2021

# MAP6014

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HOMEWORK_0

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Dear Sir,

I implemented the 3 part of the homework using Python. In the following you can see my result report. Also to see the full details you could see the code on GitHub.

Sincerely yours,

Cyrus Kalantarpour

## Implementation of Linear Congruential Algorithm

To implementation of the Congruential Algorithm I used Python. Please See Figure 1.

```
""" x is seed.
a,b,m are the parameters ansd l is the lenght of sample."""
def lcg(x, a, b, m, l):
  temp =[]
  for i in range(l):
    x = (a * x + b) % m
    temp.append(x)
  return np.array(temp)
```

```
#Example
lcg(1, 947,3937,5241, 10)

array([4884, 1282, 2079, 2134, 1809, 3253, 2820, 1567, 4683, 4852])
```

*Figure 1: Implementation of Linear Congruential Algorithm. To do so, I defined lcg (x, a, b, m, l) where, x is seed; a, b, m are the parameters of the algorithm and  l ls the length of the sample.*

Next, I defined 'Shuff' function that takes an array (for example a congruential pseudo random generated array) and a random integer 'rand'. Then, returns a same size array with same elements that its first element and 'rand'th element is permutated. Please see Figure 2

```
[3]
# array is a 1D vector and rand is a random integer less that len(array).
def Shuff(array, rand):
  temp = array.copy()
  temp[0] = array[rand]
  temp[rand] = array[0]
  return temp
```

```
[6]
#Example
Shuff(np.arange(12), 3)

array([ 3,  1,  2,  0,  4,  5,  6,  7,  8,  9, 10, 11])
```

*Figure 2: Implementation of Shuff function. It takes an array and a random number then; it returns a permuted array according to the given random number.*

Afterwards, I computed the three 100, 109, and 500 units vectors using the Shuff function. Let's name those $Sh_{\_100}, Sh_{\_109}, Sh_{\_500}$. Please see Figure 3.

```
[7]  1 # Shuffling demanded congruential vector with length 100, 109 and 500 examples
     2 Sh_100 = Shuff(lcg(1, 947,3937,5241, 100), np.random.randint(0, 100))
     3 Sh_109 = Shuff(lcg(1, 947,3937,5241, 109), np.random.randint(0, 109))
     4 Sh_500 = Shuff(lcg(1, 947,3937,5241, 500), np.random.randint(0, 500))
```

*Figure 3: Generation of the thee demanded vectors $Sh_{\_100}, Sh_{\_109}, Sh_{\_500}$.*

In the following I will make the 5000 sample sets and conduct the comparisons (Execution time, KS test, serial correlations)

## Comparison of execution time

I used the I python magic method for time calculation that is %%time method. The execution times for $Sh_{\_100}, Sh_{\_109}, Sh_{\_500}$ are 20

Please see Figure 4

```
[77]  1 %%time
      2 S_100=[Shuff(lcg(1, 947,3937,5241, 100), np.random.randint(0, 100)) for i in range(5000)]

CPU times: user 276 ms, sys: 20.8 ms, total: 297 ms
Wall time: 266 ms
```

```
[78]  1 %%time
      2 S_109 = [Shuff(lcg(1, 947,3937,5241, 109), np.random.randint(0, 109)) for i in range(5000)]

CPU times: user 294 ms, sys: 26.7 ms, total: 320 ms
Wall time: 290 ms
```

```
      1 %%time
      2 S_500 = [Shuff(lcg(1, 947,3937,5241, 500), np.random.randint(0, 500)) for i in range(5000)]

CPU times: user 986 ms, sys: 50.2 ms, total: 1.04 s
Wall time: 962 ms
```

*Figure 4: The estimation execution time for $S_{\_100}, S_{\_109}, S_{\_500}$, using %%time method.*

## KS test for uniformality

I used statsmodels library to apply the KS test. To do so, I reshaped the 5000 sample sets to get 1D arrays. Now let assume that the null hypothesis is: $S_{\_100}, S_{\_109}, S_{\_500}$ have a uniform distribution. According to the figure 5 below which shows the result of KS tests we can conclude that the null hypothesis is accepted.

```
1 stats.kstest(S_100, 'uniform')
```

```
KstestResult(statistic=0.06548726243429037, pvalue=0.0)
```

```
[31] 1 stats.kstest(S_109, 'uniform')
```

```
KstestResult(statistic=0.06170309062647228, pvalue=0.0)
```

```
[32] 1 stats.kstest(S_500, 'uniform')
```

```
KstestResult(statistic=0.04679205652090895, pvalue=0.0)
```

*Figure 5: KS test result of $S_{-100}, S_{-109}, S_{-500}$.*

## Correlation analysis

The $S_{-100}, S_{-109}, S_{-500}$ level 1 serial correlations(autocorrelation) shows an incremental behavior. It is worth noting that by changing the seed (initialization of congruential algorithm) the correlation values between $S_{-100}, S_{-109}, S_{-500}$ change drastically. Table 1 below shows the average of the serial correlations. Also, I put one one the correlation results in the Appendix.

| | |
|---|---|
| Corr_100.mean() | 67446.65680664965 |
| Corr_109.mean() | 73492.17311432045 |
| Corr_500.mean() | 334210.29161406466 |

## Answer to the second part of the homework

Changing the seed, clearly changes the vectors while Regarding the KS test it has no meaningful statistical meaning. Also, changing the seed changes the correlations vectors but it it has to effect on incremental behavior of autocorrelations.

## Montecarlo Integral Simulation

I implemented the Montecarlo Integration simulations as follows:

```
def gaussian(x):
  return (1/(np.sqrt(2*np.pi)))*np.exp(-0.5*(x**2))
def Montecarlo(a, b, n):
  xrand=np.zeros(N)
  for i in range(len(xrand)):
    xrand[i]=random.uniform(a, b)

  integral=0
  for i in range(N):
    integral+= gaussian(xrand[i])
  return (b-a)/float(N)*integral
```

Where, a, b are integral limits and N is number of points that we want to use during the simulation.

In order to computation of z_0,05, we run the above simulation with a= -∞ anddifferent b values to get a value close to 0.05. In the below, I defined a for loop tries of the Motecarlo simulation to compute the Z_0.05.x. I assigned -10000 as -∞ .

```
Resolution = np.linspace(0,10, 10000)
for i in Resolution:
  if abs(Montecarlo(-10000,i,10000)-0.05)<0.001:
    print(i)
```

***OUTPUT= - -1.825465012***

## Appendix

Serial correlation level 1 output for Sh_500. To see all the correlations please see the code.

```
array([  0.50712748,   0.54915887,   0.53159648,   0.84523562,
         1.11403448,   1.47608662,   1.6278621 ,   1.90860127,
         2.59971916,   3.15702361,   3.19040016,   3.07216363,
         4.21190634,   4.96959783,   4.39594403,   5.18312641,
         5.34478592,   5.40742468,   5.61364841,   5.71838112,
         6.36760745,   6.58893628,   6.04130745,   6.61359281,
         7.26486455,   7.19386131,   7.06765105,   8.28487453,
         7.8187033 ,   7.99288544,   9.31639873,   8.64211447,
         9.87840084,   9.75170065,   9.40797823,   9.45024739,
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        11.57276822,  11.6132602 ,  12.25746691,  12.2574874 ,
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```

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