

$$\boxed{\Psi \vdash e : \sigma} \quad \Psi ::= \emptyset \mid \Psi, x : \sigma$$

$$\begin{array}{c}
\text{S-T-VAR} \\
\frac{x : \sigma \in \Psi}{\Psi \vdash x : \sigma} \\
\\
\text{S-T-ABS} \\
\frac{\Psi, x : \sigma_1 \vdash e : \sigma_2}{\Psi \vdash \lambda x. e : \sigma_1 \rightarrow \sigma_2} \\
\\
\text{S-T-APP} \\
\frac{\Psi \vdash e_1 : \sigma_2 \rightarrow \sigma \quad \Psi \vdash e_2 : \sigma_2}{\Psi \vdash e_1(e_2) : \sigma} \\
\\
\text{S-T-STRINGIN-I} \\
\frac{s \in \mathcal{L}\{r\}}{\Psi \vdash \text{rstr}[s] : \text{stringin}[r]} \\
\\
\text{S-T-CONCAT} \\
\frac{\Psi \vdash e_1 : \text{stringin}[r_1] \quad \Psi \vdash e_2 : \text{stringin}[r_2]}{\Psi \vdash \text{rconcat}(e_1; e_2) : \text{stringin}[r_1 \cdot r_2]} \\
\\
\text{S-T-CASE} \\
\frac{\Psi \vdash e_1 : \text{stringin}[r] \quad \Psi \vdash e_2 : \sigma \quad \Psi, x : \text{stringin}[\text{lhead}(r)], y : \text{stringin}[\text{ltail}(r)] \vdash e_3 : \sigma}{\Psi \vdash \text{strcase}(e_1; e_2; x.y.e_3) : \sigma} \\
\\
\text{S-T-REPLACE} \\
\frac{\Psi \vdash e_1 : \text{stringin}[r_1] \quad \Psi \vdash e_2 : \text{stringin}[r_2] \quad \text{lreplace}(r; r_1; r_2) = r'}{\Psi \vdash \text{rreplace}[r](e_1; e_2) : \text{stringin}[r']} \\
\\
\text{S-T-SAFE-COERCE} \\
\frac{\Psi \vdash e : \text{stringin}[r'] \quad \mathcal{L}\{r'\} \subseteq \mathcal{L}\{r\}}{\Psi \vdash \text{rcoerce}[r](e) : \text{stringin}[r]} \\
\\
\text{S-T-CHECK} \\
\frac{\Psi \vdash e_0 : \text{stringin}[r_0] \quad \Psi, x : \text{stringin}[r] \vdash e_1 : \sigma \quad \Psi \vdash e_2 : \sigma}{\Psi \vdash \text{rcheck}[r](e_0; x.e_1; e_2) : \sigma}
\end{array}$$

**Figure 1: Typing rules for  $\lambda_{RS}$ . The typing context  $\Psi$  is standard.**

$$\boxed{e \Downarrow e}$$

$$\begin{array}{c}
\text{S-E-RSTR} \\
\frac{}{\text{rstr}[s] \Downarrow \text{rstr}[s]} \\
\\
\text{S-E-CONCAT} \\
\frac{e_1 \Downarrow \text{rstr}[s_1] \quad e_2 \Downarrow \text{rstr}[s_2]}{\text{rconcat}(e_1; e_2) \Downarrow \text{rstr}[s_1 s_2]} \\
\\
\text{S-E-CASE-}\epsilon \\
\frac{e_1 \Downarrow \text{rstr}[e] \quad e_2 \Downarrow v_2}{\text{strcase}(e_1; e_2; e_3) \Downarrow v_2} \\
\\
\text{S-E-CASE-CONCAT} \\
\frac{e_1 \Downarrow \text{rstr}[ps] \quad e_3 \Downarrow x.y.e_4 \quad [p/x][s/y]e_4 \Downarrow v}{\text{strcase}(e_1; e_2; e_3) \Downarrow v} \\
\\
\text{S-E-REPLACE} \\
\frac{e_1 \Downarrow \text{rstr}[e_1] \quad e_2 \Downarrow \text{rstr}[s_2] \quad \text{subst}(r; s_1; s_2) = s}{\text{rreplace}[r](e_1; e_2) \Downarrow \text{rstr}[s]} \\
\\
\text{S-E-SAFE-COERCE} \\
\frac{e \Downarrow \text{rstr}[s]}{\text{rcoerce}[r](e) \Downarrow \text{rstr}[s]} \\
\\
\text{S-E-CHECK-OK} \\
\frac{e \Downarrow \text{rstr}[s] \quad s \in \mathcal{L}\{r\} \quad [\text{rstr}[s]/x]e_1 \Downarrow v}{\text{rcheck}[r](e; x.e_1; e_2) \Downarrow v} \\
\\
\text{S-E-CHECK-NOTOK} \\
\frac{e \Downarrow \text{rstr}[s] \quad s \notin \mathcal{L}\{r\}}{\text{rcheck}[r](e; x.e_1; e_2) \Downarrow e_2}
\end{array}$$

**Figure 2: Big step semantics for  $\lambda_{RS}$ .**

$$\boxed{\Theta \vdash \iota : \tau} \quad \Theta ::= \emptyset \mid \Theta, x : \tau$$

$$\begin{array}{c}
\text{P-T-VAR} \\
\frac{x : \tau \in \Theta}{\Theta \vdash x : \tau} \\
\\
\text{P-T-ABS} \\
\frac{\Theta, x : \tau_1 \vdash \iota_2 : \tau_2}{\Theta \vdash \lambda x. \iota_2 : \tau_1 \rightarrow \tau_2} \\
\\
\text{P-T-APP} \\
\frac{\Theta \vdash \iota_1 : \tau_2 \rightarrow \tau \quad \Theta \vdash \iota_2 : \tau_2}{\Theta \vdash \iota_1(\iota_2) : \tau} \\
\\
\text{P-T-STRING} \\
\frac{}{\Theta \vdash \text{str}[s] : \text{string}} \\
\\
\text{P-T-REGEX} \\
\frac{}{\Theta \vdash \text{rx}[r] : \text{regex}} \\
\\
\text{P-T-CONCAT} \\
\frac{\Theta \vdash \iota_1 : \text{string} \quad \Theta \vdash \iota_2 : \text{string}}{\Theta \vdash \text{concat}(\iota_1; \iota_2) : \text{string}} \\
\\
\text{P-T-CASE} \\
\frac{\Theta \vdash \iota_1 : \text{string} \quad \Theta \vdash \iota_2 : \tau \quad \Theta, x : \text{string}, y : \text{string} \vdash \iota_3 : \tau}{\Theta \vdash \text{pstrcase}(\iota_1; \iota_2; \iota_3) : \tau} \\
\\
\text{P-T-REPLACE} \\
\frac{\Theta \vdash \iota_1 : \text{regex} \quad \Theta \vdash \iota_2 : \text{string} \quad \Theta \vdash \iota_3 : \text{string}}{\Theta \vdash \text{preplace}(\iota_1; \iota_2; \iota_3) : \text{string}} \\
\\
\text{P-T-CHECK} \\
\frac{\Theta \vdash \iota_r : \text{regex} \quad \Theta \vdash \iota_1 : \text{string} \quad \Theta, x : \text{string} \vdash \iota_2 : \sigma \quad \Theta \vdash \iota_3 : \sigma}{\Theta \vdash \text{check}(\iota_r; \iota_1; x.\iota_2; \iota_3) : \sigma}
\end{array}$$

**Figure 3: Typing rules for  $\lambda_P$ . The typing context  $\Theta$  is standard.**

$$\boxed{\iota \Downarrow \iota}$$

$$\begin{array}{c}
\text{P-E-ABS} \\
\frac{}{\lambda x. e \Downarrow \lambda x. e} \\
\\
\text{P-E-APP} \\
\frac{\iota_1 \Downarrow \lambda x. \iota_3 \quad \iota_2 \Downarrow \iota'_2 \quad [\iota_3/x]\iota'_2 \Downarrow v}{\iota_1(\iota_2) \Downarrow v} \\
\\
\text{P-E-STR} \\
\frac{}{\text{str}[s] \Downarrow \text{str}[s]} \\
\\
\text{P-E-RX} \\
\frac{}{\text{rx}[r] \Downarrow \text{rx}[r]} \\
\\
\text{P-E-CONCAT} \\
\frac{\iota_1 \Downarrow \text{str}[s_1] \quad \iota_2 \Downarrow \text{str}[s_2]}{\text{concat}(\iota_1; \iota_2) \Downarrow \text{str}[s_1 s_2]} \\
\\
\text{P-E-CASE-}\epsilon \\
\frac{\iota_1 \Downarrow \text{str}[\ ] \quad \iota_2 \Downarrow \iota_2}{\text{pstrcase}(\iota_1; \iota_2; \iota_3) \Downarrow v_2} \\
\\
\text{P-E-CASE-CONCAT} \\
\frac{\iota_1 \Downarrow \text{str}[ps] \quad x.y.\iota_3 \Downarrow \iota_4 \quad [p/x][s/y]\iota_4 \Downarrow v}{\text{pstrcase}(\iota_1; \iota_2; \iota_3) \Downarrow v} \\
\\
\text{P-E-REPLACE} \\
\frac{\iota_1 \Downarrow \text{rx}[r] \quad \iota_2 \Downarrow \text{str}[s_2] \quad \iota_3 \Downarrow \text{str}[s_3] \quad \text{subst}(r; s_2; s_3) = s}{\text{preplace}(\iota_1; \iota_2; \iota_3) \Downarrow \text{str}[s]} \\
\\
\text{P-E-CHECK-OK} \\
\frac{\iota \Downarrow \text{str}[s] \quad s \in \mathcal{L}\{r\} \quad [\text{str}[s]/x]\iota_1 \Downarrow \iota_3}{\text{check}(\text{rx}[r]; \iota; x.\iota_1; \iota_2) \Downarrow \iota_3} \\
\\
\text{P-E-CHECK-NOTOK} \\
\frac{\iota \Downarrow \text{str}[s] \quad s \notin \mathcal{L}\{r\}}{\text{check}(\text{rx}[r]; \iota; x.\iota_1; \iota_2) \Downarrow \iota_2}
\end{array}$$

**Figure 4: Big step semantics for of  $\lambda_P$**

|   |   |   |   |
|---|---|---|---|
| $\Psi \vdash \llbracket e \rrbracket = \iota$   |   |   |   |
| TR-VAR<br>$\frac{}{\Psi \vdash \llbracket x \rrbracket = x}$  | TR-ABS<br>$\frac{\Psi \vdash \llbracket e \rrbracket = \iota}{\Psi \vdash \llbracket \lambda x. e \rrbracket = \lambda x. \iota}$   | TR-APP<br>$\frac{\Psi \vdash \llbracket e_1 \rrbracket = \iota_1 \quad \Psi \vdash \llbracket e_2 \rrbracket = \iota_2}{\Psi \vdash \llbracket e_1(e_2) \rrbracket = \iota_1(\iota_2)}$   | TR-CASE<br>$\frac{\Psi \vdash \llbracket e_1 \rrbracket = \iota_1 \quad \Psi \vdash \llbracket e_2 \rrbracket = \iota_2 \quad \Psi \vdash \llbracket e_3 \rrbracket = \iota_3}{\Psi \vdash \llbracket \text{strcase}(e_1; e_2; e_3) \rrbracket = \text{pstrcase}(\iota_1; \iota_2; \iota_3)}$ |
| TR-STRING<br>$\frac{}{\Psi \vdash \llbracket \text{rstr}[s] \rrbracket = \text{str}[s]}$  | TR-CONCAT<br>$\frac{\Psi \vdash \llbracket e_1 \rrbracket = \iota_1 \quad \Psi \vdash \llbracket e_2 \rrbracket = \iota_2}{\Psi \vdash \llbracket \text{rconcat}(e_1; e_2) \rrbracket = \text{concat}(\iota_1; \iota_2)}$ | TR-SUBST<br>$\frac{\Psi \vdash \llbracket e_1 \rrbracket = \iota_1 \quad \Psi \vdash \llbracket e_2 \rrbracket = \iota_2}{\Psi \vdash \llbracket \text{rreplace}[r](e_1; e_2) \rrbracket = \text{replace}(\text{rx}[r]; \iota_1; \iota_2)}$   |   |
| TR-SAFE COERCE<br>$\frac{\Psi \vdash \llbracket e \rrbracket = \iota}{\Psi \vdash \llbracket \text{rcoerce}[r'](e) \rrbracket = \iota}$ |   | TR-CHECK<br>$\frac{\Psi \vdash \llbracket e \rrbracket = \iota_1 \quad \Psi \vdash \llbracket e_1 \rrbracket = \iota_1 \quad \Psi \vdash \llbracket e_2 \rrbracket = \iota_2}{\Psi \vdash \llbracket \text{rcheck}[r](e; x.e_1; e_2) \rrbracket = \text{check}(\text{rx}[r]; \iota; x.\iota_1; \iota_2)}$ |   |

**Figure 5: Translation from source terms ( $e$ ) to target terms ( $\iota$ ). The translation is type-directed.**

Note that S-E-ABS and S-E-APP are excluded from Figure 1 due to space constraints. However, they resemble the corresponding rules for  $\lambda_P$ ; see the paper for a full definition.