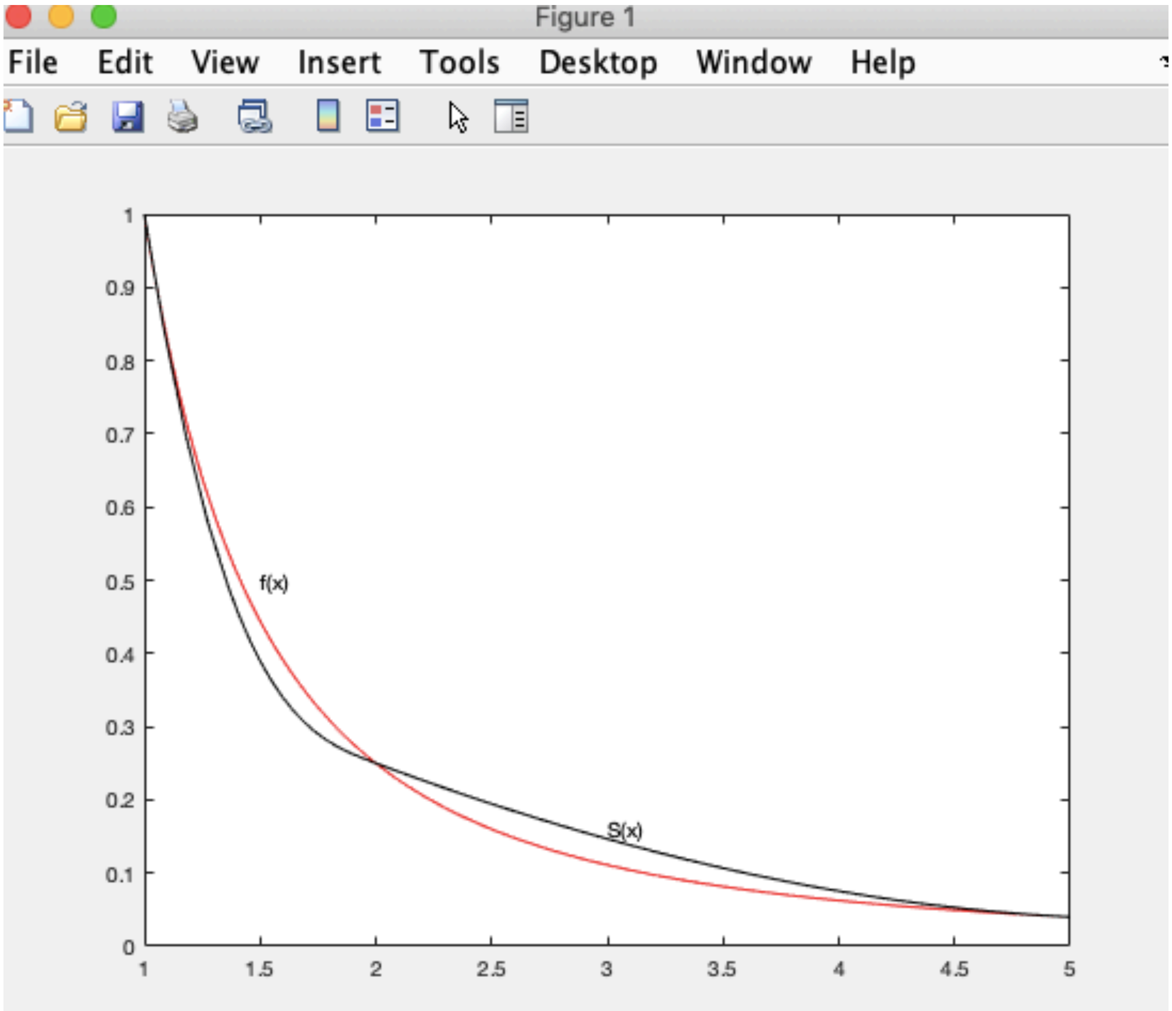
Q1

The plot of f and S for x=1:5 are shown as follows. The red line is the true function f, and the black line is the natural cubic spline S. The code is uploaded separately.



a1, a2, b1, b2, c1, c2, d1, d2 are as follow.

$$\Rightarrow \begin{cases} a_1 = 1 \\ a_2 = \frac{1}{4} = 0.25 \\ b_1 = -2 \\ b_2 = -0.118 \\ c_1 = 1.868 \\ c_2 = 0.014 \\ d_1 = -0.618 \\ d_2 = 6.667 \times 10^{-4} \end{cases}$$

The discussions and calculations are shown below.

1: Since S interpolates f at knots 1, 2, 5

$$\therefore \begin{cases} S_1(2) = f(2) \\ S_1(1) = f(1) \\ S_2(5) = f(5) \end{cases}$$

where $S_1(x) = a_1 + b_1(x-1) + c_1(x-1)^2 + d_1(x-1)^3$

$S_2(x) = a_2 + b_2(x-2) + c_2(x-2)^2 + d_2(x-2)^3$

Since S is order-4 spline, we have to have continuous derivatives up to order 2.

$\{ S_1(2) = S_2(2)$

$$\begin{cases} S_1'(2) = S_2'(2) \\ S_1''(2) = S_2''(2) \end{cases}$$

Again, to enforce continuity at boundary knots 1 and 5, we have

$$\begin{cases} S_1'(1) = f'(1) \\ S_2'(5) = f'(5) \end{cases}$$

$$\Rightarrow \begin{cases} \boxed{\begin{matrix} a_1 = 1 \\ a_2 = \frac{1}{4} \end{matrix}} \\ S_1'(x) = b_1 + 2c_1(x-1) + 3d_1(x-1)^2 \Rightarrow S_1'(1) = \boxed{b_1 = f'(1) = -2} \\ S_2'(x) = b_2 + 2c_2(x-2) + 3d_2(x-2)^2 \Rightarrow S_2'(2) = b_2 = S_1'(2) = 2c_1 + 3d_1 - 2 \\ S_1(2) = c_1 + d_1 - 1 = S_2(2) = \frac{1}{4} \end{cases}$$

$$\Rightarrow \boxed{\begin{cases} c_1 + d_1 = \frac{5}{4} \\ 2c_1 + 3d_1 = b_2 + 2 \end{cases}}$$

$$\begin{cases} S_1''(x) = 2c_1 + 6d_1(x-1) \Rightarrow S_1''(2) \boxed{= 2c_1 + 6d_1 = S_2''(2) = 2c_2} \\ S_2'(5) = \boxed{b_2 + 6c_2 + 27d_2 = f'(5) = \frac{-2}{125}} \\ S_2(5) = f(5) \Rightarrow 3b_2 + 9c_2 + 27d_2 = \frac{-2}{100} \\ \Rightarrow \boxed{b_2 + 3c_2 + 9d_2 = -\frac{7}{100}} \end{cases}$$

$$\Rightarrow \begin{cases} a_1 = 1 \\ a_2 = \frac{1}{4} = 0.25 \\ b_1 = -2 \\ b_2 = -0.118 \\ c_1 = 1.868 \\ c_2 = 0.014 \\ d_1 = -0.618 \\ d_2 = 6.667 \times 10^{-4} \end{cases}$$

In [ ]: