Q4

Part 1

The confusion matrix and accuracy for cubic B-spline(implemented in Python) are shown as follows. With sklearn, NuSVC better classifies the groups than SVC, with 90% accuracy.

```
Confusion Matrix - cubic B-spline
         Predicted 1  Predicted 2  Predicted 3  Predicted 4
Class 1            5            0            0            0
Class 2            0            7            0            1
Class 3            0            0            2            0
Class 4            0            2            0           13
Accuracy:0.9
```

The confusion matrix for SVM via cubic B-spline(implemented in Matlab) are shown as follows.

```
>> Q4P1

table =

     5     0     0     0
     0     8     0     1
     0     1     1     2
     0     1     0    11
```

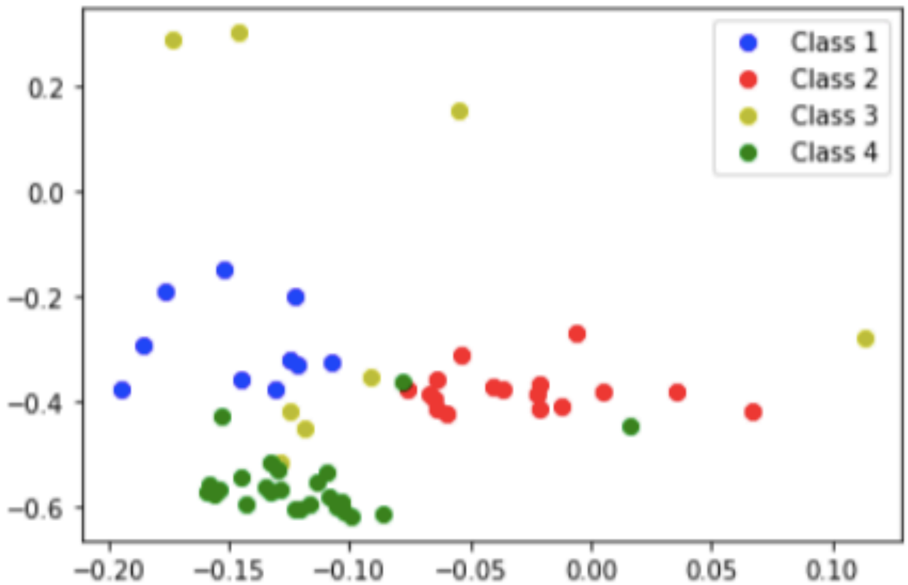
The accuracy of SVM via cubic B-spline with Matlab is 0.83.

The confusion matrix with SVC in sklearn looks:

```
Confusion Matrix - cubic B-spline
         Predicted 1  Predicted 2  Predicted 3  Predicted 4
Class 1            0            4            0            2
Class 2            0            6            0            0
Class 3            0            4            0            0
Class 4            0            1            0           13
Accuracy:0.6333333333333333
```

Part 2

The accuracy with top 2 harmonics is 0.9. The confusion matrix shown below.



Confusion Matrix - FPCA

	Predicted 1	Predicted 2	Predicted 3	Predicted 4
Class 1	4	0	0	0
Class 2	0	9	0	0
Class 3	0	0	1	2
Class 4	0	1	0	13

Accuracy:0.9

The accuracy with top 5 harmonics is 0.76. The confusion matrix shown below.

Confusion Matrix - FPCA

	Predicted 1	Predicted 2	Predicted 3	Predicted 4
Class 1	0	3	0	1
Class 2	0	9	0	0
Class 3	0	0	1	2
Class 4	0	1	0	13

Accuracy:0.7666666666666667

The accuracy with top 8 harmonics is 0.5. The confusion matrix shown below.

Confusion Matrix - FPCA

	Predicted 1	Predicted 2	Predicted 3	Predicted 4
Class 1	0	1	0	3
Class 2	0	0	0	9
Class 3	0	0	1	2
Class 4	0	0	0	14

Accuracy:0.5

The accuracy with top 10 harmonics is 0.5. The confusion matrix shown below.

```
Confusion Matrix - FPCA
         Predicted 1  Predicted 2  Predicted 3  Predicted 4
Class 1            0            1            0            3
Class 2            0            0            0            9
Class 3            0            0            1            2
Class 4            0            0            0           14
Accuracy:0.5
```

In [45]:

```python
#import modules

from scipy.ndimage import gaussian_filter
from matplotlib import cm
from sklearn.metrics import confusion_matrix
from sklearn.model_selection import train_test_split
import pandas as pd
import numpy as np
from scipy.interpolate import BSpline
from sklearn.svm import SVC
from sklearn.svm import NuSVC
from sklearn.svm import LinearSVC
import matplotlib.pyplot as plt
%matplotlib inline

#helper function

def BSplineBasis(x: np.array, knots: np.array, degree: int) -> np.arra
y:
    '''Return B-Spline basis. Python equivalent to bs in R or the spma
k/spval combination in MATLAB.
    This function acts like the R command bs(x,knots=knots,degree=degr
ee, intercept=False)
    Arguments:
        x: Points to evaluate spline on, sorted increasing
        knots: Spline knots, sorted increasing
        degree: Spline degree.
    Returns:
        B: Array of shape (x.shape[0], len(knots)+degree+1).
    Note that a spline has len(knots)+degree coefficients. However, be
cause the intercept is missing
    you will need to remove the last 2 columns. It's being kept this w
ay to retain compatibility with
    both the matlab spmak way and how R's bs works.

    If K = length(knots) (includes boundary knots)
    Mapping this to R's bs: (Props to Nate Bartlett )
    bs(x,knots,degree,intercept=T)[,2:K+degree] is same as BSplineBasi
s(x,knots,degree)[:,:-2]
    BF = bs(x,knots,degree,intercept=F) drops the first column so BF[,
1:K+degree] == BSplineBasis(x,knots,degree)[:,:-2]
```

```
    '''
    nKnots = knots.shape[0]
    lo = min(x[0], knots[0])
    hi = max(x[-1], knots[-1])
    augmented_knots = np.append(
        np.append([lo]*degree, knots), [hi]*degree)
    DOF = nKnots + degree +1 # DOF = K+M, M = degree+1
    spline = BSpline(augmented_knots, np.eye(DOF),
                     degree, extrapolate=False)
    B = spline(x)
    return B
```

In [5]:

```
#part 1
data = pd.read_csv('Question4.csv',header=None)
y_true = np.array(data.iloc[:,-1]) #of shape(60)
X = data.iloc[:,:-1].to_numpy() #of shape(60,570)
x = np.linspace(0,1,570)
knots = np.linspace(0, 1, 70)
B = BSplineBasis(x, knots, degree=3)[:,:-2]
Bcoef = np.linalg.lstsq(B, X.T)[0].T #shape(60,72) #Bcoef is the extra
cted features of 72 dims
indices = np.arange(60)
X_trg, X_tst, Y_trg, Y_tst, i_trg, i_tst = train_test_split(
    X, y_true, indices, train_size=0.5, random_state=111) #111
clf = SVC(gamma='auto',kernel='sigmoid')
clf.fit(Bcoef[i_trg,:],Y_trg)
pred = clf.predict(Bcoef[i_tst,:])
conf = confusion_matrix(Y_tst, pred)
conf = pd.DataFrame(conf, index=['Class 1', 'Class 2', 'Class 3', 'Cla
ss 4'], columns=[
                    'Predicted 1', 'Predicted 2', 'Predicted 3', 'Pred
icted 4'])
print('Confusion Matrix - cubic B-spline\n', conf)
print(f'Accuracy:{clf.score(Bcoef[i_tst,:],Y_tst)}')
```

```
Confusion Matrix - cubic B-spline
          Predicted 1  Predicted 2  Predicted 3  Predicted 4
Class 1            0            4            0            2
Class 2            0            6            0            0
Class 3            0            4            0            0
Class 4            0            1            0           13
Accuracy:0.6333333333333333
```

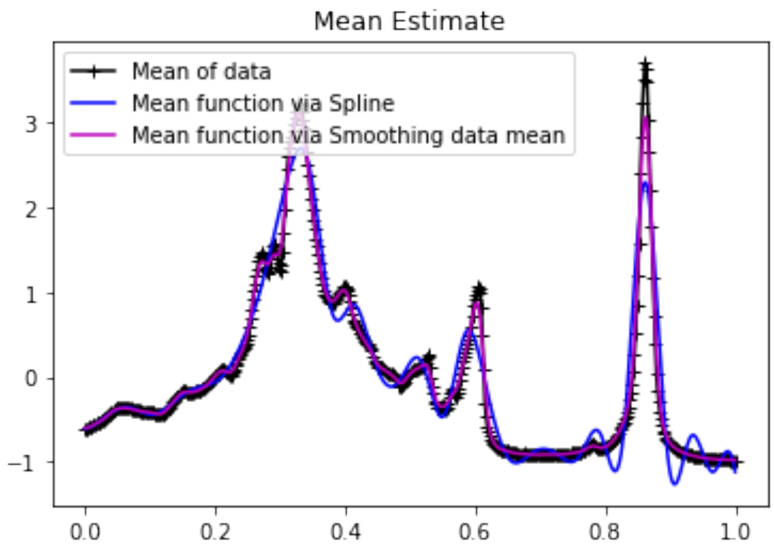
```
/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:8: Futu
reWarning: `rcond` parameter will change to the default of machine p
recision times ``max(M, N)`` where M and N are the input matrix dime
nsions.
To use the future default and silence this warning we advise to pass
`rcond=None`, to keep using the old, explicitly pass `rcond=-1`.
```

In [18]:
```
#part 1
data = pd.read_csv('Question4.csv',header=None)
y_true = np.array(data.iloc[:,-1]) #of shape(60)
X = data.iloc[:,:-1].to_numpy() #of shape(60,570)
x = np.linspace(0,1,570)
knots = np.linspace(0, 1, 70)
B = BSplineBasis(x, knots, degree=3)[:,:-2]
Bcoef = np.linalg.lstsq(B, X.T)[0].T #shape(60,72) #Bcoef is the extra
cted features of 72 dims
indices = np.arange(60)
X_trg, X_tst, Y_trg, Y_tst, i_trg, i_tst = train_test_split(
    X, y_true, indices, train_size=0.5, random_state=39) #111,29,30,39
clf = NuSVC()
clf.fit(Bcoef[i_trg,:],Y_trg)
pred = clf.predict(Bcoef[i_tst,:])
conf = confusion_matrix(Y_tst, pred)
conf = pd.DataFrame(conf, index=['Class 1', 'Class 2', 'Class 3', 'Cla
ss 4'], columns=[
                    'Predicted 1', 'Predicted 2', 'Predicted 3', 'Pred
icted 4'])
print('Confusion Matrix - cubic B-spline\n', conf)
print(f'Accuracy:{clf.score(Bcoef[i_tst,:],Y_tst)}')
```

```
Confusion Matrix - cubic B-spline
          Predicted 1  Predicted 2  Predicted 3  Predicted 4
Class 1             5            0            0            0
Class 2             0            7            0            1
Class 3             0            0            2            0
Class 4             0            2            0           13
Accuracy:0.9

/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:8: Futu
reWarning: `rcond` parameter will change to the default of machine p
recision times ``max(M, N)`` where M and N are the input matrix dime
nsions.
To use the future default and silence this warning we advise to pass
`rcond=None`, to keep using the old, explicitly pass `rcond=-1`.
```

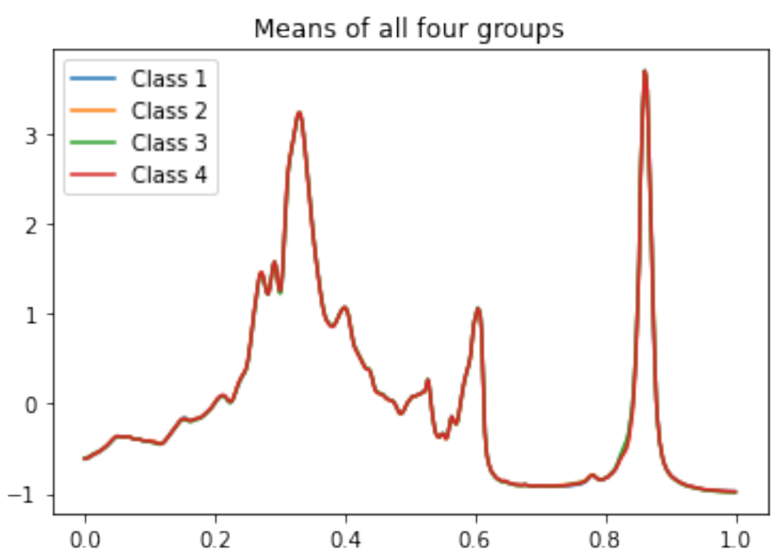
In [20]:
```
#part 2
#fpca for feature extraction

data = pd.read_csv('Question4.csv',header=None)
y_true = np.array(data.iloc[:,-1]) #of shape(60)
X = data.iloc[:,:-1].to_numpy() #of shape(60,570)
indices = np.arange(60)
g1 = X[y_true==1,:]
g2 = X[y_true==2,:]
g3 = X[y_true==3,:]
g4 = X[y_true==4,:]
m1= g1.mean(0)
m2= g2.mean(0)
m3= g3.mean(0)
m4= g4.mean(0)
X_trg, X_tst, Y_trg, Y_tst, i_trg, i_tst = train_test_split(
    X, y_true, indices, train_size=0.5, random_state=123)

x = np.linspace(0,1,570) #explanatory variables x
knots = np.linspace(0, 1, 30)
B = BSplineBasis(x, knots, degree=3)[:,:-2] #basis matrix B of shape(5
70,10)

#step 1
#estimate mean function via one dimensional kernel regression
B_stacked = np.tile(B.T, 60).T
X_stacked = X.ravel()
beta = np.linalg.lstsq(B_stacked, X_stacked)[0]
mu_hat = B.dot(beta)  # Mean function via B-Spline #of shape(570,1)
mu_hat2 = gaussian_filter(X.mean(0),3) #3 is the standard deviation fo
r gaussian kernel

plt.plot(x, X.mean(0), 'k+-', label='Mean of data')
plt.plot(x, mu_hat, 'b', label='Mean function via Spline')
plt.plot(x, mu_hat2, 'm', label='Mean function via Smoothing data mean
')
plt.legend()
plt.title("Mean Estimate")
plt.show()
```

```
/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:27: Fut
ureWarning: `rcond` parameter will change to the default of machine
precision times ``max(M, N)`` where M and N are the input matrix dim
ensions.
To use the future default and silence this warning we advise to pass
`rcond=None`, to keep using the old, explicitly pass `rcond=-1`.
```

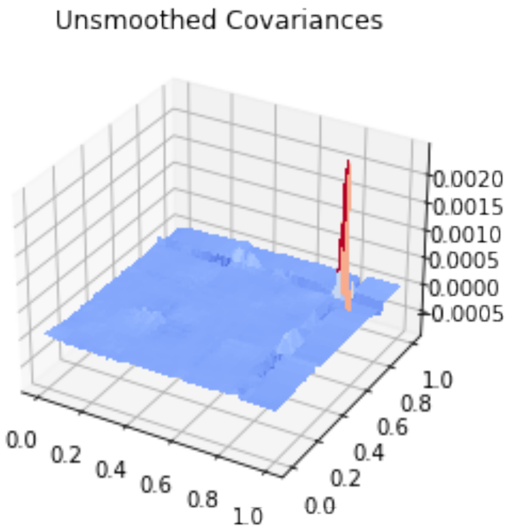


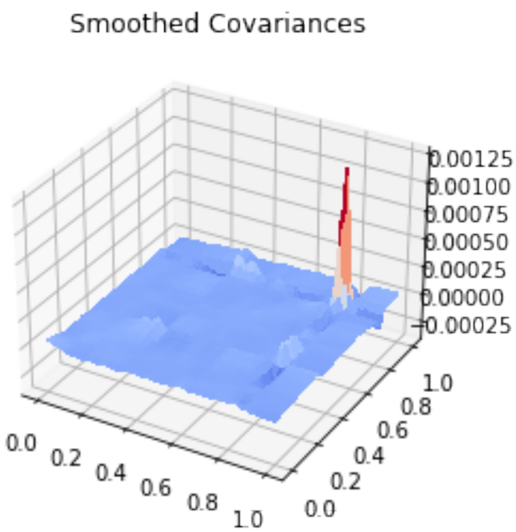
In [21]:

```
plt.plot(x, m1,  label='Class 1')
plt.plot(x, m2,  label='Class 2')
plt.plot(x, m3,  label='Class 3')
plt.plot(x, m4,  label='Class 4')
plt.legend()
plt.title("Means of all four groups")
plt.show()
```



In [22]:
```
#step 2
#estimate a smoothed functional covariance surface via two dimensional
kernel regression

diffs = X-mu_hat #of shape(60,570)
Cov = np.cov(diffs.T) #np.cov takes input with shape(num of variables,
num of observations)
grids = np.meshgrid(x, x)
fig = plt.figure()
ax = fig.gca(projection='3d')
ax.plot_surface(grids[0], grids[1], Cov, cmap=cm.coolwarm,
                linewidth=0, antialiased=False)
plt.title('Unsmoothed Covariances')
plt.show()
# additional Cov smoothing:
Cov = gaussian_filter(Cov, sigma=7)
fig = plt.figure()
ax = fig.gca(projection='3d')
ax.plot_surface(grids[0], grids[1], Cov, cmap=cm.coolwarm,
                linewidth=0, antialiased=False)
plt.title('Smoothed Covariances')
plt.show()
```

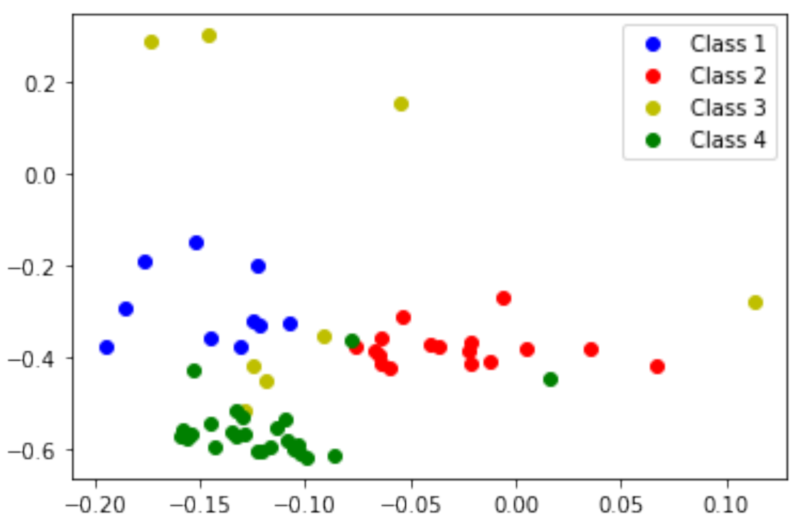

Unsmoothed Covariances
0.0020
0.0015
0.0010
0.0005
0.0000
−0.0005
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.2
0.4
0.6
0.8
1.0


Smoothed Covariances
0.00125
0.00100
0.00075
0.00050
0.00025
0.00000
−0.00025
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.2
0.4
0.6
0.8
1.0

In [23]:
```
#step 3
#solve eigen-functions

l, psi = np.linalg.eigh(Cov) #eigen-values and eigen-functions in asce
nding order
PC2 = psi[:, -2]
PC5 = psi[:, -5]
PC8 = psi[:, -8]
PC10 = psi[:, -10]
PCs = np.column_stack([PC2,PC5,PC8,PC10])
FPC_scores = diffs.dot(PCs) #principal components of the original data
matrix

clf = SVC()
clf.fit(FPC_scores[i_trg, :], Y_trg)
pred = clf.predict(FPC_scores[i_tst, :])
conf = confusion_matrix(Y_tst, pred)
conf = pd.DataFrame(conf, index=['Class 1', 'Class 2', 'Class 3', 'Cla
ss 4'], columns=[
                    'Predicted 1', 'Predicted 2', 'Predicted 3', 'Pred
icted 4'])
print('Confusion Matrix - FPCA\n', conf)
print(f'Accuracy:{clf.score(FPC_scores[i_tst, :],Y_tst)}')
```

```
Confusion Matrix - FPCA
          Predicted 1  Predicted 2  Predicted 3  Predicted 4
Class 1             0            0            0            4
Class 2             0            9            0            0
Class 3             0            1            0            2
Class 4             0            1            0           13
Accuracy:0.7333333333333333
```

In [27]:
```
PCs = psi[:, -2:]
FPC_scores = diffs.dot(PCs) #principal components of the original data matrix
plt.plot(FPC_scores[y_true==1,0],FPC_scores[y_true==1,1],'bo',label='Class 1')
plt.plot(FPC_scores[y_true==2,0],FPC_scores[y_true==2,1],'ro',label='Class 2')
plt.plot(FPC_scores[y_true==3,0],FPC_scores[y_true==3,1],'yo',label='Class 3')
plt.plot(FPC_scores[y_true==4,0],FPC_scores[y_true==4,1],'go',label='Class 4')
plt.legend()
plt.show()
clf = SVC()
clf.fit(FPC_scores[i_trg,:], Y_trg)
pred = clf.predict(FPC_scores[i_tst,:])
conf = confusion_matrix(Y_tst, pred)
conf = pd.DataFrame(conf, index=['Class 1', 'Class 2', 'Class 3', 'Class 4'], columns=[
                    'Predicted 1', 'Predicted 2', 'Predicted 3', 'Predicted 4'])
print('Confusion Matrix - FPCA\n', conf)
print(f'Accuracy:{clf.score(FPC_scores[i_tst, :],Y_tst)}')
```




```
Confusion Matrix - FPCA
          Predicted 1  Predicted 2  Predicted 3  Predicted 4
Class 1            4            0            0            0
Class 2            0            9            0            0
Class 3            0            0            1            2
Class 4            0            1            0           13
Accuracy:0.9
```

In [43]:
```
PCs = psi[:, -5:]
FPC_scores = diffs.dot(PCs) #principal components of the original data matrix

clf = SVC()
clf.fit(FPC_scores[i_trg,:], Y_trg)
pred = clf.predict(FPC_scores[i_tst,:])
conf = confusion_matrix(Y_tst, pred)
conf = pd.DataFrame(conf, index=['Class 1', 'Class 2', 'Class 3', 'Class 4'], columns=[
                    'Predicted 1', 'Predicted 2', 'Predicted 3', 'Predicted 4'])
print('Confusion Matrix - FPCA\n', conf)
print(f'Accuracy:{clf.score(FPC_scores[i_tst, :],Y_tst)}')
```

```
Confusion Matrix - FPCA
          Predicted 1  Predicted 2  Predicted 3  Predicted 4
Class 1            0            3            0            1
Class 2            0            9            0            0
Class 3            0            0            1            2
Class 4            0            1            0           13
Accuracy:0.7666666666666667
```

In [34]:
```
PCs = psi[:, -8:]
FPC_scores = diffs.dot(PCs) #principal components of the original data matrix

clf = SVC()
clf.fit(FPC_scores[i_trg,:], Y_trg)
pred = clf.predict(FPC_scores[i_tst,:])
conf = confusion_matrix(Y_tst, pred)
conf = pd.DataFrame(conf, index=['Class 1', 'Class 2', 'Class 3', 'Class 4'], columns=[
                    'Predicted 1', 'Predicted 2', 'Predicted 3', 'Predicted 4'])
print('Confusion Matrix - FPCA\n', conf)
print(f'Accuracy:{clf.score(FPC_scores[i_tst, :],Y_tst)}')
```

```
Confusion Matrix - FPCA
          Predicted 1  Predicted 2  Predicted 3  Predicted 4
Class 1            0            1            0            3
Class 2            0            0            0            9
Class 3            0            0            1            2
Class 4            0            0            0           14
Accuracy:0.5
```

In [44]:
```
PCs = psi[:, -10:]
FPC_scores = diffs.dot(PCs) #principal components of the original data matrix

clf = SVC()
clf.fit(FPC_scores[i_trg,:], Y_trg)
pred = clf.predict(FPC_scores[i_tst,:])
conf = confusion_matrix(Y_tst, pred)
conf = pd.DataFrame(conf, index=['Class 1', 'Class 2', 'Class 3', 'Class 4'], columns=[
                    'Predicted 1', 'Predicted 2', 'Predicted 3', 'Predicted 4'])
print('Confusion Matrix - FPCA\n', conf)
print(f'Accuracy:{clf.score(FPC_scores[i_tst, :],Y_tst)}')
```

```
Confusion Matrix - FPCA
          Predicted 1  Predicted 2  Predicted 3  Predicted 4
Class 1             0            1            0            3
Class 2             0            0            0            9
Class 3             0            0            1            2
Class 4             0            0            0           14
Accuracy:0.5
```

In [ ]: