



## Trilateration

Using trilateration algorithm, the Voronoi site agent estimates the target's location through three distance measurements (local station infers the thief's position with collected information).

From: Consensus Tracking of Multi-Agent Systems with Switching Topologies, 2020

Related terms:

Intersections, Software-Defined Networking, Global Positioning System, Basestation, Localisation, Received Signal Strength

### A 3D-Location System Optimized for Simple Static and Mobile Devices

Herbert Schweinzer, Peter Krammer, in Fieldbus Systems and Their Applications 2005, 2006

#### 2.4 Trilateration

Using **trilateration** for locating a device with the unknown spatial position  $(x, y, z)$ , the distances  $d_i$  to well-known locations  $(a_i, b_i, c_i)$  are given with:

$$d_i = \sqrt{(x - a_i)^2 + (y - b_i)^2 + (z - c_i)^2} \quad (2)$$

Presupposing a minimum of three given locations,  $x$ ,  $y$ , and  $z$  can be computed based on the equation above. Multilateration uses more than three reference locations, which can be used to improve the accuracy. Distances  $d_i$  usually are measured by TOF. Another possibility is *pseudo-ranging* where time-intervals between received signals are measured instead of TOFs. Analogue to GPS systems, pseudo-ranging uses an asynchronous receiver without knowledge about the start of signal transmissions. Assuming a synchronous start of all transmissions, distances can be measured with a common unknown offset  $d_0$

$$d_i = \sqrt{(x - a_i)^2 + (y - b_i)^2 + (z - c_i)^2} - d_0 \quad (3)$$

Using at least four transmitters on fixed locations,  $d_0$  can be eliminated and  $x$ ,  $y$ ,  $z$  of the receiving device can be computed.

In Hazas and Ward (2003) a comparison of the two methods is presented where the accuracy of pseudo-ranging is shown to be worse by a factor of about 5. Moreover, the vertical and horizontal accuracy depend on the geometric arrangement of receiver and ceiling transmitters.

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### Mobile Robot Localization and Mapping

Spyros G. Tzafestas, in Introduction to Mobile Robot Control, 2014

### 12.6.2 Localization by Trilateration

In **trilateration**, the location of a WMR is determined using distance measurements to known active beacon,  $B_i$ . Typically, three or more transmitters are used and one receiver on the robot. The locations of transmitters (beacons) must be known. Conversely, the robot may have one transmitter onboard and the receivers can be placed at known positions in the environment (e.g., on the walls of a room). Two examples of **localization** by trilateration are the beacon systems that use **ultrasonic sensors** (Figure 12.5) and the GPS system (Figure 4.24b).

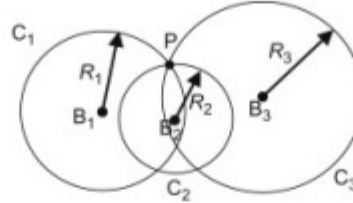


Figure 12.5. Graphical illustration of localization by trilateration. The robot  $P$  lies at the intersection of the three circles  $C_i$  centered at the beacons  $B_i$  with radii  $R_i (i = 1, 2, 3)$ .

Ultrasonic-based trilateration localization systems are appropriate for use in relatively small area environments (because of the short range of ultrasound), where no obstructions that interfere with the wave transmission exist. If the environment is large, the installation of multiple networked beacons throughout the operating area is of increased complexity, a fact that reduces the applicability a sonar-based trilateration. The two alternative designs are:

- Use of a single **transducer** transmitting from the robot with many receivers at fixed positions.
- Use of a single listening receiver on the robot, with multiple **fixed transmitters** serving as beacons (this is analogous to the GPS concept).

The position  $P(x, y)$  of the robot in the world coordinate system can be found by least-squares estimation as follows. In Figure 12.5, let  $(x_i, y_i), i = 1, 2, 3, \dots, N (N \geq 3)$  be the known world coordinates of the beacons  $B_i, i = 1, 2, 3, \dots, N$  and  $R_i, i = 1, 2, 3, \dots, N$  the corresponding robot-beacon distances (radii of the intersecting circles). Then, we have:

$$(x_i - x)^2 + (y_i - y)^2 = R_i^2 \quad (i = 1, 2, 3, \dots, N)$$

Expanding and rearranging gives:

$$x^2 + y^2 + (-2x_i)x + (-2y_i)y + x_i^2 + y_i^2 = R_i^2$$

To eliminate the squares of the unknown variables  $x$  and  $y$ , we pairwise subtract the above equations, for example, we subtract the  $k$ th equation from the  $i$ th equation to get:

$$2(x_k - x_i)x + 2(y_k - y_i)y = b_i$$

where:

$$b_i = R_i^2 - R_k^2 + (x_k^2 + y_k^2) - (x_i^2 + y_i^2)$$

Overall, we obtain the following **overdetermined system** of **linear equations** with two unknowns:

$$Ax = b, \quad x = [x, y]^T, \quad b = [b_1, b_2, \dots, b_N]^T$$

where  $A$  is the matrix:

$$\mathbf{A} = 2 \begin{bmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \\ \vdots & \vdots \\ (x_N - x_{N-1}) & (y_N - y_{N-1}) \end{bmatrix}$$

The solution of this linear algebraic system is given by (see Eqs. (2.7) and (2.8a)Eq. (2.7)Eq. (2.8a)):

$$\mathbf{x} = \mathbf{A}^\dagger \mathbf{B}, \quad \mathbf{A}^\dagger = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T$$

under the assumption that the matrix  $\mathbf{A}^T \mathbf{A}$  is invertible.

An alternative way of computing  $\mathbf{x} = [\mathbf{x}, \mathbf{y}]^T$  is to apply the general *iterative least-squares* technique for nonlinear estimation. To this end, we expand the nonlinear function:

$$F_i = (x_i - x)^2 + (y_i - y)^2 - R_i^2 = 0 \quad (i = 1, 2, \dots, N)$$

in Taylor series about an *a priori* position estimate

$\hat{\mathbf{x}}_q = [\hat{x}_q, \hat{y}_q]^T (q = 0)$ , and keep only first-order terms, namely:

$$\mathbf{F}(\mathbf{x}) = \mathbf{F}(\hat{\mathbf{x}}_q) + \mathbf{A}(\hat{\mathbf{x}}_q) \Delta \mathbf{x}_q = 0, \quad q = 0, 1, 2, \dots$$

where:

$$\Delta \mathbf{x}_q = \mathbf{x} - \hat{\mathbf{x}}_q, \quad \mathbf{F}(\mathbf{x}) = [F_1(\mathbf{x}), F_2(\mathbf{x}), \dots, F_N(\mathbf{x})]^T$$

and  $\mathbf{A}(\hat{\mathbf{x}}_q)$  is the Jacobian matrix of  $\mathbf{F}(\mathbf{x})$  evaluated at  $\mathbf{x} = \hat{\mathbf{x}}_q$ :

$$\begin{aligned} \mathbf{A}(\hat{\mathbf{x}}_q) &= \left[ \frac{\partial \mathbf{F}(\mathbf{x})}{\partial \mathbf{x}} \right]_{\mathbf{x}=\hat{\mathbf{x}}_q} \\ &= 2 \begin{bmatrix} (\hat{x}_q - x_1) & (\hat{y}_q - y_1) \\ (\hat{x}_q - x_2) & (\hat{y}_q - y_2) \\ \vdots & \vdots \\ (\hat{x}_q - x_N) & (\hat{y}_q - y_N) \end{bmatrix} \end{aligned}$$

Therefore, we get the following iterative equation for updating the

$$\hat{\mathbf{x}}_q$$

estimate  $\llbracket \rrbracket$ :

$$\hat{\mathbf{x}}_{q+1} = \hat{\mathbf{x}}_q - \mathbf{A}^\dagger(\hat{\mathbf{x}}_q) \mathbf{F}(\hat{\mathbf{x}}_q), \quad q = 0, 1, 2, \dots$$

$$\hat{\mathbf{x}}_0$$

where  $\llbracket \rrbracket$  is known.

The iteration is repeated until  $\|\hat{\mathbf{x}}_{q+1} - \hat{\mathbf{x}}_q\|$  becomes smaller than a predetermined small number  $\epsilon$  or the number of iterations arrives at a maximum number  $q_{\max} = Q$ .

In practice, due to measurement errors in the positions of the beacons and the radii of the circles, the circles do not intersect at a single point but overlap in a small region. The position of the robot is somewhere in this region. Each pair of circles gives two intersection points, and so with three circles (beacons) we have six intersection points. Three of these points are clustered together more closely than the other three points. A simple procedure for determining the smallest intersection (clustering) region is as follows:

- Compute the distance between each pair of circle intersection points.
- Select the two closest intersection points as the initial cluster.
-

Compute the centroid of this cluster (The  $x,y$  coordinates of the centroid are obtained by the averages of the  $x,y$  coordinates of the points in the cluster).

- Find the circle intersection point which is closest to the cluster centroid.
- Add this intersection point to the cluster and compute again the cluster centroid.
- Continue in the same manner until  $N$  intersection points have been added to the cluster, where  $N$  is the number of beacons (circles).
- The location of the robot is given by the centroid of the final cluster.

As an exercise, the reader may consider the case of two beacons (circles) and compute geometrically the positions of their intersection points. Also, the conditions under which the solution is nonsingular must be determined.

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## Fundamentals of Airborne Acoustic Positioning Systems

Fernando J. Álvarez Franco, in [Geographical and Fingerprinting Data to Create Systems for Indoor Positioning and Indoor/Outdoor Navigation](#), 2019

### 4.1 Spherical Lateration

Also known as trilateration, it can be defined as the method of determining the relative position of points by measuring absolute distances and using the geometry of spheres (see Fig. 4A). As mentioned before, absolute distance from the receiver to the  $i$ th beacon ( $d_i$ ) can be easily obtained from the Time-of-Arrival of the signal emitted by this beacon ( $TOA_i$ ) as,

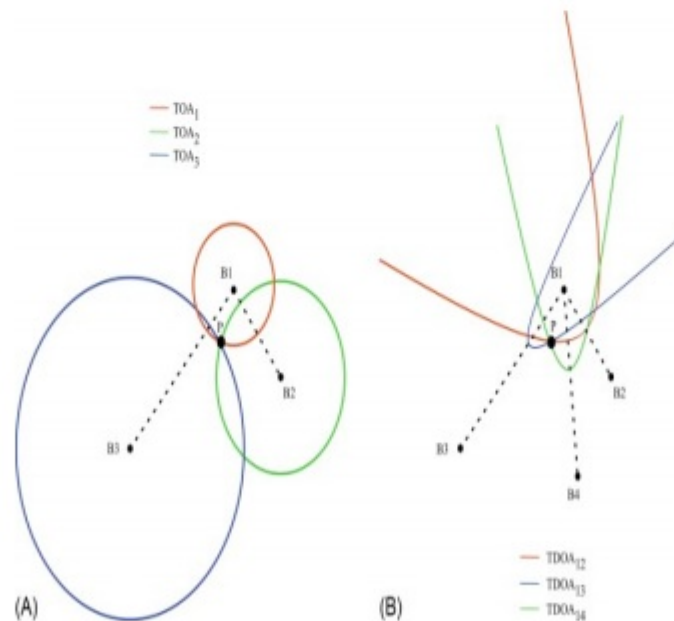


Fig. 4. Planar representation of the spherical (A) and hyperbolic (B) lateration techniques.

$$d_i = c \cdot (TOA_i - t_{rx}), \quad (8)$$

where  $t_{TX}$  represents the instant of emission. Location is then obtained as the intersection of the spherical surfaces defined by these distances,

$$d_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2}, \quad (9)$$

where  $(x_i, y_i, z_i)$  and  $(x, y, z)$  are the coordinates of the  $i$ th beacon and the receiver respectively. In three-dimensional geometry, a minimum of three TOAs (three beacons) are necessary to narrow the possible locations down to two, from which only one is usually coherent with the geometry of the problem. Two families of methods are usually employed to solve the system of nonlinear equations represented in Eq. (9):

- Closed solution: the system is linearized by squaring the equations and introducing new variables. These methods are faster but less precise because the noise contribution is also squared. Besides, they require the participation of additional beacons to deal with the increased dimensionality of the problem. A representative example of this family is the Bancroft method (Bancroft, 1985).
- Iterative solution: equations are linearized by performing a Taylor expansion around an initial position estimate. The system is then iteratively solved to update the position until a final solution within a certain tolerance margin is obtained. These methods are slower than the previous ones and they also require an initial estimate, but in return they are more precise. A good example of this family is the Gauss-Newton algorithm (Bjorck, 1996).

The main inconvenient of spherical lateration is that the receiver must know the precise instant of emission ( $t_{TX}$ ), and the emitters and receiver clocks must be synchronized, although this latter problem is not in acoustic systems as critical as it is in RF-based systems.

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## Sensors Used in Mobile Systems

Gregor Klančar, ... Igor Škrjanc, in Wheeled Mobile Robotics, 2017

Global Navigation Satellite System

The most often used localization system using the trilateration principle is the Global Navigation Satellite System (GNSS). Satellites present active markers that transmit coded signals to the GNSS receiver station whose position needs to be estimated by trilateration. Satellites have very accurate atomic clocks and their positions are known (computed using Kepler elements and other TLE [two-line element set] parameters). There are several GNSS systems: Navstar GPS from ZDA, Glonass from Russia, and Galileo from Europe. The Navstar GPS system is designed to operate with at least 24 satellites that encircle Earth twice a day at a height of 20,200 km.

GNSS seems to be a very convenient sensor system for localization but it has some limitations that need to be considered when designing mobile systems. Its use is not possible where obstacles (e.g., trees, hills, buildings, indoors, etc.) can block the GNSS signal. Multiple signal reflections lead to multipath problems (wrong distance estimate). The expected accuracy of the system is approximately 5 m for single receiver system and approximately 1 cm for referenced systems (with additional receiver in reference station).



A simplified explanation of GNSS localization is as follows. GNSS receivers measure the traveling time of a GNSS signal from a certain satellite. Traveling time is the difference of the reception time  $t_r$  and transmission time  $t_t$ . Knowing that the signal travels at the speed of light  $c$  the distance between the receiver and satellite is computed. However, the receiver clock is not as accurate as the atomic clock on satellites. Therefore, some time bias or distance error appears, which is unknown but equal for all distances to satellites. The GNSS receiver therefore needs to estimate four parameters, namely its 3D position ( $x, y$ , and  $z$ ) and time bias  $t_b$ .

If around each received satellite a sphere with measured distance is drawn, the following can be concluded. The intersection of two spheres is a circle and the intersection of three spheres are two points where the receiver can be located. Therefore, we need at least four spheres to reliably estimate the position of the receiver. If the receiver is on the Earth's surface then Earth can be considered as the fourth sphere to isolate a correct point obtained from the intersection of three satellite' spheres. Ideally only three satellite receptions would be enough. But as already mentioned the receiver clock is inaccurate and this causes unknown time bias  $t_b$ . The intersection of the four spheres (three from satellites and the fourth from Earth) is therefore not a point but an area. To estimate  $t_b$  and lower localization error also the reception of the fourth satellite is required. Therefore, a minimum of four satellite receptions is required for GNSS localization as shown in Fig. 5.16. GNSS localization therefore is needed to solve the following set of equations:

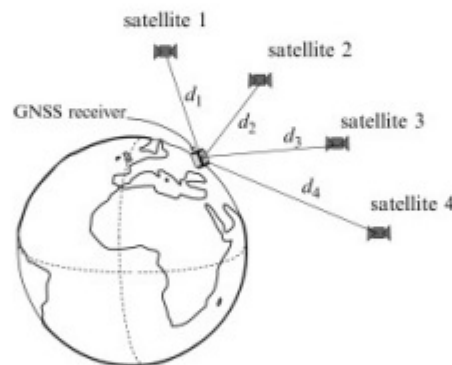


Fig. 5.16. GNSS localization requires reception of minimally four satellites to estimate the GNSS receiver position and the time drift.

$$\begin{aligned}
 d_1 &= c(t_{r1} - t_{t1} - t_d) = \sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2} \\
 d_2 &= c(t_{r2} - t_{t2} - t_d) = \sqrt{(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2} \\
 d_3 &= c(t_{r3} - t_{t3} - t_d) = \sqrt{(x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2} \\
 d_4 &= c(t_{r4} - t_{t4} - t_d) = \sqrt{(x_4 - x)^2 + (y_4 - y)^2 + (z_4 - z)^2}
 \end{aligned}
 \tag{5.81}$$

where unknowns are the receiver position  $x, y, z$  and the receiver time drift  $t_d$ . For  $i$ th satellite the known values are position  $(x_i, y_i, z_i)$ , reception time  $t_{ri}$  and transmission time  $t_{ti}$ , and speed of light  $c$ .

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Francesco Dramis, ... Antonello Cestari, in Developments in Earth Surface Processes, 2011

#### 4.1 Global Positioning System

The GPS can provide accurate measurements of the latitude, longitude and elevation of a survey/sampling point by means of geometric trilateration (a method for determining the intersections of three sphere surfaces given their centres and radii) of a constellation of geostationary satellites (Leick, 1995). For this reason, GPS has become more and more widespread among field geomorphologists (Cornelius et al., 2006), particularly for active processes (Coe et al., 2003). Voženílek (2000) compared GPS-aided geomorphological mapping and conventional surveying techniques, highlighting the utility of the tool in terms of accuracy and data management.

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URL: <https://www.sciencedirect.com/science/article/pii/B9780444534460000033>

### Cooperative relay tracking strategy for multiagent systems with assistance of Voronoi diagrams

Lijing Dong, Sing Kiong Nguang, in Consensus Tracking of Multi-Agent Systems with Switching Topologies, 2020

#### 6.6 Conclusion

This chapter has proposed a cooperative relay tracking strategy to capture targets intruding a domain monitored by a large number of smart agents. Trilateration algorithm is used to calculate the location of a target, the key information for designing a tracking controller. Since a target is tracked by several tracking agents, the tracking agents need to cooperate and communicate with each other. The domain is divided into many Voronoi cells with assistance of Voronoi diagram. In the tracking process the tracking agent which is able to access the location of a target is relayed by a series of Voronoi sites. When a target is moving into a new Voronoi cell, the Voronoi site replaces one of the tracking agents, which results in a switching topology. The switching problem of topologies is solved by modeling it as Markov chain process, and the relay of tracking agents is reflected by the jump of tracking errors. Simulation results prove the advantage of proposed relay tracking algorithm.

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### Location Estimation Methods

Shahin Farahani, in ZigBee Wireless Networks and Transceivers, 2008

#### 7.2.4 Cooperative Location Estimation

In cooperative location estimation, not only are the distances from the tracked node to the anchor nodes measured but also the relative distances of the tracked nodes to each other are used as part of location estimation. Figure 7.6 highlights the difference between the basic trilateration method and the cooperative technique. In Figure 7.6a, the location of the tracked node A is determined using range estimation between the node A and the anchor nodes 1 to 4. The other tracked nodes with unknown locations do not participate in determining the location of the tracked node A. Every time a node

needs to determine its own location using trilateration, only the tracked node itself and the nearby anchor nodes will participate in locationing.

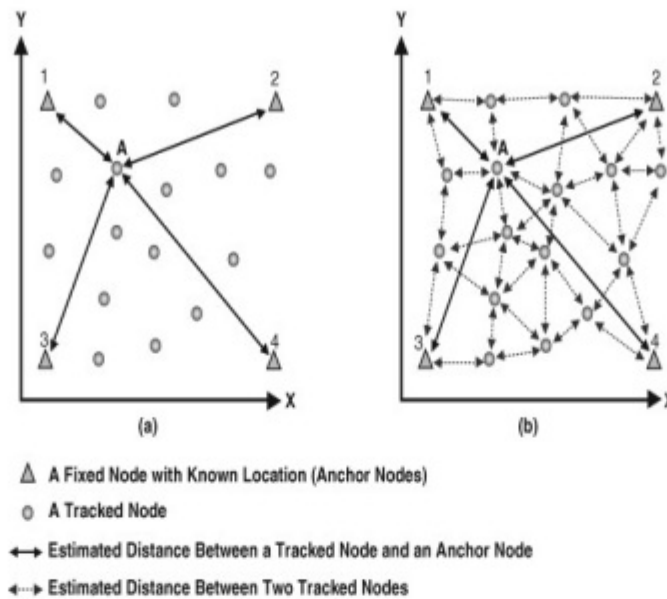


Figure 7.6. (a) Basic Trilateration Method and (b) Cooperative Location Estimation

In the cooperative method shown in Figure 7.6b, in contrast, the location of several tracked nodes can be determined concurrently using an iterative method. First, the RSSI measurements at the anchor nodes provide an estimate for the location of the tracked nodes participating in cooperative locationing. Then each tracked node determines its approximate distance to the neighboring tracked nodes using the RSSI. The approximate distance between the tracked nodes is the additional information available in the cooperative method, which helps refine the location-estimation accuracy beyond the achievable accuracy in a basic trilateration method. The cooperative location-estimation involves nonlinear optimization and can become computationally intensive. In the cooperative method, the estimated locations of the tracked nodes are iteratively adjusted until the optimum result is achieved.

In a trilateration method, increasing the number of anchor nodes in a given area results in an improvement in location accuracy. But increasing the number of tracked nodes does not have any positive effect on accuracy of the trilateration technique. In the cooperative method, on the other hand, increasing either the number of anchor nodes or the number of tracked nodes can result in improvement in location-estimation accuracy.

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## Location Information Processing

David Munoz, ... Rogerio Enriquez, in [Position Location Techniques and Applications](#), 2009

### 3.2.1 Geometric Multilateration Based on TOA Measurements

To determine the range between the NOI and a land reference, the TOA technique exploits the knowledge of the propagation speed of a wave (acoustic or electromagnetic) in a specific medium such as air or water. In a 2D scenario, three land references are required to perform a trilateration process as illustrated in Figure 3.2. If the



estimated distance between the NOI and the first land reference is denoted as  $d_1$ , then the mobile will be located on a circle of radius  $d_1$  centered at the reference coordinates. Note that the use of a single land reference would yield an NOI position ambiguity equivalent to the circumference of the aforementioned circle. This position ambiguity will be reduced to the area created by the intersection of two circles when another land reference is added to the system. Finally, in a noise-free scenario, a third land reference will allow the intersection of a third circle that will cause the position ambiguity to be reduced to a single point that should correspond to the true position of the NOI.

TOA-based trilateration methods require knowledge of absolute propagation times; thus, clock synchronization between the NOI and the land references is essential to avoid errors in distance computation and location estimates. Assuming that the NOI initiates transmission of a signal at time  $t_0$ , and that this signal is received at the  $i$ -th land reference at time  $t_i$ , the range or distance estimate may be found as

$$d_i = c(t_i - t_0), \quad (3.5)$$

where  $c$  is the wave propagation speed (equal to the speed of light  $c = 3 \times 10^8$  m/s for the case of an electromagnetic wave propagating in the air). Ranges from three land references to the NOI are estimated and are used in the trilateration process, which, considering the scenario shown in Figure 3.2, may be formulated using the following system of nonlinear equations:

$$\begin{aligned} d_1^2 &= x^2 + y^2, \\ d_2^2 &= (x_2 - x)^2 + (y_2 - y)^2 = (D - x)^2 + y^2, \\ d_3^2 &= (x_3 - x)^2 + (y_3 - y)^2 = \frac{1}{2}(D - 2x)^2 + (D\sqrt{3} - 2y)^2. \end{aligned} \quad (3.6)$$

Note that this system of equations consists of three circles centered at each of the reference points. With this system of equations, we can find two different solutions for  $(x, y)$  that correspond to the two intersection points of the circles centered at their corresponding land references. The third equation in (3.6) allows us to solve the two-solution ambiguity by selecting the one that is closest to the range estimate given by the circle of radius  $d_3$ . One of the drawbacks of this method is that the third equation is not used in an optimal way to find the position location.

In a noise-free scenario, the three circles will intersect at exactly one point. Note, however, that in the case of noisy distance measurements, the circles will be displaced and their intersection will yield a polygon with an area that will correspond to a position ambiguity. It is common practice to use the final NOI position estimate as the centroid of this resulting polygon. Further, whenever more than three land references are available, one may obtain the NOI position estimate as the average, or centroid, of the estimated coordinates obtained from all possible trilateration realizations calculated with various triplet combinations of land references.

Although geometric trilateration procedures are simple to implement and rather computationally inexpensive, they are limited in the sense that they do not consider the reduction of noise effects and the optimal combination of multiple (more than three) range measurements. In the following sections, we will discuss basic techniques that help to overcome such problems by using statistical location-estimation methods such as least squares and maximum likelihood (ML), among others.

We can extend the TOA trilateration algorithm to the case of a 3D scenario as shown in Figure 3.3. In this scenario, each  $i$ -th reference with known position  $(x_i, y_i, z_i)$  has a range estimate based on TOA that defines a sphere of radius  $R_i$  around reference  $i$  within which the NOI must be located. The ambiguity, in this case, is defined not only by two points of intersection but by curves of intersection between two spheres. This ambiguity can be resolved by using the third range estimate obtained from the remaining land reference. The system of equations for this scenario is given by

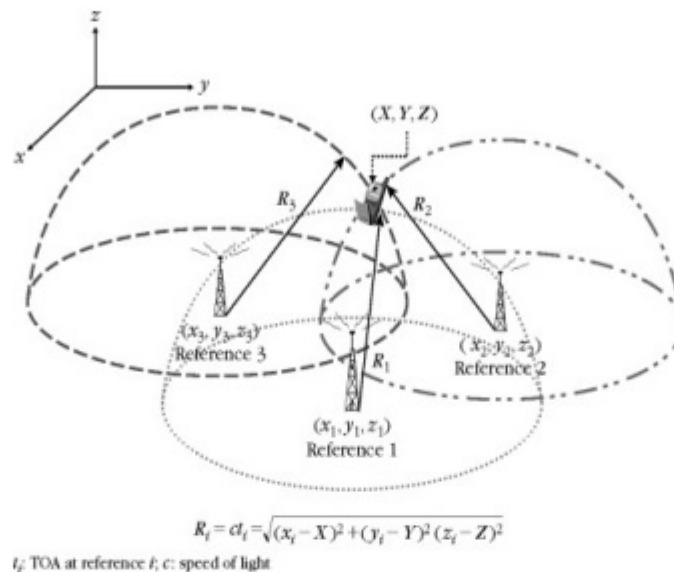


FIGURE 3.3. TOA in 3D.

$$R_i^2 = (X - x_i)^2 + (Y - y_i)^2 + (Z - z_i)^2, \quad i = 1, 2, 3. \quad (3.7)$$

We can see that each of the equations in (3.7) is a sphere of radius  $R_i$ ,  $i = 1, 2, 3$ , centered at each of the references.

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## Autonomous Guided Vehicles

Gregor Klančar, ... Igor Škrjanc, in *Wheeled Mobile Robotics*, 2017

### 7.4.4 Localization and Mapping

AGVs in manufacturing typically need to operate in large facilities. They can apply many features to solve localization and navigation. Quite often a robust solution is sensing induction from the electric wire in the floor or sensing magnetic tape glued to the floor. Currently the most popular solution is the usage of markers (active or passive) on known location and then AGVs localize by triangulation or trilateration. The latter is usually solved by a laser range finder and special reflecting markers. Other solutions may include wall following by range sensors or camera- or ceiling-mounted markers. All of the mentioned approaches are usefully combined with odometry. However, the recent modern solutions apply algorithms for SLAM which make them more flexible and easier to use in new and/or dynamically changing environments. They use sensors to locate usually natural features in the environment (e.g., flat surfaces, border lines, etc.). From features that were already observed and are stored in the existing map the unit can localize, while newly observed features extend the map.

Obtained maps are then used also for path planning.

## Heuristic Approaches to the Position Location Problem

David Munoz, ... Rogerio Enriquez, in *Position Location Techniques and Applications*, 2009

### 4.1.1 Range-Free Location Estimation Systems

Location determination is only possible in relation to an agreed-on reference. In many cases, the location process invokes multiple observation sites that supply information for centralized information processing, and it is assumed that the node can be reached from observation sites in a single step. These observations imply range-related measurements such as field intensity or delay differences that are followed by **trilateration** processes that consider intersection of conic forms. These trilateration processes require at least three observation points, although it may not be possible to ensure that **propagation conditions** are equally adequate to and from all observation points. For instance, if the node is in **close proximity** to a **base station**, it tends to be well apart from other observation points. (Good visibility to and from other distant **base stations** may require higher transmission power that also impacts higher interference levels.)

Technologic advances in antenna technology allow determining angular location and this scheme permits the use of only two observation sites. If coverage of the region cannot be guaranteed, some technologies provide for the introduction of supplementary location mobile units (LMUs) [1, 2] (see Figure 4.1a). Assuming that the base station/AP and an LMU are located at coordinates  $(-D, 0)$  and  $(D, 0)$ , they will provide for the node at  $P(x, y)$  angular observations

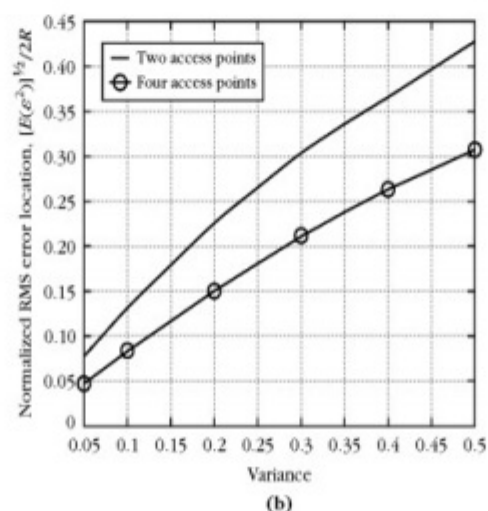
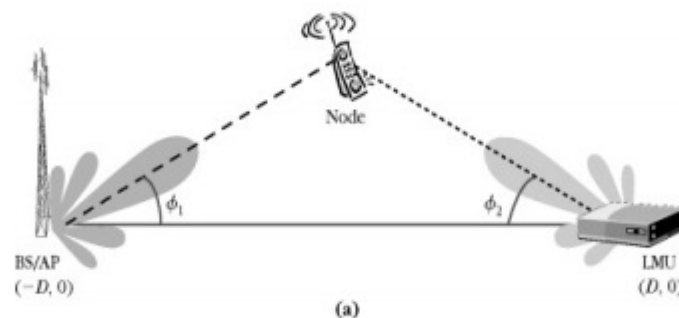


FIGURE 4.1. Angular range-free location. (a) Angular location scenario. (b) Error performance versus angular error variance.

$$\phi_1 = \tan^{-1} \left( \frac{y}{D+x} \right)$$

and

$$\phi_2 = \tan^{-1} \left( \frac{y}{D-x} \right);$$

the node is assumed to be at the intersection of lines

$$y = [\tan \phi_1](x + D) \text{ and } y = [\tan \phi_2](D - x).$$

Location will be uncertain when the node is on the line connecting the AP and the LMU. Precise angular observations are prevented by multipath phenomena and the location error sensitivity to angular errors will strongly depend on the  $(x, y)$  location. Figure 4.1b shows mean square error compared to angular error. Gaussian error distribution has been assumed.

Other range schemes may relate location information to quality-of-service figures. For instance, systems like WiMax or LTE are able to dynamically assign throughputs depending on the perceived carrier-to-interference ratio (CIR). That is, quality-of-service parameters may be associated with location areas of subscriber units as illustrated in Figure 4.2. In general, since CIRs tend to monotonically decrease as separation distance between subscriber unit and the base station increases, we can say that for a  $\text{CIR} > \zeta_i$ ,  $n_0$  will belong to the region

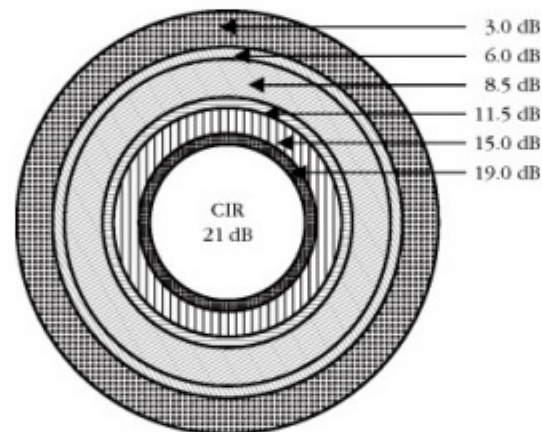


FIGURE 4.2. CIR location regions.

$$R_i = \left\{ (x, y) \mid \sqrt{x^2 + y^2} \leq \rho_i \right\}$$

and for  $\zeta_i < \text{CIR} = \zeta_{i+1}$ ,  $n_0$  will be said to be in the ring  $R_i \cap R_{i+1}^c$ . If quality parameters associated with different base stations are available, the subscriber unit is said to be in the intersection of the corresponding rings.

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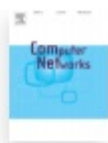


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