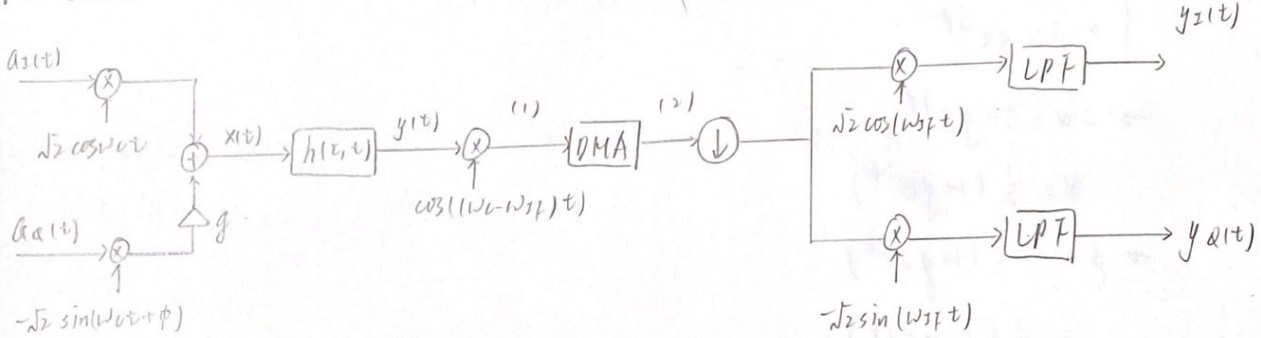
P.12 Exercise



$$x(t) = a_I(t)\cdot\sqrt{2}\cos W_c t + a_Q(t)\cdot(-\sqrt{2}\sin(W_c t+\phi))\cdot g$$

$$= \sqrt{2}\left[a_I(t)\cos W_c t - g\, a_Q(t)\sin(W_c t+\phi)\right].$$

$$y(t) = x(t)$$

$$(1) = y(t)\cdot\cos((W_c - W_{IF})t)$$

$$= \sqrt{2}\cdot\left[a_I(t)\cdot\frac{\cos W_{IF}t + \cos(2W_c t - W_{IF}t)}{2} - g\cdot a_Q(t)\cdot\frac{\sin(W_{IF}t+\phi)+\sin(2W_c t+\phi-W_{IF}t)}{2}\right].$$

$$(2) = \text{LPF}\{(1)\} = \sqrt{2}\cdot\left[a_I(t)\cdot\frac{\cos W_{IF}t}{2} - g\cdot a_Q(t)\cdot\frac{\sin(W_{IF}t+\phi)}{2}\right]$$

$$y_I(t) = \text{LPF}\{(2)\cdot\sqrt{2}\cos(W_{IF}t)\}$$

$$= \text{LPF}\left\{2\cdot\left[a_I(t)\cdot\frac{1}{2}\cdot\frac{1+\cos 2W_{IF}t}{2} - g\, a_Q(t)\cdot\frac{1}{2}\cdot\frac{\sin(\phi)+\sin(2W_{IF}t+\phi)}{2}\right]\right\}.$$

$$= \text{LPF}\left\{a_I(t)\cdot\frac{1+\cos 2W_{IF}t}{2} - g\, a_Q(t)\cdot\frac{\sin\phi+\sin(2W_{IF}t+\phi)}{2}\right\}.$$

$$= \frac{1}{2}a_I(t) - g\cdot\frac{1}{2}\cdot a_Q(t)\cdot\sin\phi$$

$$y_Q(t) = \text{LPF}\{(2)\cdot(-\sqrt{2}\sin(W_{IF}t))\}.$$

$$= \text{LPF}\left\{-2\cdot\left[a_I(t)\cdot\frac{1}{2}\cos W_{IF}t\sin W_{IF}t - g\, a_Q(t)\cdot\frac{1}{2}\cdot\frac{\cos(\phi)-\cos(2W_{IF}t+\phi)}{2}\right]\right\}.$$

$$= \text{LPF}\left\{-a_I(t)\cdot\frac{1}{2}\sin 2W_{IF}t + \frac{1}{2}g\, a_Q(t)\cdot[\cos\phi - \cos(2W_{IF}t+\phi)]\right\}.$$

$$= \frac{1}{2}\cdot g\cdot a_Q(t)\cdot\cos\phi$$

⇒ Recovered Signal: $y_I(t) + j\, y_Q(t)$

$$= \frac{1}{2}a_I(t) - g\cdot\frac{1}{2}a_Q(t)\sin\phi + j\cdot\frac{1}{2}g\, a_Q(t)\cdot\cos\phi$$

$$= \frac{1}{2}a_I(t) + j\cdot\frac{1}{2}\cdot g\, a_Q(t)\cdot[j\sin\phi + \cos\phi]$$

$$= \frac{1}{2}\left[a_I(t) + j\, g\, a_Q(t)\, e^{j\phi}\right]$$

$$= \frac{1}{2}\left\{\alpha[a_I(t) + j\, a_Q(t)] + \beta[a_I(t) - j\, a_Q(t)]\right\}$$

$$= \frac{1}{2}\left\{\alpha\, a(t) + \beta\, a^*(t)\right\}$$