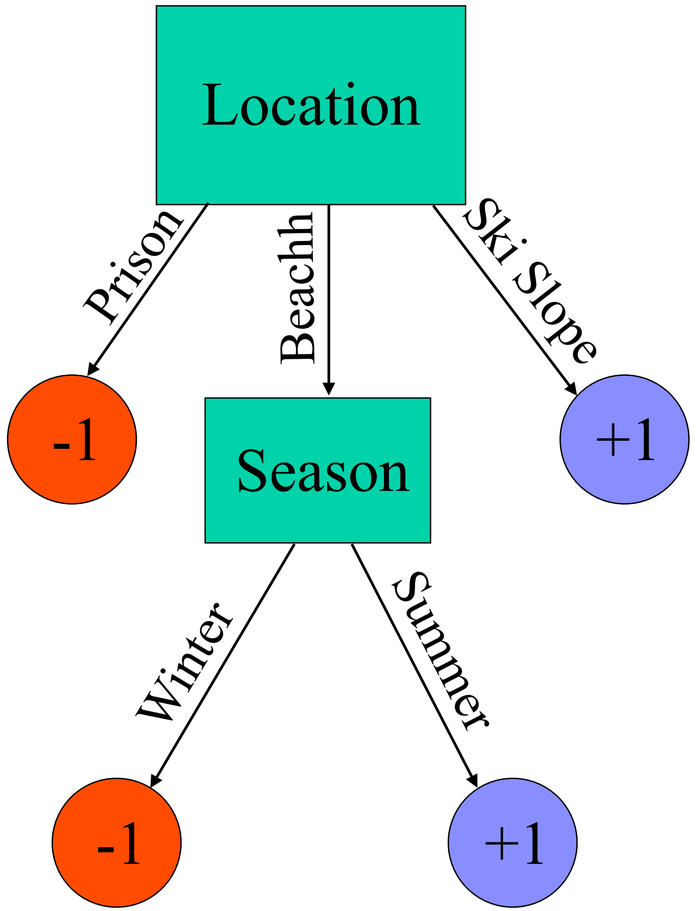
Decision trees

Decision Trees / Discrete Features

Where and when do you have fun?

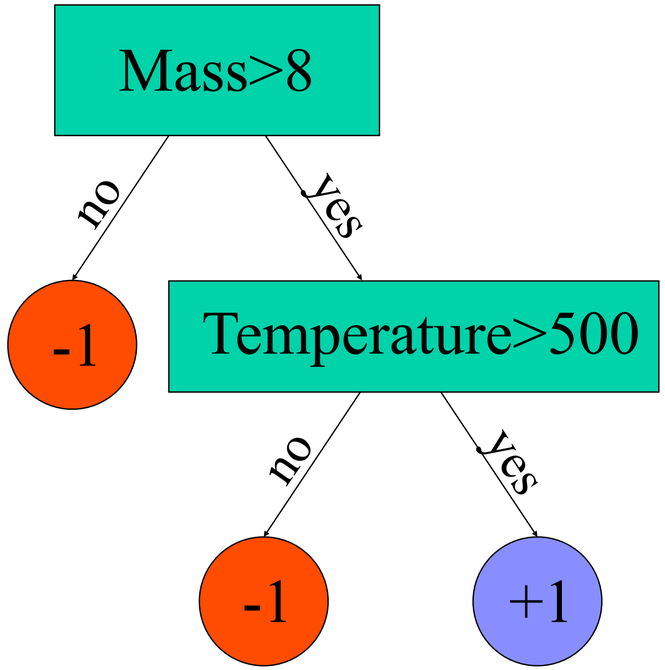
Season	Location	Fun?
summer	prison	-1
summer	beach	+1
Winter	ski-slope	+1
Winter	beach	-1



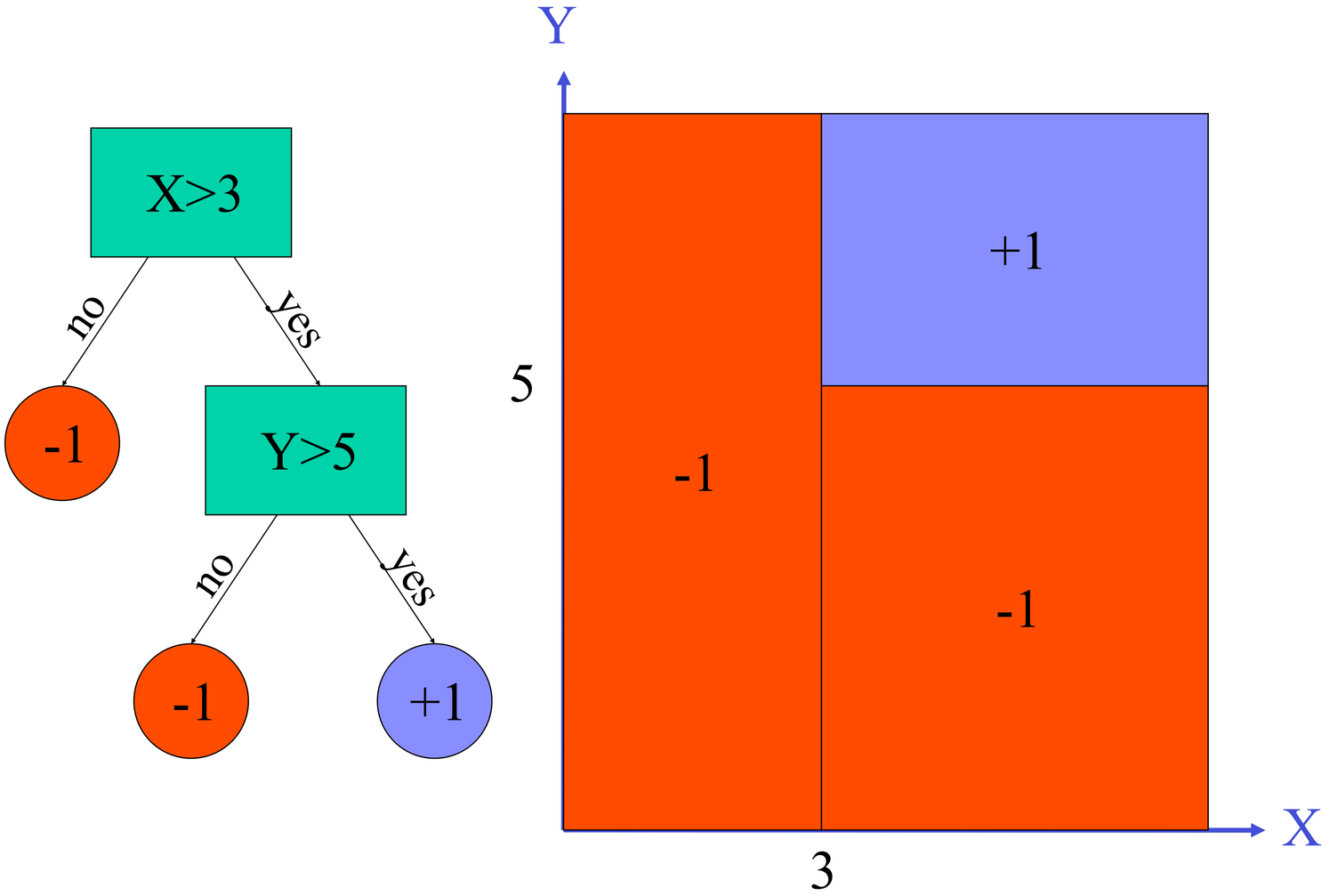
Decision Trees / Continuous Features

Will it explode?

Mass	Temperature	explosion
1	100	-1
3.4	945	-1
10	32	-1
11.5	1202	+1



Decision Trees



Decision trees

- Popular because very flexible and easy to interpret.
- Learning a decision tree = finding a tree with small error on the training set.

  1. Start with the root node.
  2. At each step split one of the leaves
  3. Repeat until a termination criterion.

Which node to split?

- We want the children to be more “pure” than the parent.
- Example:
  - Parent node is 50%+, 50%-.
  - Child nodes are (90%+,10%-),(10%+,90%-)
- How can we quantify improvement in purity?

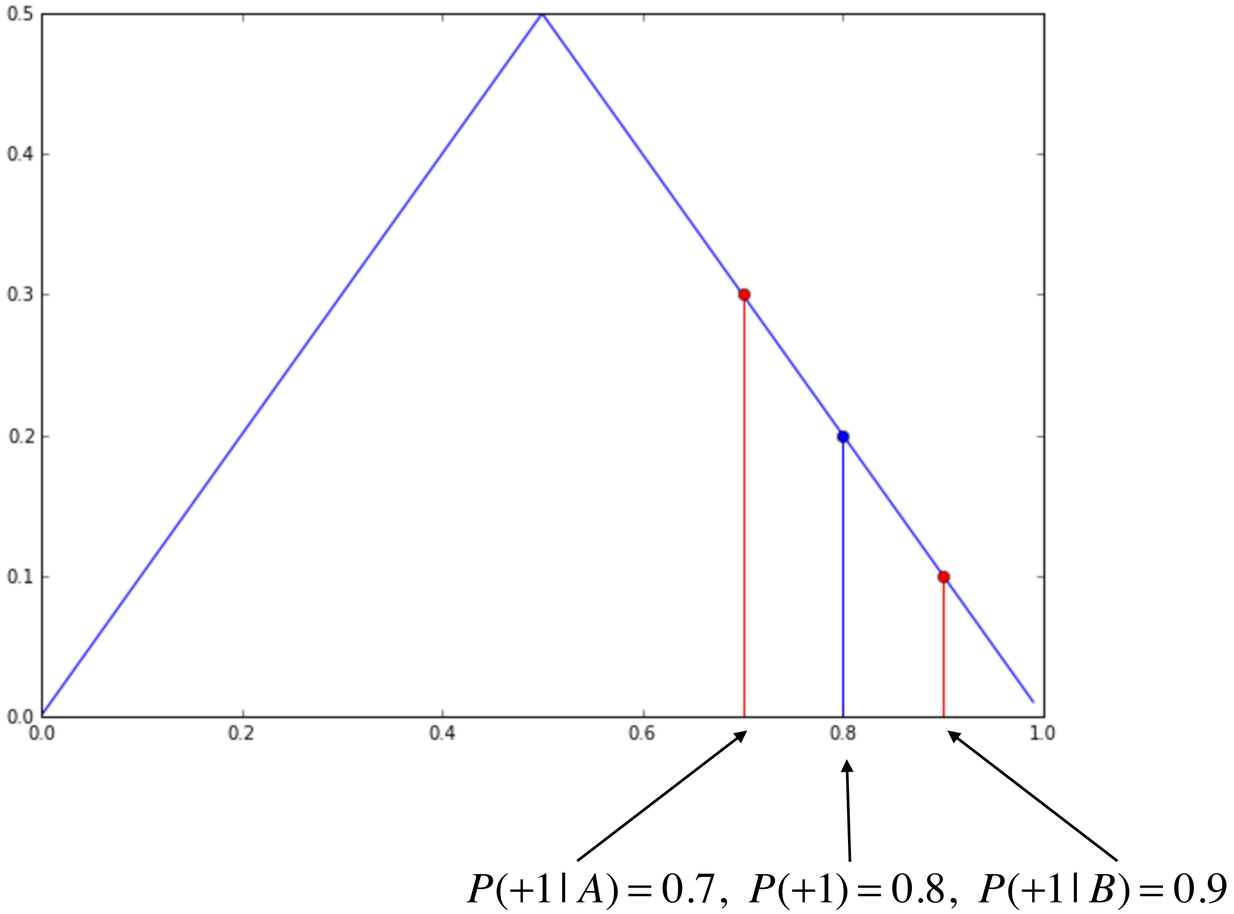
Naive approach: minimize training error

- **A good case**
- Parent node is 55%+, 45%-.
  - Predict + always: Error = 45%
- Child nodes are (90%+,10%-),(10%+,90%-)
  - Predict + on left, - on right: Error = 10%
- Clear Improvement

Naive approach: minimize training error

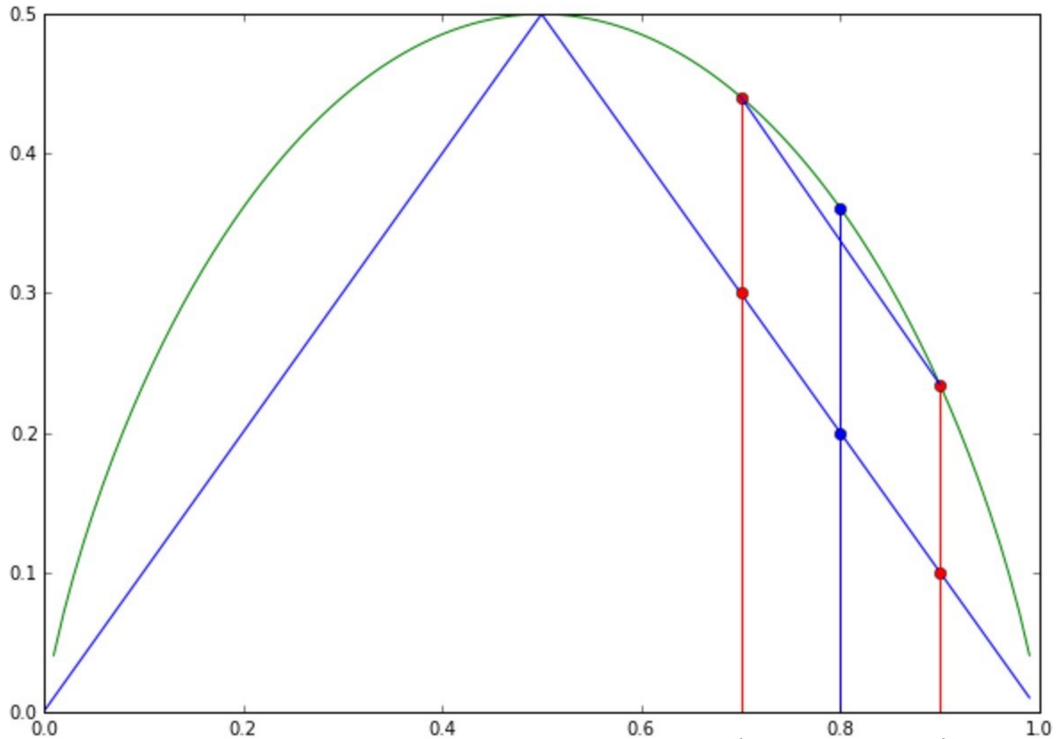
- **A bad case**
- Parent node is 80%+, 20%-.
  - Predict + always: Error = 20%
- Child nodes are (90%+,10%-),(70%+,30%-)
  - Predict + always: Error = 20%
- No improvement in training error!

The problem with classification error (pictorially)



Fixing the problem

instead of $\mathrm{err}(p) = \min(p, 1-p)$ use $\frac{H(p)}{2} = -\frac{1}{2}\left(p \log_2 p + (1-p)\log_2(1-p)\right)$



$P(+1 \mid A) = 0.7,\ P(+1) = 0.8,\ P(+1 \mid B) = 0.9$

Any strictly convex function can be used

$$H(P) =$$

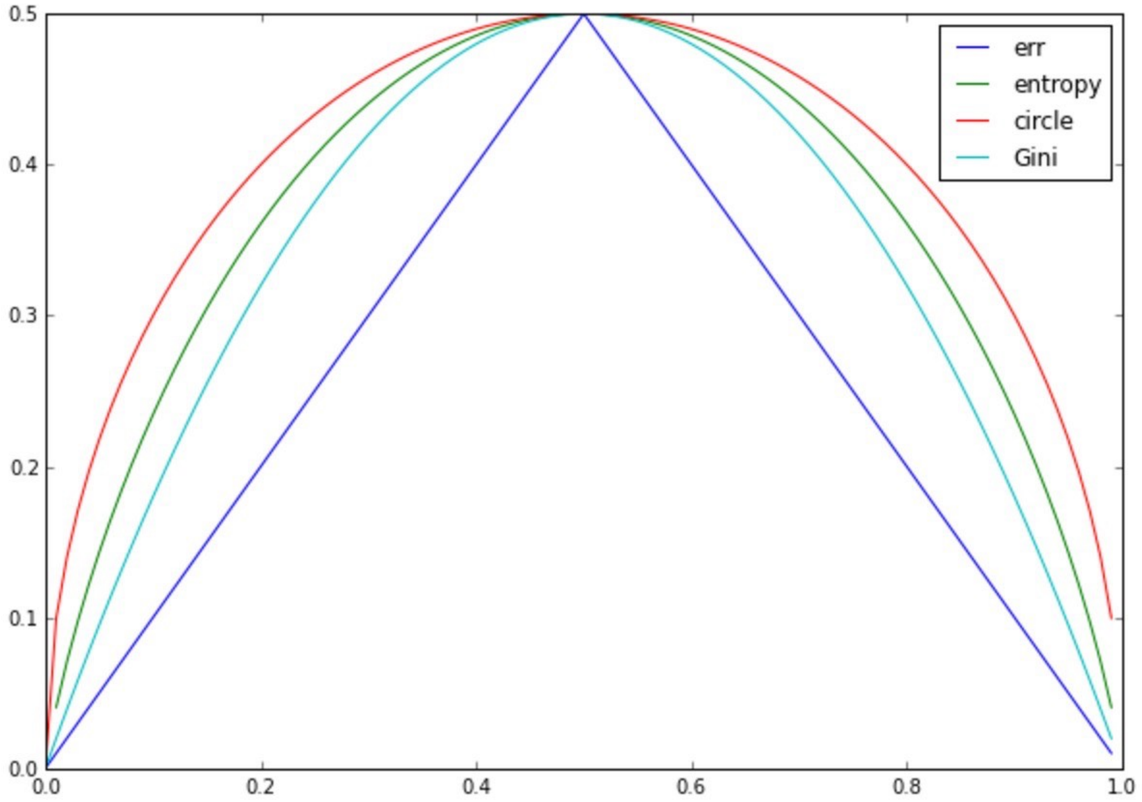
$$p \log p + (1-p)\log(1-p)$$

$$Circle(p) =$$

$$\sqrt{1/4 - (p - 1/2)^2}$$

$$Gini(p) =$$

$$p(1-p)$$



Decision tree learning algorithm

- Learning a decision tree = finding a tree with small error on the training set.

1. Start with the root node.

2. At each step split one of the leaves

3. Repeat until a termination criterion.

4. Next, How do we search for a splitting rule.

The splitting step

- Given: current tree.
- For each leaf and each feature,
  - find all possible splitting rules (finite because data is finite).
  - compute reduction in entropy
- find leaf X feature X split rule the minimizes entropy.
- Add selected rule to split selected leaf.

Enumerating splitting rules

- If feature has a fixed, small, number of values. then either:
  - Split on all values (Location is beach/prison/ski-slope)
  - or Split on equality to one value (location = beach)
- If feature is continuous (temperature) then either:
  - Sort records by feature value and search for best split.
  - or split on parcentiles: 1%,2%,….,99%

Splitting on percentiles

- Suppose data is in an RDD with 100 million examples.
- sorting according to each feature value is very expensive.
- Instead: use Sample(false,0.00001).collect() to get a sample of about 10,000 examples.
- sort the sample (small, sort in head node).
- pick examples at location 100,200,… as boundaries. Call those feature values T1,T2,T3,… ,T99
- Broadcast boundaries to all partitions.
- Each partition computes it's contribution to $P(+1|T_i \leq f \leq T_{i+1})$

Summary

- The splitting criteria is a strictly concave function that bounds the training error.
- The search for a splitting rule tests all possible splits of current tree.
- When the feature is continuous and the data large - split on percentiles instead of splitting on each example.
- Next, Performance on the test set and reducing over-fitting .

Pruning trees

- Trees are very flexible.
- A “fully grown” tree is one where all leaves are “pure”, i.e. each leaf contains all + labeled examples or all - labeled examples.
- A fully grown tree has training error zero.
- If the tree is large and the data is limited, the test error of the tree is likely to be high = the tree overfits the data.
- Statisticians say that trees are “high variance” or “unstable”
- One way to reduce overfitting is “pruning” which means that the fully grown tree is made smaller by “pruning” leaves that have have few examples and contribute little to the training set performance.

Bagging

- Bagging, invented by Leo Breiman in the 90s, is a different way to reduce the variance of trees.
- Instead of pruning the tree, we generate many trees, using randomly selected subsets of the training data.
- We predict using the majority vote over the trees.
- A more sophisticated method to reduce variance, that is currently very popular, is “Random Forests” about which we will talk in a later lesson.

Summary

- Fully grown trees over-fit the data.
- Two ways to reduce over-fitting:
  - Pruning: remove leaves to make the tree smaller.
  - Bagging: take the majority vote over many trees.