Homework 4

Discussion of Question 1:

$X=\begin{bmatrix}1\\-1\end{bmatrix}$:

$$\phi\left(\begin{bmatrix}5\\-1\\2\end{bmatrix}^T\begin{bmatrix}1\\1\\-1\end{bmatrix}\right)=2,\ \phi\left(\begin{bmatrix}2\\-0.5\\-0.1\end{bmatrix}^T\begin{bmatrix}1\\1\\-1\end{bmatrix}\right)=1.6,\ y=\phi\left(\begin{bmatrix}0\\0.5\\-0.7\end{bmatrix}^T\begin{bmatrix}1\\2\\1.6\end{bmatrix}\right)=0$$

$X=\begin{bmatrix}-2\\-3\end{bmatrix}$:

$$\phi\left(\begin{bmatrix}5\\-1\\2\end{bmatrix}^T\begin{bmatrix}1\\-2\\-3\end{bmatrix}\right)=1,\ \phi\left(\begin{bmatrix}2\\-0.5\\-0.1\end{bmatrix}^T\begin{bmatrix}1\\-2\\-3\end{bmatrix}\right)=3.3,\ y=\phi\left(\begin{bmatrix}0\\0.5\\-0.7\end{bmatrix}^T\begin{bmatrix}1\\1\\3.3\end{bmatrix}\right)=0$$

$X=\begin{bmatrix}-5\\-1\end{bmatrix}$:

$$\phi\left(\begin{bmatrix}5\\-1\\2\end{bmatrix}^T\begin{bmatrix}1\\-5\\-1\end{bmatrix}\right)=8,\ \phi\left(\begin{bmatrix}2\\-0.5\\-0.1\end{bmatrix}^T\begin{bmatrix}1\\-5\\-1\end{bmatrix}\right)=4.6,\ y=\phi\left(\begin{bmatrix}0\\0.5\\-0.7\end{bmatrix}^T\begin{bmatrix}1\\8\\4.6\end{bmatrix}\right)=0.78$$

$X=\begin{bmatrix}-2\\2\end{bmatrix}$:

$$\phi\left(\begin{bmatrix}5\\-1\\2\end{bmatrix}^T\begin{bmatrix}1\\-2\\2\end{bmatrix}\right)=11,\ \phi\left(\begin{bmatrix}2\\-0.5\\-0.1\end{bmatrix}^T\begin{bmatrix}1\\-2\\2\end{bmatrix}\right)=2.8,\ y=\phi\left(\begin{bmatrix}0\\0.5\\-0.7\end{bmatrix}^T\begin{bmatrix}1\\11\\2.8\end{bmatrix}\right)=3.54$$

$X=\begin{bmatrix}6\\-2\end{bmatrix}$:

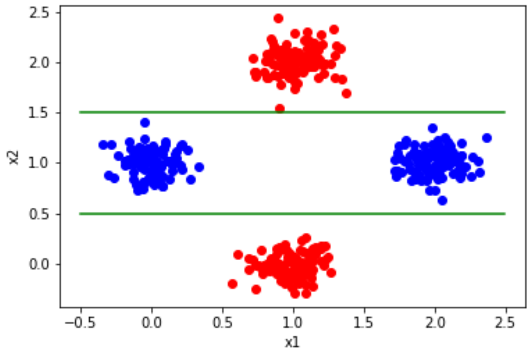
$$\phi\left(\begin{bmatrix}5\\-1\\2\end{bmatrix}^T\begin{bmatrix}1\\6\\-2\end{bmatrix}\right)=0,\ \phi\left(\begin{bmatrix}2\\-0.5\\-0.1\end{bmatrix}^T\begin{bmatrix}1\\6\\-2\end{bmatrix}\right)=0,\ y=\phi\left(\begin{bmatrix}0\\0.5\\-0.7\end{bmatrix}^T\begin{bmatrix}1\\0\\0\end{bmatrix}\right)=0$$

$X=\begin{bmatrix}4\\3\end{bmatrix}$:

$$\phi\left(\begin{bmatrix}5\\-1\\2\end{bmatrix}^T\begin{bmatrix}1\\4\\3\end{bmatrix}\right)=7,\ \phi\left(\begin{bmatrix}2\\-0.5\\-0.1\end{bmatrix}^T\begin{bmatrix}1\\4\\3\end{bmatrix}\right)=0,\ y=\phi\left(\begin{bmatrix}0\\0.5\\-0.7\end{bmatrix}^T\begin{bmatrix}1\\7\\0\end{bmatrix}\right)=3.5$$

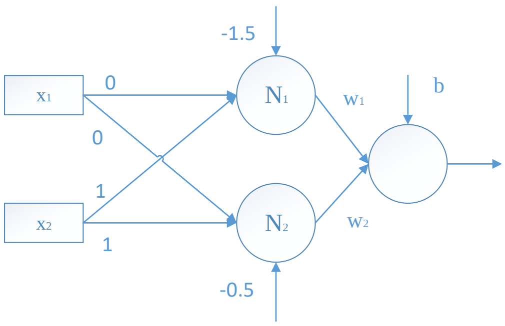
Discussion of Part 1, Question 2:

For dataset1, Network 1



Step 1: For N1, the boundary line is $x_2 - 1.5 = 0$; For N2, the boundary line is $x_2 - 0.5 = 0$, thus if a data points is subject to $x_2 > 1.5$ or $x_2 < 0.5$, the data point belongs red cluster. If it is subject to $0.5 \leq x_2 \leq 1.5$, the data point belongs to blue cluster.

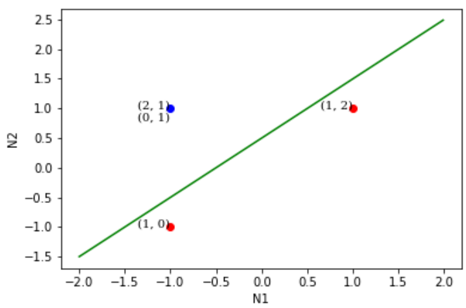
Step 2: Thus, we can have



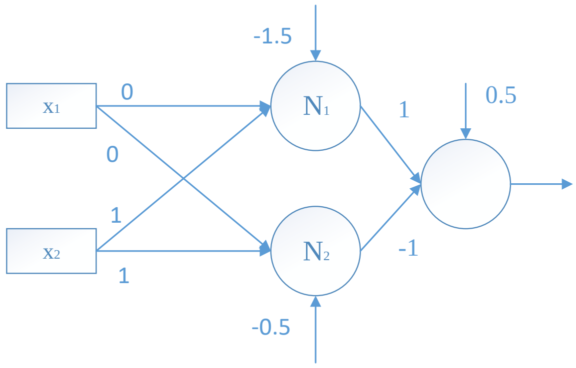
Step 3: Set activator function

$$\sigma(v) = \begin{cases} 1 & v \geq 0 \\ -1 & v < 0 \end{cases}$$

Step 4: Draw the boundary line in the hidden layer.

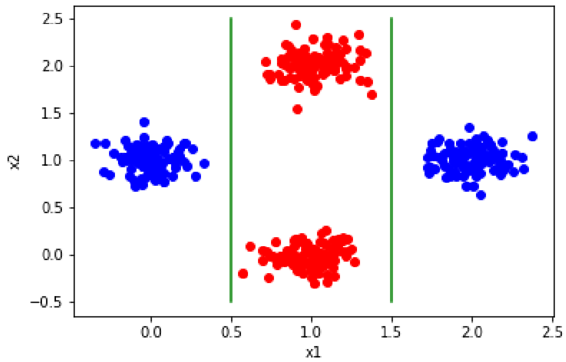


Step 5: The boundary line is $N_1 - N_2 + 0.5 = 0$, thus the final network is



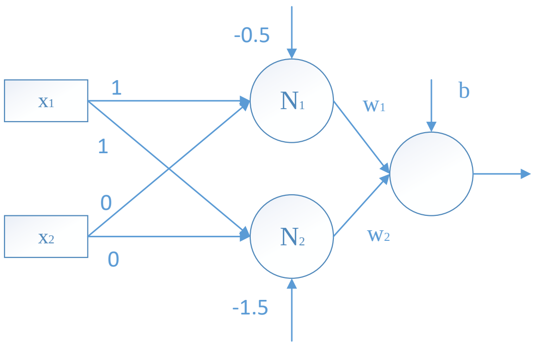
If the output of the network is 1, the input data point belongs to red cluster. If the output of the network is -1, the input data point belongs to blue cluster.

For dataset1, Network 2



Step 1: For N_1, the boundary line is $x_1 - 0.5 = 0$; For N_2, the boundary line is $x_1 - 1.5 = 0$, thus if a data points is subject to $x_1 > 1.5$ or $x_1 < 0.5$, the data point belongs blue cluster. If it is subject to $0.5 \le x_1 \le 1.5$, the data point belongs to red cluster.

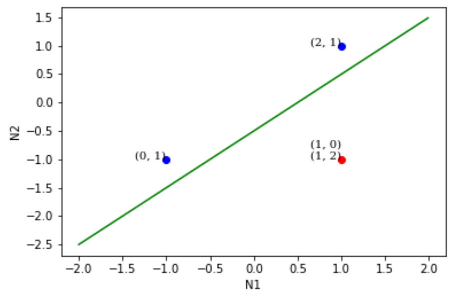
Step 2: Thus, we can have



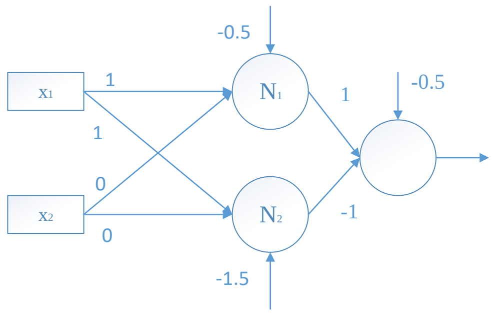
Step 3: Set activator function

$$\sigma(v) = \begin{cases} 1 & v \ge 0 \\ -1 & v < 0 \end{cases}$$

Step 4: Draw the boundary line in the hidden layer.

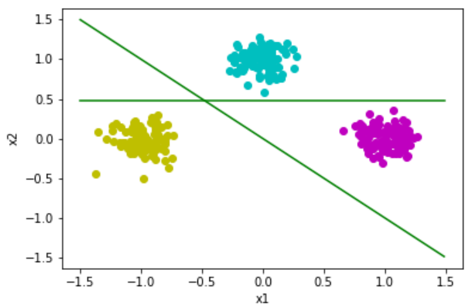


Step 5: The boundary line is $N_1 - N_2 - 0.5 = 0$, thus the final network is



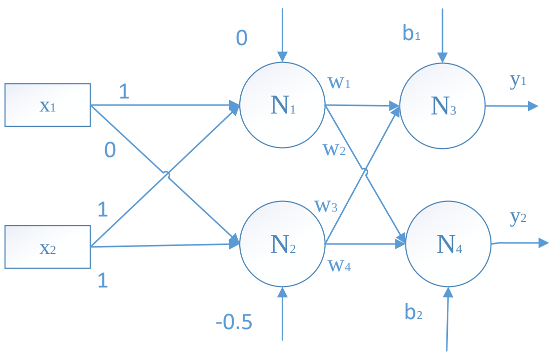
If the output of the network is 1, the input data point belongs to red cluster. If the output of the network is -1, the input data point belongs to blue cluster.

For dataset2, Network 1



Step 1: For N₁, the boundary line is $x_1 + x_2 = 0$; For N₂, the boundary line is $x_2 - 0.5 = 0$, thus if a data points is subject to $x_1 + x_2 < 0$, the data point belongs yellow cluster. If it is subject to $x_2 > 0.5$, the data point belongs to blue cluster. If it is subject to $x_2 < 0.5$ and $x_1 + x_2 > 0$, the data point belongs to purple cluster.

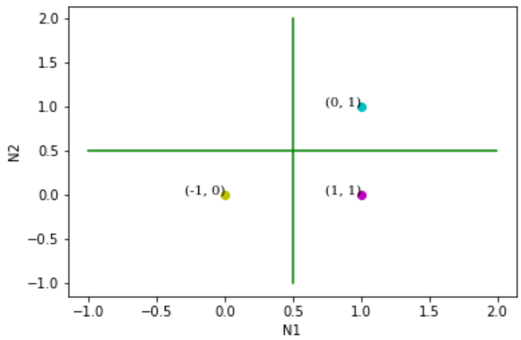
Step 2: Thus, we can have



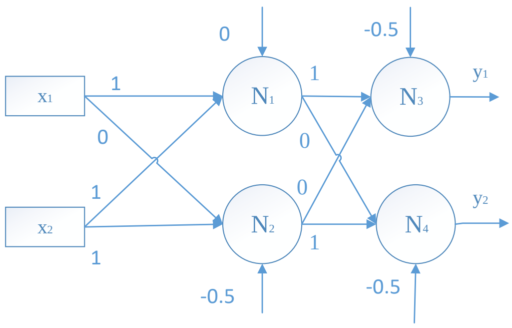
Step 3: Set activator function

$$\sigma(v)=\begin{cases}1 & v>0\\ 0 & v\le 0\end{cases}$$

Step 4: Draw the boundary line in the hidden layer.

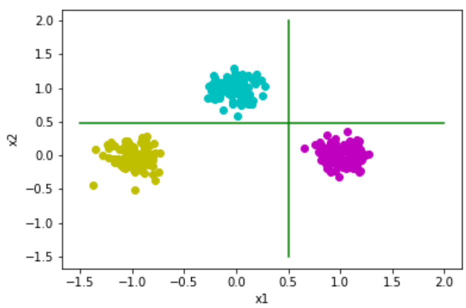


Step 5: The boundary lines are $N_1-0.5=0$ and $N_2-0.5=0$, thus the final network is



If the output y_1 of the network is 0, the input data point belongs to yellow cluster. If the output y_1 of the network is greater than 0, and the output y_2 of the network is 0, the input data point belongs to purple cluster. If the output y_2 of the network is greater than 0, the input data point belongs to blue cluster.

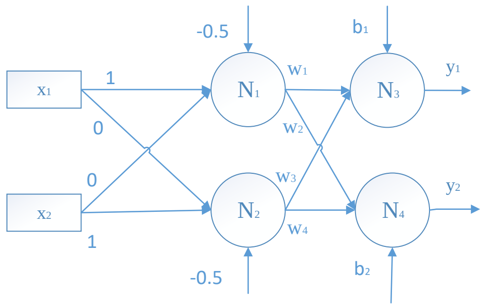
For dataset2, Network 2



Step 1: For N1, the boundary line is $x_1-0.5=0$; For N2, the boundary line is $x_2-0.5=0$, thus if a data points is subject to $x_1>0.5$, the data point belongs blue cluster. If it is subject to $x_2>0.5$,

the data point belongs to blue cluster. If it is subject to $x_1 < 0.5$ and $x_2 < 0.5$, the data point belongs to yellow cluster.

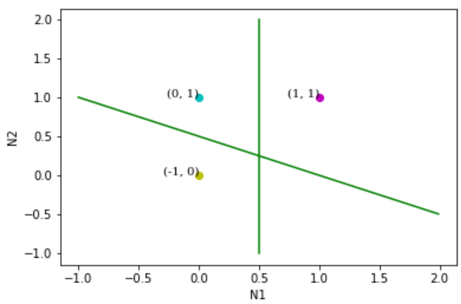
Step 2: Thus, we can have



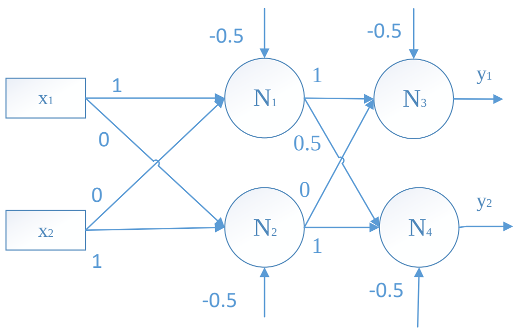
Step 3: Set activator function

$$\sigma(v) = \begin{cases} 1 & v \geq 0 \\ 0 & v < 0 \end{cases}$$

Step 4: Draw the boundary line in the hidden layer.

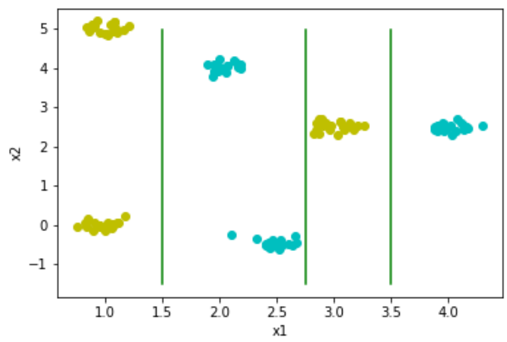


Step 5: The boundary lines are $N_1 - 0.5 = 0$ and $0.5N_1 + N_2 - 0.5 = 0$, thus the final network is



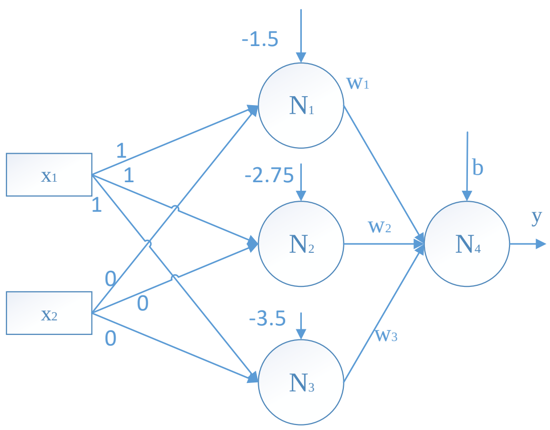
If the output y_1 of the network is 1, the input data point belongs to purple cluster. If the output y_1 of the network is 0, and the output y_2 of the network is 1, the input data point belongs to blue cluster. If the output y_2 of the network is 0, the input data point belongs to yellow cluster.

For dataset3, Network 1



Step 1: For N_1, the boundary line is $x_1 - 1.5 = 0$; For N_2, the boundary line is $x_1 - 2.75 = 0$; For N_3, the boundary line is $x_1 - 3.5 = 0$, thus if a data points is subject to $2.75 < x_1 < 3.5$ or $x_1 < 1.5$, the data point belongs yellow cluster. If it is subject to $1.5 \le x_1 \le 2.75$ or $x_1 > 3.5$, the data point belongs to blue cluster.

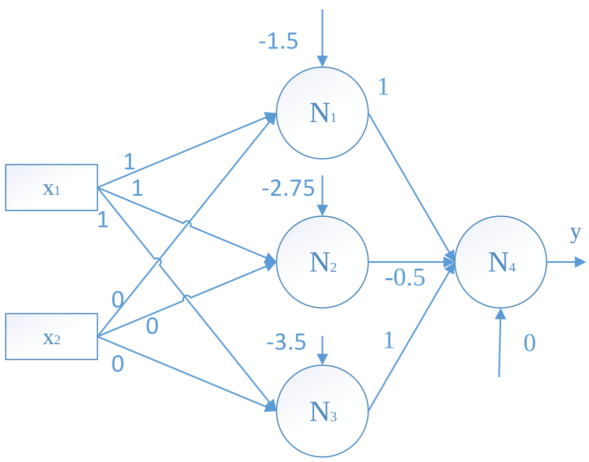
Step 2: Thus, we can have



Step 3: Set activator function

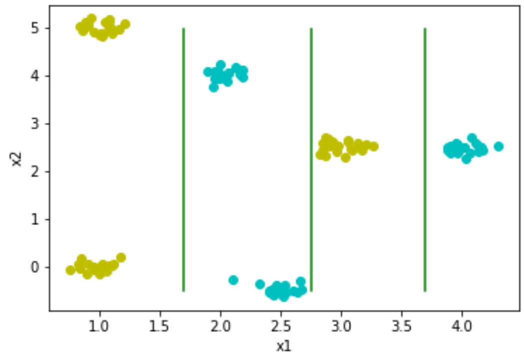
$$\sigma(v) = \begin{cases} 1 & v \ge 0 \\ -1 & v < 0 \end{cases}$$

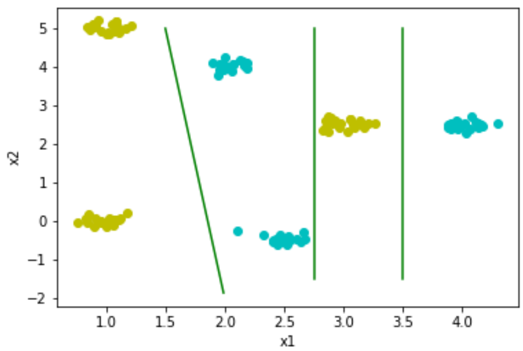
Step 4: One possible boundary line is $N_1 - 0.5N_2 + N_3 = 0$, thus the final network is



If the output of the network is 1, the input data point belongs to blue cluster. If the output of the network is -1, the input data point belongs to yellow cluster.

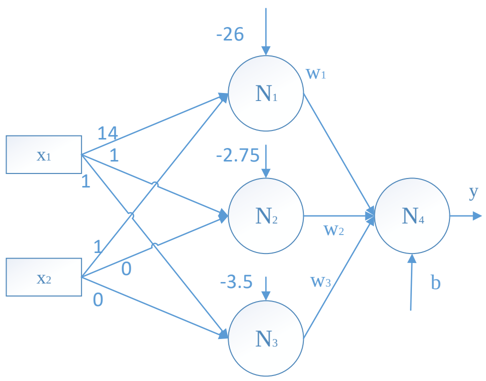
For dataset3, Network 2





Step 1: For N_1, the boundary line is $14x_1 + x_2 - 26 = 0$; For N_2, the boundary line is $x_1 - 2.75 = 0$; For N_3, the boundary line is $x_1 - 3.5 = 0$, thus if a data points is subject to $2.75 < x_1 < 3.5$ or $14x_1 + x_2 - 26 < 0$, the data point belongs yellow cluster. If it is subject to $14x_1 + x_2 - 26 > 0, x_2 < 2.75$ or $x_1 > 3.5$, the data point belongs to blue cluster.

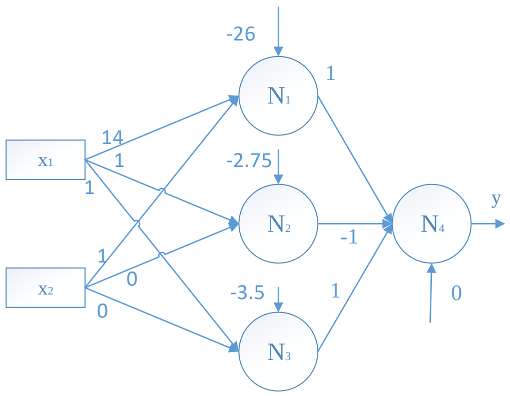
Step 2: Thus, we can have



Step 3: Set activator function

$$\sigma(v) = \begin{cases} 1 & v \geq 0 \\ -1 & v < 0 \end{cases}$$

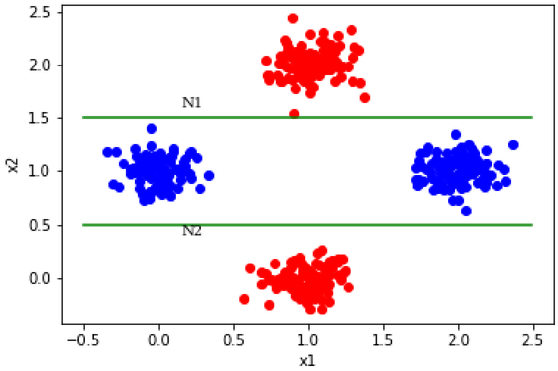
Step 4: One possible boundary line is $N_1 - N_2 + N_3 = 0$, thus the final network is

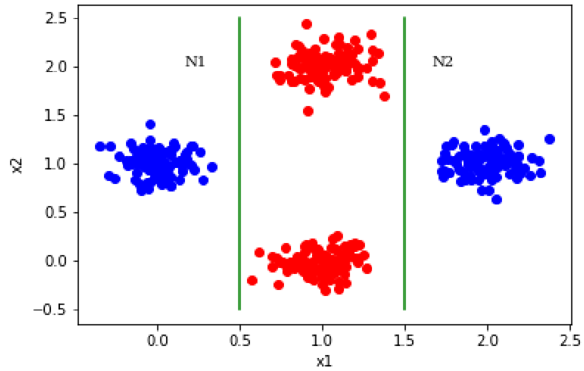


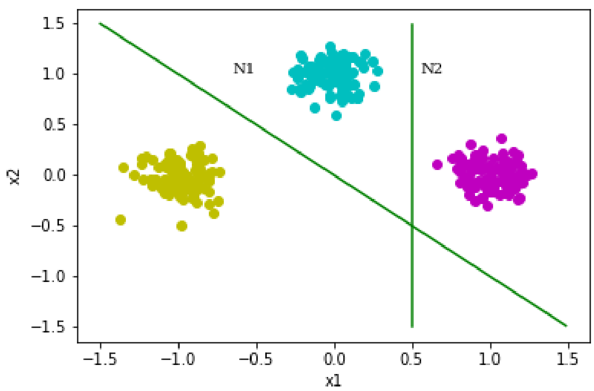
If the output of the network is 1, the input data point belongs to blue cluster. If the output of the network is -1, the input data point belongs to yellow cluster.

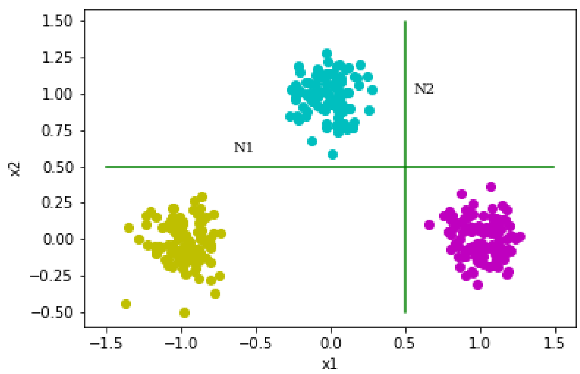
Discussion of Part 2, Question 2:

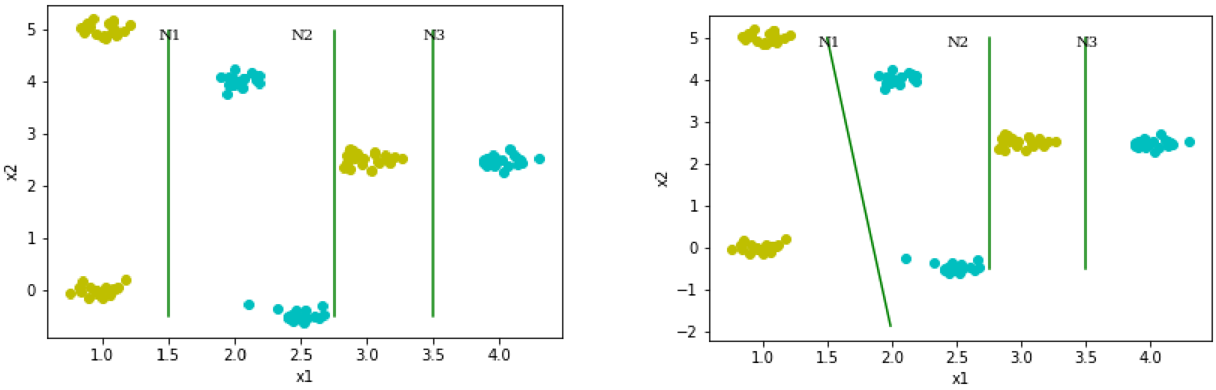
The result of output value y can be found after running my code. The plots of classifying datasets are shown as the following:










N1
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