

$$\begin{array}{c}
\boxed{\Psi \vdash e : \sigma} \quad \Psi ::= () \mid \Psi, x : \sigma \\
\\
\text{S-T-STRING-}I \\
\frac{}{s \in \mathcal{L}\{r\}} \\
\Psi \vdash \text{rstr}[s] : \text{string}[r] \\
\\
\text{S-T-CASE} \\
\frac{\Psi \vdash e_1 : \text{string}[r] \quad \Psi, x : \text{string}[\text{head}(r)], y : \text{string}[\text{tail}(r)] \vdash e_3 : \sigma}{\Psi \vdash \text{rstrcase}(e_1; e_2; x, y, e_3) : \sigma} \\
\\
\text{S-T-REPLACE} \\
\frac{\Psi \vdash e_1 : \text{string}[r_1] \quad \Psi \vdash e_2 : \text{string}[r_2] \quad \text{replace}(r; r_1; r_2) = r'}{\Psi \vdash \text{rreplace}[r](e_1; e_2) : \text{string}[r']} \\
\\
\text{S-T-SAFE}COERCE \\
\frac{\Psi \vdash e : \text{string}[r'] \quad \mathcal{L}\{r'\} \subseteq \mathcal{L}\{r\}}{\Psi \vdash \text{rcoerce}[r](e) : \text{string}[r]} \\
\\
\text{S-T-CHECK} \\
\frac{\Psi \vdash e_0 : \text{string}[r_0] \quad \Psi, x : \text{string}[r] \vdash e_1 : \sigma \quad \Psi \vdash e_2 : \sigma}{\Psi \vdash \text{rcheck}[r](e_0; x.e_1; e_2) : \sigma}
\end{array}$$

$$\begin{array}{c}
\boxed{\Theta \vdash \iota : \tau} \quad \Theta ::= \emptyset \quad | \quad \Theta, x : \tau \\
\\
\text{P-T-STRING} \quad \Theta \vdash \text{str}[s] : \text{string} \\
\\
\text{P-T-REGEX} \quad \Theta \vdash \text{rx}[r] : \text{regex} \\
\\
\text{P-T-CONCAT} \quad \frac{\Theta \vdash \iota_1 : \text{string} \quad \Theta \vdash \iota_2 : \text{string}}{\Theta \vdash \text{concat}(\iota_1; \iota_2) : \text{string}} \\
\\
\text{P-T-CASE} \quad \frac{\Theta \vdash \iota_1 : \text{string} \quad \Theta \vdash \iota_2 : \tau \quad \Theta, x : \text{string}, y : \text{string} \vdash \iota_3 : \tau}{\Theta \vdash \text{strcase}(\iota_1; \iota_2; x, y, \iota_3) : \tau} \\
\\
\text{P-T-REPLACE} \quad \frac{\Theta \vdash \iota_1 : \text{regex} \quad \Theta \vdash \iota_2 : \text{string} \quad \Theta \vdash \iota_3 : \text{string}}{\Theta \vdash \text{replace}(\iota_1; \iota_2; \iota_3) : \text{string}} \\
\\
\text{P-T-CHECK} \quad \frac{\Theta \vdash \iota_r : \text{regex} \quad \Theta \vdash \iota_1 : \text{string} \quad \Theta \vdash \iota_2 : \sigma \quad \Theta \vdash \iota_3 : \sigma}{\Theta \vdash \text{check}(\iota_r; \iota_1; \iota_2; \iota_3) : \sigma}
\end{array}$$

$\iota \Downarrow \psi$		
<b>P-E-ABS</b>	<b>P-E-APP</b>	<b>P-E-STR</b>
$\lambda x.e \Downarrow \lambda x.e$	$\iota_1 \Downarrow \lambda x.\iota_3 \quad \iota_2 \Downarrow \psi_2 \quad [\dot{\psi}_2/x]\iota_3 \Downarrow \psi_3$	$\text{str}[s] \Downarrow \text{str}[s]$
<b>P-E-RX</b>	<b>P-E-CONCAT</b>	<b>P-E-CASE-<math>\epsilon</math></b>
$\text{rx}[r] \Downarrow \text{rx}[r]$	$\iota_1 \Downarrow \text{str}[s_1] \quad \iota_2 \Downarrow \text{str}[s_2]$ $\text{concat}(\iota_1; \iota_2) \Downarrow \text{str}[s_1 s_2]$	$\iota_1 \Downarrow \text{str}[\epsilon] \quad \iota_2 \Downarrow \psi_2$ $\text{strcase}(\iota_1; \iota_2; x, y.\iota_3) \Downarrow \psi_2$
	<b>P-E-CASE-CONCAT</b>	
	$\iota_1 \Downarrow \text{str}[a s] \quad [\text{str}[a], \text{str}[s]/x, y]\iota_3 \Downarrow \psi$ $\text{strcase}(\iota_1; \iota_2; x, y.\iota_3) \Downarrow \psi$	
	<b>P-E-REPLACE</b>	
$\iota_1 \Downarrow \text{rx}[r] \quad \iota_2 \Downarrow \text{str}[s_2] \quad \iota_3 \Downarrow \text{str}[s_3] \quad \text{subst}(r; s_2; s_3) = s$		
	$\text{replace}(\iota_1; \iota_2; \iota_3) \Downarrow \text{str}[s]$	
	<b>P-E-CHECK-OK</b>	
$\iota_r \Downarrow \text{rx}[r] \quad \iota \Downarrow \text{str}[s] \quad s \in \mathcal{L}\{r\} \quad \iota_1 \Downarrow \psi_1$		
	$\text{check}(\iota_r; \iota; \iota_1; \iota_2) \Downarrow \psi_1$	
	<b>P-E-CHECK-NOTOK</b>	
$\iota_r \Downarrow \text{rx}[r] \quad \iota \Downarrow \text{str}[s] \quad s \notin \mathcal{L}\{r\} \quad \iota_2 \Downarrow \psi_2$		
	$\text{check}(\iota_r; \iota; \iota_1; \iota_2) \Downarrow \psi_2$	

$\boxed{e \Downarrow v}$		
$\text{S-E-ABS}$	$\text{S-E-APP}$	$\text{S-E-RSTR}$
$\lambda x. e \Downarrow \lambda x. e$	$\frac{e_1 \Downarrow \lambda x. e_3 \quad e_2 \Downarrow v_2 \quad [v_2/x]e_3 \Downarrow v}{e_1(e_2) \Downarrow v}$	$\frac{}{\text{rstr}[s] \Downarrow \text{rstr}[s]}$
$\text{S-E-CONCAT}$	$\text{S-E-CASE-}\epsilon$	
$\frac{e_1 \Downarrow \text{rstr}[s_1] \quad e_2 \Downarrow \text{rstr}[s_2]}{\text{rconcat}(e_1; e_2) \Downarrow \text{rstr}[s_1 s_2]}$	$\frac{e_1 \Downarrow \text{rstr}[e] \quad e_2 \Downarrow v_2}{\text{rstrcase}(e_1; e_2; x, y. e_3) \Downarrow v_2}$	
$\text{S-E-CASE-CONCAT}$		
$\frac{e_1 \Downarrow \text{rstr}[as] \quad [\text{rstr}[a], \text{rstr}[s]/x, y]e_3 \Downarrow v_3}{\text{rstrcase}(e_1; e_2; x, y. e_3) \Downarrow v_3}$		
$\text{S-E-REPLACE}$	$\text{S-E-SAFE-COERCE}$	
$\frac{e_1 \Downarrow \text{rstr}[s_1] \quad e_2 \Downarrow \text{rstr}[s_2] \quad \text{subst}(r; s_1; s_2) = s}{\text{replace}[r](e_1; e_2) \Downarrow \text{rstr}[s]}$	$\frac{}{\text{rcoerce}[r](e) \Downarrow \text{rstr}[s]}$	
$\text{S-E-CHECK-OK}$		
$\frac{e \Downarrow \text{rstr}[s] \quad s \in \mathcal{L}\{r\} \quad [\text{rstr}[s]/x]e_1 \Downarrow v}{\text{rcheck}[r](e; x.e_1; e_2) \Downarrow v}$		
$\text{S-E-CHECK-NOTOK}$		
$\frac{e \Downarrow \text{rstr}[s] \quad s \notin \mathcal{L}\{r\} \quad e_2 \Downarrow v}{\text{rcheck}[r](e; x.e_1; e_2) \Downarrow v}$		

$\boxed{[\sigma] = \tau}$		
$\frac{\text{TR-T-STRING}}{[\text{stringin}[r]] = \text{string}}$	$\frac{\text{TR-T-ARROW}}{[\sigma_1] = \tau_1 \quad [\sigma_2] = \tau_2}$	
	$[\sigma_1 \rightarrow \sigma_2] = \tau_1 \rightarrow \tau_2$	
$\boxed{[\Psi] = \Theta}$		
$\frac{\text{TR-T-CONTEXT-EMP}}{[\emptyset] = \emptyset}$	$\frac{\text{TR-T-CONTEXT-EXT}}{[\Psi] = \Theta \quad [\sigma] = \tau}$	
	$[\Psi, x : \sigma] = \Theta, x : \tau$	
$\boxed{[e] = \iota}$		
$\frac{\text{TR-CASE}}{[\text{rstrcase}(e_1; e_2; x, y.e_3)] = \text{strcase}(\iota_1; \iota_2; x, y.\iota_3)}$	$\frac{\text{TR-STRING}}{[\text{rstr}[s]] = \text{str}[s]}$	
$\frac{\text{TR-CONCAT}}{[\text{rconcat}(e_1; e_2)] = \text{concat}(\iota_1; \iota_2)}$	$\frac{\text{TR-SUBST}}{[\text{rreplace}[r](e_1; e_2)] = \text{replace}(\text{rx}[r]; \iota_1; \iota_2)}$	
$\frac{\text{TR-SAFE-COERCE}}{[\text{rcoerce}[r'](e)] = \iota}$	$\frac{\text{TR-CHECK}}{[\text{rcheck}[r](e; x.e_1; e_2)] = \text{check}(\text{rx}[r]; \iota; (\lambda x.\iota_1)(\iota); \iota_2)}$	