Question 1: Image Denoising (40 points)

Read and display image "theAlps.jpg".

a) Convert the image into grayscale.
b) Define two new images $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$ by adding noise to the gray scale image.
   1- Zero-mean Gaussian white noise with variance of 0.01
   2- Salt-and-pepper noise, affecting approximately 5% of pixels. This noise can be caused by sharp and sudden disturbances in the image signal. It presents itself as sparsely occurring white and black pixels.
c) Apply the following denoising techniques to $\boldsymbol{J}_1$and $\boldsymbol{J}_2$ and display the results. You can consider kernel size 3× 3.
   1- Gaussian filter $h_g = \frac{1}{2\pi\sigma^2} exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$ and $\sigma = 2$.
   2- Median filtering. The median filtering method sorts the intensities in the $m \times n$ neighborhood window of the reference pixel and calculates the median value of the sorted data. The original value at the reference pixel is then replaced by the median value. Figure 1. illustrates an example calculation.

123	125	126	130	140
122	124	126	127	135
118	120	150	125	134
119	115	119	123	133
111	116	110	120	130

Neighborhood values are:

115,119, 120, 123, **124**, 125, 126, 127, 150

Median is 124

Figure 1: Median value of a local pixel neighborhood in a 3×3 window mask.

   3- Arithmetic mean filter.
   4- Geometric mean filter. It is based on the mathematic geometric mean. The output image $\hat{f}(x, y)$ of a geometric mean is given by

$$\hat{f}(x, y) = \left[\prod_{(r,c)\in S_{xy}} g(r, c)\right]^{1/mn}$$

   S_{xy} represents the set of coordinates in a rectangular subimage window (neighborhood) of size $m \times n$, centered on point (x, y). Each restored pixel is given by the product of all the pixels in the subimage area, raised to the power $1/_{mn}$.
   5- Harmonic mean filter. The output image $\hat{f}(x, y)$ of a harmonic mean is given by

$$\hat{f}(x,y) = \frac{mn}{\sum_{(r,c)\in S_{xy}} \frac{1}{g(r,c)}}$$

6- Contraharmonic mean filter with $Q = -1, 0$ and 1.

$$\hat{f}(x,y) = \frac{\sum_{(r,c)\in S_{xy}} g(r,c)^{Q+1}}{\sum_{(r,c)\in S_{xy}} g(r,c)^{Q}}$$

Q is called the order of the filter.

7- Minimum filter

$$\hat{f}(x,y) = \min_{(r,c)\in S_{xy}} \{g(r,c)\}$$

8- Maximum filter

$$\hat{f}(x,y) = \max_{(r,c)\in S_{xy}} \{g(r,c)\}$$

9- Midpoint filter. The midpoint filter computes the midpoint between the maximum and minimum values in the area encompassed by the filter:

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(r,c)\in S_{xy}} \{g(r,c)\} + \min_{(r,c)\in S_{xy}} \{g(r,c)\}\right]$$

d) What denoising technique do you recommend for removing Gaussian noise? What denoising technique do you recommend for removing salt-and-pepper noise?

Question 2: Edge detection (30 points)

Read image "Eagle.jpg" and convert the image into grayscale and display it. In this question, we compare the following edge detection techniques:

1- Compute the first partial derivatives of the image in the x and y directions using Sobel operator. Using these derivatives, compute the gradient magnitude of the image. Show how thresholding can affect the gradient image.

2- Compute the first partial derivatives of the image in the x and y directions using Prewitt operator. Using these derivatives, compute the gradient magnitude of the image. Show how thresholding can affect the gradient image.

3- Detect edges using Laplacian of Gaussian (LoG). Compute $\nabla^2 G$ for the function $G(x,y) = \frac{1}{2\pi\sigma^2}\, exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$ and $\sigma = 1$. Construct a 5×5 LoG kernel and convolve it with the image. (Hint: $\nabla^2 G = \frac{\partial^2 G(x,y)}{\partial x^2} + \frac{\partial^2 G(x,y)}{\partial y^2}$)

4- Detect edges using extended Difference of Gaussians (DoG) operator. DoG is a bandpass filter that attenuates all frequencies between the cut-off frequencies of the two Gaussians. DoG is defined as:

$$D(\sigma,k,\gamma) = G(\sigma) - \gamma\, G(k\sigma)$$

Display output image $T_{\varepsilon,\varphi}(D(\sigma,k,\gamma) * \text{Image})$.

$$T_{\varepsilon,\varphi}(u) = \begin{cases} 1 & u \geq \varepsilon \\ 1 + \tanh\big(\varphi\,(u - \varepsilon)\big) & \text{otherwise} \end{cases}$$

Where ‘*’ denotes convolution, $k = 1.6$, $\varepsilon = -0.1$, $\gamma = 0.98$, $\varphi = 200$ and $\sigma = 0.8$.

Question 3: Image Segmentation (30 points)

Read and display “RoseonIce.jpg”.

1- Use K-means clustering to segment the image into 2, 3, 4 and 5 clusters.
2- Convert the image into grayscale and plot the histogram.
3- Use Otsu’s method to determine the optimal threshold to convert the image into black and white. Report the optimal threshold value.
4- Report the images obtained and the corresponding histograms. Perform log-transformation: use $c = 40$. Display the image and its histogram.
5- Perform power-law transformation: use $c = 0.1$ and γ= 1.4. Display the image and its histogram.