main_content

In this document, we demonstrate the programming to reproduce the result we have in the main content.

```
library(parallel)
source(here::here('src/optimization/w_optimization.R'))
```

Figure 1

```
eq_mps <- list(rep(0.9, 3),
               rep(0.8, 3),
               rep(0.7, 3))
grid_corr <- head(seq(0, 1, by = 0.001), -1)
```

```
#' This cell will run the global optimizer for 3000 times with
#' parallel package, which might be time consuming if the number
#' of core is small. So `eval` is set to FALSE by default, and
#' data from previous run will be loaded

opt_disj_pow <- data.frame(
  mp = numeric(),
  corr = numeric(),
  opt_disj = numeric(),
  non_zero = numeric()
)

for (mp in eq_mps) {
    res_list <- mclapply(grid_corr, function(x)
      go_optim_w_dp(alpha=0.025, mp=mp, rho=x))
    optim_values <- unlist(lapply(res_list, function(x) x[1, "optimal_value"]))
    non_zeros <- unlist(lapply(res_list, function(x) sum(x[1, c("w1", "w2", "w3")] != 0)))
    opt_disj_pow <- opt_disj_pow %>%
      add_row(mp = mp[1],
              corr = grid_corr,
              opt_disj = optim_values,
              non_zero = non_zeros)
}
```

```
opt_disj_pow <- readRDS(here::here("data/dat_f1.rds"))

ggplot(opt_disj_pow, aes(x=corr, y=opt_disj)) +
  geom_line() +
  facet_grid(rows=vars(mp), scales = "free")
```

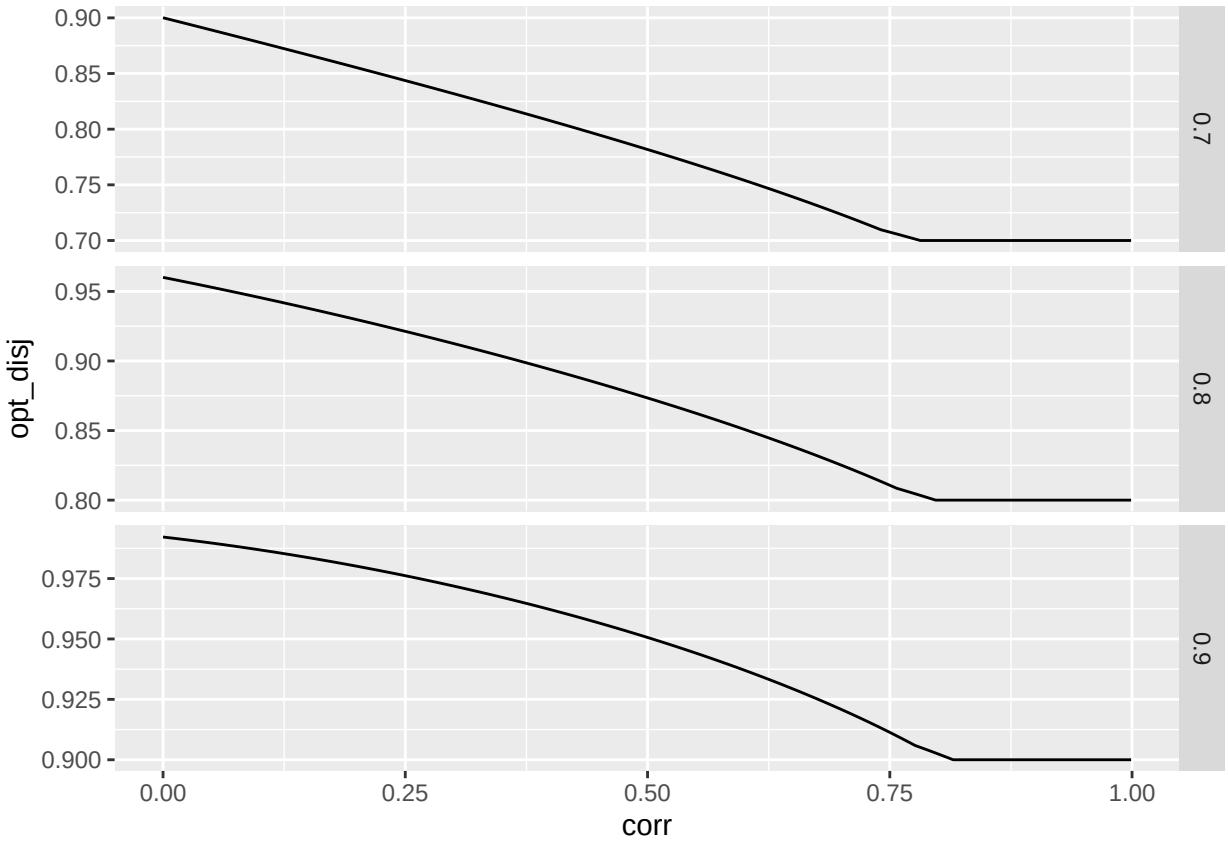

Table 1

```
list_mp <- list(rep(0.9, 3),
                rep(0.8, 3),
                rep(0.7, 3),
                c(0.9, 0.75, 0.6),
                c(0.9, 0.75, 0.3),
                c(0.9, 0.75, 0.1),
                c(0.9, 0.5, 0.5),
                c(0.9, 0.1, 0.1))

res_list <- mclapply(list_mp, function(x)
  optim_w_dp(alpha=0.025, mp=x))

opt_ws <- unlist(lapply(res_list,
                        function(x) paste0(round(x$w, 3), collapse = ", ")))
opt_pows <- unlist(lapply(res_list,
                          function(x) -x$optima$objective))
mps <- unlist(lapply(list_mp,
                     function(x) paste0(x, collapse = ", ")))


opt_disj_ind_3 <- cbind(mp = mps,
```

```
                        opt_w = opt_ws,
                        opt_pow = round(opt_pows*100, 3))
opt_disj_ind_3
```

```
##      mp               opt_w                 opt_pow
## [1,] "0.9, 0.9, 0.9"  "0.333, 0.333, 0.333" "99.22"
## [2,] "0.8, 0.8, 0.8"  "0.333, 0.333, 0.333" "96.007"
## [3,] "0.7, 0.7, 0.7"  "0.333, 0.333, 0.333" "90.011"
## [4,] "0.9, 0.75, 0.6" "0.536, 0.299, 0.164" "95.74"
## [5,] "0.9, 0.75, 0.3" "0.626, 0.353, 0.021" "94.694"
## [6,] "0.9, 0.75, 0.1" "0.639, 0.361, 0"     "94.629"
## [7,] "0.9, 0.5, 0.5"  "0.715, 0.142, 0.142" "92.53"
## [8,] "0.9, 0.1, 0.1"  "1, 0, 0"             "90"
```

Table 2

```
list_mp <- list(rep(0.9, 3),
                rep(0.8, 3),
                rep(0.7, 3))

list_corr <- list(0.9, 0.78, 0.7, 0.5)
list_para <- asplit(expand.grid(corr = list_corr, mp = list_mp), 1)

res_list <- mclapply(list_para,
                     function(x)
                       go_optim_w_dp(alpha=0.025, mp=x$mp, rho=x$corr))
bhmk_list <- mclapply(list_para,
                      function(x)
                        disjunctive_power_corr(w=rep(1/3, 3), alpha=0.025,
                                               mp=x$mp, rho=x$corr))

mps <- unlist(lapply(list_para,
                     function(x) paste0(x$mp, collapse = ", ")))
corrs <- unlist(lapply(list_para,
                       function(x) paste0(x$corr, collapse = ", ")))
opt_ws <- unlist(lapply(res_list,
                        function(x) paste0(round(x[1, c("w1", "w2", "w3")], 3), collapse = ", ")))
opt_pows <- unlist(lapply(res_list,
                          function(x)
                            x[1, "optimal_value"]))

opt_disj_dep_eq <- rbind(cbind(mp = mps,
                               corr = corrs,
                               opt_w = opt_ws,
                               disj_pow = round(opt_pows * 100, 3)),
                         cbind(mp = mps,
                               corr = corrs,
                               w = "1/3, 1/3, 1/3",
                               disj_pow = round(unlist(bhmk_list) * 100, 3)
                               )
```

```
                        ) %>%
  as.data.frame() %>%
  arrange(desc(mp), desc(corr))
opt_disj_dep_eq
```

```
##               mp corr               opt_w disj_pow
## 1  0.9, 0.9, 0.9  0.9             1, 0, 0       90
## 2  0.9, 0.9, 0.9  0.9       1/3, 1/3, 1/3   87.301
## 3  0.9, 0.9, 0.9 0.78         0.5, 0.5, 0   90.535
## 4  0.9, 0.9, 0.9 0.78       1/3, 1/3, 1/3   90.507
## 5  0.9, 0.9, 0.9  0.7 0.333, 0.333, 0.333   92.082
## 6  0.9, 0.9, 0.9  0.7       1/3, 1/3, 1/3   92.082
## 7  0.9, 0.9, 0.9  0.5 0.333, 0.333, 0.333   95.062
## 8  0.9, 0.9, 0.9  0.5       1/3, 1/3, 1/3   95.062
## 9  0.8, 0.8, 0.8  0.9             1, 0, 0       80
## 10 0.8, 0.8, 0.8  0.9       1/3, 1/3, 1/3   75.502
## 11 0.8, 0.8, 0.8 0.78         0.5, 0.5, 0   80.373
## 12 0.8, 0.8, 0.8 0.78       1/3, 1/3, 1/3    80.13
## 13 0.8, 0.8, 0.8  0.7 0.333, 0.333, 0.333   82.512
## 14 0.8, 0.8, 0.8  0.7       1/3, 1/3, 1/3   82.512
## 15 0.8, 0.8, 0.8  0.5 0.333, 0.333, 0.333   87.342
## 16 0.8, 0.8, 0.8  0.5       1/3, 1/3, 1/3   87.342
## 17 0.7, 0.7, 0.7  0.9             1, 0, 0       70
## 18 0.7, 0.7, 0.7  0.9       1/3, 1/3, 1/3   64.272
## 19 0.7, 0.7, 0.7 0.78         0.5, 0.5, 0   70.034
## 20 0.7, 0.7, 0.7 0.78       1/3, 1/3, 1/3   69.554
## 21 0.7, 0.7, 0.7  0.7 0.333, 0.333, 0.333   72.338
## 22 0.7, 0.7, 0.7  0.7       1/3, 1/3, 1/3   72.338
## 23 0.7, 0.7, 0.7  0.5 0.333, 0.333, 0.333   78.174
## 24 0.7, 0.7, 0.7  0.5       1/3, 1/3, 1/3   78.174
```

Table 3

```
list_mp <- list(c(0.9, 0.75, 0.6),
                c(0.9, 0.75, 0.3),
                c(0.9, 0.5, 0.5))

list_corr <- list(0.8,
                  0.4,
                  matrix(c(1, 0.8, 0.4,
                           0.8, 1, 0.2,
                           0.4, 0.2, 1), nrow=3),
                  matrix(c(1, 0.2, 0.4,
                           0.2, 1, 0.8,
                           0.4, 0.8, 1), nrow=3))

list_para <- asplit(expand.grid(corr = list_corr, mp = list_mp), 1)

res_list <- mclapply(list_para,
                     function(x)
```

```
                       go_optim_w_dp(alpha=0.025, mp=x$mp, rho=x$corr))

bhmk_list <- mclapply(list_para,
                      function(x) {
                        bhmk_w = optim_w_dp(alpha=0.025, mp=x$mp)$w
                        bhmk_pow = disjunctive_power_corr(w=bhmk_w, alpha=0.025,
                                                          mp=x$mp, rho=x$corr)
                        list(w = bhmk_w, pow = bhmk_pow)
                        }
                      )

mps <- unlist(lapply(list_para,
                     function(x) paste0(x$mp, collapse = ", ")))
corrs <- unlist(lapply(list_para,
                       function(x) {
                         if (length(x$corr) == 1) {
                           paste0(rep(x$corr, 3), collapse = ", ")
                         }
                         else {
                           paste0(c(x$corr[1, 2], x$corr[1, 3], x$corr[2, 3]), collapse = ", ")
                         }
                       }
                       ))
opt_ws <- unlist(lapply(res_list,
                        function(x) paste0(round(x[1, c("w1", "w2", "w3")], 3), collapse = ", ")))
bhmk_ws <- unlist(lapply(bhmk_list,
                         function(x) paste0(round(x$w, 3), collapse = ", ")))
bhmk_pow <- unlist(lapply(bhmk_list,
                          function(x) x$pow))
opt_pows <- unlist(lapply(res_list,
                          function(x)
                            x[1, "optimal_value"]))

opt_disj_dep_uneq <- rbind(cbind(mp = mps,
                                 corr = corrs,
                                 opt_w = opt_ws,
                                 disj_pow = round(opt_pows * 100, 3)),
                           cbind(mp = mps,
                                 corr = corrs,
                                 w = bhmk_ws,
                                 disj_pow = round(bhmk_pow * 100, 3))) %>%
  as.data.frame() %>%
  arrange(desc(mp), desc(corr))
opt_disj_dep_uneq
```

```
##                mp          corr               opt_w disj_pow
## 1  0.9, 0.75, 0.6 0.8, 0.8, 0.8            1, 0, 0       90
## 2  0.9, 0.75, 0.6 0.8, 0.8, 0.8 0.536, 0.299, 0.164   85.795
## 3  0.9, 0.75, 0.6 0.8, 0.4, 0.2     0.953, 0, 0.047    90.16
## 4  0.9, 0.75, 0.6 0.8, 0.4, 0.2 0.536, 0.299, 0.164   87.659
## 5  0.9, 0.75, 0.6 0.4, 0.4, 0.4 0.782, 0.201, 0.018   91.295
## 6  0.9, 0.75, 0.6 0.4, 0.4, 0.4 0.536, 0.299, 0.164   90.722
## 7  0.9, 0.75, 0.6 0.2, 0.4, 0.8     0.697, 0.303, 0   92.925
```

```
## 8  0.9, 0.75, 0.6 0.2, 0.4, 0.8 0.536, 0.299, 0.164   91.802
## 9  0.9, 0.75, 0.3 0.8, 0.8, 0.8             1, 0, 0       90
## 10 0.9, 0.75, 0.3 0.8, 0.8, 0.8 0.626, 0.353, 0.021   87.092
## 11 0.9, 0.75, 0.3 0.8, 0.4, 0.2             1, 0, 0       90
## 12 0.9, 0.75, 0.3 0.8, 0.4, 0.2 0.626, 0.353, 0.021   87.144
## 13 0.9, 0.75, 0.3 0.4, 0.4, 0.4     0.792, 0.208, 0   91.259
## 14 0.9, 0.75, 0.3 0.4, 0.4, 0.4 0.626, 0.353, 0.021   90.939
## 15 0.9, 0.75, 0.3 0.2, 0.4, 0.8     0.697, 0.303, 0   92.925
## 16 0.9, 0.75, 0.3 0.2, 0.4, 0.8 0.626, 0.353, 0.021   92.765
## 17  0.9, 0.5, 0.5 0.8, 0.8, 0.8             1, 0, 0       90
## 18  0.9, 0.5, 0.5 0.8, 0.8, 0.8 0.715, 0.142, 0.142   87.345
## 19  0.9, 0.5, 0.5 0.8, 0.4, 0.2     0.994, 0, 0.006   90.011
## 20  0.9, 0.5, 0.5 0.8, 0.4, 0.2 0.715, 0.142, 0.142   88.198
## 21  0.9, 0.5, 0.5 0.4, 0.4, 0.4 0.989, 0.005, 0.005    90.02
## 22  0.9, 0.5, 0.5 0.4, 0.4, 0.4 0.715, 0.142, 0.142   88.902
## 23  0.9, 0.5, 0.5 0.2, 0.4, 0.8       0.92, 0.08, 0   90.421
## 24  0.9, 0.5, 0.5 0.2, 0.4, 0.8 0.715, 0.142, 0.142   89.319
```

Table 4

```
list_mp <- list(rep(0.9, 3),
                rep(0.8, 3),
                rep(0.7, 3),
                c(0.9, 0.75, 0.6),
                c(0.9, 0.75, 0.3),
                c(0.9, 0.75, 0.1),
                c(0.9, 0.5, 0.5),
                c(0.9, 0.1, 0.1))

res_list <- mclapply(list_mp, function(x)
  optim_w_cp(alpha=0.025, mp=x))

opt_ws <- unlist(lapply(res_list,
                        function(x) paste0(round(x$w, 3), collapse = ", ")))
opt_pows <- unlist(lapply(res_list,
                          function(x) -x$optima$objective))
mps <- unlist(lapply(list_mp,
                     function(x) paste0(x, collapse = ", ")))


opt_conj_ind_3 <- cbind(mp = mps,
                        opt_w = opt_ws,
                        opt_pow = round(opt_pows*100, 3))
opt_conj_ind_3
```

```
##      mp               opt_w                 opt_pow
## [1,] "0.9, 0.9, 0.9"  "0.333, 0.333, 0.333" "51.518"
## [2,] "0.8, 0.8, 0.8"  "0.333, 0.333, 0.333" "28.517"
## [3,] "0.7, 0.7, 0.7"  "0.333, 0.333, 0.333" "15.4"
## [4,] "0.9, 0.75, 0.6" "0.208, 0.338, 0.454" "21.268"
## [5,] "0.9, 0.75, 0.3" "0.167, 0.268, 0.565" "9.194"
```

```
## [6,] "0.9, 0.75, 0.1" "0.137, 0.219, 0.644" "2.706"
## [7,] "0.9, 0.5, 0.5"  "0.166, 0.417, 0.417" "9.566"
## [8,] "0.9, 0.1, 0.1"  "0.102, 0.449, 0.449" "0.198"
```

Table 5

```
list_mp <- list(c(0.9, 0.75, 0.6),
                c(0.9, 0.75, 0.3),
                c(0.9, 0.5, 0.5))

list_corr <- list(0.8,
                  0.4,
                  matrix(c(1, 0.8, 0.4,
                           0.8, 1, 0.2,
                           0.4, 0.2, 1), nrow=3),
                  matrix(c(1, 0.2, 0.4,
                           0.2, 1, 0.8,
                           0.4, 0.8, 1), nrow=3))

list_para <- asplit(expand.grid(corr = list_corr, mp = list_mp), 1)

res_list <- mclapply(list_para,
                     function(x)
                       optim_w_cp(alpha=0.025, mp=x$mp, rho=x$corr))

bhmk_list <- mclapply(list_para,
                      function(x) {
                        bhmk_w = optim_w_cp(alpha=0.025, mp=x$mp)$w
                        bhmk_pow = conjunctive_power_corr(w=bhmk_w, alpha=0.025,
                                                          mp=x$mp, rho=x$corr)
                        list(w = bhmk_w, pow = bhmk_pow)
                        }
                      )

mps <- unlist(lapply(list_para,
                     function(x) paste0(x$mp, collapse = ", ")))
corrs <- unlist(lapply(list_para,
                       function(x) {
                         if (length(x$corr) == 1) {
                           paste0(rep(x$corr, 3), collapse = ", ")
                         }
                         else {
                           paste0(c(x$corr[1, 2], x$corr[1, 3], x$corr[2, 3]), collapse = ", ")
                         }
                       }
                       ))
opt_ws <- unlist(lapply(res_list,
                        function(x) paste0(round(x$w, 3), collapse = ", ")))
opt_pows <- unlist(lapply(res_list,
                          function(x)
                            -x$optima$objective))
```

```
bhmk_ws <- unlist(lapply(bhmk_list,
                         function(x) paste0(round(x$w, 3), collapse = ", ")))
bhmk_pow <- unlist(lapply(bhmk_list,
                          function(x) x$pow))

opt_conj_dep_uneq <- rbind(cbind(mp = mps,
                                 corr = corrs,
                                 opt_w = opt_ws,
                                 conj_pow = round(opt_pows * 100, 3)),
                           cbind(mp = mps,
                                 corr = corrs,
                                 w = bhmk_ws,
                                 conj_pow = round(bhmk_pow * 100, 3))) %>%
  as.data.frame() %>%
  arrange(desc(mp), desc(corr))
opt_conj_dep_uneq
```

```
##                mp          corr               opt_w conj_pow
## 1  0.9, 0.75, 0.6 0.8, 0.8, 0.8 0.121, 0.314, 0.565   42.451
## 2  0.9, 0.75, 0.6 0.8, 0.8, 0.8 0.208, 0.338, 0.454   41.525
## 3  0.9, 0.75, 0.6 0.8, 0.4, 0.2 0.109, 0.347, 0.545   31.577
## 4  0.9, 0.75, 0.6 0.8, 0.4, 0.2 0.208, 0.338, 0.454   30.693
## 5  0.9, 0.75, 0.6 0.4, 0.4, 0.4 0.172, 0.331, 0.497   31.079
## 6  0.9, 0.75, 0.6 0.4, 0.4, 0.4 0.208, 0.338, 0.454   30.966
## 7  0.9, 0.75, 0.6 0.2, 0.4, 0.8  0.225, 0.295, 0.48   36.184
## 8  0.9, 0.75, 0.6 0.2, 0.4, 0.8 0.208, 0.338, 0.454   36.074
## 9  0.9, 0.75, 0.3 0.8, 0.8, 0.8 0.055, 0.153, 0.791   23.769
## 10 0.9, 0.75, 0.3 0.8, 0.8, 0.8 0.167, 0.268, 0.565   21.502
## 11 0.9, 0.75, 0.3 0.8, 0.4, 0.2 0.066, 0.253, 0.682   15.525
## 12 0.9, 0.75, 0.3 0.8, 0.4, 0.2 0.167, 0.268, 0.565    14.77
## 13 0.9, 0.75, 0.3 0.4, 0.4, 0.4  0.115, 0.225, 0.66   16.088
## 14 0.9, 0.75, 0.3 0.4, 0.4, 0.4 0.167, 0.268, 0.565   15.804
## 15 0.9, 0.75, 0.3 0.2, 0.4, 0.8  0.15, 0.149, 0.702   20.049
## 16 0.9, 0.75, 0.3 0.2, 0.4, 0.8 0.167, 0.268, 0.565   19.139
## 17  0.9, 0.5, 0.5 0.8, 0.8, 0.8 0.064, 0.468, 0.468   27.486
## 18  0.9, 0.5, 0.5 0.8, 0.8, 0.8 0.166, 0.417, 0.417   26.526
## 19  0.9, 0.5, 0.5 0.8, 0.4, 0.2  0.05, 0.464, 0.486   16.986
## 20  0.9, 0.5, 0.5 0.8, 0.4, 0.2 0.166, 0.417, 0.417   15.951
## 21  0.9, 0.5, 0.5 0.4, 0.4, 0.4 0.118, 0.441, 0.441   17.576
## 22  0.9, 0.5, 0.5 0.4, 0.4, 0.4 0.166, 0.417, 0.417   17.435
## 23  0.9, 0.5, 0.5 0.2, 0.4, 0.8 0.174, 0.431, 0.394   22.719
## 24  0.9, 0.5, 0.5 0.2, 0.4, 0.8 0.166, 0.417, 0.417   22.696
```