STRUCTURAL MECHANICS - II
CE 331

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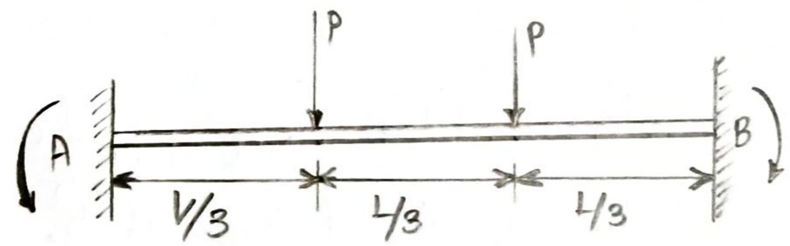
DATE : 2/09/2023

SIGNATURE :

ASSIGNMENT - 2

Question 1 : Obtain fixed end moment for following propped cantilever and fixed beams :

(a)



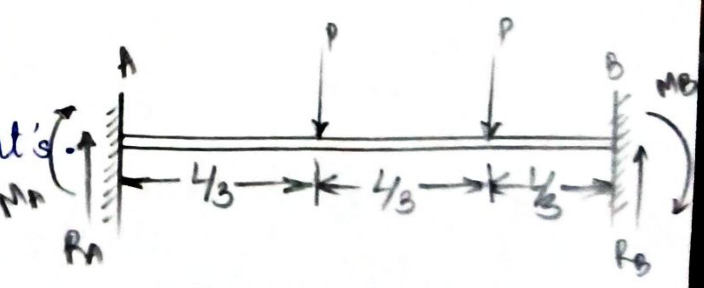
Solution :- Neglecting Axial force's, $(\sum F_x = 0)$

So, Degree of Indeterminance = no of Unknown Rx^n - No. of Equilibrium Eq^n.

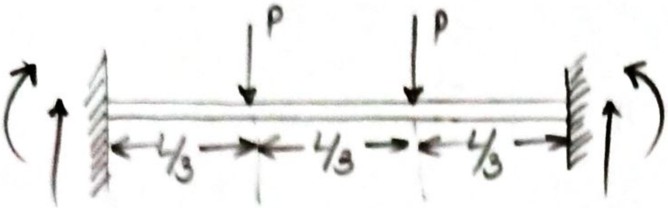
$= 4 - 2$

D.O.I. $= 2$

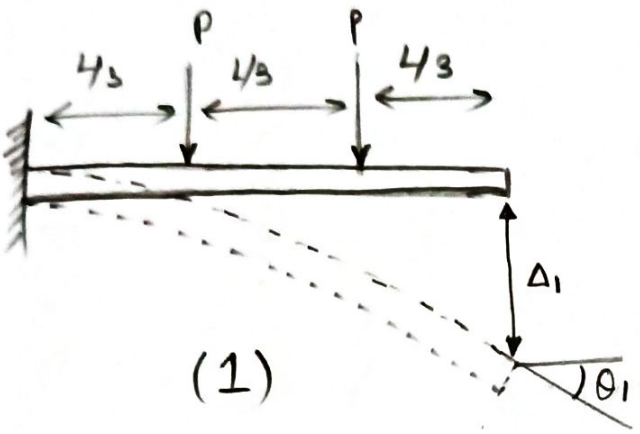
So, consider M_B & R_B as Redundant's.



Now, Assuming side B to be open, and treated whole spane as cantilever.

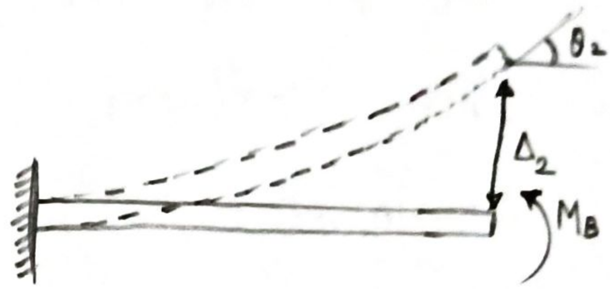


$\cong$



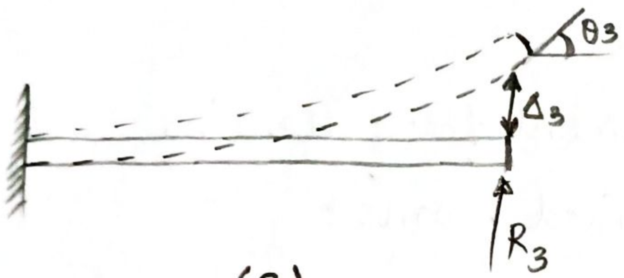
(1)

$\oplus$



(2)

$\oplus$

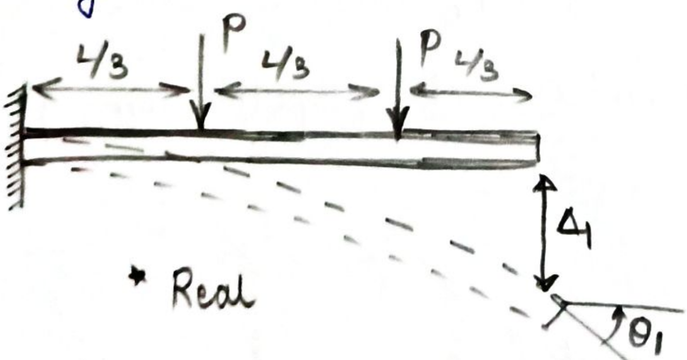


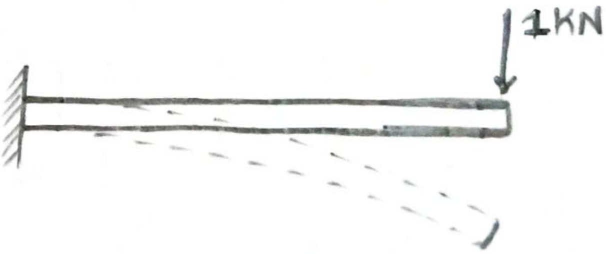
(3)

so, compatibility Equation's, $\theta_1 = \theta_2 + \theta_3$ } Net slope & Deflection

$\Delta_1 = \Delta_2 + \Delta_3$ } zero at 'B'.

Solving (1)



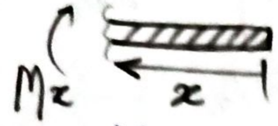


* Using Virtual Work method :-

Taking section's of beam from free end at Distance of x,

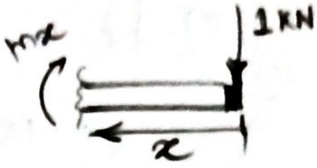
Real beam section's

→ $0 \leq x \leq L/3$



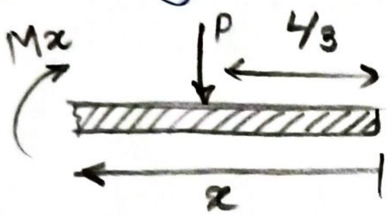
$M_x = 0 \; ; \; \sum M_x = 0$

Virtual beam section's.



$m_x = -x \; ; \; \sum m_x = 0$

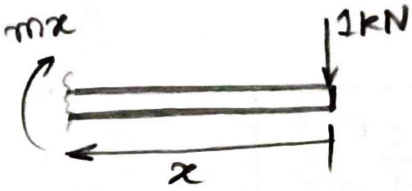
⇒ $L/3 \leq x \leq \frac{2L}{3}$



$\circlearrowleft + \sum M_x = 0$

$M_x + P(x - L/3) = 0$

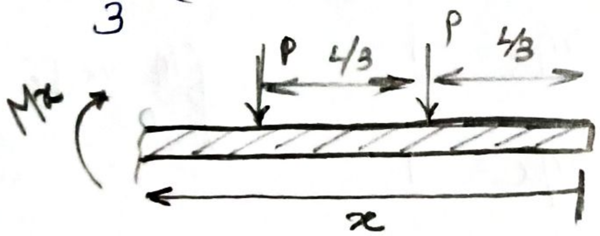
$M_x = -P(x - L/3)$



$\circlearrowleft + \sum M_x = 0$

$m_x = -x$

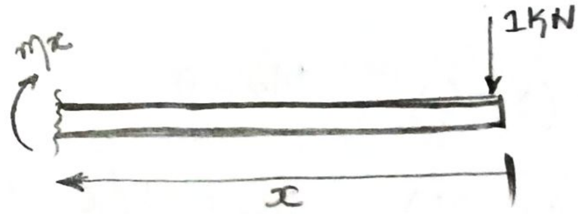
⇒ $\frac{2L}{3} \leq x \leq L$



$\circlearrowleft + \sum M_x = 0,$

$M_x + P\left(x - \frac{L}{3}\right) + P\left(x - \frac{2L}{3}\right) = 0$

$M_x = -P(2x - L)$



$\circlearrowleft + \sum M_x = 0$

$m_x = -x$

$$\because \quad \Delta = \oint \frac{M_x m_x}{EI} dx$$

$$\Delta_1 = \int_0^{L/3} \frac{0 \times (-x)}{EI} dx + \int_{L/3}^{2L/3} \frac{-P(x - L/3)(-x)}{EI} dx + \int_{2L/3}^{0} \frac{-[P(x-L/3) + P(x - 2L/3)](-x)}{EI} dx$$

$$\Delta_1 = \int_{L/3}^{2L/3} \frac{P}{EI}\left(x^2 - \frac{Lx}{3}\right) dx + \int_{2L/3}^{0} \frac{P}{EI}(2x - L)(x)\, dx$$

$$* \quad \boxed{\Delta_1 = \frac{2}{9}\frac{PL^3}{EI}}$$

For slope θ_1

Real Virtual

$x \in (0, L/3)$ $\quad M_x = 0$ $\quad m_x = -1$ $\qquad \because \oint \frac{M_x m_x}{EI} dx = \theta$

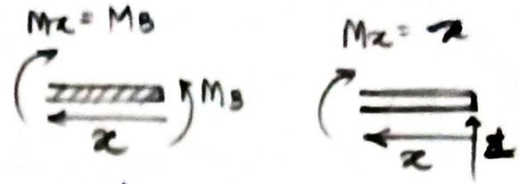
$x \in (L/3, \frac{2L}{3})$ $\quad M_x = -P(x - L/3)$ $\quad m_x = -1$

$x \in (\frac{2L}{3}, L)$ $\quad M_x = -P(2x - L)$ $\quad m_x = -1$

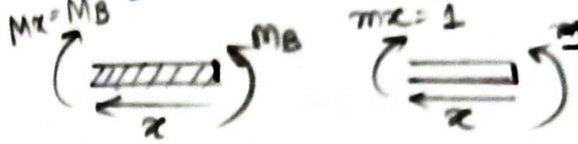
$$\theta_1 = \int_0^{L/3} \frac{0 \times -1\, dx}{EI} + \int_{L/3}^{2L/3} \frac{P(x - L/3)\, dx}{EI} + \int_{2L/3}^{L} \frac{P(2x - L)\, dx}{EI}$$

$$* \quad \boxed{\theta_1 = \frac{5PL^2}{18EI}}$$

solving (2), using Virtual Work method.



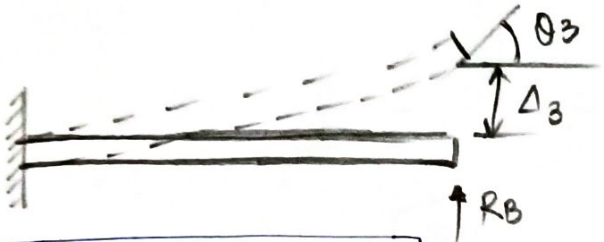
$$\Delta_2 = \int_0^L \frac{M_B x}{EI} dx = \frac{M_B L^2}{2EI}$$

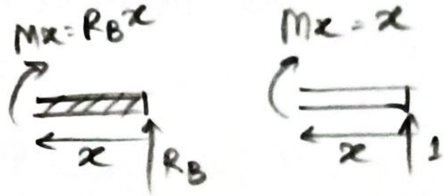


★ $$\boxed{\Delta_2 = \frac{M_B L^2}{2EI}, \quad \theta_2 = \frac{M_B L}{EI}}$$

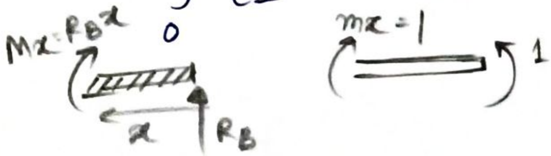
$$\theta_2 = \int_0^L \frac{M_B \, dx}{EI} = \frac{M_B L}{EI}$$

Solving (3), using Virtual Work method,





$$\Delta_3 = \int_0^L \frac{R_B x^2}{EI} dx = \frac{R_B L^3}{3EI}$$



★ $$\boxed{\Delta_3 = \frac{R_B L^3}{3EI}, \quad \theta_3 = \frac{R_B L^2}{2EI}}$$

$$\theta_3 = \int_0^L \frac{R_B x \, dx}{EI} = \frac{R_B L^2}{2EI}$$

Applying, Compatibility. Equation,

$$\Delta_1 = \Delta_2 + \Delta_3$$

$$\frac{2}{9}\frac{PL^3}{EI} = \frac{M_B L^2}{2EI} + \frac{R_B L^3}{3EI}$$

$$\theta_1 = \theta_2 + \theta_3$$

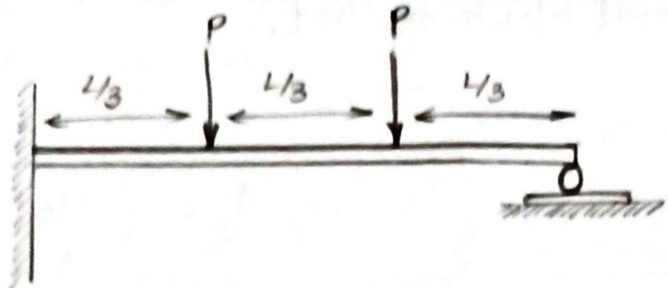
$$\frac{5PL^2}{18EI} = \frac{M_B L}{EI} + \frac{R_B L^2}{2EI}$$

solving Both,

★ $$\boxed{M_B = -\frac{2PL}{9}}, \quad ★ \boxed{R_B = P}$$

★ $$\boxed{\text{End Moment's are } = \pm \frac{2PL}{9}}$$

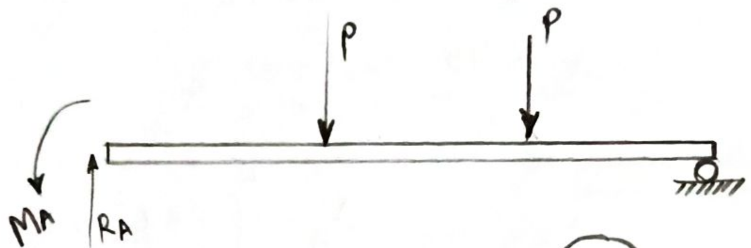
(b)



solution :-

Degree of Indeterminancy = 3 - 2 = 1

⇒ Take M_A as Redundant



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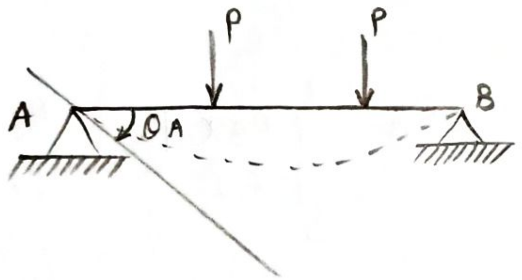

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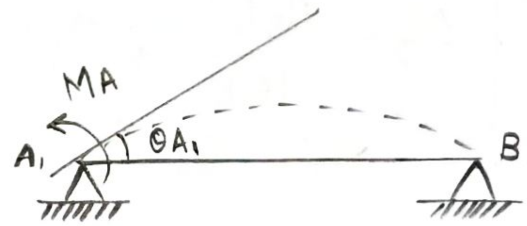
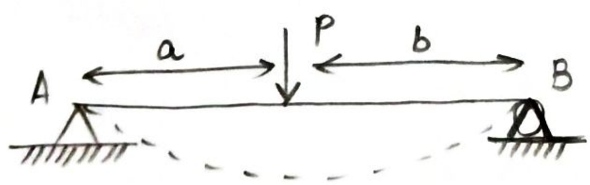


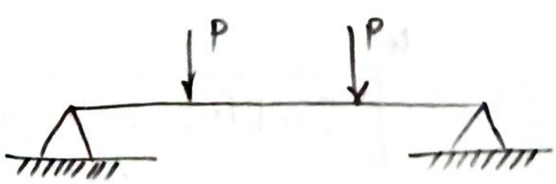
For Slope at A = 0

$$\theta_A + \theta_{A_1} = 0$$

Solving θ_A



$$\theta_A = -\frac{Pab(L+b)}{6LEI}$$



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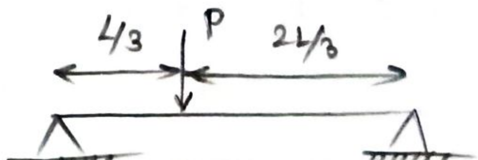

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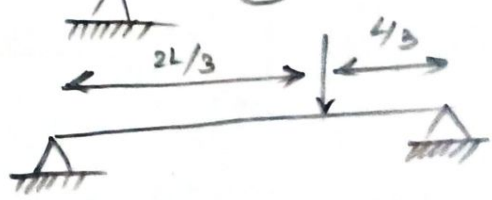


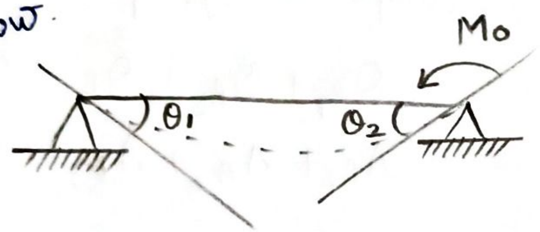
$$\theta_{A_1} = \frac{-P(L/3)\left(\frac{2L}{3}\right)(L+2L/3)}{6LEI}$$

$$\theta_{A_2} = \frac{-P\left(\frac{2L}{3}\right)\left(\frac{L}{3}\right)\left(L+\frac{L}{3}\right)}{6LEI}$$

$$\theta_A = \theta_{A_1} + \theta_{A_2} = -\frac{PL^2}{9EI}$$

solving for θ_{A_1},

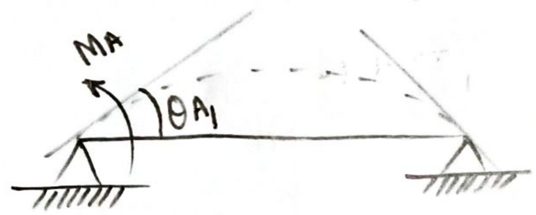
we know.



$$\theta_1 = \frac{M_oL}{3EI} \qquad \because \theta\ \circlearrowright$$

$$\theta_2 = -\frac{M_oL}{6EI}$$

so,



$$\theta_{A_1} = \frac{M_AL}{3EI}$$

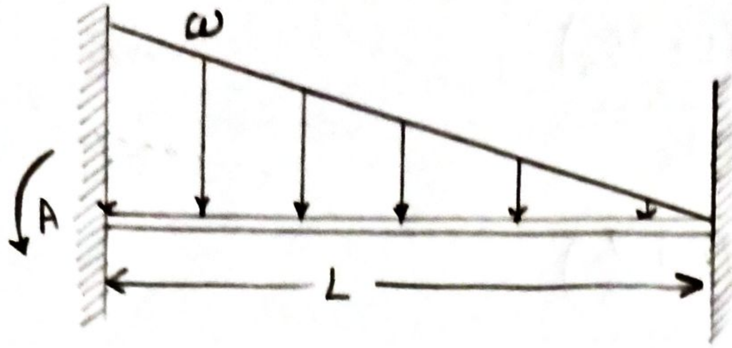
Appling compalibility Eqn

$$\theta_A + \theta_{A_1} = 0$$

$$-\frac{PL^2}{9EI} + \frac{M_AL}{3EI} = 0$$

$$\star\ \boxed{M_A = \frac{PL}{3}}$$

(C)



Solution :–

Degree of static Indeterminancy ⇒ $4-2=2$

Taking M_A & M_B as redundant Rxⁿ.

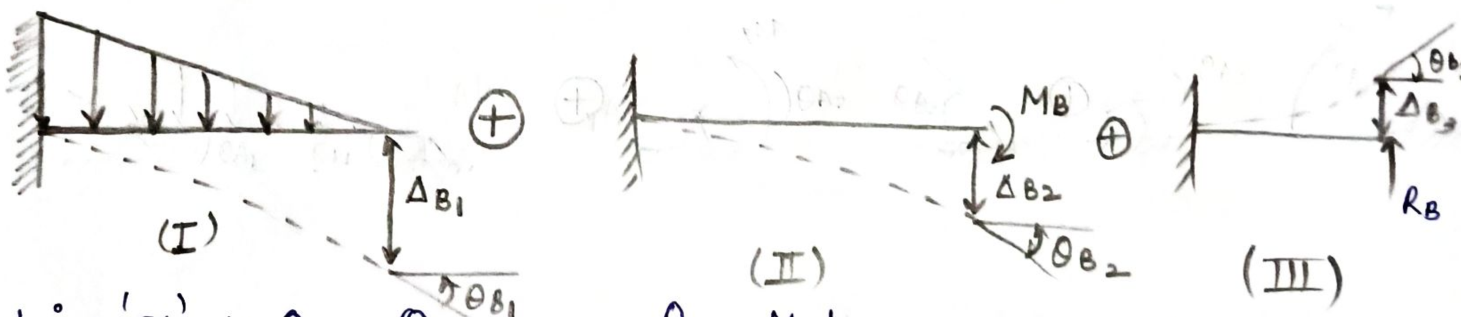
So, it can be drawn as,

compatibility eqⁿ

$$\theta_{B_1} + \theta_{B_2} + \theta_{B_3} = 0$$

$$\Delta_{B_1} + \Delta_{B_2} + \Delta_{B_3} = 0$$

≅



solving 'I' to θ_{A_1}, θ_{B_1}

$$\theta_{B_2} = \frac{M_B L}{EI}$$

$$\Delta_{B_2} = \frac{-M_B L^2}{2EI}$$

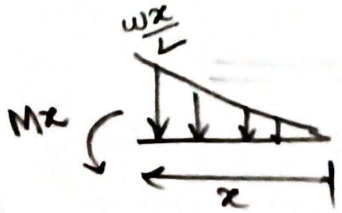
$$\theta_{B_3} = \frac{-R_B L^2}{2EI}$$

$$\theta_{B_3} = \frac{R_B L^3}{3EI}$$

Virtual Work

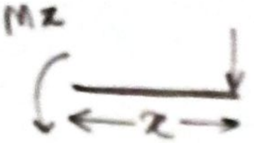
using ~~Double~~ - ~~Integration~~ Method

$$\circlearrowleft \sum M_x = 0$$



$$M_x = \frac{wx}{L} \cdot \frac{x}{2} \cdot \frac{x}{3} = \frac{wx^3}{6L}$$

M_x



$m_x = x$

Virtual Loading.

m_θ

$m_\theta = 1$

Virtual Moment Loading.

$\because$ $$1 \cdot \Delta_{B_1} = \oint \frac{M_x m_x}{EI} dx$$

$$\Delta_{B_1} = \int_0^L \frac{Wx^3}{6L} \cdot x \cdot \frac{1}{EI} dx \quad ; \quad \Delta_{B_1} = \frac{WL^4}{30EI}$$

$\because$ $$1 \cdot \theta_{B_1} = \oint \frac{M_x m_\theta \, dx}{EI}$$

$$\theta_{B_1} = \int_0^L \frac{Wx^3}{6L} \cdot \frac{1}{EI} dx = \frac{WL^3}{24EI}$$

Putting value's in compatibility condition :-

$$\theta_{B_1} + \theta_{B_2} + \theta_{B_3} = 0 \qquad \Delta_{B_1} + \Delta_{B_2} + \Delta_{B_3} = 0$$

$$\frac{WL^3}{24EI} + \frac{M_B L}{EI} - \frac{R_B L^2}{2EI} = 0 \qquad \frac{-WL^4}{30EI} - \frac{M_B L^2}{2EI} + \frac{R_B L^3}{3EI} = 0$$

solving both eqn

$$M_B = \frac{WL^2}{30} \;, \quad R_B = \frac{3WL}{20}$$

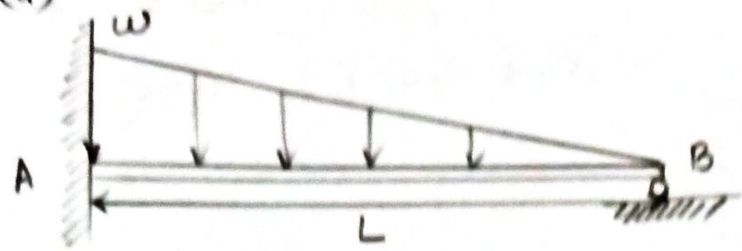
Now, Taking moment about A in beam AB,

$$\sum M_A = 0 \Rightarrow -M_A + M_B - R_B L + \frac{WL}{2} \cdot \frac{L}{3} = 0$$

$$M_A = \frac{WL^2}{20}$$

$\star$ $$\boxed{M_A = \frac{WL^2}{20} \;, \quad M_B = \frac{WL^2}{30}}$$

(d)



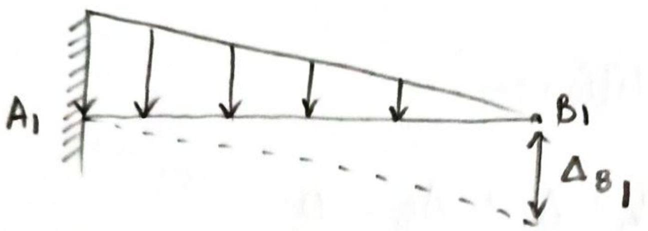
Solution :-

Degree of static interdminacy = 1

No of redundant R_x^n = 1

Taking R_B as redundant

considering the given beam as :-



$$\therefore \quad \Delta_{B_1} = \frac{-wL^4}{30EI}$$

(from Q : 1(c))

+

$$\Delta_{B_2} = \frac{R_B L^3}{3EI}$$

(from Q : 1 (b))

Compatibility condition's :-

$$\Delta_B = 0$$

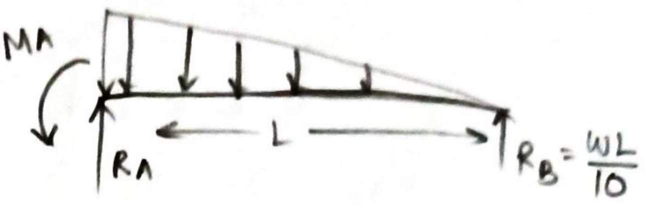
$$\Delta_{B_1} + \Delta_{B_2} = 0$$

$$-\frac{wL^4}{30EI} + \frac{R_B L^3}{3EI} = 0$$

$$\star \quad \therefore \boxed{R_B = \frac{wL}{10}}$$

Taking moment about A in beam AB,

$\circlearrowleft^{+} \sum M_A = 0$

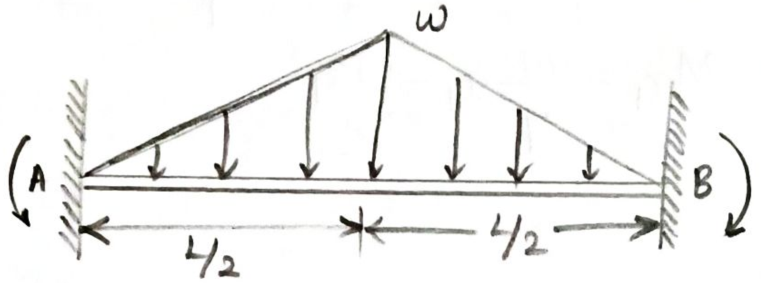


$$-M_A + \frac{WL}{2} \cdot \frac{L}{3} - R_B L = 0$$

$$M_A = \frac{WL^2}{6} - \frac{WL^2}{10} = \frac{WL^2}{15}$$

★ $\boxed{M_A = \frac{WL^2}{15}}$

(c)

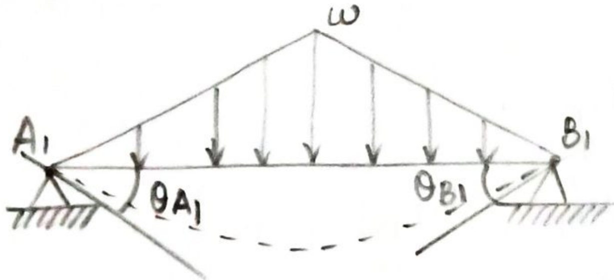


solution :- Degree of static Indeterminacy = 2

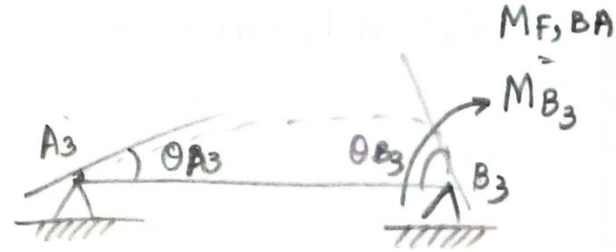
No of redundant reactions = 2

Taking M_A and M_B as redundant.

Now, the given beam can be considered as



⊕



$$\theta_{A_3} = \frac{M_{F,BA} \cdot L}{6EI}$$

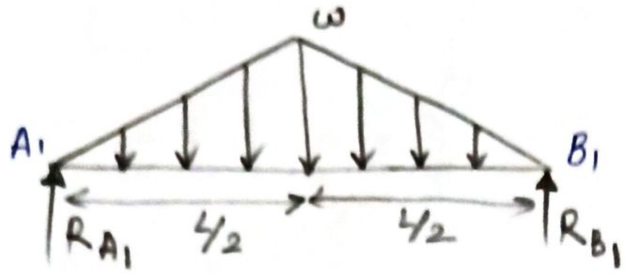
$$\theta_{B_3} = \frac{M_{F,BA} \cdot L}{3EI}$$

⊕

$$\theta_{A_2} = \frac{M_{F,AB} \cdot L}{3EI}$$

$$\theta_{B_2} = \frac{M_{F,AB} \cdot L}{6EI}$$

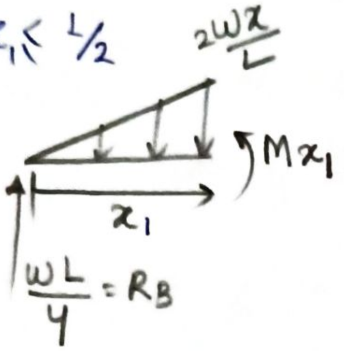
Using Virtual-work method for beam $A_1 B_1$



$$R_{A_1} = R_{B_1} = \frac{wL}{4}$$

Taking x, from Left (A_1) to B_1

★ $0 \leq x_1 \leq L/2$



$$\circlearrowleft^{+} \sum M_x = 0$$

$$M_{x_1} + \frac{2wx_1}{L} \cdot \frac{x_1}{2} \cdot \frac{x_1}{3} - \frac{wLx_1}{4} = 0$$

$$M_{x_1} = \frac{wL}{4} x_1 - \frac{wx_1^3}{3L}$$

★ $\frac{L}{2} \leq x_2 \leq L$

similarly taking x_2 from right B_1 to A_1

Due to symmetry,

$$M_{x_2} = \frac{wLx_2}{4} - \frac{wx_2^3}{3L}$$

Virtual beam :—

$$\sum F_y = 0$$

$$R_{A_1}' = -R_{B_1}'$$

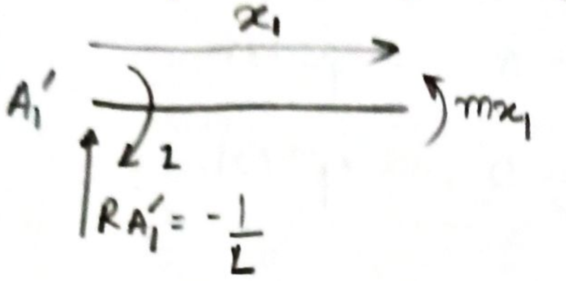
$$\circlearrowleft^{+} \sum M_{A_1'} = 0$$

$$-1 + R_{B_1}'(L) = 0$$

$$R_{B_1}' = 1/L$$

$$R_{A_1}' = \frac{1}{L}$$

At $x = x_1$ from A (Left to Right) $\quad \circlearrowright \sum M_{x_1} = 0$



$$m_{x_1} - \left(-\frac{1}{L}\right)(x) - 1 = 0$$

* $\boxed{m_{x_1} = 1 - \frac{x_1}{L}}$

At $x = x_2$ from B (Right to Left) $\quad \circlearrowleft + \sum M_{x_2} = 0$

$$m_{x_2} - \left(\frac{1}{L}\right)(x) = 0$$

* $\boxed{m_{x_2} = \frac{x_2}{L}}$

Now, we know

$$1.\ \theta_{A_1} = \oint \frac{mM\,dx}{EI}$$

$$\theta_{A_1} = \frac{1}{EI}\left[\int_0^{L/2} m_{x_1} M_{x_1}\,dx_1 + \int_0^{L/2} m_{x_2} M_{x_2}\,dx_2\right]$$

$$\theta_{A_1} = \frac{1}{EI}\left[\int_0^{L/2} \left(1 - \frac{x_1}{L}\right)\left(\frac{wLx_1}{4} - \frac{wx_1^3}{3L}\right)dx_1 + \int_0^{L/2} \frac{x_2}{L}\left(\frac{wLx_2}{4} - \frac{wx_2^3}{3L}\right)dx_2\right]$$

$$\theta_{A_1} = \frac{1}{EI}\left[\int_0^{L/2} \left(\frac{wLx}{4} - \frac{wx^3}{3L}\right)dx\right]$$

* $\boxed{\theta_{A_1} = \frac{1}{EI} \cdot \frac{5wL^3}{192}}$

By symmetry $\boxed{\theta_{B_2} = \frac{5wL^3}{192EI}}$

Compatibility Conditions

$\theta_A = 0 \quad ; \quad \theta_{A_1} + \theta_{A_2} + \theta_{A_3} = 0$ — eqn (1)

$\theta_B = 0 \quad ; \quad \theta_{B_1} + \theta_{B_2} + \theta_{B_3} = 0$ — eqn (2)

By eqn (1),

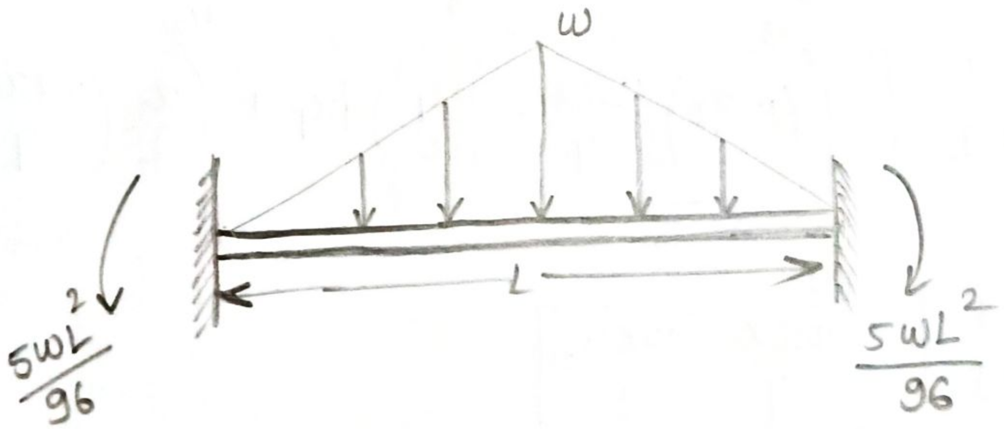
$$-\frac{5wL^3}{192EI} + \frac{M_{F,AB}}{3EI} + \frac{M_{F,BA}}{6EI} = 0 \quad \text{— (3)}$$

by eqn (2)

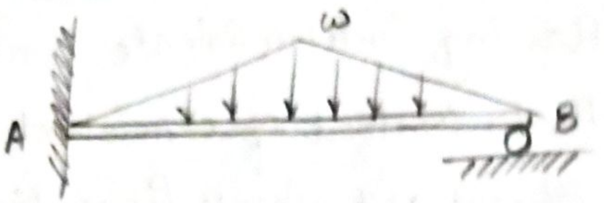
$$-\frac{5wL^3}{192EI} + \frac{M_{F,AB}}{6EI} + \frac{M_{F,BA}}{3EI} = 0 \quad \text{— (4)}$$

solving both eqn

$$\boxed{M_{F,AB} = M_{F,BA} = \frac{5wL^2}{96}}$$



(f)



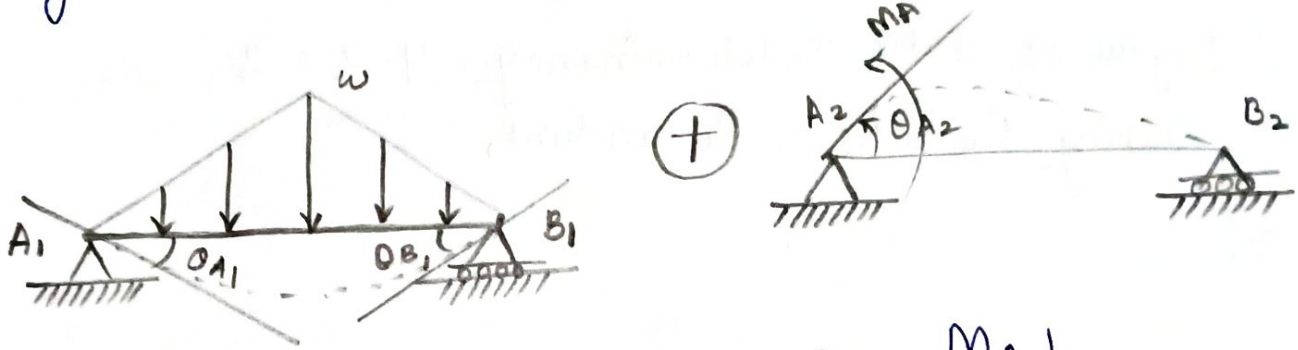
solution :-

Degree of Indeterminacy = 1

No. of redundant reaction = 1

Taking M_A, as redundant

The given beam can be considered as.



$$\theta_{A_1} = -\frac{5wL^3}{192EI}$$

(from Q : 1(e))

$$\theta_{A_2} = \frac{M_A \cdot L}{3EI}$$

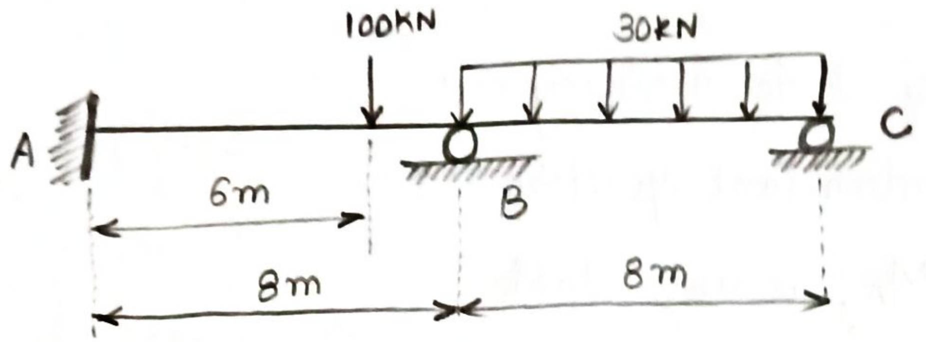
compatability condition.

$$\theta_A = 0 \; ; \; \theta_{A_1} + \theta_{A_2} = 0$$

$$-\frac{5wL^3}{192EI} + \frac{M_A L}{3EI} = 0$$

$$\boxed{M_A = \frac{5wL^2}{64}}$$

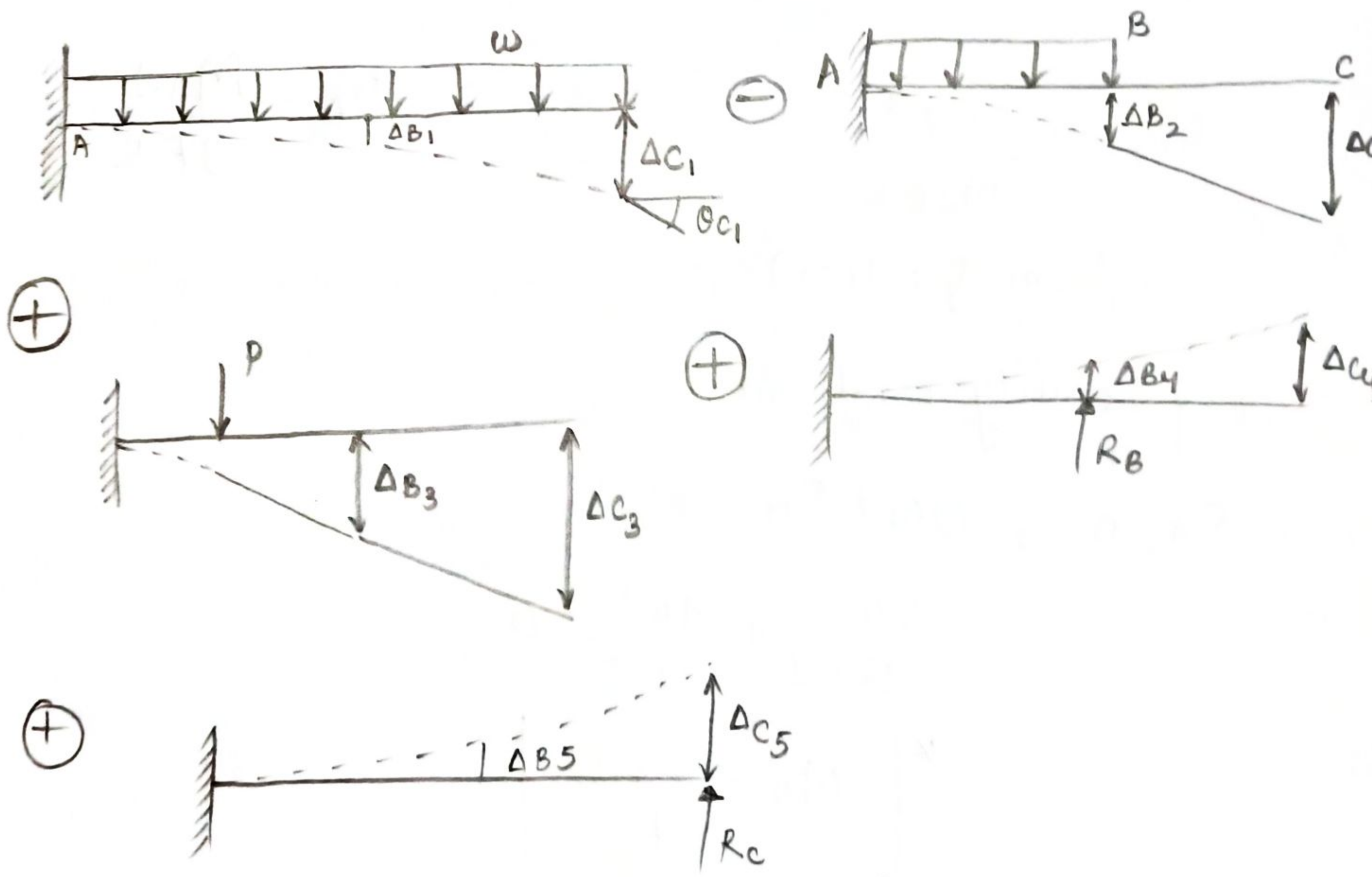
Question 2 : Analyse the following indeterminate continous beam using force method (method of consistent deformation). Draw bending moment and shear force diagram for the beam.



solution :—

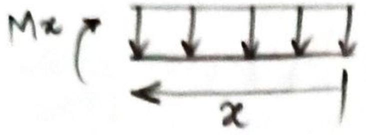
Degree of static Indeterminancy = 4 - 2 = 2
Taking R_B & R_C as Redundant,

so,



Calculating Δ_{B_1} & Δ_{C_1}

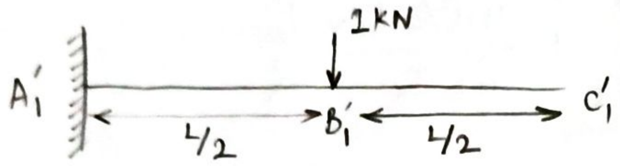
Taking x from free end C_1

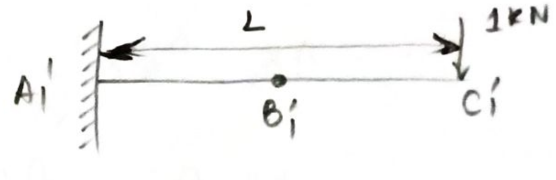


$\circlearrowleft^{+} \sum M_x = 0$

$$\boxed{M_x = -\frac{Wx^2}{2}} \qquad x \in (0, L)$$

Virtual Beam :—





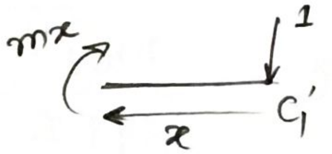
Taking x from right to left

$0 < x < L/2$

$\circlearrowleft^{+} \sum M_x = 0$

$m_x = 0$

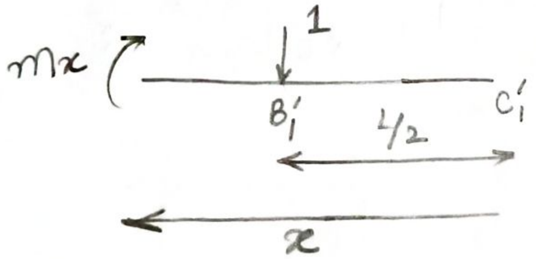
$0 \le x \le L$



$\circlearrowleft^{+} \sum M_x = 0$

$m_x = -x$

$L/2 < x < L$



$\circlearrowleft^{+} \sum M_x = 0$

$m_x + (x - L/2) = 0$

$$\boxed{m_x = -(x - L/2)}$$

$$\Delta_{C_1} = \oint_L \frac{M_x m_x \, dx}{EI}$$

$$\Delta_{C_1} = \int_0 \frac{\left(\frac{-wx^2}{2}\right)(-x)}{EI} dx$$

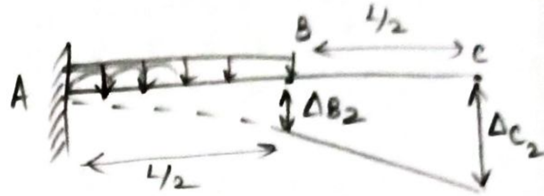
$$\boxed{\Delta_{C_1} = \frac{wL^4}{8EI}} \downarrow$$

1. $\Delta_{B_1} = \oint_{L/2} \frac{M_x m_x \, dx}{EI}$

$$\Delta_{B_1} = \int_0^{L/2} \frac{\frac{-wx^2}{2}(0)\,dx}{EI} + \int_{L/2}^{L} \frac{\frac{-wx^2}{2} \cdot (x - L/2)}{EI} dx = \frac{w}{2EI} \int_{L/2}^{L} \left(x^3 - \frac{x^2 L}{2}\right) dx$$

* $$\boxed{\Delta_{B_1} = \frac{17wL^4}{384EI}} \downarrow$$

Calculating Δ_{B_2} & Δ_{C_2}



$$\Delta_{B_2} = \frac{w(L/2)^4}{8EI} = \boxed{\frac{wL^4}{128EI}} \downarrow$$

$$\Delta C_2 = \Delta B_2 + \theta_B \times \frac{L}{2} = \frac{wL^4}{128EI} + \frac{w(L/2)^3}{6EI} \times \frac{L}{2}$$

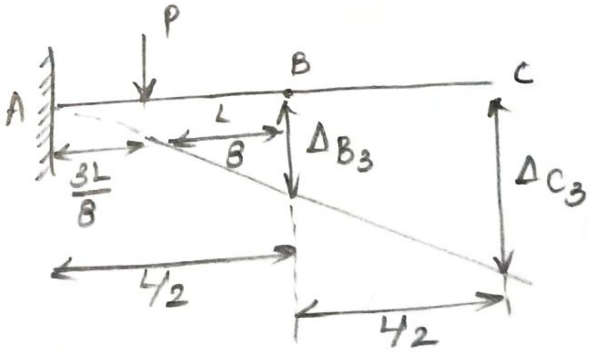
$$\because \theta_B = \frac{w(L/2)^2}{6EI}$$

* $$\boxed{\Delta C_2 = \frac{7wL^4}{384EI}} \downarrow$$

Calculating Δ_{B_3} & Δ_{C_3}

$$\Delta_{B_3} = \Delta_P + \theta_P \times \frac{L}{8} = \frac{P(3L/8)^3}{3EI} + \frac{P(3L/8)^2}{2EI} \times \frac{L}{8}$$

* $$\boxed{\Delta_{B_3} = \frac{27PL^3}{1024EI}} \downarrow$$

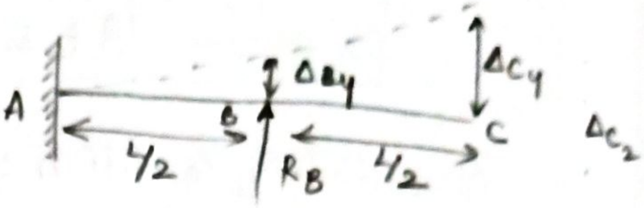


$$\Delta_{C_3} = \Delta_P + \theta_P\left(\frac{5L}{8}\right)$$

$$\Delta_{C_3} = \frac{P\left(\frac{3L}{8}\right)^3}{3EI} + \frac{P(3L/8)^2}{2EI} \cdot \frac{5L}{8} = \boxed{\frac{63PL^3}{1024EI}} \downarrow = \Delta_{CB}$$

Calculating Δ_{By} & Δ_{Cy}

$$* \quad \boxed{\Delta_{By} = \frac{R_B (L/2)^3}{3EI} = \frac{R_B L^3}{24EI} \uparrow}$$



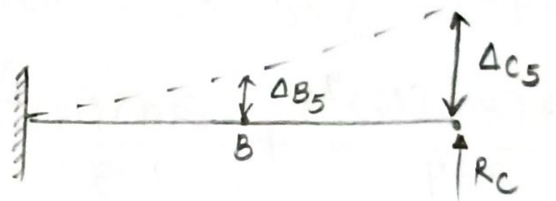
$$\Delta_{Cy} = \Delta_{By} + \theta_{By} \times \frac{L}{2}$$

$$\Delta_{Cy} = \frac{R_B L^3}{24EI} + \frac{R_B (L/2)^2}{2EI} \cdot \frac{L}{2}$$

$$\boxed{\Delta_{Cy} = \frac{5BL^3}{48EI}} \uparrow$$

Calculating Δ_{B5} & Δ_{C5}

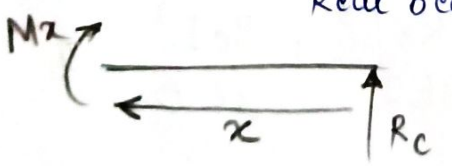
$$\boxed{\Delta_{C5} = \frac{R_C L^3}{3EI} \uparrow}$$



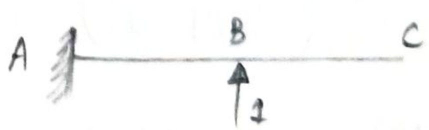
Using Virtual-Work method for calculation of Δ_{B5}

Taking x from right to left

Real beam



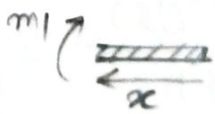
Virtual Beam



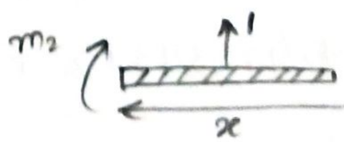
$\circlearrowleft + \sum M_x = 0$

$M_x - R_C(x) = 0$

$M_x = R_C x \qquad x \in (0, L)$



$m_1 = 0$



$m_2 = x - L/2$

$$\Delta_{B5} = \int_{L/2}^{L} \frac{(x - L/2) R_C x}{EI} dx$$

$$\star \quad \boxed{\Delta_{B5} = \frac{R_C \cdot 5L^3}{48EI} \uparrow}$$

Compatibility condition :-

$\Delta_B = 0$; $\Delta_{B1} - \Delta_{B2} + \Delta_{B3} + \Delta_{B4} + \Delta_{B5} = 0$ — eqⁿ (1)

$\Delta_C = 0$; $\Delta_{C1} - \Delta_{C2} + \Delta_{C3} + \Delta_{C4} + \Delta_{C5} = 0$ — eqⁿ (2)

putting all value's in eqⁿ 1

$$\frac{-17wL^4}{384EI} - \left(\frac{-wL^4}{128EI}\right) - \frac{27PL^3}{1024EI} + \frac{R_B L^3}{24EI} + \frac{5R_C L^3}{48EI} = 0$$

$$\frac{-17(30)(16)^1}{384} + \frac{(30)(16)^1}{128} - \frac{27(100)}{1024} + \frac{R_B}{24} + \frac{5R_C}{48} = 0$$

$$-20.13671875 + \frac{R_B}{24} + \frac{5R_C}{48} = 0$$

$$2R_B + 5R_C = 966.5625 \quad \text{— eq}^n \ (3)$$

By putting value's in eqⁿ (2)

$$\frac{-wL^4}{8EI} - \left(-\frac{7wL^4}{384EI}\right) - \frac{63PL^3}{1024EI} + \frac{5R_B L^3}{48EI} + \frac{R_C L^3}{3EI} = 0$$

$$-\frac{30(16)}{8} + \frac{7(30)(16)}{384} - \frac{63(100)}{1024} + \frac{5R_B}{48} + \frac{R_C}{3} = 0$$

$$-57.40234375 + \frac{5R_B}{48} + \frac{R_C}{3} = 0$$

$$2755.3125 = 5R_B + 16R_C \quad \text{— eq}^n \ (4)$$

From eqⁿ (3) and (4)

$R_B \approx 241.138\ kN = 241.205357\ kN$

$R_C = 96.83035714\ kN$

Now, In beam AB,

$\sum F_y = 0$

$R_A + R_B + R_C - wL/2 - P = 0$

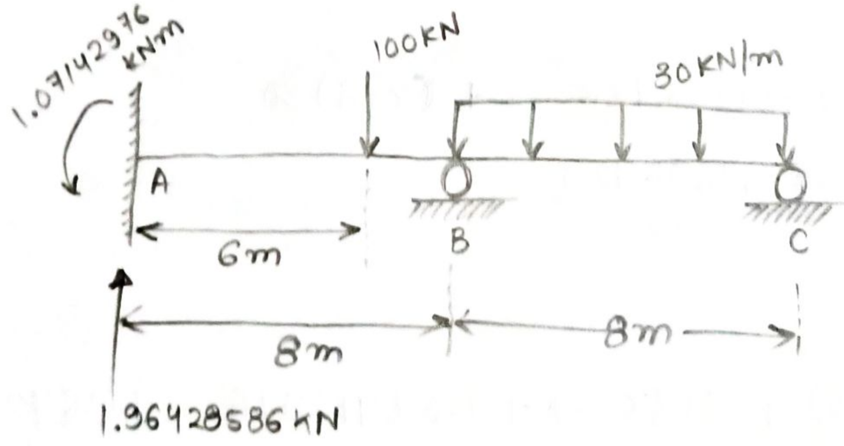
$R_A + 241.205357 + 96.83035714 - 30(8) - 100 = 0$

$R_A = 1.96428586\ KN$

$\sum M_A = 0$

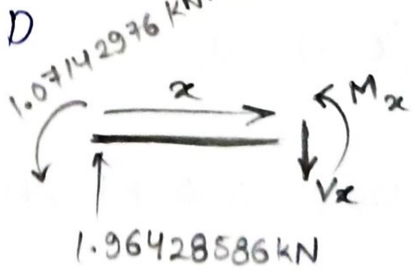
$-M + 100(6) + 30 \times 12 \times 8 - R_B \times 8 - R_C \times 16 = 0$

$M = 1.07142976\ kNm$



Making SFD & BMD

when $0 < x < 6$



$+\downarrow \sum F_y = 0$

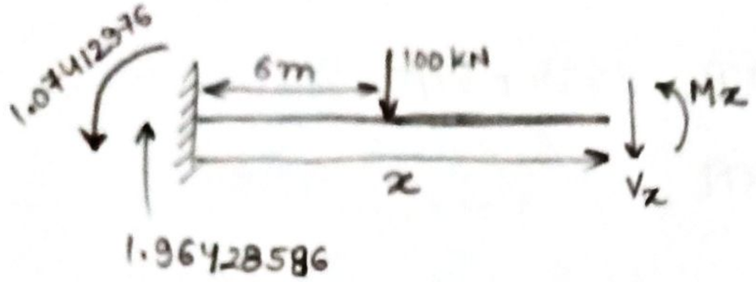
$V_x = 1.96428586\ KN$

$\circlearrowleft + \sum M_x = 0$

$M_x + 1.07142976 - 1.96428586x = 0$

$* \quad M_x = 1.96428586x - 1.07142976$

when $6<x<8$



$\curvearrowleft + \sum M_x = 0$

$M_x + 1.96428586(-x) + 1.07412976 + (x-6)100 = 0$

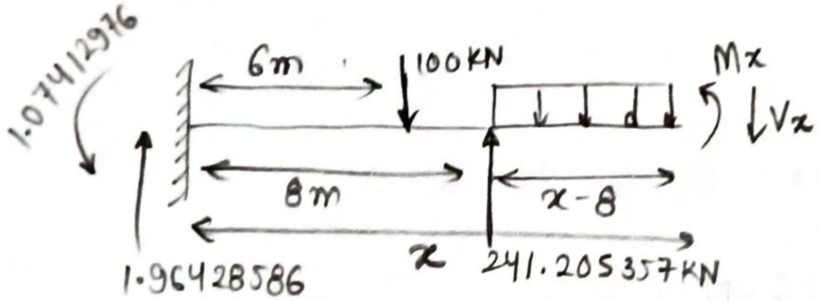
$M_x = -98.03571414x + 598.9258702$

$+\uparrow \sum F_y = 0$

$-V_x - 100 + 1.96428586 = 0$

$V_x = 98.03571414 \text{ KN}$

when $8<x<16$



$\downarrow + \sum F_y = 0$

$V_x + 100 - 241.205357 - 1.96428586 + (x-8)30$

$V_x = -30x + 383.1696429$

$\curvearrowleft + \sum M_x = 0$

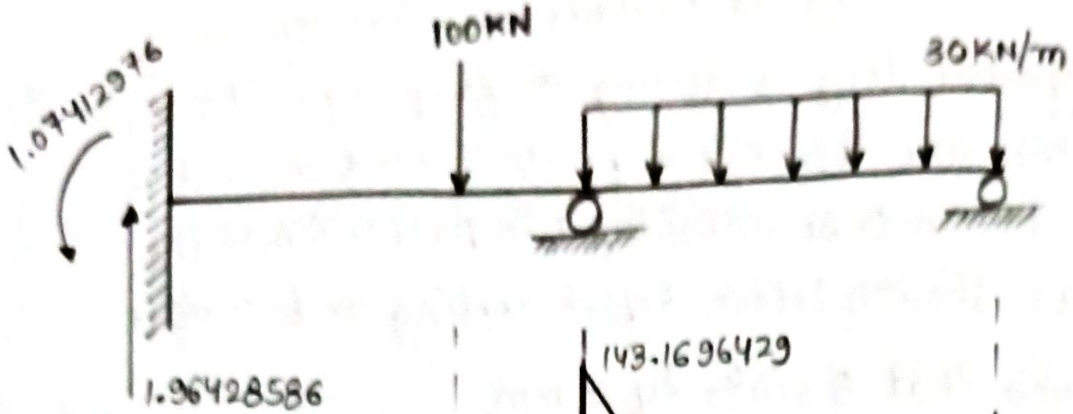
$M_x + 30(x-8)\frac{(x-8)}{2} + 100(x-6) + 1.07412976 - 1.96428586(x)$

$- 241.205357(x-8) = 0$

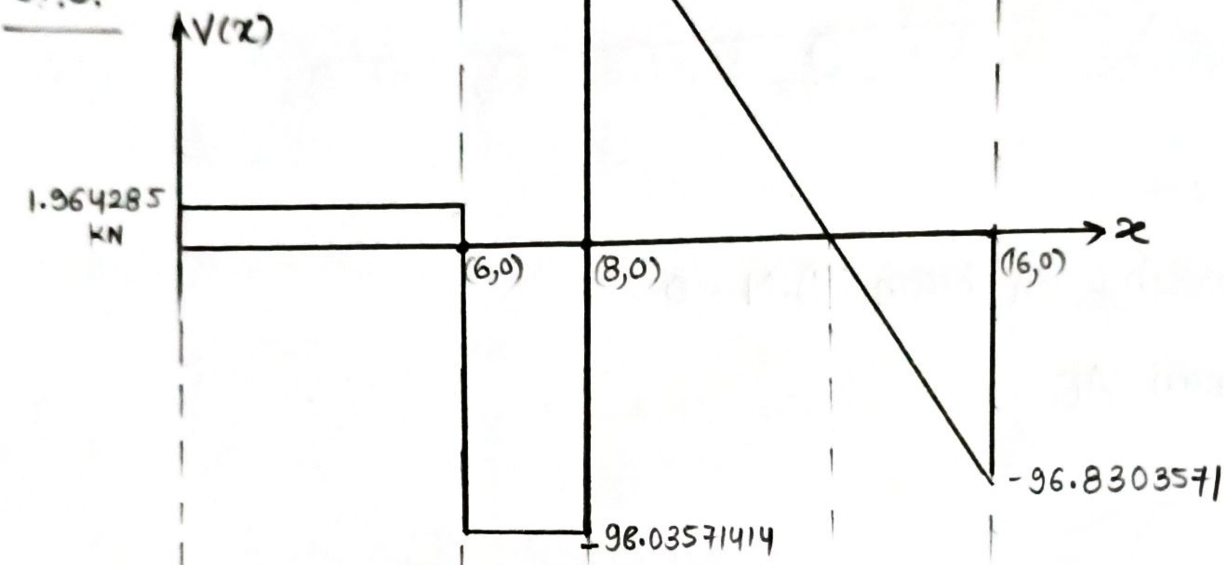
$M_x + 15(x-8)^2 + 100x - 600 + 1.07412976 - 1.96428586x$

$- 241.205357x + 1929.642856 = 0$

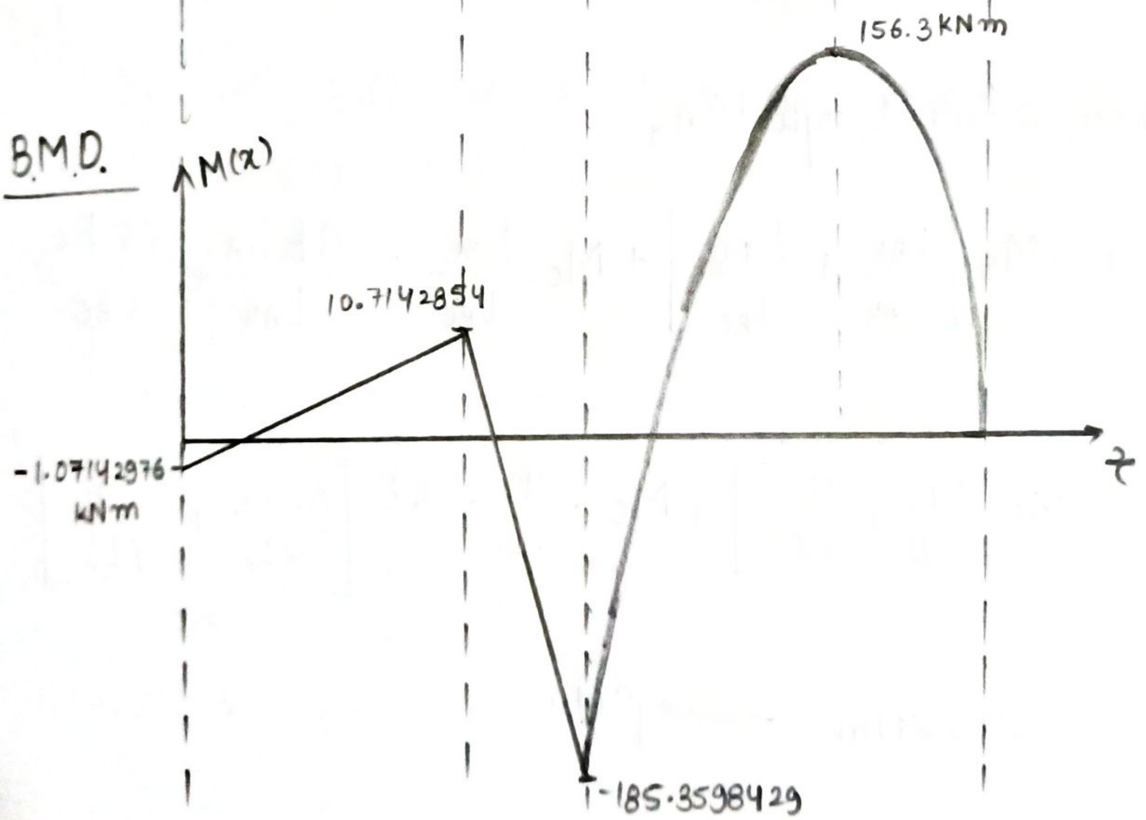
$M_x = -15x^2 + 383.1696429x - 2290.716986$


100KN
30KN/m
1.07412976
1.96428586

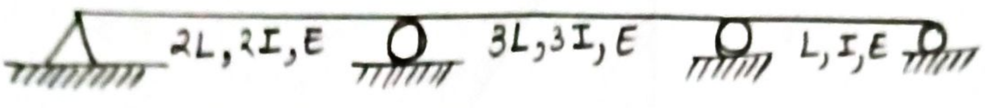
S.F.D.


V(x)
1.964285 KN
(6,0)
(8,0)
(16,0)
143.1696429
-96.8303571
-98.03571414
x

B.M.D.


M(x)
156.3 KNm
10.7142854
-1.07142976 kNm
-185.3598429
x

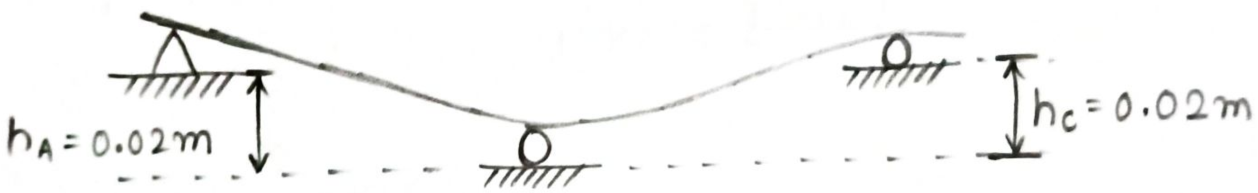
Question 3 : The fallauing continuous beam has spans of different lengths resting on four supports A, B, C and D. The structure designer supports second moment of area of cross-section to be considered in proportion to the span length as shown below. Before appling and service load it observes that B sinks by 20mm.



Solution :-

No loading on beam B.M = 0

Four Span AC



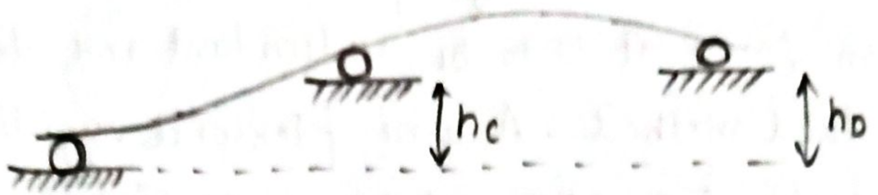
Using Three moment equation,

$$M_A \frac{L_{AB}}{I_{AB}} + 2M_B \left[\frac{L_{AB}}{I_{AB}} + \frac{L_{BC}}{I_{BC}}\right] + M_C \cdot \frac{L_{BC}}{I_{BC}} = \frac{6Eh_A}{L_{AB}} + \frac{6Eh_C}{L_{BC}}$$

$$M_A \frac{2L}{2I} + 2M_B \left[\frac{2L}{2I} + \frac{3L}{3I}\right] + M_C \cdot \frac{3L}{3I} = 6E\left[\frac{0.02}{2L} + \frac{0.02}{3L}\right]$$

$4M_B + M_C = 240\text{ KNm}$ —— $eq^n (1)$

For span BD



$$h_C = h_D = 0.02 m$$

$$M_B \cdot \frac{3L}{3I} + 2M_C\left[\frac{3L}{3I} + \frac{L}{I}\right] + M_D \frac{L}{I} = -6E\left(\frac{0.02}{3L}\right)$$

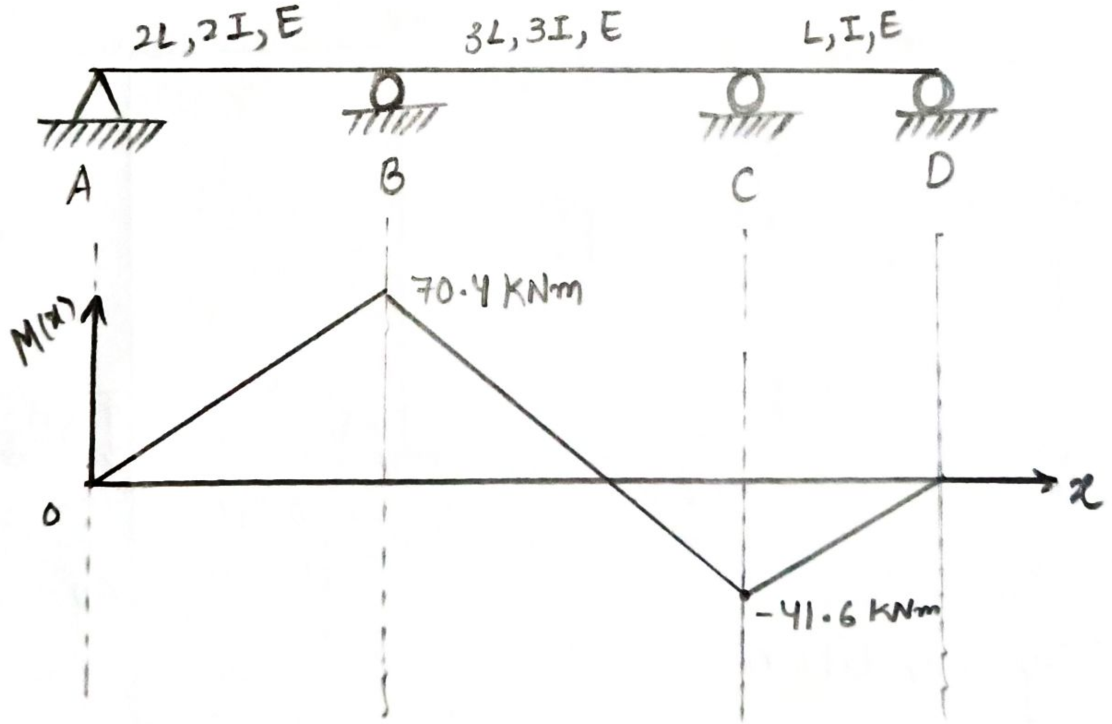
$$M_B + 4M_C = -96 \text{ KNm} \quad \text{— eq}^n \ (2)$$

On solving Both eq^n

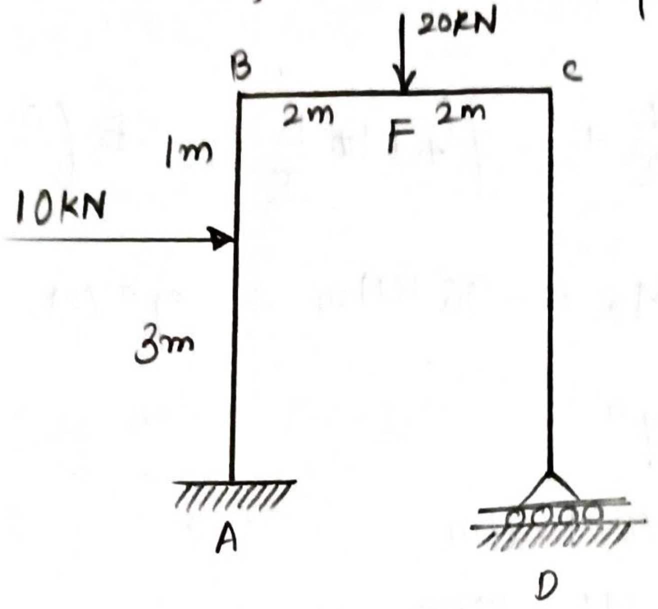
$$M_B = 70.4 \text{ KNm}$$

$$M_C = -41.6 \text{ KNm}$$

BMD of beam AD



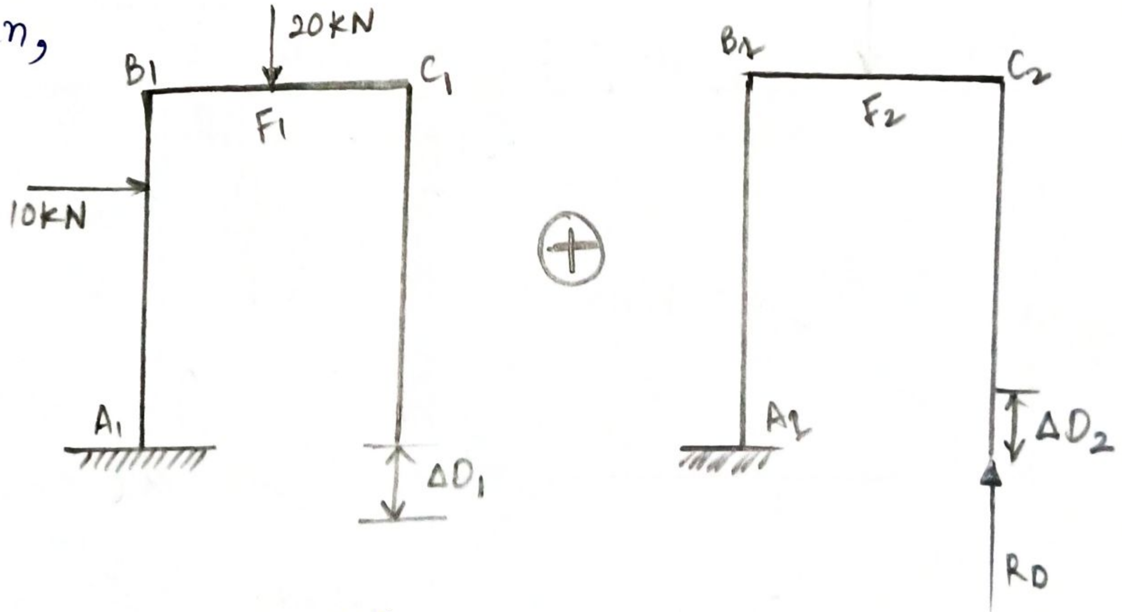
Question 4: The frame shown is indeterminate with degree of static indeterminacy equal to 1. Consider the vertical reaction force at D to be redundant and determine it using unit load method. Assume flexural rigidity of AB, BC and CD to be EI, $2EI$ and EI respectively.



Solution :-

Take R_D as redundand reaction,

Then,

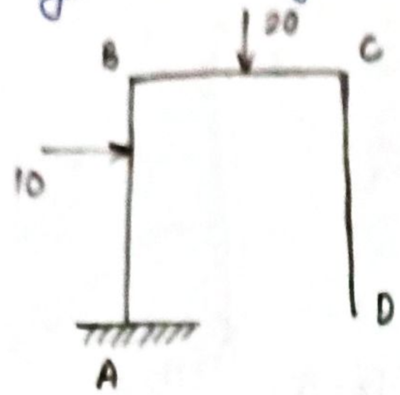


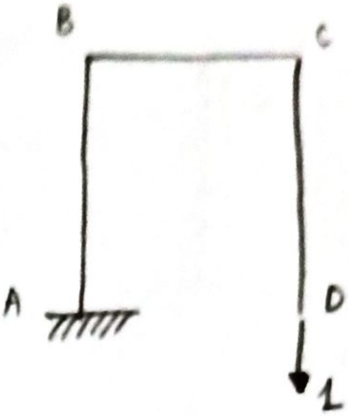
compatibility condition

$$\Delta D = 0$$

$$\Delta_{D_1} + \Delta_{D_2} = 0 \quad \text{— eq}^n \ (1)$$

Solving Δ_{D1} using unit load Method.

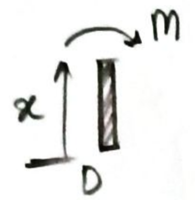


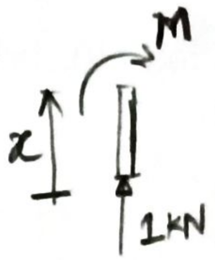


Real Beam

unit load beam

Take x from D to C $x(0,4)$





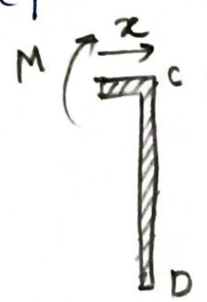
$\sum M_x = 0$

$m = 0$

$\circlearrowleft^{+} \sum M_x = 0$

$M = 0$

Take x from C to B $(0,2) \in x$



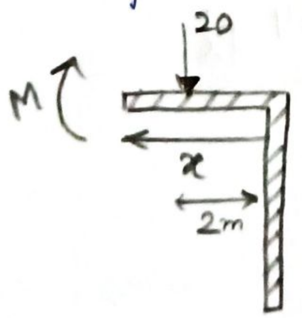
$\sum M_x = 0$

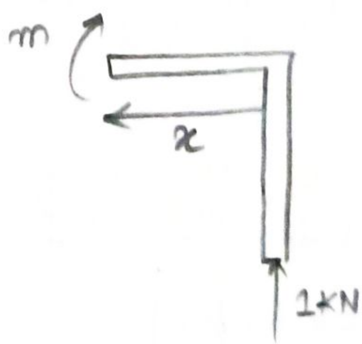
$m = x$

$\circlearrowleft^{+} \sum M_x = 0$

$M = 0$

Take x from C to between F,B





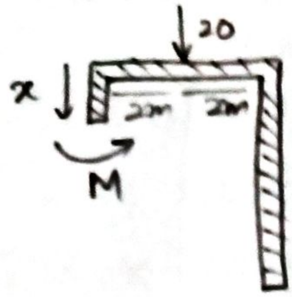
$\sum M_x = 0$

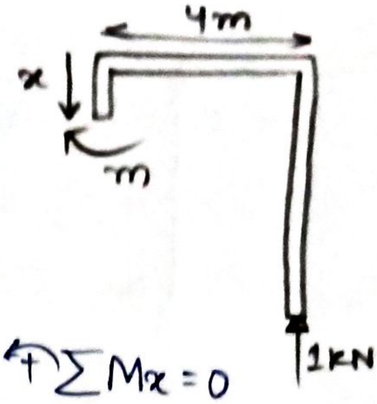
$m = x$

$\circlearrowleft^{+} \sum M_x = 0$

$M = -20(x-2)$

From B to Downward's





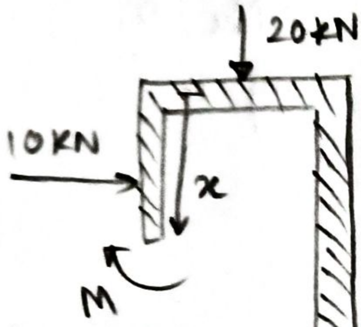
$\circlearrowleft + \sum M_x = 0$

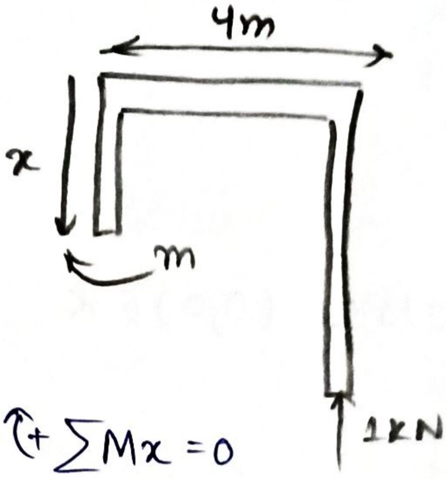
$M = -40 \text{ KNm}$

$\circlearrowleft + \sum M_x = 0$

$m = 4 \text{ KNm}$

From 10KN to Downward's





$\circlearrowleft + \sum M_x = 0$

$M = -10(x-1) - 40$

$\circlearrowleft + \sum M_x = 0$

$m = 4 \text{KNm}$

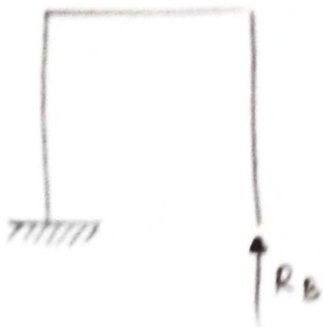
$$\text{So, } \Delta D_1 = \int_0^4 \frac{0 \times 0}{EI} dx + \int_1^2 \frac{0 \times x}{EI} dx + \int_2^4 \frac{20(2-x)x}{EI} dx$$

$$+ \int_0^4 \frac{-40 \times 4}{EI} dx + \int_1^4 \frac{(10(1-x)-40)4}{EI} dx$$

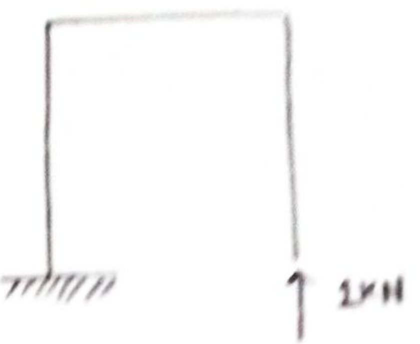
$$\Delta D_1 = \frac{-200}{3EI} - \frac{160}{EI} - \frac{660}{EI}$$

$$\boxed{\Delta_{D_1} = -\frac{886.667}{EI}}$$

Solving Δ_{D_2} using unit load method:-



* Real beam



* Virtual beam

$EI, (0,4) \quad M_x = 0 \qquad m_x = 0$

$2EI, (0,4) \quad M_x = R_D x \qquad m_x = x$

$EI, (0,4) \quad M_x = R_D 4 \qquad m_x = 4$

$$\Delta_{D_2} = \int_0^4 \frac{0 \times 0}{EI} dx + \int_0^4 \frac{R_D x \cdot x}{2EI} dx + \int_0^4 \frac{4 \cdot R_D \cdot 4}{EI} dx$$

$$\Delta_{D_2} = \left[\frac{R_D x^3}{2EI \cdot 3}\right]_0^4 + \left[\frac{R_D \cdot 16 \cdot x}{EI}\right]_0^4$$

$$\Delta_{D_2} = \frac{74.669 V_D}{EI}$$

by compatibility,

$$-\frac{886.67}{EI} + \frac{74.669 V_D}{EI} = 0 \qquad ; \ \Delta_{D_1} + \Delta_{D_2} = 0$$

* $\boxed{V_D = 11.875 \text{ KN}}$