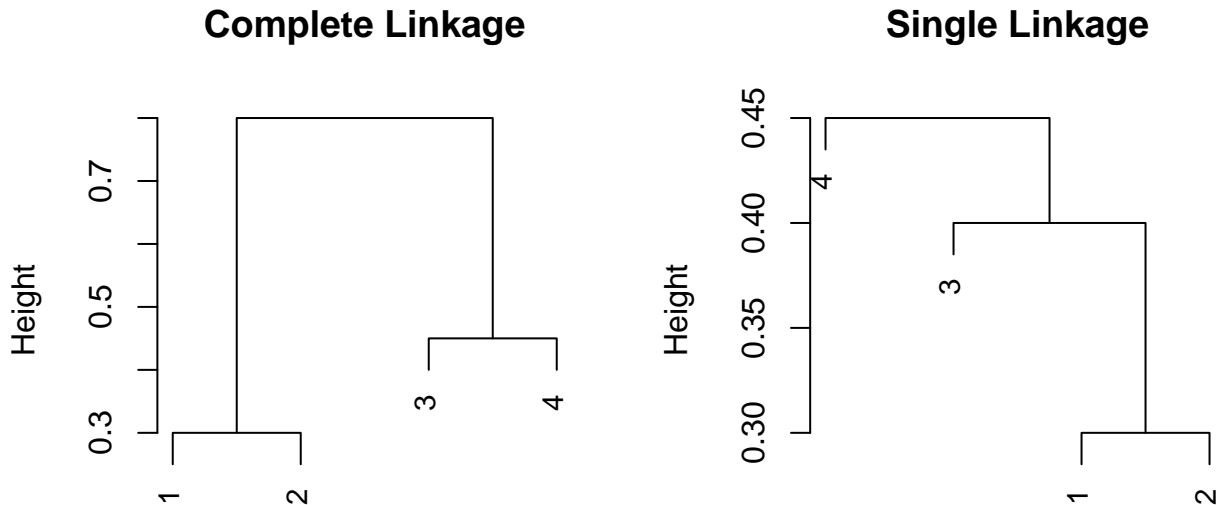


# STATS 415 HW11

Wenfei Yan (wenfeiy). GSI: April Cho.

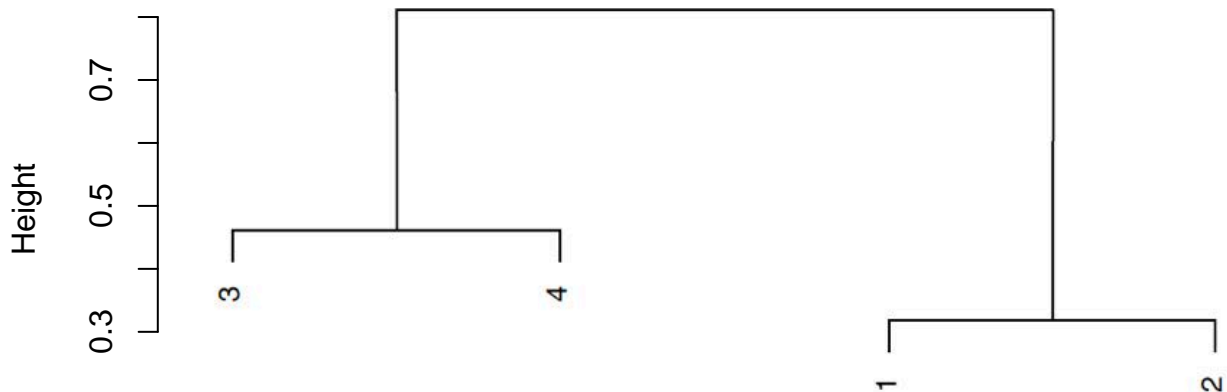
1. (a)(b) Denote the four observations by 1, 2, 3, 4 from left to right in the dissimilarity matrix. The dendrograms using complete linkage and single linkage are shown in the following graph.

```
distance = matrix(c(0, 0.3, 0.4, 0.7, 0.3, 0, 0.5, 0.8, 0.4, 0.5, 0, 0.45, 0.7, 0.8, 0.45, 0), nrow = 4)
hc.complete=hclust(as.dist(distance), method="complete")
hc.single=hclust(as.dist(distance), method="single")
par(mfrow=c(1,2))
plot(hc.complete,main="Complete Linkage", xlab="", sub="", cex=.9)
plot(hc.single, main="Single Linkage", xlab="", sub="", cex=.9)
```



- (c) One cluster is (1, 2), and the other is (3, 4).  
 (d) One cluster is (1, 2, 3), and the other is (4).

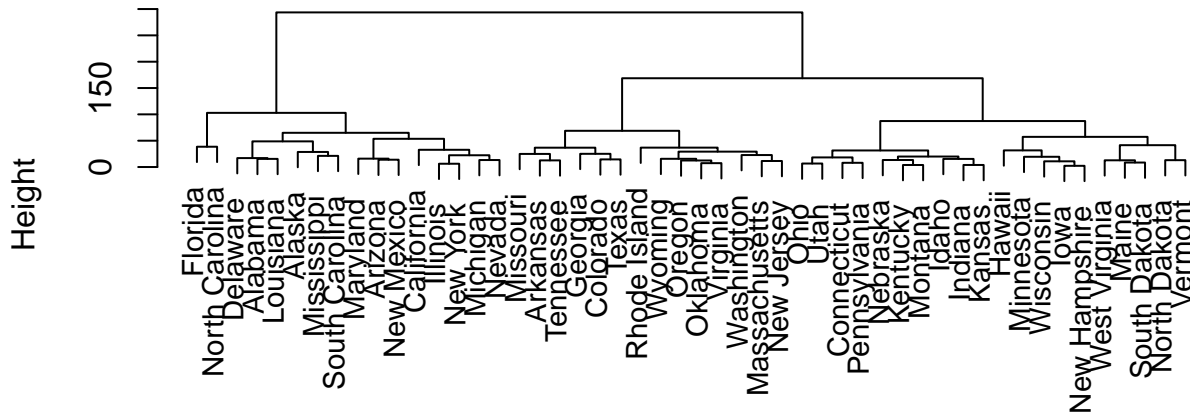
## Complete Linkage



- (e)  
 2. (a)

```
library(ISLR)
X = as.matrix(USArrests)
hc.complete=hclust(dist(X), method="complete")
plot(hc.complete, xlab="", sub="", cex=.9)
```

## Cluster Dendrogram



(b)

```
hc.clusters.complete = cutree(hc.complete,3)
table(hc.clusters.complete, rownames(USArrests))
```

Cluster 1: Alabama, Alaska, Arizona, California, Delaware, Florida, Illinois, Louisiana, Maryland, Michigan, Mississippi, Nevada, New Mexico, New York, North Carolina, South Carolina.

Cluster 2: Arkansas, Colorado, Georgia, Massachusetts, Missouri, New Jersey, Oklahoma, Oregon, Rhode Island, Tennessee, Texas, Virginia, Washington, Wyoming.

Cluster 3: Connecticut, Hawaii, Idaho, Indiana, Iowa, Kansas, Kentucky, Maine, Minnesota, Montana, Nebraska, New Hampshire, North Dakota, Ohio, Pennsylvania, South Dakota, Utah, Vermont, West Virginia, Wisconsin.

```
library(cluster)
plot(silhouette(hc.clusters.complete, dist = dist(X)))
```

## Silhouette plot of (x = hc.clusters.complete, dist = dist(X))

n = 50

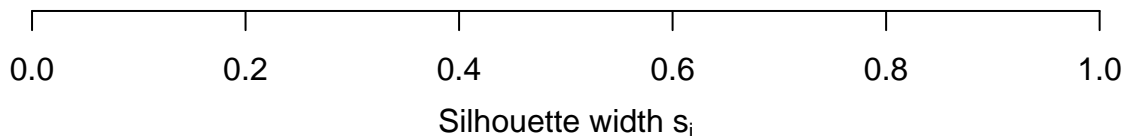
3 clusters  $C_j$

$j : n_j \mid \text{ave}_{i \in C_j} s_i$

1 : 16 | 0.54

2 : 14 | 0.54

3 : 20 | 0.52

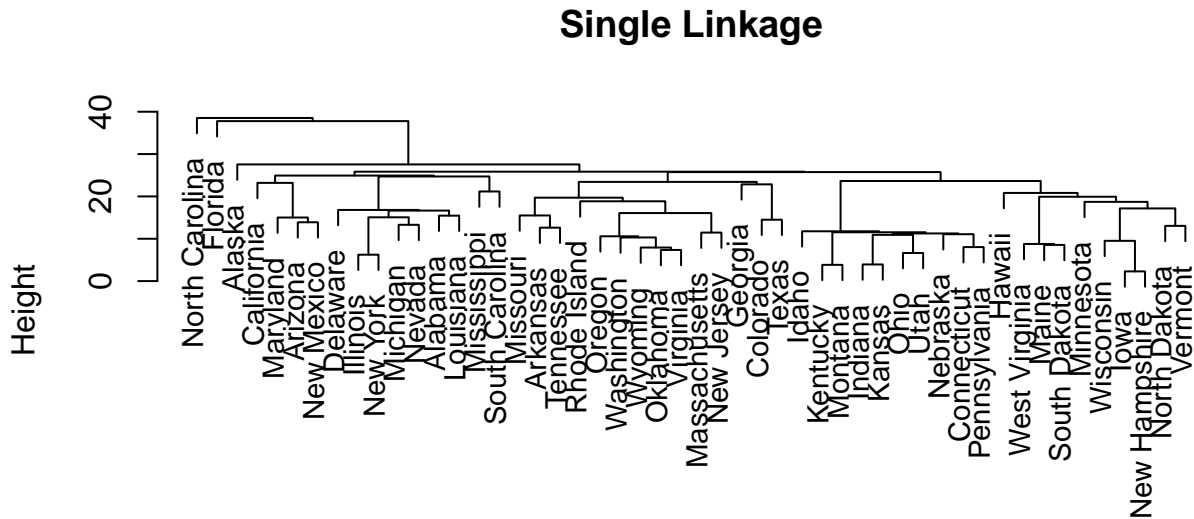


Average silhouette width : 0.53

The silhouette widths of each cluster are very close and the sizes of each cluster are also similar, which means the qualities of each cluster are similar. Also there's no point with negative silhouette coefficients, which means the clustering is pretty good in terms of the silhouette measure.

(c)

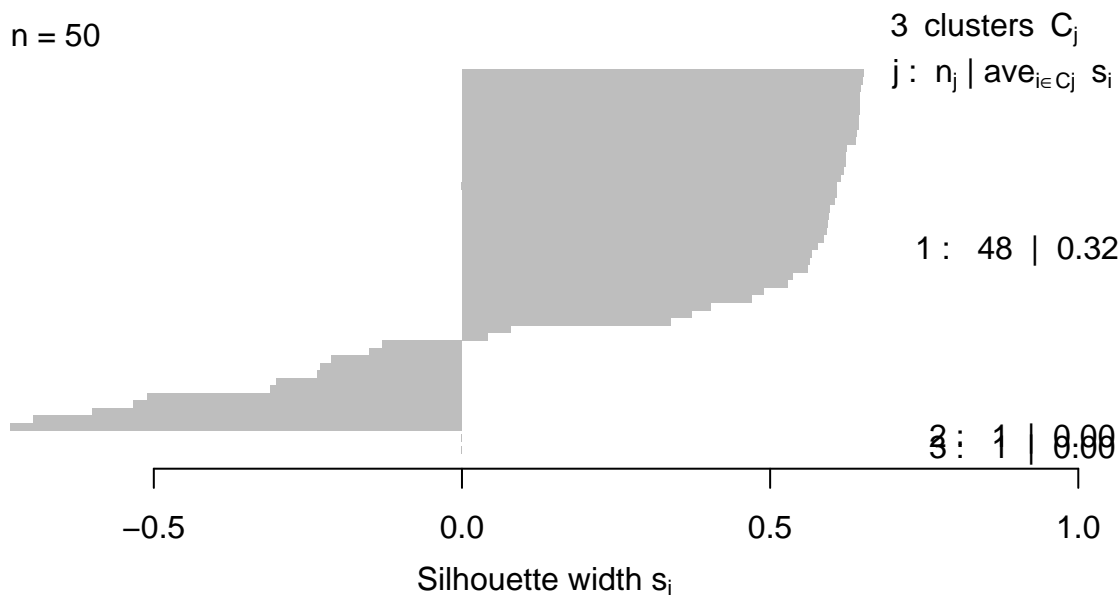
```
hc.single=hclust(dist(X), method="single")
plot(hc.single,main="Single Linkage", xlab="", sub="", cex=.9)
```



```
plot(silhouette(cutree(hc.single, 3), dist = dist(X)))
```

### Silhouette plot of (x = cutree(hc.single, 3), dist = dist(X))

n = 50



Average silhouette width : 0.3

```
table(cutree(hc.single, 3), rownames(USArrests))
```

Cluster 1: Alabama, Alaska, Arizona, California, Delaware, Illinois, Louisiana, Maryland, Michigan, Mississippi, Nevada, New Mexico, New York, South Carolina, Arkansas, Colorado, Georgia, Massachusetts, Missouri, New Jersey, Oklahoma, Oregon, Rhode Island, Tennessee, Texas, Virginia, Washington, Wyoming, Connecticut, Hawaii, Idaho, Indiana, Iowa, Kansas, Kentucky, Maine, Minnesota, Montana, Nebraska, New Hampshire, North Dakota, Ohio, Pennsylvania, South Dakota, Utah, Vermont, West Virginia, Wisconsin. Cluster 2: Florida. Cluster 3: North Carolina.

The clusters are extremely skewed. Most points are in cluster 1, and there's only one in cluster 2 and another one in cluster 3. The clustering result with single linkage is not very ideal.

(d)

```
set.seed(1111)
km.out=kmeans(dist(X), 3, nstart=20)
km.clusters=km.out$cluster
table(km.clusters, hc.clusters.complete)
```

```
##          hc.clusters.complete
## km.clusters  1  2  3
##          1 16  0  0
##          2  0 14  0
##          3  0  0 20
```

Here I assign `nstart=20` to use multiple random assignments for initial clusters, and the `kmeans()` function will report only the best result with smallest within cluster sum of squares. The clustering results of the k-means clustering and the hierarchical clustering with complete linkage are exactly the same, so I omit the result of “reporting which states belong to which clusters is”.

```
plot(silhouette(km.clusters, dist(X)), main="Silhouette plot from K-means")
```

## Silhouette plot from K-means

$n = 50$

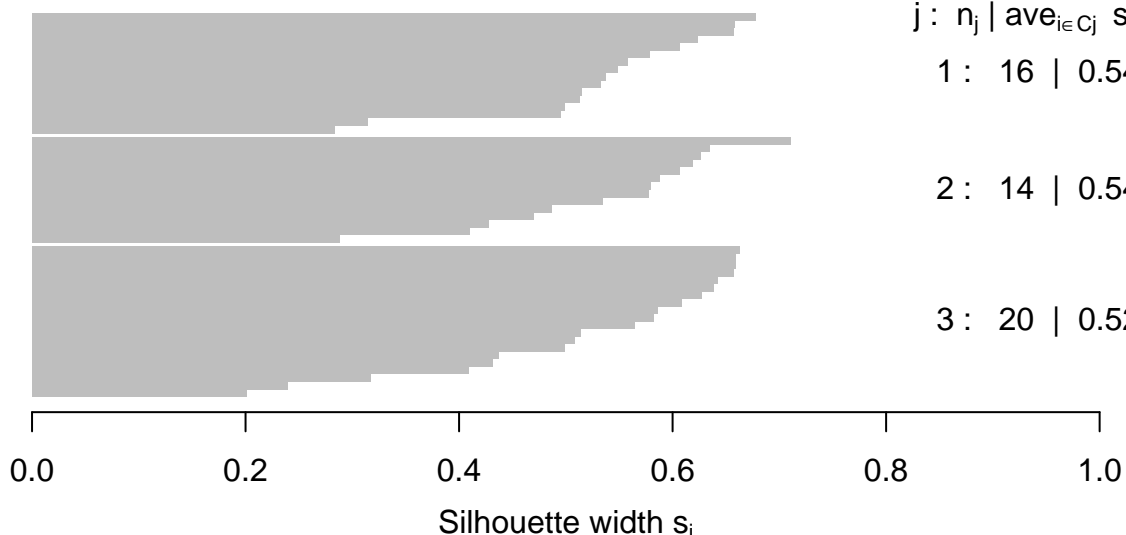
3 clusters  $C_j$

$j : n_j \mid \text{ave}_{i \in C_j} s_i$

1 : 16 | 0.54

2 : 14 | 0.54

3 : 20 | 0.52



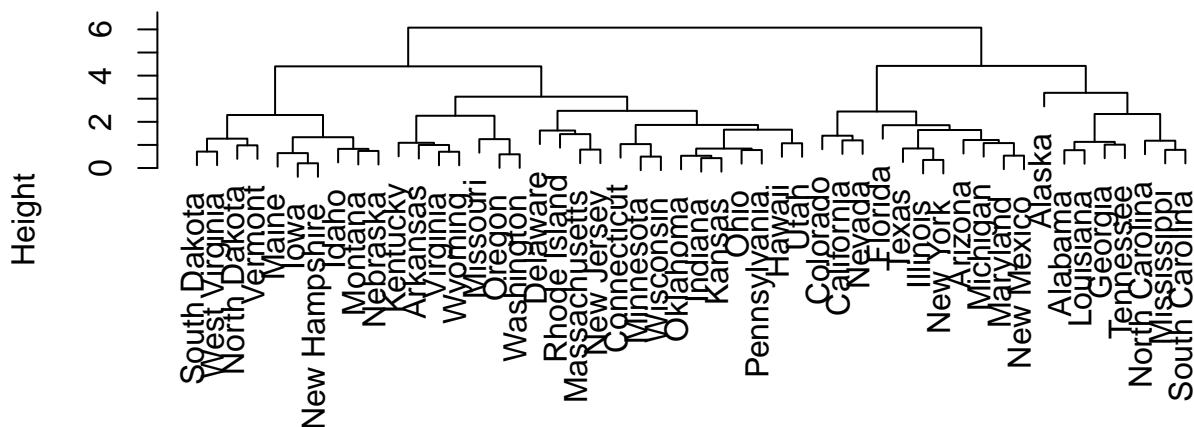
Average silhouette width : 0.53

(e) I scale the variables by the `scale()` function, and I omit other codes, which is the same as the previous, except that I use `dist(sdX)` instead of `dist(X)`.

```
sdX = scale(X)
```

Hierarchical:

## Complete Linkage



Complete linkage clusters are

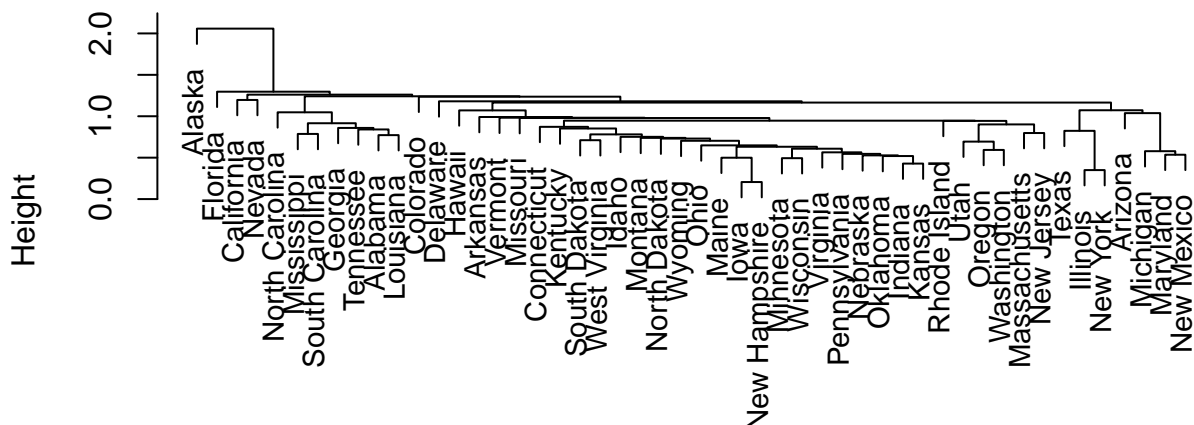
Cluster 1: Alabama, Alaska, Georgia, “Louisiana” “Mississippi” “North Carolina” “South Carolina” Tennessee.

Cluster 2: “Arizona” “California” “Colorado” “Florida” “Illinois” “Maryland” “Michigan” “Nevada” “New Mexico” “New

York" "Texas".

Cluster 3: "Arkansas" "Connecticut" "Delaware" "Hawaii" "Idaho" "Indiana" "Iowa" "Kansas" "Kentucky" "Maine" "Massachusetts" "Minnesota" "Missouri" "Montana" "Nebraska" "New Hampshire" "New Jersey" "North Dakota" "Ohio" "Oklahoma" "Oregon" "Pennsylvania" "Rhode Island" "South Dakota" "Utah" "Vermont" "Virginia" "Washington" "West Virginia" "Wisconsin" "Wyoming".

### Single Linkage



Single linkage clusters are

Cluster 1: Alabama, North Carolina, Arizona, California, Delaware, Illinois, Louisiana, Maryland, Michigan, Mississippi, Nevada, New Mexico, New York, South Carolina, Arkansas, Colorado, Georgia, Massachusetts, Missouri, New Jersey, Oklahoma, Oregon, Rhode Island, Tennessee, Texas, Virginia, Washington, Wyoming, Connecticut, Hawaii, Idaho, Indiana, Iowa, Kansas, Kentucky, Maine, Minnesota, Montana, Nebraska, New Hampshire, North Dakota, Ohio, Pennsylvania, South Dakota, Utah, Vermont, West Virginia, Wisconsin.

Cluster 2: Alaska. Cluster 3: Florida.

### Silhouette plot of (x = hc.clusters.complete, dist = dist(sdX))

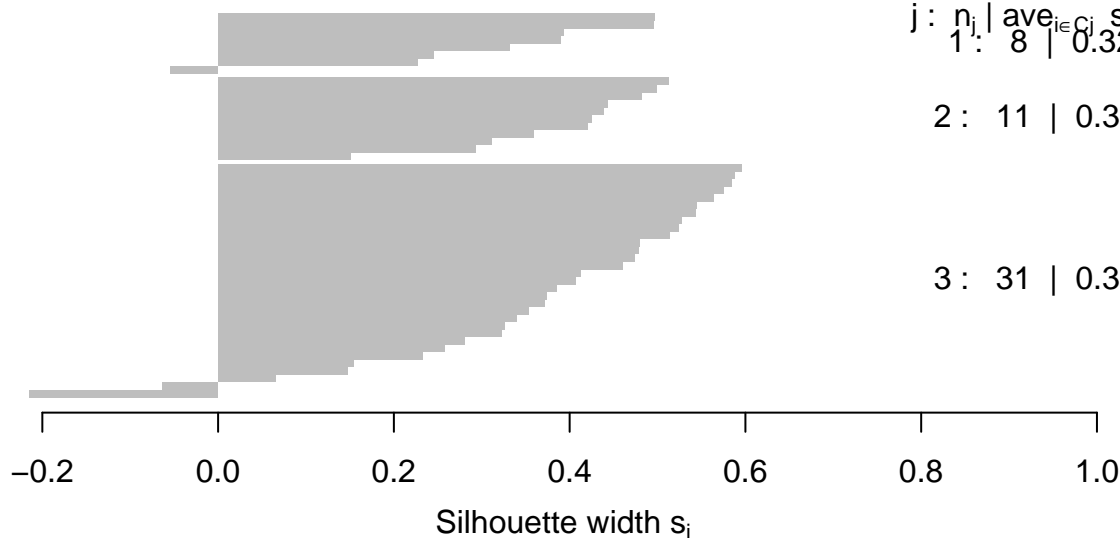
n = 50

3 clusters  $C_j$

$j : n_j | \text{ave}_{i \in C_j} s_i$   
1 : 8 | 0.32

2 : 11 | 0.39

3 : 31 | 0.37



Average silhouette width : 0.37

## Silhouette plot of (x = cutree(hc.single, 3), dist = dist(sdX))

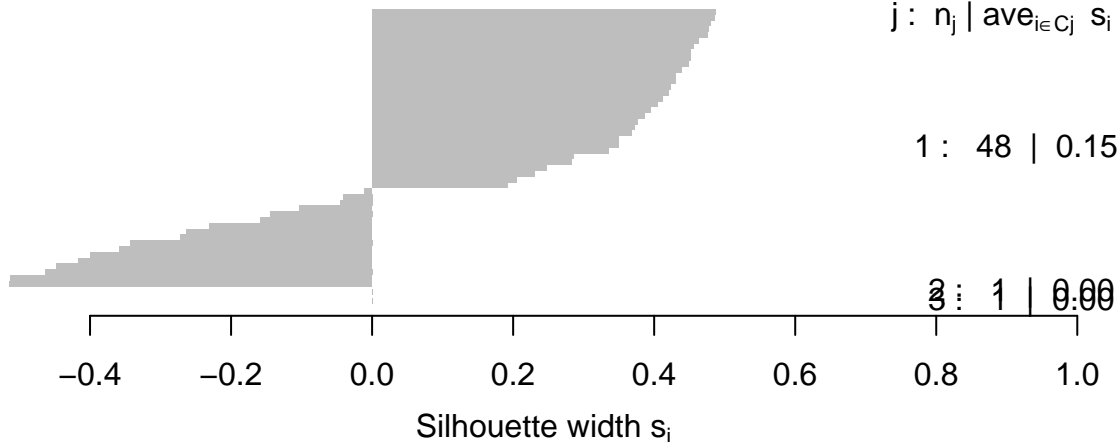
n = 50

3 clusters  $C_j$

$j : n_j \mid \text{ave}_{i \in C_j} s_i$

1 : 48 | 0.15

3 : 1 | 0.00



Average silhouette width : 0.15

### K-means:

Cluster 1: Alabama, Alaska, Arizona, California, Colorado, Florida, Georgia, Illinois, Louisiana, Maryland, Michigan, Mississippi, Nevada, New Mexico, New York, North Carolina, South Carolina, Tennessee, Texas.

Cluster 2: Arkansas, Connecticut, Delaware, Hawaii, Indiana, Kansas, Kentucky, Massachusetts, Missouri, Montana, Nebraska, New Jersey, Ohio, Oklahoma, Oregon, Pennsylvania, Rhode Island, Utah, Virginia, Washington, Wyoming.

Cluster 3: Idaho, Iowa, Maine, Minnesota, New Hampshire, North Dakota, South Dakota, Vermont, West Virginia, Wisconsin.

## Silhouette plot from K-means

n = 50

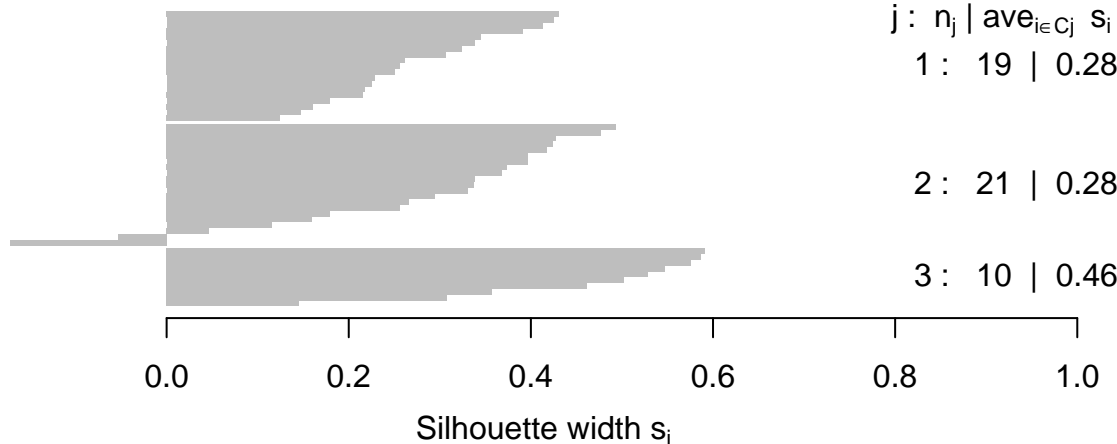
3 clusters  $C_j$

$j : n_j \mid \text{ave}_{i \in C_j} s_i$

1 : 19 | 0.28

2 : 21 | 0.28

3 : 10 | 0.46



Average silhouette width : 0.31

- (f) The scaling changes the cluster assignments of each points. For hierarchical clustering with complete linkage, the sizes of each cluster becomes different, and there also appears points with negative silhouette coefficients, while previous there're no such points. The quality of the clustering seems to be worse in terms of the silhouette measure. For hierarchical clustering with single linkage, the clustering becomes even worse. The average Silhouette width decreases, and there's more points with negative silhouette coefficients. For k-means clustering, the result is no longer the same as the hierarchical clustering using complete linkage. Also, there appear two points with negative silhouette width. The clustering quality also drops.

Yes, if we initially don't have any preference on any variables. If we don't scale the variables, the number of assaults is likely to dominate the clustering.