

# Modularly Programmable Syntax

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## Abstract

Full-scale functional programming languages often make *ad hoc* choices in the design of their concrete syntax. For example, while nearly all major functional languages build in derived forms for lists, introducing derived forms for other library constructs, e.g. for HTML trees or regular expressions, requires forming new syntactic dialects of these languages. Unfortunately, programmers have no way to modularly combine such syntactic dialects in general, limiting the choices ultimately available to them. In this work, we introduce and formally specify language primitives that mitigate the need for syntactic dialects by giving library providers the ability to programmatically control syntactic expansion in a safe, hygienic and modular manner.

TODO: expand on this



## Acknowledgments

TODO: Acknowledgments





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# Chapter 1

## Introduction

*The recent development of programming languages suggests that the simultaneous achievement of simplicity and generality in language design is a serious unsolved problem.*

— John Reynolds (1970) [33]

### 1.1 Motivation

Functional programming languages come in many sizes. The smallest functional languages – often referred to as “lambda calculi” – allow researchers and language designers to study the mathematical properties of language primitives of interest in isolation. These “small-scale” languages inform the design of “full-scale” languages, which combine several such primitives and also define various derived syntactic forms (colloquially, “syntactic sugar”) that decrease the syntactic cost of selected idioms. For example, Standard ML (SML) [17, 26], OCaml [24] and Haskell [22] build in derived forms that decrease the syntactic cost of working with lists. For example, in these languages, the form `[1, 2, 3, 4, 5]` desugars to:

```
Cons(1, Cons(2, Cons(3, Cons(4, Cons(5, Nil)))))
```

The hope amongst many language designers is that a limited number of derived forms like these will suffice to produce a “general-purpose” programming language, i.e. one that satisfies programmers working in a wide variety of application domains. Unfortunately, a stable language design that fully achieves this ideal has yet to emerge, as evidenced by the diverse array of *syntactic dialects* – dialects that introduce only new derived forms – that continue to proliferate around all major contemporary languages. For example, Ur/Web is a syntactic dialect of Ur (an ML-like full-scale language [8]) that builds in derived forms for SQL queries, HTML elements and other datatypes used in the domain of web programming [9]. We will consider a large number of other examples of syntactic dialects in Sec. 2.1. Tools like Camlp4 [24], Sugar\* [11, 12] and Racket’s preprocessor [13], which we will discuss in Sec. 2.2, have decreased the engineering costs of constructing syntactic dialects, further contributing to their proliferation.

### 1.1.1 Dialects Considered Harmful

Some view this proliferation of dialects as harmless or even as desirable, arguing that programmers can simply choose the right dialect for the job at hand [40]. However, this “dialect-oriented” approach is, in an important sense, anti-modular: programmers cannot always “combine” different dialects when they want to use the primitives that they feature together within a single program. For example, a programmer might have access to a syntactic dialect featuring HTML syntax and one featuring regular expression syntax, but it is not always straightforward to construct a dialect featuring both. Such a dialect might be desirable for constructing, for example, a web-based bioinformatics tool.

In some cases, coming up with a combined dialect is difficult because the dialects are specified using different formalisms. In other cases, a common formalism has been used, but it does not operationalize the notion of dialect combination (e.g. Racket’s preprocessor [13]). But even if we restrict our interest to dialects specified using a common formalism that does operationalize some notion of dialect combination, there may still be a problem: the formalism may not guarantee that the combined dialect will conserve important properties that can be established about the dialects in isolation. For example, consider two syntactic dialects specified using Camlp4, one specifying derived syntax for finite mappings, the other specifying overlapping syntax for *ordered* finite mappings. Though each dialect has a deterministic grammar, when these grammars are naïvely combined, syntactic ambiguities will arise. We are aware of only one formalism that guarantees that determinism is conserved when syntactic dialects are combined [34], but it has limited expressive power, as we will discuss in Sec. 2.2.2.

### 1.1.2 Large Languages Considered Harmful

Dialects sometimes have a less direct influence on large-scale software development: they can help convince the designers in control of comparatively popular languages, like OCaml and Scala, to include some variant of the primitives that they feature into backwards-compatible language revisions. This *ad hoc* approach is unsustainable, for three main reasons. First, as we will discuss in Sec. 2.1, there are simply too many potentially useful such primitives, and many of these are only relevant in relatively narrow application domains. It is unreasonable to expect language designers to be able to evaluate all of these use cases in a timely and informed manner. Second, primitives introduced earlier in a language’s lifespan can end up monopolizing finite “syntactic resources”, forcing subsequent primitives to use ever more esoteric forms. And third, primitives that prove after some time to be flawed in some way cannot be removed or modified without breaking backwards compatibility. For all these reasons, language designers are justifiably reticent to add new primitives to major programming languages.

### 1.1.3 Toward More General Primitives

If, as we’ve argued, language designers should strive both to avoid dialect formation and to keep general-purpose languages small, stable and free of *ad hoc* primitives, this leaves two possible paths forward. One, exemplified (arguably) by SML, is to simply eschew “niche” embellishments and settle on the existing design, which might be considered to sit at a “sweet spot” in the overall language design space (accepting that in some circumstances, this leads to high syntactic cost). The other path forward is to search for a small number of highly general primitives that allow us degrade many of the constructs that are built primitively into languages and their dialects today instead to modular library constructs. Encouragingly, primitives of this sort do occasionally arise. For example, a recent revision of OCaml added support for generalized algebraic data types (GADTs), based on research on guarded recursive datatype constructors [41]. Using GADTs, OCaml was able to move some of the *ad hoc* machinery for typechecking operations that use format strings, like `sprintf`, out of the language and into a library. Syntactic machinery related to `sprintf`, however, remains built in.

## 1.2 Overview of Contributions

Our aim in the work being proposed is to take further steps down the second path just described by introducing primitive language constructs that reduce the need for syntactic dialects and *ad hoc* derived syntactic forms. In particular, we plan to introduce the following primitives:

1. **Typed syntax macros** (TSMs) give library providers programmatic control over the parsing and expansion of literal forms at compile-time. We introduce TSMs first in the context of a simple language of expressions and types in Chapter 3, then add support for pattern matching in Chapter 4 and parameterized types and modules in Chapter 5.
2. **Type-specific languages** (TSLs), described in Chapter 6, further reduce syntactic cost by allowing library providers to associate a TSM with a type declaration and then rely on a local type inference scheme to invoke that TSM and apply its parameters implicitly.

As vehicles for this work, we will specify a small-scale typed lambda calculus in each of the chapters just mentioned. For the sake of examples, we will also describe (but not formally specify) a “full-scale” functional language called VerseML.<sup>1</sup> VerseML is, as its name suggests, a dialect of ML. It diverges from other dialects of ML that have a similar underlying type structure, like Standard ML and OCaml, in that it uses a local type inference scheme [32] (like, for example, Scala [27]) for reasons that have to do with the mechanisms described in Chapter 6. The reason we will not follow Standard ML [26]

<sup>1</sup>We distinguish VerseML from Wyvern, which is the language referred to in prior publications about some of the work that we will describe, because Wyvern is a group effort evolving independently in some important ways.



in giving a complete formal specification of VerseML here is both to emphasize that the primitives we introduce can be considered for inclusion in a variety of language designs (not exclusively dialects of ML), and to avoid distracting the reader with specifications for primitives that are already well-understood in the literature and that are orthogonal to those we introduce here.

The primitives we introduce perform *static code generation* (also sometimes called *static* or *compile-time metaprogramming*), meaning that the relevant rules in the static semantics of the language call for the evaluation of *static functions* that generate term encodings. Static functions are functions that are evaluated statically, i.e. during typing. The design we are proposing also has conceptual roots in earlier work on *active libraries*, which similarly envisioned using compile-time computation to give library providers more control over various aspects of a programming system (but did not take an approach rooted in the study of type systems) [39].

The main challenge in the design of the primitives we introduce will come in ensuring that they are metatheoretically well-behaved. If we are not careful, many of the problems that arise when combining language dialects, discussed earlier, could simply shift into the semantics of these primitives.<sup>2</sup> Our main technical contributions will be to rigorously address these problems in a principled manner. In particular, we will maintain:

- a *type discipline*, meaning that the language is type safe, and that programmers reading a well-typed expression can determine its type without examining its expansion;
- a *hygiene discipline* (a.k.a. a *binding discipline*), meaning that the expansion logic does not make any assumptions about the names of variables in the context surrounding the expansion, nor does it introduce “hidden bindings” into subexpressions; and
- *modular reasoning principles*, meaning that library providers will have the ability to reason about the constructs that they have defined in isolation, and clients will be able to use them safely in any combination, without the possibility of conflict.<sup>3</sup>

We will make these notions more precise as we continue.

## Thesis Statement

In summary, we will defend the following thesis statement:

A functional programming language can give library providers the ability to express new syntactic expansions while maintaining a type discipline, a hygiene discipline and modular reasoning principles.

<sup>2</sup>This is why languages like VerseML are often called “extensible languages”, though this is somewhat of a misnomer. The defining characteristic of an extensible language is that it *doesn’t* need to be extended in situations where other languages would need to be extended. We will avoid this terminology.

<sup>3</sup>This is not quite true – simple naming conflicts can arise. We will tacitly assume that they are being avoided extrinsically, e.g. by using a URI-based naming scheme as in the Java ecosystem.

## 1.3 Disclaimers

Before we continue, it may be useful to explicitly acknowledge that completely eliminating the need for dialects would indeed be asking for too much: certain language design decisions are fundamentally incompatible with others or require coordination across a language design. We aim only to decrease the need for syntactic dialects in this work. We will not consider situations that require modifications to the underlying type structure of the language in this work.

It may also be useful to explicitly acknowledge that library providers could leverage the primitives we introduce to define constructs that are in “poor taste”. We expect that in practice, VerseML will come with a standard library defining an expertly curated collection of standard constructs, as well as guidelines for advanced users regarding when it would be sensible to use the mechanisms we introduce (following the example of languages that support operator overloading or type classes [16], which also have some potential for “abuse” or “overuse”).



# Chapter 2

## Background

### 2.1 Motivating Examples

To further motivate our work, we will now provide a number of examples of derived syntactic forms that decrease the syntactic cost of working with various data structures. We cover the first two examples – regular expressions and lists – in substantial detail. We will refer back to these examples throughout this work. We then more concisely survey a number of other examples, grouped into categories, to establish the broad applicability of our contributions.

#### 2.1.1 Regular Expressions

Let us take the perspective of a regular expression library provider (we assume the reader has some familiarity with regular expressions [38]). The abstract syntax of regexes,  $r$ , over strings,  $s$ , is specified below:

$$r ::= \text{empty} \mid \text{str}(s) \mid \text{seq}(r;r) \mid \text{or}(r;r) \mid \text{star}(r)$$

**Recursive Sums** One way to express this abstract syntax is by defining a recursive sum type [18]. In VerseML, a recursive labeled sum type can be defined like this:

```
type Rx = Empty | Str of string | Seq of Rx * Rx |  
          Or of Rx * Rx | Star of Rx | Group of Rx
```

Figure 2.1: Definition of the recursive sum type Rx

The abstract syntax of regexes is too verbose to be practical in all but the most trivial examples, so the POSIX standard specifies a more concise concrete syntax [3]. A number of programming languages support derived syntax for regular expressions based on this standard, e.g. Perl [10]. Let us consider a hypothetical dialect of ML called ML+Rx (perhaps constructed using a tool like Camlp4, discussed in Sec. 2.2.2)

that similarly builds in derived forms for regexes (we will compare VerseML to ML+Rx in later chapters). ML+Rx supports *regex literals*, e.g.

```
/A|T|G|C/
```

desugars to:

```
Or(Str "A", Or(Str "T", Or(Str "G", Str "C")))
```

ML+Rx also supports *spliced subexpressions* in regex literals. For example, the function `example_rx` shown below constructs a regex by splicing in a string, `name`, and another regex, `ssn`:

```
let ssn = /\d\d\d-\d\d-\d\d\d\d/
fun example_rx(name : string) => /@name: %ssn/
```

Figure 2.2: An example of syntax that supports spliced subexpressions.

The prefix `@` indicates that `name` should be spliced in as a string, and the prefix `%` indicates that `ssn` should be spliced in as a regex. The body of `example_rx` desugars to the following:

```
Seq(Str(name), Seq(Str ":", ssn))
```

Notice that `name` appears wrapped in the constructor `Str` because it was prefixed by `@`, whereas `ssn` appears unadorned because it was prefixed by `%`.

To splice in an expression that does not take the form of a variable, e.g. a function call, we can delimit it with parentheses:

```
/@(capitalize name): %ssn/
```

Finally, ML+Rx allows us to pattern match on a value of type `Rx` using derived pattern syntax. For example:

```
fun read_example_rx(r : Rx) =>
  match r with
  | /@name: %ssn/ => Some (name, ssn)
  | _ => None
```

Figure 2.3: An example of derived pattern syntax.

This expression desugars to:

```
fun read_example_rx(r : Rx) =>
  match r with
  | Seq(Str(name), Seq(Str ":", ssn)) => Some (name, ssn)
  | _ => None
```

**Abstract Types** Encoding regexes as values of type `Rx` is straightforward, but there are reasons why one might not wish to expose this encoding to clients directly. First, regexes are usually identified up to their reduction to a normal form. For example, `seq(empty, r)` has normal form `r`. It can be useful for regexes with the same normal form

to be indistinguishable from the perspective of client code. Second, it can be useful for performance reasons to maintain additional data alongside regexes (e.g. a corresponding finite automaton), but one would not want to expose this “implementation detail” to clients. In fact, there may be many ways to represent regular expression patterns, each with different performance trade-offs, so we would like to provide clients with a choice of implementations. For these reasons, another approach in VerseML, as in ML, is to abstract over the choice of representation using the module system’s support for abstract types. In particular, we can define the *module signature* `RX` where the type of patterns, `t`, is held abstract:

```
signature RX = sig {
  type t
  val Empty : t
  val Str : string -> t
  val Seq : t * t -> t
  val Or : t * t -> t
  val Star : t -> t
  val Group : t -> t
  val case : (
    t -> {
      Empty : 'a,
      Str : string -> 'a,
      Seq : t * t -> 'a,
      Or : t * t -> 'a,
      Star : t -> 'a,
      Group : t -> 'a
    } -> 'a
  )
}
```

Figure 2.4: Definition of the `RX` signature.

Clients of any module `R` that has been sealed against `RX`, written `R :> RX`, manipulate patterns as values of the type `R.t` using the interface described by this signature. The identity of the type `R.t` is held abstract outside the module during typechecking (i.e. it acts as a newly generated type). As a result, the burden of proving that there is no way to use the case analysis function to distinguish patterns with the same normal form is local to the module, and implementation details do not escape (and can thus evolve freely).

TODO: talk about module-parameterized derived syntactic forms for this

TODO: talk about pattern matching over values of abstract type

## 2.1.2 Lists

TODO: write this (Spring 2016)

## 2.1.3 Sets, Maps, Vectors and Other Containers

TODO: write this (Spring 2016)

## 2.1.4 HTML and Other Web Languages

TODO: write this; cite Ur/Web (Spring 2016)

## 2.1.5 Dates, URLs and Other Standardized Formats

TODO: write this (Spring 2016)

## 2.1.6 Query Languages

The language of regular expressions can be considered a query language over strings. There are many other query languages that focus on different types of data, e.g. XQuery for XML trees, or that are associated with different database technologies, e.g. SQL for relational databases. TODO: finish this (Spring 2016)

## 2.1.7 Monadic Commands

TODO: write this; cite Bob's blog (Spring 2016)

## 2.1.8 Quasiquote and Object Language Syntax

TODO: write this (Spring 2016)

## 2.1.9 Grammars

TODO: write this (Spring 2016)

## 2.1.10 Mathematical and Scientific Notations

SMILES: Chemical Notation

TODO: write this; cite SMILES [https://en.wikipedia.org/wiki/Simplified\\_molecular-input\\_line-entry\\_system](https://en.wikipedia.org/wiki/Simplified_molecular-input_line-entry_system) (Spring 2016)

T<sub>E</sub>X Mathematical Formula Notation

TODO: write this (Spring 2016)

## 2.2 Existing Approaches

TODO: revise, reformat and extend (Spring 2016 / as needed)

## 2.2.1 Dynamic String Parsing

To expose this more concise concrete syntax for regular expression patterns to clients, the most common approach is to provide a function that parses strings to produce patterns. Because, as just mentioned, there may be many implementations of the RX signature, the usual approach is to define a parameterized module (a.k.a. a *functor* in SML) defining utility functions like this abstractly:

```
module RXUtil(R : RX) => mod {  
  fun parse(s : string) : R.t => (* ... regex parser here ... *)  
}
```

This allows a client of any module `R : RX` to use the following definitions:

```
let module RUtil = RXUtil(R)  
let val rxparse = RUtil.parse
```

to construct patterns like this:

```
rxparse "A|T|G|C"
```

Unfortunately, this approach is imperfect for several reasons:

1. First, there are syntactic conflicts between string escape sequences and pattern escape sequences. For example, the following is not a well-formed term:

```
let val ssn = rxparse "\d\d\d-\d\d-\d\d\d\d"
```

When compiling an expression like this, the client would see an error message like `error: illegal escape character1`, because `\d` is not a valid string escape sequence. In a small lab study, we observed that this class of error often confused even experienced programmers if they had not used regular expressions recently [28]. One workaround has higher syntactic cost – we must double all backslashes:

```
let val ssn = rxparse "\\d\\d\\d-\\d\\d-\\d\\d\\d\\d"
```

Some languages, anticipating such modes of use, build in alternative string forms that leave escape sequences uninterpreted. For example, OCaml supports the following, which has only a constant syntactic cost:

```
let val ssn = rxparse {rx|\d\d\d-\d\d-\d\d\d\d|rx}
```

2. The next problem is that dynamic string parsing mainly decreases the syntactic cost of complete patterns. Patterns constructed compositionally cannot easily benefit from this technique. For example, consider the following function from strings to patterns:

```
fun example(name : string) =>  
  R.Seq(R.Str(name), R.Seq(rxparse ":" , ssn)) (* ssn as above *)
```

Had we built derived syntax for regular expression patterns into the language primitively (following Unix conventions of using forward slashes as delimiters), we could have used *splicing syntax*:

<sup>1</sup>This is the error message that `javac` produces. When compiling an analogous expression using SML of New Jersey (SML/NJ), we encounter a more confusing error message: `Error: unclosed string`.



```
fun example_shorter(name : string) => /@name: %ssn/
```

An identifier (or parenthesized expression, not shown) prefixed with an @ is a spliced string, and one prefixed with a % is a spliced pattern.

It is difficult to capture idioms like this using dynamic string parsing, because strings cannot contain sub-expressions directly.

3. For functions like `example` where we are constructing patterns on the basis of data of type `string`, using strings coincidentally to introduce patterns tempts programmers to use string concatenation in subtly incorrect ways. For example, consider the following seemingly more readable definition of `example`:

```
fun example_bad(name : string) =>
  rxparse (name ^ {rx| : \d\d\d-\d\d-\d\d\d\d|rx})
```

Both `example` and `example_bad` have the same type and behave identically at many inputs, particularly “typical” inputs (i.e. alphabetic names). It is only when the input name contains special characters that have meaning in the concrete syntax of patterns that a problem arises.

In applications that query sensitive data, mistakes like this lead to *injection attacks*, which are among the most common and catastrophic security threats on the web today [4]. These are, of course, a consequence of the programmer making a mistake in an effort to decrease syntactic cost, but proving that mistakes like this have not been made involves reasoning about complex run-time data flows, so it is once again notoriously difficult to automate. If our language supported derived syntax for patterns, this kind of mistake would be substantially less common (because `example_shorter` has lower syntactic cost than `example_bad`).

4. The next problem is that pattern parsing does not occur until the pattern is evaluated. For example, the following malformed pattern will only trigger an exception when this expression is evaluated during the full moon:

```
case (moon_phase) {
  Full => rxparse "(GC" (* malformedness not statically detected *)
  | _ => (* ... *)
}
```

Though malformed patterns can sometimes be discovered dynamically via testing, empirical data gathered from large open source projects suggests that there remain many malformed regular expression patterns that are not detected by a project’s test suite “in the wild” [36].

Statically verifying that pattern formation errors will not dynamically arise requires reasoning about arbitrary dynamic behavior. This is an undecidable verification problem in general and can be difficult to even partially automate. In this example, the verification procedure would first need to be able to establish that the variable `rxparse` is equal to the parse function `RUtil.parse`. If the string argument had not been written literally but rather computed, e.g. as `"(G" ^ "C"` where `^` is the string concatenation function applied in infix style, it would also need to be able to establish that this expression is equivalent to the string `"(GC"`. For patterns that are

dynamically constructed based on input to a function, evaluating the expression statically (or, more generally, in some earlier “stage” of evaluation [21]) also does not suffice.

Of course, asking the client to provide a proof of well-formedness would defeat the purpose of lowering syntactic cost.

In contrast, were our language to primitively support derived pattern syntax, pattern parsing would occur at compile-time and so malformed patterns would produce a compile-time error.

5. Dynamic string parsing also necessarily incurs dynamic cost. Regular expression patterns are common when processing large datasets, so it is easy to inadvertently incur this cost repeatedly. For example, consider mapping over a list of strings:

```
map exmpl_list (fn s => rxmatch (rxparse "A|T|G|C") s)
```

To avoid incurring the parsing cost for each element of `exmpl_list`, the programmer or compiler must move the parsing step out of the closure (for example, by eta-reduction in this simple example).<sup>2</sup> If the programmer must do this, it can (in more complex examples) increase syntactic cost and cognitive cost by moving the pattern itself far away from its use site. Alternatively, an appropriately tuned memoization (i.e. caching) strategy could be used to amortize some of this cost, but it is difficult to reason compositionally about performance using such a strategy.

In contrast, were our language to primitively support derived pattern syntax, the expansion would be computed at compile-time and incur no dynamic cost.

The problems above are not unique to regular expression patterns. Whenever a library encourages the use of dynamic string parsing to address the issue of syntactic cost (which is, fundamentally, not a dynamic issue), these problems arise. This fact has motivated much research on reducing the need for dynamic string parsing [6]. Existing alternatives can be broadly classified as being based on either *direct syntax extension* or *static term rewriting*. We describe these next, in Secs. 2.2.2 and 2.2.3 respectively.

## 2.2.2 Direct Syntax Extension

One tempting alternative to dynamic string parsing is to use a system that gives the users of a language the power to directly extend its concrete syntax with new derived forms.

The simplest such systems are those where the elaboration of each new syntactic form is defined by a single rewrite rule. For example, Gallina, the “external language” of the Coq proof assistant, supports such extensions [25]. A formal account of such a system has been developed by Griffin [15]. Unfortunately, a single equation is not enough to allow us to express pattern syntax following the usual conventions. For example, a system like Coq’s cannot handle escape characters, because there is no way to programmatically examine a form when generating its expansion.

<sup>2</sup>Anecdotally, in major contemporary compilers, this optimization is not automatic.

Other syntax extension systems are more flexible. For example, many are based on context-free grammars, e.g. Sugar\* [11] and Camlp4 [24] (amongst many others). Other systems give library providers direct programmatic access to the parse stream, like Common Lisp’s *reader macros* [37] (which are distinct from its term-rewriting macros, described in Sec. 2.2.3 below) and Racket’s preprocessor [13]. All of these would allow us to add pattern syntax into our language’s grammar, perhaps following Unix conventions and supporting splicing syntax as described above:

```
let val ssn = /\d\d\d-\d\d-\d\d\d\d/
fun example_shorter(name : string) => /@name: %ssn/
```

We sidestep the problems of dynamic string parsing described above when we directly extend the syntax of our language using any of these systems. Unfortunately, direct syntax extension introduces serious new problems. First, the systems mentioned thus far cannot guarantee that syntactic conflicts between such extensions will not arise. As stated directly in the Coq manual: “mixing different symbolic notations in [the] same text may cause serious parsing ambiguity”. If another library provider used similar syntax for a different implementation or variant of regular expressions, or for some other unrelated construct, then a client could not simultaneously use both libraries at the same time. So properly considered, every combination of extensions introduced using these mechanisms creates a *de facto* syntactic dialect of our language. The benefit of these systems is only that they lower the implementation cost of constructing syntactic dialects.

In response to this problem, Schwerdfeger and Van Wyk developed a modular analysis that accepts only context-free grammar extensions that begin with an identifying starting token and obey certain constraints on the follow sets of base language’s non-terminals [34]. Extensions that specify distinct starting tokens and that satisfy these constraints can be used together in any combination without the possibility of syntactic conflict. However, the most natural starting tokens like *rx* cannot be guaranteed to be unique. To address this problem, programmers must agree on a convention for defining “globally unique identifiers”, e.g. the common URI convention used on the web and by the Java packaging system. However, this forces us to use a more verbose token like *edu\_cmu\_VerseML\_rx*. There is no simple way for clients of our extension to define scoped abbreviations for starting tokens because this mechanism operates purely at the level of the context-free grammar.

Putting this aside, we must also consider another modularity-related question: which particular module should the expansion use? Clearly, simply assuming that some module identified as *R* matching *RX* is in scope is a brittle solution. In fact, we should expect that the system actively prevents such capture of specific variable names to ensure that variables (here, module variables) can be freely renamed. Such a *hygiene discipline* is well-understood only when performing term-to-term rewriting (discussed below) or in simple language-integrated rewrite systems like those found in Coq. For mechanisms that operate strictly at the level of context-free grammars or the parse stream, it is not clear how one could address this issue. The onus is then on the library provider to make no assumptions about variable names and instead require that the client explicitly

identify the module they intend to use as an “argument” within the newly introduced form:

```
let val ssn = edu_cmu_VerseML_rx R /\d\d\d-\d\d-\d\d\d\d/
```

Another problem with the approach of direct syntax extension is that, given an unfamiliar piece of syntax, there is no straightforward method for determining what type it will have, causing difficulties for both humans (related to code comprehension) and tools.

**TODO: Related work I haven’t mentioned yet:**

- Fan: <http://zhanghongbo.me/fan/start.html>
- Well-Typed Islands Parse Faster:  
<http://www.ccs.neu.edu/home/ejs/papers/tfp12-island.pdf>
- User-defined infix operators
- SML quote/unquote
- That Modularity paper
- Template Haskell and similar

### 2.2.3 Term Rewriting

An alternative approach is to leave the concrete syntax of the language fixed, but repurpose it for novel ends using a *local term-rewriting system*. The LISP macro system [19] is perhaps the most prominent example of such a system. Early variants of this system suffered from the problem of unhygienic variable capture just described, but later variants, notably in the Scheme dialect of LISP, brought support for enforcing hygiene [23]. In languages with a richer static type discipline, variants of macros that restrict rewriting to a particular type and perform the rewriting statically have also been studied [14, 20] and integrated into languages, e.g. MacroML [14] and Scala [7].

The most immediate problem with using these for our example is that we are not aware of any such statically-typed macro system that integrates cleanly with an ML-style module system. In other words, macros cannot be parameterized by modules. However, let us imagine such a macro system. We could use it to repurpose string syntax as follows:

```
let val ssn = rx R {rx|\d\d\d-\d\d-\d\d\d\d|rx}
```

The definition of the macro `rx` might look like this:

```
1 macro rx[Q : RX](e) at Q.t {  
2   static fun f(e : Exp) : Exp => case(e) {  
3     StrLit(s) => (* regex parser here *)  
4     | BinOp(Caret, e1, e2) => 'Q.Seq(Q.Str(%e1), %(f e2))'  
5     | BinOp(Plus, e1, e2) => 'Q.Seq(%(f e1), %(f e2))'  
6     | _ => raise Error  
7   }  
8 }
```

Here, `rx` is a macro parameterized by a module matching `rx` (we identify it as `Q` to emphasize that there is nothing special about the identifier `R`) and taking a single argument, identified as `e`. The macro specifies a type annotation, `at Q.t`, which imposes the constraint that the expansion the macro statically generates must be of type `Q.t` for the provided parameter `Q`. This expansion is generated by a *static function* that examines the syntax tree of the provided argument (syntax trees are of a type `Exp` defined in the standard library; cf. SML/NJ's visible compiler [2]). If it is a string literal, as in the example above, it statically parses the literal body to generate an expansion (the details of the parser, elided on line 3, would be entirely standard). By parsing the string statically, we avoid the problems of dynamic string parsing for statically-known patterns.

For patterns that are constructed compositionally, we need to get more creative. For example, we might repurpose the infix operators that are normally used for other purposes to support string and pattern splicing, e.g. as follows:

```
fun example_using_macro(name : string) =>
  rx R (name ^ ":" + ssn)
```

The binary operator `^` is repurposed to indicate a spliced string and `+` is repurposed to indicate a spliced pattern. The logic for handling these forms can be seen above on lines 4 and 5, respectively. We assume that there is derived syntax available at the type `Exp`, i.e. *quasiquote* syntax as in Lisp [5] and Scala [35], here delimited by backticks and using the prefix `%` to indicate a spliced value (i.e. `unquote`).

Having to creatively repurpose existing forms in this way limits the effect a library provider can have on syntactic cost (particularly when it would be desirable to express conventions that are quite different from the conventions adopted by the language). It also can create confusion for readers expecting parenthesized expressions to behave in a consistent manner. However, this approach is preferable to direct syntax extension because it avoids many of the problems discussed above: there cannot be syntactic conflicts (because the syntax is not extended at all), we can define macro abbreviations because macros are integrated into the language, there is a hygiene discipline that guarantees that the expansion will not capture variables inadvertently, and by using a typed macro system, programmers need not examine the expansion to know what type the expansion produced by a macro must have.

# Chapter 3

## Unparameterized Expression TSMs

We now introduce a new primitive – the **typed syntax macro** (TSM) – that combines much of the syntactic flexibility of syntax extensions with the reasoning guarantees of typed macros. This chapter focuses on the simplest case: TSMs that generate expressions of a single specified type (*unparameterized expression TSMs*). We will consider pattern matching in Chapter 4 and parameterized families of types in Chapter 5.

We begin in Sec. 3.1 by describing expression TSMs in VerseML by example. In particular, we will show a TSM for introducing values of the type  $Rx$  defined in Figure 2.1. We then formally specify unparameterized expression TSMs with a lambda calculus,  $\text{miniVerse}_{\mathcal{U}}$ , in Sec. 3.2.

### 3.1 Expression TSMs By Example

#### 3.1.1 Usage

Consider the following concrete VerseML expression:

```
$rx /A|T|G|C/
```

We apply a TSM, identified as  $\$rx$ , to a *literal form*, `/A|T|G|C/`. Literal forms are left unparsed when concrete expressions are first parsed by a VerseML compiler. A number of literal forms, shown in Figure 3.1, are available in VerseML’s concrete syntax. Though certain TSMs may by convention call for the use of particular literal forms, any literal form can be used with any TSM, e.g. we could have written  $\$rx$  ‘`A|T|G|C`’ above (this would actually be convenient if we wanted to write a regex containing forward slashes but not backticks). Because the set of literal forms is fixed by VerseML, TSMs cannot introduce syntactic conflicts by construction.

During the typechecking process, the TSM parses the *body* of the provided literal form, i.e. the characters in blue, to generate a *candidate expansion*. The language then *validates* the candidate expansion, according to criteria that we will establish in Sec. 3.1.4. If validation succeeds, the language generates the *final expansion* (or more concisely, simply the *expansion*) of the expression. The expansion of the expression above, written concretely, is:



```

1 'body cannot contain an apostrophe'
2 'body cannot contain a backtick'
3 [body cannot contain unmatched square brackets]
4 {body cannot contain an unmatched curly brace}
5 /body cannot contain a forward slash/
6 \body cannot contain a backslash\
7 42 (* numeric forms *)
8 42px (* numeric forms with suffixes *)

```

Figure 3.1: Literal forms available for use with TSMs (and TSLs, cf. Chapter 6) in VerseML’s concrete syntax. The characters in blue are the literal bodies. In this figure, each line describes how the body is constrained by the form shown on that line. The Wyvern language specifies additional forms, including whitespace-delimited forms [29] and multipart forms [30], but for simplicity we leave these out of VerseML.

```
Or(Str "A", Or(Str "T", Or(Str "G", Str "C")))
```

The constructors in the expansion above are those of the type `Rx` that was defined in Figure 2.1.

### 3.1.2 Definition

The definition of the TSM `$rx` shown in use above has the following form:

```

syntax $rx at Rx {
  static fn(body : Body) : ParseResultExp => (* regex parser here *)
}

```

This TSM definition first identifies the TSM as `$rx`. VerseML requires that all TSM variable identifiers begin with a dollar sign (to clearly distinguish TSMs from functions). A similar convention is also enforced by the Rust macro system [1].

The TSM definition then specifies a *type annotation*, `at Rx`, and a *parse function*, within curly braces. The parse function is a *static function* that parses the literal body to generate an encoding of the candidate expansion, or an error if one cannot be generated (e.g. when the body is ill-formed according to the syntactic specification that the TSM implements). Static functions are functions that will be evaluated statically (and they cannot refer to surrounding variable bindings). The parse function has type `Body -> ParseResult`. These types are defined in the VerseML *prelude*, which is a set of definitions available ambiently. The input type, `Body`, gives the parse function access to the body of the provided literal form, which can be represented as a string:

```
type Body = string
```

The output type, `ParseResultExp`, is a labeled sum type that distinguishes between successful parses and parse errors:

```

type ParseResultExp = Success of CExp
                      | ParseError of {msg : string, loc : IndexRange}

```

Successful parses, constructed by `Success`, generate candidate expansions, which are encoded as values of type `CExp`, shown in Figure 3.2. The elided constructors in Figure

```

type CETyp = TyVar of var_t
                | Arrow of CETyp * CETyp
                | ...
                | Spliced of IndexRange

type CEEExp = Var of var_t
                | Fn of var_t * CETyp * CEEExp
                | App of CEEExp * CEEExp
                | ...
                | Spliced of IndexRange

```

Figure 3.2: Abbreviated definitions of CETyp and CEEExp in VerseML. We assume some suitable type `var_t` exists, not shown.

3.2 encode the abstract syntax of VerseML expressions and types (as in the SML visible compiler [2]). We discuss the constructors labeled `Spliced` in Sec. 3.1.3. To decrease the syntactic cost of working with the types defined in Figure 3.2, the prelude implements *quasiquote syntax* using TSMs. We will discuss these TSMs in more detail in Sec. 7.1.1. The definitions in Figure 3.2 are recursive labeled sum types for simplicity, but alternative encodings of abstract syntax, e.g. based on abstract binding trees [18], could also have been chosen with only minor modification to the semantics.

If the parse function determines that a candidate expansion cannot be generated, i.e. there is a parse error in the literal body, it returns a value constructed by `ParseError`. It must provide an error message and indicate the location of the error within the body of the literal form as a value of type `IndexRange`:

```

type IndexRange = {startIndex : nat, endIndex : nat} (* inclusive *)

```

The error message and error location can be used by VerseML compilers when reporting errors to the programmer.

### 3.1.3 Splicing

To support spliced subexpressions, like we described in Sec. 2.1.1, the parse function must be able to parse subexpressions out of the supplied literal body. For example, consider the code snippet from Figure 2.2, rewritten using the `$rx` TSM:

```

val ssn = $rx /\d\d\d-\d\d-\d\d\d\d/
fun example_rx_tsm(name: string) => $rx /@name: %ssn/

```

The subexpressions `name` and `ssn` on the second line appear directly in the body of the literal form, so we call them *spliced subexpressions*. When the parse function determines that a subsequence of the body should be treated as a spliced subexpression (here, by recognizing the characters `@` or `%` followed by a variable or parenthesized expression), it can refer to it within the candidate expansion it generates using the `Spliced` constructor of the `CEEExp` type shown in Figure 3.2. Spliced subexpressions are referred to indirectly by their position within the literal body (i.e. with a value of type `IndexRange`, defined above) to prevent TSMs from forging a spliced subexpression (i.e. claiming that an



expression is a spliced subexpression, even though it does not appear in the body of the literal form). Expressions can contain type expressions, so one can also mark spliced type expressions in an analogous manner.

The candidate expansion generated by `$rx` for the body of `example_rx_tsm`, if written in a hypothetical concrete syntax for candidate expansions where spliced subexpressions are written `spliced<startIdx, endIndex>`, is:

```
Seq(Str(spliced<1, 4>), Seq(Str ":", spliced<8, 10>))
```

Here, `spliced<1, 4>` refers to the subexpression name by position and `spliced<8, 10>` refers to the subexpression `ssn` by position.

### 3.1.4 Typing

The language *validates* candidate expansions. The first check involves checking the candidate expansion against the type annotation specified by the TSM, i.e. the type `Rx` in the example above. This maintains a type discipline: if a programmer encounters a TSM being applied when reading a well-typed program, they need only look up the TSM's type annotation to determine the type of that expression. They do not need to examine the expansion directly.

The spliced subexpressions that the candidate expansion refers to (by their position within the literal body, cf. above) are recursively parsed, typed and expanded during this process.

### 3.1.5 Hygiene

To maintain a useful binding discipline, the validation process checks two additional properties: **context independent expansion** and **expansion independent splicing**. These are collectively referred to as the *hygiene properties*.

**Context Independent Expansion** Programmers expect to be able to choose variable identifiers freely, i.e. without “hidden constraints” imposed by the TSMs they are using. For this reason, context-dependent candidate expansions, i.e. those with free variables, are deemed invalid (even at application sites where those variables happen to be bound). An example of a “bad” TSM that generates context-dependent expansions is shown below:

```
syntax $bad1 at Rx {
  static fn(body : Body) : ParseResultExp => Success (Var 'x')
}
```

The candidate expansion this TSM generates would be well-typed only when there is a binding `x : Rx` in the application site typing context, so it is deemed invalid (even in such a context).

Application site bindings are, however, available when recursively typing and expanding the spliced subexpressions referred to in the candidate expansion because spliced subexpressions are authored by the TSM client and appear at the application site. We

saw examples of spliced subexpressions that referred to variables bound at the application site in Sec. 3.1.3.

In the examples in Sec. 3.1.1 and Sec. 3.1.3, the expansion used constructors associated with the  $Rx$  type, e.g. `Seq` and `Str`. This might appear to violate context independence. However, this is not the case because in VerseML, constructor labels are not variables. Syntactically, they must begin with a capital letter (as in Haskell). Different labeled sum types can use common constructor labels without conflict because the type the term is being checked against – e.g.  $Rx$ , due to the type ascription on  $\$rx$  – determines which type of value will be constructed.

**Expansion Independent Splicing** Spliced subexpressions have access to only those variables that were bound at the application site, i.e. a TSM cannot introduce new bindings into spliced subexpressions. For example, consider the following hypothetical candidate expansion (written concretely as above):

```
fn(x : Rx) => spliced<0, 4>
```

The variable  $x$  would not be available when typing the indicated spliced subexpression, nor would it shadow any bindings of  $x$  at the application site.

The benefit of maintaining this property is that the TSM client can determine which bindings are available in a spliced subexpression without examining the expansion it appears within. There are no “hidden variables”. The trade-off is that this prevents library providers from defining alternative binding idioms. For example, Haskell’s derived form for monadic commands (i.e. `do`-notation) supports binding the result of executing a command to a variable that is then available in the subsequent commands in a command sequence. We will show an alternative formulation of Haskell’s syntax for monadic commands that uses VerseML’s anonymous function syntax to bind variables in Sec. 7.1.2. We will discuss mechanisms that might allow us to relax this restriction while retaining client control over variable naming as future work in Sec. 7.3.3.

## Final Expansion

After checking that the candidate expansion is valid, the semantics generates the *final expansion* by replacing the references to spliced subexpressions with their final expansions. For example, the final expansion of the body of `example_rx_tsm` is:

```
Seq(Str(name), Seq(Str ":", " ", ssn))
```

## 3.2 miniVerse<sub>U</sub>

To make the intuitions developed in the previous section precise, we will now introduce a reduced calculus with support for unparameterized expression TSMs called `miniVerseU`. We specify all formal systems in this document within the metatheoretic framework detailed in *PFPL* [18], and assume familiarity of fundamental background

Sort	Abstract Form	Stylized Form	Description
Typ $\tau ::=$	$t$	$t$	variable
	$\text{parr}(\tau; \tau)$	$\tau \multimap \tau$	partial function
	$\text{all}(t.\tau)$	$\forall t.\tau$	polymorphic
	$\text{rec}(t.\tau)$	$\mu t.\tau$	recursive
	$\text{prod}(\{i \mapsto \tau_i\}_{i \in L})$	$\langle \{i \mapsto \tau_i\}_{i \in L} \rangle$	labeled product
EExp $e ::=$	$\text{sum}(\{i \mapsto \tau_i\}_{i \in L})$	$[\{i \mapsto \tau_i\}_{i \in L}]$	labeled sum
	$x$	$x$	variable
	$\text{elam}(\tau; x.e)$	$\lambda x:\tau.e$	abstraction
	$\text{eap}(e; e)$	$e(e)$	application
	$\text{etlam}(t.e)$	$\Lambda t.e$	type abstraction
	$\text{etap}(e; \tau)$	$e[\tau]$	type application
	$\text{efold}(t.\tau; e)$	$\text{fold}(e)$	fold
	$\text{eunfold}(e)$	$\text{unfold}(e)$	unfold
	$\text{etpl}(\{i \mapsto e_i\}_{i \in L})$	$\langle \{i \mapsto e_i\}_{i \in L} \rangle$	labeled tuple
	$\text{epr}[\ell](e)$	$e \cdot \ell$	projection
	$\text{ein}[\ell](\{i \mapsto \tau_i\}_{i \in L}; e)$	$\ell \cdot e$	injection
	$\text{ecase}(e; \{i \mapsto x_i.e_i\}_{i \in L})$	$\text{case } e \{i \mapsto x_i.e_i\}_{i \in L}$	case analysis

Figure 3.3: Syntax of types and expanded expressions in  $\text{miniVerse}_{\text{U}}$ . Metavariable  $x$  ranges over variables,  $t$  ranges over type variables,  $\ell$  ranges over labels and  $L$  ranges over sets of labels. The forms  $\{i \mapsto e_i\}_{i \in L}$ ,  $\{i \mapsto \tau_i\}_{i \in L}$  and  $\{i \mapsto x_i.e_i\}_{i \in L}$  indicate finite mappings from each label  $i \in L$  to an expression, type or binder over an expression, respectively. The label set is omitted for concision when writing particular finite mappings, e.g.  $\ell_1 \mapsto e_1, \ell_2 \mapsto e_2$ . We write  $\{i \mapsto \tau_i\}_{i \in L} \otimes \ell \mapsto \tau$  when  $\ell \notin L$  for the extension of  $\{i \mapsto \tau_i\}_{i \in L}$  with a mapping from  $\ell$  to  $\tau$ . When we use the stylized forms  $\text{fold}(e)$  and  $\ell \cdot e$ , we assume that the reader can infer the type information from context.

concepts (e.g. abstract binding trees, substitution, implicit identification of terms up to  $\alpha$ -equivalence, structural induction and rule induction) covered therein.

### 3.2.1 Types and Expanded Expressions

At the “semantic core” of  $\text{miniVerse}_{\text{U}}$  are *types*,  $\tau$ , and *expanded expressions*,  $e$ . Their syntax is specified by the syntax chart in Figure 3.3. Types and expanded expressions form a language with support for partial functions, quantification over types, recursive types, and labeled product and sum types. The reader can consult *PFPL* [18] (or another text on typed programming languages, e.g. *TAPL* [31]) for a detailed account of these constructs (or closely related variants thereof). For our purposes, it suffices to recall the following facts and definitions.

## Statics of Expanded Expressions

The *statics of expanded expressions* is specified by generic hypothetical judgements of the following form:

Judgement Form	Description
$\Delta \vdash \tau \text{ type}$	$\tau$ is a well-formed type under $\Delta$
$\Delta \vdash \Gamma \text{ ctx}$	$\Gamma$ is a well-formed typing context under $\Delta$
$\Delta \Gamma \vdash e : \tau$	$e$ has type $\tau$ under $\Delta$ and $\Gamma$

*Type formation contexts*,  $\Delta$ , are finite sets of hypotheses of the form  $t \text{ type}$  and *typing contexts*,  $\Gamma$ , consist of hypotheses of the form  $x : \tau$  and operate as finite mappings from variables to types. Syntactically, we write contexts as comma-separated sequences of hypotheses identified up to exchange and contraction. We write empty contexts as  $\emptyset$  (or omit them entirely).

The type formation judgement,  $\Delta \vdash \tau \text{ type}$ , ensures that all free type variables in  $\tau$  are tracked by  $\Delta$ . It is inductively defined by the following rules:

$$\frac{}{\Delta, t \text{ type} \vdash t \text{ type}} \quad (3.1a)$$

$$\frac{\Delta \vdash \tau_1 \text{ type} \quad \Delta \vdash \tau_2 \text{ type}}{\Delta \vdash \text{parr}(\tau_1; \tau_2) \text{ type}} \quad (3.1b)$$

$$\frac{\Delta, t \text{ type} \vdash \tau \text{ type}}{\Delta \vdash \text{all}(t.\tau) \text{ type}} \quad (3.1c)$$

$$\frac{\Delta, t \text{ type} \vdash \tau \text{ type}}{\Delta \vdash \text{rec}(t.\tau) \text{ type}} \quad (3.1d)$$

$$\frac{\{\Delta \vdash \tau_i \text{ type}\}_{i \in L}}{\Delta \vdash \text{prod}(\{i \hookrightarrow \tau_i\}_{i \in L}) \text{ type}} \quad (3.1e)$$

$$\frac{\{\Delta \vdash \tau_i \text{ type}\}_{i \in L}}{\Delta \vdash \text{sum}(\{i \hookrightarrow \tau_i\}_{i \in L}) \text{ type}} \quad (3.1f)$$

Premises of the form  $\{\mathcal{J}_i\}_{i \in L}$  mean that for each  $i \in L$ , the judgement  $\mathcal{J}_i$  must hold.

The typing context formation judgement,  $\Delta \vdash \Gamma \text{ ctx}$ , ensures that all types in the typing context are well-formed according to Rules (3.1). It is inductively defined by the following rules:

$$\frac{}{\Delta \vdash \emptyset \text{ ctx}} \quad (3.2a)$$

$$\frac{\Delta \vdash \Gamma \text{ ctx} \quad \Delta \vdash \tau \text{ type}}{\Delta \vdash \Gamma, x : \tau \text{ ctx}} \quad (3.2b)$$

The typing judgement,  $\Delta \Gamma \vdash e : \tau$ , assigns types to expressions. It is inductively defined by the following rules:

$$\frac{}{\Delta \Gamma, x : \tau \vdash x : \tau} \quad (3.3a)$$

$$\frac{\Delta \vdash \tau \text{ type} \quad \Delta \Gamma, x : \tau \vdash e : \tau'}{\Delta \Gamma \vdash \text{elam}(\tau; x.e) : \text{parr}(\tau; \tau')} \quad (3.3b)$$

$$\frac{\Delta \Gamma \vdash e_1 : \text{parr}(\tau; \tau') \quad \Delta \Gamma \vdash e_2 : \tau}{\Delta \Gamma \vdash \text{eap}(e_1; e_2) : \tau'} \quad (3.3c)$$

$$\frac{\Delta, t \text{ type} \Gamma \vdash e : \tau}{\Delta \Gamma \vdash \text{etlam}(t.e) : \text{all}(t.\tau)} \quad (3.3d)$$

$$\frac{\Delta \Gamma \vdash e : \text{all}(t.\tau) \quad \Delta \vdash \tau' \text{ type}}{\Delta \Gamma \vdash \text{etap}(e; \tau') : [\tau'/t]\tau} \quad (3.3e)$$

$$\frac{\Delta, t \text{ type} \vdash \tau \text{ type} \quad \Delta \Gamma \vdash e : [\text{rec}(t.\tau)/t]\tau}{\Delta \Gamma \vdash \text{efold}(t.\tau; e) : \text{rec}(t.\tau)} \quad (3.3f)$$

$$\frac{\Delta \Gamma \vdash e : \text{rec}(t.\tau)}{\Delta \Gamma \vdash \text{eunfold}(e) : [\text{rec}(t.\tau)/t]\tau} \quad (3.3g)$$

$$\frac{\{\Delta \Gamma \vdash e_i : \tau_i\}_{i \in L}}{\Delta \Gamma \vdash \text{etpl}(\{i \hookrightarrow e_i\}_{i \in L}) : \text{prod}(\{i \hookrightarrow \tau_i\}_{i \in L})} \quad (3.3h)$$

$$\frac{\Delta \Gamma \vdash e : \text{prod}(\{i \hookrightarrow \tau_i\}_{i \in L} \otimes \ell \hookrightarrow \tau)}{\Delta \Gamma \vdash \text{epr}[\ell](e) : \tau} \quad (3.3i)$$

$$\frac{\{\Delta \vdash \tau_i \text{ type}\}_{i \in L} \quad \Delta \vdash \tau \text{ type} \quad \Delta \Gamma \vdash e : \tau}{\Delta \Gamma \vdash \text{ein}[\ell](\{i \hookrightarrow \tau_i\}_{i \in L} \otimes \ell \hookrightarrow \tau; e) : \text{sum}(\{i \hookrightarrow \tau_i\}_{i \in L} \otimes \ell \hookrightarrow \tau)} \quad (3.3j)$$

$$\frac{\Delta \Gamma \vdash e : \text{sum}(\{i \hookrightarrow \tau_i\}_{i \in L}) \quad \{\Delta \Gamma, x_i : \tau_i \vdash e_i : \tau\}_{i \in L}}{\Delta \Gamma \vdash \text{ecase}(e; \{i \hookrightarrow x_i.e_i\}_{i \in L}) : \tau} \quad (3.3k)$$

The rules given above validate the following standard lemmas.

The Weakening Lemma expresses the intuition that extending a context with unused variables preserves well-formedness and typing.

**Lemma 3.1** (Weakening). *All of the following hold:*

1. If  $\Delta \vdash \tau \text{ type}$  then  $\Delta, t \text{ type} \vdash \tau \text{ type}$ .
2. If  $\Delta \vdash \Gamma \text{ ctx}$  then  $\Delta, t \text{ type} \vdash \Gamma \text{ ctx}$ .
3. If  $\Delta \Gamma \vdash e : \tau$  then  $\Delta, t \text{ type} \Gamma \vdash e : \tau$ .
4. If  $\Delta \Gamma \vdash e : \tau$  and  $\Delta \vdash \tau' \text{ type}$  then  $\Delta \Gamma, x : \tau' \vdash e : \tau$ .

*Proof Sketch.*

1. By rule induction on Rules (3.1).
2. By rule induction on Rules (3.2).
3. By rule induction on Rules (3.3).
4. By rule induction on Rules (3.3).

□

The Substitution Lemma expresses the intuition that substitution of a type for a type variable, or an expression of the appropriate type for an expression variable, preserves well-formedness and typing.

**Lemma 3.2** (Substitution). *All of the following hold:*

1. If  $\Delta, t \text{ type} \vdash \tau \text{ type}$  and  $\Delta \vdash \tau' \text{ type}$  then  $\Delta \vdash [\tau'/t]\tau \text{ type}$ .
2. If  $\Delta, t \text{ type} \vdash \Gamma \text{ ctx}$  and  $\Delta \vdash \tau' \text{ type}$  then  $\Delta \vdash [\tau'/t]\Gamma \text{ ctx}$ .
3. If  $\Delta, t \text{ type} \vdash e : \tau$  and  $\Delta \vdash \tau' \text{ type}$  then  $\Delta [\tau'/t]\Gamma \vdash [\tau'/t]e : [\tau'/t]\tau$ .
4. If  $\Delta \Gamma, x : \tau' \vdash e : \tau$  and  $\Delta \Gamma \vdash e' : \tau'$  then  $\Delta \Gamma \vdash [e'/x]e : \tau$ .

*Proof Sketch.* In each case, by rule induction on the derivation of the first assumption. □

The Decomposition Lemma is the converse of the Substitution Lemma.

**Lemma 3.3** (Decomposition). *All of the following hold:*

1. If  $\Delta \vdash [\tau'/t]\tau \text{ type}$  and  $\Delta \vdash \tau' \text{ type}$  then  $\Delta, t \text{ type} \vdash \tau \text{ type}$ .
2. If  $\Delta \vdash [\tau'/t]\Gamma \text{ ctx}$  and  $\Delta \vdash \tau' \text{ type}$  then  $\Delta, t \text{ type} \vdash \Gamma \text{ ctx}$ .
3. If  $\Delta [\tau'/t]\Gamma \vdash [\tau'/t]e : [\tau'/t]\tau$  and  $\Delta \vdash \tau' \text{ type}$  then  $\Delta, t \text{ type} \vdash e : \tau$ .
4. If  $\Delta \Gamma \vdash [e'/x]e : \tau$  and  $\Delta \Gamma \vdash e' : \tau'$  then  $\Delta \Gamma, x : \tau' \vdash e : \tau$ .

*Proof Sketch.*

1. Type formation of  $[\tau'/t]\tau$  does not depend on the structure of  $\tau'$ .
2. Context formation of  $[\tau'/t]\Gamma$  does not depend on the structure of  $\tau'$ .
3. The derivation of  $\Delta [\tau'/t]\Gamma \vdash [\tau'/t]e : [\tau'/t]\tau$  does not depend on the structure of  $\tau'$ .
4. Typing of  $[e'/x]e$  depends only on the type of  $e'$  wherever it occurs, if at all.

□

The Regularity Lemma expresses the intuition that the type assigned to an expression under a well-formed typing context is well-formed.

**Lemma 3.4** (Regularity). *If  $\Delta \vdash \Gamma \text{ ctx}$  and  $\Delta \Gamma \vdash e : \tau$  then  $\Delta \vdash \tau \text{ type}$ .*

*Proof Sketch.* By rule induction on Rules (3.3) and inversion of Rule (3.2b). □

## Dynamics of Expanded Expressions

The *dynamics* of expanded expressions is specified as a structural dynamics (a.k.a. structural operational semantics) by judgements of the following form:

Judgement Form	Description
$e \mapsto e'$	$e$ transitions to $e'$
$e \text{ val}$	$e$ is a value

We also define auxiliary judgements for *iterated transitions*,  $e \mapsto^* e'$ , and *evaluation*,  $e \Downarrow e'$ .

**Definition 3.5** (Iterated Transition). *Iterated transition,  $e \mapsto^* e'$ , is the reflexive, transitive closure of the transition judgement.*

**Definition 3.6** (Evaluation). *Evaluation,  $e \Downarrow e'$ , is derivable iff  $e \mapsto^* e'$  and  $e' \text{ val}$ .*

Our subsequent developments do not require making reference to particular rules in the dynamics (because TSMs operate statically), so for concision, we do not produce the rules here. Instead, it suffices to state certain conditions.

The Canonical Forms condition characterizes well-typed values. We assume an *eager* (i.e. *by-value*) formulation of the dynamics.

**Condition 3.7** (Canonical Forms). *If  $\vdash e : \tau$  and  $e$  val then:*

1. *If  $\tau = \text{parr}(\tau_1; \tau_2)$  then  $e = \text{elam}(\tau_1; x.e')$  and  $x : \tau_1 \vdash e' : \tau_2$ .*
2. *If  $\tau = \text{all}(t.\tau')$  then  $e = \text{etlam}(t.e')$  and  $t \text{ type} \vdash e' : \tau'$ .*
3. *If  $\tau = \text{rec}(t.\tau')$  then  $e = \text{efold}(t.\tau'; e')$  and  $\vdash e' : [\text{rec}(t.\tau')/t]\tau'$ .*
4. *If  $\tau = \text{prod}(\{i \hookrightarrow \tau_i\}_{i \in L})$  then  $e = \text{etpl}(\{i \hookrightarrow e_i\}_{i \in L})$  and for each  $i \in L$  we have that  $\vdash e_i : \tau_i$  and  $e_i$  val.*
5. *If  $\tau = \text{sum}(\{i \hookrightarrow \tau_i\}_{i \in L})$  then  $e = \text{ein}[\ell](\{i \hookrightarrow \tau_i\}_{i \in L \setminus \ell} \otimes \ell \hookrightarrow \tau'; e')$  and  $\ell \in L$  and  $\vdash e' : \tau'$  and  $e'$  val.*

We also require that the statics and dynamics be coherent. This is expressed in the standard way by Preservation and Progress conditions (together, these constitute the Type Safety Condition).

**Condition 3.8** (Preservation). *If  $\vdash e : \tau$  and  $e \mapsto e'$  then  $\vdash e' : \tau$ .*

**Condition 3.9** (Progress). *If  $\vdash e : \tau$  then either  $e$  val or there exists an  $e'$  such that  $e \mapsto e'$ .*

### 3.2.2 Unexpanded Expressions and Typed Expansion

Programs evaluate as expanded expressions, but programmers author *unexpanded expressions*,  $\hat{e}$ . The syntax of unexpanded expressions is specified by the chart in Figure 3.4.

Unexpanded expressions are typed and expanded simultaneously according to the *typed expansion judgement*.

#### Judgement Form    Description

$\Delta \Gamma \vdash_{\Sigma} \hat{e} \rightsquigarrow e : \tau$      $\hat{e}$  expands to  $e$  at type  $\tau$  under  $\Delta$  and  $\Gamma$  with macro environment  $\Sigma$

The following rules govern the typed expansion judgement:

$$\frac{}{\Delta \Gamma, x : \tau \vdash_{\Sigma} x \rightsquigarrow x : \tau} \quad (3.4a)$$

$$\frac{\Delta \vdash \tau \text{ type} \quad \Delta \Gamma, x : \tau \vdash_{\Sigma} \hat{e} \rightsquigarrow e : \tau'}{\Delta \Gamma \vdash_{\Sigma} \text{ulam}(\tau; x.\hat{e}) \rightsquigarrow \text{elam}(\tau; x.e) : \text{parr}(\tau; \tau')} \quad (3.4b)$$

$$\frac{\Delta \Gamma \vdash_{\Sigma} \hat{e}_1 \rightsquigarrow e_1 : \text{parr}(\tau; \tau') \quad \Delta \Gamma \vdash_{\Sigma} \hat{e}_2 \rightsquigarrow e_2 : \tau}{\Delta \Gamma \vdash_{\Sigma} \text{uap}(\hat{e}_1; \hat{e}_2) \rightsquigarrow \text{eap}(e_1; e_2) : \tau'} \quad (3.4c)$$

$$\frac{\Delta, t \text{ type} \Gamma \vdash_{\Sigma} \hat{e} \rightsquigarrow e : \tau}{\Delta \Gamma \vdash_{\Sigma} \text{utlam}(t.\hat{e}) \rightsquigarrow \text{etlam}(t.e) : \text{all}(t.\tau)} \quad (3.4d)$$

$$\frac{\Delta \Gamma \vdash_{\Sigma} \hat{e} \rightsquigarrow e : \text{all}(t.\tau) \quad \Delta \vdash \tau' \text{ type}}{\Delta \Gamma \vdash_{\Sigma} \text{utap}(\hat{e}; \tau') \rightsquigarrow \text{etap}(e; \tau') : [\tau'/t]\tau} \quad (3.4e)$$

Sort	Abstract Form	Stylized Form	Description
UExp	$\hat{e} ::= x$	$x$	variable
	$\text{ulam}(\tau; x.\hat{e})$	$\lambda x:\tau.\hat{e}$	abstraction
	$\text{uap}(\hat{e}; \hat{e})$	$\hat{e}(\hat{e})$	application
	$\text{utlam}(t.\hat{e})$	$\Lambda t.\hat{e}$	type abstraction
	$\text{utap}(\hat{e}; \tau)$	$\hat{e}[\tau]$	type application
	$\text{ufold}(t.\tau; \hat{e})$	$\text{fold}(\hat{e})$	fold
	$\text{uunfold}(\hat{e})$	$\text{unfold}(\hat{e})$	unfold
	$\text{utpl}(\{i \hookrightarrow \hat{e}_i\}_{i \in L})$	$\langle \{i \hookrightarrow \hat{e}_i\}_{i \in L} \rangle$	labeled tuple
	$\text{upr}[\ell](\hat{e})$	$\hat{e} \cdot \ell$	projection
	$\text{uin}[\ell](\{i \hookrightarrow \tau_i\}_{i \in L}; \hat{e})$	$\ell \cdot \hat{e}$	injection
	$\text{ucase}(\hat{e}; \{i \hookrightarrow x_i.\hat{e}_i\}_{i \in L})$	$\text{case } \hat{e} \{i \hookrightarrow x_i.\hat{e}_i\}_{i \in L}$	case analysis
	$\text{usyntax}(\tau; \hat{e}; a.\hat{e})$	$\text{syntax } a \text{ at } \tau \{ \hat{e} \} \text{ in } \hat{e}$	macro definition
	$\text{utsmap}[b][a]$	$a / b /$	macro application

Figure 3.4: Syntax of unexpanded expressions in miniVerse<sub>U</sub>. Metavariable  $a$  ranges over macro identifiers and  $b$  ranges over literal bodies. Literal bodies might contain unparsed subexpressions, so variable renaming and substitution cannot be defined in the usual manner over unexpanded expressions.

$$\frac{\Delta, t \text{ type} \vdash \tau \text{ type} \quad \Delta \Gamma \vdash_{\Sigma} \hat{e} \rightsquigarrow e : [\text{rec}(t.\tau)/t]\tau}{\Delta \Gamma \vdash_{\Sigma} \text{ufold}(t.\tau; \hat{e}) \rightsquigarrow \text{efold}(t.\tau; e) : \text{rec}(t.\tau)} \quad (3.4f)$$

$$\frac{\Delta \Gamma \vdash_{\Sigma} \hat{e} \rightsquigarrow e : \text{rec}(t.\tau)}{\Delta \Gamma \vdash_{\Sigma} \text{uunfold}(\hat{e}) \rightsquigarrow \text{eunfold}(e) : [\text{rec}(t.\tau)/t]\tau} \quad (3.4g)$$

$$\frac{\{\Delta \Gamma \vdash_{\Sigma} \hat{e}_i \rightsquigarrow e_i : \tau_i\}_{i \in L}}{\Delta \Gamma \vdash_{\Sigma} \text{utpl}(\{i \hookrightarrow \hat{e}_i\}_{i \in L}) \rightsquigarrow \text{etpl}(\{i \hookrightarrow e_i\}_{i \in L}) : \text{prod}(\{i \hookrightarrow \tau_i\}_{i \in L})} \quad (3.4h)$$

$$\frac{\Delta \Gamma \vdash_{\Sigma} \hat{e} \rightsquigarrow e : \text{prod}(\{i \hookrightarrow \tau_i\}_{i \in L} \otimes \ell \hookrightarrow \tau)}{\Delta \Gamma \vdash_{\Sigma} \text{upr}[\ell](\hat{e}) \rightsquigarrow \text{epr}[\ell](e) : \tau} \quad (3.4i)$$

$$\frac{\left\{ \begin{array}{c} \Delta \vdash \tau_i \text{ type} \\ \Delta \Gamma \vdash_{\Sigma} \text{uin}[\ell](\{i \hookrightarrow \tau_i\}_{i \in L} \otimes \ell \hookrightarrow \tau; \hat{e}) \rightsquigarrow \\ \text{ein}[\ell](\{i \hookrightarrow \tau_i\}_{i \in L} \otimes \ell \hookrightarrow \tau; e) : \text{sum}(\{i \hookrightarrow \tau_i\}_{i \in L} \otimes \ell \hookrightarrow \tau) \end{array} \right\}}{\Delta \Gamma \vdash_{\Sigma} \hat{e} \rightsquigarrow e : \text{sum}(\{i \hookrightarrow \tau_i\}_{i \in L})} \quad (3.4j)$$

$$\frac{\Delta \Gamma \vdash_{\Sigma} \hat{e} \rightsquigarrow e : \text{sum}(\{i \hookrightarrow \tau_i\}_{i \in L}) \quad \{\Delta \Gamma, x_i : \tau_i \vdash_{\Sigma} \hat{e}_i \rightsquigarrow e_i : \tau_i\}_{i \in L}}{\Delta \Gamma \vdash_{\Sigma} \text{ucase}(\hat{e}; \{i \hookrightarrow x_i.\hat{e}_i\}_{i \in L}) \rightsquigarrow \text{ecase}(e; \{i \hookrightarrow x_i.e_i\}_{i \in L}) : \tau} \quad (3.4k)$$

$$\frac{\Delta \vdash \tau \text{ type} \quad \emptyset \emptyset \vdash_{\emptyset} \hat{e}_{\text{parse}} \rightsquigarrow e_{\text{parse}} : \text{parr}(\text{Body}; \text{ParseResultExp}) \quad a \notin \text{dom}(\Sigma) \quad \Delta \Gamma \vdash_{\Sigma, a \hookrightarrow \text{syntax}(\tau; e_{\text{parse}})} \hat{e} \rightsquigarrow e : \tau'}{\Delta \Gamma \vdash_{\Sigma} \text{usyntax}(\tau; \hat{e}_{\text{parse}}; a.\hat{e}) \rightsquigarrow e : \tau'} \quad (3.4l)$$

$$\frac{b \downarrow e_{\text{body}} \quad e_{\text{parse}}(e_{\text{body}}) \Downarrow \text{Success} \cdot e_{\text{cand}} \quad e_{\text{cand}} \uparrow_{\text{CEExp}} \hat{e} \quad \emptyset \emptyset \vdash_{\Delta; \Gamma; \Sigma, a \hookrightarrow \text{syntax}(\tau; e_{\text{parse}}); b} \hat{e} \rightsquigarrow e : \tau}{\Delta \Gamma \vdash_{\Sigma, a \hookrightarrow \text{syntax}(\tau; e_{\text{parse}})} \text{utsmap}[b][a] \rightsquigarrow e : \tau} \quad (3.4m)$$



Notice that each form of expanded expression (Figure 3.3) corresponds to a form of unexpanded expression (Figure 3.4). For each typing rule in Rules (3.3), there is a corresponding typed expansion rule – Rules (3.4a) through (3.4k) – where the unexpanded and expanded forms correspond. The premises also correspond – if a typing judgement appears as a premise of a typing rule, then the corresponding premise in the corresponding typed expansion rule is the corresponding typed expansion judgement. The macro environment is not extended or inspected by these rules (it is only “threaded through” them opaquely).

There are two forms of unexpanded expressions that do not correspond to forms of expanded expressions: the macro definition form, and the macro application form. These two forms are the topic of the next two subsections, respectively.

### 3.2.3 Macro Definition

The *macro definition form*:

$$\text{syntax } a \text{ at } \tau \{ \hat{e}_{\text{parse}} \} \text{ in } \hat{e}$$

allows the programmer to introduce a macro identified as  $a$  at type  $\tau$  with *unexpanded parse function*  $\hat{e}_{\text{parse}}$  into the macro environment of  $\hat{e}$ . The abstract form corresponding to this stylized form is  $\text{usyntax}(\tau; \hat{e}_{\text{parse}}; a.\hat{e})$ . The premises of Rule (3.4l), which governs this form, can be understood as follows, in order:

1. The first premise ensures that the type annotation specifies a well-formed type,  $\tau$ .
2. The second premise types and expands the *unexpanded parse function*,  $\hat{e}_{\text{parse}}$ , to produce the *expanded parse function*,  $e_{\text{parse}}$ . Notice that this occurs under empty contexts, i.e. parse functions cannot refer to the surrounding variable bindings. This is because parse functions are evaluated when a macro is applied during the typed expansion process (as we will discuss momentarily), not during evaluation of the program they appear within. Parse functions must be of type

$$\text{parr}(\text{Body}; \text{ParseResultExp})$$

where  $\text{ParseResultExp}$  abbreviates the following labeled sum type<sup>1</sup>:

$$\text{ParseResultExp} \triangleq [\text{Success} \hookrightarrow \text{CEExp}, \text{ParseError} \hookrightarrow \langle \rangle]$$

and  $\text{Body}$  and  $\text{CEExp}$  abbreviate types that we will characterize below.

3. The third premise of Rule (3.4l) checks that there is not already a macro identified as  $a$  in the macro environment,  $\Sigma$ .

Macro environments are finite mappings from macro identifiers to *expanded macro definitions*,  $\text{syntax}(\tau; e_{\text{parse}})$ , where  $\tau$  is the macro’s type annotation and  $e_{\text{parse}}$  is the macro’s *expanded parse function*. The *macro environment formation judgement*,

<sup>1</sup>In VerseML, the `ParseError` constructor required an error message and an error location, but we omit these in our formalization for simplicity

$\Delta \vdash \Sigma \text{ menv}$ , ensures that the type annotations are well-formed under  $\Delta$  and the parse functions are of type  $\text{parr}(\text{Body}; \text{ParseResultExp})$ .

Judgement Form	Description
----------------	-------------

$\Delta \vdash \Sigma \text{ menv}$	Macro environment $\Sigma$ is well-formed under $\Delta$ .
-------------------------------------	--

This judgement is inductively defined by the following rules:

$$\frac{}{\Delta \vdash \emptyset \text{ menv}} \quad (3.5a)$$

$$\frac{\Delta \vdash \tau \text{ type} \quad \emptyset \vdash e_{\text{parse}} : \text{parr}(\text{Body}; \text{ParseResultExp})}{\Delta \vdash \Sigma, a \hookrightarrow \text{syntax}(\tau; e_{\text{parse}}) \text{ menv}} \quad (3.5b)$$

4. The fourth premise of Rule (3.4l) extends the macro environment with the newly determined expanded macro definition and proceeds to produce a type,  $\tau'$ , and expansion,  $e$ , for  $\hat{e}$ .

The conclusion of the rule gives  $\tau'$  and  $e$  as the type and expansion of the expression as a whole.

### 3.2.4 Macro Application

The last form of unexpanded expression is the form for applying a macro identified as  $a$  to a literal form with literal body  $b$ :

$$a / b /$$

The abstract form corresponding to this stylized form is  $\text{utsmap}[b][a]$ , i.e. for each literal body,  $b$ , there is an operator  $\text{utsmap}[b]$ , indexed by the macro identifier  $a$  and taking no arguments. The premises of Rule (3.4m), which governs this form, can be understood as follows, in order:

1. The *body encoding judgement*  $b \downarrow e_{\text{body}}$  specifies a mapping from the literal body,  $b$ , to an expanded value,  $e_{\text{body}}$ , of type  $\text{Body}$ . An inverse mapping is specified by the *body decoding judgement*  $e_{\text{body}} \uparrow b$ .

Judgement Form	Description
$b \downarrow e$	$b$ encodes to $e$
$e \uparrow b$	$e$ decodes to $b$

Rather than picking a particular definition of  $\text{Body}$  and inductively defining these judgements against it, we simply state the following sufficient conditions, which establish an isomorphism between literal bodies and values of type  $\text{Body}$ .

**Condition 3.10 (Body Encoding).** *For every literal body  $b$ , we have that  $b \downarrow e_{\text{body}}$  and  $\vdash e_{\text{body}} : \text{Body}$  and  $e_{\text{body}} \text{ val}$ .*

**Condition 3.11** (Body Decoding). If  $\vdash e_{\text{body}} : \text{Body}$  and  $e_{\text{body}} \text{ val}$  then  $e_{\text{body}} \uparrow b$  for some  $b$ .

**Condition 3.12** (Body Encoding Inverse). If  $b \downarrow e_{\text{body}}$  then  $e_{\text{body}} \uparrow b$ .

**Condition 3.13** (Body Decoding Inverse). If  $\vdash e_{\text{body}} : \text{Body}$  and  $e_{\text{body}} \text{ val}$  and  $e_{\text{body}} \uparrow b$  then  $b \downarrow e_{\text{body}}$ .

**Condition 3.14** (Body Encoding Uniqueness). If  $b \downarrow e_{\text{body}}$  and  $b \downarrow e'_{\text{body}}$  then  $e_{\text{body}} = e'_{\text{body}}$ .

**Condition 3.15** (Body Decoding Uniqueness). If  $\vdash e_{\text{body}} : \text{Body}$  and  $e_{\text{body}} \text{ val}$  and  $e_{\text{body}} \uparrow b$  and  $e_{\text{body}} \uparrow b'$  then  $b = b'$ .

2. The second premise applies the expanded parse function  $e_{\text{parse}}$  associated with  $a$  in the macro environment to  $e_{\text{body}}$ . If parsing succeeds, i.e. a value of the (stylized) form  $\text{Success} \cdot e_{\text{cand}}$  results from evaluation, then  $e_{\text{cand}}$  will be a value of type  $\text{CEExp}$  (assuming a well-formed macro environment and by the definition of evaluation, transitive application of the Preservation Assumption 3.8 and the Canonical Forms Assumption 3.7). We call  $e_{\text{cand}}$  the *encoding of the candidate expansion*.

If the parse function produces a value labeled `ParseError`, then typed expansion fails. No rule is necessary to handle this case.

3. The judgement  $e_{\text{cand}} \uparrow_{\text{CEExp}} \hat{e}$  decodes the encoding of the candidate expansion, i.e. it maps  $e_{\text{cand}}$  onto a *candidate expansion expression*,  $\hat{e}$ . The syntax of candidate expansion expressions and *candidate expansion types*,  $\hat{\tau}$ , which can occur within candidate expansion expressions, is specified in Figure 3.5. The inverse mapping is specified by the judgement  $\hat{e} \downarrow_{\text{CEExp}} e_{\text{cand}}$ .

Judgement Form	Description
$e \uparrow_{\text{CEExp}} \hat{e}$	$e$ decodes to $\hat{e}$
$\hat{e} \downarrow_{\text{CEExp}} e$	$\hat{e}$ encodes to $e$

As with type `Body`, rather than defining the type `CEExp` explicitly, and these judgements inductively, we give the following sufficient conditions. These establish an isomorphism between values of type `CEExp` and candidate expansion expressions.

**Condition 3.16** (Candidate Expansion Decoding). If  $\vdash e_{\text{cand}} : \text{CEExp}$  and  $e_{\text{cand}} \text{ val}$  then  $e_{\text{cand}} \uparrow_{\text{CEExp}} \hat{e}$  for some  $\hat{e}$ .

**Condition 3.17** (Candidate Expansion Encoding). For every  $\hat{e}$ , we have  $\hat{e} \downarrow_{\text{CEExp}} e_{\text{cand}}$  such that  $\vdash e_{\text{cand}} : \text{CEExp}$  and  $e_{\text{cand}} \text{ val}$ .

**Condition 3.18** (Candidate Expansion Decoding Inverse). If  $\vdash e_{\text{cand}} : \text{CEExp}$  and  $e_{\text{cand}} \text{ val}$  and  $e_{\text{cand}} \uparrow_{\text{CEExp}} \hat{e}$  then  $\hat{e} \downarrow_{\text{CEExp}} e_{\text{cand}}$ .

**Condition 3.19** (Candidate Expansion Encoding Inverse). If  $\hat{e} \downarrow_{\text{CEExp}} e_{\text{cand}}$  then  $\hat{e} \uparrow_{\text{CEExp}} e_{\text{cand}}$ .

**Condition 3.20** (Candidate Expansion Decoding Uniqueness). If  $\vdash e_{\text{cand}} : \text{CEExp}$  and  $e_{\text{cand}} \text{ val}$  and  $e_{\text{cand}} \uparrow_{\text{CEExp}} \hat{e}$  and  $e_{\text{cand}} \uparrow_{\text{CEExp}} \hat{e}'$  then  $\hat{e} = \hat{e}'$ .

**Condition 3.21** (Candidate Expansion Encoding Uniqueness). If  $\hat{e} \downarrow_{\text{CEExp}} e_{\text{cand}}$  and  $\hat{e} \downarrow_{\text{CEExp}} e'_{\text{cand}}$  then  $e_{\text{cand}} = e'_{\text{cand}}$ .

Sort	Abstract Form	Stylized Form	Description
CETyp $\tau ::=$	$t$	$t$	variable
	$\text{ceparr}(\tau; \tau)$	$\tau \rightarrow \tau$	partial function
	$\text{ceall}(t, \tau)$	$\forall t. \tau$	polymorphic
	$\text{cerec}(t, \tau)$	$\mu t. \tau$	recursive
	$\text{ceprod}(\{i \hookrightarrow \tau_i\}_{i \in L})$	$\langle \{i \hookrightarrow \tau_i\}_{i \in L} \rangle$	labeled product
	$\text{cesum}(\{i \hookrightarrow \tau_i\}_{i \in L})$	$[\{i \hookrightarrow \tau_i\}_{i \in L}]$	labeled sum
	$\text{cesplicedt}[m; n]$	$\text{spliced}\langle m, n \rangle$	spliced
CEExp $e ::=$	$x$	$x$	variable
	$\text{celam}(\tau; x. e)$	$\lambda x: \tau. e$	abstraction
	$\text{ceap}(e; e)$	$e(e)$	application
	$\text{cetlam}(t. e)$	$\Lambda t. e$	type abstraction
	$\text{cetap}(e; \tau)$	$e[\tau]$	type application
	$\text{cefold}(t, \tau; e)$	$\text{fold}(e)$	fold
	$\text{ceunfold}(e)$	$\text{unfold}(e)$	unfold
	$\text{cetpl}(\{i \hookrightarrow e_i\}_{i \in L})$	$\langle \{i \hookrightarrow e_i\}_{i \in L} \rangle$	labeled tuple
	$\text{cepr}[\ell](e)$	$e \cdot \ell$	projection
	$\text{cein}[\ell](\{i \hookrightarrow \tau_i\}_{i \in L}; e)$	$\ell \cdot e$	injection
	$\text{cecase}(e; \{i \hookrightarrow x_i. e_i\}_{i \in L})$	$\text{case } e \{i \hookrightarrow x_i. e_i\}_{i \in L}$	case analysis
	$\text{cesplicede}[m; n]$	$\text{spliced}\langle m, n \rangle$	spliced

Figure 3.5: Syntax of candidate expansion types,  $\tau$  (pronounced “grave  $\tau$ ”), and expressions,  $e$  (pronounced “grave  $e$ ”), in  $\text{miniVerse}_{\text{U}}$ . Metavariables  $m$  and  $n$  range over natural numbers.

4. The final premise of Rule (3.4m) *validates* the candidate expansion and simultaneously generates the *final expansion*,  $e$ , according to the *candidate expansion expression validation judgement*, described next.

### 3.2.5 Candidate Expansion Validation

The *candidate expansion validation judgements* validate candidate expansion types,  $\tau$ , and expressions,  $e$ , and simultaneously generate their final expansions.

Judgement Form	Description
$\Delta \vdash^{\mathcal{S}} \tau \rightsquigarrow \tau \text{ type}$	Candidate expansion type $\tau$ expands to $\tau$ under $\Delta$ and splicing scene $\mathcal{S}$ .
$\Delta \Gamma \vdash^{\mathcal{S}} e \rightsquigarrow e : \tau$	Candidate expansion expression $e$ expands to $e$ at type $\tau$ under $\Delta, \Gamma$ and splicing scene $\mathcal{S}$ .

*Splicing scenes*,  $\mathcal{S}$ , are of the form  $\Delta; \Gamma; \Sigma; b$ . They consist of the type formation context,  $\Delta$ , the typing context,  $\Gamma$ , the macro environment,  $\Sigma$ , and the literal body,  $b$ , from the macro application site (cf. Rule (3.4m)).

## Candidate Expansion Type Validation

The *candidate expansion type validation judgement*,  $\Delta \vdash^S \tilde{\tau} \rightsquigarrow \tau \text{ type}$ , is inductively defined by the following rules:

$$\begin{array}{c}
 \frac{}{\Delta, t \text{ type} \vdash^S t \rightsquigarrow t \text{ type}} \quad (3.6a) \\
 \frac{\Delta \vdash^S \tilde{\tau}_1 \rightsquigarrow \tau_1 \text{ type} \quad \Delta \vdash^S \tilde{\tau}_2 \rightsquigarrow \tau_2 \text{ type}}{\Delta \vdash^S \text{ceparr}(\tilde{\tau}_1; \tilde{\tau}_2) \rightsquigarrow \text{parr}(\tau_1; \tau_2) \text{ type}} \quad (3.6b) \\
 \frac{\Delta, t \text{ type} \vdash^S \tilde{\tau} \rightsquigarrow \tau \text{ type}}{\Delta \vdash^S \text{ceall}(t.\tilde{\tau}) \rightsquigarrow \text{all}(t.\tau) \text{ type}} \quad (3.6c) \\
 \frac{\Delta, t \text{ type} \vdash^S \tilde{\tau} \rightsquigarrow \tau \text{ type}}{\Delta \vdash^S \text{cerrec}(t.\tilde{\tau}) \rightsquigarrow \text{rec}(t.\tau) \text{ type}} \quad (3.6d) \\
 \frac{\{\Delta \vdash^S \tilde{\tau}_i \rightsquigarrow \tau_i \text{ type}\}_{i \in L}}{\Delta \vdash^S \text{ceprod}(\{i \hookrightarrow \tilde{\tau}_i\}_{i \in L}) \rightsquigarrow \text{prod}(\{i \hookrightarrow \tau_i\}_{i \in L}) \text{ type}} \quad (3.6e) \\
 \frac{\{\Delta \vdash^S \tilde{\tau}_i \rightsquigarrow \tau_i \text{ type}\}_{i \in L}}{\Delta \vdash^S \text{cesum}(\{i \hookrightarrow \tilde{\tau}_i\}_{i \in L}) \rightsquigarrow \text{sum}(\{i \hookrightarrow \tau_i\}_{i \in L}) \text{ type}} \quad (3.6f) \\
 \frac{\text{parseTyp}(\text{subseq}(b; m; n)) = \tau \quad \Delta \cap \Delta_S = \emptyset \quad \Delta_S \vdash \tau \text{ type}}{\Delta \vdash^{\Gamma_S; \Delta_S; \Sigma_S; b} \text{cesplicedt}[m; n] \rightsquigarrow \tau \text{ type}} \quad (3.6g)
 \end{array}$$

Each form of type,  $\tau$ , corresponds to a form of candidate expansion type,  $\tilde{\tau}$  (compare Figures 3.3 and 3.5). For each type formation rule in Rules (3.1), there is a corresponding candidate expansion type validation rule – Rules (3.6a) to (3.6f) – where the candidate expansion type and the final expansion correspond. The premises also correspond.

The only form of candidate expansion type that does not correspond to a form of type is  $\text{cesplicedt}[m; n]$ , which is a *reference to a spliced type*, i.e. it indicates that a type should be parsed out from the literal body,  $b$ , beginning at position  $m$  and ending at position  $n$ . Rule (3.6g) governs this form. The first premise extracts the indicated subsequence of  $b$  (using the metafunction  $\text{subseq}(b; m; n)$ ) and parses it (using the metafunction  $\text{parseTyp}(b)$ ) to produce the spliced type,  $\tau$ . We assume sensible definitions of these metafunctions.

The third premise of Rule (3.6g) checks that  $\tau$  is well-formed under the application site type formation context,  $\Delta_S$ . The second premise requires that the expansion's type formation context,  $\Delta$ , be disjoint from the application site type formation context,  $\Delta_S$ . This can always be achieved by alpha-varying the candidate expansion type that the reference to the spliced type appears within. Such a change in bound variable names cannot “leak into” spliced types because the hypotheses in  $\Delta$  are not made available to the spliced type  $\tau$ . This achieves the *expansion independent splicing* property described in Sec. 3.1.3 for type variables.

Notice also that Rule (3.6g) is the only rule where  $\Delta_S$  appears. This achieves the *context-independent expansion* property described in Sec. 3.1.3 for type variables.

## Candidate Expansion Expression Validation

The *candidate expansion expression validation judgement*,  $\Delta \Gamma \vdash^S \hat{e} \rightsquigarrow e : \tau$ , is inductively defined by the following rules:

$$\begin{array}{c}
 \frac{}{\Delta \Gamma, x : \tau \vdash^S x \rightsquigarrow x : \tau} \quad (3.7a) \\
 \\
 \frac{\Delta \vdash^S \tilde{\tau} \rightsquigarrow \tau \text{ type} \quad \Delta \Gamma, x : \tau \vdash^S \hat{e} \rightsquigarrow e : \tau'}{\Delta \Gamma \vdash^S \text{celam}(\tilde{\tau}; x.\hat{e}) \rightsquigarrow \text{elam}(\tau; x.e) : \text{parr}(\tau; \tau')} \quad (3.7b) \\
 \\
 \frac{\Delta \Gamma \vdash^S \hat{e}_1 \rightsquigarrow e_1 : \text{parr}(\tau; \tau') \quad \Delta \Gamma \vdash^S \hat{e}_2 \rightsquigarrow e_2 : \tau}{\Delta \Gamma \vdash^S \text{ceap}(\hat{e}_1; \hat{e}_2) \rightsquigarrow \text{eap}(e_1; e_2) : \tau'} \quad (3.7c) \\
 \\
 \frac{\Delta, t \text{ type } \Gamma \vdash^S \hat{e} \rightsquigarrow e : \tau}{\Delta \Gamma \vdash^S \text{cetlam}(t.\hat{e}) \rightsquigarrow \text{etlam}(t.e) : \text{all}(t.\tau)} \quad (3.7d) \\
 \\
 \frac{\Delta \Gamma \vdash^S \hat{e} \rightsquigarrow e : \text{all}(t.\tau) \quad \Delta \vdash^S \tilde{\tau}' \rightsquigarrow \tau' \text{ type}}{\Delta \Gamma \vdash^S \text{cetap}(\hat{e}; \tilde{\tau}') \rightsquigarrow \text{etap}(e; \tau') : [\tau' / t]\tau} \quad (3.7e) \\
 \\
 \frac{\Delta, t \text{ type } \vdash^S \tilde{\tau} \rightsquigarrow \tau \text{ type} \quad \Delta \Gamma \vdash^S \hat{e} \rightsquigarrow e : [\text{rec}(t.\tau) / t]\tau}{\Delta \Gamma \vdash^S \text{cefold}(t.\hat{e}; \hat{e}) \rightsquigarrow \text{efold}(t.\tau; e) : \text{rec}(t.\tau)} \quad (3.7f) \\
 \\
 \frac{\Delta \Gamma \vdash^S \hat{e} \rightsquigarrow e : \text{rec}(t.\tau)}{\Delta \Gamma \vdash^S \text{ceunfold}(\hat{e}) \rightsquigarrow \text{eunfold}(e) : [\text{rec}(t.\tau) / t]\tau} \quad (3.7g) \\
 \\
 \frac{\{\Delta \Gamma \vdash^S \hat{e}_i \rightsquigarrow e_i : \tau_i\}_{i \in L}}{\Delta \Gamma \vdash^S \text{cetpl}(\{i \hookrightarrow \hat{e}_i\}_{i \in L}) \rightsquigarrow \text{etpl}(\{i \hookrightarrow e_i\}_{i \in L}) : \text{prod}(\{i \hookrightarrow \tau_i\}_{i \in L})} \quad (3.7h) \\
 \\
 \frac{\Delta \Gamma \vdash^S \hat{e} \rightsquigarrow e : \text{prod}(\{i \hookrightarrow \tau_i\}_{i \in L} \otimes \ell \hookrightarrow \tau)}{\Delta \Gamma \vdash^S \text{cepr}[\ell](\hat{e}) \rightsquigarrow \text{epr}[\ell](e) : \tau} \quad (3.7i) \\
 \\
 \frac{\{\Delta \vdash^S \tilde{\tau}_i \rightsquigarrow \tau_i \text{ type}\}_{i \in L} \quad \Delta \vdash^S \tilde{\tau} \rightsquigarrow \tau \text{ type} \quad \Delta \Gamma \vdash^S \hat{e} \rightsquigarrow e : \tau}{\left\{ \begin{array}{l} \Delta \Gamma \vdash_\Sigma \text{cein}[\ell](\{i \hookrightarrow \tilde{\tau}_i\}_{i \in L} \otimes \ell \hookrightarrow \tilde{\tau}; \hat{e}) \\ \rightsquigarrow \\ \text{ein}[\ell](\{i \hookrightarrow \tau_i\}_{i \in L} \otimes \ell \hookrightarrow \tau; e) : \text{sum}(\{i \hookrightarrow \tau_i\}_{i \in L} \otimes \ell \hookrightarrow \tau) \end{array} \right\}} \quad (3.7j) \\
 \\
 \frac{\Delta \Gamma \vdash^S \hat{e} \rightsquigarrow e : \text{sum}(\{i \hookrightarrow \tau_i\}_{i \in L}) \quad \{\Delta \Gamma, x_i : \tau_i \vdash^S \hat{e}_i \rightsquigarrow e_i : \tau\}_{i \in L}}{\Delta \Gamma \vdash^S \text{cecase}(\hat{e}; \{i \hookrightarrow x_i.\hat{e}_i\}_{i \in L}) \rightsquigarrow \text{ecase}(e; \{i \hookrightarrow x_i.e_i\}_{i \in L}) : \tau} \quad (3.7k) \\
 \\
 \frac{\text{parseUExp}(\text{subseq}(b; m; n)) = \hat{e} \quad \Delta \cap \Delta_S = \emptyset \quad \text{dom}(\Gamma) \cap \text{dom}(\Gamma_S) = \emptyset \quad \Delta_S \Gamma_S \vdash_{\Sigma_S} \hat{e} \rightsquigarrow e : \tau}{\Delta \Gamma \vdash_{\Delta_S; \Gamma_S; \Sigma_S; b} \text{cespliced}[m; n] \rightsquigarrow e : \tau} \quad (3.7l)
 \end{array}$$

Each form of expanded expression,  $e$ , corresponds to a form of candidate expansion expression,  $\hat{e}$  (compare Figures 3.3). For each typing rule in Rules 3.3, there is a corresponding candidate expansion expression validation rule – Rules (3.7a) to (3.7k) –

where the candidate expansion expression and expanded expression correspond. The premises also correspond.

The only form of candidate expansion expression that does not correspond to a form of expanded expression is  $\text{cesplicede}[m; n]$ , which is a *reference to a spliced unexpanded expression*, i.e. it indicates that an unexpanded expression should be parsed out from the literal body,  $b$ , beginning at position  $m$  and ending at position  $n$ . Rule (3.71) governs this form. The first premise extracts the indicated subsequence of  $b$  (using the metafunction  $\text{subseq}(b; m; n)$ ) and parses it (using the metafunction  $\text{parseUExp}(b)$ ) to produce the spliced unexpanded expression,  $\hat{e}$ . We assume sensible definitions of these metafunctions.

The final premise of Rule (3.71) types and expands the spliced unexpanded expression  $\hat{e}$  under the application site contexts,  $\Delta_S$  and  $\Gamma_S$ , and macro environment,  $\Sigma_S$ . The second premise requires that the expansion's type formation context,  $\Delta$ , be disjoint from the application site type formation context,  $\Delta_S$ . Similarly, the third premise requires that the expansion's typing context,  $\Gamma$ , be disjoint from the application site typing context,  $\Gamma_S$ . These requirements can always be satisfied by alpha-varying the candidate expansion expression that the reference to the spliced unexpanded expression appears within. Such a change in bound variable names cannot “leak into” spliced unexpanded expressions because the hypotheses in  $\Delta$  and  $\Gamma$  are not available to the spliced unexpanded expression  $\hat{e}$ . This achieves the *expansion independent splicing* property described in Sec. 3.1.3 for variables, type variables and macro identifiers.

Notice also that Rule (3.71) is the only rule where  $\Delta_S$ ,  $\Gamma_S$  and  $\Sigma_S$  appear. This achieves the context-independent expansion property described in Sec. 3.1.3 for variables, type variables and macro identifiers.

### 3.2.6 Metatheory

For the rules we have given to be sensible, we must have that typed expansion produces an expansion of the expected type. Formally, this can be stated as the following theorem.

**Theorem 3.22** (Typed Expansion). *If  $\Delta \Gamma \vdash_{\Sigma} \hat{e} \rightsquigarrow e : \tau$  and  $\Delta \vdash \Sigma \text{ menv}$  then  $\Delta \Gamma \vdash e : \tau$ .*

*Proof.* By rule induction over Rules (3.4).

**Case (3.4a).** We have

- |  |                |
|--|----------------|
| (1) $\hat{e} = x$                              | by assumption  |
| (2) $e = x$                                    | by assumption  |
| (3) $\Gamma = \Gamma', x : \tau$               | by assumption  |
| (4) $\Delta \Gamma', x : \tau \vdash x : \tau$ | by Rule (3.3a) |

**Case (3.4b).** We have

- |   |               |
|---|---------------|
| (1) $\hat{e} = \text{ulam}(\tau_1; x.\hat{e}')$ | by assumption |
| (2) $e = \text{elam}(\tau_1; x.e')$             | by assumption |
| (3) $\tau = \text{parr}(\tau_1; \tau_2)$        | by assumption |



- |   |                               |
|---|-------------------------------|
| (4) $\Delta \vdash \tau_1$ type   | by assumption                 |
| (5) $\Delta \Gamma, x : \tau_1 \vdash_{\Sigma} \hat{e}' \rightsquigarrow e' : \tau_2$ | by assumption                 |
| (6) $\Delta \Gamma, x : \tau_1 \vdash e' : \tau_2$                                    | by IH on (5)                  |
| (7) $\Delta \Gamma \vdash \text{elam}(\tau_1; x.e') : \text{parr}(\tau_1; \tau_2)$    | by Rule (3.3b) on (4) and (6) |

**Case (3.4c).** We have

- |  |                               |
|--|-------------------------------|
| (1) $\hat{e} = \text{uap}(\hat{e}_1; \hat{e}_2)$   | by assumption                 |
| (2) $e = \text{eap}(e_1; e_2)$   | by assumption                 |
| (3) $\Delta \Gamma \vdash_{\Sigma} \hat{e}_1 \rightsquigarrow e_1 : \text{parr}(\tau_1; \tau)$ | by assumption                 |
| (4) $\Delta \Gamma \vdash_{\Sigma} \hat{e}_2 \rightsquigarrow e_2 : \tau_1$                    | by assumption                 |
| (5) $\Delta \Gamma \vdash e_1 : \text{parr}(\tau_1; \tau)$                                     | by IH on (3)                  |
| (6) $\Delta \Gamma \vdash e_2 : \tau_1$  | by IH on (4)                  |
| (7) $\Delta \Gamma \vdash \text{eap}(e_1; e_2) : \tau$   | by Rule (3.3c) on (5) and (6) |

**Case (3.4d) through (3.4k).** These cases follow analogously.

**Case (3.4l).** We have

- |   |                                    |
|---|------------------------------------|
| (1) $\hat{e} = \text{usyntax}(\tau'; \hat{e}_{\text{parse}}; a.\hat{e}')$   | by assumption                      |
| (2) $\Delta \vdash \tau'$ type  | by assumption                      |
| (3) $\emptyset \emptyset \vdash_{\emptyset} \hat{e}_{\text{parse}} \rightsquigarrow e_{\text{parse}} : \text{parr}(\text{Body}; \text{ParseResultExp})$ | by assumption                      |
| (4) $\Delta \Gamma \vdash_{\Sigma, a \hookrightarrow \text{syntax}(\tau'; e_{\text{parse}})} \hat{e}' \rightsquigarrow e : \tau$                        | by assumption                      |
| (5) $\Delta \vdash \Sigma$ menv   | by assumption                      |
| (6) $\emptyset \vdash \emptyset$ menv   | by Rule (3.5a)                     |
| (7) $\emptyset \emptyset \vdash e_{\text{parse}} : \text{parr}(\text{Body}; \text{ParseResultExp})$   | by IH on (3) and (6)               |
| (8) $\Delta \vdash \Sigma, a \hookrightarrow \text{syntax}(\tau'; e_{\text{parse}})$ menv   | by Rule (3.5b) on (5), (2) and (7) |
| (9) $\Delta \Gamma \vdash e : \tau$   | by IH on (4) and (8)               |

**Case (3.4m).** We have

- |  |                                |
|--|--------------------------------|
| (1) $\hat{e} = \text{utsmmap}[b][a]$   | by assumption                  |
| (2) $\Sigma = \Sigma', a \hookrightarrow \text{syntax}(\tau; e_{\text{parse}})$                | by assumption                  |
| (3) $b \downarrow e_{\text{body}}$   | by assumption                  |
| (4) $e_{\text{parse}}(e_{\text{body}}) \Downarrow \text{Success} \cdot e_{\text{cand}}$        | by assumption                  |
| (5) $e_{\text{cand}} \uparrow_{\text{CEExp}} \hat{e}$  | by assumption                  |
| (6) $\emptyset \emptyset \vdash_{\Delta; \Gamma; \Sigma; b} \hat{e} \rightsquigarrow e : \tau$ | by assumption                  |
| (7) $\Delta \vdash \Sigma$ menv  | by assumption                  |
| (8) $\Delta \Gamma \vdash e : \tau$  | by Theorem 3.23 on (6) and (7) |



□

A typed expansion judgement appears as a premise in Rule (3.7l), and a candidate expansion expression validation judgement appears as a premise in Rule (3.4m), so Rules (3.4) and Rules (3.7) must be mutually defined. We must also define the following theorem about candidate expansion expression validation mutually with Theorem 3.22.

**Theorem 3.23** (Candidate Expansion Expression Validation). *If  $\Delta \Gamma \vdash^{\Delta_S; \Gamma_S; \Sigma_S; b} \dot{e} \rightsquigarrow e : \tau$  and  $\Delta_S \vdash \Sigma_S \text{ menv}$  then  $\Delta \cup \Delta_S \Gamma \cup \Gamma_S \vdash e : \tau$ .*

*Proof.* By rule induction over Rules (3.7).

**Case (3.7a).** We have

- |  |  |
|--|--|
| (1) $\dot{e} = x$  | by assumption                          |
| (2) $e = x$  | by assumption                          |
| (3) $\Gamma = \Gamma', x : \tau$   | by assumption                          |
| (4) $\Delta \cup \Delta_S \Gamma', x : \tau \vdash x : \tau$               | by Rule (3.3a)                         |
| (5) $\Delta \cup \Delta_S \Gamma', x : \tau \cup \Gamma_S \vdash x : \tau$ | by Lemma 3.1 to (4)<br>over $\Gamma_S$ |

**Case (3.7b).** We have

- |  |                                       |
|--|---------------------------------------|
| (1) $\dot{e} = \text{celam}(\dot{\tau}_1; x.\dot{e}')$   | by assumption                         |
| (2) $e = \text{elam}(\tau_1; x.e')$  | by assumption                         |
| (3) $\tau = \text{parr}(\tau_1; \tau_2)$   | by assumption                         |
| (4) $\Delta \vdash^{\Delta_S; \Gamma_S; \Sigma_S; b} \dot{\tau}_1 \rightsquigarrow \tau_1 \text{ type}$        | by assumption                         |
| (5) $\Delta \Gamma, x : \tau_1 \vdash^{\Delta_S; \Gamma_S; \Sigma_S; b} \dot{e}' \rightsquigarrow e' : \tau_2$ | by assumption                         |
| (6) $\Delta \cup \Delta_S \vdash \tau_1 \text{ type}$  | by Lemma 3.26 on (4)                  |
| (7) $\Delta \cup \Delta_S \Gamma, x : \tau_1 \cup \Gamma_S \vdash e' : \tau_2$                                 | by IH on (5)                          |
| (8) $\Delta \cup \Delta_S \Gamma \cup \Gamma_S, x : \tau_1 \vdash e' : \tau_2$                                 | by exchange over $\Gamma_S$<br>on (7) |
| (9) $\Delta \cup \Delta_S \Gamma \cup \Gamma_S \vdash \text{elam}(\tau_1; x.e') : \text{parr}(\tau_1; \tau_2)$ | by Rule (3.3b) on (6)<br>and (8)      |

**Case (3.7c).** We have

- |   |                                  |
|---|----------------------------------|
| (1) $\dot{e} = \text{ceap}(\dot{e}_1; \dot{e}_2)$   | by assumption                    |
| (2) $e = \text{eap}(e_1; e_2)$  | by assumption                    |
| (3) $\Delta \Gamma \vdash^{\Delta_S; \Gamma_S; \Sigma_S; b} \dot{e}_1 \rightsquigarrow e_1 : \text{parr}(\tau_1; \tau)$ | by assumption                    |
| (4) $\Delta \Gamma \vdash^{\Delta_S; \Gamma_S; \Sigma_S; b} \dot{e}_2 \rightsquigarrow e_2 : \tau_1$                    | by assumption                    |
| (5) $\Delta \cup \Delta_S \Gamma \cup \Gamma_S \vdash e_1 : \text{parr}(\tau_1; \tau)$                                  | by IH on (3)                     |
| (6) $\Delta \cup \Delta_S \Gamma \cup \Gamma_S \vdash e_2 : \tau_1$   | by IH on (4)                     |
| (7) $\Delta \cup \Delta_S \Gamma \cup \Gamma_S \vdash \text{eap}(e_1; e_2) : \tau$                                      | by Rule (3.3c) on (5)<br>and (6) |

Case (3.7d) through (3.7k). These cases follow analogously.

Case (3.7l). We have

- |   |                                |
|---|--------------------------------|
| (1) $\hat{e} = \text{cesplicede}[m; n]$                                     | by assumption                  |
| (2) $\text{parseUExp}(\text{subseq}(b; m; n)) = \hat{e}$                    | by assumption                  |
| (3) $\Delta_S \Gamma_S \vdash_{\Sigma_S} \hat{e} \rightsquigarrow e : \tau$ | by assumption                  |
| (4) $\Delta_S \vdash \Sigma_S \text{menv}$                                  | by assumption                  |
| (5) $\Delta_S \Gamma_S \vdash e : \tau$                                     | by Theorem 3.22 on (3) and (4) |
| (6) $\Delta \cup \Delta_S \Gamma \cup \Gamma_S \vdash e : \tau$             | by Lemma 3.1 on (5)            |

□

The mutual induction used to prove Theorems 3.22 and 3.23 is justified by showing that the following numeric metric on the subjects of the rule induction is decreasing:

$$\begin{aligned} \|\Delta \Gamma \vdash_{\Sigma} \hat{e} \rightsquigarrow e : \tau\| &= \|\hat{e}\| \\ \|\Delta \Gamma \vdash^{\Delta_S; \Gamma_S; \Sigma_S; b} \hat{e} \rightsquigarrow e : \tau\| &= \|b\| \end{aligned}$$

Here,  $\|b\|$  is the length of the literal body  $b$  and  $\|\hat{e}\|$  is the sum of the lengths of the bodies in  $\hat{e}$ , i.e.

$$\begin{aligned} \|x\| &= 0 \\ \|\text{ulam}(\tau; x.\hat{e})\| &= \|\hat{e}\| \\ \|\text{uap}(\hat{e}_1; \hat{e}_2)\| &= \|\hat{e}_1\| + \|\hat{e}_2\| \\ \|\text{utlam}(t.\hat{e})\| &= \|\hat{e}\| \\ \|\text{utap}(\hat{e}; \tau)\| &= \|\hat{e}\| \\ \|\text{ufold}(t.\tau; \hat{e})\| &= \|\hat{e}\| \\ \|\text{uunfold}(\hat{e})\| &= \|\hat{e}\| \\ \|\text{utpl}(\{i \hookrightarrow \hat{e}_i\}_{i \in L})\| &= \sum_{i \in L} \|\hat{e}_i\| \\ \|\text{upr}[\ell](\hat{e})\| &= \|\hat{e}\| \\ \|\text{uin}[\ell](\{i \hookrightarrow \tau_i\}_{i \in L}; \hat{e})\| &= \|\hat{e}\| \\ \|\text{ucase}(\hat{e}; \{i \hookrightarrow x_i.\hat{e}_i\}_{i \in L})\| &= \|\hat{e}\| + \sum_{i \in L} \|\hat{e}_i\| \\ \|\text{usyntax}(\tau; \hat{e}_{\text{parse}}; a.\hat{e})\| &= \|\hat{e}\| \\ \|\text{utsmmap}[b][a]\| &= \|b\| \end{aligned}$$

The only case in the proof of Theorem 3.22 that invokes Theorem 3.23 is Case (3.4m). There, we have that the metric remains stable:

$$\|\emptyset \emptyset \vdash^{\Delta; \Gamma; \Sigma; b} \hat{e} \rightsquigarrow e : \tau\| = \|\Delta \Gamma \vdash_{\Sigma} \text{utsmmap}[b][a] \rightsquigarrow e : \tau\|$$

i.e., by the definitions above,

$$\|b\| = \|b\|$$

The only case in the proof of Theorem 3.23 that invokes Theorem 3.22 is Case (3.71). There, we have that  $\text{parseUExp}(\text{subseq}(b; m; n)) = \hat{e}$  and Theorem 3.22 is invoked on the judgement  $\Delta_S \Gamma_S \vdash_{\Sigma_S} \hat{e} \rightsquigarrow e : \tau$ , so we must have that:

$$\|\Delta_S \Gamma_S \vdash_{\Sigma_S} \hat{e} \rightsquigarrow e : \tau\| < \|\Delta \Gamma \vdash^{\Delta_S; \Gamma_S; \Sigma_S; b} \text{cesplicede}[m; n] \rightsquigarrow e : \tau\|$$

i.e.

$$\|\hat{e}\| < \|b\|$$

This is established by the following two conditions. The first condition simply states that subsequences of  $b$  are no longer than  $b$ .

**Condition 3.24** (Body Subsequences). *If  $\text{subseq}(b; m; n) = b'$  then  $\|b'\| \leq \|b\|$ .*

The second condition states that an unexpanded expression constructed by parsing a literal body  $b$  is smaller than the length of  $b$  (because some characters need to be used to invoke a TSM on each literal body).

**Condition 3.25** (Body Parsing). *If  $\text{parseUExp}(b) = \hat{e}$  then  $\|\hat{e}\| < \|b\|$ .*

Because the metric is strictly decreasing in the proof of Theorem 3.23 and non-increasing in the proof of Theorem 3.22, the mutual induction is well-founded.

Proving Theorem 3.23 also requires the following lemma about candidate expansion type validation.

**Lemma 3.26** (Candidate Expansion Type Validation). *If  $\Delta \vdash^{\Delta_S; \Gamma_S; \Sigma_S; b} \hat{\tau} \rightsquigarrow \tau$  type then  $\Delta \cup \Delta_S \vdash \tau$  type.*

*Proof.* By rule induction over Rules (3.6).

**Case (3.6a).** We have

- |   |  |
|---|--|
| (1) $\Delta = \Delta', t$ type                      | by assumption  |
| (2) $\hat{\tau} = t$                                | by assumption  |
| (3) $\tau = t$                                      | by assumption  |
| (4) $\Delta', t$ type $\vdash t$ type               | by Rule (3.1a)   |
| (5) $\Delta', t$ type $\cup \Delta_S \vdash t$ type | by iterated application of Lemma 3.1 over $\Delta_S$ to (4). |

**Case (3.6b).** We have

- |   |               |
|---|---------------|
| (1) $\hat{\tau} = \text{ceparr}(\hat{\tau}_1; \hat{\tau}_2)$                                    | by assumption |
| (2) $\tau = \text{parr}(\tau_1; \tau_2)$  | by assumption |
| (3) $\Delta \vdash^{\Delta_S; \Gamma_S; \Sigma_S; b} \hat{\tau}_1 \rightsquigarrow \tau_1$ type | by assumption |
| (4) $\Delta \vdash^{\Delta_S; \Gamma_S; \Sigma_S; b} \hat{\tau}_2 \rightsquigarrow \tau_2$ type | by assumption |
| (5) $\Delta \cup \Delta_S \vdash \tau_1$ type   | by IH on (3)  |
| (6) $\Delta \cup \Delta_S \vdash \tau_2$ type   | by IH on (4)  |

(7)  $\Delta \cup \Delta_S \vdash \text{parr}(\tau_1; \tau_2)$  type

by Rule (3.1b) on (5)  
and (6)

**Case (3.6c).** We have

- (1)  $\dot{\tau} = \text{ceall}(t.\dot{\tau}')$
- (2)  $\tau = \text{all}(t.\tau')$
- (3)  $\Delta, t \text{ type} \vdash^{\Delta_S; \Gamma_S; \Sigma_S; b} \dot{\tau}' \rightsquigarrow \tau' \text{ type}$
- (4)  $\Delta, t \text{ type} \cup \Delta_S \vdash \tau' \text{ type}$
- (5)  $\Delta \cup \Delta_S, t \text{ type} \vdash \tau' \text{ type}$
- (6)  $\Delta \cup \Delta_S \vdash \text{all}(t.\tau') \text{ type}$

by assumption  
by assumption  
by assumption  
by IH on (3)  
by iterated exchange  
over  $\Delta_S$  on (4)  
by Rule (3.1c) on (5)

**Case (3.6d).** We have

- (1)  $\dot{\tau} = \text{cerec}(t.\dot{\tau}')$
- (2)  $\tau = \text{rec}(t.\tau')$
- (3)  $\Delta, t \text{ type} \vdash^{\Delta_S; \Gamma_S; \Sigma_S; b} \dot{\tau}' \rightsquigarrow \tau' \text{ type}$
- (4)  $\Delta, t \text{ type} \cup \Delta_S \vdash \tau' \text{ type}$
- (5)  $\Delta \cup \Delta_S, t \text{ type} \vdash \tau' \text{ type}$
- (6)  $\Delta \cup \Delta_S \vdash \text{rec}(t.\tau') \text{ type}$

by assumption  
by assumption  
by assumption  
by IH on (3)  
by iterated exchange  
over  $\Delta_S$  on (4)  
by Rule (3.1d) on (5)

**Case (3.6e).** We have

- (1)  $\dot{\tau} = \text{ceprod}(\{i \hookrightarrow \dot{\tau}_i\}_{i \in L})$
- (2)  $\tau = \text{prod}(\{i \hookrightarrow \tau_i\}_{i \in L})$
- (3)  $\{\Delta \vdash^{\Delta_S; \Gamma_S; \Sigma_S; b} \dot{\tau}_i \rightsquigarrow \tau_i \text{ type}\}_{i \in L}$
- (4)  $\{\Delta \cup \Delta_S \vdash \tau_i \text{ type}\}_{i \in L}$
- (5)  $\Delta \cup \Delta_S \vdash \text{prod}(\{i \hookrightarrow \tau_i\}_{i \in L}) \text{ type}$

by assumption  
by assumption  
by assumption  
by IH on (3)<sub>i</sub> for each  
 $i \in L$   
by Rule (3.1e) on (4)

**Case (3.6f).** We have

- (1)  $\dot{\tau} = \text{cesum}(\{i \hookrightarrow \dot{\tau}_i\}_{i \in L})$
- (2)  $\tau = \text{sum}(\{i \hookrightarrow \tau_i\}_{i \in L})$
- (3)  $\{\Delta \vdash^{\Delta_S; \Gamma_S; \Sigma_S; b} \dot{\tau}_i \rightsquigarrow \tau_i \text{ type}\}_{i \in L}$
- (4)  $\{\Delta \cup \Delta_S \vdash \tau_i \text{ type}\}_{i \in L}$
- (5)  $\Delta \cup \Delta_S \vdash \text{sum}(\{i \hookrightarrow \tau_i\}_{i \in L}) \text{ type}$

by assumption  
by assumption  
by assumption  
by IH on (3)<sub>i</sub> for each  
 $i \in L$   
by Rule (3.1f) on (4)

**Case (3.6g).** We have

- (1)  $\dot{\tau} = \text{cesplicedt}[m; n]$

by assumption

- (2)  $\text{parseType}(\text{subseq}(b; m; n)) = \tau$
- (3)  $\Delta_S \vdash \tau \text{ type}$
- (4)  $\Delta \cup \Delta_S \vdash \tau \text{ type}$

by assumption  
 by assumption  
 by iterated  
 application of Lemma  
 3.1 and exchange  
 over (3)

□

# Chapter 4

## Unparameterized Pattern TSMs

### 4.1 Pattern TSMs By Example

TSMs as we have described them so far decrease the syntactic cost of introducing a value at a specified type. In full-scale functional languages like ML, one typically deconstructs a value using *nested pattern matching*. For example, let us return to the definition of the datatype `Rx` shown at the beginning of Sec. ?? . We can pattern match over a value, `r`, of type `Rx` using VerseML's `match` construct like this:

```
match r with
  Seq(Str(name), Seq(Str ": ", ssn)) => display name ssn
| _ => raise Invalid
```

In a functional language with primitive support for regular expression pattern syntax, we would expect to be able to write this example more concisely using the splicing forms discussed in Sec. 3.1:

```
match r with
  /@name: %ssn/ => display name ssn
| _ => raise Invalid
```

Patterns are not expressions, so we cannot simply use a TSM defined at type `Rx` in a pattern. To address this, we must extend our language with support for typed pattern syntax macros (TPSMs). TPSMs are entirely analogous to TSMs, differing primarily in that the expansions they generate are patterns, rather than expressions. Assuming the abstract syntax of patterns is encoded by the type `Pat` (analogous to `Exp`), we can define a TPSM at type `Rx` as follows:

```
pattern syntax rx at Rx {
  static fn (body : Body) : Pat =>
    (* regex pattern parser here *)
}
```

Using this TPSM, we can rewrite our example as follows:

```
match r with
  rx /@name: %ssn/ => display name ssn
| _ => raise Invalid
```

To ensure that the client of the TPSM need not “guess at” what variables are bound by the pattern, variables (e.g. `name` and `ssn` here) can only appear in spliced subpatterns (just as variables bound at the use site can only appear in spliced subexpressions when using TSMs). We leave a formal account of TPSMs (in a reduced calculus that features simple pattern matching) as work that remains to be completed (see Sec. ??).

ML does not presently support pattern matching over values of an abstract data type. However, there have been proposals for adding support for pattern matching over abstract data types defined by modules having a “datatype-like” shape, e.g. those that define a case analysis function like the one specified by `RX`, shown in Sec. ?. We leave further discussion of such a facility and of parameterized TPSMs also as remaining work (see Sec. ??).

#### **4.1.1 Usage**

#### **4.1.2 Definition**

#### **4.1.3 Splicing and Binding**

#### **4.1.4 Validation**

### **4.2 miniVerse<sub>UP</sub>**

#### **4.2.1 Types, Expanded Patterns and Expanded Expressions**

#### **4.2.2 Macro Expansion and Validation**

#### **4.2.3 Metatheory**

# Chapter 5

## Parameterized TSMs

### 5.1 Parameterized TSMs By Example

#### 5.1.1 Value Parameters By Example

#### 5.1.2 Type Parameters By Example

#### 5.1.3 Module Parameters By Example

### 5.2 miniVerse<sub>∀</sub>

#### 5.2.1 Signatures, Types and Expanded Expressions

#### 5.2.2 Parameter Application and Deferred Substitution

#### 5.2.3 Macro Expansion and Validation

#### 5.2.4 Metatheory





# Chapter 6

## Type-Specific Languages (TSLs)

With TSMs, library providers can control the expansion of arbitrary syntax that appears between delimiters, but clients must explicitly identify the TSM and provide the required type and module parameters at each use site. To further lower the syntactic cost of using TSMs, so that it compares to the syntactic cost of derived syntax built in primitively (e.g. list syntax), we will now discuss how VerseML allows library providers to define *type-specific languages* (TSLs) by associating a TSM directly with an abstract type or datatype. When the type system encounters a delimited form not prefixed by a TSM name, it applies the TSM associated with the type it is being analyzed against implicitly.

### 6.1 TSLs By Example

For example, a module `P` can associate the TSM `rx` defined in the previous section with the abstract type `R.t` by qualifying the definition of the sealed module it is defined by as follows:

```
module R = mod {  
  type t = (* ... *)  
          (* ... *)  
} :> RX with syntax rx
```

More generally, when sealing a module expression against a signature, the programmer can specify, for each abstract type that is generated, at most one previously defined TSMs. This TSM must take as its first parameter the module being sealed.

The following function has the same expansion as `example_using_tsm` but, by using the TSL just defined, it is more concise. Notice the return type annotation, which is necessary to ensure that the TSL can be unambiguously determined:

```
fun example_using_tsl(name : string) : R.t => /@name: %ssn/
```

As another example, let us consider the standard list datatype. We can use TSLs to express derived list syntax, for both expressions and patterns:

```
datatype list('a) { Nil | Cons of 'a * list('a) } with syntax {  
  static fn (body : Body) =>  
    (* ... comma-delimited spliced exps ... *)
```

```

} with pattern syntax {
  static fn (body : Body) : Pat =>
    (* ... list pattern parser ... *)
}

```

Together with the TSL for regular expression patterns, this allows us to write lists like this:

```

let val x : list(R.t) = [/\\d/, /\\d\\d/, /\\d\\d\\d/]

```

From the client's perspective, it is essentially as if the language had built in derived syntax for lists and regular expression patterns directly.

## 6.2 Parameterized Modules

TSLs can be associated with abstract types that are generated by parameterized modules (i.e. generative functors in Standard ML) as well. For example, consider a trivially parameterized module that creates modules sealed against RX:

```

module F() => mod {
  type t = (* ... *)
  (* ... *)
} :> RX with syntax rx

```

Each application of F generates a distinct abstract type. The semantics associates the appropriately parameterized TSM with each of these as they are generated:

```

module F1 = F() (* F1.t has TSL rx(F1) *)
module F2 = F() (* F2.t has TSL rx(F2) *)

```

As a more complex example, let us define two signatures, A and B, a TSM \$G and a parameterized module G : A -> B:

```

signature A = sig { type t; val x : t }
signature B = sig { type u; val y : u }
syntax $G(M : A)(G : B) at G.u { (* ... *) }
module G(M : A) => mod {
  type u = M.t; val y = M.x } :> B with syntax $G(M)

```

Both G and \$G take a parameter M : A. We associate the partially applied TSM \$G(M) with the abstract type that G generates. Again, this satisfies the requirement that one must be able to apply the TSM being associated with the abstract type to the module being sealed.

Only fully abstract types can have TSLs associated with them. Within the definition of G, type u does not have a TSL available to it because it is synonymous to M.t. More generally, TSL lookup respects type equality, so any synonyms of a type with a TSL will also have that TSL. We can see this in the following example, where the type u has a different TSL associated with it inside and outside the definition of the module N:

```

module M : A = mod { type t = int; val x = 0 }
module G1 = G(M) (* G1.t has TSL $G(M), per above *)
module N = mod {
  type u = G1.t (* u = G1.t in this scope, so u also has TSL $G(M) *)

```

```

val y = /asdf/ (* we can use it to create a value of that type *)
} :> B (* did not specify a TSL for N.u at the point where it is sealed,
        so N.u has no TSL in the outer scope *)
val z : N.u = /asdf/ (* ERROR: no TSL for type N.u *)

```

### 6.3 miniVerse<sub>TSL</sub>

A formal specification of TSLs in a language that supports only non-parametric datatypes is available in a paper published in ECOOP 2014 [29]. We will add support for parameterized TSLs in the dissertation (see Sec. ??).



# Chapter 7

## Discussion & Future Directions

### 7.1 Interesting Applications

#### 7.1.1 TSMs For Defining TSMs

Static functions can also make use of TSMs. In this section, we will show how quasiquotation syntax and grammar-based parser generators can be expressed using TSMs. These TSMs are quite useful for writing other TSMs.

##### Quasiquotation

TSMs must generate values of type `CEExp`. Doing so explicitly can have high syntactic cost. To decrease the syntactic cost of constructing values of this type, the prelude includes a TSM that provides quasiquotation syntax (cf. Sec. 2.1.8):

```
syntax $qqexp at CEExp {  
  static fn(body : Body) : ParseResult => (* expression parser here *)  
}  
  
syntax $qqtype at CETyp {  
  static fn(body : Body) : ParseResult => (* type parser here *)  
}
```

For example, the following expression:

```
let gx = $qqexp 'g(x)'
```

is more concise than its expansion:

```
let gx = App(Var 'g', Var 'x')
```

The full concrete syntax of the language can be used. Anti-quotation, i.e. splicing in an expression of type `MarkedExp`, is indicated by the prefix `%`:

```
let fgx = $qqexp 'f(%gx)'
```

The expansion of this expression is:

```
let fgx = App(Var 'f', gx)
```

## Parser Generators

TODO: grammars, compile function, TSM for grammar, example of IP address

### 7.1.2 Monadic Commands

## 7.2 Summary

TODO: Write summary

## 7.3 Future Directions

### 7.3.1 Mechanically Verifying TSM Definitions

Finally, VerseML is not designed for advanced theorem proving tasks where languages like Coq, Agda or Idris might be used today. That said, we conjecture that the primitives we describe could be integrated into languages like Gallina (the “external language” of the Coq proof assistant [25]) with modifications, but do not plan to pursue this line of research here.

In such a setting, you could verify TSM definitions TODO: finish writing this

### 7.3.2 Improved Error Reporting

### 7.3.3 Controlled Binding

### 7.3.4 Type-Aware Splicing

### 7.3.5 Integration With Code Editors

### 7.3.6 Resugaring

TODO: Cite recent work at PLDI (?) and ICFP from Brown

### 7.3.7 Non-Textual Display Forms

TODO: Talk about active code completion work and future ideas

# Bibliography

TODO (Later): List conference abbreviations.

TODO (Later): Remove extraneous nonsense from entries.

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