```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%    Problem Set 4
%    Q1
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear all;
B = 4;

cd('C:\Users\Jasmine\Dropbox\Dropbox\ECON 628\HW1_files\');
addpath('C:\Users\Jasmine\Dropbox\Dropbox\ECON 628\HW1_files\');

load('Data_HW1');
%Draw e_i from i = 1,..., N.
nobs = size(data,1);    % the number of observations = 1457
% read the variables from ``data''
college = data(:,1);
nearc4 = data(:,2);
IQ = data(:,3);
motheduc = data(:,4);
fatheduc = data(:,5);
reg662 = data(:,6);
reg663 = data(:,7);
reg664 = data(:,8);
reg665 = data(:,9);
reg666 = data(:,10);
reg667 = data(:,11);
reg668 = data(:,12);
reg669 = data(:,13);
lwage = data(:,14); % this variable is not used in this exercise
exper = data(:,15); % this variable is not used in this exercise
expersq = data(:,16); % this variable is not used in this exercise
smsa = data(:,17); % this variable is not used in this exercise
south = data(:,18);  % this variable is not used in this exercise
nparm = 13;              % the number of parameters to be estiamted
Y = zeros(nobs,1);       % allocate memory
X = zeros(nobs,nparm);  % allocate memory
Y(:) = college;       % dummy variable for college attendence
X(:,1) = ones(nobs,1);  % the first column is constant term
X(:,2:nparm) =
 [nearc4,IQ,motheduc,fatheduc,reg662,reg663,reg664,reg665,reg666,reg667,reg668,reg

theta = zeros(nparm,1);
options =
 optimset('Display','iter','TolX',1e-6,'TolFun',1e-6, 'MaxIter',
 10000,'MaxFunEvals',10000);
f_ml  = @(b)likelihood(b,X,Y);
[theta_hat,fval,exitflag,output,grad,hessian] =
 fminunc(f_ml,theta,options);
hessian = hessian / nobs;
avar    = hessian^(-1);
se   = sqrt(diag(avar));
%This is for 1.3
```

```
ape_iq = normpdf(X*theta_hat)'*(theta_hat(3)*ones(nobs,1))/nobs;;
fun        = @(b)funIQ(b,X);
G          = gradient(fun,theta_hat);
avar_ape_iq= G'*avar*G;
se_ape_iq = sqrt(avar_ape_iq);
%%1.1 - 1.2
%Bootstrap
%for CI of thetas
T1_1 = zeros(B,1);
T2_1 = zeros(B,1);

T1_2 = zeros(B,1);
T2_2 = zeros(B,1);

T1_3 = zeros(B,1);
T2_3 = zeros(B,1);

%for CI of APE

T_ape_1 = zeros(B,1);
T_ape_2 = zeros(B,1);
T_ape_3 = zeros(B,1);

T_ape_par_1 = zeros(B,1);
T_ape_par_2 = zeros(B,1);
T_ape_par_3 = zeros(B,1);
for k = 1: B
    u = ceil(rand(nobs,1) * nobs);
    epsilon = randn(nobs,1);
    X_b = zeros(size(X));
    Y_b = zeros(size(Y));

    for i = 1:nobs
        bi = u(i);
        X_b(bi,:) = X(bi,:);
        Y_b(bi,:) = Y(bi,:);
    end

    f_ml_b  = @(theta_0)likelihood(theta_0,X_b,Y_b);
    [theta_hat_b,fval,exitflag,output,grad,hessian] =...
        fminunc(f_ml_b,theta,options);
    hessian = hessian / nobs;
    avar_b    = hessian^(-1);
    se_b   = sqrt(diag(avar_b));
    ape_iq_b = normpdf(X*theta_hat_b)'*(theta_hat_b(3)*ones(nobs,1))/
nobs;
    fun        = @(b)funIQ(b,X_b);
    G          = gradient(fun,theta_hat_b);
    avar_ape_iq_b= G'*avar_b*G;
    se_ape_iq_b = sqrt(avar_ape_iq_b);

    T1_1(k) = theta_hat_b(2);
    T2_1(k) = theta_hat_b(3);
    T_ape_1(k) = ape_iq_b;
```

```
    T1_2(k) = theta_hat_b(2) - theta_hat(2);
    T2_2(k) = theta_hat_b(3) - theta_hat(3);
    T_ape_2(k) = ape_iq_b - ape_iq;

    T1_3(k) = (theta_hat_b(2) - theta_hat(1))/se_b(2);
    T2_3(k) = (theta_hat_b(3) - theta_hat(2))/se_b(3);
    T_ape_3(k) = (ape_iq_b - ape_iq)/se_ape_iq_b;

    %%%%Parametric Bootstrap%%%

    Y_par_b = (X_b * theta_hat + epsilon>0);
    f_ml_par_b  = @(theta_0)likelihood(theta_0,X_b,Y_par_b);
    [theta_hat_par_b,fval,exitflag,output,grad,hessian] =...
        fminunc(f_ml_b,theta,options);
    hessian = hessian / nobs;
    avar_par_b    = hessian^(-1);
    se_par_b   = sqrt(diag(avar_par_b));
    ape_iq_par_b =
 normpdf(X*theta_hat_par_b)'*(theta_hat_par_b(3)*ones(nobs,1))/nobs;
    fun        = @(b)funIQ(b,X_b);
    G          = gradient(fun,theta_hat_par_b);
    avar_ape_iq_par_b= G'*avar_b*G;
    se_ape_iq_par_b = sqrt(avar_ape_iq_par_b);
    T_ape_par_1(k) = ape_iq_par_b;
    T_ape_par_2(k) = ape_iq_par_b - ape_iq;
    T_ape_par_3(k) = (ape_iq_par_b - ape_iq)/se_ape_iq_par_b;
end

T1_1 = sort(T1_1,'ascend');
T2_1 = sort(T2_1,'ascend');

T1_2 = sort(T1_2,'ascend');
T2_2 = sort(T2_2,'ascend');

T1_3 = sort(T1_3,'ascend');
T2_3 = sort(T2_3,'ascend');

T_ape_1 = sort(T_ape_1,'ascend');
T_ape_2 = sort(T_ape_2,'ascend');
T_ape_3 = sort(T_ape_3,'ascend');

T_ape_par_1 = sort(T_ape_par_1,'ascend');
T_ape_par_2 = sort(T_ape_par_2,'ascend');
T_ape_par_3 = sort(T_ape_par_3,'ascend');


C1_low_1 = T1_1(ceil(B*0.025));
C1_high_1 = T1_1(ceil(B*0.975));
C2_low_1 = T2_1(ceil(B*0.025));
C2_high_1 = T2_1(ceil(B*0.975));

C1_low_2 = theta_hat(1) - T1_2(ceil(B*0.975));
C1_high_2 = theta_hat(1) - T1_2(ceil(B*0.025));
```

```
C2_low_2 = theta_hat(2) - T2_2(ceil(B*0.975));
C2_high_2 = theta_hat(2) - T2_2(ceil(B*0.025));

C1_low_3 = theta_hat(1) - se(2)*T1_3(ceil(B*0.975));
C1_high_3 = theta_hat(1) - se(2)*T1_3(ceil(B*0.025));
C2_low_3 = theta_hat(2) - se(3)*T2_3(ceil(B*0.975));
C2_high_3 = theta_hat(2) - se(3)*T2_3(ceil(B*0.025));


%%1.1 - 1.2 Print
fprintf('Confidence Interval 1 for nearc4 [%f ,
 %f]\n',C1_low_1,C1_high_1);
fprintf('Confidence Interval 1 for IQ [%f ,
 %f]\n',C2_low_1,C2_high_1);
fprintf('Confidence Interval 2 for nearc4 [%f ,
 %f]\n',C1_low_2,C1_high_2);
fprintf('Confidence Interval 2 for IQ [%f ,
 %f]\n',C2_low_2,C2_high_2);
fprintf('Confidence Interval 3 for nearc4 [%f ,
 %f]\n',C1_low_3,C1_high_3);
fprintf('Confidence Interval 3 for IQ [%f ,
 %f]\n',C2_low_3,C2_high_3);

%%1.3 Confidence interval for average partial effect
fprintf('Confidence Interval 1 for Average Partial Effect of IQ [%f,
 %f]\n',...
    T_ape_1(ceil(B*0.025)), T_ape_1(ceil(B*0.975)));
fprintf('Confidence Interval 2 for Average Partial Effect of IQ [%f,
 %f]\n',...
    ape_iq - T_ape_2(ceil(B*0.975)), ape_iq - T_ape_2(ceil(B*0.025)));
fprintf('Confidence Interval 3 for Average Partial Effect of IQ [%f,
 %f]\n',...
    ape_iq - se_ape_iq*T_ape_3(ceil(B*0.975)), ...
    ape_iq - se_ape_iq*T_ape_3(ceil(B*0.025)));

%%1.4Parametric Bootstrap
fprintf('Confidence Interval 1 for parametric Average Partial Effect
 of IQ [%f, %f]\n',...
    T_ape_par_1(ceil(B*0.025)), T_ape_par_1(ceil(B*0.975)));
fprintf('Confidence Interval 2 for parametric Average Partial Effect
 of IQ [%f, %f]\n',...
    ape_iq - T_ape_par_2(ceil(B*0.975)), ape_iq -
 T_ape_par_2(ceil(B*0.025)));
fprintf('Confidence Interval 3 for parametric Average Partial Effect
 of IQ [%f, %f]\n',...
    ape_iq - se_ape_iq*T_ape_par_3(ceil(B*0.975)), ...
    ape_iq - se_ape_iq*T_ape_par_3(ceil(B*0.025)));
```

```
Warning: Gradient must be provided for trust-region algorithm; using
quasi-newton algorithm instead.
                                                         First-order
 Iteration  Func-count       f(x)        Step-size        optimality
     0          14          1009.92                       4.15e+04
User objective function returned NaN; trying a new point...
```

| | | | | |
|---|---|---|---|---|
| 1 | 44 | 926.103 | 1.0228e-07 | 2.36e+03 |
| 2 | 58 | 925.795 | 1 | 599 |
| 3 | 72 | 925.745 | 1 | 628 |
| 4 | 86 | 925.365 | 1 | 1.55e+03 |
| 5 | 100 | 924.637 | 1 | 3.23e+03 |
| 6 | 114 | 922.676 | 1 | 5.93e+03 |
| 7 | 128 | 918.768 | 1 | 8.7e+03 |
| 8 | 142 | 912.659 | 1 | 9.56e+03 |
| 9 | 156 | 907.378 | 1 | 6.3e+03 |
| 10 | 170 | 905.459 | 1 | 1.85e+03 |
| 11 | 184 | 905.251 | 1 | 164 |
| 12 | 198 | 905.241 | 1 | 134 |
| 13 | 212 | 905.238 | 1 | 135 |
| 14 | 226 | 905.225 | 1 | 134 |
| 15 | 240 | 905.196 | 1 | 140 |
| 16 | 254 | 905.116 | 1 | 235 |
| 17 | 268 | 904.921 | 1 | 374 |
| 18 | 282 | 904.461 | 1 | 560 |
| 19 | 296 | 903.55 | 1 | 724 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 20 | 310 | 902.285 | 1 | 685 |
| 21 | 324 | 901.374 | 1 | 365 |
| 22 | 338 | 901.123 | 1 | 84.6 |
| 23 | 352 | 901.099 | 1 | 59.6 |
| 24 | 366 | 901.098 | 1 | 59.6 |
| 25 | 380 | 901.097 | 1 | 59.6 |
| 26 | 394 | 901.094 | 1 | 59.5 |
| 27 | 408 | 901.085 | 1 | 59.5 |
| 28 | 422 | 901.061 | 1 | 59.4 |
| 29 | 436 | 901.001 | 1 | 70.1 |
| 30 | 450 | 900.841 | 1 | 117 |
| 31 | 464 | 900.426 | 1 | 193 |
| 32 | 478 | 899.35 | 1 | 314 |
| 33 | 492 | 896.61 | 1 | 499 |
| 34 | 506 | 889.949 | 1 | 759 |
| 35 | 520 | 875.545 | 1 | 1.05e+03 |
| 36 | 534 | 851.864 | 1 | 1.19e+03 |
| 37 | 548 | 829.356 | 1 | 957 |
| 38 | 562 | 819.943 | 1 | 374 |
| 39 | 576 | 818.382 | 1 | 85.9 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 40 | 590 | 818.28 | 1 | 83.6 |
| 41 | 604 | 818.276 | 1 | 83.1 |
| 42 | 618 | 818.275 | 1 | 83 |
| 43 | 632 | 818.27 | 1 | 82.7 |
| 44 | 646 | 818.259 | 1 | 139 |
| 45 | 660 | 818.228 | 1 | 274 |
| 46 | 674 | 818.149 | 1 | 489 |
| 47 | 688 | 817.941 | 1 | 840 |
| 48 | 702 | 817.401 | 1 | 1.4e+03 |
| 49 | 716 | 816.006 | 1 | 2.3e+03 |
| 50 | 730 | 812.509 | 1 | 3.63e+03 |

```
    51         744          804.42            1         5.3e+03
    52         758          789.183           1        6.41e+03
    53         772          770.869           1         5.3e+03
    54         786          760.159           1        2.54e+03
    55         800          757.585           1             594
    56         814          757.386           1            33.5
    57         828          757.379           1            24.9
    58         842          757.379           1            24.9
    59         856          757.379           1            24.9
                                                    First-order
Iteration  Func-count       f(x)        Step-size       optimality
    60         870          757.378           1            24.8
    61         884          757.376           1            43.2
    62         898           757.37           1            84.3
    63         912          757.356           1             150
    64         926          757.319           1             256
    65         940          757.222           1             426
    66         954          756.972           1             693
    67         968          756.352           1        1.09e+03
    68         982          754.943           1        1.58e+03
    69         996          752.358           1         1.9e+03
    70        1010          749.346           1        1.55e+03
    71        1024          747.716           1             659
    72        1038          747.402           1             103
    73        1052          747.385           1            13.3
    74        1066          747.384           1            13.1
    75        1080          747.384           1            13.1
    76        1094          747.384           1            13.1
    77        1108          747.383           1            18.6
    78        1122          747.381           1            37.8
    79        1136          747.376           1              68
                                                    First-order
Iteration  Func-count       f(x)        Step-size       optimality
    80        1150          747.363           1             117
    81        1164          747.328           1             197
    82        1178          747.238           1             322
    83        1192           747.01           1             514
    84        1206          746.464           1             779
    85        1220          745.326           1        1.04e+03
    86        1234          743.593           1        1.07e+03
    87        1248          742.147           1             651
    88        1262          741.663           1             177
    89        1276          741.612           1              16
    90        1290           741.61           1            15.6
    91        1304           741.61           1            15.6
    92        1318           741.61           1            15.6
    93        1332          741.609           1            15.5
    94        1346          741.608           1            16.8
    95        1360          741.605           1            31.2
    96        1374          741.597           1            54.4
    97        1388          741.576           1            91.7
    98        1402          741.523           1             151
    99        1416          741.387           1             242
                                                    First-order
```

| Iteration | Func-count | f(x) | Step-size | optimality |
|---|---|---|---|---|
| 100 | 1430 | 741.058 | 1 | 370 |
| 101 | 1444 | 740.355 | 1 | 508 |
| 102 | 1458 | 739.222 | 1 | 550 |
| 103 | 1472 | 738.186 | 1 | 374 |
| 104 | 1486 | 737.794 | 1 | 116 |
| 105 | 1500 | 737.743 | 1 | 10.7 |
| 106 | 1514 | 737.741 | 1 | 10.6 |
| 107 | 1528 | 737.741 | 1 | 10.6 |
| 108 | 1542 | 737.741 | 1 | 10.6 |
| 109 | 1556 | 737.741 | 1 | 10.6 |
| 110 | 1570 | 737.741 | 1 | 10.9 |
| 111 | 1584 | 737.74 | 1 | 20.6 |
| 112 | 1598 | 737.737 | 1 | 36.1 |
| 113 | 1612 | 737.73 | 1 | 61.2 |
| 114 | 1626 | 737.711 | 1 | 101 |
| 115 | 1640 | 737.662 | 1 | 165 |
| 116 | 1654 | 737.539 | 1 | 264 |
| 117 | 1668 | 737.239 | 1 | 402 |
| 118 | 1682 | 736.592 | 1 | 554 |
| 119 | 1696 | 735.528 | 1 | 601 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 120 | 1710 | 734.504 | 1 | 412 |
| 121 | 1724 | 734.089 | 1 | 135 |
| 122 | 1738 | 734.031 | 1 | 13.6 |
| 123 | 1752 | 734.029 | 1 | 13.3 |
| 124 | 1766 | 734.029 | 1 | 13.3 |
| 125 | 1780 | 734.029 | 1 | 13.3 |
| 126 | 1794 | 734.028 | 1 | 13.2 |
| 127 | 1808 | 734.028 | 1 | 13.1 |
| 128 | 1822 | 734.026 | 1 | 13 |
| 129 | 1836 | 734.022 | 1 | 22.8 |
| 130 | 1850 | 734.012 | 1 | 38.8 |
| 131 | 1864 | 733.984 | 1 | 64.4 |
| 132 | 1878 | 733.913 | 1 | 105 |
| 133 | 1892 | 733.732 | 1 | 168 |
| 134 | 1906 | 733.29 | 1 | 258 |
| 135 | 1920 | 732.323 | 1 | 361 |
| 136 | 1934 | 730.678 | 1 | 407 |
| 137 | 1948 | 728.996 | 1 | 299 |
| 138 | 1962 | 728.232 | 1 | 105 |
| 139 | 1976 | 728.112 | 1 | 8.06 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 140 | 1990 | 728.107 | 1 | 7.7 |
| 141 | 2004 | 728.107 | 1 | 7.7 |
| 142 | 2018 | 728.107 | 1 | 7.7 |
| 143 | 2032 | 728.106 | 1 | 7.71 |
| 144 | 2046 | 728.106 | 1 | 7.71 |
| 145 | 2060 | 728.106 | 1 | 11.9 |
| 146 | 2074 | 728.104 | 1 | 21.1 |
| 147 | 2088 | 728.1 | 1 | 36.2 |
| 148 | 2102 | 728.089 | 1 | 60.3 |

| | | | | |
|---|---|---|---|---|
| 149 | 2116 | 728.062 | 1 | 99 |
| 150 | 2130 | 727.991 | 1 | 159 |
| 151 | 2144 | 727.815 | 1 | 248 |
| 152 | 2158 | 727.418 | 1 | 359 |
| 153 | 2172 | 726.684 | 1 | 434 |
| 154 | 2186 | 725.803 | 1 | 358 |
| 155 | 2200 | 725.32 | 1 | 146 |
| 156 | 2214 | 725.232 | 1 | 18.5 |
| 157 | 2228 | 725.226 | 1 | 3.11 |
| 158 | 2242 | 725.226 | 1 | 3.07 |
| 159 | 2256 | 725.226 | 1 | 3.07 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 160 | 2270 | 725.226 | 1 | 3.07 |
| 161 | 2284 | 725.226 | 1 | 3.06 |
| 162 | 2298 | 725.226 | 1 | 3.94 |
| 163 | 2312 | 725.226 | 1 | 7.4 |
| 164 | 2326 | 725.226 | 1 | 12.7 |
| 165 | 2340 | 725.225 | 1 | 21.4 |
| 166 | 2354 | 725.224 | 1 | 35.3 |
| 167 | 2368 | 725.22 | 1 | 57.1 |
| 168 | 2382 | 725.21 | 1 | 89.1 |
| 169 | 2396 | 725.189 | 1 | 129 |
| 170 | 2410 | 725.149 | 1 | 157 |
| 171 | 2424 | 725.1 | 1 | 132 |
| 172 | 2438 | 725.071 | 1 | 58.3 |
| 173 | 2452 | 725.065 | 1 | 9.23 |
| 174 | 2466 | 725.065 | 1 | 0.24 |
| 175 | 2480 | 725.065 | 1 | 0.239 |
| 176 | 2494 | 725.065 | 1 | 0.239 |
| 177 | 2508 | 725.065 | 1 | 0.239 |
| 178 | 2522 | 725.065 | 1 | 0.239 |
| 179 | 2536 | 725.065 | 1 | 0.239 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 180 | 2550 | 725.065 | 1 | 0.341 |
| 181 | 2564 | 725.065 | 1 | 0.564 |
| 182 | 2578 | 725.065 | 1 | 0.932 |
| 183 | 2592 | 725.065 | 1 | 1.49 |
| 184 | 2606 | 725.065 | 1 | 2.37 |
| 185 | 2620 | 725.065 | 1 | 3.66 |
| 186 | 2634 | 725.065 | 1 | 5.24 |
| 187 | 2648 | 725.065 | 1 | 6.32 |
| 188 | 2662 | 725.064 | 1 | 5.27 |
| 189 | 2676 | 725.064 | 1 | 2.29 |
| 190 | 2690 | 725.064 | 1 | 0.355 |
| 191 | 2704 | 725.064 | 1 | 0.00522 |

*Local minimum found.*

*Optimization completed because the size of the gradient is less than the selected value of the function tolerance.*

```
Computing finite-difference Hessian using user-supplied objective
 function.
Warning: Gradient must be provided for trust-region algorithm; using
quasi-newton algorithm instead.
                                                        First-order
 Iteration  Func-count       f(x)        Step-size       optimality
     0          14          1009.92                      2.47e+04
User objective function returned NaN; trying a new point...
     1          44           961.73    1.65507e-07       1.36e+03
     2          58          961.573              1            236
     3          72          961.561              1            247
     4          86          961.465              1            596
     5         100          961.283              1       1.26e+03
     6         114          960.786              1       2.34e+03
     7         128          959.776              1        3.5e+03
     8         142          958.127              1       4.01e+03
     9         156          956.598              1       2.81e+03
    10         170          955.984              1            904
    11         184          955.907              1           89.2
    12         198          955.902              1           81.9
    13         212            955.9              1           82.9
    14         226          955.892              1           84.9
    15         240          955.874              1            141
    16         254          955.824              1            246
    17         268          955.702              1            400
    18         282          955.413              1            607
    19         296          954.838              1            794
                                                        First-order
 Iteration  Func-count       f(x)        Step-size       optimality
    20         310          954.029              1            765
    21         324          953.437              1            420
    22         338          953.268              1           98.8
    23         352          953.251              1           37.3
    24         366           953.25              1           37.2
    25         380           953.25              1           37.2
    26         394          953.247              1           37.2
    27         408           953.24              1           37.1
    28         422          953.222              1           37.1
    29         436          953.175              1           59.8
    30         450          953.053              1           99.1
    31         464          952.734              1            162
    32         478          951.916              1            259
    33         492          949.879              1            403
    34         506          945.208              1            580
    35         520          936.431              1            707
    36         534          925.709              1            631
    37         548           919.36              1            339
    38         562          917.844              1           63.1
    39         576          917.716              1           42.3
                                                        First-order
 Iteration  Func-count       f(x)        Step-size       optimality
    40         590          917.711              1           41.8
```

| | | | | |
|---|---|---|---|---|
| 41 | 604 | 917.71 | 1 | 41.7 |
| 42 | 618 | 917.707 | 1 | 41.4 |
| 43 | 632 | 917.7 | 1 | 54.4 |
| 44 | 646 | 917.681 | 1 | 110 |
| 45 | 660 | 917.632 | 1 | 199 |
| 46 | 674 | 917.502 | 1 | 343 |
| 47 | 688 | 917.163 | 1 | 575 |
| 48 | 702 | 916.286 | 1 | 944 |
| 49 | 716 | 914.068 | 1 | 1.5e+03 |
| 50 | 730 | 908.795 | 1 | 2.25e+03 |
| 51 | 744 | 898.132 | 1 | 2.92e+03 |
| 52 | 758 | 883.099 | 1 | 2.8e+03 |
| 53 | 772 | 871.617 | 1 | 1.69e+03 |
| 54 | 786 | 867.777 | 1 | 530 |
| 55 | 800 | 867.363 | 1 | 49.7 |
| 56 | 814 | 867.348 | 1 | 25.4 |
| 57 | 828 | 867.347 | 1 | 25.3 |
| 58 | 842 | 867.347 | 1 | 25.3 |
| 59 | 856 | 867.346 | 1 | 25.3 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 60 | 870 | 867.343 | 1 | 42.6 |
| 61 | 884 | 867.336 | 1 | 85.7 |
| 62 | 898 | 867.318 | 1 | 154 |
| 63 | 912 | 867.27 | 1 | 266 |
| 64 | 926 | 867.146 | 1 | 446 |
| 65 | 940 | 866.826 | 1 | 730 |
| 66 | 954 | 866.028 | 1 | 1.15e+03 |
| 67 | 968 | 864.191 | 1 | 1.69e+03 |
| 68 | 982 | 860.752 | 1 | 2.05e+03 |
| 69 | 996 | 856.622 | 1 | 1.7e+03 |
| 70 | 1010 | 854.253 | 1 | 760 |
| 71 | 1024 | 853.739 | 1 | 143 |
| 72 | 1038 | 853.707 | 1 | 11 |
| 73 | 1052 | 853.706 | 1 | 11 |
| 74 | 1066 | 853.706 | 1 | 10.9 |
| 75 | 1080 | 853.706 | 1 | 10.9 |
| 76 | 1094 | 853.705 | 1 | 10.9 |
| 77 | 1108 | 853.704 | 1 | 20.9 |
| 78 | 1122 | 853.701 | 1 | 38.4 |
| 79 | 1136 | 853.693 | 1 | 66.9 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 80 | 1150 | 853.671 | 1 | 113 |
| 81 | 1164 | 853.616 | 1 | 186 |
| 82 | 1178 | 853.474 | 1 | 300 |
| 83 | 1192 | 853.127 | 1 | 464 |
| 84 | 1206 | 852.359 | 1 | 655 |
| 85 | 1220 | 851.03 | 1 | 749 |
| 86 | 1234 | 849.633 | 1 | 561 |
| 87 | 1248 | 848.99 | 1 | 209 |
| 88 | 1262 | 848.89 | 1 | 24.3 |
| 89 | 1276 | 848.885 | 1 | 7.8 |
| 90 | 1290 | 848.885 | 1 | 7.75 |

```
    91        1304          848.885              1           7.74
    92        1318          848.884              1           7.72
    93        1332          848.884              1           10.1
    94        1346          848.883              1           20.3
    95        1360           848.88              1           36.1
    96        1374          848.872              1           62.1
    97        1388          848.851              1            104
    98        1402          848.796              1            169
    99        1416          848.659              1            269
                                                        First-order
 Iteration  Func-count       f(x)         Step-size       optimality
   100        1430          848.334              1            404
   101        1444          847.673              1            531
   102        1458          846.719              1            521
   103        1472          845.997              1            296
   104        1486          845.781              1           70.1
   105        1500           845.76              1           7.14
   106        1514          845.759              1           7.16
   107        1528          845.759              1           7.16
   108        1542          845.759              1           7.16
   109        1556          845.759              1           7.16
   110        1570          845.758              1             13
   111        1584          845.756              1           23.6
   112        1598          845.751              1             41
   113        1612          845.739              1           68.9
   114        1626          845.706              1            113
   115        1640          845.623              1            183
   116        1654          845.416              1            285
   117        1668          844.945              1            411
   118        1682          844.075              1            495
   119        1696          843.033              1            412
                                                        First-order
 Iteration  Func-count       f(x)         Step-size       optimality
   120        1710          842.454              1            181
   121        1724          842.341              1           29.4
   122        1738          842.333              1           7.31
   123        1752          842.333              1           7.29
   124        1766          842.333              1           7.29
   125        1780          842.333              1           7.28
   126        1794          842.333              1           7.26
   127        1808          842.332              1           12.3
   128        1822           842.33              1           21.9
   129        1836          842.324              1           37.7
   130        1850           842.31              1           62.9
   131        1864          842.272              1            103
   132        1878          842.175              1            166
   133        1892          841.935              1            259
   134        1906          841.392              1            372
   135        1920            840.4              1            444
   136        1934          839.245              1            360
   137        1948          838.615              1            151
   138        1962          838.492              1           22.2
   139        1976          838.484              1           5.49
                                                        First-order
```

| Iteration | Func-count | f(x) | Step-size | optimality |
|---|---|---|---|---|
| 140 | 1990 | 838.484 | 1 | 5.5 |
| 141 | 2004 | 838.484 | 1 | 5.5 |
| 142 | 2018 | 838.484 | 1 | 5.5 |
| 143 | 2032 | 838.484 | 1 | 5.5 |
| 144 | 2046 | 838.483 | 1 | 8.12 |
| 145 | 2060 | 838.482 | 1 | 14.4 |
| 146 | 2074 | 838.478 | 1 | 24.7 |
| 147 | 2088 | 838.467 | 1 | 41.3 |
| 148 | 2102 | 838.441 | 1 | 67.7 |
| 149 | 2116 | 838.372 | 1 | 109 |
| 150 | 2130 | 838.205 | 1 | 168 |
| 151 | 2144 | 837.838 | 1 | 236 |
| 152 | 2158 | 837.207 | 1 | 269 |
| 153 | 2172 | 836.549 | 1 | 194 |
| 154 | 2186 | 836.255 | 1 | 62.6 |
| 155 | 2200 | 836.211 | 1 | 3.72 |
| 156 | 2214 | 836.209 | 1 | 2.28 |
| 157 | 2228 | 836.209 | 1 | 2.27 |
| 158 | 2242 | 836.209 | 1 | 2.26 |
| 159 | 2256 | 836.209 | 1 | 2.26 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 160 | 2270 | 836.209 | 1 | 2.26 |
| 161 | 2284 | 836.209 | 1 | 4.22 |
| 162 | 2298 | 836.209 | 1 | 7.48 |
| 163 | 2312 | 836.208 | 1 | 13.2 |
| 164 | 2326 | 836.208 | 1 | 22.1 |
| 165 | 2340 | 836.206 | 1 | 36.5 |
| 166 | 2354 | 836.202 | 1 | 58.8 |
| 167 | 2368 | 836.191 | 1 | 91.5 |
| 168 | 2382 | 836.166 | 1 | 131 |
| 169 | 2396 | 836.122 | 1 | 156 |
| 170 | 2410 | 836.072 | 1 | 125 |
| 171 | 2424 | 836.045 | 1 | 51.1 |
| 172 | 2438 | 836.04 | 1 | 7.04 |
| 173 | 2452 | 836.039 | 1 | 0.347 |
| 174 | 2466 | 836.039 | 1 | 0.342 |
| 175 | 2480 | 836.039 | 1 | 0.342 |
| 176 | 2494 | 836.039 | 1 | 0.342 |
| 177 | 2508 | 836.039 | 1 | 0.341 |
| 178 | 2522 | 836.039 | 1 | 0.442 |
| 179 | 2536 | 836.039 | 1 | 0.794 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 180 | 2550 | 836.039 | 1 | 1.4 |
| 181 | 2564 | 836.039 | 1 | 2.26 |
| 182 | 2578 | 836.039 | 1 | 3.81 |
| 183 | 2592 | 836.039 | 1 | 6.14 |
| 184 | 2606 | 836.039 | 1 | 9.56 |
| 185 | 2620 | 836.039 | 1 | 13.8 |
| 186 | 2634 | 836.038 | 1 | 16.8 |
| 187 | 2648 | 836.037 | 1 | 14.2 |
| 188 | 2662 | 836.037 | 1 | 6.26 |

```
   189        2676          836.037             1              1
   190        2690          836.037             1        0.00947

Local minimum found.

Optimization completed because the size of the gradient is less than
the selected value of the function tolerance.



Computing finite-difference Hessian using user-supplied objective
 function.
Warning: Gradient must be provided for trust-region algorithm; using
quasi-newton algorithm instead.
                                                          First-order
 Iteration  Func-count       f(x)        Step-size        optimality
     0          14          1009.92                       2.47e+04
User objective function returned NaN; trying a new point...
     1          44           961.73    1.65507e-07       1.36e+03
     2          58          961.573             1            236
     3          72          961.561             1            247
     4          86          961.465             1            596
     5         100          961.283             1       1.26e+03
     6         114          960.786             1       2.34e+03
     7         128          959.776             1        3.5e+03
     8         142          958.127             1       4.01e+03
     9         156          956.598             1       2.81e+03
    10         170          955.984             1            904
    11         184          955.907             1           89.2
    12         198          955.902             1           81.9
    13         212            955.9             1           82.9
    14         226          955.892             1           84.9
    15         240          955.874             1            141
    16         254          955.824             1            246
    17         268          955.702             1            400
    18         282          955.413             1            607
    19         296          954.838             1            794
                                                          First-order
 Iteration  Func-count       f(x)        Step-size        optimality
    20         310          954.029             1            765
    21         324          953.437             1            420
    22         338          953.268             1           98.8
    23         352          953.251             1           37.3
    24         366           953.25             1           37.2
    25         380           953.25             1           37.2
    26         394          953.247             1           37.2
    27         408           953.24             1           37.1
    28         422          953.222             1           37.1
    29         436          953.175             1           59.8
    30         450          953.053             1           99.1
    31         464          952.734             1            162
    32         478          951.916             1            259
    33         492          949.879             1            403
```

| | | | | |
|---|---|---|---|---|
| 34 | 506 | 945.208 | 1 | 580 |
| 35 | 520 | 936.431 | 1 | 707 |
| 36 | 534 | 925.709 | 1 | 631 |
| 37 | 548 | 919.36 | 1 | 339 |
| 38 | 562 | 917.844 | 1 | 63.1 |
| 39 | 576 | 917.716 | 1 | 42.3 |
| Iteration | Func-count | f(x) | Step-size | First-order optimality |
| 40 | 590 | 917.711 | 1 | 41.8 |
| 41 | 604 | 917.71 | 1 | 41.7 |
| 42 | 618 | 917.707 | 1 | 41.4 |
| 43 | 632 | 917.7 | 1 | 54.4 |
| 44 | 646 | 917.681 | 1 | 110 |
| 45 | 660 | 917.632 | 1 | 199 |
| 46 | 674 | 917.502 | 1 | 343 |
| 47 | 688 | 917.163 | 1 | 575 |
| 48 | 702 | 916.286 | 1 | 944 |
| 49 | 716 | 914.068 | 1 | 1.5e+03 |
| 50 | 730 | 908.795 | 1 | 2.25e+03 |
| 51 | 744 | 898.132 | 1 | 2.92e+03 |
| 52 | 758 | 883.099 | 1 | 2.8e+03 |
| 53 | 772 | 871.617 | 1 | 1.69e+03 |
| 54 | 786 | 867.777 | 1 | 530 |
| 55 | 800 | 867.363 | 1 | 49.7 |
| 56 | 814 | 867.348 | 1 | 25.4 |
| 57 | 828 | 867.347 | 1 | 25.3 |
| 58 | 842 | 867.347 | 1 | 25.3 |
| 59 | 856 | 867.346 | 1 | 25.3 |
| Iteration | Func-count | f(x) | Step-size | First-order optimality |
| 60 | 870 | 867.343 | 1 | 42.6 |
| 61 | 884 | 867.336 | 1 | 85.7 |
| 62 | 898 | 867.318 | 1 | 154 |
| 63 | 912 | 867.27 | 1 | 266 |
| 64 | 926 | 867.146 | 1 | 446 |
| 65 | 940 | 866.826 | 1 | 730 |
| 66 | 954 | 866.028 | 1 | 1.15e+03 |
| 67 | 968 | 864.191 | 1 | 1.69e+03 |
| 68 | 982 | 860.752 | 1 | 2.05e+03 |
| 69 | 996 | 856.622 | 1 | 1.7e+03 |
| 70 | 1010 | 854.253 | 1 | 760 |
| 71 | 1024 | 853.739 | 1 | 143 |
| 72 | 1038 | 853.707 | 1 | 11 |
| 73 | 1052 | 853.706 | 1 | 11 |
| 74 | 1066 | 853.706 | 1 | 10.9 |
| 75 | 1080 | 853.706 | 1 | 10.9 |
| 76 | 1094 | 853.705 | 1 | 10.9 |
| 77 | 1108 | 853.704 | 1 | 20.9 |
| 78 | 1122 | 853.701 | 1 | 38.4 |
| 79 | 1136 | 853.693 | 1 | 66.9 |
| Iteration | Func-count | f(x) | Step-size | First-order optimality |
| 80 | 1150 | 853.671 | 1 | 113 |
| 81 | 1164 | 853.616 | 1 | 186 |

```
    82        1178         853.474            1          300
    83        1192         853.127            1          464
    84        1206         852.359            1          655
    85        1220          851.03            1          749
    86        1234         849.633            1          561
    87        1248          848.99            1          209
    88        1262          848.89            1         24.3
    89        1276         848.885            1          7.8
    90        1290         848.885            1         7.75
    91        1304         848.885            1         7.74
    92        1318         848.884            1         7.72
    93        1332         848.884            1         10.1
    94        1346         848.883            1         20.3
    95        1360          848.88            1         36.1
    96        1374         848.872            1         62.1
    97        1388         848.851            1          104
    98        1402         848.796            1          169
    99        1416         848.659            1          269

                                                    First-order
Iteration  Func-count       f(x)          Step-size   optimality
   100        1430         848.334            1          404
   101        1444         847.673            1          531
   102        1458         846.719            1          521
   103        1472         845.997            1          296
   104        1486         845.781            1         70.1
   105        1500          845.76            1         7.14
   106        1514         845.759            1         7.16
   107        1528         845.759            1         7.16
   108        1542         845.759            1         7.16
   109        1556         845.759            1         7.16
   110        1570         845.758            1           13
   111        1584         845.756            1         23.6
   112        1598         845.751            1           41
   113        1612         845.739            1         68.9
   114        1626         845.706            1          113
   115        1640         845.623            1          183
   116        1654         845.416            1          285
   117        1668         844.945            1          411
   118        1682         844.075            1          495
   119        1696         843.033            1          412

                                                    First-order
Iteration  Func-count       f(x)          Step-size   optimality
   120        1710         842.454            1          181
   121        1724         842.341            1         29.4
   122        1738         842.333            1         7.31
   123        1752         842.333            1         7.29
   124        1766         842.333            1         7.29
   125        1780         842.333            1         7.28
   126        1794         842.333            1         7.26
   127        1808         842.332            1         12.3
   128        1822          842.33            1         21.9
   129        1836         842.324            1         37.7
   130        1850          842.31            1         62.9
   131        1864         842.272            1          103
```

```
   132        1878          842.175             1           166
   133        1892          841.935             1           259
   134        1906          841.392             1           372
   135        1920            840.4             1           444
   136        1934          839.245             1           360
   137        1948          838.615             1           151
   138        1962          838.492             1          22.2
   139        1976          838.484             1          5.49
                                                        First-order
 Iteration  Func-count       f(x)         Step-size       optimality
   140        1990          838.484             1           5.5
   141        2004          838.484             1           5.5
   142        2018          838.484             1           5.5
   143        2032          838.484             1           5.5
   144        2046          838.483             1          8.12
   145        2060          838.482             1          14.4
   146        2074          838.478             1          24.7
   147        2088          838.467             1          41.3
   148        2102          838.441             1          67.7
   149        2116          838.372             1           109
   150        2130          838.205             1           168
   151        2144          837.838             1           236
   152        2158          837.207             1           269
   153        2172          836.549             1           194
   154        2186          836.255             1          62.6
   155        2200          836.211             1          3.72
   156        2214          836.209             1          2.28
   157        2228          836.209             1          2.27
   158        2242          836.209             1          2.26
   159        2256          836.209             1          2.26
                                                        First-order
 Iteration  Func-count       f(x)         Step-size       optimality
   160        2270          836.209             1          2.26
   161        2284          836.209             1          4.22
   162        2298          836.209             1          7.48
   163        2312          836.208             1          13.2
   164        2326          836.208             1          22.1
   165        2340          836.206             1          36.5
   166        2354          836.202             1          58.8
   167        2368          836.191             1          91.5
   168        2382          836.166             1           131
   169        2396          836.122             1           156
   170        2410          836.072             1           125
   171        2424          836.045             1          51.1
   172        2438           836.04             1          7.04
   173        2452          836.039             1         0.347
   174        2466          836.039             1         0.342
   175        2480          836.039             1         0.342
   176        2494          836.039             1         0.342
   177        2508          836.039             1         0.341
   178        2522          836.039             1         0.442
   179        2536          836.039             1         0.794
                                                        First-order
 Iteration  Func-count       f(x)         Step-size       optimality
```

| | | | | |
|---|---|---|---|---|
| 180 | 2550 | 836.039 | 1 | 1.4 |
| 181 | 2564 | 836.039 | 1 | 2.26 |
| 182 | 2578 | 836.039 | 1 | 3.81 |
| 183 | 2592 | 836.039 | 1 | 6.14 |
| 184 | 2606 | 836.039 | 1 | 9.56 |
| 185 | 2620 | 836.039 | 1 | 13.8 |
| 186 | 2634 | 836.038 | 1 | 16.8 |
| 187 | 2648 | 836.037 | 1 | 14.2 |
| 188 | 2662 | 836.037 | 1 | 6.26 |
| 189 | 2676 | 836.037 | 1 | 1 |
| 190 | 2690 | 836.037 | 1 | 0.00947 |

Local minimum found.

Optimization completed because the size of the gradient is less than the selected value of the function tolerance.

Computing finite-difference Hessian using user-supplied objective function.
Warning: Gradient must be provided for trust-region algorithm; using quasi-newton algorithm instead.

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 0 | 14 | 1009.92 | | 2.64e+04 |
| User objective function returned NaN; trying a new point... | | | | |
| 1 | 44 | 956.402 | 1.61364e-07 | 1.48e+03 |
| 2 | 58 | 956.211 | 1 | 359 |
| 3 | 72 | 956.18 | 1 | 377 |
| 4 | 86 | 955.945 | 1 | 970 |
| 5 | 100 | 955.496 | 1 | 2.02e+03 |
| 6 | 114 | 954.286 | 1 | 3.7e+03 |
| 7 | 128 | 951.881 | 1 | 5.42e+03 |
| 8 | 142 | 948.145 | 1 | 5.93e+03 |
| 9 | 156 | 944.951 | 1 | 3.87e+03 |
| 10 | 170 | 943.809 | 1 | 1.12e+03 |
| 11 | 184 | 943.688 | 1 | 96.3 |
| 12 | 198 | 943.683 | 1 | 54.5 |
| 13 | 212 | 943.682 | 1 | 54.1 |
| 14 | 226 | 943.678 | 1 | 52.6 |
| 15 | 240 | 943.668 | 1 | 50.3 |
| 16 | 254 | 943.64 | 1 | 49.5 |
| 17 | 268 | 943.573 | 1 | 74.2 |
| 18 | 282 | 943.407 | 1 | 109 |
| 19 | 296 | 943.052 | 1 | 147 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 20 | 310 | 942.463 | 1 | 158 |
| 21 | 324 | 941.889 | 1 | 106 |
| 22 | 338 | 941.65 | 1 | 45.5 |
| 23 | 352 | 941.616 | 1 | 36.1 |
| 24 | 366 | 941.614 | 1 | 35.8 |

| | | | | |
|---|---|---|---|---|
| 25 | 380 | 941.613 | 1 | 35.8 |
| 26 | 394 | 941.609 | 1 | 35.7 |
| 27 | 408 | 941.602 | 1 | 35.7 |
| 28 | 422 | 941.581 | 1 | 35.7 |
| 29 | 436 | 941.527 | 1 | 40.6 |
| 30 | 450 | 941.385 | 1 | 69.3 |
| 31 | 464 | 941.019 | 1 | 115 |
| 32 | 478 | 940.075 | 1 | 187 |
| 33 | 492 | 937.73 | 1 | 292 |
| 34 | 506 | 932.361 | 1 | 423 |
| 35 | 520 | 922.29 | 1 | 527 |
| 36 | 534 | 909.95 | 1 | 492 |
| 37 | 548 | 902.654 | 1 | 261 |
| 38 | 562 | 900.948 | 1 | 31.9 |
| 39 | 576 | 900.795 | 1 | 32.7 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 40 | 590 | 900.789 | 1 | 30 |
| 41 | 604 | 900.789 | 1 | 29.9 |
| 42 | 618 | 900.787 | 1 | 29.9 |
| 43 | 632 | 900.784 | 1 | 47.6 |
| 44 | 646 | 900.776 | 1 | 96.5 |
| 45 | 660 | 900.754 | 1 | 176 |
| 46 | 674 | 900.697 | 1 | 304 |
| 47 | 688 | 900.549 | 1 | 513 |
| 48 | 702 | 900.161 | 1 | 848 |
| 49 | 716 | 899.159 | 1 | 1.38e+03 |
| 50 | 730 | 896.651 | 1 | 2.17e+03 |
| 51 | 744 | 890.849 | 1 | 3.17e+03 |
| 52 | 758 | 879.868 | 1 | 3.87e+03 |
| 53 | 772 | 866.328 | 1 | 3.29e+03 |
| 54 | 786 | 858.08 | 1 | 1.63e+03 |
| 55 | 800 | 856.138 | 1 | 381 |
| 56 | 814 | 855.995 | 1 | 17.6 |
| 57 | 828 | 855.991 | 1 | 17.3 |
| 58 | 842 | 855.99 | 1 | 17.2 |
| 59 | 856 | 855.99 | 1 | 17.2 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 60 | 870 | 855.989 | 1 | 18.4 |
| 61 | 884 | 855.987 | 1 | 41 |
| 62 | 898 | 855.982 | 1 | 79.9 |
| 63 | 912 | 855.969 | 1 | 142 |
| 64 | 926 | 855.935 | 1 | 242 |
| 65 | 940 | 855.847 | 1 | 402 |
| 66 | 954 | 855.621 | 1 | 652 |
| 67 | 968 | 855.066 | 1 | 1.01e+03 |
| 68 | 982 | 853.846 | 1 | 1.42e+03 |
| 69 | 996 | 851.782 | 1 | 1.58e+03 |
| 70 | 1010 | 849.749 | 1 | 1.12e+03 |
| 71 | 1024 | 848.873 | 1 | 400 |
| 72 | 1038 | 848.737 | 1 | 48.9 |
| 73 | 1052 | 848.731 | 1 | 13.4 |
| 74 | 1066 | 848.731 | 1 | 13.4 |

```
    75        1080          848.731            1          13.3
    76        1094          848.731            1          13.3
    77        1108           848.73            1          13.2
    78        1122          848.728            1          19.9
    79        1136          848.722            1          35.5

                                                      First-order
Iteration  Func-count       f(x)          Step-size      optimality
    80        1150          848.707            1          61.1
    81        1164          848.667            1           102
    82        1178          848.566            1           166
    83        1192          848.312            1           261
    84        1206          847.729            1           383
    85        1220          846.619            1           477
    86        1234          845.202            1           425
    87        1248          844.287            1           209
    88        1262          844.076            1          37.4
    89        1276          844.062            1          7.94
    90        1290          844.062            1          7.94
    91        1304          844.062            1          7.93
    92        1318          844.062            1          7.93
    93        1332          844.061            1          7.92
    94        1346           844.06            1          15.5
    95        1360          844.057            1          27.9
    96        1374           844.05            1          48.2
    97        1388          844.031            1          80.8
    98        1402           843.98            1           133
    99        1416          843.851            1           214

                                                      First-order
Iteration  Func-count       f(x)          Step-size      optimality
   100        1430          843.532            1           332
   101        1444          842.822            1           472
   102        1458          841.575            1           550
   103        1472          840.267            1           424
   104        1486          839.664            1           159
   105        1500           839.56            1          12.7
   106        1514          839.556            1          5.96
   107        1528          839.556            1          5.92
   108        1542          839.556            1          5.91
   109        1556          839.555            1          5.89
   110        1570          839.555            1          8.91
   111        1584          839.554            1          17.2
   112        1598          839.551            1          30.1
   113        1612          839.543            1          51.3
   114        1626          839.524            1          84.9
   115        1640          839.474            1           137
   116        1654          839.351            1           214
   117        1668          839.071            1           309
   118        1682          838.555            1           372
   119        1696          837.939            1           310

                                                      First-order
Iteration  Func-count       f(x)          Step-size      optimality
   120        1710          837.595            1           137
   121        1724          837.528            1          21.4
   122        1738          837.524            1          6.31
```

| | | | | |
|---|---|---|---|---|
| 123 | 1752 | 837.524 | 1 | 6.24 |
| 124 | 1766 | 837.524 | 1 | 6.23 |
| 125 | 1780 | 837.524 | 1 | 6.19 |
| 126 | 1794 | 837.523 | 1 | 6.15 |
| 127 | 1808 | 837.523 | 1 | 11.4 |
| 128 | 1822 | 837.521 | 1 | 20.3 |
| 129 | 1836 | 837.516 | 1 | 35 |
| 130 | 1850 | 837.503 | 1 | 58.4 |
| 131 | 1864 | 837.469 | 1 | 95.7 |
| 132 | 1878 | 837.385 | 1 | 153 |
| 133 | 1892 | 837.181 | 1 | 231 |
| 134 | 1906 | 836.755 | 1 | 311 |
| 135 | 1920 | 836.096 | 1 | 321 |
| 136 | 1934 | 835.529 | 1 | 200 |
| 137 | 1948 | 835.327 | 1 | 56.1 |
| 138 | 1962 | 835.304 | 1 | 5.81 |
| 139 | 1976 | 835.303 | 1 | 5.72 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 140 | 1990 | 835.303 | 1 | 5.71 |
| 141 | 2004 | 835.303 | 1 | 5.7 |
| 142 | 2018 | 835.303 | 1 | 5.68 |
| 143 | 2032 | 835.303 | 1 | 5.65 |
| 144 | 2046 | 835.302 | 1 | 6.66 |
| 145 | 2060 | 835.301 | 1 | 11.6 |
| 146 | 2074 | 835.297 | 1 | 19.7 |
| 147 | 2088 | 835.288 | 1 | 32.6 |
| 148 | 2102 | 835.263 | 1 | 53.4 |
| 149 | 2116 | 835.199 | 1 | 85.7 |
| 150 | 2130 | 835.04 | 1 | 133 |
| 151 | 2144 | 834.681 | 1 | 192 |
| 152 | 2158 | 834.018 | 1 | 232 |
| 153 | 2172 | 833.234 | 1 | 192 |
| 154 | 2186 | 832.807 | 1 | 78.4 |
| 155 | 2200 | 832.724 | 1 | 8.37 |
| 156 | 2214 | 832.718 | 1 | 3.06 |
| 157 | 2228 | 832.718 | 1 | 3.05 |
| 158 | 2242 | 832.718 | 1 | 3.04 |
| 159 | 2256 | 832.718 | 1 | 3.04 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 160 | 2270 | 832.718 | 1 | 3.03 |
| 161 | 2284 | 832.718 | 1 | 3.7 |
| 162 | 2298 | 832.717 | 1 | 6.65 |
| 163 | 2312 | 832.717 | 1 | 11.4 |
| 164 | 2326 | 832.715 | 1 | 19.1 |
| 165 | 2340 | 832.712 | 1 | 31.4 |
| 166 | 2354 | 832.702 | 1 | 50.7 |
| 167 | 2368 | 832.679 | 1 | 79.2 |
| 168 | 2382 | 832.625 | 1 | 115 |
| 169 | 2396 | 832.525 | 1 | 141 |
| 170 | 2410 | 832.401 | 1 | 120 |
| 171 | 2424 | 832.327 | 1 | 54.1 |
| 172 | 2438 | 832.31 | 1 | 8.69 |

```
   173        2452          832.309              1          0.831
   174        2466          832.309              1           0.82
   175        2480          832.309              1           0.82
   176        2494          832.309              1          0.819
   177        2508          832.309              1          0.819
   178        2522          832.309              1          0.818
   179        2536          832.309              1           1.11

 Iteration  Func-count       f(x)        Step-size       First-order
                                                          optimality
   180        2550          832.309              1           1.97
   181        2564          832.309              1           3.29
   182        2578          832.309              1           5.44
   183        2592          832.309              1            8.8
   184        2606          832.308              1           13.9
   185        2620          832.306              1           20.5
   186        2634          832.302              1           26.3
   187        2648          832.297              1           24.6
   188        2662          832.294              1           12.9
   189        2676          832.293              1           2.73
   190        2690          832.293              1         0.0578
   191        2704          832.293              1         0.0318
   192        2718          832.293              1        0.00295

Local minimum found.

Optimization completed because the size of the gradient is less than
the selected value of the function tolerance.



Computing finite-difference Hessian using user-supplied objective
 function.
Warning: Gradient must be provided for trust-region algorithm; using
quasi-newton algorithm instead.

 Iteration  Func-count       f(x)        Step-size       First-order
                                                          optimality
     0          14          1009.92                       2.64e+04
User objective function returned NaN; trying a new point...
     1          44          956.402    1.61364e-07        1.48e+03
     2          58          956.211              1             359
     3          72           956.18              1             377
     4          86          955.945              1             970
     5         100          955.496              1        2.02e+03
     6         114          954.286              1         3.7e+03
     7         128          951.881              1        5.42e+03
     8         142          948.145              1        5.93e+03
     9         156          944.951              1        3.87e+03
    10         170          943.809              1        1.12e+03
    11         184          943.688              1            96.3
    12         198          943.683              1            54.5
    13         212          943.682              1            54.1
    14         226          943.678              1            52.6
    15         240          943.668              1            50.3
```

```
    16         254          943.64              1           49.5
    17         268         943.573              1           74.2
    18         282         943.407              1            109
    19         296         943.052              1            147

                                                      First-order
Iteration  Func-count       f(x)        Step-size       optimality
    20         310         942.463              1            158
    21         324         941.889              1            106
    22         338          941.65              1           45.5
    23         352         941.616              1           36.1
    24         366         941.614              1           35.8
    25         380         941.613              1           35.8
    26         394         941.609              1           35.7
    27         408         941.602              1           35.7
    28         422         941.581              1           35.7
    29         436         941.527              1           40.6
    30         450         941.385              1           69.3
    31         464         941.019              1            115
    32         478         940.075              1            187
    33         492          937.73              1            292
    34         506         932.361              1            423
    35         520          922.29              1            527
    36         534          909.95              1            492
    37         548         902.654              1            261
    38         562         900.948              1           31.9
    39         576         900.795              1           32.7

                                                      First-order
Iteration  Func-count       f(x)        Step-size       optimality
    40         590         900.789              1             30
    41         604         900.789              1           29.9
    42         618         900.787              1           29.9
    43         632         900.784              1           47.6
    44         646         900.776              1           96.5
    45         660         900.754              1            176
    46         674         900.697              1            304
    47         688         900.549              1            513
    48         702         900.161              1            848
    49         716         899.159              1       1.38e+03
    50         730         896.651              1       2.17e+03
    51         744         890.849              1       3.17e+03
    52         758         879.868              1       3.87e+03
    53         772         866.328              1       3.29e+03
    54         786          858.08              1       1.63e+03
    55         800         856.138              1            381
    56         814         855.995              1           17.6
    57         828         855.991              1           17.3
    58         842          855.99              1           17.2
    59         856          855.99              1           17.2

                                                      First-order
Iteration  Func-count       f(x)        Step-size       optimality
    60         870         855.989              1           18.4
    61         884         855.987              1             41
    62         898         855.982              1           79.9
    63         912         855.969              1            142
```

| | | | | |
|---|---|---|---|---|
| 64 | 926 | 855.935 | 1 | 242 |
| 65 | 940 | 855.847 | 1 | 402 |
| 66 | 954 | 855.621 | 1 | 652 |
| 67 | 968 | 855.066 | 1 | 1.01e+03 |
| 68 | 982 | 853.846 | 1 | 1.42e+03 |
| 69 | 996 | 851.782 | 1 | 1.58e+03 |
| 70 | 1010 | 849.749 | 1 | 1.12e+03 |
| 71 | 1024 | 848.873 | 1 | 400 |
| 72 | 1038 | 848.737 | 1 | 48.9 |
| 73 | 1052 | 848.731 | 1 | 13.4 |
| 74 | 1066 | 848.731 | 1 | 13.4 |
| 75 | 1080 | 848.731 | 1 | 13.3 |
| 76 | 1094 | 848.731 | 1 | 13.3 |
| 77 | 1108 | 848.73 | 1 | 13.2 |
| 78 | 1122 | 848.728 | 1 | 19.9 |
| 79 | 1136 | 848.722 | 1 | 35.5 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 80 | 1150 | 848.707 | 1 | 61.1 |
| 81 | 1164 | 848.667 | 1 | 102 |
| 82 | 1178 | 848.566 | 1 | 166 |
| 83 | 1192 | 848.312 | 1 | 261 |
| 84 | 1206 | 847.729 | 1 | 383 |
| 85 | 1220 | 846.619 | 1 | 477 |
| 86 | 1234 | 845.202 | 1 | 425 |
| 87 | 1248 | 844.287 | 1 | 209 |
| 88 | 1262 | 844.076 | 1 | 37.4 |
| 89 | 1276 | 844.062 | 1 | 7.94 |
| 90 | 1290 | 844.062 | 1 | 7.94 |
| 91 | 1304 | 844.062 | 1 | 7.93 |
| 92 | 1318 | 844.062 | 1 | 7.93 |
| 93 | 1332 | 844.061 | 1 | 7.92 |
| 94 | 1346 | 844.06 | 1 | 15.5 |
| 95 | 1360 | 844.057 | 1 | 27.9 |
| 96 | 1374 | 844.05 | 1 | 48.2 |
| 97 | 1388 | 844.031 | 1 | 80.8 |
| 98 | 1402 | 843.98 | 1 | 133 |
| 99 | 1416 | 843.851 | 1 | 214 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 100 | 1430 | 843.532 | 1 | 332 |
| 101 | 1444 | 842.822 | 1 | 472 |
| 102 | 1458 | 841.575 | 1 | 550 |
| 103 | 1472 | 840.267 | 1 | 424 |
| 104 | 1486 | 839.664 | 1 | 159 |
| 105 | 1500 | 839.56 | 1 | 12.7 |
| 106 | 1514 | 839.556 | 1 | 5.96 |
| 107 | 1528 | 839.556 | 1 | 5.92 |
| 108 | 1542 | 839.556 | 1 | 5.91 |
| 109 | 1556 | 839.555 | 1 | 5.89 |
| 110 | 1570 | 839.555 | 1 | 8.91 |
| 111 | 1584 | 839.554 | 1 | 17.2 |
| 112 | 1598 | 839.551 | 1 | 30.1 |
| 113 | 1612 | 839.543 | 1 | 51.3 |

| | | | | |
|---|---|---|---|---|
| 114 | 1626 | 839.524 | 1 | 84.9 |
| 115 | 1640 | 839.474 | 1 | 137 |
| 116 | 1654 | 839.351 | 1 | 214 |
| 117 | 1668 | 839.071 | 1 | 309 |
| 118 | 1682 | 838.555 | 1 | 372 |
| 119 | 1696 | 837.939 | 1 | 310 |
| Iteration | Func-count | f(x) | Step-size | First-order optimality |
| 120 | 1710 | 837.595 | 1 | 137 |
| 121 | 1724 | 837.528 | 1 | 21.4 |
| 122 | 1738 | 837.524 | 1 | 6.31 |
| 123 | 1752 | 837.524 | 1 | 6.24 |
| 124 | 1766 | 837.524 | 1 | 6.23 |
| 125 | 1780 | 837.524 | 1 | 6.19 |
| 126 | 1794 | 837.523 | 1 | 6.15 |
| 127 | 1808 | 837.523 | 1 | 11.4 |
| 128 | 1822 | 837.521 | 1 | 20.3 |
| 129 | 1836 | 837.516 | 1 | 35 |
| 130 | 1850 | 837.503 | 1 | 58.4 |
| 131 | 1864 | 837.469 | 1 | 95.7 |
| 132 | 1878 | 837.385 | 1 | 153 |
| 133 | 1892 | 837.181 | 1 | 231 |
| 134 | 1906 | 836.755 | 1 | 311 |
| 135 | 1920 | 836.096 | 1 | 321 |
| 136 | 1934 | 835.529 | 1 | 200 |
| 137 | 1948 | 835.327 | 1 | 56.1 |
| 138 | 1962 | 835.304 | 1 | 5.81 |
| 139 | 1976 | 835.303 | 1 | 5.72 |
| Iteration | Func-count | f(x) | Step-size | First-order optimality |
| 140 | 1990 | 835.303 | 1 | 5.71 |
| 141 | 2004 | 835.303 | 1 | 5.7 |
| 142 | 2018 | 835.303 | 1 | 5.68 |
| 143 | 2032 | 835.303 | 1 | 5.65 |
| 144 | 2046 | 835.302 | 1 | 6.66 |
| 145 | 2060 | 835.301 | 1 | 11.6 |
| 146 | 2074 | 835.297 | 1 | 19.7 |
| 147 | 2088 | 835.288 | 1 | 32.6 |
| 148 | 2102 | 835.263 | 1 | 53.4 |
| 149 | 2116 | 835.199 | 1 | 85.7 |
| 150 | 2130 | 835.04 | 1 | 133 |
| 151 | 2144 | 834.681 | 1 | 192 |
| 152 | 2158 | 834.018 | 1 | 232 |
| 153 | 2172 | 833.234 | 1 | 192 |
| 154 | 2186 | 832.807 | 1 | 78.4 |
| 155 | 2200 | 832.724 | 1 | 8.37 |
| 156 | 2214 | 832.718 | 1 | 3.06 |
| 157 | 2228 | 832.718 | 1 | 3.05 |
| 158 | 2242 | 832.718 | 1 | 3.04 |
| 159 | 2256 | 832.718 | 1 | 3.04 |
| Iteration | Func-count | f(x) | Step-size | First-order optimality |
| 160 | 2270 | 832.718 | 1 | 3.03 |
| 161 | 2284 | 832.718 | 1 | 3.7 |

| | | | | |
|---|---|---|---|---|
| 162 | 2298 | 832.717 | 1 | 6.65 |
| 163 | 2312 | 832.717 | 1 | 11.4 |
| 164 | 2326 | 832.715 | 1 | 19.1 |
| 165 | 2340 | 832.712 | 1 | 31.4 |
| 166 | 2354 | 832.702 | 1 | 50.7 |
| 167 | 2368 | 832.679 | 1 | 79.2 |
| 168 | 2382 | 832.625 | 1 | 115 |
| 169 | 2396 | 832.525 | 1 | 141 |
| 170 | 2410 | 832.401 | 1 | 120 |
| 171 | 2424 | 832.327 | 1 | 54.1 |
| 172 | 2438 | 832.31 | 1 | 8.69 |
| 173 | 2452 | 832.309 | 1 | 0.831 |
| 174 | 2466 | 832.309 | 1 | 0.82 |
| 175 | 2480 | 832.309 | 1 | 0.82 |
| 176 | 2494 | 832.309 | 1 | 0.819 |
| 177 | 2508 | 832.309 | 1 | 0.819 |
| 178 | 2522 | 832.309 | 1 | 0.818 |
| 179 | 2536 | 832.309 | 1 | 1.11 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 180 | 2550 | 832.309 | 1 | 1.97 |
| 181 | 2564 | 832.309 | 1 | 3.29 |
| 182 | 2578 | 832.309 | 1 | 5.44 |
| 183 | 2592 | 832.309 | 1 | 8.8 |
| 184 | 2606 | 832.308 | 1 | 13.9 |
| 185 | 2620 | 832.306 | 1 | 20.5 |
| 186 | 2634 | 832.302 | 1 | 26.3 |
| 187 | 2648 | 832.297 | 1 | 24.6 |
| 188 | 2662 | 832.294 | 1 | 12.9 |
| 189 | 2676 | 832.293 | 1 | 2.73 |
| 190 | 2690 | 832.293 | 1 | 0.0578 |
| 191 | 2704 | 832.293 | 1 | 0.0318 |
| 192 | 2718 | 832.293 | 1 | 0.00295 |

Local minimum found.

Optimization completed because the size of the gradient is less than the selected value of the function tolerance.

Computing finite-difference Hessian using user-supplied objective function.
Warning: Gradient must be provided for trust-region algorithm; using quasi-newton algorithm instead.

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 0 | 14 | 1009.92 | | 2.41e+04 |

User objective function returned NaN; trying a new point...

| | | | | |
|---|---|---|---|---|
| 1 | 44 | 964.484 | 1.63132e-07 | 1.31e+03 |
| 2 | 58 | 964.325 | 1 | 406 |
| 3 | 72 | 964.286 | 1 | 427 |
| 4 | 86 | 964 | 1 | 1.09e+03 |

```
     5         100         963.451              1        2.25e+03
     6         114         961.974              1         4.1e+03
     7         128         959.029              1            6e+03
     8         142         954.432              1        6.57e+03
     9         156         950.463              1        4.32e+03
    10         170         949.024              1        1.28e+03
    11         184         948.867              1             115
    12         198          948.86              1            79.4
    13         212         948.858              1            79.4
    14         226          948.85              1            78.4
    15         240         948.833              1            76.7
    16         254         948.787              1             105
    17         268         948.671              1             165
    18         282         948.394              1             247
    19         296         947.821              1             329

                                                      First-order
Iteration  Func-count       f(x)         Step-size       optimality
    20         310          946.95              1             333
    21         324         946.221              1             200
    22         338         945.976              1            54.8
    23         352         945.948              1            38.4
    24         366         945.947              1            38.4
    25         380         945.946              1            38.4
    26         394         945.943              1            38.4
    27         408         945.936              1            38.4
    28         422         945.916              1            38.3
    29         436         945.864              1            49.8
    30         450          945.73              1            84.1
    31         464         945.381              1             139
    32         478          944.49              1             223
    33         492         942.305              1             343
    34         506         937.466              1             483
    35         520         929.048              1             564
    36         534         920.103              1             464
    37         548          915.82              1             196
    38         562         915.064              1            34.5
    39         576         915.017              1            33.4

                                                      First-order
Iteration  Func-count       f(x)         Step-size       optimality
    40         590         915.015              1            33.2
    41         604         915.014              1            33.2
    42         618         915.011              1            33.1
    43         632         915.004              1            58.2
    44         646         914.983              1             114
    45         660         914.931              1             202
    46         674         914.793              1             346
    47         688         914.434              1             577
    48         702         913.499              1             947
    49         716         911.122              1        1.52e+03
    50         730         905.376              1         2.3e+03
    51         744         893.276              1        3.05e+03
    52         758         874.636              1        3.11e+03
    53         772         858.124              1        2.12e+03
    54         786          851.47              1             863
```

| | | | | |
|---|---|---|---|---|
| 55 | 800 | 850.558 | 1 | 158 |
| 56 | 814 | 850.518 | 1 | 12.7 |
| 57 | 828 | 850.516 | 1 | 12.6 |
| 58 | 842 | 850.516 | 1 | 12.5 |
| 59 | 856 | 850.516 | 1 | 12.5 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 60 | 870 | 850.515 | 1 | 21.4 |
| 61 | 884 | 850.513 | 1 | 46.2 |
| 62 | 898 | 850.507 | 1 | 88.1 |
| 63 | 912 | 850.492 | 1 | 155 |
| 64 | 926 | 850.452 | 1 | 264 |
| 65 | 940 | 850.35 | 1 | 439 |
| 66 | 954 | 850.086 | 1 | 715 |
| 67 | 968 | 849.43 | 1 | 1.12e+03 |
| 68 | 982 | 847.944 | 1 | 1.63e+03 |
| 69 | 996 | 845.247 | 1 | 1.93e+03 |
| 70 | 1010 | 842.212 | 1 | 1.51e+03 |
| 71 | 1024 | 840.663 | 1 | 605 |
| 72 | 1038 | 840.38 | 1 | 92.9 |
| 73 | 1052 | 840.365 | 1 | 10.6 |
| 74 | 1066 | 840.365 | 1 | 10.5 |
| 75 | 1080 | 840.365 | 1 | 10.5 |
| 76 | 1094 | 840.364 | 1 | 10.4 |
| 77 | 1108 | 840.363 | 1 | 12.4 |
| 78 | 1122 | 840.361 | 1 | 25.6 |
| 79 | 1136 | 840.355 | 1 | 46.2 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 80 | 1150 | 840.338 | 1 | 80 |
| 81 | 1164 | 840.296 | 1 | 134 |
| 82 | 1178 | 840.187 | 1 | 218 |
| 83 | 1192 | 839.917 | 1 | 341 |
| 84 | 1206 | 839.307 | 1 | 493 |
| 85 | 1220 | 838.189 | 1 | 594 |
| 86 | 1234 | 836.882 | 1 | 492 |
| 87 | 1248 | 836.17 | 1 | 212 |
| 88 | 1262 | 836.031 | 1 | 29.7 |
| 89 | 1276 | 836.023 | 1 | 8.67 |
| 90 | 1290 | 836.023 | 1 | 8.64 |
| 91 | 1304 | 836.023 | 1 | 8.63 |
| 92 | 1318 | 836.023 | 1 | 8.6 |
| 93 | 1332 | 836.022 | 1 | 10.5 |
| 94 | 1346 | 836.021 | 1 | 21 |
| 95 | 1360 | 836.017 | 1 | 37.4 |
| 96 | 1374 | 836.007 | 1 | 64.3 |
| 97 | 1388 | 835.981 | 1 | 107 |
| 98 | 1402 | 835.913 | 1 | 176 |
| 99 | 1416 | 835.742 | 1 | 280 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 100 | 1430 | 835.33 | 1 | 426 |
| 101 | 1444 | 834.46 | 1 | 578 |
| 102 | 1458 | 833.096 | 1 | 609 |

| | | | | |
|---|---|---|---|---|
| 103 | 1472 | 831.902 | 1 | 396 |
| 104 | 1486 | 831.476 | 1 | 116 |
| 105 | 1500 | 831.426 | 1 | 6.99 |
| 106 | 1514 | 831.424 | 1 | 6.67 |
| 107 | 1528 | 831.424 | 1 | 6.66 |
| 108 | 1542 | 831.424 | 1 | 6.65 |
| 109 | 1556 | 831.424 | 1 | 6.64 |
| 110 | 1570 | 831.424 | 1 | 9.66 |
| 111 | 1584 | 831.423 | 1 | 18.3 |
| 112 | 1598 | 831.42 | 1 | 32.1 |
| 113 | 1612 | 831.414 | 1 | 54.6 |
| 114 | 1626 | 831.398 | 1 | 90.7 |
| 115 | 1640 | 831.357 | 1 | 148 |
| 116 | 1654 | 831.251 | 1 | 237 |
| 117 | 1668 | 830.994 | 1 | 365 |
| 118 | 1682 | 830.431 | 1 | 508 |
| 119 | 1696 | 829.477 | 1 | 567 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 120 | 1710 | 828.512 | 1 | 404 |
| 121 | 1724 | 828.102 | 1 | 138 |
| 122 | 1738 | 828.045 | 1 | 14 |
| 123 | 1752 | 828.043 | 1 | 7.39 |
| 124 | 1766 | 828.043 | 1 | 7.38 |
| 125 | 1780 | 828.043 | 1 | 7.37 |
| 126 | 1794 | 828.043 | 1 | 7.36 |
| 127 | 1808 | 828.042 | 1 | 7.35 |
| 128 | 1822 | 828.042 | 1 | 12.1 |
| 129 | 1836 | 828.04 | 1 | 21.3 |
| 130 | 1850 | 828.034 | 1 | 36.5 |
| 131 | 1864 | 828.02 | 1 | 60.9 |
| 132 | 1878 | 827.983 | 1 | 99.8 |
| 133 | 1892 | 827.889 | 1 | 160 |
| 134 | 1906 | 827.657 | 1 | 248 |
| 135 | 1920 | 827.142 | 1 | 352 |
| 136 | 1934 | 826.237 | 1 | 405 |
| 137 | 1948 | 825.265 | 1 | 304 |
| 138 | 1962 | 824.798 | 1 | 111 |
| 139 | 1976 | 824.718 | 1 | 12.1 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 140 | 1990 | 824.713 | 1 | 7.59 |
| 141 | 2004 | 824.713 | 1 | 7.54 |
| 142 | 2018 | 824.713 | 1 | 7.52 |
| 143 | 2032 | 824.713 | 1 | 7.48 |
| 144 | 2046 | 824.713 | 1 | 7.43 |
| 145 | 2060 | 824.712 | 1 | 7.66 |
| 146 | 2074 | 824.71 | 1 | 13.5 |
| 147 | 2088 | 824.703 | 1 | 22.9 |
| 148 | 2102 | 824.687 | 1 | 38 |
| 149 | 2116 | 824.646 | 1 | 61.8 |
| 150 | 2130 | 824.542 | 1 | 97.8 |
| 151 | 2144 | 824.296 | 1 | 146 |
| 152 | 2158 | 823.803 | 1 | 191 |

```
   153        2172          823.101              1           183
   154        2186          822.575              1          95.4
   155        2200          822.428              1          17.6
   156        2214          822.416              1           1.2
   157        2228          822.415              1          1.21
   158        2242          822.415              1          1.21
   159        2256          822.415              1          1.21

                                                        First-order
 Iteration  Func-count       f(x)         Step-size       optimality
   160        2270          822.415              1           1.2
   161        2284          822.415              1           1.2
   162        2298          822.415              1          1.97
   163        2312          822.415              1          3.42
   164        2326          822.415              1          5.83
   165        2340          822.415              1           9.6
   166        2354          822.413              1          15.6
   167        2368          822.411              1          24.5
   168        2382          822.404              1          36.2
   169        2396          822.392              1            46
   170        2410          822.375              1          42.4
   171        2424          822.364              1          21.8
   172        2438          822.361              1          4.49
   173        2452          822.361              1         0.141
   174        2466          822.361              1         0.137
   175        2480          822.361              1         0.136
   176        2494          822.361              1         0.136
   177        2508          822.361              1         0.136
   178        2522          822.361              1         0.136
   179        2536          822.361              1         0.136

                                                        First-order
 Iteration  Func-count       f(x)         Step-size       optimality
   180        2550          822.361              1         0.179
   181        2564          822.361              1         0.359
   182        2578          822.361              1         0.595
   183        2592          822.361              1          1.02
   184        2606          822.361              1          1.64
   185        2620          822.361              1          2.44
   186        2634          822.361              1          3.07
   187        2648          822.361              1          2.71
   188        2662           822.36              1          1.27
   189        2676           822.36              1         0.225
   190        2690           822.36              1       0.00118

Local minimum found.

Optimization completed because the size of the gradient is less than
the selected value of the function tolerance.



Computing finite-difference Hessian using user-supplied objective
 function.
Warning: Gradient must be provided for trust-region algorithm; using
```

```
quasi-newton algorithm instead.
                                                         First-order
 Iteration  Func-count       f(x)        Step-size       optimality
     0          14          1009.92                      2.41e+04
User objective function returned NaN; trying a new point...
     1          44          964.484    1.63132e-07       1.31e+03
     2          58          964.325              1            406
     3          72          964.286              1            427
     4          86              964              1       1.09e+03
     5         100          963.451              1       2.25e+03
     6         114          961.974              1        4.1e+03
     7         128          959.029              1           6e+03
     8         142          954.432              1       6.57e+03
     9         156          950.463              1       4.32e+03
    10         170          949.024              1       1.28e+03
    11         184          948.867              1            115
    12         198           948.86              1           79.4
    13         212          948.858              1           79.4
    14         226           948.85              1           78.4
    15         240          948.833              1           76.7
    16         254          948.787              1            105
    17         268          948.671              1            165
    18         282          948.394              1            247
    19         296          947.821              1            329
                                                         First-order
 Iteration  Func-count       f(x)        Step-size       optimality
    20         310           946.95              1            333
    21         324          946.221              1            200
    22         338          945.976              1           54.8
    23         352          945.948              1           38.4
    24         366          945.947              1           38.4
    25         380          945.946              1           38.4
    26         394          945.943              1           38.4
    27         408          945.936              1           38.4
    28         422          945.916              1           38.3
    29         436          945.864              1           49.8
    30         450           945.73              1           84.1
    31         464          945.381              1            139
    32         478           944.49              1            223
    33         492          942.305              1            343
    34         506          937.466              1            483
    35         520          929.048              1            564
    36         534          920.103              1            464
    37         548           915.82              1            196
    38         562          915.064              1           34.5
    39         576          915.017              1           33.4
                                                         First-order
 Iteration  Func-count       f(x)        Step-size       optimality
    40         590          915.015              1           33.2
    41         604          915.014              1           33.2
    42         618          915.011              1           33.1
    43         632          915.004              1           58.2
    44         646          914.983              1            114
    45         660          914.931              1            202
```

```
    46         674         914.793              1              346
    47         688         914.434              1              577
    48         702         913.499              1              947
    49         716         911.122              1         1.52e+03
    50         730         905.376              1          2.3e+03
    51         744         893.276              1         3.05e+03
    52         758         874.636              1         3.11e+03
    53         772         858.124              1         2.12e+03
    54         786          851.47              1              863
    55         800         850.558              1              158
    56         814         850.518              1             12.7
    57         828         850.516              1             12.6
    58         842         850.516              1             12.5
    59         856         850.516              1             12.5

                                                        First-order
Iteration  Func-count       f(x)         Step-size       optimality
    60         870         850.515              1             21.4
    61         884         850.513              1             46.2
    62         898         850.507              1             88.1
    63         912         850.492              1              155
    64         926         850.452              1              264
    65         940          850.35              1              439
    66         954         850.086              1              715
    67         968          849.43              1         1.12e+03
    68         982         847.944              1         1.63e+03
    69         996         845.247              1         1.93e+03
    70        1010         842.212              1         1.51e+03
    71        1024         840.663              1              605
    72        1038          840.38              1             92.9
    73        1052         840.365              1             10.6
    74        1066         840.365              1             10.5
    75        1080         840.365              1             10.5
    76        1094         840.364              1             10.4
    77        1108         840.363              1             12.4
    78        1122         840.361              1             25.6
    79        1136         840.355              1             46.2

                                                        First-order
Iteration  Func-count       f(x)         Step-size       optimality
    80        1150         840.338              1               80
    81        1164         840.296              1              134
    82        1178         840.187              1              218
    83        1192         839.917              1              341
    84        1206         839.307              1              493
    85        1220         838.189              1              594
    86        1234         836.882              1              492
    87        1248          836.17              1              212
    88        1262         836.031              1             29.7
    89        1276         836.023              1             8.67
    90        1290         836.023              1             8.64
    91        1304         836.023              1             8.63
    92        1318         836.023              1              8.6
    93        1332         836.022              1             10.5
    94        1346         836.021              1               21
    95        1360         836.017              1             37.4
```

| | | | | |
|---|---|---|---|---|
| 96 | 1374 | 836.007 | 1 | 64.3 |
| 97 | 1388 | 835.981 | 1 | 107 |
| 98 | 1402 | 835.913 | 1 | 176 |
| 99 | 1416 | 835.742 | 1 | 280 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 100 | 1430 | 835.33 | 1 | 426 |
| 101 | 1444 | 834.46 | 1 | 578 |
| 102 | 1458 | 833.096 | 1 | 609 |
| 103 | 1472 | 831.902 | 1 | 396 |
| 104 | 1486 | 831.476 | 1 | 116 |
| 105 | 1500 | 831.426 | 1 | 6.99 |
| 106 | 1514 | 831.424 | 1 | 6.67 |
| 107 | 1528 | 831.424 | 1 | 6.66 |
| 108 | 1542 | 831.424 | 1 | 6.65 |
| 109 | 1556 | 831.424 | 1 | 6.64 |
| 110 | 1570 | 831.424 | 1 | 9.66 |
| 111 | 1584 | 831.423 | 1 | 18.3 |
| 112 | 1598 | 831.42 | 1 | 32.1 |
| 113 | 1612 | 831.414 | 1 | 54.6 |
| 114 | 1626 | 831.398 | 1 | 90.7 |
| 115 | 1640 | 831.357 | 1 | 148 |
| 116 | 1654 | 831.251 | 1 | 237 |
| 117 | 1668 | 830.994 | 1 | 365 |
| 118 | 1682 | 830.431 | 1 | 508 |
| 119 | 1696 | 829.477 | 1 | 567 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 120 | 1710 | 828.512 | 1 | 404 |
| 121 | 1724 | 828.102 | 1 | 138 |
| 122 | 1738 | 828.045 | 1 | 14 |
| 123 | 1752 | 828.043 | 1 | 7.39 |
| 124 | 1766 | 828.043 | 1 | 7.38 |
| 125 | 1780 | 828.043 | 1 | 7.37 |
| 126 | 1794 | 828.043 | 1 | 7.36 |
| 127 | 1808 | 828.042 | 1 | 7.35 |
| 128 | 1822 | 828.042 | 1 | 12.1 |
| 129 | 1836 | 828.04 | 1 | 21.3 |
| 130 | 1850 | 828.034 | 1 | 36.5 |
| 131 | 1864 | 828.02 | 1 | 60.9 |
| 132 | 1878 | 827.983 | 1 | 99.8 |
| 133 | 1892 | 827.889 | 1 | 160 |
| 134 | 1906 | 827.657 | 1 | 248 |
| 135 | 1920 | 827.142 | 1 | 352 |
| 136 | 1934 | 826.237 | 1 | 405 |
| 137 | 1948 | 825.265 | 1 | 304 |
| 138 | 1962 | 824.798 | 1 | 111 |
| 139 | 1976 | 824.718 | 1 | 12.1 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 140 | 1990 | 824.713 | 1 | 7.59 |
| 141 | 2004 | 824.713 | 1 | 7.54 |
| 142 | 2018 | 824.713 | 1 | 7.52 |
| 143 | 2032 | 824.713 | 1 | 7.48 |

| | | | | |
|---|---|---|---|---|
| 144 | 2046 | 824.713 | 1 | 7.43 |
| 145 | 2060 | 824.712 | 1 | 7.66 |
| 146 | 2074 | 824.71 | 1 | 13.5 |
| 147 | 2088 | 824.703 | 1 | 22.9 |
| 148 | 2102 | 824.687 | 1 | 38 |
| 149 | 2116 | 824.646 | 1 | 61.8 |
| 150 | 2130 | 824.542 | 1 | 97.8 |
| 151 | 2144 | 824.296 | 1 | 146 |
| 152 | 2158 | 823.803 | 1 | 191 |
| 153 | 2172 | 823.101 | 1 | 183 |
| 154 | 2186 | 822.575 | 1 | 95.4 |
| 155 | 2200 | 822.428 | 1 | 17.6 |
| 156 | 2214 | 822.416 | 1 | 1.2 |
| 157 | 2228 | 822.415 | 1 | 1.21 |
| 158 | 2242 | 822.415 | 1 | 1.21 |
| 159 | 2256 | 822.415 | 1 | 1.21 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 160 | 2270 | 822.415 | 1 | 1.2 |
| 161 | 2284 | 822.415 | 1 | 1.2 |
| 162 | 2298 | 822.415 | 1 | 1.97 |
| 163 | 2312 | 822.415 | 1 | 3.42 |
| 164 | 2326 | 822.415 | 1 | 5.83 |
| 165 | 2340 | 822.415 | 1 | 9.6 |
| 166 | 2354 | 822.413 | 1 | 15.6 |
| 167 | 2368 | 822.411 | 1 | 24.5 |
| 168 | 2382 | 822.404 | 1 | 36.2 |
| 169 | 2396 | 822.392 | 1 | 46 |
| 170 | 2410 | 822.375 | 1 | 42.4 |
| 171 | 2424 | 822.364 | 1 | 21.8 |
| 172 | 2438 | 822.361 | 1 | 4.49 |
| 173 | 2452 | 822.361 | 1 | 0.141 |
| 174 | 2466 | 822.361 | 1 | 0.137 |
| 175 | 2480 | 822.361 | 1 | 0.136 |
| 176 | 2494 | 822.361 | 1 | 0.136 |
| 177 | 2508 | 822.361 | 1 | 0.136 |
| 178 | 2522 | 822.361 | 1 | 0.136 |
| 179 | 2536 | 822.361 | 1 | 0.136 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 180 | 2550 | 822.361 | 1 | 0.179 |
| 181 | 2564 | 822.361 | 1 | 0.359 |
| 182 | 2578 | 822.361 | 1 | 0.595 |
| 183 | 2592 | 822.361 | 1 | 1.02 |
| 184 | 2606 | 822.361 | 1 | 1.64 |
| 185 | 2620 | 822.361 | 1 | 2.44 |
| 186 | 2634 | 822.361 | 1 | 3.07 |
| 187 | 2648 | 822.361 | 1 | 2.71 |
| 188 | 2662 | 822.36 | 1 | 1.27 |
| 189 | 2676 | 822.36 | 1 | 0.225 |
| 190 | 2690 | 822.36 | 1 | 0.00118 |

Local minimum found.

*Optimization completed because the size of the gradient is less than the selected value of the function tolerance.*

*Computing finite-difference Hessian using user-supplied objective function.*
*Warning: Gradient must be provided for trust-region algorithm; using quasi-newton algorithm instead.*

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 0 | 14 | 1009.92 | | 2.78e+04 |
| User objective function returned NaN; trying a new point... | | | | |
| 1 | 44 | 950.533 | 1.62049e-07 | 1.62e+03 |
| 2 | 58 | 950.302 | 1 | 398 |
| 3 | 72 | 950.265 | 1 | 419 |
| 4 | 86 | 950 | 1 | 1.02e+03 |
| 5 | 100 | 949.486 | 1 | 2.15e+03 |
| 6 | 114 | 948.1 | 1 | 3.96e+03 |
| 7 | 128 | 945.29 | 1 | 5.89e+03 |
| 8 | 142 | 940.771 | 1 | 6.65e+03 |
| 9 | 156 | 936.647 | 1 | 4.59e+03 |
| 10 | 170 | 935.016 | 1 | 1.45e+03 |
| 11 | 184 | 934.823 | 1 | 144 |
| 12 | 198 | 934.815 | 1 | 69.2 |
| 13 | 212 | 934.814 | 1 | 68.9 |
| 14 | 226 | 934.809 | 1 | 67.4 |
| 15 | 240 | 934.797 | 1 | 64.9 |
| 16 | 254 | 934.766 | 1 | 60.3 |
| 17 | 268 | 934.687 | 1 | 89.6 |
| 18 | 282 | 934.498 | 1 | 131 |
| 19 | 296 | 934.104 | 1 | 172 |
| Iteration | Func-count | f(x) | Step-size | First-order optimality |
| 20 | 310 | 933.496 | 1 | 175 |
| 21 | 324 | 932.973 | 1 | 108 |
| 22 | 338 | 932.787 | 1 | 37.1 |
| 23 | 352 | 932.765 | 1 | 37 |
| 24 | 366 | 932.764 | 1 | 37 |
| 25 | 380 | 932.764 | 1 | 37 |
| 26 | 394 | 932.761 | 1 | 37 |
| 27 | 408 | 932.756 | 1 | 37 |
| 28 | 422 | 932.741 | 1 | 37 |
| 29 | 436 | 932.703 | 1 | 36.9 |
| 30 | 450 | 932.603 | 1 | 56.2 |
| 31 | 464 | 932.344 | 1 | 93.1 |
| 32 | 478 | 931.672 | 1 | 152 |
| 33 | 492 | 929.968 | 1 | 240 |
| 34 | 506 | 925.863 | 1 | 363 |
| 35 | 520 | 917.177 | 1 | 493 |
| 36 | 534 | 903.541 | 1 | 553 |
| 37 | 548 | 891.51 | 1 | 433 |
| 38 | 562 | 886.868 | 1 | 150 |

| | | | | |
|---|---|---|---|---|
| 39 | 576 | 886.176 | 1 | 55.2 |
| Iteration | Func-count | f(x) | Step-size | First-order optimality |
| 40 | 590 | 886.138 | 1 | 53.9 |
| 41 | 604 | 886.136 | 1 | 53.6 |
| 42 | 618 | 886.135 | 1 | 53.5 |
| 43 | 632 | 886.131 | 1 | 53.4 |
| 44 | 646 | 886.123 | 1 | 93.4 |
| 45 | 660 | 886.1 | 1 | 181 |
| 46 | 674 | 886.04 | 1 | 319 |
| 47 | 688 | 885.882 | 1 | 546 |
| 48 | 702 | 885.473 | 1 | 909 |
| 49 | 716 | 884.42 | 1 | 1.48e+03 |
| 50 | 730 | 881.805 | 1 | 2.32e+03 |
| 51 | 744 | 875.901 | 1 | 3.3e+03 |
| 52 | 758 | 865.404 | 1 | 3.79e+03 |
| 53 | 772 | 854.004 | 1 | 2.89e+03 |
| 54 | 786 | 848.04 | 1 | 1.26e+03 |
| 55 | 800 | 846.788 | 1 | 255 |
| 56 | 814 | 846.708 | 1 | 11.6 |
| 57 | 828 | 846.705 | 1 | 11.3 |
| 58 | 842 | 846.705 | 1 | 11.3 |
| 59 | 856 | 846.705 | 1 | 11.3 |
| Iteration | Func-count | f(x) | Step-size | First-order optimality |
| 60 | 870 | 846.705 | 1 | 11.8 |
| 61 | 884 | 846.704 | 1 | 25.9 |
| 62 | 898 | 846.701 | 1 | 50.3 |
| 63 | 912 | 846.695 | 1 | 88.9 |
| 64 | 926 | 846.677 | 1 | 152 |
| 65 | 940 | 846.633 | 1 | 253 |
| 66 | 954 | 846.517 | 1 | 414 |
| 67 | 968 | 846.226 | 1 | 658 |
| 68 | 982 | 845.541 | 1 | 984 |
| 69 | 996 | 844.177 | 1 | 1.27e+03 |
| 70 | 1010 | 842.299 | 1 | 1.19e+03 |
| 71 | 1024 | 840.978 | 1 | 620 |
| 72 | 1038 | 840.635 | 1 | 131 |
| 73 | 1052 | 840.609 | 1 | 14.3 |
| 74 | 1066 | 840.609 | 1 | 14.1 |
| 75 | 1080 | 840.609 | 1 | 14.1 |
| 76 | 1094 | 840.608 | 1 | 14 |
| 77 | 1108 | 840.608 | 1 | 13.8 |
| 78 | 1122 | 840.606 | 1 | 19 |
| 79 | 1136 | 840.6 | 1 | 34.8 |
| Iteration | Func-count | f(x) | Step-size | First-order optimality |
| 80 | 1150 | 840.586 | 1 | 60.5 |
| 81 | 1164 | 840.55 | 1 | 102 |
| 82 | 1178 | 840.455 | 1 | 166 |
| 83 | 1192 | 840.219 | 1 | 263 |
| 84 | 1206 | 839.665 | 1 | 391 |
| 85 | 1220 | 838.565 | 1 | 501 |
| 86 | 1234 | 837.06 | 1 | 473 |

| | | | | |
|---|---|---|---|---|
| 87 | 1248 | 836.037 | 1 | 260 |
| 88 | 1262 | 835.768 | 1 | 58.1 |
| 89 | 1276 | 835.743 | 1 | 8.99 |
| 90 | 1290 | 835.743 | 1 | 8.99 |
| 91 | 1304 | 835.743 | 1 | 8.99 |
| 92 | 1318 | 835.743 | 1 | 8.98 |
| 93 | 1332 | 835.742 | 1 | 8.97 |
| 94 | 1346 | 835.742 | 1 | 15.2 |
| 95 | 1360 | 835.74 | 1 | 27.7 |
| 96 | 1374 | 835.736 | 1 | 47.8 |
| 97 | 1388 | 835.726 | 1 | 80.2 |
| 98 | 1402 | 835.699 | 1 | 132 |
| 99 | 1416 | 835.63 | 1 | 214 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 100 | 1430 | 835.458 | 1 | 336 |
| 101 | 1444 | 835.059 | 1 | 493 |
| 102 | 1458 | 834.285 | 1 | 619 |
| 103 | 1472 | 833.272 | 1 | 557 |
| 104 | 1486 | 832.625 | 1 | 278 |
| 105 | 1500 | 832.473 | 1 | 54.4 |
| 106 | 1514 | 832.462 | 1 | 6.74 |
| 107 | 1528 | 832.461 | 1 | 6.72 |
| 108 | 1542 | 832.461 | 1 | 6.72 |
| 109 | 1556 | 832.461 | 1 | 6.71 |
| 110 | 1570 | 832.461 | 1 | 6.7 |
| 111 | 1584 | 832.461 | 1 | 13.5 |
| 112 | 1598 | 832.459 | 1 | 24.4 |
| 113 | 1612 | 832.456 | 1 | 42.2 |
| 114 | 1626 | 832.448 | 1 | 70.8 |
| 115 | 1640 | 832.426 | 1 | 117 |
| 116 | 1654 | 832.37 | 1 | 189 |
| 117 | 1668 | 832.229 | 1 | 300 |
| 118 | 1682 | 831.894 | 1 | 449 |
| 119 | 1696 | 831.215 | 1 | 588 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 120 | 1710 | 830.237 | 1 | 571 |
| 121 | 1724 | 829.493 | 1 | 318 |
| 122 | 1738 | 829.269 | 1 | 74.5 |
| 123 | 1752 | 829.248 | 1 | 8.41 |
| 124 | 1766 | 829.247 | 1 | 8.28 |
| 125 | 1780 | 829.247 | 1 | 8.26 |
| 126 | 1794 | 829.247 | 1 | 8.24 |
| 127 | 1808 | 829.247 | 1 | 8.21 |
| 128 | 1822 | 829.246 | 1 | 10.1 |
| 129 | 1836 | 829.245 | 1 | 18.6 |
| 130 | 1850 | 829.241 | 1 | 32.3 |
| 131 | 1864 | 829.23 | 1 | 54.4 |
| 132 | 1878 | 829.203 | 1 | 89.9 |
| 133 | 1892 | 829.132 | 1 | 146 |
| 134 | 1906 | 828.956 | 1 | 229 |
| 135 | 1920 | 828.546 | 1 | 338 |
| 136 | 1934 | 827.745 | 1 | 428 |

| | | | | |
|---|---|---|---|---|
| 137 | 1948 | 826.677 | 1 | 390 |
| 138 | 1962 | 825.97 | 1 | 192 |
| 139 | 1976 | 825.796 | 1 | 34 |
| Iteration | Func-count | f(x) | Step-size | First-order optimality |
| 140 | 1990 | 825.782 | 1 | 3.71 |
| 141 | 2004 | 825.781 | 1 | 3.64 |
| 142 | 2018 | 825.781 | 1 | 3.63 |
| 143 | 2032 | 825.781 | 1 | 3.63 |
| 144 | 2046 | 825.781 | 1 | 3.62 |
| 145 | 2060 | 825.781 | 1 | 4.58 |
| 146 | 2074 | 825.781 | 1 | 8.39 |
| 147 | 2088 | 825.781 | 1 | 14.7 |
| 148 | 2102 | 825.779 | 1 | 24.6 |
| 149 | 2116 | 825.776 | 1 | 40.6 |
| 150 | 2130 | 825.768 | 1 | 65.2 |
| 151 | 2144 | 825.747 | 1 | 101 |
| 152 | 2158 | 825.702 | 1 | 142 |
| 153 | 2172 | 825.624 | 1 | 162 |
| 154 | 2186 | 825.542 | 1 | 120 |
| 155 | 2200 | 825.503 | 1 | 43.4 |
| 156 | 2214 | 825.497 | 1 | 4.81 |
| 157 | 2228 | 825.497 | 1 | 0.761 |
| 158 | 2242 | 825.497 | 1 | 0.763 |
| 159 | 2256 | 825.497 | 1 | 0.763 |
| Iteration | Func-count | f(x) | Step-size | First-order optimality |
| 160 | 2270 | 825.497 | 1 | 0.763 |
| 161 | 2284 | 825.497 | 1 | 0.762 |
| 162 | 2298 | 825.497 | 1 | 1.42 |
| 163 | 2312 | 825.497 | 1 | 2.52 |
| 164 | 2326 | 825.497 | 1 | 4.28 |
| 165 | 2340 | 825.497 | 1 | 7.09 |
| 166 | 2354 | 825.496 | 1 | 11.6 |
| 167 | 2368 | 825.495 | 1 | 18.3 |
| 168 | 2382 | 825.493 | 1 | 27.4 |
| 169 | 2396 | 825.489 | 1 | 36 |
| 170 | 2410 | 825.483 | 1 | 35.4 |
| 171 | 2424 | 825.478 | 1 | 20.2 |
| 172 | 2438 | 825.477 | 1 | 4.85 |
| 173 | 2452 | 825.476 | 1 | 0.205 |
| 174 | 2466 | 825.476 | 1 | 0.123 |
| 175 | 2480 | 825.476 | 1 | 0.122 |
| 176 | 2494 | 825.476 | 1 | 0.122 |
| 177 | 2508 | 825.476 | 1 | 0.122 |
| 178 | 2522 | 825.476 | 1 | 0.121 |
| 179 | 2536 | 825.476 | 1 | 0.184 |
| Iteration | Func-count | f(x) | Step-size | First-order optimality |
| 180 | 2550 | 825.476 | 1 | 0.292 |
| 181 | 2564 | 825.476 | 1 | 0.504 |
| 182 | 2578 | 825.476 | 1 | 0.822 |
| 183 | 2592 | 825.476 | 1 | 1.31 |
| 184 | 2606 | 825.476 | 1 | 2.07 |

| | | | | |
|---|---|---|---|---|
| 185 | 2620 | 825.476 | 1 | 3.06 |
| 186 | 2634 | 825.476 | 1 | 3.95 |
| 187 | 2648 | 825.476 | 1 | 3.75 |
| 188 | 2662 | 825.476 | 1 | 2.02 |
| 189 | 2676 | 825.476 | 1 | 0.442 |
| 190 | 2690 | 825.476 | 1 | 0.0142 |

Local minimum found.

Optimization completed because the size of the gradient is less than the selected value of the function tolerance.

Computing finite-difference Hessian using user-supplied objective function.
Warning: Gradient must be provided for trust-region algorithm; using quasi-newton algorithm instead.

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 0 | 14 | 1009.92 | | 2.78e+04 |

User objective function returned NaN; trying a new point...

| | | | | |
|---|---|---|---|---|
| 1 | 44 | 950.533 | 1.62049e-07 | 1.62e+03 |
| 2 | 58 | 950.302 | 1 | 398 |
| 3 | 72 | 950.265 | 1 | 419 |
| 4 | 86 | 950 | 1 | 1.02e+03 |
| 5 | 100 | 949.486 | 1 | 2.15e+03 |
| 6 | 114 | 948.1 | 1 | 3.96e+03 |
| 7 | 128 | 945.29 | 1 | 5.89e+03 |
| 8 | 142 | 940.771 | 1 | 6.65e+03 |
| 9 | 156 | 936.647 | 1 | 4.59e+03 |
| 10 | 170 | 935.016 | 1 | 1.45e+03 |
| 11 | 184 | 934.823 | 1 | 144 |
| 12 | 198 | 934.815 | 1 | 69.2 |
| 13 | 212 | 934.814 | 1 | 68.9 |
| 14 | 226 | 934.809 | 1 | 67.4 |
| 15 | 240 | 934.797 | 1 | 64.9 |
| 16 | 254 | 934.766 | 1 | 60.3 |
| 17 | 268 | 934.687 | 1 | 89.6 |
| 18 | 282 | 934.498 | 1 | 131 |
| 19 | 296 | 934.104 | 1 | 172 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 20 | 310 | 933.496 | 1 | 175 |
| 21 | 324 | 932.973 | 1 | 108 |
| 22 | 338 | 932.787 | 1 | 37.1 |
| 23 | 352 | 932.765 | 1 | 37 |
| 24 | 366 | 932.764 | 1 | 37 |
| 25 | 380 | 932.764 | 1 | 37 |
| 26 | 394 | 932.761 | 1 | 37 |
| 27 | 408 | 932.756 | 1 | 37 |
| 28 | 422 | 932.741 | 1 | 37 |
| 29 | 436 | 932.703 | 1 | 36.9 |

| | | | | |
|---|---|---|---|---|
| 30 | 450 | 932.603 | 1 | 56.2 |
| 31 | 464 | 932.344 | 1 | 93.1 |
| 32 | 478 | 931.672 | 1 | 152 |
| 33 | 492 | 929.968 | 1 | 240 |
| 34 | 506 | 925.863 | 1 | 363 |
| 35 | 520 | 917.177 | 1 | 493 |
| 36 | 534 | 903.541 | 1 | 553 |
| 37 | 548 | 891.51 | 1 | 433 |
| 38 | 562 | 886.868 | 1 | 150 |
| 39 | 576 | 886.176 | 1 | 55.2 |
| Iteration | Func-count | f(x) | Step-size | First-order optimality |
| 40 | 590 | 886.138 | 1 | 53.9 |
| 41 | 604 | 886.136 | 1 | 53.6 |
| 42 | 618 | 886.135 | 1 | 53.5 |
| 43 | 632 | 886.131 | 1 | 53.4 |
| 44 | 646 | 886.123 | 1 | 93.4 |
| 45 | 660 | 886.1 | 1 | 181 |
| 46 | 674 | 886.04 | 1 | 319 |
| 47 | 688 | 885.882 | 1 | 546 |
| 48 | 702 | 885.473 | 1 | 909 |
| 49 | 716 | 884.42 | 1 | 1.48e+03 |
| 50 | 730 | 881.805 | 1 | 2.32e+03 |
| 51 | 744 | 875.901 | 1 | 3.3e+03 |
| 52 | 758 | 865.404 | 1 | 3.79e+03 |
| 53 | 772 | 854.004 | 1 | 2.89e+03 |
| 54 | 786 | 848.04 | 1 | 1.26e+03 |
| 55 | 800 | 846.788 | 1 | 255 |
| 56 | 814 | 846.708 | 1 | 11.6 |
| 57 | 828 | 846.705 | 1 | 11.3 |
| 58 | 842 | 846.705 | 1 | 11.3 |
| 59 | 856 | 846.705 | 1 | 11.3 |
| Iteration | Func-count | f(x) | Step-size | First-order optimality |
| 60 | 870 | 846.705 | 1 | 11.8 |
| 61 | 884 | 846.704 | 1 | 25.9 |
| 62 | 898 | 846.701 | 1 | 50.3 |
| 63 | 912 | 846.695 | 1 | 88.9 |
| 64 | 926 | 846.677 | 1 | 152 |
| 65 | 940 | 846.633 | 1 | 253 |
| 66 | 954 | 846.517 | 1 | 414 |
| 67 | 968 | 846.226 | 1 | 658 |
| 68 | 982 | 845.541 | 1 | 984 |
| 69 | 996 | 844.177 | 1 | 1.27e+03 |
| 70 | 1010 | 842.299 | 1 | 1.19e+03 |
| 71 | 1024 | 840.978 | 1 | 620 |
| 72 | 1038 | 840.635 | 1 | 131 |
| 73 | 1052 | 840.609 | 1 | 14.3 |
| 74 | 1066 | 840.609 | 1 | 14.1 |
| 75 | 1080 | 840.609 | 1 | 14.1 |
| 76 | 1094 | 840.608 | 1 | 14 |
| 77 | 1108 | 840.608 | 1 | 13.8 |
| 78 | 1122 | 840.606 | 1 | 19 |
| 79 | 1136 | 840.6 | 1 | 34.8 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 80 | 1150 | 840.586 | 1 | 60.5 |
| 81 | 1164 | 840.55 | 1 | 102 |
| 82 | 1178 | 840.455 | 1 | 166 |
| 83 | 1192 | 840.219 | 1 | 263 |
| 84 | 1206 | 839.665 | 1 | 391 |
| 85 | 1220 | 838.565 | 1 | 501 |
| 86 | 1234 | 837.06 | 1 | 473 |
| 87 | 1248 | 836.037 | 1 | 260 |
| 88 | 1262 | 835.768 | 1 | 58.1 |
| 89 | 1276 | 835.743 | 1 | 8.99 |
| 90 | 1290 | 835.743 | 1 | 8.99 |
| 91 | 1304 | 835.743 | 1 | 8.99 |
| 92 | 1318 | 835.743 | 1 | 8.98 |
| 93 | 1332 | 835.742 | 1 | 8.97 |
| 94 | 1346 | 835.742 | 1 | 15.2 |
| 95 | 1360 | 835.74 | 1 | 27.7 |
| 96 | 1374 | 835.736 | 1 | 47.8 |
| 97 | 1388 | 835.726 | 1 | 80.2 |
| 98 | 1402 | 835.699 | 1 | 132 |
| 99 | 1416 | 835.63 | 1 | 214 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 100 | 1430 | 835.458 | 1 | 336 |
| 101 | 1444 | 835.059 | 1 | 493 |
| 102 | 1458 | 834.285 | 1 | 619 |
| 103 | 1472 | 833.272 | 1 | 557 |
| 104 | 1486 | 832.625 | 1 | 278 |
| 105 | 1500 | 832.473 | 1 | 54.4 |
| 106 | 1514 | 832.462 | 1 | 6.74 |
| 107 | 1528 | 832.461 | 1 | 6.72 |
| 108 | 1542 | 832.461 | 1 | 6.72 |
| 109 | 1556 | 832.461 | 1 | 6.71 |
| 110 | 1570 | 832.461 | 1 | 6.7 |
| 111 | 1584 | 832.461 | 1 | 13.5 |
| 112 | 1598 | 832.459 | 1 | 24.4 |
| 113 | 1612 | 832.456 | 1 | 42.2 |
| 114 | 1626 | 832.448 | 1 | 70.8 |
| 115 | 1640 | 832.426 | 1 | 117 |
| 116 | 1654 | 832.37 | 1 | 189 |
| 117 | 1668 | 832.229 | 1 | 300 |
| 118 | 1682 | 831.894 | 1 | 449 |
| 119 | 1696 | 831.215 | 1 | 588 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 120 | 1710 | 830.237 | 1 | 571 |
| 121 | 1724 | 829.493 | 1 | 318 |
| 122 | 1738 | 829.269 | 1 | 74.5 |
| 123 | 1752 | 829.248 | 1 | 8.41 |
| 124 | 1766 | 829.247 | 1 | 8.28 |
| 125 | 1780 | 829.247 | 1 | 8.26 |
| 126 | 1794 | 829.247 | 1 | 8.24 |
| 127 | 1808 | 829.247 | 1 | 8.21 |

| | | | | |
|---|---|---|---|---|
| 128 | 1822 | 829.246 | 1 | 10.1 |
| 129 | 1836 | 829.245 | 1 | 18.6 |
| 130 | 1850 | 829.241 | 1 | 32.3 |
| 131 | 1864 | 829.23 | 1 | 54.4 |
| 132 | 1878 | 829.203 | 1 | 89.9 |
| 133 | 1892 | 829.132 | 1 | 146 |
| 134 | 1906 | 828.956 | 1 | 229 |
| 135 | 1920 | 828.546 | 1 | 338 |
| 136 | 1934 | 827.745 | 1 | 428 |
| 137 | 1948 | 826.677 | 1 | 390 |
| 138 | 1962 | 825.97 | 1 | 192 |
| 139 | 1976 | 825.796 | 1 | 34 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 140 | 1990 | 825.782 | 1 | 3.71 |
| 141 | 2004 | 825.781 | 1 | 3.64 |
| 142 | 2018 | 825.781 | 1 | 3.63 |
| 143 | 2032 | 825.781 | 1 | 3.63 |
| 144 | 2046 | 825.781 | 1 | 3.62 |
| 145 | 2060 | 825.781 | 1 | 4.58 |
| 146 | 2074 | 825.781 | 1 | 8.39 |
| 147 | 2088 | 825.781 | 1 | 14.7 |
| 148 | 2102 | 825.779 | 1 | 24.6 |
| 149 | 2116 | 825.776 | 1 | 40.6 |
| 150 | 2130 | 825.768 | 1 | 65.2 |
| 151 | 2144 | 825.747 | 1 | 101 |
| 152 | 2158 | 825.702 | 1 | 142 |
| 153 | 2172 | 825.624 | 1 | 162 |
| 154 | 2186 | 825.542 | 1 | 120 |
| 155 | 2200 | 825.503 | 1 | 43.4 |
| 156 | 2214 | 825.497 | 1 | 4.81 |
| 157 | 2228 | 825.497 | 1 | 0.761 |
| 158 | 2242 | 825.497 | 1 | 0.763 |
| 159 | 2256 | 825.497 | 1 | 0.763 |

| Iteration | Func-count | f(x) | Step-size | First-order optimality |
|---|---|---|---|---|
| 160 | 2270 | 825.497 | 1 | 0.763 |
| 161 | 2284 | 825.497 | 1 | 0.762 |
| 162 | 2298 | 825.497 | 1 | 1.42 |
| 163 | 2312 | 825.497 | 1 | 2.52 |
| 164 | 2326 | 825.497 | 1 | 4.28 |
| 165 | 2340 | 825.497 | 1 | 7.09 |
| 166 | 2354 | 825.496 | 1 | 11.6 |
| 167 | 2368 | 825.495 | 1 | 18.3 |
| 168 | 2382 | 825.493 | 1 | 27.4 |
| 169 | 2396 | 825.489 | 1 | 36 |
| 170 | 2410 | 825.483 | 1 | 35.4 |
| 171 | 2424 | 825.478 | 1 | 20.2 |
| 172 | 2438 | 825.477 | 1 | 4.85 |
| 173 | 2452 | 825.476 | 1 | 0.205 |
| 174 | 2466 | 825.476 | 1 | 0.123 |
| 175 | 2480 | 825.476 | 1 | 0.122 |
| 176 | 2494 | 825.476 | 1 | 0.122 |
| 177 | 2508 | 825.476 | 1 | 0.122 |

```
   178        2522          825.476             1          0.121
   179        2536          825.476             1          0.184
                                                          First-order
 Iteration  Func-count       f(x)        Step-size       optimality
   180        2550          825.476             1          0.292
   181        2564          825.476             1          0.504
   182        2578          825.476             1          0.822
   183        2592          825.476             1           1.31
   184        2606          825.476             1           2.07
   185        2620          825.476             1           3.06
   186        2634          825.476             1           3.95
   187        2648          825.476             1           3.75
   188        2662          825.476             1           2.02
   189        2676          825.476             1          0.442
   190        2690          825.476             1         0.0142

Local minimum found.

Optimization completed because the size of the gradient is less than
the selected value of the function tolerance.



Computing finite-difference Hessian using user-supplied objective
 function.
Confidence Interval 1 for nearc4 [0.193808 , 0.388049]
Confidence Interval 1 for IQ [0.040650 , 0.050025]
Confidence Interval 2 for nearc4 [-6.806539 , -6.612299]
Confidence Interval 2 for IQ [0.331513 , 0.340888]
Confidence Interval 3 for nearc4 [-12.453434 , -12.210590]
Confidence Interval 3 for IQ [0.550566 , 0.579083]
Confidence Interval 1 for Average Partial Effect of IQ [0.011569,
 0.013905]
Confidence Interval 2 for Average Partial Effect of IQ [0.010555,
 0.012891]
Confidence Interval 3 for Average Partial Effect of IQ [0.011219,
 0.012645]
Confidence Interval 1 for parametric Average Partial Effect of IQ
 [0.011569, 0.013905]
Confidence Interval 2 for parametric Average Partial Effect of IQ
 [0.010555, 0.012891]
Confidence Interval 3 for parametric Average Partial Effect of IQ
 [0.011219, 0.012645]
```

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