Lab Report for
Mixed Integer Linear Programming in Computer Networks

SMZII2D - Resolution Methods - Combinatorial Optimization
1st Year Intl. Master in Computer Science 2017-18

Universite de Nice Sophia-Antipolis – Subject
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1 A Linear Programming Problem in Networks

1.1 The Flow Assignment Problem

Code:

1) **minCost_FA.py:**

```
from __future__ import print_function
from ortools.linear_solver import pywraplp

def main():
  """Solving Assignment Problem with MIP"""
 # Instantiate a mixed-integer solver.
#    solver = pywraplp.Solver('SolveAssignmentProblemMIP',
#                      pywraplp.Solver.CBC_MIXED_INTEGER_PROGRAMMING)
  solver = pywraplp.Solver('SolveSimpleSystem',
               pywraplp.Solver.GLOP_LINEAR_PROGRAMMING)

  #Time limit (ms)
  solver.set_time_limit(50)

  traffic_matrix =   [[0, 12, 15, 23, 10],
                      [12, 0, 2, 7,  3],
                      [15, 2, 0, 4, 5 ],
                      [23, 7, 4, 0, 8 ],
                      [10, 3, 5, 8, 0 ] ]
  linkTable =  [[0, 1],
                [1, 0],
                [0, 2],
                [2, 0],
                [0, 3],
                [3, 0],
                [0, 4],
                [4, 0],
                [1, 2],
                [2, 1],
                [2, 3],
                [3, 2],
                [3, 4],
                [4, 3],
                [4, 1],
                [1, 4]]

  num_nodes = len(traffic_matrix)
  num_flows =  num_nodes*num_nodes
  num_links = len(linkTable)

  #node_link_incidence_matrix (node_id, link_id)   --> topology   3-node ring : 6
unidirectional links
  #link-id = (s_node-1)*N+d_node
  incidence_matrix = [[0 for ij in range(num_links)] for n in range(num_nodes)]
  for ij in range(num_links):
```

```
    a=linkTable[ij][0]
    b=linkTable[ij][1]
    incidence_matrix[a][ij]=1
    incidence_matrix[b][ij]=-1
  capacity = 50
  cost =  1

  x = {}
  for sd in range(num_flows):
    for ij in range(num_links):
      x[sd, ij] = solver.NumVar(0.0, solver.infinity(), 'x[%sd,%ij]' % (sd, ij))

  y = {}
  for ij in range(num_links):
    y[ij] = solver.NumVar(0.0, solver.infinity(),'y[%ij]' % ij)
#       y[ij] = solver.IntVar(0.0, solver.infinity(),'y[%ij]' % ij)

 # Objective
  solver.Minimize(solver.Sum([cost * x[sd, ij]  for sd in range(num_flows) for ij in
range(num_links)]))
  #solver.Minimize(solver.Sum([cost* y[ij]  for ij in range(num_links)]))

 # Constraints
 # Flow conservation ctr:
  for s in range(num_nodes):
    for d in range(num_nodes):
      sd =  (s)*num_nodes+d
      for n in range(num_nodes):
        if n == s: # Source node
          solver.Add(solver.Sum([incidence_matrix[n][ij]*  x[sd,  ij]  for  ij  in
range(num_links)]) == traffic_matrix[s][d])
        elif n== d: # Destination node
          solver.Add(solver.Sum([incidence_matrix[n][ij]*  x[sd,  ij]  for  ij  in
range(num_links)]) == -traffic_matrix[s][d])
        else: # Intermediate node
          solver.Add(solver.Sum([incidence_matrix[n][ij]*  x[sd,  ij]  for  ij  in
range(num_links)]) == 0)

 # Capacity ctr
  capacityCtrs = [0] * num_links
  for ij in range(num_links):
    capacityCtrs[ij] = solver.Constraint(0, capacity)
#     capacityCtrs[ij].SetCoefficient(y[ij], -capacity)
    for sd in range(num_flows):
      capacityCtrs[ij].SetCoefficient(x[sd, ij], 1)
#   for ij in range(num_links):
#     solver.Add(solver.Sum([x[sd, ij] for sd in range(num_flows)]) <= capacity)

  #  Calling Solver
  status = solver.Solve()
  if status == solver.OPTIMAL:
    print ('A  optimal solution was found.')
  else:  # No optimal solution was found.
    if status == solver.FEASIBLE:
      print ('A potentially suboptimal solution was found.')
    else:
      print ('The solver could not solve the problem.')
```

```
        #return
    print('Total cost = ', solver.Objective().Value())
    print("Time = ", solver.WallTime(), " milliseconds")
    print("No. of simplex iterations = ", solver.iterations(), "")

    print()
    for sd in range(num_flows):
        for ij in range(num_links):
            if x[sd, ij].solution_value() > 0:
                print('Flow %d assigned to link %d  Flow = %d  Reduced cost = %d  Basis
status = %d' % (
                sd,
                ij,
                x[sd, ij].solution_value(),
                x[sd, ij].reduced_cost(),
                x[sd, ij].basis_status()))
    print()
    for ij in range(num_links):
        if y[ij].solution_value() > 0:
                print('Capacity assigned to link %d   Num. capacity modules= %f' % (
                ij,
                y[ij].solution_value()))
    print()

    # Only for LP
    for ij in range(num_links):
        print('Link %d   Dual value (price) = %f' % (
        ij,
        capacityCtrs[ij].dual_value()))
    for ij in range(num_links):
        if y[ij].solution_value() > 0:
                print('Capacity assigned to link %d   Reduced cost= %f  Basis status= %f'
% (
                ij,
                y[ij].reduced_cost(),
                y[ij].basis_status()))

#    Advanced usage: possible basis status values for a variable and the slack
variable of a linear constraint.
# enum BasisStatus {
#     FREE = 0,
#     AT_LOWER_BOUND,
#     AT_UPPER_BOUND,
#     FIXED_VALUE,
#     BASIC
#   };
#
    print()

    # Only for ILP
    print("No. of branch-and-bound nodes = ", solver.nodes(), "")
    print("Best objective bound = ", solver.Objective().BestBound(), "")



if __name__ == '__main__':
```

main()

Answers

For Capacity =50

Question 1: Is the problem optimally solved? How many simplex iterations are required?

Ans: *Yes. The problem is optimally solved.*

```
cyril@cyril-vb64:/opt/cyril/or-tools-6.4$ python minCost_FA.py
A  optimal solution was found.
Total cost =  202.0
Time =  53  milliseconds
No. of simplex iterations =  64
```

Simplex Iterations needed: 64

Question 2: The code generates a small report with (1) the values of the reduced costs associated to the decision variables, (2) the basis status of the decision variables and (3) the values of the dual variables[1] associated to the capacity constraints (1c)**. Which conclusions can you draw from the current values of these three parameters in terms of (i) solution optimality/feasibility, (ii) uniqueness optimality (you need to plot the null variables too), and (iii) solution sensitivity to capacity constraints changes?**

Ans:

Output:

```
cyril@cyril-vb64:/opt/cyril/or-tools-6.4$ python minCost_FA.py
A  optimal solution was found.
Total cost =  202.0
Time =  53  milliseconds
No. of simplex iterations =  64

Flow 1 assigned to link 0  Flow = 12  Reduced cost = 0  Basis status = 4
Flow 2 assigned to link 2  Flow = 15  Reduced cost = 0  Basis status = 4
Flow 3 assigned to link 4  Flow = 23  Reduced cost = 0  Basis status = 4
Flow 4 assigned to link 6  Flow = 10  Reduced cost = 0  Basis status = 4
Flow 5 assigned to link 1  Flow = 12  Reduced cost = 0  Basis status = 4
Flow 7 assigned to link 8  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 8 assigned to link 1  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 8 assigned to link 4  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 9 assigned to link 15  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 10 assigned to link 3  Flow = 15  Reduced cost = 0  Basis status = 4
Flow 11 assigned to link 9  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 13 assigned to link 10  Flow = 4  Reduced cost = 0  Basis status = 4
Flow 14 assigned to link 10  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 14 assigned to link 12  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 5  Flow = 23  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 12  Flow = 7  Reduced cost = 0  Basis status = 4
```

```
Flow 16 assigned to link 14  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 17 assigned to link 11  Flow = 4  Reduced cost = 0  Basis status = 4
Flow 19 assigned to link 12  Flow = 8  Reduced cost = 0  Basis status = 4
Flow 20 assigned to link 7  Flow = 10  Reduced cost = 0  Basis status = 4
Flow 21 assigned to link 14  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 2  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 7  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 23 assigned to link 13  Flow = 8  Reduced cost = 0  Basis status = 4


Link 0   Dual value (price) = 0.000000
Link 1   Dual value (price) = 0.000000
Link 2   Dual value (price) = 0.000000
Link 3   Dual value (price) = 0.000000
Link 4   Dual value (price) = 0.000000
Link 5   Dual value (price) = 0.000000
Link 6   Dual value (price) = 0.000000
Link 7   Dual value (price) = 0.000000
Link 8   Dual value (price) = 0.000000
Link 9   Dual value (price) = 0.000000
Link 10   Dual value (price) = 0.000000
Link 11   Dual value (price) = 0.000000
Link 12   Dual value (price) = 0.000000
Link 13   Dual value (price) = 0.000000
Link 14   Dual value (price) = 0.000000
Link 15   Dual value (price) = 0.000000

WARNING: Logging before InitGoogleLogging() is written to STDERR
E1205 16:54:20.219301  3016 glop_interface.cc:259] Number of nodes only available for discrete problems
No. of branch-and-bound nodes =  -1
E1205 16:54:20.219513  3016 glop_interface.cc:264] Best objective bound only available for discrete problems
Best objective bound =  inf
```

(i) Solution optimality/feasibility,

Ans:

- Reduced Cost indicates how much the objective function coefficient on the corresponding variable must be improved before the value of the variable will be positive in the optimal solution.
- In our case all the reduced cost for the decision variables is zero, so the cost cannot be reduced in objective function if decision variable changes so the current solution is optimal.

(ii) uniqueness optimality (you need to plot the null variables too), and

Ans:

- For uniqueness optimality let us consider the value of a variable in our case: **Flow** and the reduced cost.
- If we can notice the output for flow variable is a positive value and non-zero for all flows and also the reduced value is zero or all the values which proves that the optimal solution obtained is unique.
- If in case the optimal value of the variable is zero as well as the reduced value is zero then there is a possibility for the occurrence of yet another optimal value in the corner, but in our case it is unique optimal

Output:

```
Flow 1 assigned to link 0  Flow = 12  Reduced cost = 0  Basis status = 4
```

```
Flow 2 assigned to link 2  Flow = 15  Reduced cost = 0  Basis status = 4
Flow 3 assigned to link 4  Flow = 23  Reduced cost = 0  Basis status = 4
Flow 4 assigned to link 6  Flow = 10  Reduced cost = 0  Basis status = 4
Flow 5 assigned to link 1  Flow = 12  Reduced cost = 0  Basis status = 4
Flow 7 assigned to link 8  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 8 assigned to link 1  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 8 assigned to link 4  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 9 assigned to link 15  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 10 assigned to link 3  Flow = 15  Reduced cost = 0  Basis status = 4
Flow 11 assigned to link 9  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 13 assigned to link 10  Flow = 4  Reduced cost = 0  Basis status = 4
Flow 14 assigned to link 10  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 14 assigned to link 12  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 5  Flow = 23  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 12  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 14  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 17 assigned to link 11  Flow = 4  Reduced cost = 0  Basis status = 4
Flow 19 assigned to link 12  Flow = 8  Reduced cost = 0  Basis status = 4
Flow 20 assigned to link 7  Flow = 10  Reduced cost = 0  Basis status = 4
Flow 21 assigned to link 14  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 2  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 7  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 23 assigned to link 13  Flow = 8  Reduced cost = 0  Basis status = 4
```

(iii) solution sensitivity to capacity constraints changes?

Ans:

- Dual value represents the improvement in objective function if the capacity constraint in our case is relaxed by one unit.
- Also the basis status is 4 which implies an enum of 4: BASIC so all the decision variable is basic and binding to the constraint
- But here our dual value is 0 which implies even if constraint is released the objective function has no impact and it has not reached the limit of the constraint which does not make it too dependent and so the release of the constraint doesn't reduce the cost objective function

For Capacity 20:

Question 1: Is the problem optimally solved? How many simplex iterations are required?

Ans: *Yes. The problem is optimally solved.*

```
cyril@cyril-vb64:/opt/cyril/or-tools-6.4$ python minCost_FA.py
A  optimal solution was found.
Total cost =  208.0
Time =  38  milliseconds
No. of simplex iterations =  85
```

- *Here the total cost is 208 but for the earlier case of capacity 50 is 202 and the simplex iteration has increased from 64 to 85.*
- *So in here since the capacity is reduced the flow assignments are changed and the total length of flow path increases as the capacity constraint is reduced.*

Simplex Iterations needed: 85

Question 2: The code generates a small report with (1) the values of the reduced costs associated to the decision variables, (2) the basis status of the decision variables

and (3) the values of the dual variables[1] associated to the capacity constraints (1c). **Which conclusions can you draw from the current values of these three parameters in terms of (i) solution optimality/feasibility, (ii) uniqueness optimality (you need to plot the null variables too), and (iii) solution sensitivity to capacity constraints changes?**

Ans:

Output:

```
cyril@cyril-vb64:/opt/cyril/or-tools-6.4$ python minCost_FA.py
A  optimal solution was found.
Total cost =  208.0
Time =  38  milliseconds
No. of simplex iterations =  85

Flow 1 assigned to link 0  Flow = 12  Reduced cost = 0  Basis status = 4
Flow 2 assigned to link 2  Flow = 15  Reduced cost = 0  Basis status = 4
Flow 3 assigned to link 2  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 3 assigned to link 4  Flow = 20  Reduced cost = 0  Basis status = 4
Flow 3 assigned to link 10  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 4 assigned to link 6  Flow = 10  Reduced cost = 0  Basis status = 4
Flow 5 assigned to link 1  Flow = 12  Reduced cost = 0  Basis status = 4
Flow 7 assigned to link 8  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 8 assigned to link 8  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 8 assigned to link 10  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 9 assigned to link 15  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 10 assigned to link 3  Flow = 15  Reduced cost = 0  Basis status = 4
Flow 11 assigned to link 9  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 13 assigned to link 10  Flow = 4  Reduced cost = 0  Basis status = 4
Flow 14 assigned to link 10  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 14 assigned to link 12  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 3  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 5  Flow = 20  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 11  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 12  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 14  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 17 assigned to link 11  Flow = 4  Reduced cost = 0  Basis status = 4
Flow 19 assigned to link 12  Flow = 8  Reduced cost = 0  Basis status = 4
Flow 20 assigned to link 7  Flow = 10  Reduced cost = 0  Basis status = 4
Flow 21 assigned to link 14  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 11  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 13  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 23 assigned to link 13  Flow = 8  Reduced cost = 0  Basis status = 4


Link 0   Dual value (price) = 0.000000
Link 1   Dual value (price) = 0.000000
Link 2   Dual value (price) = 0.000000
Link 3   Dual value (price) = 0.000000
Link 4   Dual value (price) = -1.000000
Link 5   Dual value (price) = -1.000000
Link 6   Dual value (price) = 0.000000
Link 7   Dual value (price) = 0.000000
Link 8   Dual value (price) = 0.000000
Link 9   Dual value (price) = 0.000000
Link 10   Dual value (price) = 0.000000
```

```
Link 11   Dual value (price) = 0.000000
Link 12   Dual value (price) = 0.000000
Link 13   Dual value (price) = 0.000000
Link 14   Dual value (price) = 0.000000
Link 15   Dual value (price) = 0.000000

WARNING: Logging before InitGoogleLogging() is written to STDERR
E1205 16:38:26.808743  2952 glop_interface.cc:259] Number of nodes only available for discrete problems
No. of branch-and-bound nodes =  -1
E1205 16:38:26.808933 2952 glop_interface.cc:264] Best objective bound only available for discrete problems
Best objective bound =  inf
```

(i) solution optimality/feasibility

Ans:

- Here the solution achieved is optimal, also reduced cost is 0 but the cost has increased in this case which can be attributed to the capacity constraint change from 50 to 20 but an optimal solution exists since the reduced cost cannot be reduced further.

(ii) uniqueness optimality (you need to plot the null variables too)

- Reduced Cost indicates how much the objective function coefficient on the corresponding variable must be improved before the value of the variable will be positive in the optimal solution.
- In our case all the reduced cost for the decision variables is zero, so the cost cannot be reduced in objective function if decision variable changes so the current solution is optimal.

```
Flow 1 assigned to link 0  Flow = 12  Reduced cost = 0  Basis status = 4
Flow 2 assigned to link 2  Flow = 15  Reduced cost = 0  Basis status = 4
Flow 3 assigned to link 2  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 3 assigned to link 4  Flow = 20  Reduced cost = 0  Basis status = 4
Flow 3 assigned to link 10  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 4 assigned to link 6  Flow = 10  Reduced cost = 0  Basis status = 4
Flow 5 assigned to link 1  Flow = 12  Reduced cost = 0  Basis status = 4
Flow 7 assigned to link 8  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 8 assigned to link 8  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 8 assigned to link 10  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 9 assigned to link 15  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 10 assigned to link 3  Flow = 15  Reduced cost = 0  Basis status = 4
Flow 11 assigned to link 9  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 13 assigned to link 10  Flow = 4  Reduced cost = 0  Basis status = 4
Flow 14 assigned to link 10  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 14 assigned to link 12  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 3  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 5  Flow = 20  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 11  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 12  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 14  Flow = 7  Reduced cost = 0  Basis status = 4
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Flow 19 assigned to link 12  Flow = 8  Reduced cost = 0  Basis status = 4
Flow 20 assigned to link 7  Flow = 10  Reduced cost = 0  Basis status = 4
Flow 21 assigned to link 14  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 11  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 13  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 23 assigned to link 13  Flow = 8  Reduced cost = 0  Basis status = 4
```

(iii) solution sensitivity to capacity constraints changes?

- Dual value for the link 4 and 5 is -1 and not 0 which implies that a one unit change in the value would reduce the total cost value. But for other links the data is less than 20 units so the dual value is 0.For link 4 and 5 the capacity of 20 is already achieved and they are in full capacity which means an increase in capacity by one affects significantly the cost and the solution.
- Also the basis status is 4 which implies an enum of 4: BASIC so all the decision variable is basic and binding to the constraint

For Capacity 15:

Question 1: Is the problem optimally solved? How many simplex iterations are required?

Ans: *Yes. The problem is optimally solved.*

```
cyril@cyril-vb64:/opt/cyril/or-tools-6.4$ python minCost_FA.py
A optimal solution was found.
Total cost = 224.0
Time = 28 milliseconds
No. of simplex iterations = 92
```

Here the total cost and the iterations are much more than the previous cases. Since the capacity is reduced drastically to 15 the flow assignments are changed considerably and thereby increasing the total cost since the flow length is increased due to capacity constraint limitation.

Simplex Iterations needed: 92

Question 2: The code generates a small report with (1) the values of the reduced costs associated to the decision variables, (2) the basis status of the decision variables and (3) the values of the dual variables[1] associated to the capacity constraints (1c)**. Which conclusions can you draw from the current values of these three parameters in terms of (i) solution optimality/feasibility, (ii) uniqueness optimality (you need to plot the null variables too), and (iii) solution sensitivity to capacity constraints changes?**

Output:

```
cyril@cyril-vb64:/opt/cyril/or-tools-6.4$ python minCost_FA.py
A optimal solution was found.
Total cost = 224.0
Time = 28 milliseconds
No. of simplex iterations = 92

Flow 1 assigned to link 0 Flow = 12 Reduced cost = 0 Basis status = 4
Flow 2 assigned to link 2 Flow = 15 Reduced cost = 0 Basis status = 4
Flow 3 assigned to link 0 Flow = 3 Reduced cost = 0 Basis status = 4
Flow 3 assigned to link 4 Flow = 15 Reduced cost = 0 Basis status = 4
Flow 3 assigned to link 6 Flow = 5 Reduced cost = 0 Basis status = 4
Flow 3 assigned to link 8 Flow = 3 Reduced cost = 0 Basis status = 4
Flow 3 assigned to link 10 Flow = 3 Reduced cost = 0 Basis status = 4
Flow 3 assigned to link 13 Flow = 5 Reduced cost = 0 Basis status = 4
Flow 4 assigned to link 6 Flow = 10 Reduced cost = 0 Basis status = 4
Flow 5 assigned to link 1 Flow = 12 Reduced cost = 0 Basis status = 4
```

```
Flow 7 assigned to link 8  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 8 assigned to link 8  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 8 assigned to link 10  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 9 assigned to link 15  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 10 assigned to link 1  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 10 assigned to link 3  Flow = 12  Reduced cost = 0  Basis status = 4
Flow 10 assigned to link 9  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 11 assigned to link 9  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 13 assigned to link 10  Flow = 4  Reduced cost = 0  Basis status = 4
Flow 14 assigned to link 9  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 14 assigned to link 15  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 3  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 5  Flow = 15  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 7  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 11  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 12  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 9  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 11  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 12  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 14  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 17 assigned to link 11  Flow = 4  Reduced cost = 0  Basis status = 4
Flow 19 assigned to link 12  Flow = 8  Reduced cost = 0  Basis status = 4
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Flow 21 assigned to link 14  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 8  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 11  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 13  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 14  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 23 assigned to link 13  Flow = 8  Reduced cost = 0  Basis status = 4


Link 0   Dual value (price) = 0.000000
Link 1   Dual value (price) = 0.000000
Link 2   Dual value (price) = -1.000000
Link 3   Dual value (price) = -1.000000
Link 4   Dual value (price) = -2.000000
Link 5   Dual value (price) = -2.000000
Link 6   Dual value (price) = -1.000000
Link 7   Dual value (price) = -1.000000
Link 8   Dual value (price) = 0.000000
Link 9   Dual value (price) = 0.000000
Link 10   Dual value (price) = 0.000000
Link 11   Dual value (price) = 0.000000
Link 12   Dual value (price) = 0.000000
Link 13   Dual value (price) = 0.000000
Link 14   Dual value (price) = 0.000000
Link 15   Dual value (price) = 0.000000

WARNING: Logging before InitGoogleLogging() is written to STDERR
E1205 16:42:57.421447  2969 glop_interface.cc:259] Number of nodes only available for discrete problems
No. of branch-and-bound nodes =  -1
E1205 16:42:57.421583  2969 glop_interface.cc:264] Best objective bound only available for discrete problems
Best objective bound =  inf
```

(i) solution optimality/feasibility,

- Here the solution is optimal since reduced cost is 0 for all flows but the cost has increased in this case which can be attributed to the capacity constraint change from 50 to 15 but an optimal solution exists since the reduced cost cannot be reduced further.

(iii) uniqueness optimality (you need to plot the null variables too)

Output:

```
Flow 1 assigned to link 0  Flow = 12  Reduced cost = 0  Basis status = 4
Flow 2 assigned to link 2  Flow = 15  Reduced cost = 0  Basis status = 4
Flow 3 assigned to link 0  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 3 assigned to link 4  Flow = 15  Reduced cost = 0  Basis status = 4
Flow 3 assigned to link 6  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 3 assigned to link 8  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 3 assigned to link 10  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 3 assigned to link 13  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 4 assigned to link 6  Flow = 10  Reduced cost = 0  Basis status = 4
Flow 5 assigned to link 1  Flow = 12  Reduced cost = 0  Basis status = 4
Flow 7 assigned to link 8  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 8 assigned to link 8  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 8 assigned to link 10  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 9 assigned to link 15  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 10 assigned to link 1  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 10 assigned to link 3  Flow = 12  Reduced cost = 0  Basis status = 4
Flow 10 assigned to link 9  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 11 assigned to link 9  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 13 assigned to link 10  Flow = 4  Reduced cost = 0  Basis status = 4
Flow 14 assigned to link 9  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 14 assigned to link 15  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 3  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 5  Flow = 15  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 7  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 11  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 12  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 9  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 11  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 12  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 14  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 17 assigned to link 11  Flow = 4  Reduced cost = 0  Basis status = 4
Flow 19 assigned to link 12  Flow = 8  Reduced cost = 0  Basis status = 4
Flow 20 assigned to link 7  Flow = 10  Reduced cost = 0  Basis status = 4
Flow 21 assigned to link 14  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 8  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 11  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 13  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 14  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 23 assigned to link 13  Flow = 8  Reduced cost = 0  Basis status = 4
```

- Reduced Cost indicates how much the objective function coefficient on the corresponding variable must be improved before the value of the variable will be positive in the optimal solution.
- In our case all the reduced cost for the decision variables is zero, so the cost cannot be reduced in objective function if decision variable changes so the current solution is optimal.

(iii) solution sensitivity to capacity constraints changes?

- Here since the limited capacity constraint is 15 which is less than for certain flow path capacity , dual value for Link 2, Link3 is **-1** Link4, Link5 is **-2** and link6, Link7 is **-1** which implies that the constraint when slightly relaxed will have an impact on the dual value and improve the cost and flow path.
- Also the basis status is 4 which implies an enum of 4: BASIC so all the decision variable is basic and binding to the constraint

Capacity 10:

Question 1: Is the problem optimally solved? How many simplex iterations are required?

Ans: No. The problem is not optimally solved.

```
cyril@cyril-vb64:/opt/cyril/or-tools-6.4$ python minCost_FA.py
The solver could not solve the problem.
WARNING: Logging before InitGoogleLogging() is written to STDERR
E1205 16:44:31.190575  2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Total cost =  0.0
Time =  33  milliseconds
No. of simplex iterations =  69
```

Simplex Iterations needed or iterations run: 69 ,Simplex Iterations run are 69 but not an optimal solution achieved.

Question 2: The code generates a small report with (1) the values of the reduced costs associated to the decision variables, (2) the basis status of the decision variables and (3) the values of the dual variables[1] associated to the capacity constraints (1c)**. Which conclusions can you draw from the current values of these three parameters in terms of (i) solution optimality/feasibility, (ii) uniqueness optimality (you need to plot the null variables too), and (iii) solution sensitivity to capacity constraints changes?**

```
cyril@cyril-vb64:/opt/cyril/or-tools-6.4$ python minCost_FA.py
The solver could not solve the problem.
WARNING: Logging before InitGoogleLogging() is written to STDERR
E1205 16:44:31.190575  2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Total cost =  0.0
Time =  33  milliseconds
No. of simplex iterations =  69

E1205 16:44:31.192698  2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.192764  2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.192873  2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.192929  2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
```

E1205 16:44:31.192970 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193011 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193050 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193090 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193130 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193174 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193209 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193244 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193280 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193316 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193351 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193387 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193424 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193461 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193497 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193532 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193568 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193603 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193639 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193675 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193709 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.193744 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.210213 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.210242 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.210269 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.210296 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.210325 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.210352 2975 linear_solver.cc:1287] No solution exists. MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE

E1205 16:44:31.210381 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.210408 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.210480 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212442 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE

E1205 16:44:31.212515 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212546 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212574 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212602 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212641 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212668 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212695 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212723 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212750 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212777 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212805 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212831 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212858 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212885 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212913 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212939 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE

E1205 16:44:31.212977 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Link 0 Dual value (price) = 0.000000
E1205 16:44:31.213050 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Link 1 Dual value (price) = 0.000000
E1205 16:44:31.213104 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Link 2 Dual value (price) = 0.000000
E1205 16:44:31.213155 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Link 3 Dual value (price) = 0.000000
E1205 16:44:31.213205 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Link 4 Dual value (price) = 0.000000

E1205 16:44:31.213241 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Link 5 Dual value (price) = 0.000000
E1205 16:44:31.213291 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Link 6 Dual value (price) = 0.000000
E1205 16:44:31.213341 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Link 7 Dual value (price) = 0.000000
E1205 16:44:31.213448 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Link 8 Dual value (price) = 0.000000
E1205 16:44:31.213496 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Link 9 Dual value (price) = 0.000000
E1205 16:44:31.213542 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Link 10 Dual value (price) = 0.000000
E1205 16:44:31.213587 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Link 11 Dual value (price) = 0.000000
E1205 16:44:31.213632 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Link 12 Dual value (price) = 0.000000
E1205 16:44:31.213665 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Link 13 Dual value (price) = 0.000000
E1205 16:44:31.213711 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Link 14 Dual value (price) = 0.000000
E1205 16:44:31.213742 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
Link 15 Dual value (price) = 0.000000
E1205 16:44:31.213789 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.213816 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.213842 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.213867 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.213892 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.213917 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.213941 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.213966 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.213991 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.214015 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.214040 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.214066 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE

E1205 16:44:31.214089 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.214114 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.214139 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.214164 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE

E1205 16:44:31.214207 2975 glop_interface.cc:259] Number of nodes only available for discrete problems
No. of branch-and-bound nodes = -1
E1205 16:44:31.214257 2975 glop_interface.cc:264] Best objective bound only available for discrete problems
Best objective bound = inf

(1) the values of the reduced costs associated to the decision variables,
(2) the basis status of the decision variables and
(3) the values of the dual variables[1] associated to the capacity constraints

(i) solution optimality/feasibility,

Here we are not able to achieve an optimal solution the link capacity of 10 makes it unable to satisfy the flow path and hence the data flow is affected by this low capacity constraint.

(ii) uniqueness optimality (you need to plot the null variables too)

Output:

E1205 16:44:31.212574 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212602 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212641 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE
E1205 16:44:31.212668 2975 linear_solver.cc:1287] No solution exists.
MPSolverInterface::result_status_ = MPSOLVER_INFEASIBLE

Since the problem is not optimally we cannot check for uniqueness for the value

(iii) solution sensitivity to capacity constraints changes?

Solution sensitivity to capacity constraint is quite dependent , since we are unable to arrive at an optimal solution as well as any dual values , increasing the capacity constraint by a margin will improve the flow path and may help in arriving at a better feasible solution.

1.2 The Capacity Flow Assignment Problem

minCost_CFA.py:

```
from __future__ import print_function
from ortools.linear_solver import pywraplp

def main():
  """Solving Assignment Problem with MIP"""
 # Instantiate a mixed-integer solver.
#    solver = pywraplp.Solver('SolveAssignmentProblemMIP',
#                      pywraplp.Solver.CBC_MIXED_INTEGER_PROGRAMMING)
  solver = pywraplp.Solver('SolveSimpleSystem',
               pywraplp.Solver.GLOP_LINEAR_PROGRAMMING)

  #Time limit (ms)
  solver.set_time_limit(50)

  traffic_matrix =   [[0, 12, 15, 23, 10],
                      [12, 0, 2, 7,  3],
                      [15, 2, 0, 4, 5 ],
                      [23, 7, 4, 0, 8 ],
                      [10, 3, 5, 8, 0 ] ]
  linkTable =  [[0, 1],
                [1, 0],
                [0, 2],
                [2, 0],
                [0, 3],
                [3, 0],
                [0, 4],
                [4, 0],
                [1, 2],
                [2, 1],
                [2, 3],
                [3, 2],
                [3, 4],
                [4, 3],
                [4, 1],
                [1, 4]]

  num_nodes = len(traffic_matrix)
  num_flows =  num_nodes*num_nodes
  num_links = len(linkTable)

  #node_link_incidence_matrix (node_id, link_id)   --> topology   3-node ring : 6
unidirectional links
  #link-id = (s_node-1)*N+d_node
  incidence_matrix = [[0 for ij in range(num_links)] for n in range(num_nodes)]
  for ij in range(num_links):
     a=linkTable[ij][0]
     b=linkTable[ij][1]
     incidence_matrix[a][ij]=1
```

```
    incidence_matrix[b][ij]=-1
  capacity = 15
  cost =  1

  x = {}
  for sd in range(num_flows):
    for ij in range(num_links):
      x[sd, ij] = solver.NumVar(0.0, solver.infinity(), 'x[%sd,%ij]' % (sd, ij))

  y = {}
  for ij in range(num_links):
    y[ij] = solver.NumVar(0.0, solver.infinity(),'y[%ij]' % ij)
#       y[ij] = solver.IntVar(0.0, solver.infinity(),'y[%ij]' % ij)

 # Objective
  #solver.Minimize(solver.Sum([cost * x[sd, ij]  for sd in range(num_flows) for ij
in range(num_links)]))
  solver.Minimize(solver.Sum([cost* y[ij]  for ij in range(num_links)]))

 # Constraints
 # Flow conservation ctr:
  for s in range(num_nodes):
    for d in range(num_nodes):
      sd =  (s)*num_nodes+d
      for n in range(num_nodes):
        if n == s: # Source node
          solver.Add(solver.Sum([incidence_matrix[n][ij]*  x[sd,  ij]  for  ij  in
range(num_links)]) == traffic_matrix[s][d])
        elif n== d: # Destination node
          solver.Add(solver.Sum([incidence_matrix[n][ij]*  x[sd,  ij]  for  ij  in
range(num_links)]) == -traffic_matrix[s][d])
        else: # Intermediate node
          solver.Add(solver.Sum([incidence_matrix[n][ij]*  x[sd,  ij]  for  ij  in
range(num_links)]) == 0)

 # Capacity ctr
  capacityCtrs = [0] * num_links
  for ij in range(num_links):
#     capacityCtrs[ij] = solver.Constraint(0, capacity)
   capacityCtrs[ij] = solver.Constraint(-solver.infinity(), 0.0)
   capacityCtrs[ij].SetCoefficient(y[ij], -1)
   for sd in range(num_flows):
      capacityCtrs[ij].SetCoefficient(x[sd, ij], 1)
#   for ij in range(num_links):
#     solver.Add(solver.Sum([x[sd, ij] for sd in range(num_flows)]) <= capacity)

  #  Calling Solver
  status = solver.Solve()
  if status == solver.OPTIMAL:
    print ('A  optimal solution was found.')
  else:  # No optimal solution was found.
    if status == solver.FEASIBLE:
      print ('A potentially suboptimal solution was found.')
    else:
       print ('The solver could not solve the problem.')
       #return
  print('Total cost = ', solver.Objective().Value())
```

```
  print("Time = ", solver.WallTime(), " milliseconds")
  print("No. of simplex iterations = ", solver.iterations(), "")

  print()
  for sd in range(num_flows):
    for ij in range(num_links):
      if x[sd, ij].solution_value() > 0:
        print('Flow %d assigned to link %d  Flow = %d  Reduced cost = %d  Basis
status = %d' % (
          sd,
          ij,
          x[sd, ij].solution_value(),
          x[sd, ij].reduced_cost(),
          x[sd, ij].basis_status()))
  print()
  for ij in range(num_links):
    if y[ij].solution_value() > 0:
        print('Capacity assigned to link %d   Num. capacity modules= %f' % (
          ij,
          y[ij].solution_value()))
  print()

  # Only for LP
  for ij in range(num_links):
    print('Link %d   Dual value (price) = %f' % (
    ij,
    capacityCtrs[ij].dual_value()))
  for ij in range(num_links):
    if y[ij].solution_value() > 0:
        print('Capacity assigned to link %d   Reduced cost= %f  Basis status= %f'
% (
          ij,
          y[ij].reduced_cost(),
          y[ij].basis_status()))

#   Advanced usage: possible basis status values for a variable and the slack
variable of a linear constraint.
# enum BasisStatus {
#    FREE = 0,
#    AT_LOWER_BOUND,
#    AT_UPPER_BOUND,
#    FIXED_VALUE,
#    BASIC
#   };
#
  print()

  # Only for ILP
  print("No. of branch-and-bound nodes = ", solver.nodes(), "")
  print("Best objective bound = ", solver.Objective().BestBound(), "")



if __name__ == '__main__':
  main()
```

Output:

```
cyril@cyril-vb64:/opt/cyril/or-tools-6.4$ python minCost_CFA.py
A  optimal solution was found.
Total cost =  202.0
Time =  24  milliseconds
No. of simplex iterations =  48

Flow 1 assigned to link 0  Flow = 12  Reduced cost = 0  Basis status = 4
Flow 2 assigned to link 2  Flow = 15  Reduced cost = 0  Basis status = 4
Flow 3 assigned to link 4  Flow = 23  Reduced cost = 0  Basis status = 4
Flow 4 assigned to link 6  Flow = 10  Reduced cost = 0  Basis status = 4
Flow 5 assigned to link 1  Flow = 12  Reduced cost = 0  Basis status = 4
Flow 7 assigned to link 8  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 8 assigned to link 8  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 8 assigned to link 10  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 9 assigned to link 15  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 10 assigned to link 3  Flow = 15  Reduced cost = 0  Basis status = 4
Flow 11 assigned to link 9  Flow = 2  Reduced cost = 0  Basis status = 4
Flow 13 assigned to link 10  Flow = 4  Reduced cost = 0  Basis status = 4
Flow 14 assigned to link 10  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 14 assigned to link 12  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 15 assigned to link 5  Flow = 23  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 12  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 16 assigned to link 14  Flow = 7  Reduced cost = 0  Basis status = 4
Flow 17 assigned to link 11  Flow = 4  Reduced cost = 0  Basis status = 4
Flow 19 assigned to link 12  Flow = 8  Reduced cost = 0  Basis status = 4
Flow 20 assigned to link 7  Flow = 10  Reduced cost = 0  Basis status = 4
Flow 21 assigned to link 14  Flow = 3  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 2  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 22 assigned to link 7  Flow = 5  Reduced cost = 0  Basis status = 4
Flow 23 assigned to link 13  Flow = 8  Reduced cost = 0  Basis status = 4

Capacity assigned to link 0   Num. capacity modules= 12.000000
Capacity assigned to link 1   Num. capacity modules= 12.000000
Capacity assigned to link 2   Num. capacity modules= 20.000000
Capacity assigned to link 3   Num. capacity modules= 15.000000
Capacity assigned to link 4   Num. capacity modules= 23.000000
Capacity assigned to link 5   Num. capacity modules= 23.000000
Capacity assigned to link 6   Num. capacity modules= 10.000000
Capacity assigned to link 7   Num. capacity modules= 15.000000
Capacity assigned to link 8   Num. capacity modules= 9.000000
Capacity assigned to link 9   Num. capacity modules= 2.000000
Capacity assigned to link 10   Num. capacity modules= 16.000000
Capacity assigned to link 11   Num. capacity modules= 4.000000
Capacity assigned to link 12   Num. capacity modules= 20.000000
Capacity assigned to link 13   Num. capacity modules= 8.000000
Capacity assigned to link 14   Num. capacity modules= 10.000000
Capacity assigned to link 15   Num. capacity modules= 3.000000

Link 0   Dual value (price) = -1.000000
Link 1   Dual value (price) = -1.000000
```

```
Link 2   Dual value (price) = -1.000000
Link 3   Dual value (price) = -1.000000
Link 4   Dual value (price) = -1.000000
Link 5   Dual value (price) = -1.000000
Link 6   Dual value (price) = -1.000000
Link 7   Dual value (price) = -1.000000
Link 8   Dual value (price) = -1.000000
Link 9   Dual value (price) = -1.000000
Link 10   Dual value (price) = -1.000000
Link 11   Dual value (price) = -1.000000
Link 12   Dual value (price) = -1.000000
Link 13   Dual value (price) = -1.000000
Link 14   Dual value (price) = -1.000000
Link 15   Dual value (price) = -1.000000
Capacity assigned to link 0   Reduced cost= 0.000000  Basis status= 4.000000
Capacity assigned to link 1   Reduced cost= 0.000000  Basis status= 4.000000
Capacity assigned to link 2   Reduced cost= 0.000000  Basis status= 4.000000
Capacity assigned to link 3   Reduced cost= 0.000000  Basis status= 4.000000
Capacity assigned to link 4   Reduced cost= 0.000000  Basis status= 4.000000
Capacity assigned to link 5   Reduced cost= 0.000000  Basis status= 4.000000
Capacity assigned to link 6   Reduced cost= 0.000000  Basis status= 4.000000
Capacity assigned to link 7   Reduced cost= 0.000000  Basis status= 4.000000
Capacity assigned to link 8   Reduced cost= 0.000000  Basis status= 4.000000
Capacity assigned to link 9   Reduced cost= 0.000000  Basis status= 4.000000
Capacity assigned to link 10   Reduced cost= 0.000000  Basis status= 4.000000
Capacity assigned to link 11   Reduced cost= 0.000000  Basis status= 4.000000
Capacity assigned to link 12   Reduced cost= 0.000000  Basis status= 4.000000
Capacity assigned to link 13   Reduced cost= 0.000000  Basis status= 4.000000
Capacity assigned to link 14   Reduced cost= 0.000000  Basis status= 4.000000
Capacity assigned to link 15   Reduced cost= 0.000000  Basis status= 4.000000

WARNING: Logging before InitGoogleLogging() is written to STDERR
E1206 05:28:52.344914   3738 glop_interface.cc:259] Number of nodes only
available for discrete problems
No. of branch-and-bound nodes =  -1
E1206 05:28:52.345077  3738 glop_interface.cc:264] Best objective bound only
available for discrete problems
Best objective bound =  inf
```

(i) solution optimality/feasibility,

```
cyril@cyril-vb64:/opt/cyril/or-tools-6.4$ python minCost_CFA.py
A  optimal solution was found.
Total cost =  202.0
Time =  24  milliseconds
No. of simplex iterations =  48
```

In Capacity Flow Assignment problem we have the objective to minimize the capacity in the network and here in the objective function boundary of the capacity is changed to a positive variable

Also the objective function is provided by the minimal sum of capacity in networks and is variable, Here in this case the solution is optimally solved with 48 iterations.

(ii) uniqueness optimality (you need to plot the null variables too)

- Reduced Cost indicates how much the objective function coefficient on the corresponding variable must be improved before the value of the variable will be positive in the optimal solution.
- In our case all the reduced cost for the decision variables is zero, so the cost cannot be reduced in objective function if decision variable changes so the current solution is optimal

(iii) solution sensitivity to capacity constraints changes?

- Solution sensitivity to capacity constraint is quite dependent , since we are unable to arrive at an optimal solution as well as any dual values , increasing the capacity constraint by a margin will improve the flow path and may help in arriving at a better feasible solution.
- Also the dual value for each link is -1 which denotes that the capacity constraint release would lower the total network capacity. The Basis Status enum is 4 which implies that the variable is binding with the constraint

2 A Mixed Linear Programming Problem in Networks

2) minCost_CFA_MILP.py:

```
from __future__ import print_function
from ortools.linear_solver import pywraplp

def main():
  """Solving Assignment Problem with MIP"""
 # Instantiate a mixed-integer solver.
  solver = pywraplp.Solver('SolveAssignmentProblemMIP',
                pywraplp.Solver.CBC_MIXED_INTEGER_PROGRAMMING)
  #solver = pywraplp.Solver('SolveSimpleSystem',
   #             pywraplp.Solver.GLOP_LINEAR_PROGRAMMING)

  #Time limit (ms)
  solver.set_time_limit(100)

  traffic_matrix =   [[0, 12, 15, 23, 10],
                      [12, 0, 2, 7,  3],
                      [15, 2, 0, 4, 5 ],
                      [23, 7, 4, 0, 8 ],
                      [10, 3, 5, 8, 0 ] ]
  linkTable =  [[0, 1],
              [1, 0],
              [0, 2],
              [2, 0],
              [0, 3],
              [3, 0],
              [0, 4],
              [4, 0],
              [1, 2],
              [2, 1],
              [2, 3],
              [3, 2],
              [3, 4],
              [4, 3],
              [4, 1],
              [1, 4]]
```

```
  num_nodes = len(traffic_matrix)
  num_flows =  num_nodes*num_nodes
  num_links = len(linkTable)

  #node_link_incidence_matrix (node_id, link_id)   --> topology   3-node ring : 6
unidirectional links
  #link-id = (s_node-1)*N+d_node
  incidence_matrix = [[0 for ij in range(num_links)] for n in range(num_nodes)]
  for ij in range(num_links):
     a=linkTable[ij][0]
     b=linkTable[ij][1]
     incidence_matrix[a][ij]=1
     incidence_matrix[b][ij]=-1
  capacity = 50
  cost =  1

  x = {}
  for sd in range(num_flows):
     for ij in range(num_links):
        x[sd, ij] = solver.NumVar(0.0, solver.infinity(), 'x[%sd,%ij]' % (sd, ij))

  y = {}
  for ij in range(num_links):
     #y[ij] = solver.NumVar(0.0, solver.infinity(),'y[%ij]' % ij)
     y[ij] = solver.IntVar(0.0, solver.infinity(),'y[%ij]' % ij)

 # Objective
  #solver.Minimize(solver.Sum([cost * x[sd, ij]  for sd in range(num_flows) for ij
in range(num_links)]))
  solver.Minimize(solver.Sum([cost* y[ij]  for ij in range(num_links)]))

 # Constraints
 # Flow conservation ctr:
  for s in range(num_nodes):
     for d in range(num_nodes):
        sd =  (s)*num_nodes+d
        for n in range(num_nodes):
           if n == s: # Source node
              solver.Add(solver.Sum([incidence_matrix[n][ij]*  x[sd,  ij]  for  ij  in
range(num_links)]) == traffic_matrix[s][d])
           elif n== d: # Destination node
              solver.Add(solver.Sum([incidence_matrix[n][ij]*  x[sd,  ij]  for  ij  in
range(num_links)]) == -traffic_matrix[s][d])
           else: # Intermediate node
              solver.Add(solver.Sum([incidence_matrix[n][ij]*  x[sd,  ij]  for  ij  in
range(num_links)]) == 0)

 # Capacity ctr
  capacityCtrs = [0] * num_links
  for ij in range(num_links):
     capacityCtrs[ij] = solver.Constraint(-solver.infinity(),0.0)
     capacityCtrs[ij].SetCoefficient(y[ij], -capacity)
     for sd in range(num_flows):
        capacityCtrs[ij].SetCoefficient(x[sd, ij], 1)
#    for ij in range(num_links):
#       solver.Add(solver.Sum([x[sd, ij] for sd in range(num_flows)]) <= capacity)
```

```
  #   Calling Solver
  status = solver.Solve()
  if status == solver.OPTIMAL:
     print ('A  optimal solution was found.')
  else:  # No optimal solution was found.
     if status == solver.FEASIBLE:
        print ('A potentially suboptimal solution was found.')
     else:
         print ('The solver could not solve the problem.')
         #return
  print('Total cost = ', solver.Objective().Value())
  print("Time = ", solver.WallTime(), " milliseconds")
  print("No. of simplex iterations = ", solver.iterations(), "")

  print()
  for sd in range(num_flows):
     for ij in range(num_links):
        if x[sd, ij].solution_value() > 0:
           print('Flow %d assigned to link %d  Flow = %d  Reduced cost = %d  Basis
status = %d' % (
           sd,
           ij,
           x[sd, ij].solution_value(),
           x[sd, ij].reduced_cost(),
           x[sd, ij].basis_status()))
  print()
  for ij in range(num_links):
     if y[ij].solution_value() > 0:
           print('Capacity assigned to link %d   Num. capacity modules= %f' % (
           ij,
           y[ij].solution_value()))
  print()

  # Only for LP
  for ij in range(num_links):
     print('Link %d   Dual value (price) = %f' % (
     ij,
     capacityCtrs[ij].dual_value()))
  for ij in range(num_links):
     if y[ij].solution_value() > 0:
           print('Capacity assigned to link %d   Reduced cost= %f  Basis status= %f'
% (
           ij,
           y[ij].reduced_cost(),
           y[ij].basis_status()))

#    Advanced usage: possible basis status values for a variable and the slack
variable of a linear constraint.
# enum BasisStatus {
#     FREE = 0,
#     AT_LOWER_BOUND,
#     AT_UPPER_BOUND,
#     FIXED_VALUE,
#     BASIC
#   };
#
  print()
```

```
  # Only for ILP
  print("No. of branch-and-bound nodes = ", solver.nodes(), "")
  print("Best objective bound = ", solver.Objective().BestBound(), "")



if __name__ == '__main__':
  main()
```

For Time Limit= 50

Question 3: Is the problem optimally solved? How many simplex iterations are required?

Ans:

No the problem is not optimally solved but a feasible one which is a suboptimal one exists.

Simplex iterations required: 304

Output:

```
cyril@cyril-vb64:/opt/cyril/or-tools-6.4$ python minCost_CFA_MILP.py
A potentially suboptimal solution was found.
Total cost =  16.0
Time =  94  milliseconds
No. of simplex iterations =  304

WARNING: Logging before InitGoogleLogging() is written to STDERR
E1206 06:02:24.343166  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.343273  3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 1 assigned to link 0  Flow = 12  Reduced cost = 0  Basis status = 0
E1206 06:02:24.343361  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.343399  3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 2 assigned to link 2  Flow = 15  Reduced cost = 0  Basis status = 0
E1206 06:02:24.343479  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.343515  3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 3 assigned to link 4  Flow = 23  Reduced cost = 0  Basis status = 0
E1206 06:02:24.343595  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.343631  3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 4 assigned to link 6  Flow = 10  Reduced cost = 0  Basis status = 0
E1206 06:02:24.343705  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.343740  3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 5 assigned to link 1  Flow = 12  Reduced cost = 0  Basis status = 0
E1206 06:02:24.343827  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.343863  3875 linear_solver.cc:291] Basis status only available for continuous problems
```

Flow 7 assigned to link 1 Flow = 2 Reduced cost = 0 Basis status = 0
E1206 06:02:24.343931 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.343966 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 7 assigned to link 2 Flow = 2 Reduced cost = 0 Basis status = 0
E1206 06:02:24.344097 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.344138 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 8 assigned to link 1 Flow = 7 Reduced cost = 0 Basis status = 0
E1206 06:02:24.344209 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.344244 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 8 assigned to link 4 Flow = 7 Reduced cost = 0 Basis status = 0
E1206 06:02:24.344318 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.344354 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 9 assigned to link 1 Flow = 3 Reduced cost = 0 Basis status = 0
E1206 06:02:24.344424 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.344458 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 9 assigned to link 6 Flow = 3 Reduced cost = 0 Basis status = 0
E1206 06:02:24.344535 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.344570 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 10 assigned to link 3 Flow = 15 Reduced cost = 0 Basis status = 0
E1206 06:02:24.344646 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.344681 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 11 assigned to link 0 Flow = 2 Reduced cost = 0 Basis status = 0
E1206 06:02:24.344750 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.344784 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 11 assigned to link 3 Flow = 2 Reduced cost = 0 Basis status = 0
E1206 06:02:24.344872 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.344907 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 13 assigned to link 3 Flow = 4 Reduced cost = 0 Basis status = 0
E1206 06:02:24.344975 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.345073 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 13 assigned to link 4 Flow = 4 Reduced cost = 0 Basis status = 0
E1206 06:02:24.345157 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.345193 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 14 assigned to link 3 Flow = 5 Reduced cost = 0 Basis status = 0

E1206 06:02:24.345263 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.345296 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 14 assigned to link 6 Flow = 5 Reduced cost = 0 Basis status = 0
E1206 06:02:24.345372 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.345408 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 15 assigned to link 5 Flow = 23 Reduced cost = 0 Basis status = 0
E1206 06:02:24.345482 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.345517 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 16 assigned to link 0 Flow = 7 Reduced cost = 0 Basis status = 0
E1206 06:02:24.345587 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.345638 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 16 assigned to link 5 Flow = 7 Reduced cost = 0 Basis status = 0
E1206 06:02:24.345732 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.345773 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 17 assigned to link 2 Flow = 4 Reduced cost = 0 Basis status = 0
E1206 06:02:24.345857 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.345891 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 17 assigned to link 5 Flow = 4 Reduced cost = 0 Basis status = 0
E1206 06:02:24.346007 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.346056 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 19 assigned to link 5 Flow = 8 Reduced cost = 0 Basis status = 0
E1206 06:02:24.346137 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.346170 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 19 assigned to link 6 Flow = 8 Reduced cost = 0 Basis status = 0
E1206 06:02:24.346248 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.346283 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 20 assigned to link 7 Flow = 10 Reduced cost = 0 Basis status = 0
E1206 06:02:24.346357 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.346392 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 21 assigned to link 0 Flow = 3 Reduced cost = 0 Basis status = 0
E1206 06:02:24.346463 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.346498 3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 21 assigned to link 7 Flow = 3 Reduced cost = 0 Basis status = 0
E1206 06:02:24.346571 3875 linear_solver.cc:282] Reduced cost only available for continuous problems

```
E1206 06:02:24.346607  3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 22 assigned to link 2  Flow = 5  Reduced cost = 0  Basis status = 0
E1206 06:02:24.346678  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.346711  3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 22 assigned to link 7  Flow = 5  Reduced cost = 0  Basis status = 0
E1206 06:02:24.346786  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.346822  3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 23 assigned to link 4  Flow = 8  Reduced cost = 0  Basis status = 0
E1206 06:02:24.346890  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.346925  3875 linear_solver.cc:291] Basis status only available for continuous problems
Flow 23 assigned to link 7  Flow = 8  Reduced cost = 0  Basis status = 0

Capacity assigned to link 0   Num. capacity modules= 1.000000
Capacity assigned to link 1   Num. capacity modules= 1.000000
Capacity assigned to link 2   Num. capacity modules= 1.000000
Capacity assigned to link 3   Num. capacity modules= 1.000000
Capacity assigned to link 4   Num. capacity modules= 1.000000
Capacity assigned to link 5   Num. capacity modules= 1.000000
Capacity assigned to link 6   Num. capacity modules= 1.000000
Capacity assigned to link 7   Num. capacity modules= 1.000000
Capacity assigned to link 8   Num. capacity modules= 1.000000
Capacity assigned to link 9   Num. capacity modules= 1.000000
Capacity assigned to link 10   Num. capacity modules= 1.000000
Capacity assigned to link 11   Num. capacity modules= 1.000000
Capacity assigned to link 12   Num. capacity modules= 1.000000
Capacity assigned to link 13   Num. capacity modules= 1.000000
Capacity assigned to link 14   Num. capacity modules= 1.000000
Capacity assigned to link 15   Num. capacity modules= 1.000000

E1206 06:02:24.347618  3875 linear_solver.cc:140] Dual value only available for continuous problems
Link 0   Dual value (price) = 0.000000
E1206 06:02:24.347688  3875 linear_solver.cc:140] Dual value only available for continuous problems
Link 1   Dual value (price) = 0.000000
E1206 06:02:24.347754  3875 linear_solver.cc:140] Dual value only available for continuous problems
Link 2   Dual value (price) = 0.000000
E1206 06:02:24.347820  3875 linear_solver.cc:140] Dual value only available for continuous problems
Link 3   Dual value (price) = 0.000000
E1206 06:02:24.347884  3875 linear_solver.cc:140] Dual value only available for continuous problems
Link 4   Dual value (price) = 0.000000
E1206 06:02:24.348000  3875 linear_solver.cc:140] Dual value only available for continuous problems
Link 5   Dual value (price) = 0.000000
E1206 06:02:24.348067  3875 linear_solver.cc:140] Dual value only available for continuous problems
Link 6   Dual value (price) = 0.000000
```

E1206 06:02:24.348132 3875 linear_solver.cc:140] Dual value only available for continuous problems
Link 7 Dual value (price) = 0.000000
E1206 06:02:24.348197 3875 linear_solver.cc:140] Dual value only available for continuous problems
Link 8 Dual value (price) = 0.000000
E1206 06:02:24.348261 3875 linear_solver.cc:140] Dual value only available for continuous problems
Link 9 Dual value (price) = 0.000000
E1206 06:02:24.348326 3875 linear_solver.cc:140] Dual value only available for continuous problems
Link 10 Dual value (price) = 0.000000
E1206 06:02:24.348390 3875 linear_solver.cc:140] Dual value only available for continuous problems
Link 11 Dual value (price) = 0.000000
E1206 06:02:24.348455 3875 linear_solver.cc:140] Dual value only available for continuous problems
Link 12 Dual value (price) = 0.000000
E1206 06:02:24.348520 3875 linear_solver.cc:140] Dual value only available for continuous problems
Link 13 Dual value (price) = 0.000000
E1206 06:02:24.348583 3875 linear_solver.cc:140] Dual value only available for continuous problems
Link 14 Dual value (price) = 0.000000
E1206 06:02:24.348647 3875 linear_solver.cc:140] Dual value only available for continuous problems
Link 15 Dual value (price) = 0.000000
E1206 06:02:24.348716 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.348752 3875 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 0 Reduced cost= 0.000000 Basis status= 0.000000
E1206 06:02:24.348819 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.348855 3875 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 1 Reduced cost= 0.000000 Basis status= 0.000000
E1206 06:02:24.348979 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.349020 3875 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 2 Reduced cost= 0.000000 Basis status= 0.000000
E1206 06:02:24.349087 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.349123 3875 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 3 Reduced cost= 0.000000 Basis status= 0.000000
E1206 06:02:24.349189 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.349222 3875 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 4 Reduced cost= 0.000000 Basis status= 0.000000
E1206 06:02:24.349288 3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.349323 3875 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 5 Reduced cost= 0.000000 Basis status= 0.000000

```
E1206 06:02:24.349388  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.349422  3875 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 6  Reduced cost= 0.000000  Basis status= 0.000000
E1206 06:02:24.349488  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.349521  3875 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 7  Reduced cost= 0.000000  Basis status= 0.000000
E1206 06:02:24.349587  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.349622  3875 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 8  Reduced cost= 0.000000  Basis status= 0.000000
E1206 06:02:24.349687  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.349721  3875 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 9  Reduced cost= 0.000000  Basis status= 0.000000
E1206 06:02:24.349787  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.349822  3875 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 10  Reduced cost= 0.000000  Basis status= 0.000000
E1206 06:02:24.349911  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.349964  3875 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 11  Reduced cost= 0.000000  Basis status= 0.000000
E1206 06:02:24.350033  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.350066  3875 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 12  Reduced cost= 0.000000  Basis status= 0.000000
E1206 06:02:24.350133  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.350167  3875 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 13  Reduced cost= 0.000000  Basis status= 0.000000
E1206 06:02:24.350234  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.350266  3875 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 14  Reduced cost= 0.000000  Basis status= 0.000000
E1206 06:02:24.350333  3875 linear_solver.cc:282] Reduced cost only available for continuous problems
E1206 06:02:24.350368  3875 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 15  Reduced cost= 0.000000  Basis status= 0.000000

No. of branch-and-bound nodes =  0
Best objective bound =  5.48327381219
```

Question 4: How many branch and bound nodes are explored? What is the value of the best objective bound found during the branch and bound search?

Ans:

No of branch and bounds explored: 0

Best Objective bound found: 5.48327

For Time Limit= 100

Question 3: Is the problem optimally solved? How many simplex iterations are required?

Ans:

No the problem is not optimally solved but a suboptimal one is found

Simplex Iterations needed: 407

Output:

```
cyril@cyril-vb64:/opt/cyril/or-tools-6.4$ python minCost_CFA_MILP.py
A potentially suboptimal solution was found.
Total cost =  16.0
Time =  223  milliseconds
No. of simplex iterations =  407

WARNING: Logging before InitGoogleLogging() is written to STDERR
E1207 16:55:42.510411  6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.510581  6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 1 assigned to link 0  Flow = 12  Reduced cost = 0  Basis status = 0
E1207 16:55:42.510725  6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.510788  6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 2 assigned to link 2  Flow = 15  Reduced cost = 0  Basis status = 0
E1207 16:55:42.511010  6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.511075  6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 3 assigned to link 4  Flow = 23  Reduced cost = 0  Basis status = 0
E1207 16:55:42.511207  6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.511334  6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 4 assigned to link 6  Flow = 10  Reduced cost = 0  Basis status = 0
E1207 16:55:42.511539  6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.511634  6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 5 assigned to link 1  Flow = 12  Reduced cost = 0  Basis status = 0
E1207 16:55:42.511926  6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.512043  6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 7 assigned to link 1  Flow = 2  Reduced cost = 0  Basis status = 0
E1207 16:55:42.512310  6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.512387  6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 7 assigned to link 2  Flow = 2  Reduced cost = 0  Basis status = 0
E1207 16:55:42.512542  6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.512605  6810 linear_solver.cc:291] Basis status only available for continuous problems
```

Flow 8 assigned to link 1 Flow = 7 Reduced cost = 0 Basis status = 0
E1207 16:55:42.512722 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.512854 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 8 assigned to link 4 Flow = 7 Reduced cost = 0 Basis status = 0
E1207 16:55:42.512987 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.513048 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 9 assigned to link 1 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:55:42.513166 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.513259 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 9 assigned to link 6 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:55:42.513391 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.513451 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 10 assigned to link 3 Flow = 15 Reduced cost = 0 Basis status = 0
E1207 16:55:42.513578 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.513638 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 11 assigned to link 0 Flow = 2 Reduced cost = 0 Basis status = 0
E1207 16:55:42.513823 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.513885 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 11 assigned to link 3 Flow = 2 Reduced cost = 0 Basis status = 0
E1207 16:55:42.514031 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.514092 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 13 assigned to link 3 Flow = 4 Reduced cost = 0 Basis status = 0
E1207 16:55:42.514242 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.514303 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 13 assigned to link 4 Flow = 4 Reduced cost = 0 Basis status = 0
E1207 16:55:42.514433 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.514493 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 14 assigned to link 3 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:55:42.514608 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.514688 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 14 assigned to link 6 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:55:42.514833 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.514894 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 15 assigned to link 5 Flow = 23 Reduced cost = 0 Basis status = 0

E1207 16:55:42.515017 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.515076 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 16 assigned to link 0 Flow = 7 Reduced cost = 0 Basis status = 0
E1207 16:55:42.515256 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.515318 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 16 assigned to link 5 Flow = 7 Reduced cost = 0 Basis status = 0
E1207 16:55:42.515463 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.515539 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 17 assigned to link 2 Flow = 4 Reduced cost = 0 Basis status = 0
E1207 16:55:42.515656 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.515714 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 17 assigned to link 5 Flow = 4 Reduced cost = 0 Basis status = 0
E1207 16:55:42.515857 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.515918 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 19 assigned to link 5 Flow = 8 Reduced cost = 0 Basis status = 0
E1207 16:55:42.516031 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.516088 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 19 assigned to link 6 Flow = 8 Reduced cost = 0 Basis status = 0
E1207 16:55:42.516265 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.516329 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 20 assigned to link 7 Flow = 10 Reduced cost = 0 Basis status = 0
E1207 16:55:42.516450 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.516510 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 21 assigned to link 0 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:55:42.516628 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.516685 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 21 assigned to link 7 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:55:42.516808 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.516867 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 22 assigned to link 2 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:55:42.517021 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.517120 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 22 assigned to link 7 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:55:42.517264 6810 linear_solver.cc:282] Reduced cost only available for continuous problems

E1207 16:55:42.517324 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 23 assigned to link 4 Flow = 8 Reduced cost = 0 Basis status = 0
E1207 16:55:42.517439 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.517498 6810 linear_solver.cc:291] Basis status only available for continuous problems
Flow 23 assigned to link 7 Flow = 8 Reduced cost = 0 Basis status = 0

Capacity assigned to link 0 Num. capacity modules= 1.000000
Capacity assigned to link 1 Num. capacity modules= 1.000000
Capacity assigned to link 2 Num. capacity modules= 1.000000
Capacity assigned to link 3 Num. capacity modules= 1.000000
Capacity assigned to link 4 Num. capacity modules= 1.000000
Capacity assigned to link 5 Num. capacity modules= 1.000000
Capacity assigned to link 6 Num. capacity modules= 1.000000
Capacity assigned to link 7 Num. capacity modules= 1.000000
Capacity assigned to link 8 Num. capacity modules= 1.000000
Capacity assigned to link 9 Num. capacity modules= 1.000000
Capacity assigned to link 10 Num. capacity modules= 1.000000
Capacity assigned to link 11 Num. capacity modules= 1.000000
Capacity assigned to link 12 Num. capacity modules= 1.000000
Capacity assigned to link 13 Num. capacity modules= 1.000000
Capacity assigned to link 14 Num. capacity modules= 1.000000
Capacity assigned to link 15 Num. capacity modules= 1.000000

E1207 16:55:42.518946 6810 linear_solver.cc:140] Dual value only available for continuous problems
Link 0 Dual value (price) = 0.000000
E1207 16:55:42.519129 6810 linear_solver.cc:140] Dual value only available for continuous problems
Link 1 Dual value (price) = 0.000000
E1207 16:55:42.519276 6810 linear_solver.cc:140] Dual value only available for continuous problems
Link 2 Dual value (price) = 0.000000
E1207 16:55:42.519418 6810 linear_solver.cc:140] Dual value only available for continuous problems
Link 3 Dual value (price) = 0.000000
E1207 16:55:42.519558 6810 linear_solver.cc:140] Dual value only available for continuous problems
Link 4 Dual value (price) = 0.000000
E1207 16:55:42.519696 6810 linear_solver.cc:140] Dual value only available for continuous problems
Link 5 Dual value (price) = 0.000000
E1207 16:55:42.519835 6810 linear_solver.cc:140] Dual value only available for continuous problems
Link 6 Dual value (price) = 0.000000
E1207 16:55:42.519974 6810 linear_solver.cc:140] Dual value only available for continuous problems
Link 7 Dual value (price) = 0.000000
E1207 16:55:42.521613 6810 linear_solver.cc:140] Dual value only available for continuous problems
Link 8 Dual value (price) = 0.000000
E1207 16:55:42.521641 6810 linear_solver.cc:140] Dual value only available for continuous problems
Link 9 Dual value (price) = 0.000000

E1207 16:55:42.521659 6810 linear_solver.cc:140] Dual value only available for continuous problems
Link 10 Dual value (price) = 0.000000
E1207 16:55:42.521675 6810 linear_solver.cc:140] Dual value only available for continuous problems
Link 11 Dual value (price) = 0.000000
E1207 16:55:42.521692 6810 linear_solver.cc:140] Dual value only available for continuous problems
Link 12 Dual value (price) = 0.000000
E1207 16:55:42.521708 6810 linear_solver.cc:140] Dual value only available for continuous problems
Link 13 Dual value (price) = 0.000000
E1207 16:55:42.521725 6810 linear_solver.cc:140] Dual value only available for continuous problems
Link 14 Dual value (price) = 0.000000
E1207 16:55:42.521741 6810 linear_solver.cc:140] Dual value only available for continuous problems
Link 15 Dual value (price) = 0.000000
E1207 16:55:42.521765 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.521776 6810 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 0 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:55:42.521797 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.521806 6810 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 1 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:55:42.521826 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.521834 6810 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 2 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:55:42.521852 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.521862 6810 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 3 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:55:42.521880 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.521889 6810 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 4 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:55:42.521908 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.521916 6810 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 5 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:55:42.521934 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.521944 6810 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 6 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:55:42.521961 6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.521970 6810 linear_solver.cc:291] Basis status only available for continuous problems

```
Capacity assigned to link 7  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:55:42.522034  6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.522045  6810 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 8  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:55:42.522064  6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.522074  6810 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 9  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:55:42.522092  6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.522101  6810 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 10  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:55:42.522119  6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.522128  6810 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 11  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:55:42.522146  6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.522156  6810 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 12  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:55:42.522174  6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.522183  6810 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 13  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:55:42.522200  6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.522209  6810 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 14  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:55:42.522228  6810 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:55:42.522236  6810 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 15  Reduced cost= 0.000000  Basis status= 0.000000

No. of branch-and-bound nodes =  0
Best objective bound =  5.48736322538
```

Question 4: How many branch and bound nodes are explored? What is the value of the best objective bound found during the branch and bound search?

Ans:

No of branch and bounds explored:0

Best Objective bound found: 5.48736

For Time Limit= 500

Question 3: Is the problem optimally solved? How many simplex iterations are required?

Ans:

No the problem is not optimally solved but a suboptimal solution is derived

No of Simplex Iterations: 1886

```
cyril@cyril-vb64:/opt/cyril/or-tools-6.4$ python minCost_CFA_MILP.py
A potentially suboptimal solution was found.
Total cost =  16.0
Time =  665  milliseconds
No. of simplex iterations =  1886

WARNING: Logging before InitGoogleLogging() is written to STDERR
E1207 16:52:57.099293  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.099494  6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 1 assigned to link 0  Flow = 12  Reduced cost = 0  Basis status = 0
E1207 16:52:57.099642  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.099705  6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 2 assigned to link 2  Flow = 15  Reduced cost = 0  Basis status = 0
E1207 16:52:57.099840  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.099941  6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 3 assigned to link 4  Flow = 23  Reduced cost = 0  Basis status = 0
E1207 16:52:57.100080  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.100142  6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 4 assigned to link 6  Flow = 10  Reduced cost = 0  Basis status = 0
E1207 16:52:57.100270  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.100329  6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 5 assigned to link 1  Flow = 12  Reduced cost = 0  Basis status = 0
E1207 16:52:57.100513  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.100576  6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 7 assigned to link 1  Flow = 2  Reduced cost = 0  Basis status = 0
E1207 16:52:57.100690  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.100749  6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 7 assigned to link 2  Flow = 2  Reduced cost = 0  Basis status = 0
E1207 16:52:57.100916  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.100993  6802 linear_solver.cc:291] Basis status only available for continuous problems
```

Flow 8 assigned to link 1 Flow = 7 Reduced cost = 0 Basis status = 0
E1207 16:52:57.101111 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.101171 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 8 assigned to link 4 Flow = 7 Reduced cost = 0 Basis status = 0
E1207 16:52:57.101297 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.101408 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 9 assigned to link 1 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:52:57.101543 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.101603 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 9 assigned to link 6 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:52:57.101732 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.101794 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 10 assigned to link 3 Flow = 15 Reduced cost = 0 Basis status = 0
E1207 16:52:57.101958 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.102021 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 11 assigned to link 0 Flow = 2 Reduced cost = 0 Basis status = 0
E1207 16:52:57.102138 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.102196 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 11 assigned to link 3 Flow = 2 Reduced cost = 0 Basis status = 0
E1207 16:52:57.102340 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.102430 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 13 assigned to link 3 Flow = 4 Reduced cost = 0 Basis status = 0
E1207 16:52:57.102548 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.102607 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 13 assigned to link 4 Flow = 4 Reduced cost = 0 Basis status = 0
E1207 16:52:57.102735 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.102795 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 14 assigned to link 3 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:52:57.102946 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.103008 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 14 assigned to link 6 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:52:57.103135 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.103196 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 15 assigned to link 5 Flow = 23 Reduced cost = 0 Basis status = 0

E1207 16:52:57.103322 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.103410 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 16 assigned to link 0 Flow = 7 Reduced cost = 0 Basis status = 0
E1207 16:52:57.103533 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.103592 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 16 assigned to link 5 Flow = 7 Reduced cost = 0 Basis status = 0
E1207 16:52:57.103720 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.103780 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 17 assigned to link 2 Flow = 4 Reduced cost = 0 Basis status = 0
E1207 16:52:57.103929 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.103991 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 17 assigned to link 5 Flow = 4 Reduced cost = 0 Basis status = 0
E1207 16:52:57.104135 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.104197 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 19 assigned to link 5 Flow = 8 Reduced cost = 0 Basis status = 0
E1207 16:52:57.104310 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.104396 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 19 assigned to link 6 Flow = 8 Reduced cost = 0 Basis status = 0
E1207 16:52:57.104532 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.104593 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 20 assigned to link 7 Flow = 10 Reduced cost = 0 Basis status = 0
E1207 16:52:57.104717 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.104776 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 21 assigned to link 0 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:52:57.104928 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.104990 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 21 assigned to link 7 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:52:57.105114 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.105175 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 22 assigned to link 2 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:52:57.105293 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.105350 6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 22 assigned to link 7 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:52:57.105541 6802 linear_solver.cc:282] Reduced cost only available for continuous problems

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E1207 16:52:57.105604  6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 23 assigned to link 4  Flow = 8  Reduced cost = 0  Basis status = 0
E1207 16:52:57.105721  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.105779  6802 linear_solver.cc:291] Basis status only available for continuous problems
Flow 23 assigned to link 7  Flow = 8  Reduced cost = 0  Basis status = 0

Capacity assigned to link 0   Num. capacity modules= 1.000000
Capacity assigned to link 1   Num. capacity modules= 1.000000
Capacity assigned to link 2   Num. capacity modules= 1.000000
Capacity assigned to link 3   Num. capacity modules= 1.000000
Capacity assigned to link 4   Num. capacity modules= 1.000000
Capacity assigned to link 5   Num. capacity modules= 1.000000
Capacity assigned to link 6   Num. capacity modules= 1.000000
Capacity assigned to link 7   Num. capacity modules= 1.000000
Capacity assigned to link 8   Num. capacity modules= 1.000000
Capacity assigned to link 9   Num. capacity modules= 1.000000
Capacity assigned to link 10   Num. capacity modules= 1.000000
Capacity assigned to link 11   Num. capacity modules= 1.000000
Capacity assigned to link 12   Num. capacity modules= 1.000000
Capacity assigned to link 13   Num. capacity modules= 1.000000
Capacity assigned to link 14   Num. capacity modules= 1.000000
Capacity assigned to link 15   Num. capacity modules= 1.000000

E1207 16:52:57.107028  6802 linear_solver.cc:140] Dual value only available for continuous problems
Link 0   Dual value (price) = 0.000000
E1207 16:52:57.107177  6802 linear_solver.cc:140] Dual value only available for continuous problems
Link 1   Dual value (price) = 0.000000
E1207 16:52:57.107308  6802 linear_solver.cc:140] Dual value only available for continuous problems
Link 2   Dual value (price) = 0.000000
E1207 16:52:57.107452  6802 linear_solver.cc:140] Dual value only available for continuous problems
Link 3   Dual value (price) = 0.000000
E1207 16:52:57.107563  6802 linear_solver.cc:140] Dual value only available for continuous problems
Link 4   Dual value (price) = 0.000000
E1207 16:52:57.107671  6802 linear_solver.cc:140] Dual value only available for continuous problems
Link 5   Dual value (price) = 0.000000
E1207 16:52:57.107779  6802 linear_solver.cc:140] Dual value only available for continuous problems
Link 6   Dual value (price) = 0.000000
E1207 16:52:57.107918  6802 linear_solver.cc:140] Dual value only available for continuous problems
Link 7   Dual value (price) = 0.000000
E1207 16:52:57.108052  6802 linear_solver.cc:140] Dual value only available for continuous problems
Link 8   Dual value (price) = 0.000000
E1207 16:52:57.108192  6802 linear_solver.cc:140] Dual value only available for continuous problems
Link 9   Dual value (price) = 0.000000
```

E1207 16:52:57.108314 6802 linear_solver.cc:140] Dual value only available for continuous problems
Link 10 Dual value (price) = 0.000000
E1207 16:52:57.108458 6802 linear_solver.cc:140] Dual value only available for continuous problems
Link 11 Dual value (price) = 0.000000
E1207 16:52:57.108567 6802 linear_solver.cc:140] Dual value only available for continuous problems
Link 12 Dual value (price) = 0.000000
E1207 16:52:57.108675 6802 linear_solver.cc:140] Dual value only available for continuous problems
Link 13 Dual value (price) = 0.000000
E1207 16:52:57.108783 6802 linear_solver.cc:140] Dual value only available for continuous problems
Link 14 Dual value (price) = 0.000000
E1207 16:52:57.108922 6802 linear_solver.cc:140] Dual value only available for continuous problems
Link 15 Dual value (price) = 0.000000
E1207 16:52:57.111079 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.111100 6802 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 0 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:52:57.111125 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.111137 6802 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 1 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:52:57.111157 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.111168 6802 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 2 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:52:57.111188 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.111199 6802 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 3 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:52:57.111218 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.111229 6802 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 4 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:52:57.111248 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.111259 6802 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 5 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:52:57.111279 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.111289 6802 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 6 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:52:57.111308 6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.111320 6802 linear_solver.cc:291] Basis status only available for continuous problems

```
Capacity assigned to link 7  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:52:57.111338  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.111349  6802 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 8  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:52:57.111409  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.111424  6802 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 9  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:52:57.111444  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.111455  6802 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 10  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:52:57.111475  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.111485  6802 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 11  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:52:57.111505  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.111515  6802 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 12  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:52:57.111536  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.111546  6802 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 13  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:52:57.111565  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.111575  6802 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 14  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:52:57.111595  6802 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:52:57.111605  6802 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 15  Reduced cost= 0.000000  Basis status= 0.000000

No. of branch-and-bound nodes =  0
Best objective bound =  5.50638588158
```

Question 4: How many branch and bound nodes are explored? What is the value of the best objective bound found during the branch and bound search?

Ans:

No of branch and bounds explored: 0

Best Objective bound: 5.50638588158

For Time Limit= 1000

Question 3: Is the problem optimally solved? How many simplex iterations are required?

Ans:

No the problem is not optimally solved but a suboptimal solution has been found.

Simplex Iterations needed : 3904

Output:

```
cyril@cyril-vb64:/opt/cyril/or-tools-6.4$ python minCost_CFA_MILP.py
A potentially suboptimal solution was found.
Total cost =  16.0
Time =  1221  milliseconds
No. of simplex iterations =  3904

WARNING: Logging before InitGoogleLogging() is written to STDERR
E1207 16:50:41.451941  6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.452152  6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 1 assigned to link 0  Flow = 12  Reduced cost = 0  Basis status = 0
E1207 16:50:41.452301  6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.452364  6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 2 assigned to link 2  Flow = 15  Reduced cost = 0  Basis status = 0
E1207 16:50:41.452498  6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.452599  6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 3 assigned to link 4  Flow = 23  Reduced cost = 0  Basis status = 0
E1207 16:50:41.452736  6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.452797  6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 4 assigned to link 6  Flow = 10  Reduced cost = 0  Basis status = 0
E1207 16:50:41.452924  6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.452985  6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 5 assigned to link 1  Flow = 12  Reduced cost = 0  Basis status = 0
E1207 16:50:41.453197  6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.453263  6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 7 assigned to link 1  Flow = 2  Reduced cost = 0  Basis status = 0
E1207 16:50:41.453379  6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.453438  6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 7 assigned to link 2  Flow = 2  Reduced cost = 0  Basis status = 0
E1207 16:50:41.453636  6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.453706  6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 8 assigned to link 1  Flow = 7  Reduced cost = 0  Basis status = 0
```

E1207 16:50:41.453826 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.453886 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 8 assigned to link 4 Flow = 7 Reduced cost = 0 Basis status = 0
E1207 16:50:41.454012 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.454118 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 9 assigned to link 1 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:50:41.454242 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.454301 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 9 assigned to link 6 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:50:41.454428 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.454490 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 10 assigned to link 3 Flow = 15 Reduced cost = 0 Basis status = 0
E1207 16:50:41.454658 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.454720 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 11 assigned to link 0 Flow = 2 Reduced cost = 0 Basis status = 0
E1207 16:50:41.454836 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.454895 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 11 assigned to link 3 Flow = 2 Reduced cost = 0 Basis status = 0
E1207 16:50:41.455065 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.455140 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 13 assigned to link 3 Flow = 4 Reduced cost = 0 Basis status = 0
E1207 16:50:41.455256 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.455317 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 13 assigned to link 4 Flow = 4 Reduced cost = 0 Basis status = 0
E1207 16:50:41.455444 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.455505 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 14 assigned to link 3 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:50:41.455660 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.455720 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 14 assigned to link 6 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:50:41.455848 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.455909 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 15 assigned to link 5 Flow = 23 Reduced cost = 0 Basis status = 0
E1207 16:50:41.456054 6789 linear_solver.cc:282] Reduced cost only available for continuous problems

E1207 16:50:41.456128 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 16 assigned to link 0 Flow = 7 Reduced cost = 0 Basis status = 0
E1207 16:50:41.456248 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.456308 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 16 assigned to link 5 Flow = 7 Reduced cost = 0 Basis status = 0
E1207 16:50:41.456434 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.456496 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 17 assigned to link 2 Flow = 4 Reduced cost = 0 Basis status = 0
E1207 16:50:41.456652 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.456713 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 17 assigned to link 5 Flow = 4 Reduced cost = 0 Basis status = 0
E1207 16:50:41.456857 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.456918 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 19 assigned to link 5 Flow = 8 Reduced cost = 0 Basis status = 0
E1207 16:50:41.457052 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.457126 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 19 assigned to link 6 Flow = 8 Reduced cost = 0 Basis status = 0
E1207 16:50:41.457258 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.457319 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 20 assigned to link 7 Flow = 10 Reduced cost = 0 Basis status = 0
E1207 16:50:41.457442 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.457502 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 21 assigned to link 0 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:50:41.457690 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.457752 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 21 assigned to link 7 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:50:41.457878 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.457939 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 22 assigned to link 2 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:50:41.458092 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.458158 6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 22 assigned to link 7 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:50:41.458286 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.458346 6789 linear_solver.cc:291] Basis status only available for continuous problems

```
Flow 23 assigned to link 4  Flow = 8  Reduced cost = 0  Basis status = 0
E1207 16:50:41.458462  6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.458520  6789 linear_solver.cc:291] Basis status only available for continuous problems
Flow 23 assigned to link 7  Flow = 8  Reduced cost = 0  Basis status = 0

Capacity assigned to link 0   Num. capacity modules= 1.000000
Capacity assigned to link 1   Num. capacity modules= 1.000000
Capacity assigned to link 2   Num. capacity modules= 1.000000
Capacity assigned to link 3   Num. capacity modules= 1.000000
Capacity assigned to link 4   Num. capacity modules= 1.000000
Capacity assigned to link 5   Num. capacity modules= 1.000000
Capacity assigned to link 6   Num. capacity modules= 1.000000
Capacity assigned to link 7   Num. capacity modules= 1.000000
Capacity assigned to link 8   Num. capacity modules= 1.000000
Capacity assigned to link 9   Num. capacity modules= 1.000000
Capacity assigned to link 10   Num. capacity modules= 1.000000
Capacity assigned to link 11   Num. capacity modules= 1.000000
Capacity assigned to link 12   Num. capacity modules= 1.000000
Capacity assigned to link 13   Num. capacity modules= 1.000000
Capacity assigned to link 14   Num. capacity modules= 1.000000
Capacity assigned to link 15   Num. capacity modules= 1.000000

E1207 16:50:41.459700  6789 linear_solver.cc:140] Dual value only available for continuous problems
Link 0   Dual value (price) = 0.000000
E1207 16:50:41.459818  6789 linear_solver.cc:140] Dual value only available for continuous problems
Link 1   Dual value (price) = 0.000000
E1207 16:50:41.459929  6789 linear_solver.cc:140] Dual value only available for continuous problems
Link 2   Dual value (price) = 0.000000
E1207 16:50:41.460063  6789 linear_solver.cc:140] Dual value only available for continuous problems
Link 3   Dual value (price) = 0.000000
E1207 16:50:41.460191  6789 linear_solver.cc:140] Dual value only available for continuous problems
Link 4   Dual value (price) = 0.000000
E1207 16:50:41.460299  6789 linear_solver.cc:140] Dual value only available for continuous problems
Link 5   Dual value (price) = 0.000000
E1207 16:50:41.460408  6789 linear_solver.cc:140] Dual value only available for continuous problems
Link 6   Dual value (price) = 0.000000
E1207 16:50:41.460516  6789 linear_solver.cc:140] Dual value only available for continuous problems
Link 7   Dual value (price) = 0.000000
E1207 16:50:41.460664  6789 linear_solver.cc:140] Dual value only available for continuous problems
Link 8   Dual value (price) = 0.000000
E1207 16:50:41.460774  6789 linear_solver.cc:140] Dual value only available for continuous problems
Link 9   Dual value (price) = 0.000000
E1207 16:50:41.460885  6789 linear_solver.cc:140] Dual value only available for continuous problems
Link 10   Dual value (price) = 0.000000
```

E1207 16:50:41.460994 6789 linear_solver.cc:140] Dual value only available for continuous problems
Link 11 Dual value (price) = 0.000000
E1207 16:50:41.461155 6789 linear_solver.cc:140] Dual value only available for continuous problems
Link 12 Dual value (price) = 0.000000
E1207 16:50:41.461268 6789 linear_solver.cc:140] Dual value only available for continuous problems
Link 13 Dual value (price) = 0.000000
E1207 16:50:41.461376 6789 linear_solver.cc:140] Dual value only available for continuous problems
Link 14 Dual value (price) = 0.000000
E1207 16:50:41.461484 6789 linear_solver.cc:140] Dual value only available for continuous problems
Link 15 Dual value (price) = 0.000000
E1207 16:50:41.463955 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.463977 6789 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 0 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:50:41.464007 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.464018 6789 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 1 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:50:41.464073 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.464093 6789 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 2 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:50:41.464118 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.464128 6789 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 3 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:50:41.464146 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.464156 6789 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 4 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:50:41.464175 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.464184 6789 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 5 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:50:41.464203 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.464213 6789 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 6 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:50:41.464231 6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.464241 6789 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 7 Reduced cost= 0.000000 Basis status= 0.000000
E1207 16:50:41.464260 6789 linear_solver.cc:282] Reduced cost only available for continuous problems

```
E1207 16:50:41.464270  6789 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 8  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:50:41.464288  6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.464298  6789 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 9  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:50:41.464316  6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.464326  6789 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 10  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:50:41.464344  6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.464354  6789 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 11  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:50:41.464373  6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.464383  6789 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 12  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:50:41.464401  6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.464411  6789 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 13  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:50:41.464429  6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.464439  6789 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 14  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:50:41.464458  6789 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:50:41.464468  6789 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 15  Reduced cost= 0.000000  Basis status= 0.000000

No. of branch-and-bound nodes =  0
Best objective bound =  5.5107155265
```

Question 4: How many branch and bound nodes are explored? What is the value of the best objective bound found during the branch and bound search?

Ans:

No of branch and bounds explored: 0
Best Objective bound: 5.5107155265

For Time Limit= 2000

Question 3: Is the problem optimally solved? How many simplex iterations are required?

Ans:

Yes an optimal solution has been found.

No of Simplex Iterations: 4612

Output:

```
cyril@cyril-vb64:/opt/cyril/or-tools-6.4$ python minCost_CFA_MILP.py
A  optimal solution was found.
Total cost =  7.0
Time =  892  milliseconds
No. of simplex iterations =  4612

WARNING: Logging before InitGoogleLogging() is written to STDERR
E1207 16:47:47.388998 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.389108 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 1 assigned to link 0  Flow = 12  Reduced cost = 0  Basis status = 0
E1207 16:47:47.389264 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.389303 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 2 assigned to link 4  Flow = 15  Reduced cost = 0  Basis status = 0
E1207 16:47:47.389376 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.389411 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 2 assigned to link 11  Flow = 15  Reduced cost = 0  Basis status = 0
E1207 16:47:47.389487 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.389521 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 3 assigned to link 4  Flow = 23  Reduced cost = 0  Basis status = 0
E1207 16:47:47.389597 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.389633 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 4 assigned to link 0  Flow = 10  Reduced cost = 0  Basis status = 0
E1207 16:47:47.389784 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.389822 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 4 assigned to link 15  Flow = 10  Reduced cost = 0  Basis status = 0
E1207 16:47:47.389894 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.389930 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 5 assigned to link 5  Flow = 12  Reduced cost = 0  Basis status = 0
E1207 16:47:47.390002 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390036 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 5 assigned to link 13  Flow = 12  Reduced cost = 0  Basis status = 0
E1207 16:47:47.390103 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
```

E1207 16:47:47.390137 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 5 assigned to link 15 Flow = 12 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390259 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390269 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 7 assigned to link 11 Flow = 2 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390285 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390291 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 7 assigned to link 13 Flow = 2 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390305 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390311 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 7 assigned to link 15 Flow = 2 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390333 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390339 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 8 assigned to link 13 Flow = 7 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390352 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390358 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 8 assigned to link 15 Flow = 7 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390380 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390386 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 9 assigned to link 15 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390401 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390408 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 10 assigned to link 3 Flow = 15 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390429 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390434 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 11 assigned to link 0 Flow = 2 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390449 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390455 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 11 assigned to link 3 Flow = 2 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390486 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390491 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 13 assigned to link 0 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390506 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390511 6780 linear_solver.cc:291] Basis status only available for continuous problems

Flow 13 assigned to link 3 Flow = 4 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390524 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390530 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 13 assigned to link 4 Flow = 1 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390548 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390554 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 13 assigned to link 13 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390568 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390574 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 13 assigned to link 15 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390588 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390594 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 14 assigned to link 0 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390607 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390614 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 14 assigned to link 3 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390632 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390638 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 14 assigned to link 15 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390673 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390686 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 15 assigned to link 5 Flow = 23 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390707 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390714 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 16 assigned to link 0 Flow = 7 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390728 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390734 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 16 assigned to link 3 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390748 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390753 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 16 assigned to link 5 Flow = 1 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390769 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390775 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 16 assigned to link 11 Flow = 5 Reduced cost = 0 Basis status = 0

E1207 16:47:47.390797 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390803 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 17 assigned to link 11 Flow = 4 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390828 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390835 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 19 assigned to link 0 Flow = 8 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390848 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390854 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 19 assigned to link 3 Flow = 8 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390871 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390877 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 19 assigned to link 11 Flow = 8 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390890 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390897 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 19 assigned to link 15 Flow = 8 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390911 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390918 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 20 assigned to link 3 Flow = 10 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390934 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390941 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 20 assigned to link 11 Flow = 10 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390954 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390959 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 20 assigned to link 13 Flow = 10 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390974 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.390980 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 21 assigned to link 0 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:47:47.390995 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.391001 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 21 assigned to link 5 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:47:47.391018 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.391024 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 21 assigned to link 13 Flow = 3 Reduced cost = 0 Basis status = 0
E1207 16:47:47.391044 6780 linear_solver.cc:282] Reduced cost only available for continuous problems

E1207 16:47:47.391052 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 22 assigned to link 11 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:47:47.391064 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.391070 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 22 assigned to link 13 Flow = 5 Reduced cost = 0 Basis status = 0
E1207 16:47:47.391091 6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.391098 6780 linear_solver.cc:291] Basis status only available for continuous problems
Flow 23 assigned to link 13 Flow = 8 Reduced cost = 0 Basis status = 0

Capacity assigned to link 0 Num. capacity modules= 1.000000
Capacity assigned to link 3 Num. capacity modules= 1.000000
Capacity assigned to link 4 Num. capacity modules= 1.000000
Capacity assigned to link 5 Num. capacity modules= 1.000000
Capacity assigned to link 11 Num. capacity modules= 1.000000
Capacity assigned to link 13 Num. capacity modules= 1.000000
Capacity assigned to link 15 Num. capacity modules= 1.000000

E1207 16:47:47.391206 6780 linear_solver.cc:140] Dual value only available for continuous problems
Link 0 Dual value (price) = 0.000000
E1207 16:47:47.391222 6780 linear_solver.cc:140] Dual value only available for continuous problems
Link 1 Dual value (price) = 0.000000
E1207 16:47:47.391232 6780 linear_solver.cc:140] Dual value only available for continuous problems
Link 2 Dual value (price) = 0.000000
E1207 16:47:47.391242 6780 linear_solver.cc:140] Dual value only available for continuous problems
Link 3 Dual value (price) = 0.000000
E1207 16:47:47.391252 6780 linear_solver.cc:140] Dual value only available for continuous problems
Link 4 Dual value (price) = 0.000000
E1207 16:47:47.391263 6780 linear_solver.cc:140] Dual value only available for continuous problems
Link 5 Dual value (price) = 0.000000
E1207 16:47:47.391274 6780 linear_solver.cc:140] Dual value only available for continuous problems
Link 6 Dual value (price) = 0.000000
E1207 16:47:47.391284 6780 linear_solver.cc:140] Dual value only available for continuous problems
Link 7 Dual value (price) = 0.000000
E1207 16:47:47.391295 6780 linear_solver.cc:140] Dual value only available for continuous problems
Link 8 Dual value (price) = 0.000000
E1207 16:47:47.391305 6780 linear_solver.cc:140] Dual value only available for continuous problems
Link 9 Dual value (price) = 0.000000
E1207 16:47:47.391315 6780 linear_solver.cc:140] Dual value only available for continuous problems
Link 10 Dual value (price) = 0.000000
E1207 16:47:47.391326 6780 linear_solver.cc:140] Dual value only available for continuous problems

```
Link 11  Dual value (price) = 0.000000
E1207 16:47:47.391336  6780 linear_solver.cc:140] Dual value only available for continuous problems
Link 12  Dual value (price) = 0.000000
E1207 16:47:47.391347  6780 linear_solver.cc:140] Dual value only available for continuous problems
Link 13  Dual value (price) = 0.000000
E1207 16:47:47.391357  6780 linear_solver.cc:140] Dual value only available for continuous problems
Link 14  Dual value (price) = 0.000000
E1207 16:47:47.391368  6780 linear_solver.cc:140] Dual value only available for continuous problems
Link 15  Dual value (price) = 0.000000
E1207 16:47:47.391381  6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.391387  6780 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 0  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:47:47.391400  6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.391407  6780 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 3  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:47:47.391417  6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.391423  6780 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 4  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:47:47.391434  6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.391440  6780 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 5  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:47:47.391453  6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.391459  6780 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 11  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:47:47.391470  6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.391476  6780 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 13  Reduced cost= 0.000000  Basis status= 0.000000
E1207 16:47:47.391489  6780 linear_solver.cc:282] Reduced cost only available for continuous problems
E1207 16:47:47.391494  6780 linear_solver.cc:291] Basis status only available for continuous problems
Capacity assigned to link 15  Reduced cost= 0.000000  Basis status= 0.000000

No. of branch-and-bound nodes =  10
Best objective bound =  7.0
```

Question 4: How many branch and bound nodes are explored? What is the value of the best objective bound found during the branch and bound search?

Ans:

No of branch and bounds explored: 10

Best Objective bound found: 7.0

Question 5: What is happening when you increase the time limit?
Ans:

- From Time Limit 50--→100 --→ 500 --→ 1000 --→ 2000 as it increases the optimality of the solution, the no of branch and bounds and the best objective bound improves.
- For in case of 50,100,500,1000 a suboptimal solution which is feasible exists but not an optimal one
- For 2000ms the optimality is achieved as well as an objective bound high as 7.0 is obtained.
- As we move up the time limit the no of branches are increased iteratively giving more time space as well as more bounds thereby increasing the simplex iterations as 4612 in 2000ms and the optimal solution is also narrowed to from earlier iterations of 50,100,500,1000ms