.0
                 6
   Ε
        {\tt NaN}
                 7
   F
        NaN
                 8
Outer Merge:
 key value_x value_y
O A
       1.0
             NaN
1 B
        2.0
                5.0
2 C
        3.0
               NaN
3 D
        4.0
               6.0
               7.0
4 E
        NaN
5 F
        {\tt NaN}
                8.0
```

# 2.10.38 .sort\_values()

```
data = {
   'Name': ['Alice', 'Bob', 'Charlie', 'David', 'Emma'],
   'Age': [25, 35, 30, 28, 40],
   'Salary': [50000, 60000, 55000, 52000, 65000]
df = pd.DataFrame(data)
df.sort_values(by='Age', inplace=True)
Name Age Salary
   Alice 25 50000
   David 28 52000
3
2 Charlie 30 55000
    Bob 35 60000
1
    Emma 40 65000
4
```

# 2.10.39 .iloc()

The dataframe used to showcase is the following:

Access row:

```
B 100
C
    1000
Name: 0, dtype: int64
   A B C
  20 200 2000
1
2 30 300 3000
        400
              4000
3
   40
         В
    Α
   10
        100
               1000

    2
    30
    300
    3000

    4
    50
    500
    5000

2
```

### 2.10.40 .ewm() (Exponentially Weighted Moving Average)

Access column:

```
range_of_columns = df.iloc[:, 1:4] # Accessing a range of columns
0
  10
   20
1
   30
2
3
  40
4
  50
Name: A, dtype: int64
A C
 20 2000
 30 3000
3 40 4000
4 50 5000
  B C
0 100 1000
1 200 2000
2 300 3000
3 400 4000
4 500 5000
```

Access specific element:

```
3 40 4000
```

#### 2.10.41 MACD

```
short_ema = df['Close'].ewm(span=short_window, adjust=False).mean() # ewm =
    exponentially weighted .mean() to turn it into EMA, adjust=False stands for
    equal weight, otherwise recent observations have more weight
long_ema = df['Close'].ewm(span=long_window, adjust=False).mean() # if adjust = True
    weights are adjusted to account for 1 when summed, e.g. SMA of 20 days on the
    first 5 days will be lower
macd = short_ema - long_ema
signal_line = macd.ewm(span=signal_window, adjust=False).mean()
macd_histogram = macd - signal_line
```

#### 2.11 YFinance

Import module

```
import yfinance as yf
```

Gather stock price data with time interval

```
# Define symbol
stock_symbols = ["AAPL"]

# Define start and end date
start_date = "2022-01-01"
end_date = "2023-11-30"

# Fetch stock data from Yahoo Finance
stock_data = yf.download(stock_symbols, start=start_date, end=end_date)

# format it to a Pandas DataFrame
stock_df = pd.DataFrame(stock_data)
```

```
[******************100%***************
                                                   1 of 1 completed
                 Open 
                             High
                                          Low
                                                    Close
                                                            Adi Close
                                                                          Volume
Date
2021-12-31 178.089996 179.229996 177.259995 177.570007 175.564056
                                                                        64062300
2022-01-03
          177.830002
                       182.880005
                                   177.710007 182.009995
                                                          179.953888
                                                                      104487900
           182.630005
                       182.940002
                                   179.119995
                                              179.699997
                                                           177.669998
                                                                        99310400
2022-01-04
2022-01-05
           179.610001
                       180.169998
                                   174.639999
                                               174.919998
                                                           172.943985
                                                                        94537600
2022-01-06
           172.699997
                       175.300003
                                   171.639999
                                               172.000000
                                                           170.056961
                                                                        96904000
                       186.029999
                                   184.210007
                                               184.800003
2023-11-13 185.820007
                                                           184.800003
                                                                       43627500
```

Figure 13: YFinance results for scraping AAPL stock prices

# 3 Major APIs

### 3.1 Kraken

# 3.1.1 Required modules

```
import requests
import json
```

#### 3.1.2 Pair info

### 3.1.3 OHLC data

URL parameters are: (to be separated with &)

- Pair: seems obvious
- Interval: time interval in minutes

#### 3.1.4 Historical dataframe

```
csv_file_path = "C:\\Users\\cyril\\University\\projects\\QFT\\datasets\\
   Kraken_OHLCVT\\TIAUSD_1.csv"
ohlc_data = pd.read_csv(csv_file_path, header=0, names=["DateTime", "Open", "High",
   "Low", "Close", "Volume", "Trades"])
ohlc_data.head()
```

### 3.1.5 Pair price dataframe

```
pair = "TIAUSD"
url = f'https://api.kraken.com/0/public/OHLC?pair={pair}&interval=1'
response = requests.get(url)
data = response.json()
```

```
data = data['result']['TIAUSD']
df = pd.DataFrame(data, columns=['Time', 'Open', 'High', 'Low', 'Close', 'VWAP', '
   Volume', 'Count']) # Convert to DataFrame
print(df)
High
                                Close
                                         VWAP
Time
        Open
                        I.ow
                                                 Volume
                                                        Count
0
    1709040360
               17.3824
                       17.3824
                                17.3130
                                        17.3130
                                                17.3635
                                                         692.20434
                                                                      27
1
    1709040420
               17.3255
                       17.3255
                                17.3062
                                        17.3062
                                                17.3202
                                                         11.47762
                                                                       3
    1709040480
               17.2873
                                                                       7
                       17.3330
                                17.2819
                                        17.3220
                                                17.2846
                                                         526.38073
                                        17.2806
    1709040540
               17.2807
                       17.2807
                                17.2806
                                                17.2806
                                                         24.57475
    1709040600 17.3506
                       17.3721
                                17.3506
                                        17.3663
                                                17.3666
                                                         309.78807
                                                                       3
              16.9507
    1709083260
                       16.9507
                                                 0.0000
715
                                16.9507
                                        16.9507
                                                          0.00000
                                                                       0
    1709083320 16.9589 16.9601
                               16.9589
                                        16.9601
                                                         235.80317
716
                                                16.9597
                                                                       .3
    1709083380 16.9601 16.9601
                               16.9601
                                        16.9601
717
                                                 0.0000
                                                          0.00000
                                                                       0
718 1709083440 16.9601 16.9601
                               16.9601
                                        16.9601
                                                0.0000
                                                           0.00000
                                                                       0
719 1709083500 16.9389 16.9389 16.9389 16.9389 16.9389
                                                           2.36142
                                                                       1
```

#### 3.1.6 Status code

Get the status code of the api call (prey for 200)

```
response = requests.get(url)
response.status_code
```

# 4 Algorithms

#### 4.1 Hashing

#### 4.1.1 Hash function

Mathematical function that takes an input/key and produces a fixed-size string of byte. The output (a number) is called hash code/value.

#### 4.1.2 Hash function characteristics

Deterministic because the same input produces the same output, the output is of fixed size, it must be collision resistant because it is hard to find different inputs that produce the same output, must be prone to avalanche effect, in other words a small change in input changes drastically the output.

### 4.1.3 Hash buckets

# 5 Finance

#### 5.1 Drawdown

```
df['cumulative_max'] = df['values'].cummax()
df['cumulative_sum'] = df['values'].cumsum()
df['drawdown'] = df['cumulative_max'] - df['cumulative_sum']
```

#### 5.2 Technical Indicators

#### 5.2.1 SMA

```
dataset['short_mavg'] = dataset['Close'].rolling(window=10, min_periods=1, center=
    False).mean()
```

#### 5.2.2 EMA

```
def EMA(df, n):
EMA = pd.Series(df['Close'].ewm(span=n, min_periods=n).mean(), name='EMA_' + str(n))
return EMA

dataset['EMA10'] = EMA(dataset, 10)
dataset['EMA30'] = EMA(dataset, 30)
dataset['EMA200'] = EMA(dataset, 200)
```

#### 5.2.3 ROC (Rate of Change)

```
#calculation of rate of change
def ROC(df, n):
    M = df.diff(n - 1)
    N = df.shift(n - 1)
    ROC = pd.Series(((M / N) * 100), name = 'ROC_' + str(n))
    return ROC

dataset['ROC10'] = ROC(dataset['Close'], 10)
dataset['ROC30'] = ROC(dataset['Close'], 30)
```

#### 5.2.4 BBands (Bollinger Bands)

#### 5.2.5 RSI

```
avg_gain_values = [] # track avg gains for visualization purposes
avg_loss_values = [] # track avg losses for visualization purposes
rsi_values = [] # track computed RSI values
last_price = 0 # current_price - last_price > 0 => gain. current_price -
last_price < 0</pre>
for close_price in prices:
    if last_price == 0:
        last_price = close_price
    gain_history.append(max(0, close_price - last_price))
    loss_history.append(max(0, last_price - close_price))
    last_price = close_price
    if len(gain_history) > time_period: # maximum observations is equal to lookback
       period
        del (gain_history[0])
        del (loss_history[0])
    avg_gain = stats.mean(gain_history) # average gain over lookback period
    avg_loss = stats.mean(loss_history) # average loss over lookback period
    avg_gain_values.append(avg_gain)
    avg_loss_values.append(avg_loss)
    rs = 0
    if avg_loss > 0: # to avoid division by 0, which is undefined
        rs = avg_gain / avg_loss
    rsi = 100 - (100 / (1 + rs))
    rsi_values.append(rsi)
```

#### 5.2.6 MOM (MOMentum)

```
prices = df['High']
momentum = []

for i in range(1, len(prices)):
    momentum.append(prices[i]-prices[i-1])
```

### 5.2.7 STOK & STOD

```
def STOK(close, low, high, n):
    STOK = ((close - low.rolling(n).min()) / (high.rolling(n).max() - \
    low.rolling(n).min())) * 100
    return STOK

def STOD(close, low, high, n):
    STOK = ((close - low.rolling(n).min()) / (high.rolling(n).max() - \
    low.rolling(n).min())) * 100
    STOD = STOK.rolling(3).mean()
    return STOD

dataset['%K10'] = STOK(dataset['Close'], dataset['Low'], dataset['High'], 10)
    dataset['%D10'] = STOD(dataset['Close'], dataset['Low'], dataset['High'], 30)
    dataset['%D30'] = STOK(dataset['Close'], dataset['Low'], dataset['High'], 30)
    dataset['%K200'] = STOK(dataset['Close'], dataset['Low'], dataset['High'], 200)
```

# 6 Data Science

### 6.1 Lifecycle

#### 6.1.1 Stages

- Ask a question: must narrow a broad question into one that can be answered with data, amongst four categories (descriptive (e.g. how have house prices changed over time), exploratory (e.g. which aspects of a house drive its price), inferential, predictive)
- ♦ Obtain the data: most of the time expensive/hard to gather
- ♦ Understand the data: plots to uncover patterns, summarize visually
- ♦ Understand the world:

### 6.1.2 Data scope

Range of questions that can be answered is limited with a given amount of data.

#### 6.1.3 Target population, access frame, sample

Target population: collection of elements we intend to describe/draw conclusion about (of which one element is called an atom/unit)

Access frame: collection of elements accessible for measurement and observation (e.g. in a vote, asking by phone to someone who doesn't vote is in the frame but trying to phone someone that doesn't answer to unknown numbers is out of frame) Sample: subset of units taken from the access frame to observe and measure.

#### 6.1.4 Protocols

Instrument being used to take the measurements and the procedure for taking them.

### 6.1.5 Accuracy

Bias:

Precision:

### 6.2 Bias

### 6.2.1 Coverage bias

Target frame doesn't include everytarget population (units) (e.g. survey by phone can't reach those without phones).

### 6.2.2 Selection bias

Mechanism used to choose units for the sample tends to select more of certain units rather than others (e.g. self selection with volunteers to participate in the study).

### 6.2.3 Nonresponse bias

Unit: someone from selected sample is unwilling to participate (e.g. someone who doesn't answer phone calls) Item: someone is willing to participate but not to answer (e.g. someone answeres the phone but doesn't answer)

### 6.2.4 Measurement bias

Instrument systematically misses the target (in the same direction otherwise correction occurs) (e.g. can occur with sensors but also when making a survey if people are not at ease giving honest answers or even just questions wrongly formulated).

#### 6.3 Variations

#### 6.3.1 Sampling variation

Using chance to select a sample.

### 6.3.2 Assignment variation

When splitting units into different treatment groups. Thus changing the way we split into treatment groups will affect results.

#### 6.3.3 Measurement variation

If an instrument commits errors that are normally distributed around the truth, the result is also centered around the truth.

# 6.4 Sampling

#### 6.4.1 Stratified sampling

Divide the population into nonoverlapping groups (named strata of which one group is named stratum)

#### 6.4.2 Cluster sampling

Divide the population into nonoverlapping subgroups

### 6.5 Distributions

#### 6.5.1 Hypergeometric distribution

Drawing without replacement.

### 6.5.2 Multivariate Hypergeometric distribution

For example a poll with 3 possible votes.

### 6.5.3 Binomial distribution

Drawing with replacement.

# 7 Machine Learning

### 7.1 Lingo

- ♦ Collaborative filtering: guess what user might like based on what other similar user liked.
- ♦ Dataset: row = sample, colump = properties/features
- ♦ Deterministic: can be perfectly determined based on initial conditions, no stochasticity implied
- ♦ Feature = Predictor: input data (column wise)
- ♦ Holdout set: test set
- ♦ Validation set: intermediate holdout set
- ♦ Inference time: time required to make a prediction
- ♦ Label: output data
- ♦ Lazy learner: no learning required (such as KNN)
- ♦ Nested vs flat dataset: nested is a dataset that contains datasets itself and a flat one is one that does not
- ♦ Multiclass: pick one choice amongst multiple classes to belong to (e.g. a digit can be a 3 or a 2)
- ♦ Multilabel: pick multiple choices amongst multiple classes to belong to (e.g. photo containing cat or dog)

#### **7.2** Bias

#### 7.2.1 Data snooping bias

Adapting machine learning model to specific spotted pattern, thus exposing ourselves to overfitting. In other words using data multiple times and pretend the data is independent.

#### 7.2.2 Selection bias

Certain group of individuals systematically excluded from sample in non-random manner. Can happen by self-selection where individuals want or do not want to bring data to the dataset. Stratified sampling (taking random sample representative of each part of possible sample) solves this problematic issue.

#### 7.2.3 Sampling bias

Selected sample overrepresents or underrepresents group of individuals. Stratified sampling (taking random sample representative of each part of possible sample) solves this problematic issue.

### 7.3 Classification vs. Regression

#### 7.3.1 Classification

Binary if two possibles classes, multiclass if more than two. Discrete prediction problems.

# 7.3.2 Multiclass classification

Model picks on group amongst many.

#### One-vs-one

**One-vs-rest** Suppose we have multiple labels such as [11, 12, 13], one-vs-rest consists of taking each label and creating a model to ask wether it is or not the said label (here: 11 or not, 12 or not, 13 or not). Thus allowing binary models to solve multiclass classification problems.

### 7.3.3 Singlelabel vs. Multilabel

Singlelabel Each instance assigned to only one class.

 ${\bf Multilabel} \quad {\bf Instance \ belongs \ to \ multiple \ classes}.$ 

#### 7.3.4 Regression

Continuous data to predict.

# 7.4 Supervised vs. Unsupervised

#### 7.4.1 Supervised

Goal is to minimize cost function Learn from input/output pairs, easier to measure performance ex: zip code reading on envelope, determine if tumor is benign, fraudulent activity in credit card transactions

#### 7.4.2 Unsupervised

Only input is known, harder to evaluate ex: stock market mood, grouping customers by preferences and potential purchases, ...

# 7.5 Supervised Models

Data is labeled (i.e. there is an expected answer), most common supervised tasks are regression and classification. Clustering if expected labels are unknown but classification if those are known.

Supervised models are either about regression or about classification. While classification problems are probability, regression problems on the other hand output a continuous set of possible outcomes.

### 7.5.1 Logistic regression

Primarily a classification model, despite confusing name. Fits an equation to the distribution of data.

### 7.5.2 LDA (Linear Discriminant Analysis)

### 7.5.3 Polynomial regression

Import:

```
from sklearn.preprocessing import PolynomialFeatures
```

Fit:

```
poly_features = PolynomialFeatures(degree=2, include_bias=False)
X_poly = poly_features.fit_transform(X)
```

### 7.5.4 DecisionTreeRegressor

Works on non-linear data relationships. Can be used both for regression and classification

Import:

```
from sklearn.tree import DecisionTreeRegressor
```

Build model:

```
tree_regressor = DecisionTreeRegressor()
```

Fit:

```
tree_regressor.fit(X, Y)
```

Predict: input X values outputs y

```
prediction = tree_regressor.predict(X)
```

# 7.6 Semisupervised Models

# 7.6.1 deep belief networks (DBNs)

Based on unsupervised restricted Boltzmann machines (RBMs) stack on top of one another then system as a whole is fine-tuned using supervised learning techniques.

# 7.7 Unsupervised Models

Data is unlabeled (i.e. there is no expected answer), most of the data in today's world are this way, we have X label but not y label. Common unsupervised tasks include clustering, visualization, dimensionality reduction, and association rule learning. Permits to save a lot of time (e.g. a factory gets pictures of thousands of manufactured pictures but are not labeled wether or not they are defective)

Clustering Group similar instances together into clusters. Commonly used in finance to find homogenous classes of assets.

**Anomaly Detection** Learn what normal is and then based on this "normal" template detect outliers/abnormal samples.

**Density Estimation** Task is to estimate the PDF (Probability Density Function) of the random process that generated the dataset (commonly used for anomaly detection, low probability instances might be outliers).

#### 7.7.1 K-means

For clustering. Lloyd-Forgy method, bad for non-spherical clusters, works better with similar-size clusters. Finds K centroids and assigns each data point to exactly one cluster, the goal of the model is to minimize the within-cluster variance (inertia) based on euclidean distance.

**Overview** Initialization is made with random centroids selection. Each data point is assigned to a cluster's center (put simply a centroid). Cluster centers are updates to the mean of the assigned points. Process is repeated until stabilization is acquired. Advantages are fast convergence and simplicity as well as scalability to large dataset and construction of evenly sized clusters. Only default is knowing the number of clusters.

### Hyperparapeters

- ⋄ number of clusters:
- ♦ maximum iterations: if doesn't converge to solution must stop somewhere
- ♦ number initial: number of times the algorithm will be run with different centroids seeds (default is at least 10 iterations).

### Finding optimal number of clusters in kmeans

- Elbow method: Based on the sum of squared errors (SSE) within clusters.
- Silhouette method: Based on the silhouette score. Measures similarity between a point and its cluster (cohesion) vs other clusters (separation).

Hard clustering Assigns to a single cluster

**Soft clustering** Assigns a score per cluster (e.g. score can be distance to centroid, GRBF (Gaussian Radial Basis Function))

### 7.7.2 Hierarchical Clustering

Clusters have predominant ordering from top to bottom tf does it even mean?. No need to provide the number of clusters.

```
from sklearn.cluster import KMeans
kmeans = KMeans(n_clusters=5)
y_pred = kmeans.fit_predict(X)
kmeans.cluster_centers_
```

Disadvantage is having to explicitly specify the number of clusters to be found. Then to make predictions the following can be used:

```
kmeans.predict(X_new)
```

Returns the distance to all the cluster centroids.

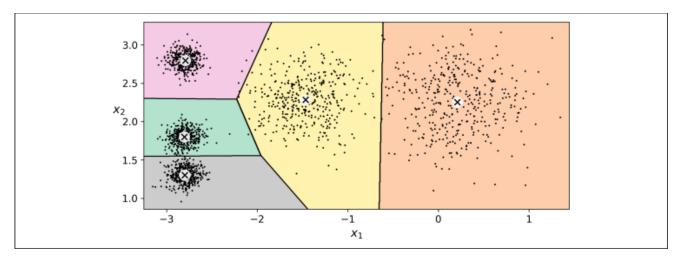


Figure 14: KMeans Clustering, where rounded x represents centroids

Define initial centroids

```
good_init = np.array([[-3, 3], [-3, 2], [-3, 1], [-1, 2], [0, 2]])
kmeans = KMeans(n_clusters=5, init=good_init, n_init=1)
```

init = "random" can be used to avoid doing this step.

Charles Elkan proposed to avoid some distance calculations and this model is used by default but original model can be used by setting hyperparameter algorithm="full".

### 7.7.3 K-Means mini-batches

David Sculley, useful if dataset does not fit in memory.

```
from sklearn.cluster import MiniBatchKMeans
minibatch_kmeans = MiniBatchKMeans(n_clusters=5)
minibatch_kmeans.fit(X)
```

### 7.7.4 Agglomerative Clustering

Most common type, group instances based on similarity. Algorithms works by creating N clusters (where each instances is a cluster itself (to put simply N is the number of instances)), then the two closest data points are taken and merged into one bigger cluster (proceeding to agglomeration).

```
from sklearn.cluster import AgglomerativeClustering
from scipy.cluster.hierarchy import dendrogram, linkage
import matplotlib.pyplot as plt

# Fit the model
model = AgglomerativeClustering(n_clusters=4, linkage='ward')
clust_labels1 = model.fit_predict(dataset[-50:])

# Generate linkage matrix
```

```
linkage_matrix = linkage(dataset, method='ward', metric='euclidean') #//# ward, We
    use ward as the method since it minimizes the variance of distances between the
    clusters

# Plot dendrogram
plt.figure(figsize=(10, 7))
dendrogram(linkage_matrix)
plt.title('Hierarchical Clustering Dendrogram')
plt.xlabel('sample index')
plt.ylabel('distance')
plt.show()
```

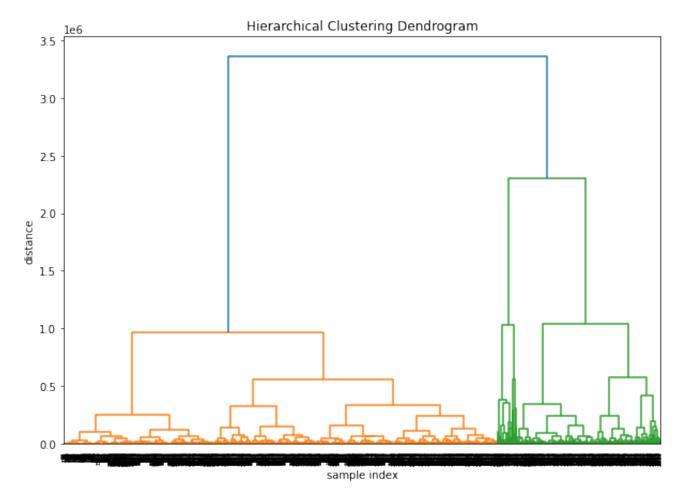


Figure 15: Same dataset rotated 45° shows high sensitivity

Divisive hierarchial clustering Oppostive of agglomerative.

# 7.7.5 AffinityPropagation

```
from sklearn.cluster import AffinityPropagation
# Initialize the algorithm and set the number of PC's
ap = AffinityPropagation()
ap.fit(dataset)
```

### 7.7.6 DBSCAN (Density-Based Spatial Clustering of Applications with Noise)

Clustering for arbitrary shape. The algorithm takes a point/instance and sees the neighborhood  $\epsilon$ , if the  $\epsilon$  has at least min-samples instances in its neighborhood then  $\epsilon$  is considered core instance (thus they are located in dense regions). Works well is clusters are dense enough and separated by low density regions. eps is considered the maximum distance between two instances to be considered of the same class. Based on density of points.

```
from sklearn.cluster import DBSCAN
dbscan = DBSCAN(eps=0.05, min_samples=5)
dbscan.fit(X)
dbscan.labels_
```

```
dbscan.core_sample_indices_
808
```

```
>>> dbscan.core_sample_indices_
array([ 0, 4, 5, 6, 7, 8, 10, 11, ..., 992, 993, 995, 997, 998, 999])
```

#### 7.7.7 HDBSCAN (Hierarchical DBSCAN)

https://github.com/scikit-learn-contrib/hdbscan/

### 7.7.8 Agglomeration Clustering

Small bubbles attached to one another until it forms a bigger cluster (like merging a tree's branches).

#### 7.7.9 BIRCH (Balanced Iterative Reducing and Clustering using Hierarchies)

Designed for big datasets with few features (i20).

#### 7.7.10 Mean-Shift

This algorithm starts by placing a circle centered on each instance; then for each circle it computes the mean of all the instances located within it, and it shifts the circle so that it is centered on the mean. Next, it iterates this mean-shifting step until all the circles stop moving (i.e., until each of them is centered on the mean of the instances it contains)

### 7.7.11 Affinity Propagation

Voting system, does not require number of clusters to be known beforehand.

#### 7.7.12 Spectral Clustering

Similarity matrix between instances to create low-dimensional matrix of it then uses other clustering method in the low-dimensional space.

#### 7.7.13 GMM (Gaussian Mixtures Model)

Assumption that instances were generated from a mixture of several Gaussian distributions. Anomaly detection using GMM can be performed before training another model on the cleaned data. To do so density treshold is defined and instances in the low density region are kicked. MLE (Maximum Likelihood Estimation), MAP (Maximum A-Posteriori), BIC, AIC with gm.big() and gm.aic() Advance is that shape can be imposed.

```
from sklearn.mixture import GaussianMixture
gm = GaussianMixture(n_components=3, n_init=10)
gm.fit(X)
>>> gm.weights_
array([0.20965228, 0.4000662 , 0.39028152])
>>> gm.means_
array([[ 3.39909717, 1.05933727],
    [-1.40763984, 1.42710194],
    [ 0.05135313, 0.07524095]])
>>> gm.covariances_
array([[[ 1.14807234, -0.03270354],
    [-0.03270354, 0.95496237]],
    [[ 0.63478101, 0.72969804],
    [ 0.72969804, 1.1609872 ]],
    [[ 0.68809572, 0.79608475],
    [ 0.79608475, 1.21234145]]])
```

Shape can be imposed: "spherical" all clusters must be spherical, but they can have different diameters (i.e., different variances). "diag" clusters can take on any ellipsoidal shape of any size, but the ellipsoid's axes must be parallel to the coordinate axes (i.e., the covariance matrices must be diagonal). "tied" All clusters must have the same ellipsoidal shape, size, and orientation (i.e., all clusters share the same covariance matrix). Outliers identification

```
densities = gm.score_samples(X)
density_threshold = np.percentile(densities, 4)
anomalies = X[densities < density_threshold]</pre>
```

- 7.7.14 Fast-MCD (Fast-Minimum Covariance Determinant)
- 7.7.15 Isolation forest
- 7.7.16 LOF (Local Outlier Factor)
- 7.7.17 One-Class SVM
- 7.7.18 Hierarchical Cluster Analysis (HCA)

Clustering

#### 7.7.19 One-class SVM

Anomaly detection and novelty detection

### 7.7.20 Isolation Forest

Anomaly detection and novelty detection

### 7.7.21 Apriori

Association rule learning

### 7.7.22 Eclat

Association rule learning

# 7.8 Ensemble Learning

"there is wisdom in the crowd"

#### 7.8.1 Voting classifiers

Suppose we have 4 models, each with 80% accuracy in their predictions, comparing their predictions and keeping the one occurring the most often is called hard voting (majority-vote). Accuracy somehow often reaches higher level than with the best individual model. Even a sufficient list of weak learners (less precize than random) can become a strong learner if enough models are combined (cf. law of large numbers), for example a list of models with each an accuracy of 51% can expect a global accuracy of around 75% (nevertheless models must be independent to make uncorrelated errors). Soft voting can also be used if all models have a predict\_proba method, soft-voting often leads to higher accuracy predictions because it gives more weight to highly confident votes, to set up soft proba, hyperparameters arguments must be the following: voting="soft" and probability=True (to have probability between 0 and 1).

### 7.8.2 Bagging (Bootstrap Aggregating)

Used for models with high variance. Training same algorithm on different subsets of the training set then combining as if they were different predictors (can be done in parallel). Resulting algorithm has an equivalent bias but lower variance. Another advantage is that each small subset can be trained on a separated core/server. Automatically uses soft voting is probability is available (unlike voting classifiers)

```
from sklearn.ensemble import BaggingClassifier
bag_clf = BaggingClassifier(
    DecisionTreeClassifier(), n_estimators=500,
    max_samples=100, bootstrap=True, n_jobs=-1)
```

Where  $n_i obs$  sets the number of cores to use, -1 uses as many cores as available.

#### 7.8.3 Boosting

Emphasis on learning the examples the previous model got wrong. Residual models (correcting the error) are typically not very complicated.

#### 7.8.4 AdaBoost (Adaptive Boosting)

Boosting method (referenced in 7.8.3) algorithm. Correct precessor by paying attention to instances it underfitted and tweaking weights to adjust result. Default is it doesn't scale well. (sklearn uses SAMME (Stagewise Additive Modeling using a Multiclass Exponential loss function) AdaBoost algorithm) mathematics behind: negative weights allowed?

### 7.8.5 Gradient Boosting

Boosting method (referenced in 7.8.3) algorithm. Same principle as AdaBoost but instead fits to residual errors made by the previous predictor GBRT is supervised and makes use of this but other algorithms also do, which?

#### 7.8.6 XGBoost

Boosting method (referenced in 7.8.3) algorithm.

```
import xgboost

# train an XGBoost model
X, y = shap.datasets.california()
model = xgboost.XGBRegressor().fit(X, y)
```

#### 7.8.7 Pasting

i believe there is a misconception with bagging

#### 7.8.8 Stacking (Stacked Generalization)

Combines output of a collection of models to make a prediction. All models are trained then a model is trained using output of all models as feature. The upper-level model's goal is to learn how to best combine the outcomes of the initial models. No hard-voting but model is trained to perform best aggregation possible. The way it works is by gathering all predictions and blending them into a single output prediction.

#### 7.8.9 Out-of-Bag (oob)

With bagging some instances of training set can be used multiple times (or not at all). Set oob\_score=True to get score without having to create separated training/testing set.

### 7.8.10 Random patches

too complicated for now but need to learn about it

#### 7.8.11 Random subspaces

too complicated for now but need to learn about it

- 7.9 Ensemble Models
- 7.9.1 GBRT (Gradient Boosted Regression Trees)
- 7.9.2 Voting Classifier

```
from sklearn.ensemble import RandomForestClassifier
from sklearn.ensemble import VotingClassifier
from sklearn.linear_model import LogisticRegression
from sklearn.svm import SVC
log_clf = LogisticRegression()
rnd_clf = RandomForestClassifier()
svm_clf = SVC()
voting_clf = VotingClassifier(
   estimators=[('lr', log_clf), ('rf', rnd_clf), ('svc', svm_clf)],
   voting='hard')
voting_clf.fit(X_train, y_train)
from sklearn.metrics import accuracy_score
for clf in (log_clf, rnd_clf, svm_clf, voting_clf):
   clf.fit(X_train, y_train)
   y_pred = clf.predict(X_test)
   print(clf.__class__.__name__, accuracy_score(y_test, y_pred))
LogisticRegression 0.87
RandomForestClassifier 0.825
SVC 0.875
VotingClassifier 0.875
```

**Hyperparameters** Set voting to *soft* for probability assigned to each class and hard to go binary.

- 7.9.3 GDM (Gradient Boosting Machine)
- 7.9.4 AdaBoost (Adaptive Boosting)

```
from sklearn.ensemble import AdaBoostClassifier
ada_clf = AdaBoostClassifier(
    DecisionTreeClassifier(max_depth=1), n_estimators=200,
    algorithm="SAMME.R", learning_rate=0.5)
```

```
ada_clf.fit(X_train, y_train)
ada_clf.fit(X_train, y_train)
predictions = ada_clf.predict(X_test)
```

### 7.10 Batch Learning vs. Online Learning

#### ♦ Batch:

Incapable of learning incrementally, trained then used. Called offline learning. Therefore this solution is not advised for usecases where fast reaction is required such as for stock prices prediction. Useful when training everyday is not sustainable or carrying a lot of data to train on the go is impossible e.g. a rover on mars or a smartphone.

#### ♦ Online (incremental learning):

Consists of training model incrementally with mini-batches (advantage is that learning step is fast and cheap). To save space this method allows throwing data away once used to train.

Allows usage of "out-of-core" learning where algo runs training step on slice of data and repeat until it has trained on all the data, practical to handle vast quantities of data a machine can't handle alone.

Learning rate can be set to decide how fast or slow model must learn, thus to be less or more sensitive to suddent outliers.

# 7.11 Instance-Based vs. Model-Based Learning

Inference: make predictions on new cases.

#### ♦ Instance-Based:

Based on measure of similarity it makes prediction. e.g. the count of words in a mail

#### ♦ Model-Based Learning:

Search for an optimal value for the model parameters such that the model will generalize well to new instances. We usually train such systems by minimizing a cost function that measures how bad the system is at making predictions on the training data, plus a penalty for model complexity if the model is regularized. To make predictions, we feed the new instance's features into the model's prediction function, using the parameter values found by the learning algorithm.

### 7.12 Performance Evaluation

#### 7.12.1 No Free Lunch (NFL) theorem

If we make no assumption about the data, nothing happens, that is why we must try to think how the data behaves and thus use the most convenient model (e.g. the data evolves in a linear way)

#### 7.12.2 Bias

Low performance betcause model is **not fitting well**.

### 7.12.3 Variance

Due to model's excessive adaptation to small variations in training data (high degree polynomial regression **tend to overfit** thus have more variance).

### 7.12.4 Bias-variance trade-off

Increasing model complexity will increase bias and reduce variance but cannot get both at the same time.

#### 7.12.5 Testing and validating

Using split of data around 80/20 ratio, can show how model performs on "never seen before data". Few errors on training but a lot in testing is an indicator of overfitting. holdout validation

#### 7.12.6 Cross-validation

Comes at cost of training multiple times.

# 7.12.7 Irreducible error

Due to inherent noise in data, only way to reduce it is to clean the data (remove outliers, ...).

#### 7.12.8 Data drift

Current data with previously fitted model doesn't work anymore because data changed.

# 7.12.9 Cost/Loss Function

Used to calculate error committed by the model.

#### 7.12.10 Evaluation Metrics

evaluation metrics

### 7.13 Cost/Loss Function Algorithms

Calculations for cost functions are detailled in 7.12.9

# 7.13.1 MSE (Mean Squared Error)

More sensitive to unusually large errors.

```
from sklearn.metrics import mean_squared_error
mse = mean_squared_error(y_test, y_pred)
```

And is calculated as follows:

$$L(\theta, \mathbf{y}) = \frac{1}{n} \sum_{i=1}^{n} (y_i - \theta)^2$$

Where  $y_i$  is the given output and  $\theta$  is the expected value.

#### 7.13.2 RMSE (Root Mean Squared Error)

Typical for regression problems, higher weight given to large errors. Or simply Euclidean norm, commonly noted  $\|\mathbf{x}\|$ .

$$RMSE(X,h) = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (h(x^{(i)}) - y^{(i)})^2}$$

Where m is the number of instances (e.g. 2000 cars data for estimating car values),  $x^{(i)}$  is the feature vector of the  $i^{th}$  instance of the dataset and  $y^{(i)}$  the expected output.

Feature vector is written like mathematical vector, indeed is vertical like this:

$$\mathbf{x^{(i)}} = \begin{pmatrix} 118.29\\ 33.91\\ 1833.29 \end{pmatrix}$$

As well as expected output is (e.g):

$$y^{(i)} = 156$$

The X matrix is the matrix of all feature vectors

$$X = \begin{pmatrix} (x^{(1)})^T \\ (x^{(2)})^T \\ \vdots \\ (x^{(n)})^T \end{pmatrix}$$

And finally, h being the prediction function, indeed the output of  $h(x^{(i)})$  is noted  $\hat{y}^{(i)}$ .

```
from sklearn.metrics import mean_squared_error
mse = mean_squared_error(y_test, y_pred)
rmse = np.sqrt(mse)
```

### 7.13.3 MAE (Mean Absolute Error)

Less sensitive to unusually large errors.

Also called average absolute deviation. Less dependencies to outliers. Nicknamed Manhattan error, gives sum of errors but doesn't indicate if over or under fitting.

$$MAE(\theta, \mathbf{y}) = \frac{1}{n} \sum_{i=1}^{n} |y_i - \theta|$$

Where  $\theta$  is the expected value and  $y_i$  is the ith output given by the model in a sample of n predictions. The minimizing value is the median.

#### 7.13.4 Huber loss

$$L(y, \hat{y}) = \begin{cases} \frac{1}{2}(y - \hat{y})^2 & \text{for } |y - \hat{y}| \le \delta \\ \delta(|y - \hat{y}| - \frac{1}{2}\delta) & \text{otherwise} \end{cases}$$

Where y is the true value,  $\hat{y}$  is the predicted value delta is the threshold

#### 7.14 Distance Measurement

Calculations for cost functions are detailled in 7.12.9, distance measurement is used for cost functions/metrics calculations but various exist.

#### 7.14.1 Euclidean

#### 7.14.2 Manhattan

#### 7.14.3 Minkowski

### 7.14.4 Cosine similarity

Mathematical mean to compute the similarity between pairs of vectors (in machine learning refers to feature vector). In the example of a sentence, works by computing one more dimension per type of word and is not impacted by the length of a line but the position between different vectors. More similarity implies **closer to 1**. Basically it is a transformation of the scalar product of two vectors:

$$A.B = \parallel A \parallel \parallel B \parallel cos(\theta)$$

And then isolate  $\theta$ 

$$\frac{A.B}{\parallel A \parallel \parallel B \parallel} = cos(\theta)$$

Now it is only needed to isolate  $\theta$  by using arcos

# 7.15 Optimization Algorithms

Calculations for cost functions are detailled in 7.12.9

# 7.15.1 GD (Gradient Descent)

Work by (repeats the following until sufficiently optimized):

- intializing (guess parameters)
- ♦ calculate gradient of cost function
- ♦ update parameters

Step size can be defined in hyperparameters as "learning rate" (equilibrium between to much time to get trained and too fast to adapt). Other issues encountered with GD are plateau and local minimum which might lead GD to stop somewhere where minimum was not reached.

Fortunately MSE is a convex function (a function is convex is taking two random points on the graph, the segment joining both points is over the graph). Indeed, approaching global minimum can be guaranteed. Default is converging to local minimum instead of global minimum, both can be done. Types of GD:

• Batch:

$$\frac{\partial}{\partial \theta_j} MSE(\boldsymbol{\theta}) = \frac{2}{m} \sum_{i=1}^m (\boldsymbol{\theta}^T \boldsymbol{x}^i - y^i) x_j^{(i)}$$

As a reminder but  $\theta_j$  is the  $j^{th}$  parameter.

• Stochastic (SGD): Random step size but if iterates a lot will never converge so step size is reduced over time.

Function defining learning rate based on number of iterations is called the "learning schedule".

• Mini-Batch:

# 7.16 Evaluation Metrics

Details on evaluation metrics at section 7.12.10

### 7.16.1 Confusion matrix

- TP (True Positive): predicted positive, is positive
- FP (False Positive): predicts positive, not positive
- TN (True Negative): predicts negative, is negative
- FN (False Negative): predcits negative, not negative

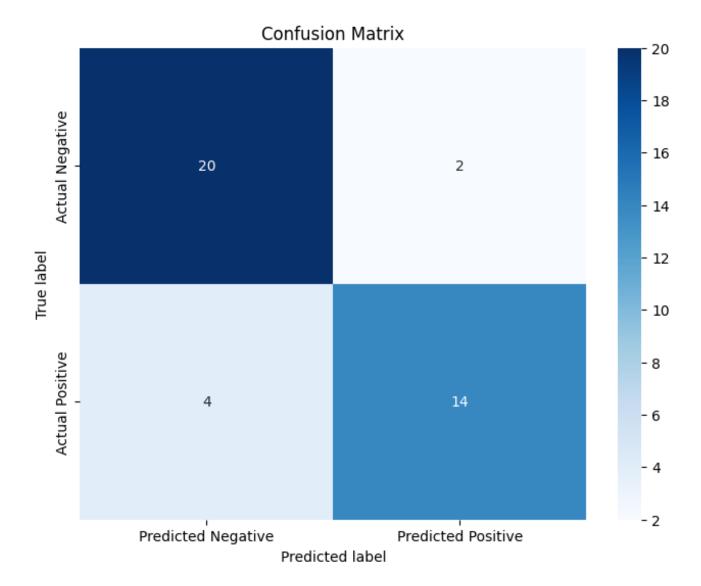


Figure 16: Signal count

Newly supported confusion matrix method:

### 7.16.2 Accuracy

Ratio of correct predictions amongst all predictions made.

$$Accuracy = \frac{TP + TN}{Total}$$

# 7.16.3 Precision (sensitivity)

Positive instances amongst predicted positive instances.

$$Precision = \frac{TP}{ActualResults} = \frac{TP}{TP + FP}$$

### 7.16.4 Recall (specificity)

$$Recall = \frac{TP}{PredictiveResults} = \frac{TP}{TP + FN}$$

In other words TP + FN are all the positive cases, predicted or not.

```
from sklearn.metrics import recall_score
score = recall_score(y_test, y_pred, pos_label=0)
```

Where pos\_label represents the positive label or the label for which we want to know the recall.

#### 7.16.5 F1

Optimal scoring at 1

$$F1 = 2*\frac{precision*recall}{precision+recall}$$

### 7.16.6 Classification report

Provides an overview of precision/recall/f1-score/support

```
import numpy as np
from sklearn.datasets import load_iris
from sklearn.model_selection import train_test_split
from sklearn.ensemble import RandomForestClassifier
from sklearn.metrics import classification_report
# Load the Iris dataset
data = load_iris()
X = data.data
y = data.target
# Split the dataset into training and testing sets
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3,
   random_state=42)
# Train a Random Forest Classifier
classifier = RandomForestClassifier(n_estimators=100, random_state=42)
classifier.fit(X_train, y_train)
# Make Predictions
y_pred = classifier.predict(X_test)
# Generate the Classification Report
report = classification_report(y_test, y_pred, target_names=data.target_names)
print(report)
```

	precision	recall	f1-score	support
setosa	1.00	1.00	1.00	19
versicolor	1.00	1.00	1.00	13
virginica	1.00	1.00	1.00	13
accuracy			1.00	45
macro avg	1.00	1.00	1.00	45
weighted avg	1.00	1.00	1.00	45

Figure 17: Classification report

### **7.16.7** R-squared

Commonly used **for regression problems**, provides an indication of the "goodness of fit", between 0 and 1 (for not fit or perfect fit)

```
from sklearn.metrics import r2_score
r2_score(y_val_test, y_pred_test)
```

And is calculated with:

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$$

With  $SS_{res} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$ , where  $y_i$  the actual value and  $\hat{y}_i$  is the predicted one, we also have and  $SS_{tot} = \sum_{i=1}^{n} (y_i - \bar{y})^2$  where  $y_i$  is the actual value and  $\bar{y}$  the mean of actual values.

### 7.16.8 adjusted R-squared

 $Adj - R^2$  is calculated by:

$$R_{adj}^2 = 1 - \frac{(1 - R^2)(n - 1)}{n - k - 1}$$

Where n is the number of observations and k the number of predictors. R-squared close to 1 means a perfect fit.

### 7.16.9 ROC curve (Receiver Operating Characteristic curve)

Plots the TPR (True Positive Rate) against the FPR (False Positive Rate). Model is **most accurate when curves arches toward upper-left corner**.

# 7.16.10 AUC (Area Under Curve)

Used with 7.16.9, varies between 0 and 1.

#### 7.16.11 OOB (Out-of-Bag)

Specifically used for Random Forests and related algorithms. Technique to estimate the performance of a model without the need for an additional validation set. Often used for random forests.

```
score = fitted_model.oob_score_
```

### 7.16.12 mAP (mean Average Precision)

the actual performance does), but it often contains an elbow.

#### 7.16.13 K-Means scoring

Inertia:

Inertia not good metric to find optimal k (number of clusters) because it decreases at each iteration (whatever

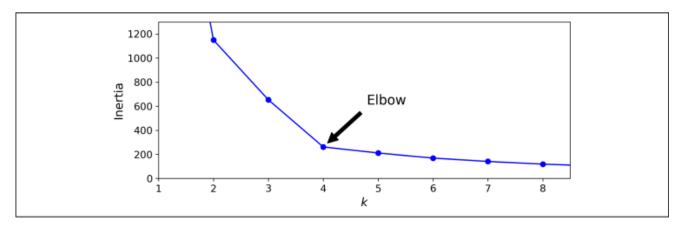


Figure 18: KMeans inertia elbow

Silhouette:

```
from sklearn.metrics import silhouette_score
silhouette_score(X, kmeans.labels_), a is the mean intra-cluster distance
and b is the mean near-cluster distance, varies between [-1:1]
```

# 7.17 Overfitting

May detect patterns where there is none e.g. country's name in data might consider the name with letter 'w' in the name to be richer without real causation. Solving overfitting can be done this in multiple ways through what is called regularization:

- ♦ simplify model (select less features)
- ♦ gather more data
- ♦ reduce noise in training data (remove outliers, fix errors, ...)

#### 7.17.1 Regularization

# 7.18 Regularization Algorithms

Regularization itself is detailed in 7.17.1

# 7.18.1 Lasso (L1)

$$CostFunction = RSS + \lambda * \sum_{j=1}^{p} |\beta_j|$$

Where the larger the  $\lambda$  the more parameters tend to approach 0. A  $\lambda$  value of 0 produces a basic linear regression. Only creates high weights (low weights are turned into 0 thus creating a sparse matrix).

Import: import module

```
from sklearn import linear_model
```

Initiate model:

```
lasso = linear_model.Lasso(alpha=0.1)
```

Fit:

```
lasso.fit(X, Y)
```

Predict:

```
prediction = lasso.predict(X)
```

# 7.18.2 Ridge (L2)

$$CostFunction = RSS + \lambda * \sum_{j=1}^{p} \beta_j^2$$

Import: import module

```
from sklearn.linear_model import Ridge
```

Initiate model:

```
ridge = linear_model.Ridge(alpha=10000)
```

Fit:

```
ridge.fit(X, Y)
```

Predict:

```
prediction = ridge.predict(X)
```

#### 7.18.3 ElasticNet

Combination of Ridge and Lasso regression.

Import:

```
from sklearn.linear_model import ElasticNet
```

Initiate model:

```
elastic_net = ElasticNet(alpha=0.1, l1_ratio=0.5)
```

Fit:

```
elastic_net.fit(X, y)
```

Predict:

```
elastic_net.predict(X)
```

# 7.19 Hyperparameters Tuning

### 7.19.1 Grid search

Try a set of parameters and keep bets results, to apply using scikit-learn, refer to 7.35.2.

### 7.19.2 Random search

Try random set of hyperparameters.

#### 7.19.3 Bayesian optimization

Train model as few times as possible as it is costly. By making use of a **surrogate function** surrogate/objective functions?

#### 7.19.4 Genetic algorithms

Highest performing is kept. Based on Darwin's theory where the fittest survives. We thus need to define a **fitness** function

# 7.20 Distributed Training

# 7.20.1 Map reduce

To parallelize training. spark?

# 7.21 Feature Engineering

Process of using domain knowledge to create new features.

# 7.21.1 Feature selection

Selecting the most useful features to train on among existing features.

#### 7.21.2 Feature extraction

Feature extraction: combining features to create a more useful one (e.g. age of a car and mileage)

#### 7.21.3 Feature cross

Feature engineering technique based on the idea of creating a synthetic feature with two or more categorical features by capturing the interaction between them.

#### 7.21.4 LabelEncoder

```
from sklearn.preprocessing import LabelEncoder
encoder = LabelEncoder()
df['Gender'] = encoder.fit_transform(df['Gender'])
```





(b) After OHE

Figure 19: Comparison of images before and after OHE

But can also be applied to a list.

```
from sklearn.preprocessing import LabelEncoder

le = LabelEncoder()
labels = ['cat', 'dog', 'bird', 'bird', 'dog', 'cat']
labels = np.array(labels)
encoded_labels = le.fit_transform(labels) # => array([1, 2, 0, 0, 2, 1])
```

#### 7.21.5 One-Hot encoding

Represent categorical data in numerical format (e.g. small is 0 and tall is 1). Used over LabelEncoding (7.21.4) because gives equal weight to each value. Doesn't work for very variable data such as id because leads to sparse matrix.

#### 7.21.6 Data augmentation

Artificially increase size of the training dataset by generating realistic variants of each training instance. Making in a very regularization technique. By realistic it is meant that augmented data (images/...) should not be distinguished from original data.

#### 7.21.7 Dimensionality reduction

The main purpose of DR is to simplify the data for computing without losing much information, to speed up training, this can also remove noise and redundant features, to make the model perform better or even saving space. Therefore it adds a certain level of complexity to the pipeline and turns the dataset into more of a blackbox. Dimensionality Reduction (DR) models are in fact unsupervised models. Main problem with dimensionality reduction is the fact that it makes data less easily interpretable.

Curse of dimensionality each instance of training dataset contains sometimes millions of features, thus making it much harder to find a good solution, dropping features or changing them might be a good solution to solve this problem (e.g. sides of picture are always white, dropping these pixels might be a good solution, other solution to calculate average of two neighbor pixels)

Manifold learning 2D or higher dimensional figure that can be twisted into a higher dimension-dimensional space. Relies therefore on the manifold hypothesis that states that most real-world high-dimensional datasets lie close to a lower-dimension space (e.g. MNIST dataset most letters are connected, of black color, ...).

# Variance preserved after scaling

```
# Truncated svd to save as much data as possible
from sklearn.decomposition import TruncatedSVD

ncomps = 4
svd = TruncatedSVD(n_components=ncomps)
svd_fit = svd.fit(dataset)
Y_pred = svd.fit_transform(dataset)
ax = pd.Series(svd_fit.explained_variance_ratio_.cumsum()).plot(kind='line', figsize = (10, 3))
ax.set_xlabel("Eigenvalues")
ax.set_ylabel("Percentage Explained")
print('Variance preserved by first 5 components == {:.2%}'.format(svd_fit.explained_variance_ratio_.cumsum()[-1]))
```

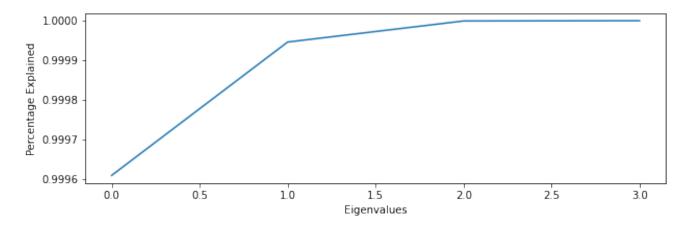


Figure 20: Variance preserved with number of features

### 7.21.8 Data scaling

One of the most important transformations, ML algorithms usually don't perform well when the numerical attributes have very different scales. Most parametric models require parametrization to avoid giving weights to certain features but non-parametric ones do not require such thing.

### 7.22 Dimensionality Reduction Algorithms

Dimensionality reduction itself is detailed in section 7.21.7

#### 7.22.1 Principal Component Analysis (PCA)

DR (Dimensionality Reduction) model. Reduce dimensionality of dataset with many variables while retainin as much variance as possible. PCA finds new variables through a linear combination called PC (Principal Components), those are orthogonal (or independent). The end goal is to find the directions that preserve the most variance, the two most common approaches are Eigen Decomposition and SVD (referred in the subsection about

dimensionality reduction) PCA (Principal Component Analysis): Works by projecting data on the hyperplane that lies closest to the data. Decision trees are sensitive to rotation, PCA can help solving this

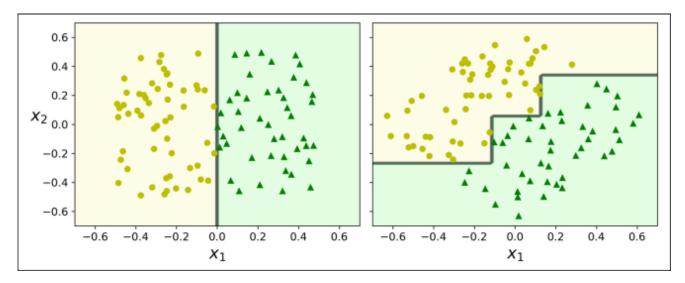


Figure 21: Same dataset rotated 45° shows high sensitivity

Major issue with PCA is not being able to apply to nonlinear data.

```
from sklearn.decomposition import PCA
pca = PCA(n_components = 2)
X2D = pca.fit_transform(X)

pca.explained_variance_ratio_
```

hyperparameters explained: svd\_solver="randomized", n\_components = output dimension

### 7.22.2 IPCA (Incremental PCA)

```
from sklearn.decomposition import IncrementalPCA
n_batches = 100
inc_pca = IncrementalPCA(n_components=154)
for X_batch in np.array_split(X_train, n_batches):
    inc_pca.partial_fit(X_batch)
X_reduced = inc_pca.transform(X_train)
```

# 7.22.3 t-distributed Stochastic Neighbor Embedding (t-SNE)

#### 7.22.4 kernel PCA

Solves major issue with PCA by handling nonlinearity.

```
from sklearn.decomposition import KernelPCA
rbf_pca = KernelPCA(n_components = 2, kernel="rbf", gamma=0.04)
X_reduced = rbf_pca.fit_transform(X)
```

hyperparameters explained: svd, n\_components, kernel, gamma

#### 7.22.5 sparse PCA

nonlinear data?

## 7.22.6 ED (Eigen Decomposition)

Steps are as follow: covariance matrix is created for the features then eigenvectors of the covariance matrix are created (directions of maximum variance), eigeinvalues are finally created, those define the magnitude of the principal components.

## 7.22.7 SVD (Singular Value Decomposition)

Goal is to get a lower rank matrix.

$$A = U \Sigma V^T$$

Where A is a mxn matrix, U is a mxn orthogonal matrix and  $\Sigma$  is a mxn diagonal matrix. And V is also an orthogonal matrix.

## 7.22.8 TSVD (Truncated SVD)

SVD but only for large singular values.

### 7.22.9 LLE (Locally Linear Embedding)

```
from sklearn.manifold import LocallyLinearEmbedding
lle = LocallyLinearEmbedding(n_components=2, n_neighbors=10)
X_reduced = lle.fit_transform(X)
```

#### 7.22.10 K-Means

# verbose?

# 7.22.11 t-SNE (t-Distributed Stochastic Neighbor Embedding)

Reduces dimension by modeling the probability of neighbors (set of points around a given point) around each point.

```
from sklearn.manifold import TSNE
X_valid_compressed = stacked_encoder.predict(X_valid)
tsne = TSNE()
X_valid_2D = tsne.fit_transform(X_valid_compressed)
```

Scatter classification:

```
plt.scatter(X_valid_2D[:, 0], X_valid_2D[:, 1], c=y_valid, s=10, cmap="tab10")
```

## 7.22.12 LLE (Locally Linear Embedding)

Visualization and dimensionality reduction

### 7.22.13 Random projection

## 7.22.14 Manifold learning

#### 7.22.15 Isomap

#### 7.22.16 Other unsupervised algorithms

The following algorithms will be mentioned as they are unsupservised classification algorithms but won't be detailed here but rather in the section dedicated to unsupervised algorithms.

Hierarchical clustering, DBSCAN

## 7.23 Data Scaling Algorithms

Further details on data transformation in section 7.21.8

## 7.23.1 Standard scaling

Consists of turning dataset into dataset with mean of 0 and variance of 1.

$$standardization = \frac{x - mean}{\sigma}$$

Where  $\sigma$  represents the standard deviation, resulting distribution has unit variance.

```
from sklearn.preprocessing import StandardScaler
scaler = StandardScaler().fit(x)
StandardisedX = pd.DataFrame(scaler.fit_transform(X))
```

#### 7.23.2 MinMaxScaler

```
from sklearn.preprocessing import MinMaxScaler
scaler = MinMaxScaler(feature_range=(0, 1))
rescaledX = pd.DataFrame(scaler.fit_transform(X))
```

Between 0 and 1

$$s = \frac{x - x_{min}}{x_{max} - x_{min}}$$

Between -1 and 1

$$s = \frac{2x - x_{max} - x_{min}}{x_{max} - x_{min}}$$

## 7.23.3 Clipping

To reduce outlier effect, consider MinMaxScaling and use -1 or 1 for outliers.

#### 7.23.4 Z-score normalization

$$s = \frac{x - \bar{x}}{\sigma}$$

Where  $\sigma$  is the standard deviation and  $\bar{x}$  is the mean. Lies between -1 and 1 the majority of the time but not always.

#### 7.23.5 LRN (Local Response Normalization)

Based on observations made on biological neurons, the most strongly activated neuron inhibit other neurons located at the same position in neighboring feature maps.

## 7.24 Feature Selection Algorithms

Reference on feature selection at 7.21.1

## 7.24.1 Chi-squared $\chi^2$

Used to find the best features.

```
from sklearn.feature_selection import chi2
selector = SelectKBest(score_func=chi2, k=2)
X_new = selector.fit_transform(X, y)
```

Where k is the number of features to keep.

## 7.25 Embedding

#### 7.25.1 Word embedding

Turn words into numbers and get similar words to have similar numbers. Same word with different meanings is assigned different numbers (e.g. "it's great we could see each other" or "my phone is broken, great"). Neurons weights are vector components.

## 7.25.2 Word2Vec

Word embedding (7.25.1) by making use of neural networks. Introduced by Tomas Mikolov et al. at Google in 2013.

- ♦ Continuous Bag of Words (CBOW): Predicts the current word given its context (surrounding words). It aims to maximize the probability of the target word given the context words. (more context on BOW in section 7.33.7)
- ♦ Skip-gram: Predicts surrounding words given the current word. It aims to maximize the probability of context words given the target word.

Negative sampling: ignore unconsidered weights.

# 7.26 Cascade

Machine learning task where output of one model is input of the other.

## 7.27 Rebalancing

Used when only one class is represented in a dataset (consider fraud vs non fraud with 1% fraud and 99% non-fraud). Solutions are:

### 7.27.1 Downsampling

Throw away instances of majority class. Models might work with 75/25 ratio but some require a net 50/50 ratio.

#### 7.27.2 Upsampling

Generating synthetic examples. **SMOTE?** 

## 7.28 Model Explainability

## 7.28.1 Attribution values

Tells how much features influence predictions.

## 7.29 Shap

Created for model explainability.

learn more about it https://github.com/shap/shap?tab=readme-ov-file

## 7.29.1 Feature importance

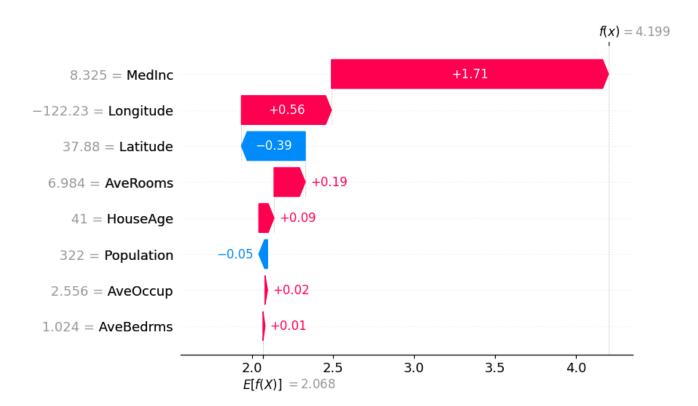


Figure 22: Feature importance in house price prediction

## 7.30 TPOT

Used for genetic algorithm

```
import numpy as np
from sklearn.model_selection import train_test_split
from sklearn.datasets import load_digits
from tpot import TPOTClassifier
# Load dataset
digits = load_digits()
X, y = digits.data, digits.target
# Split the dataset into training and testing sets
X_{train}, X_{test}, y_{train}, y_{test} = train_test_split(X, y, test_size=0.2,
   random_state=42)
# Initialize TPOT
tpot = TPOTClassifier(generations=5, population_size=20, verbosity=2, random_state
# Fit the TPOT classifier on the training data
tpot.fit(X_train, y_train)
# Evaluate the model on the test data
print(tpot.score(X_test, y_test))
# Export the optimized pipeline
tpot.export('tpot_optimized_pipeline.py')
```

## 7.31 Resilient Serving

Little human intervention required.

## 7.32 Distributed Learning

## 7.32.1 Synchronous training

Workers train on different slices of input data in parallel and values are aggregated.

## 7.32.2 Asynchronous training

Workers train on different slices of input data independently and weights are updated asynchronously.

## 7.33 Natural Language Processing

## 7.33.1 Vectorization

Converting text into numbers.

### 7.33.2 CountVectorizer

Dictionary from the corpus of words in the training text and generates a matrix of word counts.

## 7.33.3 Hashing Vectorizer

Uses word hashes to be more memory efficient

#### 7.33.4 TfidVectorizer

Dictionary containing frequency of words (term frequency-inverse document frequency (TFIDF)).

# 7.33.5 Stemming

(e.g. equals = equal when text has been stemmed)

#### 7.33.6 Lemmatizing

#### 7.33.7 Bag of words

Based on normalized text (e.g. awesome = AweSoME = Awesome).

## 7.34 Recommender Systems

## 7.34.1 Popularity based systems

Presents what is popular right now.

### 7.34.2 Collaborative systems

Recommendations based on what others have selected.

#### 7.34.3 Content-based systems

If you liked this content you might like this other content.

#### 7.35 Scikit-learn

Design principles are well built to use their API. Those are the following: Consistency:

- ♦ Estimators: estimate parameters based on dataset (e.g. imputer), uses fit() method
- ♦ Transformers: returns a transformed dataset, by relying on transformed dataset
- Predictors: predicts using the predict() method and has a score() method to evalute it's performance, in other words it's an algorithm

Inspection: estimators's hyperparameters can be accessed through public instances
Nonproliferation of classes: datasets are numpy classes or scipy matrices instead of homemade classes
Composition: reuse building blocks as much as possible, by creating pipeline
Sensible defaults: default values for most parameters, making it quick to create a baseline working system

get model fit scores and so on (calculate own error function, ...)

## 7.35.1 .train\_test\_split()

Train and test set to verify that the model learned and generalizes the data well.

```
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,
    random_state=42)
```

### 7.35.2 GridSearchCV (GridSearch Cross Validation)

```
model = <model_name>
param_grid = {'fit_intercept': [True, False], 'n_estimators': [100, 150, 200]}
grid_search = GridSearchCV(model, param_grid, cv=3, scoring='r2')
grid_search.fit(X_train, y_train)
print(grid_search.best_params_)
```

Where scoring is the metric to be optimized. Not a good idea on big datasets as it might lead to a **combinatorial** explosion.

#### 7.35.3 KFold

```
from sklearn.model_selection import KFold
from sklearn.model_selection import train_test_split
from sklearn.datasets import load_iris
from sklearn.svm import SVC
# Load a sample dataset (for demonstration purposes)
iris = load_iris()
X, y = iris.data, iris.target
# Initialize the KFold cross-validation object
kf = KFold(n_splits=5, shuffle=True, random_state=42) # Define number of splits (k)
   , shuffle data, and random state
# Loop over each fold and train/test the model
for fold, (train_index, test_index) in enumerate(kf.split(X)):
    print(f'Fold {fold+1}')
    X_train, X_test = X[train_index], X[test_index] # Split data into train and
       test sets
    y_train, y_test = y[train_index], y[test_index]
    # Initialize and train your model (for demonstration purposes, using Support
       Vector Classifier)
    model = SVC(kernel='linear')
    model.fit(X_train, y_train)
    # Evaluate the model on the test set
    accuracy = model.score(X_test, y_test)
    print(f'Accuracy: {accuracy:.4f}\n')
```

#### 7.35.4 .score()

### 7.35.5 cross\_val\_score()

Uses  $R^2$  score, detailled in 7.16.7.

#### 7.35.6 .summary()

```
model.summary()
```

```
Layer (type)
                              Output Shape
                                                         Param #
                              (None, 784)
 flatten 3 (Flatten)
 dense 10 (Dense)
                              (None, 300)
                                                        235500
 dense 11 (Dense)
                              (None, 100)
                                                         30100
 dense 12 (Dense)
                              (None, 10)
                                                         1010
Total params: 266610 (1.02 MB)
Trainable params: 266610 (1.02 MB)
Non-trainable params: 0 (0.00 Byte)
```

Figure 23: Model summary after compiling

## 7.35.7 accuracy\_score(actual, predicted)

```
accuracy_train = accuracy_score(Y_train, knn.predict(X_train))
accuracy_test = accuracy_score(Y_test, knn.predict(X_test))
```

## 7.35.8 make\_classification()

Create a classification dataset.

```
from sklearn.datasets import make_classification
# Generate pseudo-random dataset
X, y = make_classification(n_samples=25, n_features=2, n_informative=2,
                       n_redundant=0, n_classes=2, random_state=42)
print(X)
print("========"")
print(y)
[[ 1.51892967 2.06893996]
 [-0.63261556 -1.36951734]
[-1.38312222 1.68462133]
 [-1.12450988 -0.75541669]
 [-1.86225266 1.80856609]
 [-2.76108267 0.87454572]
[ 0.02320915  0.76512646]
[ 0.45424212 -2.71686178]
[-1.40610823 1.39801155]
[-2.78712335 0.70177458]
[ 0.0213132 -0.02560141]]
______
```

## 7.35.9 Feature importance bar chart

```
model = RandomForestRegressor(n_estimators= 200,n_jobs=-1)
model.fit(X_train,Y_train)
#use inbuilt class feature_importances of tree based classifiers
feat_importances = pd.Series(model.feature_importances_, index=X.columns)
feat_importances.nlargest(10).plot(kind='barh', color='r')
```

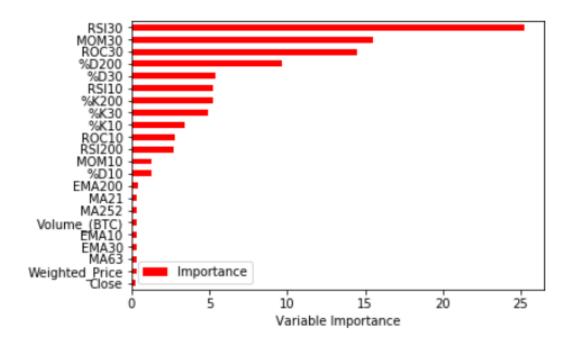


Figure 24: Different graphs stacked on matplotlib

## 7.36 TensorFlow

# **7.37** Keras

High-level TensorFlow api for building neural networks.

# 7.37.1 Convolutional layer

```
Conv2D(filters=16, kernel_size=3, activation='relu', input_shape=(28,28,3))
```

Input images divided into 3x3 chunks before passing through a max pooling layer.

### 7.38 Generate Random Data

#### 7.38.1 Blobs

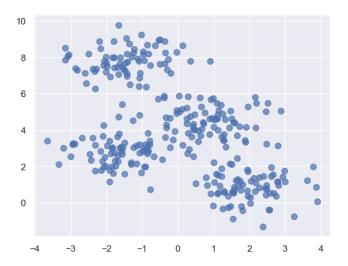


Figure 25: Blobs scatter plot

# 7.38.2 Moons

```
from sklearn.datasets import make_moons
x, y = make_moons(n_samples=200, noise=0.3, random_state=42)
```

## 7.38.3 Regression

```
from sklearn.datasets import make_regression
x, y = make_regression(n_samples=300, n_features=1, noise=20, random_state=0)
```

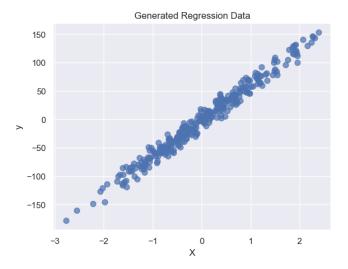


Figure 26: Regression data from sklearn

# 7.39 Linear regression

## 7.39.1 OLS (Ordinary Least Squares)

Based on assumption that target variable is a linear combination of feature values, the goal is the find a function as such:

$$f: \mathbb{R} \to \mathbb{R}, \quad y = \alpha + \beta x$$

Where this linear function is found by minimizing error which is calculated with distance to the expected value.

$$\min_{\alpha,\beta} \frac{1}{N} \sum_{n=1}^{N} (y_n - f(x_n))^2$$

In this expression  $y_n$  is the expected value and  $f(x_n)$  is the value of the function, to minimize the global value of the error we use the following calculation:

$$\beta^* = \frac{Cov(x, y)}{Var(x)}$$

$$\alpha^* = \bar{y} - \beta \bar{x}$$

Normal equation permits to immediatly calculate the searched terms.

$$\hat{\theta} = (X^T X)^{-1}$$

 $\hat{\theta}$  is the value that minimizes cost function and y vector of target values.

Import: import module

import sklearn.linear\_model

Fit: X are features y is expected value

```
model = sklearn.linear_model.LinearRegression()
model.fit(X, y)
```

Predict: input X values outputs y

```
model.predict(X_new)
```

## RSS (Sum of Squared Residuals)

$$RSS = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

# 7.40 KNN (K-Nearest Neighbors)

Number of neighbors too low overfits and too high underfits the data.

#### 7.40.1 Regressor

```
# Modules import
import sklearn.neighbors
from sklearn.datasets import make_regression
import matplotlib.pyplot as plt

# Generate regression data
x, y = make_regression(n_samples=200, n_features=1, noise=20, random_state=0)

# Display data
model = sklearn.neighbors.KNeighborsRegressor(n_neighbors=3)
model.fit(x, y)
predictions = model.predict(x)
plt.scatter(x, predictions, alpha=0.7, c="blue")
plt.scatter(x, y, alpha=0.5, c="red")
```

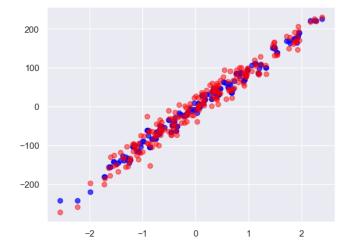


Figure 27: KNN regression fitting

#### 7.40.2 Classifier

```
# Modules import
from sklearn.neighbors import KNeighborsClassifier
model = KNeighborsClassifier(n_neighbors=10) # n_neighbors default to 5
model.fit(x_train, y_train)
model.score(x_test, y_test) # => accuracy: correct predictions/total predictions
model.predict([[15.2, 3.5, 1.2, 6.51]])
```

### 7.41 K-means

#### 7.41.1 Description

Unsupervised learning algorithm, requires to provide number of clusters desired in predictions.

#### 7.41.2 Fit clusters

```
from sklearn.cluster import KMeans
model = KMeans(n_clusters=4, random_state=0)
model.fit(points)
predicted_cluster_indexes = model.predict(points)
plt.scatter(x, y, c=predicted_cluster_indexes, s=50, alpha=0.7, cmap='viridis')
```

#### 7.41.3 Cluster centers

```
centers = model.cluster_centers_
plt.scatter(centers[:, 0], centers[:, 1], c='red', s=100)
```

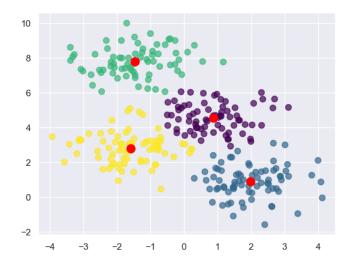


Figure 28: Clusters and respective centroids

#### 7.41.4 Elbow method for number of clusters

```
inertias = []
for i in range(1, 10):
    kmeans = KMeans(n_clusters=i, random_state=0)
    kmeans.fit(points)
    inertias.append(kmeans.inertia_)
plt.plot(range(1, 10), inertias)
plt.xlabel('Number of clusters')
plt.ylabel('Inertia')
```

#### 7.41.5 Inertia calculation

Also known as the within-cluster sum of squares, measures how close points are to their centroid.

$$\sum_{i=1}^{k} \sum_{x \in C_i} (\parallel x - \mu_i \parallel)^2$$

For each cluster, take every "point" inside and calculate the distance to the centroid of cluster  $C_i$ .

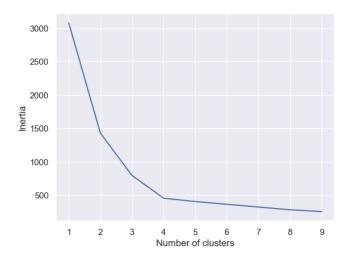


Figure 29: Elbow method describing inertia with number of clusters

# 7.42 RNC (Radius Neighbors Classifier)

### 7.42.1 Modle usecase

```
from sklearn.neighbors import RadiusNeighborsClassifier
model = RadiusNeighborsClassifier(radius=1.0)
model.fit(x_train, y_train)
model.score(x_test, y_test)
```

#### 7.42.2 Predict class probability

## 7.43 Agglomerative Clustering

## 7.43.1 Silhouette score

Optimal number of clusters maximizes the silhouette score.

```
cluster_range = range(2, 10)
silhouette_scores = []

for i in cluster_range:
    model = AgglomerativeClustering(n_clusters=i)
    cluster_labels = model.fit_predict(points) # Fit and predict in one step
    silhouette_avg = silhouette_score(points, cluster_labels)
    silhouette_scores.append(silhouette_avg)

plt.plot(range(2, 10), silhouette_scores)
```

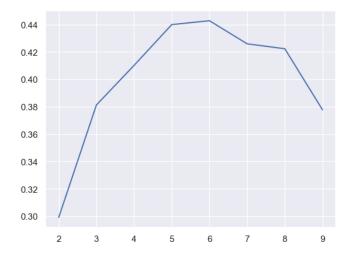


Figure 30: Silhouette score

## 7.44 DBSCAN

#### 7.44.1 Description

Unsupervised learning model for classification that doesn't require number of clusters.

# 7.44.2 Fit

```
from sklearn.cluster import DBSCAN

model = DBSCAN(eps=0.01, min_samples=50)
predicted_cluster_indexes = model.fit_predict(points)
plt.scatter(x, y, c=predicted_cluster_indexes, s=50, alpha=0.7, cmap='viridis')
```

## 7.45 Decision trees

Useful both for classification and regression tasks. Root node (depth 0) set binary criterion to classify (e.g. sepal length over 2.45cm for a flower) and proceeds to binary classification until there is no more child nodes to make predictions about. Unlike linear regression and other models, no assumptions are made by the human side but rather automatically by the model adapting itself to the data (nonparametric model, despite having hyperparameters), thus producing either a linear or nonlinear model which might be a cause of overfitting, this is partly due to the fact that the model has no pre-determined number of parameters so its degree of freedom is

unlimited (unlike linear regression, therefore a good balance must be found, setting the depth is an example of regularization that can be easily implemented). Very little data preparation is required and feature scaling is not required at all. Optimal depth is calculated with number of leaves (suppose m leaves), thus the optimal depth is calculated by  $log_2(m)^2$  rounded up. Computational complexity is O(nxmlog(m)). To work the model counts number of instances of similar feature have the same attributes (e.g. Iris Virginica has a sepal length of 2.45cm) then counts number of other instances with same feature, no outlier means gini = 0 (e.g.  $1 - (0/54)^2 - (43/54)^2 \dots$  is used to establish gini impurity). Probability is based on number of instances with current feature. When doing a regression with a decision tree output won't be continuous because leaf node (last one) will be the regression value, in a similar classification problem, that node will be the class.

- 7.45.1 DTR (Decision Tree Regressor)
- 7.45.2 DTC (Decision Tree Classifier)

### 7.45.3 CART (Classification and Regression Trees)

Decision trees both for classification as well as for regression are based on this algorithm to make predictions. Scikit-learn uses binary trees because it makes use of the CART algorithm, thus nonleaf nodes always have two children. The CART algorithm minimizes the following cost function:

$$J = \frac{m_{left}}{m}G_{left} + \frac{m_{right}}{m}G_{right}$$

Where J is the cost function, G is the gini measure of impurity (discussed in: 7.45.4) and  $m_{left/right}$  is the number of instances respectively of the left and right subsets, m indeed is the total number of instances.

#### 7.45.4 Gini measure of impurity

$$G = 1 - \sum_{i=1}^{m} p_i^2$$

Where  $p_i$  is the probability/proportion of a given class, consider the following example: class A, B and C with respectively 50%, 30% and 20% as frequency. Thus gini can be calculated by:  $1 - 0.5^2 - 0.3^2 - 0.2^2 = 0.62$ .

## ID3 comparison to CART (because ID3 can have more than two children)

Chi-squared test Is used to estimate the probability that improvement in predictions made by tree leafs are based on statistical chances not real improvements, thus we can remove them. Probability here is qualified as p-value, threshold is set (usually around 5%) and if values goes over then node is considered unnecessary and children are deleted. Pruning continues until all unnecessary nodes have been pruned.

### Gini Impurity

$$G_i = 1 - \sum_{k=1}^{n} p_{i,k}^2$$

Where  $p_{i,k}$  is the number of instances of class k among training instances of node i. Gini is the default binary classifier but Entropy can also be used, both often lead to similar results but Gini is slightly faster, therefore entropy produces more balanced trees.

**Entropy** Same idea as in thermodynamics, where order is almost perfect when entropy approaches 0. Entropy function is evaluated as such:

$$H_i = \sum_{k=1}^n p_{i,k} log_2(p_{i,k})$$

Import: import module

```
from sklearn.tree import DecisionTreeClassifier
```

Initiate model:

```
tree = DecisionTreeClassifier(max_depth=1)
```

Fit:

```
tree.fit(X_train, y_train)
```

Predict:

```
tree.predict([X])
```

Predict proba:

```
tree.predict_proba(X)
```

```
from sklearn.datasets import make_moons
from sklearn.model_selection import train_test_split
{\tt from \ sklearn.svm \ import \ SVC}
import numpy as np
import matplotlib.pyplot as plt
from sklearn.tree import DecisionTreeClassifier
# Generate moon dataset
X, y = make_moons(n_samples=200, noise=0.4)
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=42, test_size
   =0.2)
# Decision tree
tree = DecisionTreeClassifier(max_depth=1)
{\tt tree.fit(X\_train,\ y\_train)}
# Plotting functions
def plot_dataset(X, y, axes):
    plt.plot(X[:, 0][y==0], X[:, 1][y==0], "bs") # Plot different classes
    {\tt plt.plot(X[:, \ 0][y==1], \ X[:, \ 1][y==1], \ "g^")}
    plt.axis(axes)
    plt.grid(True, which='both')
    plt.xlabel(r"$x_1$", fontsize=20)
    plt.ylabel(r"$x_2$", fontsize=20, rotation=0)
def plot_predictions(clf, axes):
    x0s = np.linspace(axes[0], axes[1], 100)
    x1s = np.linspace(axes[2], axes[3], 100)
    x0, x1 = np.meshgrid(x0s, x1s)
    X = np.c_[x0.ravel(), x1.ravel()]
```

```
# Obtain probabilities instead of predictions
y_proba = clf.predict_proba(X)[:, 1].reshape(x0.shape)
# Plot probability estimates as a color gradient
plt.contourf(x0, x1, y_proba, cmap=plt.cm.brg, alpha=0.2)

# Plot predictions
plot_predictions(tree, [-1.5, 2.5, -1, 1.5])

# Plot dataset
plot_dataset(X_train, y_train, [-1.5, 2.5, -1, 1.5])

# Show the plot
plt.show()
```

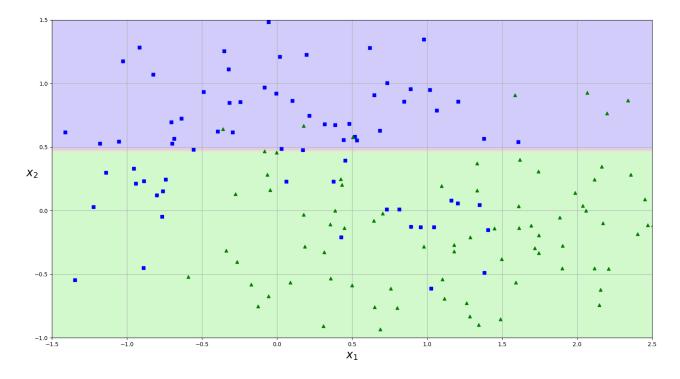
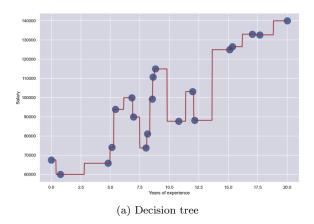


Figure 31

## 7.46 Random forest

## 7.46.1 Description

Decision trees are too much prone to overfitting thus random forests solves this problem. Number of predictors (trees) for each split is usually the square root of the total number of predictors (features). When the model makes a prediction it is the average of all the trees (each one trained on random rows of the dataframe fed as input). Because the trees are constructed independently, they can each be trained (thus random forests supports parallelized training). Prevents quite well overfitting compared to a single decision tree.



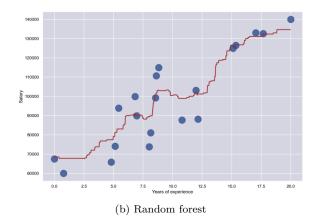


Figure 32: Comparison of decision tree versus random forest

#### 7.46.2 RandomForestRegressor

Training many Decision Trees on random features subsets and averaging out predictions.

#### 7.46.3 RandomForestClassifier

```
from sklearn.ensemble import RandomForestClassifier
forest = RandomForestClassifier(n_estimators=500, max_leaf_nodes=16, n_jobs=-1)
forest.fit(X_train, y_train)
forest.predict(X)
forest.predict_proba(X)
```

## Hyperparameters

```
n\_estimators: the number of trees 100, 200, ..., criterion: gini or entropy min\_impurity\_decrease: 0.0, 0.1, 0.2, 0.3 max\_leaf\_nodes: [16, 32, 48, 64] max\_depth = maximum number of children/branches for each tree max\_samples = maximum number of rows of the dataset that can be seen by a single tree n\_jobs = number of cpu cores for parallel processing, set to -1 to maximize
```

## 7.47 Extra trees (Extremly Randomized Trees)

Instances trained on a random sample of the training data,

```
clf = ExtraTreesClassifier(n_estimators=10, max_depth=2, random_state=0)
clf.fit(X, y)
```

Classification:

Import:

```
from sklearn.ensemble import ExtraTreesClassifier
```

Build model:

```
model = ExtraTreesClassifier()
```

Fit:

```
model.fit(X, Y)
```

Regression

Import:

```
from sklearn.ensemble import ExtraTreesRegressor
```

Build model:

```
model = ExtraTreesRegressor()
```

Fit:

```
model.fit(X, Y)
```

- 7.48 GBDT (Gradient Boosting Decision Trees)
- 7.48.1 Gradient Boosting Machines (GBM)
- 7.48.2 AdaBoost (Adaptive Boosting)
- 7.48.3 SGB (Stochastic Gradient Boosting)
- 7.49 GBR (Gradient Boosting Regressor)

```
from sklearn.ensemble import GradientBoostingRegressor
gbr = GradientBoostingRegressor(max_depth=2, n_estimators=3, learning_rate=1.0)
cross_val_score(gbr, x, y, cv=5).mean
gbr.fit(X, y)
```

Hyperparameters max\_depth=2, n\_estimators=3, learning\_rate=1.0

## 7.50 SVM (Support Vector Machines)

## 7.50.1 Description

Mainly used for classification whether linear or non-linear, regression and outlier detection. Well suited for mid-size or lower datasets. Purpose of SVM is to fit widest possible "street" between classes (thus it is called large margin classification for that reason). Adding more training instances "off the street" doesn't affect the decision boundary but the model is sensitive to feature scaling, indeed is known to be parametric. Purpose of SVM is to find compromise between largest possible street and perfectly separating both classes. Support vectors are instances located on the "street", as long as new instances are not on the support vector they won't affect the model.

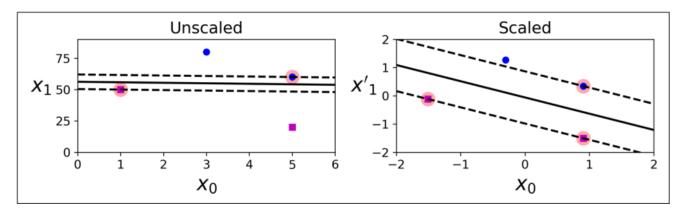


Figure 33: Sensitivity of SVM to feature scaling

To understand this sensitivity, compare vertical scale to horizontal scale (y axis vs x axis).

The output of SVM models are not probabilities but rather what class the model supposes it is. Another fact about sensitivity of this model is that small instances tend to be neglected if not scaled. However distance between decision boundary and instance can be used as confidence score.

To represent model's hyparameters  $\theta$  is commonly used, the bias term is representated using  $\theta_0$  (intercept term, value added to weighted sum of input features as a constant term allow to make predictions when all input features are zeros) and input feature weights  $\theta_1$  and  $\theta_0$ .

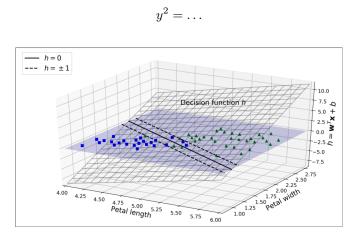


Figure 34: SVM decision function

If over the separation function 1 otherwise 0. Dashed line represents points where decision function is equal to 1 or -1 (those are parallel at equal distance to decision boundary). The purpose of training the model is to find optimal values of w and b to make the margin -1/1 as wide as possible while avoiding margin violations (hard margin) or limiting them (soft margin).

- ♦ Linear SVM Classification
- $\diamond$  Nonlinear SVM Classification To handle nonlinear data separation (because some datasets are not linearly separable) the creation of new features can be very useful, like here where a new feature is created  $x_2$  comes from  $(x_1)^2$ .

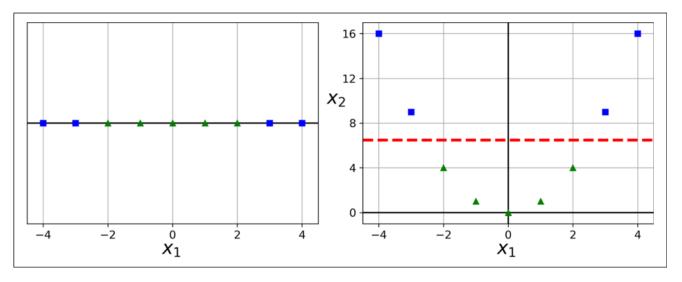


Figure 35: Linear data separation creating new features

- ♦ SVM Regression
- $\diamond$  online SVM

SVM hyperparameters:

- ♦ c: margin violation to generalize better, to avoid overfitting reduce C (regularization indeed)
- $\diamond \gamma$ : gamma? rbf?

Reducing c and  $\gamma$  can help reducing overfitting.

Import: import module

```
from sklearn.svm import SVC
```

Initiate model:

```
svc = SVC(kernel='linear', C=1.0)
```

Fit:

```
svc.fit(X_train, Y_train)
```

Predict:

```
prediction = lasso.predict(X)
```

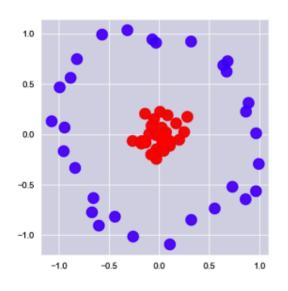
Class	Time complexity	Out-of-core support	Scaling required	Kernel trick
LinearSVC	$O(m \times n)$	No	Yes	No
SGDClassifier	$O(m \times n)$	Yes	Yes	No
SVC	$O(m^2 \times n)$ to $O(m^3 \times n)$	No	Yes	Yes

Figure 36: Computational complexity of different SVM models

## Computational Complexity

## 7.50.2 Kernel tricks

Adds dimensions to data.



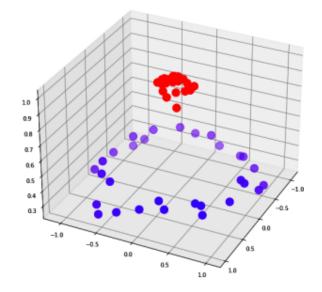


Figure 37: Kernel tricks

Always use linear kernel first (Linear SVC much faster than SVC(kernel="linear")) then switch to Gaussian RBF if dataset is not too large. Kernel trick?, different types of kernels (linear, polynomial, gaussian rbf, sigmoid)

# 7.50.3 Linear kernel

from sklearn.svm import LinearSVC

- 7.50.4 Polynomial kernel
- 7.50.5 Gaussian kernel
- 7.50.6 Gaussian RBF kernel (Gaussian Radial Basis Function kernel)

Gaussian RBF low gamma values, increasing gamma makes bell-shaped curve narrower ()

## 7.50.7 RBF kernel

Classification algorithm. Submodel of SVC.

Import: import module

```
from sklearn.svm import SVC
```

Initiate model:

```
rbf_kernel_svm_clf = Pipeline([
          ("scaler", StandardScaler()),
          ("svm_clf", SVC(kernel="rbf", gamma=5, C=1))
])
```

Fit:

```
rbf_kernel_svm_clf.fit(X, y)
```

Predict:

Predict proba:

```
from sklearn.datasets import make_classification
from sklearn.model_selection import train_test_split
from sklearn.svm import SVC
# Generate synthetic data
X, y = make_classification(n_samples=100, n_features=2, n_classes=2,
   random_state=42)
# Split data into training and testing sets
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,
   random_state=42)
# Create SVC model with RBF kernel and probability estimates
svm_rbf = SVC(kernel='rbf', gamma='auto', probability=True)
# Train the model
svm_rbf.fit(X_train, y_train)
# Obtain probability estimates for the test set
probabilities = svm_rbf.predict_proba(X_test)
print("Probability estimates for the first 5 samples in the test set:")
print(probabilities[:5])
```

```
X, y = make_moons(n_samples=20, noise=0.2, random_state=42)

rbf_kernel_svm_clf = Pipeline([
    ("scaler", StandardScaler()),
    ("svm_clf", SVC(kernel="rbf", gamma=5, C=1))
])
```

```
rbf_kernel_svm_clf.fit(X, y)
def plot_dataset(X, y, axes):
   plt.plot(X[:, 0][y==1], X[:, 1][y==1], "g^")
   plt.axis(axes)
   plt.grid(True, which='both')
   plt.xlabel(r"$x_1$", fontsize=20)
plt.ylabel(r"$x_2$", fontsize=20, rotation=0)
def plot_predictions(clf, axes):
    x0s = np.linspace(axes[0], axes[1], 100)
    x1s = np.linspace(axes[2], axes[3], 100)
   x0, x1 = np.meshgrid(x0s, x1s)
   X = np.c_[x0.ravel(), x1.ravel()]
   \mbox{\tt\#} clf.predict(X) is under that format [1 1 1 ... 1 1 1]
   y_pred = clf.predict(X).reshape(x0.shape)
   # changed format to [[1 1 0 1 0 ... 1 0]]
   y_decision = clf.decision_function(X).reshape(x0.shape)
   plt.contourf(x0, x1, y_pred, cmap=plt.cm.brg, alpha=0.2) #//# contourf?
   plt.contourf(x0, x1, y_decision, cmap=plt.cm.brg, alpha=0.1)
plot_predictions(rbf_kernel_svm_clf, [-1.5, 2.5, -1, 1.5])
plot_dataset(X, y, [-1.5, 2.5, -1, 1.5])
plt.show()
```

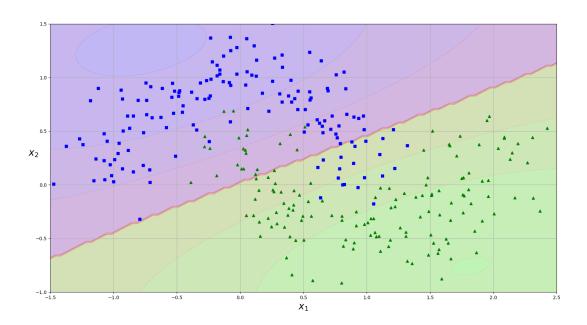


Figure 38

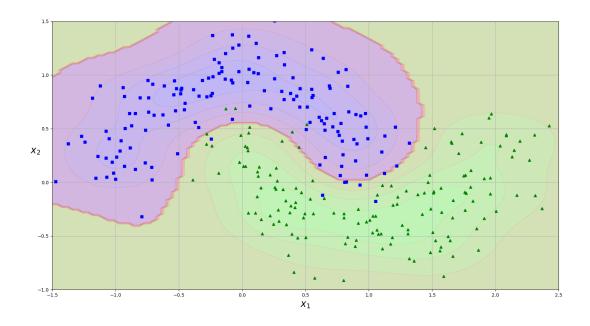


Figure 39

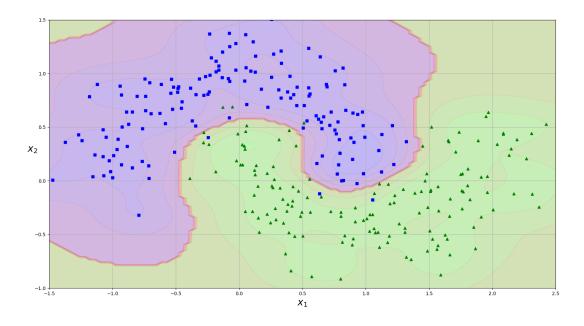


Figure 40

```
from sklearn.datasets import make_moons
from sklearn.model_selection import train_test_split
from sklearn.svm import SVC
import numpy as np
import matplotlib.pyplot as plt
from sklearn.pipeline import Pipeline
from sklearn.preprocessing import StandardScaler

# Generate moon dataset
```

```
X, y = make_moons(n_samples=200, noise=0.2, random_state=42)
# Create an SVM classifier with RBF kernel
rbf_kernel_svm_clf = Pipeline([
    ("scaler", StandardScaler()),
    ("svm_clf", SVC(kernel="rbf", gamma=5, C=1, probability=True)) # Enable
       probability estimates
])
rbf_kernel_svm_clf.fit(X, y)
# Plotting functions
def plot_dataset(X, y, axes):
    plt.plot(X[:, 0][y==0], X[:, 1][y==0], "bs") # Plot different classes
    plt.plot(X[:, 0][y==1], X[:, 1][y==1], "g^")
    plt.axis(axes)
    plt.grid(True, which='both')
    plt.xlabel(r"$x_1$", fontsize=20)
    plt.ylabel(r"$x_2$", fontsize=20, rotation=0)
def plot_predictions(clf, axes):
    x0s = np.linspace(axes[0], axes[1], 100)
    x1s = np.linspace(axes[2], axes[3], 100)
    x0, x1 = np.meshgrid(x0s, x1s)
    X = np.c_[x0.ravel(), x1.ravel()]
    \# Obtain probabilities instead of predictions
    y_proba = clf.predict_proba(X)[:, 1].reshape(x0.shape)
    # Plot probability estimates as a color gradient
    plt.contourf(x0, x1, y_proba, cmap=plt.cm.brg, alpha=0.2)
# Plot predictions
plot_predictions(rbf_kernel_svm_clf, [-1.5, 2.5, -1, 1.5])
# Plot dataset
plot_dataset(X, y, [-1.5, 2.5, -1, 1.5])
# Show the plot
plt.show()
```

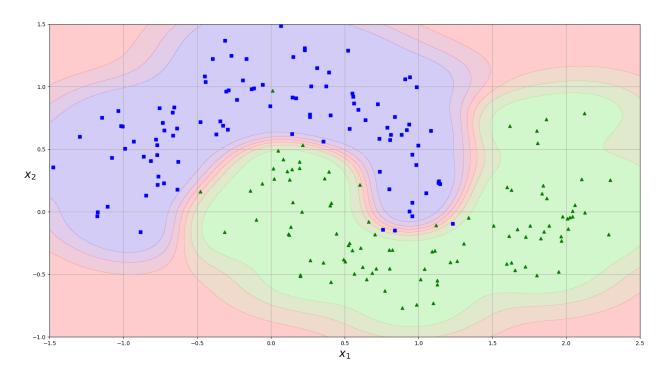


Figure 41

#### 7.50.8 Sigmoid kernel

## 7.50.9 SVR (Support Vector Regression)

```
svm_reg = SVR()
```

## 7.50.10 SVC (Support Vector Classification)

```
# Required Libraries
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_moons
from sklearn.svm import SVC
from mpl_toolkits.mplot3d import Axes3D
# Generate a 2D dataset using make_moons
x, y = make_moons(n_samples=200, noise=0.3, random_state=42)
# Plot the 2D dataset
plt.figure(figsize=(8, 6))
plt.scatter(x[:, 0], x[:, 1], c=y, cmap='cool', edgecolor='k', s=50)
plt.title('2D Dataset')
plt.xlabel('Feature 1')
plt.ylabel('Feature 2')
plt.show()
# Define an RBF kernel transformation function
def rbf_kernel(X, gamma=1.0):
    # Compute the pairwise squared Euclidean distances
    pairwise_sq_dists = np.square(X[:, np.newaxis] - X).sum(axis=2)
    # Apply the RBF kernel
    return np.exp(-gamma * pairwise_sq_dists)
```

```
# Transform the data into a higher-dimensional space using the RBF kernel
Z = rbf_kernel(X)

# Plot the transformed dataset in 3D
fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')
ax.scatter(Z[:, 0], Z[:, 1], Z[:, 2], c=y, cmap='cool', edgecolor='k', s=50)
ax.set_title('3D Dataset')
ax.set_xlabel('RBF Feature 1')
ax.set_ylabel('RBF Feature 2')
ax.set_zlabel('RBF Feature 3')
plt.show()
```

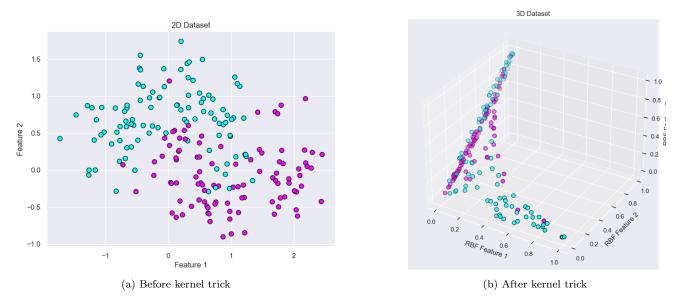


Figure 42: Dataframe values extraction

## 7.50.11 SMV (Soft Margin Classification)

Allows for misclassifications to achieve better generalization by introducing a margin of violation.

## 7.50.12 Polynomial SVM

Classification algorithm.

Make moons classification:

```
plt.plot(X[:, 0][y==1], X[:, 1][y==1], "g^")
    plt.axis(axes)
    plt.grid(True, which='both')
    plt.xlabel(r"$x_1$", fontsize=20) #//# x label
    plt.ylabel(r"$x_2$", fontsize=20, rotation=0) #//# y label
def plot_predictions(clf, axes):
    x0s = np.linspace(axes[0], axes[1], 100)
    x1s = np.linspace(axes[2], axes[3], 100)
    x0, x1 = np.meshgrid(x0s, x1s)
    X = np.c_[x0.ravel(), x1.ravel()] #//# c_? & ravel?
    y_pred = clf.predict(X).reshape(x0.shape)
    y_decision = clf.decision_function(X).reshape(x0.shape)
    plt.contourf(x0, x1, y_pred, cmap=plt.cm.brg, alpha=0.2) #//# contourf?
    \verb|plt.contourf(x0, x1, y_decision, cmap=plt.cm.brg, alpha=0.1)|\\
plot_predictions(polynomial_svm_clf, [-1.5, 2.5, -1, 1.5])
plot_dataset(X, y, [-1.5, 2.5, -1, 1.5])
plt.show()
```

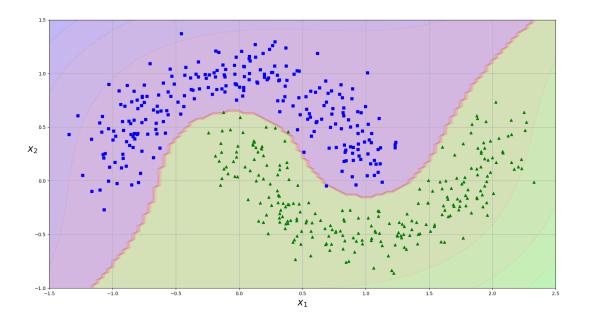


Figure 43

After introducing noisy data:

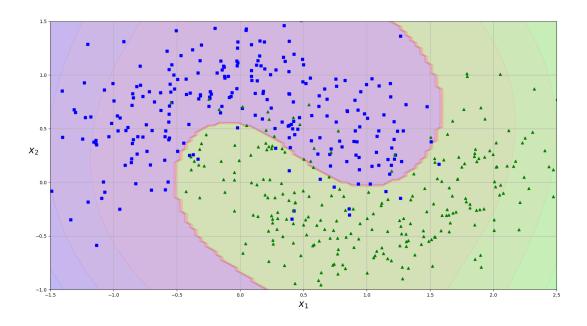


Figure 44

```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_classification
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import PolynomialFeatures, StandardScaler
from sklearn.pipeline import Pipeline
from sklearn.svm import SVC
from sklearn.metrics import accuracy_score
# Generate pseudo-random dataset with 3 informative features
X, y = make_classification(n_samples=500, n_features=3, n_informative=3,
                            n_redundant=0, n_classes=2, random_state=42)
# Split dataset into training and testing sets
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,
   random_state=42)
# Create Polynomial SVM classifier pipeline
polynomial_svm_clf = Pipeline([
    ("poly_features", PolynomialFeatures(degree=10)),
    ("scaler", StandardScaler()),
    ("svm_clf", SVC(kernel="poly", degree=10, C=10000))
])
# Train the classifier
polynomial_svm_clf.fit(X_train, y_train)
# Make predictions
y_pred = polynomial_svm_clf.predict(X_test)
# Calculate accuracy
accuracy = accuracy_score(y_test, y_pred)
print("Accuracy:", accuracy)
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
```

```
# Generate grid points for the features
x0_min, x0_max = X[:, 0].min() - 1, X[:, 0].max() + 1
x1_{min}, x1_{max} = X[:, 1].min() - 1, X[:, 1].max() + 1
x0, x1 = np.meshgrid(np.arange(x0_min, x0_max, 0.1),
                        np.arange(x1_min, x1_max, 0.1))
# Flatten the grid points and create a third feature with zeros
x2 = np.zeros_like(x0)
X_grid = np.c_[x0.ravel(), x1.ravel(), x2.ravel()]
# Predict the class labels for the grid points
y_grid = polynomial_svm_clf.predict(X_grid)
y_grid = y_grid.reshape(x0.shape)
# Create the 3D scatter plot
fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')
# Plot the data points
ax.scatter(X[:, 0], X[:, 1], X[:, 2], c=y, cmap=plt.cm.Paired)
# Plot the decision boundary
ax.plot_surface(x0, x1, y_grid, alpha=0.2)
ax.set_xlabel('Feature 1')
ax.set_ylabel('Feature 2')
ax.set_zlabel('Feature 3')
ax.set_title('Polynomial SVM Classification')
plt.show()
```

## 7.51 Naive Bayes

Often used for text classification.

## 7.52 Deep Learning

### 7.52.1 Whazit?

Neural networks, Deep Boltzmann Machines (DBM).

## 7.53 ANN (Artificial Neural Networks)

## 7.53.1 Description

Neural networks are 'cells' linked by mathematical weight to pass information. **DNN** (aka deep neural networks) are neural networks with multiple layers,

### 7.53.2 Activation functions

Input is a value between 0 and 1. documentation here: https://keras.io/api/layers/activations/ try a few

#### 7.53.3 Optimizers

Update weights parameters to reduce loss function. <a href="https://keras.io/api/optimizers/">https://keras.io/api/losses/</a>

#### 7.53.4 Weights and biases

## 7.53.5 Backpropagation

i know the word at least ... backpropagation uses reverse-mode autodiff

#### 7.53.6 Neurons weights

Initialized at random.

## 7.53.7 Weight-initialisation

Generated at random

keras layers Dense 10 activation relu kernel initializer henormal

#### 7.53.8 Training loop

Consider an input vector  $\mathbf{x}$  (a sample from the dataset), a matrix  $\mathbf{w}$  (containing the weights of a model, those being between 0 and 1) and a target value  $\mathbf{y}$ .

#### **Batch**

Draw a batch of training samples.

#### Forward pass

Run the model to obtain predictions.

Loss

Compute the loss to update weights based on the loss from the batch used in forward pass.

### Backward pass

Come back to update the weights.

#### **7.53.9** Dropout

At each iteration a random subset of neurons are dropped out (they return 0). Forces network not to rely on the same networks. Tends to slow down convergence to solution but builds stronger models. Slows down training time (by a factor of 2 or around but has no impact on inference time).

## 7.53.10 Learning rate

Update weights based on the gradient but slower to reduce bouncing.

 $w = learning\_rate * gradient$ 

## 7.53.11 Vanishing vs. exploding gradient

When performing backpropagation and going from output neuron to input neuron weights are updated using cost function of each neuron in a Gradient Descent step.

Vanishing Lower layer's weights virtually unchanged during backpropagation.

**Exploding** Algorithm diverges during backpropagation because of excessive weights.

## 7.53.12 Gradient clipping

To reduce exploding gradient problem by setting threshold.

#### 7.53.13 Transfer learning

In order to save time and computing power, reusing models already able to do things might be considerable.

### 7.53.14 Sparse models

purpose? other than speed of prediction To get one, zero out tiny weights, apply L1 regularization, use TensorFlow model optimization toolkit.

#### 7.53.15 Loss functions

### 7.54 Activation functions

Reference to 7.53.2

## 7.54.1 ReLU (Rectified Linear Unit)

$$ReLU(z) = max(0, z)$$

## 7.54.2 RReLU (Randomized leaky ReLU)

# 7.54.3 PRELU (Parametric leaky ReLU)

Outperforms ReLU on large datasets but on small datasets tends to overfit.

## 7.54.4 Sigmoid

$$\sigma(z) = \frac{1}{1 + exp(-z)}$$

## 7.54.5 Hyperbolic Tangent

$$tanh(z) = 2\sigma(2z) - 1 = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Where  $\sigma(z)$  is still the sigmoid function.

### 7.54.6 Softmax

 $\frac{1}{1}$ 

detail?

### 7.54.7 Softplus

$$softplus(z) = log(1 + exp(z))$$

## 7.54.8 ELU (Exponential Linear Unit)

# 7.54.9 SELU (Scaled ELU)

A few advantages over ReLU such as: it can take on negative values, always has a nonzero derivative, ensures the model is self-normalized to solve exploding/vanishing gradients

## 7.55 Optimizers

Reference to 7.53.3.

## 7.55.1 SGD (Stochastic Gradient Descent)

To calculate optimal weight update. Stochastic refers to random batch sampling.

```
from sklearn.linear_model import SGDRegressor

# ... (build model)

optimizer = keras.optimizers.SGD(lr=0.001, momentum=0.9, nesterov=True)
model.compile(optimizer=optimizer, loss='sparse_categorical_crossentropy', metrics=[
    'accuracy'])
```

capivalue, lr, momentum, nesterov

#### 7.55.2 Mini-batch SGD

Using SGD algorithm (mentioned in 7.55.1), on a sample of the dataset.

#### 7.55.3 Batch SGD

Using SGD algorithm (mentioned in 7.55.1), on the whole dataset.

#### 7.55.4 Momentum

Momentum: compare previous change in gradient, if they move in the same direction will take more speed (bigger than previous step).

### 7.55.5 NAG (Nesterov Accelerated Gradient)

#### 7.55.6 AdaGrad

Risk of slowing too fast, small updates with features occurring often and big ones with features occurring often.

## 7.55.7 RMSProp (Root Mean Square Propagation)

Different learning rate for each parameter.

```
optimizer = keras.optimizers.RMSprop(lr=0.001, rho=0.9)
```

## rho, lr

### 7.55.8 Adam (ADAptive Moment estimation)

Combination of momentum and RMSProp with AdaGrad

#### 7.55.9 AdaMax

## 7.55.10 Nadam (Nesterov Adam)

```
optimizer = keras.optimizers.Nadam(learning_rate=0.001)
```

### 7.56 Loss Functions

Reference to 7.53.15

#### 7.56.1 Binary crossentropy

# 7.57 RNNs (Recurring Neural Networks)

"The batter hits the ball. The outfielder immediately starts running, anticipating the ball's trajectory. He tracks it, adapts his movements, and finally catches it (under a thunder of applause). Predicting the future is something you do all the time, whether you are finishing a friend's sentence or anticipating the smell of coffee at breakfast. In this chapter we will discuss recurrent neural networks (RNNs), a class of nets that can predict the future (well, up to a point, of course). They can analyze time series data such as stock prices, and tell you when to buy or sell. In autonomous driving systems, they can anticipate car trajectories and help avoid accidents. More generally, they can work on sequences of arbitrary lengths, rather than on fixed-sized inputs like all the nets we have considered so far. For example, they can take sentences, documents, or audio samples as input, making them extremely useful for natural language processing applications such as automatic translation or speech-to-text." - Aurelien Geron, "Hands-on ML".

To summarize, RNNs are a good fit for time series time series forecasting. By that it is meant that this neural network model might be a good fit to predict future values or intermediate missing values (e.g. number of active users on a website is known before February 13th and after February 15th but not for the 14th, time series can help finding this missing value, this process is called imputation). Note that tanh(x) function is preferred to ReLU in RNNs. Two main problems to be faced are unstable gradients and limited memory.

#### 7.57.1 Unstable gradients

Recurrent dropouts and recurrent layer normalization. Neuron sending to istelf cumulates (e.g. 1.05 change on and on makes  $1.05^{1}0$  if ten times)

### 7.57.2 Memory cells

Input is output from the same cell at time t-1, indeed this behavior is consider as the one of a memory. cell initialization (distributing data? and forward knowledge?) Typically neurons can memorize as much as 10 steps of data for most of them but some are able to memorize up to 100. Particularity is to get sequence as input and sequence as output which is indeed a good fit for time series forecasting.

### 7.57.3 Limited memory

## 7.57.4 Encoder/decoder

Encoder: sequence-to-vector
 Decoder: vector-to-sequence

Useful for translation.

#### 7.57.5 Stateless

Network's hidden layer is reset after processing each input sequence.

## 7.58 CNNs (Convolutional Neural Networks)

#### 7.58.1 Description

Useful for visual perception, voice recognition and natural language processing. Scientists (David H. Hubel and Torsten Wiesel) noticed that neurons dedicated to vision spark only when a specific slice of the eye's view has input, in other words they only react in a limited region of the visual cortex. They noticed too that certain neurons only react with specific images features such as horizontal images, different orientation images and so on. Originally CNN were called neocognitron (specificity was not being sensitive to image shift). Issue with CNN is the memory requirement and ressources consumption making it hard to proceed to search grid to find optimal parameters (such as number of filters and so on). Basic idea is to take tiles from images.

#### 7.58.2 Architectures

Stack of a few convolutional layers each one generally followed by a ReLU layer, then a pooling layer then another convolutional layer with ReLU then another pooling layer and so on. The purpose of layering like this is to emphasis on smaller and smaller parts of the image as we advance in the network.

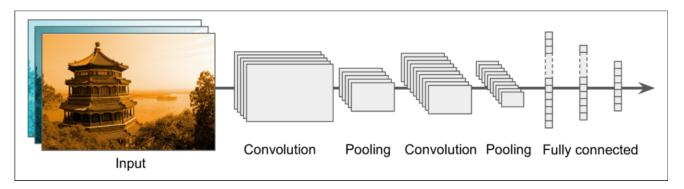


Figure 45: CNN layers with rectangular receptive fields

#### 7.58.3 Convolutional layers

Neurons in the first convolutional layer are not connected with every pixel on the input image.

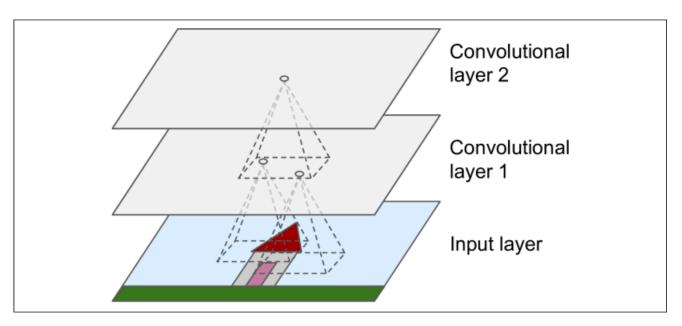


Figure 46: CNN layers with rectangular receptive fields

# 7.58.4 Pooling layers

Purpose is to subsample the input image to limit the risk of overfitting. Here image pixels take the most intensive one.

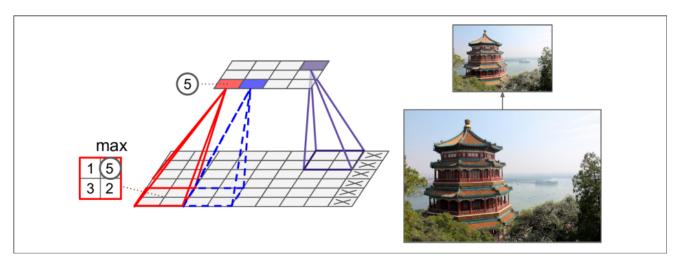


Figure 47: Max pooling layer (2 x 2 pooling kernel, stride 2, no padding)

Size of final image is half the original one (rounded down because there is no padding as can be seen on the side cases)

# 7.58.5 MaxPooling

# **7.58.6** Filters

# vertical/horizontal lines

# 7.58.7 Transfer learning

When not enough training data is available, might be smart to use lower layers of a pretrained model.

## 7.58.8 Localization

## 7.58.9 Object detection

# 7.59 GANs (Generative Adversial Networks)

Useful to generate artificial data. (e.g. train on faces to generate fake faces). Composed of two neural networks, one to generate data and the other to discriminate (between true data and generated data).

# 7.60 ANN (Artificial Neural Network) models & configuration

Those can be either supervised or unsupervised but are often both. sequential, RNN (Recurring Neural Networks) not a specific model but some ANNs work this way, DNN (Deep Neural Networks)

## 7.60.1 Import layers

```
from tensorflow.keras.layers import LSTM, Dense
```

#### 7.60.2 Import models

```
from tensorflow.keras.models import Sequential
```

#### 7.60.3 Input layer

```
input = keras.layers.Input(shape=(<shape_x>, <shape_y>))
```

shape: input table shape

# 7.60.4 Multimodal Input

# 7.60.5 layers.Flatten(input\_shape(x,y))

Input image for example with dimensions of 28 by 28:

```
import numpy as np
from tensorflow.keras import Sequential, layers

# Generate data
random_matrix = np.random.rand(4, 4)

# Create model
model = Sequential()
model.add(layers.Flatten(input_shape=(4, 4)))
```

With the matrix:  $\operatorname{array}([[0.60872335, 0.99446746, 0.28128227, 0.64169977], [0.14730914, 0.90998328, 0.33204108, 0.85083919], [0.80004922, 0.29794655, 0.10482782, 0.30146641], [0.18502237, 0.0376533, 0.44252062, 0.02067095]])$  For colored images:

```
import numpy as np
from tensorflow.keras import Sequential, layers

# Generate data
random_rgb_matrix = np.random.rand(4, 4, 3)

# Create model
model = Sequential()
model.add(layers.Flatten(input_shape=(4, 4, 3)))
model.summary()
```

where the matrix looks like this:array([[[0.63192061, 0.18524456, 0.83385466], [0.79491155, 0.92942268, 0.5088006], [0.5981006, 0.56562488, 0.88500774], [0.79289695, 0.9659789, 0.1461061]],

[[0.24680856, 0.75861799, 0.64804456], [0.76327029, 0.29416709, 0.15149997], [0.40057238, 0.99504103, 0.19209781], [0.54279693, 0.55907711, 0.75802485]]

[[0.75123018, 0.52956408, 0.2450316], [0.09417945, 0.04883064, 0.34909757], [0.71825547, 0.54400082, 0.81305528], [0.63590995, 0.39435362, 0.47630061]],

 $[[0.7187529\,,\,0.2487458\,,\,0.64103506],\,[0.26499111,\,0.92787077,\,0.46668161],\,[0.9789001\,,\,0.53370229,\,0.70885125],\\ [0.43296265,\,0.37520249,\,0.71286807]]])\ ,\ because each pixel has 3 components, note that for 3D greyscaled images it works decently well but for colorful images it becomes less efficient because it is hard for the model to learn about details of images.$ 

## 7.60.6 Dense layer

```
hidden = keras.layers.Dense(30, activation="relu")(input_)
```

#### **7.60.7** LSTM layer

```
hidden = keras.layers.LSTM(30, activation="relu")(input_)
```

#### 7.60.8 Output layer

```
output = keras.layers.Dense(1)(concat)
```

## 7.60.9 Compile model

Use different weights for loss functions with the hyperparameter  $loss_w eights$ .

#### 7.60.10 Fit model

```
history = model.fit(X_train, y_train, epochs=20, validation_data=X_test, y_valid))
```

#### 7.60.11 Plot fit results

```
pd.DataFrame(history.history).plot(figsize=(8, 5))
plt.grid(True)
plt.gca().set_ylim(0, 1)
plt.show()
```

```
pyplot.plot(<model > . history['loss'], label='train', )
pyplot.plot(<model > . history['val_loss'], '--',label='test',)
```

## 7.60.12 Save model to HDF5

```
model.save("<file_path.h5>")
```

#### 7.60.13 Load model

```
model = keras.models.load_model(<file_path.h5>)
```

## 7.60.14 Checkpoints

```
checkpoint_cb = keras.callbacks.ModelCheckpoint(<file_path.h5>)
history = model.fit(X_train, y_train, epochs=10, callbacks=[checkpoint_cb])
```

callbacks is the destination at which to save models. Set  $save_best_only = True$  to only save the best models (might be useful when gradient explodes, to avoid model from being saved at that moment).

#### 7.60.15 Checkpoints versioned

```
class SaveModelCallback(keras.callbacks.Callback):
    def __init__(self, model_index):
        super().__init__()
        self.model_index = model_index

def on_epoch_end(self, epoch, logs=None):
        model_filename = f"model_{self.model_index}.h5"
        self.model.save(model_filename)
        print(f"Model {self.model_index} saved as {model_filename}")

checkpoint_callback = SaveModelCallback(model_index=i)
history = model.fit(X_train, y_train, epochs=10, callbacks=[checkpoint_callback])
```

#### 7.60.16 Adaptive learning rate

Reduce on plateau:

```
lr_scheduler = keras.callbacks.ReduceLROnPlateau(factor=0.5, patience=5)
```

## factor, patience

Exponential decay:

```
learning_rate = keras.optimizers.schedules.ExponentialDecay(0.01, s, 0.1)
optimizer = keras.optimizers.SGD(learning_rate)
```

## 7.60.17 Transfer learning

```
model_A = keras.models.load_model("my_model_A.h5")
model_B_on_A = keras.models.Sequential(model_A.layers[:-1])
model_B_on_A.add(keras.layers.Dense(1, activation="sigmoid"))

Use this code to do not save in A when saving B:
model_A_clone = keras.models.clone_model(model_A)
model_A_clone.set_weights(model_A.get_weights())
```

#### 7.60.18 Early stopping

patience what exactly, restore\_best\_weights signification?

## 7.60.19 Regularization techniques

L2 (Ridge):

```
layer = keras.layers.Dense(100, activation="elu", kernel_regularizer=keras.
regularizers.12(0.01))
```

# 0.01 but what?

Template layer:

```
from functools import partial
RegularizedDense = partial(keras.layers.Dense,
    activation="elu",
    kernel_initializer="he_normal",
    kernel_regularizer=keras.regularizers.12(0.01))
RegularizedDense(10, activation="softmax", kernel_initializer="glorot_uniform")
```

#### 7.60.20 Initializer

He normal:

```
layer = keras.layers.Dense(100, activation="elu", kernel_initializer="
he_normal")
```

glorot uniform:

```
kernel_initializer="glorot_uniform"
```

# **7.60.21** Dropout

```
keras.layers.Dropout(rate=0.2)
```

Drop weights about random neurons.

## 7.60.22 Constraint

```
layer = keras.layers.Dense(100, kernel_constraint=keras.constraints.max_norm(1.))
```

#### 7.60.23 Custom activation/initializer/regularizer/constraint

```
def my_softplus(z):
    return tf.math.log(tf.exp(z) + 1.0)

def my_glorot_initializer(shape, dtype=tf.float32):
    stddev = tf.sqrt(2. / (shape[0] + shape[1]))
    return tf.random.normal(shape, stddev=stddev, dtype=dtype)

def my_l1_regularizer(weights):
    return tf.reduce_sum(tf.abs(0.01 * weights))

def my_positive_weights(weights): # return value is just tf.nn.relu(weights)
    return tf.where(weights < 0., tf.zeros_like(weights), weights)

layer = keras.layers.Dense(30, activation=my_softplus,
    kernel_initializer=my_glorot_initializer,
    kernel_regularizer=my_l1_regularizer,
    kernel_constraint=my_positive_weights)</pre>
```

## describe activation functions

#### 7.60.24 Compile

```
model.compile(optimizer=optimizer, loss=loss, metrics=metrics)
```

loss = 'sparse\_categorical\_crossentropy'

7.60.25

#### 7.60.26 RNN

Import RNN layer.

```
from keras.layers import SimpleRNN
```

```
model = keras.models.Sequential([
    keras.layers.SimpleRNN(20, input_shape=[None, 1])
    keras.layers.SimpleRNN(20),
    keras.layers.SimpleRNN(1)
])
```

#### 7.60.27 Autoencoders

Neural networks designed for dimensionality reduction, image denoising, anomaly detection, feature extraction and data generation.

## 7.60.28 Hopfield network

7.60.29 RBM (Restricting Boltzmann techniques)

neither do i know what this is

# 7.60.30 GAN (Generative Adversial Network)

oh yeah ian's thing

#### 7.60.31 LSTM (Long Short-Term Memory)

Unsupervised & supervised, designed to avoid long-term dependencies problems. RNN based.

- Forget: output between 0 and 1, decides whether past should be forgotten or kept
- Input: chooses which data need to be stored
- Output: decides what will yield out of each cell

Converges faster and detects long-term behavior with more accuracy.

#### 7.60.32 WaveNet

## 7.60.33 Gru (Gated Recurrent Unit) cells

#### 7.60.34 LeNet-5

Mainly used in the banking system to recognize handwritten digits, introduced by Yann LeCun.

#### 7.60.35 AlexNet

Convolutional layers directly on top of one another instead of stacking a pooling layer on top of each convolutional layer. To reduce overfitting two major regularization techniques were used (dropout at rate of 50% and data augmentation with random image shift/offsets/flipping horizontally/light dimming/...).

#### 7.60.36 ZF Net

Subclass of AlexNet network, developed by Matthew Zeiler and Rob Fergus (won ILSVRC 2013). Basically it is an AlexNet with a few tweaked hyperparameters such as number of feature maps/kernel size/stride/...

### 7.60.37 GoogLeNet

Christian Szegedy et al. from Google (won ILSVRC 2014). Makes use of a subnetwork called "inception module" using thus 10 times less hyperparameters (roughly 6 millions instead of 60 millions). Insensitive to the position of the object on the image. Later versions of their inception function were proposed over time.

# 7.60.38 VGGNet (Visual Geometry Group Net)

Developed by Karen Simonyan and Andrew Zisserman (won ILSVRC 2014) works based on a classical architecture, with 2 or 3 convolutional layers and a pooling layer, then again 2 or 3 convolutional layers and a pooling layer, and so on (reaching a total of just 16 or 19 convolutional layers, depending on the VGG variant), plus a final dense network with 2 hidden layers and the output layer. It used only 3x3 filters, but many filters.

## 7.60.39 ResNet (Residual Network)

Extremly deep CNN of 152 layers with other variants of 34, 50 and 101 layers (won the ILSVRC 2015). Key to be able to train such networks is to skip connections using shortcuts.

Connections skip/shortcut connections The signal feeding into a layer is also added to the output of the layer located a bit higher. Up on the stack. When you initialize a regular neural network, its weights are close to zero, so the net- work just outputs values close to zero. If you add a skip connection, the resulting network just outputs a copy of its inputs; in other words, it initially models the identity function. If the target function is fairly close to the identity function (which is often the case), this will speed up training considerably. If many skip connections are added, progress can be seen without even having all the layers being trained yet (because the signals can make their way through the whole network easily).

RUs (Residual Units) The deep network with skipped and untrained connections can become a stack of residual units where each residual unit is a small neural network with a skip connection.

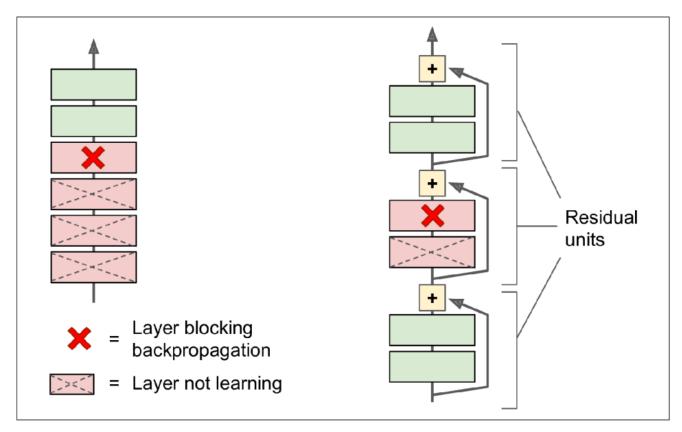


Figure 48: Regular DNN (Deep Neural Network) vs. DRN (Deep Residual Network)

# 7.60.40 XCeption (Extreme Inception)

Variant of Google's GoogLeNet, proposed in 2016 by François Chollet. Replaces the inception modules with a special type of layer called a depthwise separable convolution layer (or separable convolution layer for short). Composed of two parts (one that applies a single filter to each input feature map and the second which only searches for cross-channel patterns).

## 7.60.41 SENet (Squeeze-and-Excitation Network)

(won ILSVRC 2017)

# 7.60.42 FCN (Fully Convolutional Networks)

#### 7.60.43 YOLO (You Only Look Once)

Proposed by Joseph Redmon in 2015, so fast that it is used to run in real time on videos.

## 7.61 Reinforcement Learning

## 7.61.1 Overview

Agent makes observations and takes actions within an environment to maximize rewards. Some of these factors can be omitted (e.g. a vacuum doesn't care about its environment and cares mostly about the amount of dust it can gather, to escape a maze a robot only gets penalty at each step it makes because it is making the path longer so the end goal is only to minimize cost).

### 7.61.2 Agent

Entity that performs the action.

### 7.61.3 Environment

The world in which the agent resides.

#### 7.61.4 State

Current environment. not sure about this one

#### 7.61.5 Reward

Immediate return sent by the environment to evaluate the last action of the agent.

#### 7.61.6 Action

Things agent can do.

#### 7.61.7 Policy

Can be neural network taking observations as input and outputting the actions to take. Defined as  $\pi(S)$ , it is the behavior depending upon the current status.

#### 7.61.8 Value Function

The goal being to maximize future rewards not immediate next step can be mathematically explained by maximizing the cumulative discounted rewards which can be expressed by following equations:

$$G_t = R_{t+1} + \gamma R_{t+1} + \dots = \sum_{0}^{\infty} \gamma^k R_{t+k+1}$$

Where  $\gamma$  is the discounting factors and resides between 0 and 1.

## 7.61.9 Q-Learning

Value function based, works by picking actions depending upon Q-values with:  $a_t = max_aQ(s_t, a)$ , by applying e-greedy approach to select random action with probability  $\epsilon$ . Q-values might be hard to calculate due to the fact that states and actions spaces can be very large, thus ANNs can be used to approximate these values.

#### 7.61.10 Markov decision processes

Action leads to state

Optimal state value sum of all discounted future rewards the agent can expect on average after it reaches a state s

Partially Observable Markov Decision Process (POMDP) Not everything is observable but what is observed is a derivation of it.

## 7.61.11 Model based vs model free

Determined whether rewards and probabilities for each step are readily accessible. Model-based ends there but model-free makes use of policy and value functions.

#### 7.61.12 Model

transition probability function (P) and reward function

#### 7.61.13 Model tuning

Few parameters can be optimized but requires a lot of ressources.

Gamma (discount factor) Decaying gamma will prioritize short-term learning.

**Epsilon** Drives exploration/exploitation tendencies.

**Episodes and batch size** A higher number of episodes and larger batch size in the training set will lead to better training and a more optimal Q-value.

Number of layers and nodes of the deep learning model Better training and more optimal Q-values.

#### 7.61.14 Policy gradient methods

# 7.62 Policy Gradient Methods

More information about what is policy gradient method in section 7.61.14

#### 7.62.1 Stochastic policy

### 7.62.2 Genetic policy

Kill worst policies and keep best performing ones.

#### 7.62.3 Deep Deterministic Policy Gradient (DDPG)

## 7.62.4 Epsilon greedy policy

#### 7.62.5 SARSA

Takes its name from iterating over  $S_t$ ,  $A_t$ ,  $R_t$ ,  $S_{t+1}$ ,  $A_{t+1}$ .

#### 7.62.6 Credit assignment problem

If last action caused failure of the system in the environment last action will be taken as responsible but surely is not completely behind failure.

**Discout factor** Solution was found in evaluating action based on the sum of all rewards that come after it with a discount factor  $\gamma$ .

**Action advantage** If a good action is follow by very bad actions, discount factor will creditate lower than deserved, action advantages tries to solve this. To perform this method normalization is used amongst many episodes.

# 7.63 Reinforcement Learning models

## 7.63.1 REINFORCE

## 7.63.2 Q-Learning

Adaptation of the Q-Value Iteration algorithm to the situation where the transition probabilities and the rewards are initially unknown. Q-Learning works by watching an agent play (e.g., randomly) and gradually improving its estimates of the Q-Values accurate Q-Value estimates (or close enough), then the optimal policy is choosing the action that has the highest Q-Value (i.e., the greedy policy). Advantage is being able to learn from agent acting randomly. Takes it's name from the function Q(s,a) that returns expected reward based on the state s and by executing action a.

# 7.63.3 Deep Q-Networks (DQN)

#### 7.63.4 Double DQN

Based on the observation that the target network is prone to overestimating Q-Values.

- 7.63.5 DDQN (Dueling DQN)
- 7.63.6 Proximal Policy Optimization (PPO)
- 7.63.7 Asynchronous Advantage Actor-Critic (A3C)
- 7.63.8 Advantage Advantage Actor-Critic (A2C)
- 7.63.9 Soft Actor-Critic (SAC)
- 7.63.10 Trust Region Policy Optimization (TRPO)
- 7.63.11 Proximal Policy Optimization (PPO)
- 7.63.12 Curiosity-based exploration

# 8 Statistics Appendix (Math 001)

## 8.1 Mean

$$m = \frac{(x_1 + x_2 + \dots + x_n)}{N}$$

## 8.2 Median

$$M =$$

## 8.3 Mode

Value with highest frequency.

#### 8.4 Variance

Measurement of how much points are scattered from the mean.

$$Var(x) = \sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

Where  $\bar{x}$  is the average and the whole thing is squarred because it is a measure of distance.  $\sigma$  itself is the standard-deviation, which is detailled further in 8.7 whi n-1?

## 8.5 Covariance

$$Covar(x) = \frac{1}{n-1} \sum_{i} (x_i - \bar{x})(y_i - \bar{y})$$

## 8.6 Beta

$$\beta = \frac{cov(x, y)}{var(x)}$$

# 8.7 Standard-Deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

Where n is the size of the sample,  $x_i$  is an individual element,  $\bar{x}$  is the average value.

## 8.8 Standard Error

$$SE = \frac{s}{\sqrt{n}}$$

Where s is the sample standard deviation, n is the sample size.

#### 8.9 Relative Standard Error

Used for instruments, calculated as the standard deviation as a percentage of the mean.

$$RSE = \frac{SE}{\theta} * 100$$

SE is the standard error, theta is the sample mean. Lower the RSE, the more precise the measurement.

### 8.10 Pearson Correlation

$$Corr(x,y) = \frac{Cov(x,y)}{\sqrt{Var(x)Var(y)}}$$

- 8.11 Standard Correlation Coefficient Pearson's r
- 8.12 Convolution
- 8.13 Fourier Transform
- 8.14 Laplace Transform

#### 8.15 Autocovariance

Autocovariance: Autocovariance is a special case of covariance where both variables are from the same time series, but they are at different time points. It measures the degree of linear dependence between values of a time series at different lags. In other words, it quantifies the relationship between a time series and a lagged version of itself.

$$ACov = (X_t, X_{t+k}) = \frac{1}{N} \sum_{t=1}^{N-k} (X_t - \bar{X})(X_{t+k} - \bar{X})$$

Where:

 $X_t$  = value at time t

 $X_t + k = \text{value at time t} + k$ , where k is the lag (period)

 $\bar{X}$  = average value

## 8.16 Gaussian Distribution

The "68-95-99.7" rule. Empirical distribution.

- $\diamond$  **68% Rule**: Approximately 68% of the data falls within 1 standard deviation ( $\sigma$ ) from the mean ( $\mu$ ).
- $\diamond$  95% Rule: Approximately 95% of the data falls within 2 standard deviations (2 $\sigma$ ) from the mean ( $\mu$ ).
- $\diamond$  99.7% Rule: Approximately 99.7% of the data falls within 3 standard deviations (3 $\sigma$ ) from the mean ( $\mu$ ).

# 8.17 Time Series

time serie = collection of data points at constant time intervals Time series properties:

- ♦ constant mean
- ♦ constant variance

#### $\diamond$ time-independant autocovariance

If time series is strictly stationary without dependencies amongst values, regular linear regression can be used to forecast values. If dependency amongst values exists, ARIMA (AutoRegressionIntegrated Moving Averages) is another choice of statistical model that can be used

# 9 Probability Appendix (Math 002)

# 9.1 Picking Probability

#### 9.1.1 Clusters

Probability to pick an item in a cluster is the same as picking that cluster, consider (A, B), (C, D), (E, F, G), then:

$$P(A) = P((A, B))$$

# 9.2 Martingale

Gains and losses accumulated by player during a game, concept of probability.

# 9.3 Bayes's Theorem

$$P(A|B) = \frac{P(B|A).P(A)}{P(B)}$$

# 9.4 Laplace Smoothing

# 9.5 PDF (Probability Density Function)

# 10 Algebra Appendix (Math 003)

Mapping function = function

## 10.1 Matrices

#### 10.1.1 Sparse Matrix:

- Contains mostly zero values.
- Efficient for representing matrices with a large number of zero elements.
- Requires less memory compared to dense matrices.
- Suitable for sparse data structures and applications where memory usage is critical.

#### 10.1.2 Dense Matrix:

- Contains mostly non-zero values.
- Requires more memory compared to sparse matrices.
- Suitable for dense data structures and operations where all elements are significant.

## 10.2 Tensors

# 10.2.1 Axes (rank)

Dimensions of a tensor are called axis, defined by key attributes.

- 10.2.2 Shape
- 10.2.3 Scalars (rank-0 tensors)
- 10.2.4 Vectors (rank-1 tensors)

$$\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{pmatrix}$$

10.2.5 Matrices (rank-2 tensors)

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \\ a_{51} & a_{52} & a_{53} \end{pmatrix}$$

10.2.6 Higher rank (rank-3 and more)

$$\mathbf{A} = \begin{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \\ a_{51} & a_{52} & a_{53} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \\ a_{51} & a_{52} & a_{53} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \\ a_{51} & a_{52} & a_{53} \end{pmatrix}$$

- 10.3 Useful Calculations
- 10.3.1 Harmonic Mean:

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}}$$

10.3.2 Distances

♦ Euclidean:

$$d(a,b) = \sqrt{\sum_{i=1}^{n} (a_i - b_i)^2}$$

♦ Manhattan:

$$\sum_{i=1}^{n} |x_i - y_i|$$

♦ Geodesic: shortest number of nodes between two points

Lipschitz continuous convex function

# 10.4 Boolean Algebra

# 10.4.1 Logical Operators

 $\diamond$  AND operator:  $\land$ 

 $\diamond$  OR operator:  $\vee$ 

♦ NOT operator: ¬

 $\diamond$  XOR (exclusive OR) operator:  $\oplus$ 

## 10.4.2 Truth Tables

# 11 Analysis Appendix (Math 004)

# 11.1 Bellman equations

Used in dynamic programming, states that optimal policy (sequence of decisions) has the property that the decisions are independent of the initial state. In other words, if we have the solution to a small problem, we can find the solution to a larger problem.

$$V^{*}(s) = \max_{a} \left( R(s, a) + \gamma \sum_{s'} P(s'|s, a) V^{*}(s') \right)$$

Where: R(s, a) is the immediate reward obtained after taking action a in state s. P(s'|s, a) is the probability of transitioning to state s' given that action a is taken in state s  $\gamma$  is the discount factor, representing the importance of future rewards relative to immediate rewards.  $max_a$  denotes the maximum over all possible actions.