$\boxed{\Psi \vdash e : \sigma}$ $\quad \Psi ::= \emptyset \mid \Psi, x : \sigma$

S-T-Var
$$\frac{x : \sigma \in \Psi}{\Psi \vdash x : \sigma}$$

S-T-Abs
$$\frac{\Psi, x : \sigma_1 \vdash e : \sigma_2}{\Psi \vdash \lambda x.e : \sigma_1 \to \sigma_2}$$

S-T-App
$$\frac{\Psi \vdash e_1 : \sigma_2 \to \sigma \qquad \Psi \vdash e_2 : \sigma_2}{\Psi \vdash e_1(e_2) : \sigma}$$

S-T-Stringin-I
$$\frac{s \in \mathcal{L}\{r\}}{\Psi \vdash \mathsf{rstr}[s] : \mathsf{stringin}[r]}$$

S-T-Concat
$$\frac{\Psi \vdash e_1 : \mathsf{stringin}[r_1] \qquad \Psi \vdash e_2 : \mathsf{stringin}[r_2]}{\Psi \vdash \mathsf{rconcat}(e_1; e_2) : \mathsf{stringin}[r_1 \cdot r_2]}$$

S-T-Case
$$\frac{\begin{array}{c}\Psi \vdash e_1 : \mathsf{stringin}[r] \qquad \Psi \vdash e_2 : \sigma \\ \Psi, x : \mathsf{stringin}[\mathsf{lhead}(r)], y : \mathsf{stringin}[\mathsf{ltail}(r)] \vdash e_3 : \sigma\end{array}}{\Psi \vdash \mathsf{strcase}(e_1; e_2; x.y.e_3) : \sigma}$$

S-T-Replace
$$\frac{\begin{array}{c}\Psi \vdash e_1 : \mathsf{stringin}[r_1] \qquad \Psi \vdash e_2 : \mathsf{stringin}[r_2] \\ \mathsf{lreplace}(r; r_1; r_2) = r'\end{array}}{\Psi \vdash \mathsf{rreplace}[r](e_1; e_2) : \mathsf{stringin}[r']}$$

S-T-SafeCoerce
$$\frac{\Psi \vdash e : \mathsf{stringin}[r'] \qquad \mathcal{L}\{r'\} \subseteq \mathcal{L}\{r\}}{\Psi \vdash \mathsf{rcoerce}[r](e) : \mathsf{stringin}[r]}$$

S-T-Check
$$\frac{\Psi \vdash e_0 : \mathsf{stringin}[r_0] \qquad \Psi, x : \mathsf{stringin}[r] \vdash e_1 : \sigma \qquad \Psi \vdash e_2 : \sigma}{\Psi \vdash \mathsf{rcheck}[r](e_0; x.e_1; e_2) : \sigma}$$

Figure 1: Typing rules for λ_{RS}. The typing context Ψ is standard.

$\boxed{e \Downarrow e}$

S-E-RStr
$$\frac{}{\mathsf{rstr}[s] \Downarrow \mathsf{rstr}[s]}$$

S-E-Concat
$$\frac{e_1 \Downarrow \mathsf{rstr}[s_1] \qquad e_2 \Downarrow \mathsf{rstr}[s_2]}{\mathsf{rconcat}(e_1; e_2) \Downarrow \mathsf{rstr}[s_1 s_2]}$$

S-E-Case-ϵ
$$\frac{e_1 \Downarrow \mathsf{rstr}[\epsilon] \qquad e_2 \Downarrow v_2}{\mathsf{strcase}(e_1; e_2; e_3) \Downarrow v_2}$$

S-E-Case-Concat
$$\frac{e_1 \Downarrow \mathsf{rstr}[ps] \qquad e_3 \Downarrow x.y.e_4 \qquad [p/x][s/y]e_4 \Downarrow v}{\mathsf{strcase}(e_1; e_2; e_3) \Downarrow v}$$

S-E-Replace
$$\frac{e_1 \Downarrow \mathsf{rstr}[e_1] \qquad e_2 \Downarrow \mathsf{rstr}[s_2] \qquad \mathsf{subst}(r; s_1; s_2) = s}{\mathsf{rreplace}[r](e_1; e_2) \Downarrow \mathsf{rstr}[s]}$$

S-E-SafeCoerce
$$\frac{e \Downarrow \mathsf{rstr}[s]}{\mathsf{rcoerce}[r](e) \Downarrow \mathsf{rstr}[s]}$$

S-E-Check-Ok
$$\frac{e \Downarrow \mathsf{rstr}[s] \qquad s \in \mathcal{L}\{r\} \qquad [\mathsf{rstr}[s]/x]e_1 \Downarrow v}{\mathsf{rcheck}[r](e; x.e_1; e_2) \Downarrow v}$$

S-E-Check-NotOk
$$\frac{e \Downarrow \mathsf{rstr}[s] \qquad s \notin \mathcal{L}\{r\}}{\mathsf{rcheck}[r](e; x.e_1; e_2) \Downarrow e_2}$$

Figure 2: Big step semantics for λ_{RS}.

$\boxed{\Theta \vdash \iota : \tau}$ $\quad \Theta ::= \emptyset \mid \Theta, x : \tau$

P-T-Var
$$\frac{x : \tau \in \Theta}{\Theta \vdash x : \tau}$$

P-T-Abs
$$\frac{\Theta, x : \tau_1 \vdash \iota_2 : \tau_2}{\Theta \vdash \lambda x.\iota_2 : \tau_1 \to \tau_2}$$

P-T-App
$$\frac{\Theta \vdash \iota_1 : \tau_2 \to \tau \qquad \Theta \vdash \iota_2 : \tau_2}{\Theta \vdash \iota_1(\iota_2) : \iota}$$

P-T-String
$$\frac{}{\Theta \vdash \mathsf{str}[s] : \mathsf{string}}$$

P-T-Regex
$$\frac{}{\Theta \vdash \mathsf{rx}[r] : \mathsf{regex}}$$

P-T-Concat
$$\frac{\Theta \vdash \iota_1 : \mathsf{string} \qquad \Theta \vdash \iota_2 : \mathsf{string}}{\Theta \vdash \mathsf{concat}(\iota_1; \iota_2) : \mathsf{string}}$$

P-T-Case
$$\frac{\Theta \vdash \iota_1 : \mathsf{string} \qquad \Theta \vdash \iota_2 : \tau \qquad \Theta, x : \mathsf{string}, y : \mathsf{string} \vdash \iota_3 : \tau}{\Theta \vdash \mathsf{pstrcase}(\iota_1; \iota_2; \iota_3) : \tau}$$

P-T-Replace
$$\frac{\Theta \vdash \iota_1 : \mathsf{regex} \qquad \Theta \vdash \iota_2 : \mathsf{string} \qquad \Theta \vdash \iota_3 : \mathsf{string}}{\Theta \vdash \mathsf{preplace}(\iota_1; \iota_2; \iota_3) : \mathsf{string}}$$

P-T-Check
$$\frac{\begin{array}{c}\Theta \vdash \iota_r : \mathsf{regex} \\ \Theta \vdash \iota_1 : \mathsf{string} \qquad \Theta, x : \mathsf{string} \vdash \iota_2 : \sigma \qquad \Theta \vdash \iota_3 : \sigma\end{array}}{\Theta \vdash \mathsf{check}(\iota_r; \iota_1; x.\iota_2; \iota_3) : \sigma}$$

Figure 3: Typing rules for λ_P. The typing context Θ is standard.

$\boxed{\iota \Downarrow \iota}$

P-E-Abs
$$\frac{}{\lambda x.e \Downarrow \lambda x.e}$$

P-E-App
$$\frac{\iota_1 \Downarrow \lambda x.\iota_3 \qquad \iota_2 \Downarrow \iota_2' \qquad [\iota_3/x]\iota_2' \Downarrow v}{\iota_1(\iota_2) \Downarrow v}$$

P-E-Str
$$\frac{}{\mathsf{str}[s] \Downarrow \mathsf{str}[s]}$$

P-E-Rx
$$\frac{}{\mathsf{rx}[r] \Downarrow \mathsf{rx}[r]}$$

P-E-Concat
$$\frac{\iota_1 \Downarrow \mathsf{str}[s_1] \qquad \iota_2 \Downarrow \mathsf{str}[s_2]}{\mathsf{concat}(\iota_1; \iota_2) \Downarrow \mathsf{str}[s_1 s_2]}$$

P-E-Case-ϵ
$$\frac{\iota_1 \Downarrow \mathsf{str}[\ ] \qquad \iota 2 \Downarrow \iota_2}{\mathsf{pstrcase}(\iota_1; \iota_2; \iota_3) \Downarrow v_2}$$

P-E-Case-Concat
$$\frac{\iota_1 \Downarrow \mathsf{str}[ps] \qquad x.y.\iota_3 \Downarrow \iota_4 \qquad [p/x][s/y]\iota_4 \Downarrow v}{\mathsf{pstrcase}(\iota_1; \iota_2; \iota_3) \Downarrow v}$$

P-E-Replace
$$\frac{\iota_1 \Downarrow \mathsf{rx}[r] \qquad \iota_2 \Downarrow \mathsf{str}[s_2] \qquad \iota_3 \Downarrow \mathsf{str}[s_3] \qquad \mathsf{subst}(r; s_2; s_3) = s}{\mathsf{preplace}(\iota_1; \iota_2; \iota_3) \Downarrow \mathsf{str}[s]}$$

P-E-Check-OK
$$\frac{\iota \Downarrow \mathsf{str}[s] \qquad s \in \mathcal{L}\{r\} \qquad [\mathsf{str}[s]/x]\iota_1 \Downarrow \iota_3}{\mathsf{check}(\mathsf{rx}[r]; \iota; x.\iota_1; \iota_2) \Downarrow \iota_3}$$

P-E-Check-NotOK
$$\frac{\iota \Downarrow \mathsf{str}[s] \qquad s \notin \mathcal{L}\{r\}}{\mathsf{check}(\mathsf{rx}[r]; \iota; x.\iota_1; \iota_2) \Downarrow \iota_2}$$

Figure 4: Big step semantics for of λ_P

$$\boxed{\Psi \vdash [\![e]\!] = \iota}$$

$$\textsc{Tr-Var}\quad \frac{}{\Psi \vdash [\![x]\!] = x}$$

$$\textsc{Tr-Abs}\quad \frac{\Psi \vdash [\![e]\!] = \iota}{\Psi \vdash [\![\lambda x.e]\!] = \lambda x.\iota}$$

$$\textsc{Tr-App}\quad \frac{\Psi \vdash [\![e_1]\!] = \iota_1 \qquad \Psi \vdash [\![e_2]\!] = \iota_2}{\Psi \vdash [\![e_1(e_2)]\!] = \iota_1(\iota_2)}$$

$$\textsc{Tr-Case}\quad \frac{\Psi \vdash [\![e_1]\!] = \iota_1 \qquad \Psi \vdash [\![e_2]\!] = \iota_2 \qquad \Psi \vdash [\![e_3]\!] = \iota_3}{\Psi \vdash [\![\mathsf{strcase}(e_1; e_2; e_3)]\!] = \mathsf{pstrcase}(\iota_1; \iota_2; \iota_3)}$$

$$\textsc{Tr-string}\quad \frac{}{\Psi \vdash [\![\mathsf{rstr}[s]]\!] = \mathsf{str}[s]}$$

$$\textsc{Tr-Concat}\quad \frac{\Psi \vdash [\![e_1]\!] = \iota_1 \qquad \Psi \vdash [\![e_2]\!] = \iota_2}{\Psi \vdash [\![\mathsf{rconcat}(e_1; e_2)]\!] = \mathsf{concat}(\iota_1; \iota_2)}$$

$$\textsc{Tr-Subst}\quad \frac{\Psi \vdash [\![e_1]\!] = \iota_1 \qquad \Psi \vdash [\![e_2]\!] = \iota_2}{\Psi \vdash [\![\mathsf{rreplace}[r](e_1; e_2)]\!] = \mathsf{replace}(\mathsf{rx}[r]; \iota_1; \iota_2)}$$

$$\textsc{Tr-SafeCoerce}\quad \frac{\Psi \vdash [\![e]\!] = \iota}{\Psi \vdash [\![\mathsf{rcoerce}[r'](e)]\!] = \iota}$$

$$\textsc{Tr-Check}\quad \frac{\Psi \vdash [\![e]\!] = \iota_1 \qquad \Psi \vdash [\![e_1]\!] = \iota_1 \qquad \Psi \vdash [\![e_2]\!] = \iota_2}{\Psi \vdash [\![\mathsf{rcheck}[r](e; x.e_1; e_2)]\!] = \mathsf{check}(\mathsf{rx}[r]; \iota; x.\iota_1; \iota_2)}$$

Figure 5: Translation from source terms (e) to target terms (ι). The translation is type-directed.

Note that S-E-Abs and S-E-App are excluded from Figure 1 due to space constraints. However, they resemble the corresponding rules for λ_P; see the paper for a full definition.