Real-Time Model for Optimal Water Allocation in Irrigation Systems during Droughts

M. Delavar[1]; M. Moghadasi[2]; and S. Morid[3]

Abstract: The agricultural sector is the main victim of drought and efficient water planning, and management is the best strategy to reduce drought pressures on this sector. One operational approach is the application of real-time models to help water managers decide on mitigation measures, such as deficient irrigation or reducing cropped areas using the new incoming information (for example, recorded inflows and precipitation). The present paper introduces a real-time modeling approach for optimal water allocation during a drought. The model includes two main components: forecasting and optimization modules. The forecasting module uses a recurrent neural network technique to forecast annual inflows that is updated as monthly climate and hydrological data are introduced to the model. The optimization module allocates water among the irrigation units and their crops by considering growing stage, sensitivity to water stress at different stages, available/forecasted water, and previous decisions about water release. The model was tested for the 1999 drought of the Zayandeh Rud irrigation system. Traditional operating procedures were shown to produce 42% loss whereas the proposed method would have reduced loss to 12%.
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Introduction

Drought is a constant threat to the agricultural sector. In areas with limited water resources, demand management is the best strategy to alleviate the consequent losses from this disaster. The dynamic behavior of irrigation systems derives from the variability of river flows and growth stages of crops; thus, developing a real-time water allocation model is an effective tool for water management. Such a model has two main components: optimization and forecasting modules. The model takes into account forecasted river inflows/dam storage and optimally allocates water through irrigation networks in consideration of the planted crops, their growth stages, and economical/strategic benefits. A literature review shows that these factors have rarely been considered together for drought management.

The most challenging part of a real-time water allocation model is stream flow forecasting, which is always uncertain. This uncertainty is the main reason that assessments must be updated during a drought. Different techniques that can be applied to forecasting and statistical models have long been used, which goes back to Gabriel and Neumann (1962). Recently, the application of data-driven models such as artificial neural networks (ANN) have shown promising results (ASCE 2000). Coulibaly et al. (2001) reported that a recurrent neural network (RNN) could be utilized for inflow forecasting. Eiji et al. (2003) proposed a method for inflow forecasting of the Karogawa Dam using ANN and applied rain data outside and inside the dam basin as inputs. The model reduced the forecasting error by approximately 30%. Chetan and Sudheer (2006) developed a hybrid linear-neural model for forecasting the flow of the Kolar River in India.

Another part of the real-time water allocation model is optimization. On the basis of progress in deficit irrigation research, Shangguan et al. (2002) developed a recurrence control model for regional optimal allocation of irrigation water resources aimed at overall maximum efficiency using the decomposing-harmonization principle for large systems. Their model consisted of three layers that optimized crop irrigation scheduling, optimally allocating water for various crops and irrigation subsystems. Ghahraman and Sepaskhah (2004) developed an efficient NLP optimization model with integrated soil-water balance to reduce water demand for the Ardak Dam in Iran. Kumar et al. (2006) compared linear programming (LP) and genetic algorithms (GA) for optimum water allocation of a single-purpose reservoir. Their results showed that the performance of the LP and GA models were not significantly different. Moghaddasi et al. (2009) developed a model for optimal allocation of water among different crops and irrigation units. This model was coupled with a reservoir operating model to activate restrictions on allocatable water to the agricultural sector in consideration of the variation of inflows and demand and expected income from allocated water among the crops and irrigation units.

The main objective of the present study is to present a real-time simulation-optimization (RTSO) model for optimal water allocation in irrigation networks. The model includes two main modules, one for forecasting river flows and one for irrigation water allocation. The forecasts are updated as new data are introduced to the model. Thus, at each stage of decision making, irrigation water is allocated on the basis of the new forecasts and previous irrigation. The 1999 drought in the Zayandeh Rud irrigation system in Iran was selected to explore the proposed methodology and modeling system.

[1]Ph.D. Student, Water Resources Eng., Tarbiat Modares Univ., Tehran, Iran.

[2]Assistant Professor, Arak Univ., Arak, Iran.

[3]Associate Professor, Tarbiat Modares Univ., Tehran, Iran (corresponding author). E-mail: morid_sa@modares.ac.ir

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Material and Methods

Study Area

The Zayandeh Rud basin with an area of 41,500 km^2 is located in central Iran (Fig. 1). The Chadegan Dam was constructed on the river in 1971 with a 1,450 million cubic meters (MCM) capacity. The availability of water resources in the basin, even after construction of the dam, was not sufficient, thus interbasin transfer was applied by diverting parts of neighboring water resources (the Karoon and Dez Rivers). Tunnels divert 300–400 MCM of water per year and two more tunnels are under construction. The network includes eight irrigation units (IU): Nekooabad LB, Nekooabad RB, Mahyar, Borkhar, Abshar LB, Abshar RB, Rudasht, and small-scale systems for a total area of approximately 205,000 ha. Wheat, barley, sugar beet, and alfalfa are the primary crops in the network. Table 1 shows the cropping calendar of the various crops within the IUs (Murray-Rust et al. 2004).

Description of RTSO Water Allocation Model

As stated, the proposed procedure for real-time water allocation includes: (1) a hydrological forecasting module; and (2) an optimal water allocation module. In the following sections, more details about the modules are presented.

Hydrological Forecasting Module

The ANN technique is selected for this module. This method represents a massively parallel-distribution information processing system composed of a number of elements (or neurons). The most interesting feature of ANNs is its ability to train from the examples. Different ANN types have been formulated. In this study, the recurrent neural network (RNN) is utilized to forecast monthly hydrological data. The main characteristics of a RNN is that at least one feedback link should be added to the static inputs (Fig. 2). The RNN allows signals to propagate in both forward and backward directions, which offers the network dynamic memories.

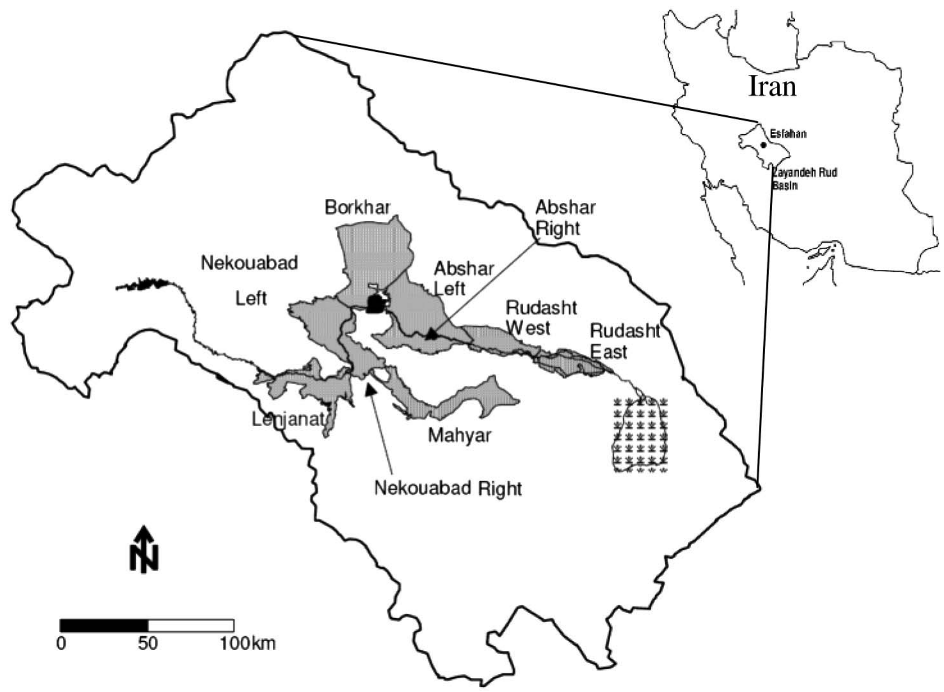

Fig. 1. Zayandeh Rud irrigation system and related infrastructures

Table 1. Typical Crop Calendars and Maximum Crop Yields in Zayandeh Rud Basin

Crop	October	November	December	January	February	March	April	May	June	July	August	September	Maximum yield (t/ha)
Wheat		*	*	*	*	*	*	*	*				9
Barley		*	*	*	*	*	*	*	*				7
Sugar beet							*	*	*	*	*	*	70
Potato						*	*	*	*	*			50
Alfalfa (1st cutting)	*	*											2.833
Alfalfa (2nd cutting)		*	*										2.833
Alfalfa (3rd cutting)							*	*					2.833
Alfalfa (4th cutting)								*	*				2.833
Alfalfa (5th cutting)									*	*			2.833
Alfalfa (6th cutting)											*	*	2.833

Note: Farshi et al. 1997.

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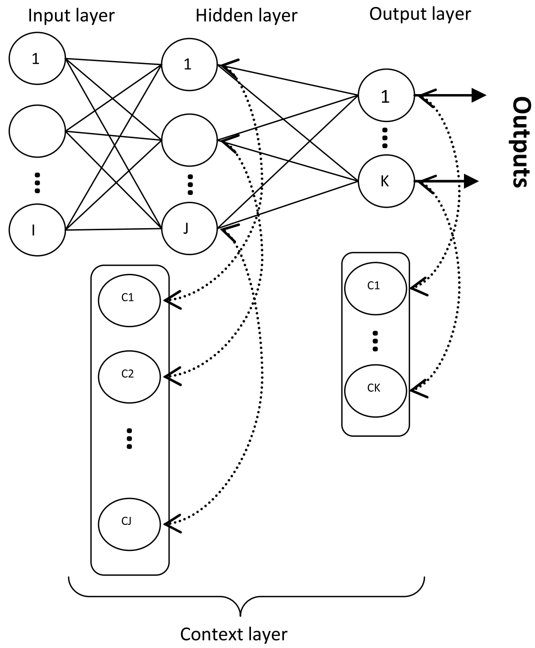

Fig. 2. Architecture of recurrent neural networks

Input selection and design of the network's architecture are of great importance when using ANNs (ASCE 2000). Applying minimum effective inputs and avoiding complex architectures from model uncertainty and overfitting is recommended (Dawson and Wibly 1998). Evaluating different inputs to predict inflows using RNN showed that the following combinations work better

$$I_{t+1} = f_1(I_t, T_t, N_t) \qquad I_{t+2} = f_2(I_t, T_t, N_t) \qquad \vdots \qquad I_{t+k} = f_k(I_t, T_t, N_t) \tag{1}$$

where K = number of month of foresight on inflows (1 to 11), t = period (month), I_t = observed inflow during period (t), T_t = observed temperature during period (t), N_t = month number during period (t), and I_{t+k} = predicted inflow during period $t+k$. The input models show how the forecast will be updated.

The input data are introduced to the model during the years 1982–2005 (70% of data used for training, 20% for validation, and 10% for testing). The performance of the forecasting module is shown in Table 2, which illustrates acceptable performance of the model for monthly predictions of reservoir inflows.

Optimal Water Allocation Module

To apply the optimal water allocation module (OPM), a top-down approach is applied such that the total allocated water for the agricultural sector is optimally distributed among the eight IUs, then among the crops and, finally, optimally circulated among the irrigation periods. The objectives for these optimization processes are to maximize crop yield and income. This method is applied by developing three submodules that utilize the methodology suggested by Moghadasi et al. (2009). The model formulation, including constraints and the objective function, are summarized in the appendix. The water distribution has a top-down path, but the related calculations are vice versa, as explained as follows.

Submodule 1 (Optimal irrigation scheduling): This model determines the irrigation scheduling for the dominant crops during the growing season on the basis of a 10-day irrigation period (the usual irrigation period in the study area). The objective function is to maximize crop production per hectare

$$\text{MAX:} \frac{Y_{ac}}{Y_{\max c}} = 1 - \sum_{g=1}^{n} Ky_g \left(1 - \frac{ETa_{c,g}}{ET_{\max_{c,g}}}\right) \tag{2}$$

where g, n, and c = growing stage, total number of growing stages, and crop type, respectively. $ET_{c.g}$ = actual crop evapotranspiration (mm/10 days), $ET_{\max_{c.g}}$ = crop maximum evapotranspiration (mm/10 days, and Ky_g = crop sensitivity coefficient (Doorenbos and Pruitt 1984), Y_{ac} = actual yield per unit area, and $Y_{\max_c}$ is maximum yield per unit area (kg/ha).

Submodule 2 (Optimal water allocation and planted acreage): This submodule maximizes the summed benefit of the crops within an IU

$$\text{MAX}\left\{\sum_{k=1}^{K} F_k(V_k) A_k Y_{\max k} P_k\right\} \tag{3}$$

where k = total number of crops, F_k (V_k) = functional relation between maximum relative yield and allocated irrigation water for crop k, $Y_{\max k}$ = maximum yield for crop k (kg/ha), A_K = acreage for crop k (ha), and P_k = marketing price per kilogram (US$) for crop k. To calculate F_k (V_k) for crop k, the first submodule needs to be run with various values of V_k.

Submodule 3 (Optimal water allocation optimization among the IUs): The third submodel optimally distributes the total release of reservoir among the IUs. The objective function maximizes the summed benefit of all IUs

$$\text{MAX}\left\{\sum_{n=1}^{N} F_n(V_n)\right\} \tag{4}$$

where Fn (V_n) = functional relation between maximum benefit and allocated water for each unit and N = total number of IUs.

Running the RTSO Model

The application of the RTSO model for water allocation involves the following steps.

1. Start real-time water allocation analysis at $t = 1$.

Table 2. Forecasting Model Performance in Real-Time Prediction of Reservoir Inflows

Prediction model	I_2[a]	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}	I_{11}	I_{12}
Month	November	December	January	February	March	April	May	June	July	August	September
MSE	569.0	798.4	891.6	812.2	1036.9	1251.4	810.0	938.1	639.1	611.2	1060.7
NMSE	0.4	0.6	0.7	0.6	0.8	1.0	0.7	0.8	0.5	0.5	0.9
MAE	18.1	21.2	23.1	20.5	24.3	26.0	22.7	26.0	17.9	17.7	24.5
Minimum absolute error	1.0	0.6	0.3	0.1	0.0	0.2	0.8	0.1	0.0	0.2	0.1
Maximumabsolute error	98.2	114.8	99.5	83.0	97.1	133.8	115.9	101.5	91.9	100.9	78.7
R	0.8	0.6	0.6	0.7	0.6	0.6	0.6	0.6	0.8	0.7	0.5

[a]I_t: predicted inflow during period t (month).

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2. Receive and renew the latest hydrological and water allocation observations, including reservoir inflow (I_t), temperature (T_t), reservoir storage (S_t), and optimal water allocation from the previous run where $t \neq 1$.
3. Run a hydrological forecasting module to forecast reservoir inflows for the next time steps ($I_{t+1}, I_{t+2}, \ldots, I_{12}$).
4. If $t \neq 1$, change the constraints of the OPM model at each time step on the basis of the previous runs. For example, for the actual evapotranspiration constrain in submodule 1, the allocated irrigation water (IR) for each crop and within $[1, t]$ [Eq. (5)] is replaced by the optimal allocated water obtained from previous runs. This constrain can be changed to Eq. (6) as follows:

$$ETa_{c,t} \leq \frac{ET_{\max_{c,t}}}{(1-P_c)(FC_c - \mathrm{PWP}_c)} \times \left[SM_{c,t} - \mathrm{PWP} + \left(\frac{IR_{c,t} - DP_{c,t}}{\mathrm{Root}_{c,t}}\right)\right] \quad \forall\ t \in [1, T] \tag{5}$$

to

$$\begin{cases} ETa_{c,t} \leq \dfrac{ET_{\max_{c,t}}}{(1-P_c)(FC_c - \mathrm{PWP}_c)} \times \left[SM_{c,t} - \mathrm{PWP} + \left(\dfrac{IR_{c,t} - DP_{c,t}}{\mathrm{Root}_{c,t}}\right)\right] & \forall\ t \in [t, T] \\ IR_{c,t-1} = IR_{c,t-1(Opt)} & \forall\ t \in [2, t] \end{cases} \tag{6}$$

5. Run the updated RTSO model formulated in step 4 and identify the new production functions. For this, submodule 1 (irrigation scheduling optimization) is executed for the crops at different water volumes and their respective yields are calculated. These are then used to define the crop production function ($F_k(V_k)$) of Eq. (3) and applied to submodule 2. Similarly, this submodule is executed for different volumes of water and the consequent incomes of the IUs are calculated to update $Fn(V_n)$ in Eq. (4). Fig. 3 illustrates updating the crop production function of the RTSO model.

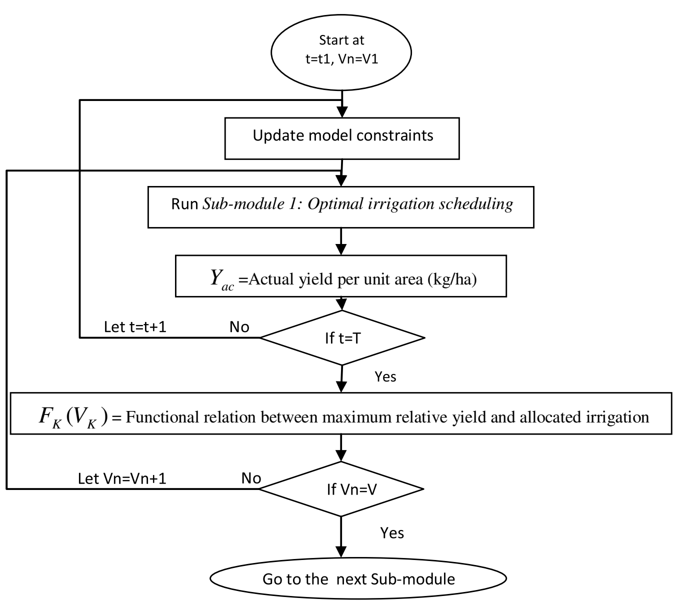

Fig. 3. Flowchart of updated crop production function for RTSO model

6. Renew and substitute the yield function at each time step.
7. Run the OPM module formulated by inputs from step 5 to obtain optimal water allocation. In this step, submodule 3 is executed using the results of submodule 2 to optimally distribute water among the IUs.

The outputs of these computations are the optimum allocated water for each IU and the optimum allocated water for each crop and planted acreage within the IUs. Optimized irrigation scheduling for the crops for a 10-day period was also calculated. Fig. 4 shows the flowchart of RSTO algorithm. Fig. 5 is a schematic of the RTSO model inputs and outputs and their interactions with a series of repetitive calculation steps.

Results and Discussion

To describe and evaluate the RTSO model, the hydro-climate of the Zayandeh Rud irrigation network during the 1999 droughts was employed. The average river inflow for the period was 1,170 MCM. Storage behind the dam was 697 MCM at the beginning of 1998 and 300 MCM at the end of 2000. The total water allocated for the city, industry, and rice was 416 MCM, meaning that 1,050 MCM was available for other crops within the IUs and the present modeling simulation.

RSTO Simulation

Tables 3 and 4 and Fig. 6 show the outputs for the irrigation of potatoes and Nekooabad IU and are useful in understanding the simulation for RSTO. Table 3 shows how the river flows were updated month by month and how the optimum irrigated water (OIW) was calculated and updated at the beginning of each month during the growing season. For instance, the OIW for June was suggested to be 372 mm at the beginning of March, but was modified to 360 mm at the beginning of May.

The crop production function (CPF) is also modified. Figs. 6(a) and 6(b) show the CPF for potatoes at the beginning of March

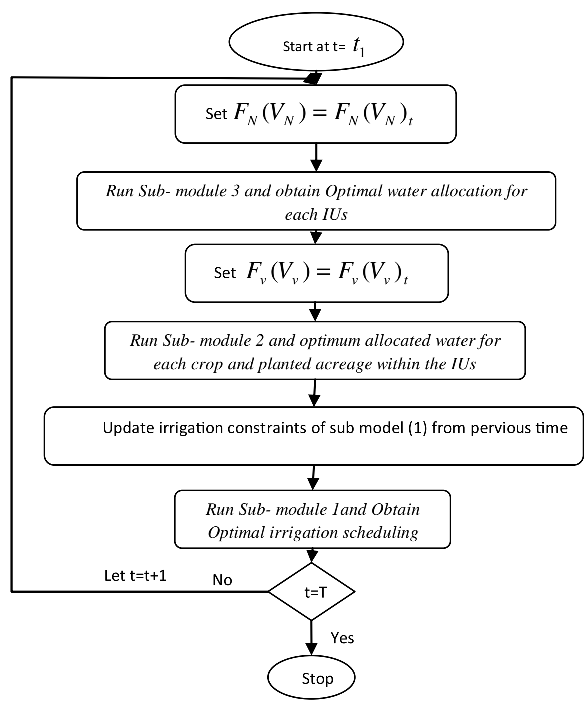

Fig. 4. Real-time optimization simulation algorithm for water allocation

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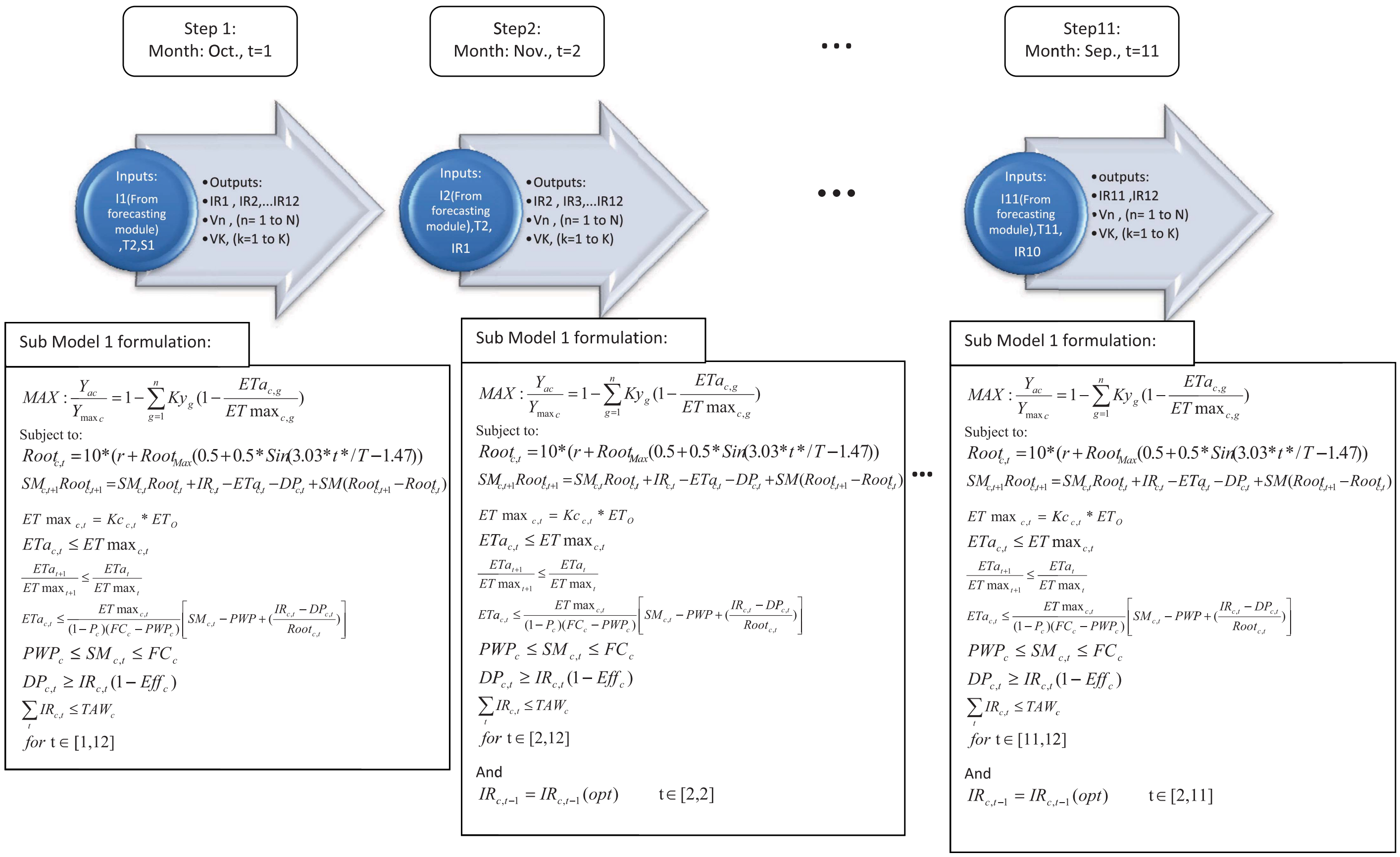

Fig. 5. Schematic view of RTSO model application by month

Table 3. Optimized Irrigation Depths for Potato Crop in Different Months in Nekooabad (LB) (mm)

Starting Month	Updated annual flow Forecast (MCM)	March	April	May	June	July
March	1,122.2	90	125	298	372	331
April	1,048.2	**90**	117	285	368	330
May	1,031.2	**90**	**117**	278	360	322
June	1,028.2	**90**	**117**	**278**	346	327
July	1,031.2	**90**	**117**	**278**	**346**	341

Note: Numbers in bold show allocated water from the previous runs that are fixed in the current RSTO run.

(before planting) and July, respectively. For each month, CPF was calculated with respect to the previously allocated water in the plan. For example, in April, a new constraint was applied to the model to indicate the actual irrigation depth applied in March.

Finally, the aforementioned calculations were repeated for other crops and aggregated to indicate optimum water allocations for each IU [see Table 4 for Nekooabad (LB) IU]. The time scale can be shortened depending on the irrigation duration.

Evaluation of the RSTO Model

For the proposed evaluation, the developed modeling system was applied in two different situations. First, the system was executed

Table 4. Real-Time Water Allocation in Nekooabad (LB) IU (MCM)

Starting month	Updated annual flow forecast (MCM)	October	November	December	January	February	March	April	May	June	July	August	September
October	1,255.2	5.87	10.45	0.55	0.00	0.62	18.22	34.28	51.31	15.14	5.68	13.39	2.92
November	1,227.2	**5.87**	8.36	0.63	0.00	0.41	15.34	27.12	41.27	10.78	4.51	10.31	1.86
December	1,222.6	**5.87**	**8.36**	0.43	0.00	0.28	17.13	28.25	38.56	11.30	3.49	9.32	1.55
January	1,197.2	**5.87**	**8.36**	**0.43**	0.00	0.38	16.31	25.14	39.62	13.93	2.47	11.35	1.85
February	1,180.2	**5.87**	**8.36**	**0.43**	**0.00**	0.48	16.18	28.31	47.53	11.76	4.35	11.49	2.77
March	1,122.2	**5.87**	**8.36**	**0.43**	**0.00**	**0.48**	17.16	24.23	39.98	11.83	3.34	11.33	2.73
April	1,048.2	**5.87**	**8.36**	**0.43**	**0.00**	**0.48**	**17.16**	22.81	43.58	9.68	2.98	9.63	2.20
May	1,031.2	**5.87**	**8.36**	**0.43**	**0.00**	**0.48**	**17.16**	**22.81**	46.41	12.88	3.42	10.29	2.85
June	1,028.2	**5.87**	**8.36**	**0.43**	**0.00**	**0.48**	**17.16**	**22.81**	**46.41**	13.92	2.47	11.34	3.81
July	1,031.2	**5.87**	**8.36**	**0.43**	**0.00**	**0.48**	**17.16**	**22.81**	**46.41**	**13.92**	3.51	13.42	3.25
August	1,055.2	**5.87**	**8.36**	**0.43**	**0.00**	**0.48**	**17.16**	**22.81**	**46.41**	**13.92**	**3.51**	12.44	3.86
September	1,050.2	**5.87**	**8.36**	**0.43**	**0.00**	**0.48**	**17.16**	**22.81**	**46.41**	**13.92**	**3.51**	**12.44**	3.01

Note: Numbers in bold show allocated water to IUs in previous runs that are fixed in the current RSTO run.

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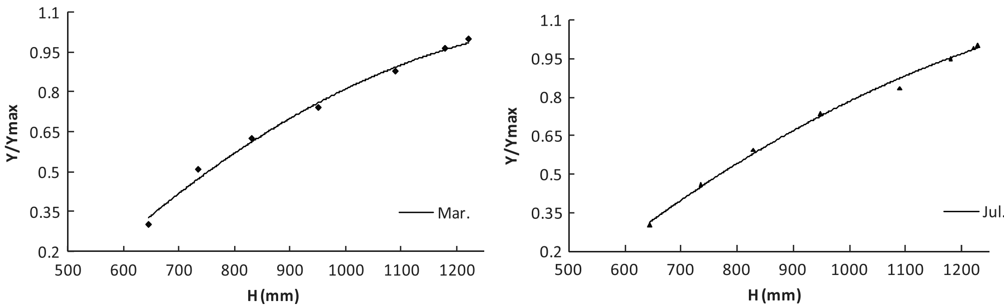

(a) (b)

Fig. 6. (a) Crop production function of potatoes at the beginning of March 1999; (b) crop production function of potatoes at the beginning of July 1999

with data from 1999 (actual data with no error in forecasts) (this run was named OPM). The next time, the model started at the beginning of 1999, followed the monthly forecasts, and updated itself as described (no use of further information), indicating the allocatable water for the IUs and crops. Table 5 shows the distribution of water among the IUs for the two runs of the model. Because of the drought and water shortage for this year, the RSTO model only applied 48 to 62% of the maximum water demand. For the OPM, this range varied from 47 to 52%, very close to the RSTO results.

Table 5. Distribution of Water among IUs for RTSO and OPM

Irrigation system	Maximum demand (V_{max})	RTSO model		OPM model	
		V	V/V_{max}	V	V/V_{max}
Nekooabad LB	260	134	0.52	130	0.5
Nekooabad RB	100	62	0.62	50	0.5
Mahyar	220	112	0.51	120	0.55
Borkhar	210	118	0.56	110	0.52
Abshar LB	260	125	0.48	130	0.5
Abshar RB	140	67	0.48	70	0.5
Rudasht	460	231	0.50	260	0.57
Small-scale systems	380	201	0.53	180	0.47

The optimized planted acreages using the two models are also shown in Table 6, which shows that the maximum reduction change is related to wheat, which varies in the different IUs from 0 to 3%.

Table 7 shows another set of output data from the models in which the calculated irrigation depths during the different growing stages of crops are illustrated. The data are only presented for the Nekooabad (LB). Table 7 shows that the maximum differences are observed in the vegetative and yield formation stages.

To evaluate the results of RSTO, the final income of IUs [total crops production (ton) multiplied by their net incomes (\$/ton)] was compared with two situations. The first situation is the traditional method of deficit irrigation that applies equitable water reduction (EWR) among the various crops. This method is described by Moghaddasi et al. (2009) and was evaluated by them for the same year (1999 drought), which resulted the total income of

Table 6. Planted Acreage (ha) Using RSTO and OPM

Irrigation network	Nekooabad LB		Nekooabad RB		Mahyar		Borkhar		Abshar LB		Abshar RB		Rudasht		Small-scale systems	
Allocation method	RSTO	OPM	RSTO	OPM	RSTO	OPM	RSTO	OPM	RSTO	OPM	RSTO	OPM	RSTO	OPM	RSTO	OPM
Wheat	12,027	11,907	5,084	5,033	15,077	15,421	10,885	10,974	13,549	13,612	7,652	7,872	28,062	28,771	19,572	19,574
Barley	3,827	3,789	2,157	2,135	5,068	5,017	1,820	1,802	2,263	2,240	989	979	7,468	7,393	12,499	12,374
Alfalfa	4,018	3,978	1,412	1,398	1,180	1,168	2,030	2,010	2,523	2,498	1,683	1,666	3,359	3,325	3,023	2,993
Sugar beet	1,206	1,194	—	—	685	678	2,254	2,231	2,802	2,774	1,114	1,103	—	—	960	950
Potato	—		—		—		—		—		—		3,905	3,866	—	

Table 7. Comparison of Optimized Water Calculated (mm) for Various Crops at Different Growing Stages Using RSTO and OPM Methods in Nekooabad (LB) IU

Crop	Vegetative		Flowering		Yield formation		Ripening	
	RSTO	OPM	RSTO	OPM	RSTO	OPM	RSTO	OPM
Wheat	14	15	241	170	254	178	78	55
Barley	18	22	28	37	168	221	77	102
Sugar beet	140	159	523	591	241	273	—	—
Potato	—	—	—	—	—	—	—	—
Alfalfa (six cutting)	587	490	—	—	294	449	—	—
Sum	759	686	792	798	957	1,121	155	157

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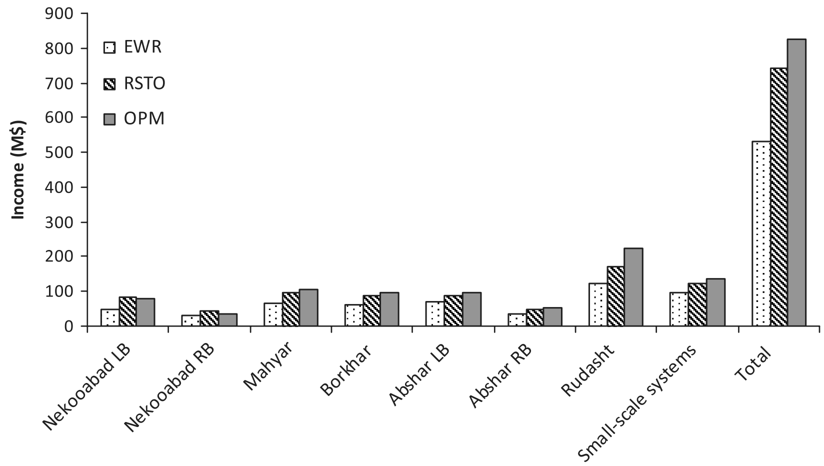

Fig. 7. Resulting income from the IUs using OPM, traditional, and RSTO approaches

$531 million for IUs. The second situation assumed that the monthly inflows from 1999 are known at the beginning of year (i.e., future inflows are known) and then water is allocated with the described OPM module. The results show that the final incomes in 1999 using RSTO and OPM are 743 and 824, respectively. The distributions of the approaches among IUs are also shown in Fig. 7 and indicate that the difference between the incomes resulting from OPM (perfect situation) and EWR is 42, and is reduced to 12% using RTSO.

Conclusions and Suggestions

This paper presents a real-time simulation-optimization algorithm for optimal water allocation of irrigation networks. The 1999 drought in the Zayandeh Rud irrigation system in Iran was selected to explore the effects of the methodology. The proposed modeling system uses hydrological forecasting and water allocation models. The following conclusions can be drawn from this study.

- The results show how a real-time modeling system can work well as a drought mitigation tool. For instance, for the 1999 drought, the model resulted in 12% loss in income of the system, whereas the traditional operation sustained 42% loss.
- The RSTO model is sensitive to the accuracy of streamflow forecast. Thus, improving the forecasting approach can improve the capability of the model.
- High uncertainty exists in any forecasting model and introducing a range of possible inflows to test the consequent decisions about allocatable water for the IUs is possible.
- The results show the importance of drought planning to alleviate loss and reveal how changes in irrigation scheduling and crop acreage can be effective mitigation measures.
- The application of systems like RTSO require more operational capability. Training programs for the basin water managers and extension learning programs for the farmers should also be considered.

Appendix. Water Allocation Optimization Module (OPM)

The proposed optimization module for water allocation is summarized in three submodels as follows:

Submodel 1: Optimal Irrigation Scheduling

$$\text{MAX}: \frac{Y_{ac}}{Y_{\max c}} = 1 - \sum_{g=1}^{n} Ky_g \left(1 - \frac{ETa_{c,g}}{ET_{\underset{c,g}{\max}}}\right) \tag{7}$$

subject to

$$\text{Root}_{c,t} = 10^*(r + \text{Root}_{\text{Max}}(0.5 + 0.5^*\text{Sin}(3.03^*t^*/T - 1.47)))$$

$$SM_{c,t+1}\text{Root}_{c,t+1} = SM_{c,t}\text{Root}_{c,t} + IR_{c,t} - ETa_{c,t} - DP_{c,t} + SM'(\text{Root}_{c,t+1} - \text{Root}_{c,t})$$

$$ET\underset{c,t}{\max} = Kc_{c,t}{}^*ET_O$$

$$ETa_{c,t} \le ET\underset{c,t}{\max}$$

$$\frac{ETa_{t+1}}{ET\underset{t+1}{\max}} \le \frac{ETa_t}{ET\underset{t}{\max}}$$

$$ETa_{c,t} \le \frac{ET\underset{c,t}{\max}}{(1-P_c)(FC_c - \text{PWP}_c)} \times \left[SM_{c,t} - \text{PWP} + \left(\frac{IR_{c,t} - DP_{c,t}}{\text{Root}_{c,t}}\right)\right]$$

$$\text{PWP}_c \le SM_{c,t} \le FC_c$$

$$DP_{c,t} \ge IR_{c,t}(1 - \text{Eff}_c)$$

$$\sum_t IR_{c,t} \le \text{TAW}_c \tag{8}$$

Submodel 2: Optimal Water Allocation and Planted Acreage

$$\text{MAX}\left\{\sum_{k=1}^{K} F_k(V_k)A_k Y_{\max k} P_k\right\} \tag{9}$$

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subject to

$$\sum_{k=1}^{K} A_k \leq A_{\text{total}}$$

$$\sum_{k=1}^{K} V_k \leq V_{\text{total}}$$

$$A_{\min k} \leq A_k \leq A_{\max k}$$

$$A_{A6} \leq A_{A5} \leq A_{A4} \leq A_{A3} \leq A_{A2} \leq A_{A1} \tag{10}$$

Submodel 3: Optimal Water Allocation Optimization among the IUS

$$\text{MAX}\left\{\sum_{n=1}^{N} F_n(V_n)\right\} \tag{11}$$

subject to

$$\sum_{n=1}^{N} V_n \leq V_{\text{total}} \qquad S_{t+1} = I_t + S_t - R_t \tag{12}$$

where A_K = acreage for each crop (ha); A_{Max} = maximum acreage for each crop (ha); $A_{\min}$ = minimum acreage for each crop (ha); DP = deep percolation (mm/10 days); $ETac, g$ = actual evapotranspiration for growth stage g of crop c in stage g (mm/10 days); $ET \max c, g$ = maximum evapotranspiration for growth stage g of crop c in stage g (mm/10 days); $ET0$ = crop reference evapotranspiration (mm); FC = field capacity (mm); FK (VK) = functional relation between maximum relative yield and allocated irrigation water; Fn (Vn) = functional relation between maximum benefit and allocated water for each unit; IR = optimal allocated irrigation water (mm/10 days); It = annual inflow to the reservoir (MCM); K = total number of crops; Kc = crop coefficient; Kyg = water sensitivity coefficient for growth stage g; n = total number of growth stages; N = total number of irrigation systems; Pk = marketing price per kilogram (US$); P = soil-water depletion; PWP = permanent wilting point; Rt = output from the reservoir for all demands; r = planting depth (for the study area this value is 7 cm for wheat and barely, 5 cm for alfalfa and sugar beet and 15 cm for potato); SM = available soil-water content (mm); SM' = constant soil-water content of deeper layers before cultivation (mm); S = reservoir storage (MCM); T = total of growth time in each of stage (day); TAW = total available water (MCM); t = operation time (one year); Yac = actual yield per unit area (kg ha_1); $Y_{\max c}$ = maximum yield per unit of area (kg ha_1); and $Y_{\max k}$ = maximum yield (kg ha_1).

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References

ASCE. (2000). "Artificial neural networks in hydrology. I: Preliminary concepts." *J. Hydrol. Eng.*, 5(2), 115–123.

Chetan, M., and Sudheer, K. P. (2006). "A hybrid linear-neural model for river flow forecasting." *Water Resour. Res.*, 42, W04402.

Coulibaly, P., Anctil, F., and Bobee, B. (2001). "Multivariate reservoir inflow forecasting using temporal neural networks." *J. Hydrol. Eng.*, 6(5), 367–376.

Dawson, C. W., and Wilby, R. (1998). "An artificial neural network approach to rainfall-runoff modeling." *Hydrol. Sci. J.*, 43(1), 47–66.

Doorenbos, J., and Pruitt, W. O. (1984). "Guidelines for predicting crop water requirements." *Irrigation and Drainage Paper 24*, Food and Agricultural Organization of the United Nations, Rome, 35–48.

Eiji, S., Kenji, M., Masanori, I., and Susumu, M. (2003). "Inflow forecasting of the dam by the neural network." *Proc., Artificial Neural Networks in Engineering Conf.*, ASME, New York, 701–706.

Farshi, A. A., Jarollahi, R., Ghasemi, M., Shahabifar, M., and Foladi, M. (1997). *Water Requirement Estimation of the Main Field Crops and Orchards*, Vol. 1, Soil and Water Research Institute, Tehran, Iran.

Gabriel, K. R., and Neumann, J. (1962). "A Markov chain model for daily rainfall occurrence quarterly." *J. Climatol.*, 88(375), 90–95.

Ghahraman, B., and Sepaskhah, A. R. (2004). "Linear and non-linear optimization models for allocation of a limited water supply." *Irrig. Drain.*, 53(1), 39–54.

Kumar, D. N., Raju, K. S., and Ashok, B. (2006). "Optimal reservoir operation for irrigation of multiple crops using genetic algorithms." *J. Irrig. Drain. Eng.*, 132(2), 123–129.

Moghaddasi, M., Morid, S., Araghinejad, S., and Aghaalikhani, M. (2009). "Assessment of irrigation water allocation based on optimization and equitable water reduction approaches to reduce agricultural drought losses: The 1999 drought in the Zayandeh Rud irrigation system (Iran)." *J. Irrig. Drain. Eng.*, 59(4), 377–387.

Murray-Rust, H., Droogers, P., and Heyadari, N. (2004). "Water for the future: Linking irrigation and water allocation in the Zayandeh Rud Basin." *IAERI-IWMI Research Rep. 6*, International Water Management Institute, Sri Lanka.

Shangguan, Z., Shao, M., and Horton, R. (2002). "A model for regional optimal allocation of irrigation water resources under deficit irrigation and its applications." *Agric. Water Manage.*, 52(2), 139–154.

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