a).

1. Sample $X \sim U(-2,2)$ $\rightarrow$ uniform distribution.

$$f(x) = \frac{1}{4} \quad (b=2,\ a=-2) \qquad \text{density is } \frac{1}{b-a}.$$

2. Let $g(x) = e^{-\frac{x^2}{2}}$ $\quad Y = g(X)$.

$$\boxed{E[g(X)]} = \int_{-2}^{2} g(x) f(x)\, dx$$

$$\overset{\text{LLN}}{=} \int_{-2}^{2} \frac{1}{4} \cdot e^{-\frac{x^2}{2}} dx = \frac{1}{4} \boxed{\int_{-2}^{2} e^{-\frac{x^2}{2}} dx}$$

$\bar{Y}_n$ as $n \to \infty$

所以用 $\bar{Y}_n$ estimate $E[g(X)]$.

$E[g(X)]$ is $\frac{1}{4}$ of $\int_{-2}^{2} e^{-\frac{x^2}{2}} dx$

所以 $4 \cdot \bar{Y}_n \Longrightarrow \int_{-2}^{2} e^{-\frac{x^2}{2}} dx$

b). $4\bar{Y}_n$ is the estimate of $\int_{-2}^{2} e^{-\frac{x^2}{2}} dx$

△问题: 为什么你们结果不一样.

① $\bar{X}_n$ is a random variable

② n 不同

所以 正因为 $\bar{Y}_n$ is Random.

$\bar{Y}_n$ 才有变化, 有变化才有 Variance.

$$Var(4\bar{Y}_n) = 16\, Var(\bar{Y}_n)$$

$Y \Leftarrow X$

$\uparrow$

-2 – 2

$$= 16 \cdot \frac{1}{n^2} \cdot n \cdot Var(Y_i)$$

$$= \sqrt{\frac{16}{n} \cdot \underbrace{Var(Y_i)}_{s^2}}$$

n is given.

$s = sd(y_i)$

$$S_{4\bar{Y}_n} = \sqrt{Var(4\bar{Y}_n)} = \boxed{4 \cdot \frac{s}{\sqrt{n}}}$$