Question 1: (25 points)

Suppose that A is a 256×512 dimensional. Let $x \epsilon \mathbb{R}^{512}$ be an S-sparse vector and $y = Ax$ be the observed system output. Use Orthogonal Matching Pursuit (OMP) algorithm for recovering the sparse signal. Let S range between 1 and 150. For each choice of S, run 10 independent numerical experiments: in each experiment, generate A such that its elements are i.i.d standard Gaussian random variables. Normalize columns of A. Generate x as a random S-sparse signal (e.g. you may generate the support of uniformly at random, with each non-zero entry drawn from the standard Gaussian distribution). An experiment is claimed successful if the solution returned by OMP obeys $\frac{\|x-\hat{x}\|_2}{\|x\|_2} < 10^{-3}$ where $\hat{x}$ is the estimate of x obtained from OMP. Report the empirical success rates (averaged over 10 experiments) for each choice of S.

Question 2: (30 points)

Solve the following optimization problem using augmented Langrangian multiplier:

$$\min_X \|X\|_* \qquad \text{subjet to} \qquad X + E = M \ , \quad P_\Omega(E) = 0$$

1- Write a pseudo code for your algorithm

Complete Algorithm: .

Input: observation samples $M_{ij}, (i,j) \in \Omega$ of matrix $M \in \mathbb{R}^{m \times n}$

Initialize: $E_0 = 0, \ Y_0 = 0, \ \ \rho = 1.1, \ \ \mu = 10^{-4}$

Output: $X \in \mathbb{R}^{m \times n}$

2- "Question2.jpg" contains matrix M and "Question2.mat" contains Ω. Code your algorithm and display output X. Plot objective function $\|X\|_*$ versus iteration.

Question 3: (30 points)

Consider the robust PCA problem

$$\min_{L,S} \ \mu\|L\|_* + \lambda\mu|S|_1 + \frac{1}{2}\|M - L - S\|_2^2 \qquad (1)$$

1- Solve (1) using proximal gradient method. Consider the gradient step size $\frac{1}{t} = 0.5$. Write the pseudo code.

Algorithm: .

Input: M, $\lambda = 0.005$, $t = 2$

Initialize: $L_0 = 0$, $S_0 = 0$, $\mu_0 = 0.99\|M\|_2$, $\bar{\mu} = 10^{-9}\|M\|_2$

while not converged

write your equation for L_k update

write your equation for S_k update

$\mu_k = \max(0.99\mu_{k-1}, \bar{\mu})$

Output: L and S

2- Use robust PCA to perform background/foreground separation. The background contains the static objects and is well modeled as a low rank matrix. Foreground contains the moving objects and is modeled as a sparse matrix. “Question3.mat” contains 500 video frames of 120×160 size. Each column of the data matrix represents a video frame. Perform background/fore ground subtraction and display your results for frames 120, 220, 260, 430.

Question 4: (15 points)

In this question, you use kernel ridge regression to perform energy efficiency analysis. “Question4.csv” contains 768 data samples. The first 8 columns are the features, and the last column is the response variable. The first 600 data samples are for training and the remaining data samples are for the test. In this question, we use polynomial kernel $k(x, y) = (x^T y)^3$. First, you need to set the free parameter λ using your training data - you should not allow the testing dataset to influence your choice of λ. Randomly split your training data into 80% training and 20% validation to tune λ. Report the optimal λ and the corresponding MSE. Evaluate the performance of your kernel in terms of mean-squared error on the test set.