PROBABILITY ASSIGNMENT

BASIC PROBABILITY.

1. Sample Space =>

(1, 1) (1, 2) (1, 3), (1,4), (1, 5), (1, 6)

(2, 1) (2, 2) (2, 3), (2, 4), (2, 5), (2, 6)

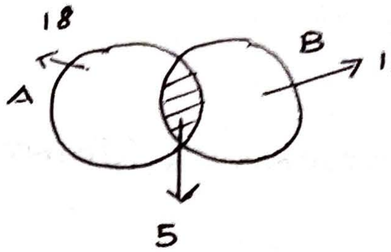
(3, 1) (3, 2) (3, 3), (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

P(sum being even) = $\frac{18}{36}$



P(one dice being 6) = $\frac{11}{36}$

P(sum being even ∩ one die being 6) = $\frac{5}{36}$.

∴ P(sum being even ∪ one die being 6 = $\frac{18}{36} + \frac{11}{36} - \frac{5}{36}$

$= \frac{24}{36} = \underline{\underline{2/3}}$

2. From the sample space.

$\frac{15}{36}$ => probability of sum of numbers being less than 7.

3. Fair coin 3 times.

$$P(\text{2 head for 3 coin}) = \frac{4}{8}$$

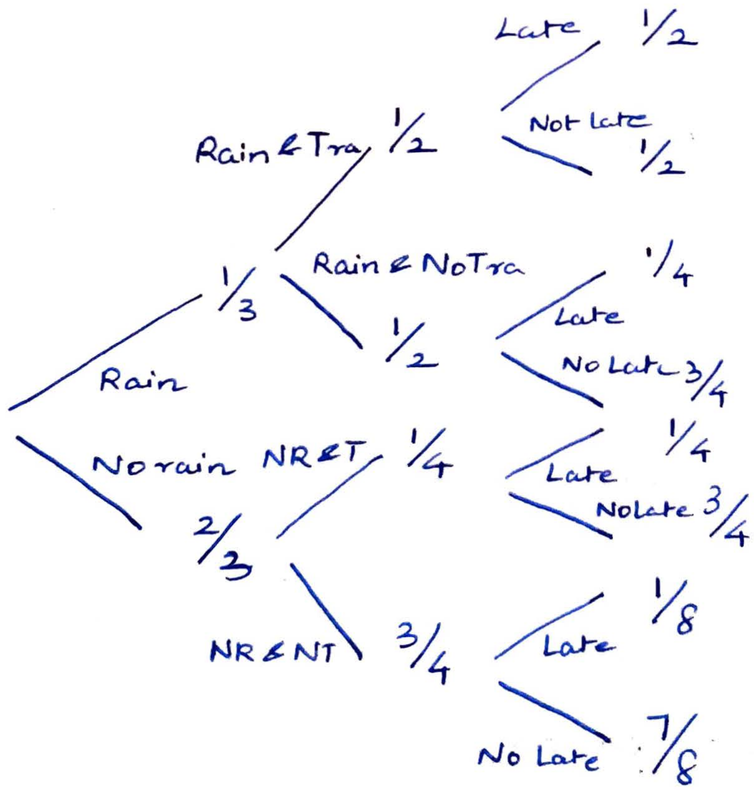
$$P(\text{1 head for 3 coin}) = \frac{7}{8}$$

$$\therefore P\left(\text{2 head} / \text{Observed 1 head}\right) = \frac{4/8}{7/8} = 4/7$$

4. The probability of other kid being a girl is $1/2$.

CONDITIONAL, JOINT AND MARGINAL PROBABILITY:

5.



i) Not raining, heavy traffic, not late. (Joint)

$$P(NR \cap T \cap NL) = \frac{3}{4} * \frac{1}{4} * \frac{2}{3} = \frac{1}{8}$$

ii] Prob of being late (Marginal)

$P(L) = P(R \cap T \cap L) + P(R \cap NT \cap L) +$

$P(NR \cap T \cap L) + P(NR \cap NT \cap L)$

$= \frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} \times \frac{1}{4} + \frac{2}{3} \times \frac{1}{4} \times \frac{1}{4}$

$+ \frac{2}{3} \times \frac{3}{4} \times \frac{1}{8}$

$= 11/48$ //

iii] Prob (rain / late to work) Conditional.

P (Rain and late to work)

$\Rightarrow P(R \cap T \cap L) + P(R \cap NT \cap L)$

$\Rightarrow \frac{1}{12} + \frac{1}{24}$

$\Rightarrow 1/8$

$\therefore P(R/L) = \frac{P(R \cap L)}{P(L)}$

$= \frac{1/8}{48/11} = 6/11$ //

6. 2 FAIR COIN , 1 (DOUBLE – HEAD COIN)

i) Probability of heads;

$$\therefore P(H) = P(\text{Fair Coin}) \cdot P(H/\text{Fair Coin})$$

$$+ F(\text{Bad Coin}) \cdot P(H/\text{Bad Coin})$$

$$= \frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times 1$$

$$P(H) = \underline{\underline{2/3}}$$

ii) Given heads fall, find prob that it is bad coin.

$$\therefore P(\text{Bad coin}/H) = \frac{P(H/\text{Bad coin}) \times P(\text{Bad coin})}{P(H)}$$

$$= \frac{1 \times 1/3}{2/3} = \underline{\underline{1/2}}$$

7. $P(\text{coffee}) = 0.7$

$P(\text{cake}) = 0.4$

$P(\text{coffee} \cap \text{cake}) = 0.2$

Find $P(\text{coffee}/\text{cake})$

$$= \frac{P(\text{coffee} \cap \text{cake})}{P(\text{cake})} = \frac{0.2}{0.4} = \frac{1}{2}$$

8. $P\left(\underset{\text{based on ball}}{\text{True}}\right) = 5/6 = P(T/W)$

$P\left(\underset{\text{based on ball}}{\text{False}}\right) = 1/6 = P(T/\overline{W})$

$P(\text{White ball}) = 1/9 = P(W)$

$P(\text{Not white ball}) = 8/9 = P(\overline{W})$

determine $P(W/T)$

$$\Rightarrow \frac{P(T/W) \times P(W)}{P(T)}$$

So true represents, that he picks a white balls and says he didn't get (or) he gets a not white ball and tells he got a white ball (false).

$$\Rightarrow \frac{5/6 \times 1/9}{5/6 \times 1/9 + 1/6 \times 8/9} = \frac{5}{13}$$

9. $P(\text{Truth based on die}) = \frac{4}{5}$

$P(6 \text{ on a die}) = 1/6$

determine $P(6/T)$

$$\Rightarrow \frac{P(T/6) \times P(6)}{P(T)} = \frac{\frac{4}{5} \times \frac{1}{6}}{\frac{4}{5} \times \frac{1}{6} + \frac{1}{5} \times \frac{5}{6}}$$

$$= 5/12$$

10. $P(\text{Science}) = 0.6$

$P(\text{Math} \cap \text{Science}) = 0.4$

$P(\text{Science}/\text{Math}) = ?$

$$= \frac{P(\text{Math} \cap \text{Science})}{P(\text{Science})} = \frac{0.4}{0.6} = 0.66$$

11. a) This is a joint probability as the question does not involve any dependency factors. It just asks for people who are male and graduate.

The answer for this can be derived from the table as; $19/100 = 0.19$

b) Probability of randomly selecting a male would be; $\frac{60}{100} \Rightarrow 0.6$

c) This is an example for marginal probability as it asks for people who are graduate which in turn would refer both Male and female. The prob $\Rightarrow \frac{31}{100} = 0.31$

d) Example for conditional probability as it states they selected PG student, determine if she's female $\Rightarrow 28/69$

BAYES THEOREM :

14. $P(\text{Swine flu}) = \frac{1}{10,000} = 0.0001 = P(S)$

$P(NS) = 0.9999$

$P(\text{False negative}/\text{Swine flu}) = 0$

$\therefore \; P(\text{Positive}/\text{swine flu}) = 1 = P(P/S)$

1 % chance of false positive

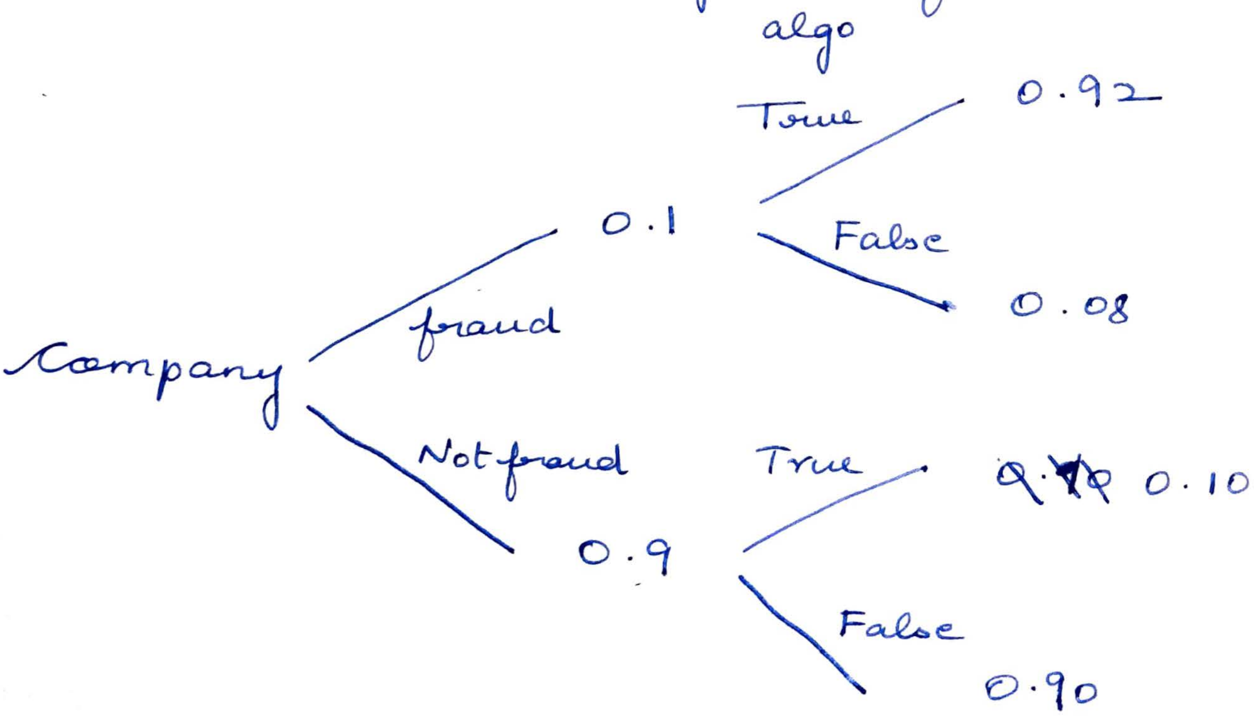
$P(P/NS) = 0.01$

$$\therefore \; P(S/P) = \frac{P(P/S)\;P(S)}{P(P/S)\,P(S) + P(P/NS)\;P(NS)}$$

$$= \frac{1 * 0.0001}{1 * 0.0001 + 0.01 * 0.9999} = \frac{0.0001}{0.010099} = 0.099$$

$\simeq$ approx 1 % of having swine flu.

12.



$P(T/F) = 0.92$

$P(F) = 0.1$

$P(NF) = 0.9$

$P(T/NF) = 0.10$

$$\therefore \quad P(F/T) = \frac{P(T/F) \times P(F)}{P(T)}$$

$$= \frac{0.92 \times 0.1}{P(F)\,P(T/F) + P(NF)\,P(T/NF)}$$

$$= \frac{0.92 \times 0.1}{0.92 \times 0.1 + 0.9 \times 0.10}$$

$$= \frac{0.092}{0.182} = 0.504$$

$\therefore$ Approx 50% the company did fraud in their filings.

13. Given,

1000 men

321 men died of renal failure

460 of 1000 had one parent with renal failure.

115 of 460 died of renal failure.

Get the prob if person dies if neither of his parents had renal failure.

$P(R)$ = death due to renal failure.

$P(RP)$ = atleast one parent suffered renal failure.

need to determine $P\left(R/_{NRP}\right)$

$$\Rightarrow \frac{P(R \cap NRP)}{P(NRP)}$$

$$P(RP) = \frac{460}{1000}$$

$$\therefore P(NRP) = 1 - \frac{460}{1000} = \frac{540}{1000}$$

$P(R \cap NRP)$ = in definition, death due to renal failure and people didn't have parents with renal failure

$$= \frac{321-115}{1000} = \frac{206}{1000}$$

$$\therefore P\left(R/_{NRP}\right) = \frac{\frac{206}{1000}}{\frac{540}{1000}} = \frac{206}{540} = \cancel{0.3754}$$

$$= 0.381$$