# MapReduce and Frequent Itemsets Mining

Yang Wang

# MapReduce (Hadoop)

Programming model designed for:

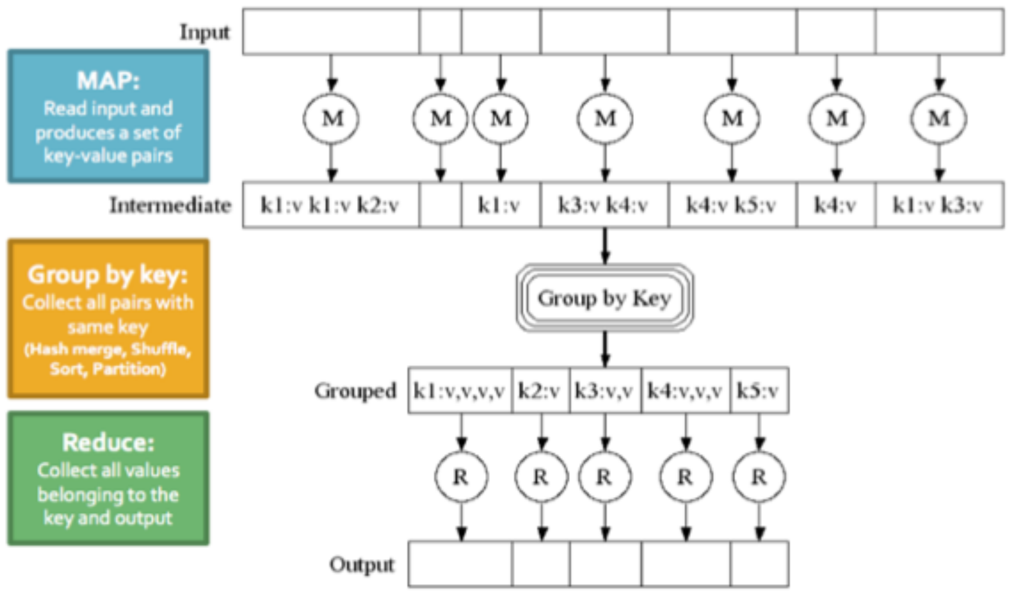
- Large Datasets (HDFS)
  - Large files broken into chunks
  - Chunks are replicated on different nodes
- Easy Parallelization
  - Takes care of scheduling
- Fault Tolerance
  - Monitors and re-executes failed tasks

# MapReduce

## 3 Steps

- Map:
  - Apply a user written map function to each input element.
  - The output of Map function is a set of key-value pairs.
- GroupByKey :
  - Sort and Shuffle : Sort all key-value pairs by key and output key-(list of value pairs)
- Reduce
  - User written reduce function applied to each key-[list of value] pairs

# Map-Reduce: A diagram



# Coping with Failure

MapReduce is designed to deal with compute nodes failing

Output from previous phases is stored. Re-execute failed tasks, not whole jobs.

**Blocking Property**: no output is used until the task is complete. Thus, we can restart a Map task that failed without fear that a Reduce task has already used some output of the failed Map task.

# Data Flow Systems

- MapReduce uses two ranks of tasks:
  - One is Map and other is Reduce
  - Data flows from first rank to second rank
- Data Flow Systems generalise this:
  - Allow any number of tasks
  - Allow functions other than Map and Reduce
- Spark is the most popular data-flow system.
  - RDD’s : Collection of records
  - Spread across clusters and read-only.

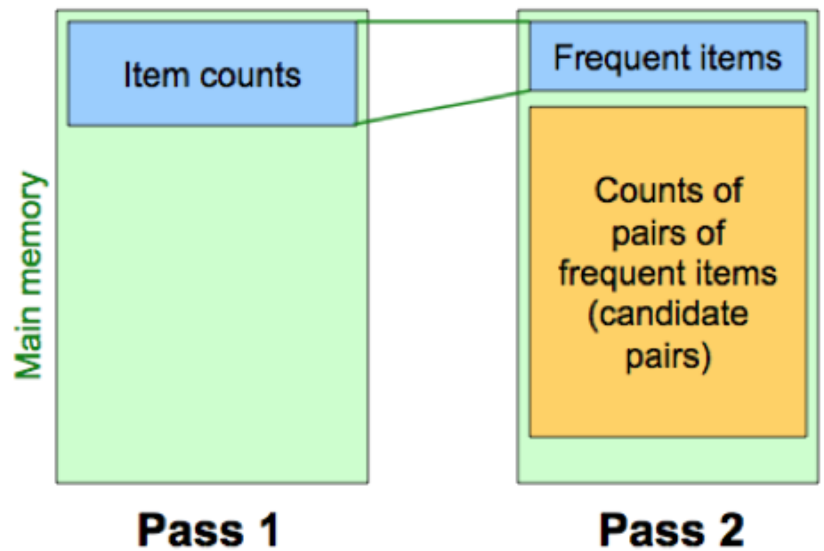
# Frequent Itemsets

- The Market-Basket Model
  - Items
  - Baskets
  - Count how many baskets contain an itemset
  - Support threshold => frequent itemsets
- Application
  - Confidence
    - Pr(D | A, B, C)

# Computation Model

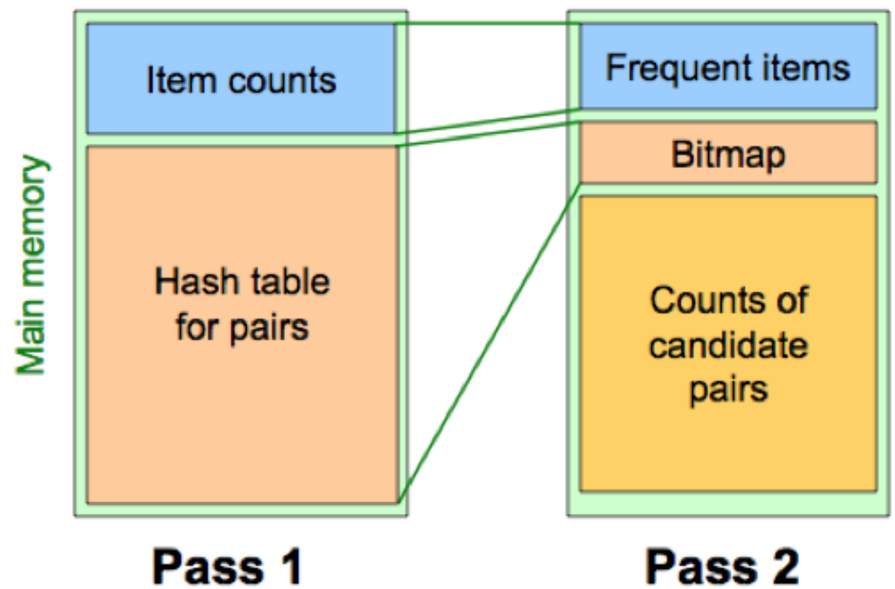
- Count frequent pairs
- Main memory is the bottleneck
- How to store pair counts?
  - Triangular matrix/Table
- Frequent pairs -> frequent items
- A-Priori Algorithm
  - Pass 1 - Item counts
  - Pass 2 - Frequent items + pair counts
- PCY
  - Pass 1 - Hash pairs into buckets
    - Infrequent bucket -> infrequent pairs
  - Pass 2 - Bitmap for buckets
    - Count pairs w/ frequent items and frequent bucket

# Main-Memory: Picture of A-Priori




Green box represents the amount of available main memory. Smaller boxes represent how the memory is used.

# Main-Memory: Picture of PCY



# All (Or Most) Frequent Itemsets

- Handle Large Datasets
- Simple Algorithm
  - Sample from all baskets
  - Run A-Priori/PCY in main memory with lower threshold
  - No guarantee
- SON Algorithm
  - Partition baskets into subsets
  - Frequent in the whole => frequent in at least one subset
- Toivonen’s Algorithm
  - Negative Border - not frequent in the sample but all immediate subsets are
  - Pass 2 - Count frequent itemsets and sets in their negative border
  - What guarantee?

# Locality Sensitive Hashing and Clustering

Hongtao Sun

# Locality-Sensitive Hashing

Main idea:

- What: hashing techniques to map similar items to the same bucket → candidate pairs
- Benefits: O(N) instead of O($N^2$): avoid comparing all pairs of items
    - Downside: false negatives and false positives
- Applications**:** similar documents, collaborative filtering, etc.

For the similar document application, the main steps are:

1. **Shingling** - converting documents to set representations
2. **Minhashing** - converting sets to short signatures using random permutations
3. **Locality-sensitive hashing** - applying the “b bands of r rows” technique on the signature matrix to an “s-shaped” curve

# Locality-Sensitive Hashing

Shingling:

- Convert documents to set representation using sequences of *k* tokens
- Example: abcabc with *k* = 2 shingle size and character tokens → {ab, bc, ca}
- Choose large enough *k* → lower probability shingle *s* appears in document
- Similar documents → similar shingles (higher Jaccard similarity)

Jaccard Similarity: $J(S_1, S_2) = |S_1 \cap S_2| / |S_1 \cup S_2|$

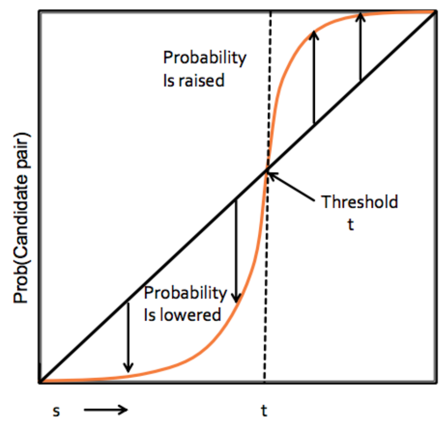
Minhashing:

- Creates summary signatures: short integer vectors that represent the sets and reflect their similarity

# Locality-Sensitive Hashing

General Theory:

- Distance measures d (similar items are "close"):
  - Ex) Euclidean, Jaccard, Cosine, Edit, Hamming
- LSH families:
  - A family of hash functions H is $(d_1, d_2, p_1, p_2)$-sensitive if for any x and y:
    - If $d(x, y) <= d_1$, $Pr [h(x) = h(y)] >= p_1$; and
    - If $d(x, y) >= d_2$, $Pr [h(x) = h(y)] <= p_2$.
- Amplification of an LSH families ("bands" technique):
  - AND construction ("rows in a band")
  - OR construction ("many bands")
  - AND-OR/OR-AND compositions



# Locality-Sensitive Hashing

Suppose that two documents have Jaccard similarity s.

Step-by-step analysis of the banding technique (b bands of r rows each)

- Probability that signatures agree in **all** rows of a particular band:
  - $s^r$
- Probability that signatures **disagree** in **at least one** row of a particular band:
  - $1 - s^r$
- Probability that signatures disagree in at least one row of **all** of the bands:
  - $(1 - s^r)^b$
- Probability that signatures **agree** in all rows of a particular band ⇒ Become candidate pair:
  - $1 - (1 - s^r)^b$

## Locality-Sensitive Hashing

A general strategy for **composing families** of minhash functions:

AND construction (over r **rows** in a single band):

- $(d_1, d_2, p_1, p_2)$-sensitive family $\Rightarrow$ $(d_1, d_2, p_1^r, p_2^r)$-sensitive family
- Lowers all probabilities

OR construction (over b **bands**):

- $(d_1, d_2, p_1, p_2)$-sensitive family $\Rightarrow$ $(d_1, d_2, 1 - (1 - p_1)^b, 1 - (1 - p_2)^b)$-sensitive family
- Makes all probabilities rise

We can try to make $p_1 \rightarrow 1$ (lower false negatives) and $p_2 \rightarrow 0$ (lower false positives), but this can require many hash functions.

# Clustering

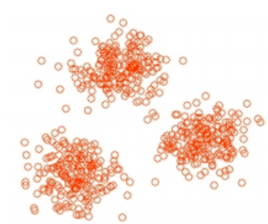
**What**: Given a **set of points** and a **distance measure**, group them into "**clusters**" so that a point is more similar to other points within the cluster compared to points in other clusters (unsupervised learning - without labels)

**How**: Two types of approaches

- **Point assignments**
  - Initialize centroids
  - **Assign** points to clusters, iteratively refine
- **Hierarchical**:
  - Each point starts in its own cluster
  - Agglomerative: repeatedly **combine** nearest clusters

# Point Assignment Clustering Approaches

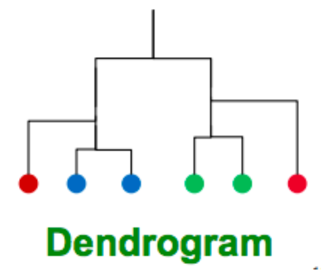
- Best for spherical/convex cluster shapes
- **k-means:** initialize cluster centroids, assign points to the nearest centroid, iteratively refine estimates of the centroids until convergence
  - Euclidean space
  - Sensitive to initialization (K-means++)
  - Good values of “k” empirically derived
  - Assumes dataset can fit in memory



- BFR algorithm: variant of k-means for very large datasets (residing on disk)
  - Keep running statistics of previous memory loads
  - Compute centroid, assign points to clusters in a second pass

## Hierarchical Clustering

- Can produce clusters with unusua
  - e.g. concentric ring-shaped clusters



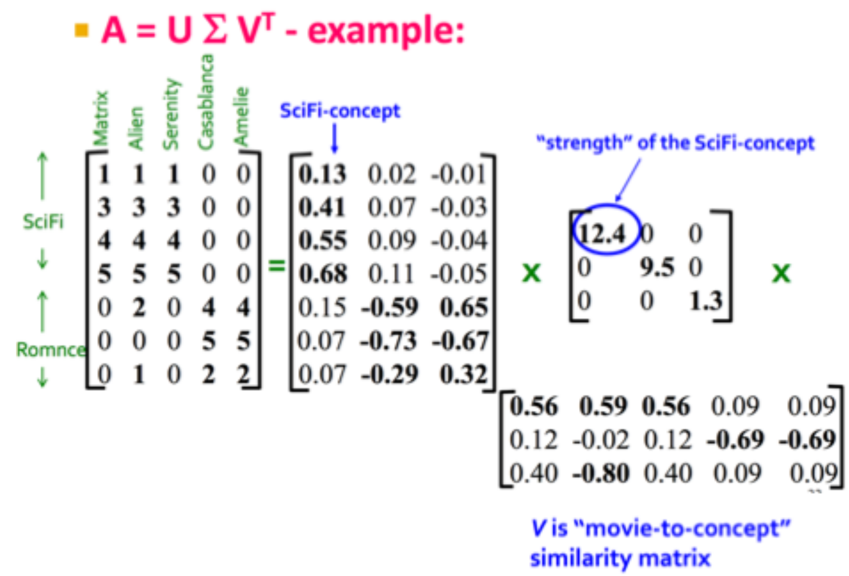

- **Agglomerative approach**:
  - Start with each point in its own cluster
  - Successively merge two "nearest" clusters until convergence
- **Differences from Point Assignment**:
  - Location of clusters: centroid in Euclidean spaces, "clustroid" in non-Euclidean spaces
  - Different intercluster distance measures: e.g. merge clusters with smallest ***max*** distance (worst case), ***min*** distance (best case), or ***average*** distance (average case) between points from each cluster
  - Which method works best depends on cluster shapes, often trial and error

# Dimensionality Reduction and Recommender Systems

Jayadev Bhaskaran

# Dimensionality Reduction

- Motivation
  - Discover hidden structure
  - Concise description
    - Save storage
    - Faster processing
- Methods
  - SVD
    - $M = U\Sigma V^T$
      - U user-to-concept matrix
      - V movie-to-concept matrix
      - Σ “strength” of each concept
  - CUR Decomposition
    - M = CUR

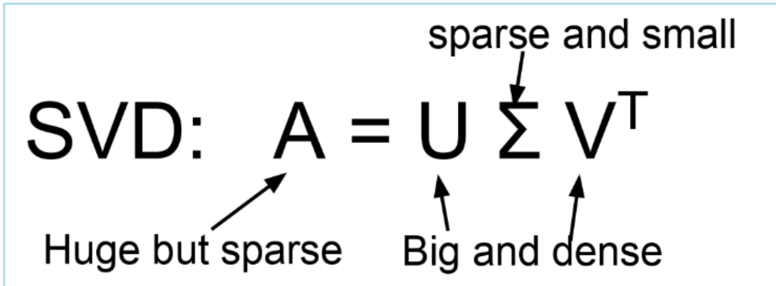


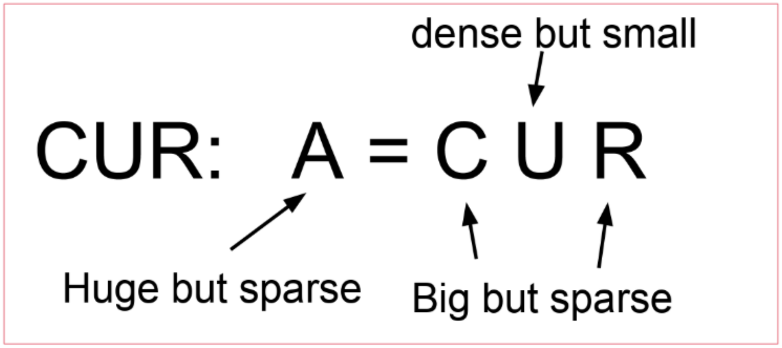
# SVD

- $M = U\Sigma V^T$
  - $U^TU = I$, $V^TV = I$, $\Sigma$ diagonal with non-negative entries
  - Best low-rank approximation (singular value thresholding)
  - Always exists for any real matrix M
- Algorithm
  - Find $\Sigma$, V
    - Find eigenpairs of $M^TM$ -> (D, V)
    - $\Sigma$ is square root of eigenvalues D
    - V is the right singular vectors
    - Similarly U can be read off from eigenvectors of $MM^T$
  - Power method: random init + repeated matrix-vector multiply (normalized) gives principal evec
  - Note: Symmetric matrices
    - $M^TM$ and $MM^T$ are both real, symmetric matrices
    - Real symmetric matrix: eigendecomposition $Q\Lambda Q^T$

# CUR

- M = CUR
- Non-uniform sampling
  - Row/Column importance proportional to norm
  - U: pseudoinverse of submatrix with sampled rows R & columns C
- Compared to SVD
  - Interpretable (actual columns & rows)
  - Sparsity preserved (U,V dense but C,R sparse)
  - May output redundant features



# Recommender Systems: Content-Based

**<u>What:</u>** Given a bunch of users, items and ratings, want to predict missing ratings

**<u>How:</u>** Recommend items to customer x similar to previous items rated highly by x

- Content-Based
  - Collect user profile ***x*** and item profile ***i***
  - Estimate utility: *u(**x**,**i**) = cos(**x**,**i**)*

users

| movies | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 1 | | 3 | | ? |
| 2 | | | 5 | 4 | |
| 3 | 2 | 4 | | 1 | 2 |
| 4 | | 2 | 4 | | 5 |
| 5 | | | 4 | 3 | 4 |
| 6 | 1 | | 3 | | 3 |

# Recommender Systems: Collaborative Filtering

- **user-user CF vs item-item CF**
  - user-user CF: estimate a user's rating based on ratings of similar users **who have rated the item**; similar definition for item-item CF
- **Similarity metrics**
  - Jaccard similarity: *binary*
  - Cosine similarity: *treats missing ratings as "negative"*
  - Pearson correlation coeff: *remove mean of non-missing ratings (standardized)*
- **Prediction of item i from user x:** ($s_{xy}$ = sim(x,y))

$$r_{xi} = \frac{\sum_{y \in N} s_{xy} \cdot r_{yi}}{\sum_{y \in N} s_{xy}}$$

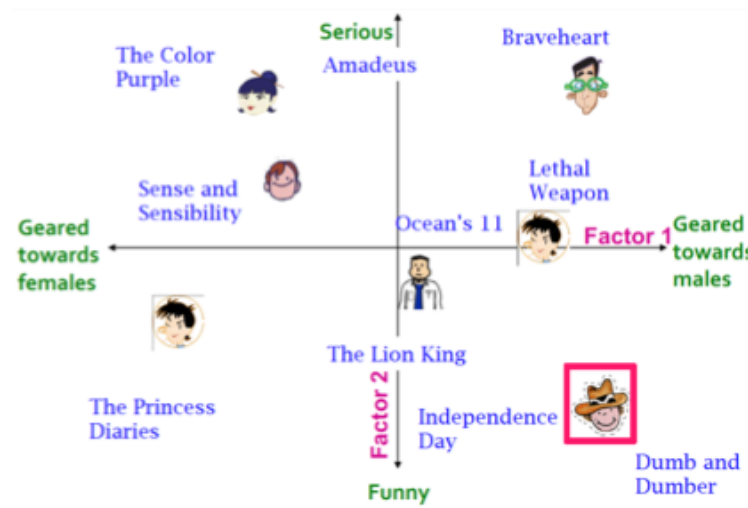
- Remove baseline estimate and only model rating deviations from baseline estimate, so that we're not affected by user/item bias

# Recommender Systems: Latent Factor Models

**Motivation:** Collaborative filtering is a local approach to predict ratings based on finding neighbors. Matrix factorization takes a more global view.

**Intuition:** Map users and movies to (lower-dimensional) latent-factor space. Make prediction based on these latent factors.

**Model:** $\hat{r}_{xi} = p_x \cdot q_i$ for user x and movie i



# Recommender Systems: Latent Factor Models

$$\min_{P,Q} \sum_{(x,i)\in\text{training}} (r_{xi} - p_x \cdot q_i)^2 + \lambda_1 \sum_x \|p_x\|^2 + \lambda_2 \sum_i \|q_i\|^2$$

- Only sum over observed ratings in the training set
- Use regularization to prevent overfitting
- Can solve via SGD (alternating update for P, Q)
- Can be extended to include biases (and temporal biases)

$$\hat{r}_{xi} = \mu + b_x + b_i + p_x \cdot q_i$$

# PageRank

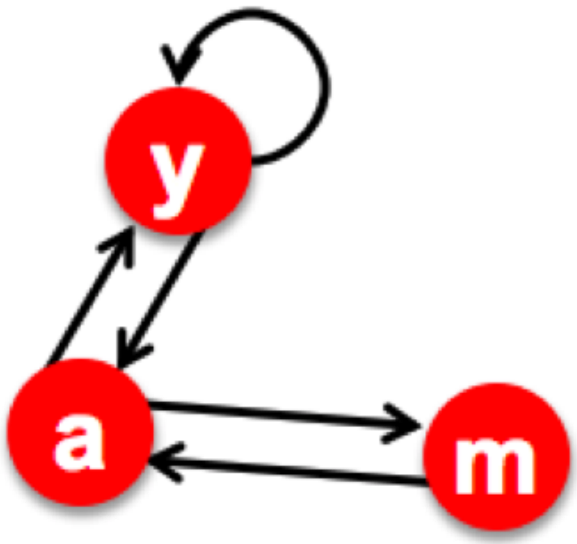
Lantao Mei

# PageRank

- PageRank is a method for determining the importance of webpages
  - Named after Larry Page
- Rank of a page depends on how many pages link to it
- Pages with higher rank get more of a vote
- The vote of a page is evenly divided among all pages that it links to

# Example

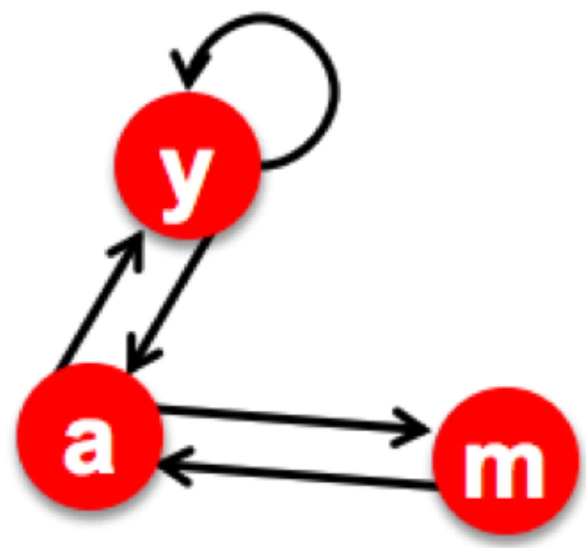
- $r_a = r_y/2 + r_m$
- $r_y = r_y/2 + r_a/2$
- $r_m = r_a / 2$



# Example

Deal with pathological situations by adding a random teleportation term

- $r_a = 0.8(r_y/2 + r_m) + 0.2/3$
- $r_y = 0.8(r_y/2 + r_a/2) + 0.2/3$
- $r_m = 0.8\ (r_a\ /\ 2) + 0.2/3$



# PageRank

If $i \rightarrow j$, then $M_{ji} = \frac{1}{d_i}$ else $M_{ji} = 0$

- **PageRank equation** [Brin-Page, '98]

$$r_j = \sum_{i \rightarrow j} \beta \frac{r_i}{d_i} + (1-\beta)\frac{1}{N}$$

- **The Google Matrix *A*:**

$[1/N]_{N \times N}$…N by N matrix where all entries are 1/N

$$A = \beta\, M + (1-\beta)\left[\frac{1}{N}\right]_{N \times N}$$

- **We have a recursive problem:** $\boldsymbol{r} = \boldsymbol{A} \cdot \boldsymbol{r}$

# Topic-specific PageRank

- Teleport can only go to a topic-specific set of "relevant" pages (teleport set)

- **To make this work all we need is to update the teleportation part of the PageRank formulation:**

$$A_{ij} = \begin{cases} \beta M_{ij} + (1-\beta)/|S| & \textbf{if } i \in S \\ \beta M_{ij} + 0 & \textbf{otherwise} \end{cases}$$

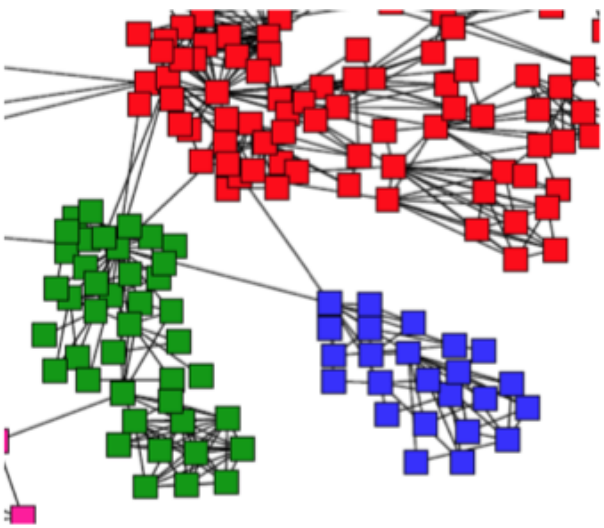
  - ***A*** is a stochastic matrix!

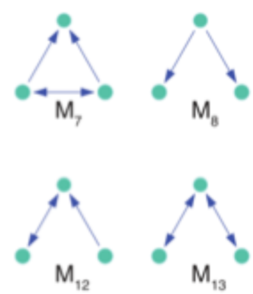
# Social Network Algorithms

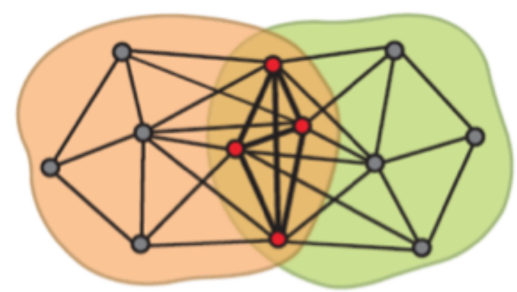
Ansh Shukla

# Graph Algorithms

- **Problem:** Finding “communities” in large graphs
  - A community is any structure we’re interested by in the graph.
  - Examples of properties we might care about: overlap, triangles, density.





# Personalized PageRank with Sweep

- Problem: Finding densely linked, non-overlapping communities.
- Intuition: Give a score to all nodes, rank nodes by score, and then partition the ranked list into clusters.
- What to know:

(Algorithm) Approximate Personalized PageRank –

  - Frame PPR in terms of lazy random walk
  - While error measure is too high
    - Run one step of lazy random walk

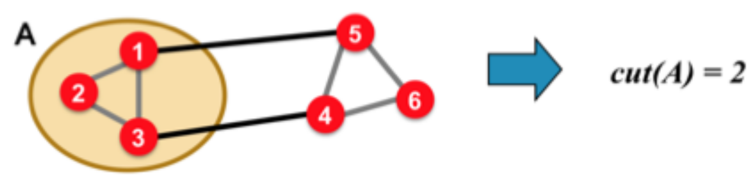
$$r_u^{(t+1)} = \frac{1}{2} r_u^{(t)} + \frac{1}{2} \sum_{i \to u} \frac{1}{d_i} r_i^{(t)}$$

$$q_u = p_u - r_u \qquad \max_{u \in V} \frac{q_u}{d_u} \geq \epsilon$$

# Personalized PageRank with Sweep

- **Problem:** Finding densely linked, non-overlapping communities.
- **Intuition:** Give a score to all nodes, rank nodes by score, and then partition the ranked list into clusters.
- **What to know:**
  - **Cut:** Set of edges with only one node in the cluster: $$cut(A) = \sum_{i \in A, j \notin A} w_{ij}$$

    **Note:** This works for weighed and unweighted (set all $w_{ij}$=1) graphs



# Personalized PageRank with Sweep

- **Problem:** Finding densely linked, non-overlapping communities.
- **Intuition:** Give a score to all nodes, rank nodes by score, and then partition the ranked list into clusters.
- **What to know:**

**Criterion: Conductance:**
Connectivity of the group to the rest of the network relative to the density of the group

$$\phi(A) = \frac{|\{(i,j) \in E; i \in A, j \notin A\}|}{\min(vol(A),\ 2m - vol(A))}$$

# Personalized PageRank with Sweep

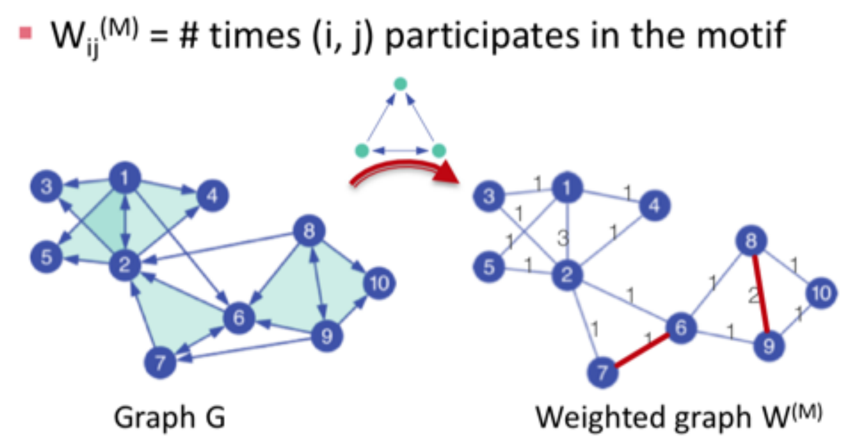
- **Problem:** Finding densely linked, non-overlapping communities.
- **Intuition:** Give a score to all nodes, rank nodes by score, and then partition the ranked list into clusters.
- **What to know:**

**Sweep:**

- Sort nodes in decreasing PPR score $r_1 > r_2 > \cdots > r_n$
- For each $\boldsymbol{i}$ compute $\boldsymbol{\phi}(\boldsymbol{A_i} = \{\boldsymbol{r_1}, \dots \boldsymbol{r_i}\})$
- Local minima of $\boldsymbol{\phi}(\boldsymbol{A_i})$ correspond to good clusters

# Motif-based spectral clustering

- **Problem:** Finding densely linked, non-overlapping communities (as before), but changing our definition of "densely linked".
- **Intuition:** Modify graph so edge weights correspond to our notion of density, modify conductance criteria, run PPR w/ Sweep.
- **What to know:**



# Motif-based spectral clustering

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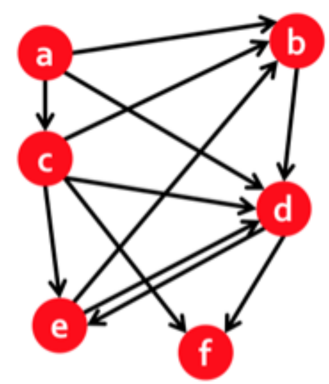
- **Intuition:** Modify graph so edge weights correspond to our notion of density, modify conductance criteria, run PPR w/ Sweep.
- **What to know:**

$$\phi_M(S) = \frac{\#(\text{motifs cut})}{\text{vol}_M(S)}$$

# Searching for small communities (trawling)

- Problem: Finding complete bipartite subgraphs $K_{s,t}$
- Intuition: Reframe the problem as one of finding frequent itemsets: think of each vertex as a basket defined by its neighbors. Run A-priori with frequency threshold s to get item sets of size t.
- What to know:

a = {b,c,d}



# Graph Embeddings

- **Problem:** Want to represent nodes in graph in vector space while capturing relevant properties like graph topology.
- **Intuition:** Define a mapping from nodes to embeddings.
- Define a node similarity function (dot product)
- Optimize the parameters of the encoder so that: similarities in one representation (graph) match similarities in another (embedding)

# Graph Embeddings

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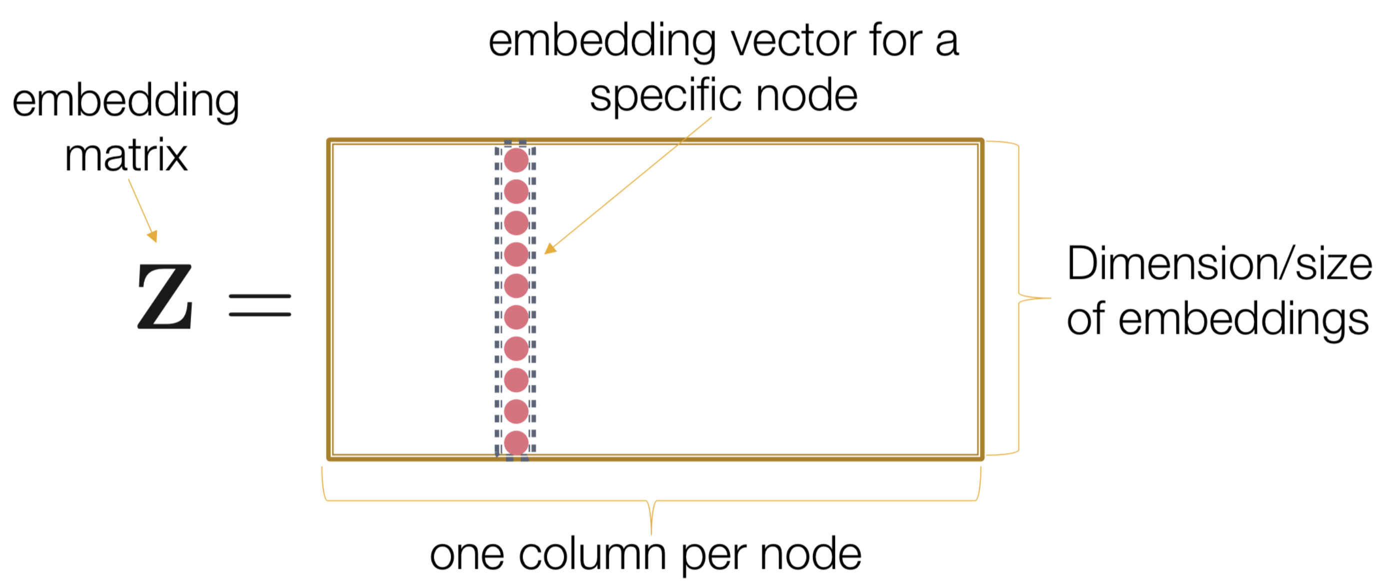
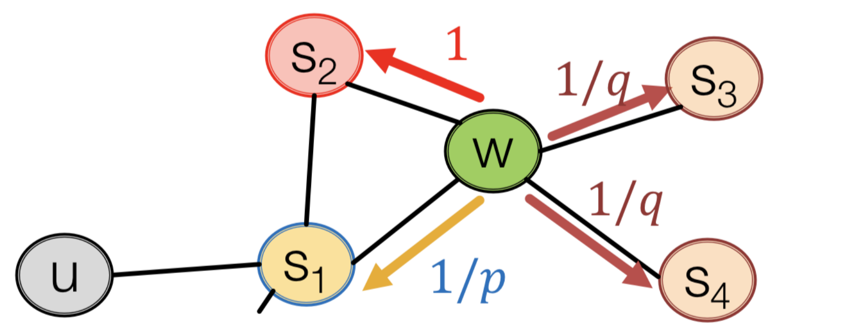
- **Intuition:** Define a mapping from nodes to embeddings.



# Graph Embeddings

- **Problem:** Want to represent nodes in graph in vector space while capturing relevant properties like graph topology.
- **Intuition:**
  - Select a random walk




- $p, q$ model transition probabilities
  - $p$ … return parameter
  - $q$ … "walk away" parameter

# Graph Embeddings

- **Problem:** Want to represent nodes in graph in vector space while capturing relevant properties like graph topology.
- **Intuition:**
  - Optimize embedding

$$\mathcal{L} = \underbrace{\sum_{u \in V}}_{\text{sum over all nodes } u} \underbrace{\sum_{v \in N_R(u)}}_{\text{sum over nodes } v \text{ seen on random walks starting from } u} -\log\left(\underbrace{\frac{\exp(\mathbf{z}_u^\top \mathbf{z}_v)}{\sum_{n \in V} \exp(\mathbf{z}_u^\top \mathbf{z}_n)}}_{\text{predicted probability of } u \text{ and } v \text{ co-occuring on random walk}}\right)$$

# Graph Embeddings

- **Problem:** Want to represent nodes in graph in vector space while capturing relevant properties like graph topology.
- **Intuition:**
  - Optimize embedding

$$\mathcal{L} = \sum_{u \in V} \sum_{v \in N_R(u)} -\log(P(v|\mathbf{z}_u))$$

# Large-Scale Machine Learning

Jerry Zhilin Jiang

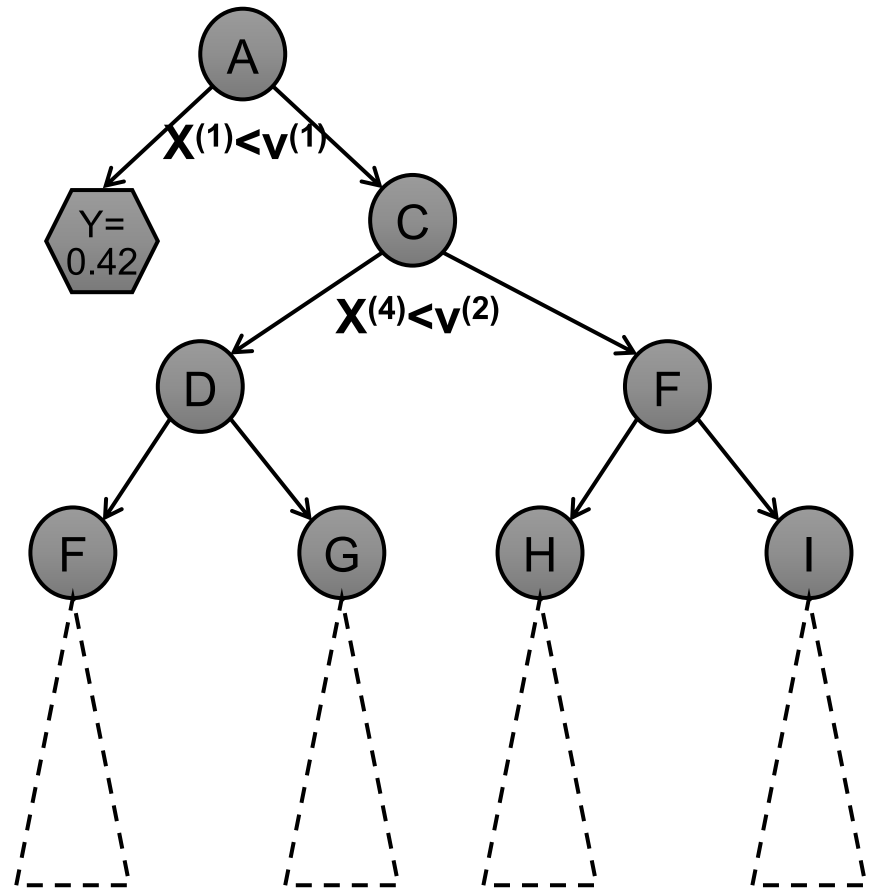
# Large-scale machine learning

## Overview

- **Supervised learning**
  - given training set with labels $(x_i, y_i)$
  - Learn the function $f$ that predicts y given x, $f(x) = y$
  - Why Hard? Need to generalize well to unseen data
- **Classification vs. Regression**
  - Classification: Label $y$ belongs to a discrete set
  - Regression: Label $y$ is continuous
- **Methods covered in this course**
  - Decision Tree
  - Support Vector Machine (SVM)

# Decision Tree

- Input: $d$ attributes (features)
  $$x^{(1)}, x^{(2)}, \dots, x^{(d)}$$
  Can be numerical or categorical
- Output: $y$ (label)
  Either numerical (regression)
  or categorical (classification)
- Given data point $\boldsymbol{x_i}$
  Start from root, “drop” it down the tree until it hits a leaf node
- Make prediction accordingly after reaching the leaf node




Three problems:

- How to split?
- When to stop?
- How to predict?

# How to split?

Measure the quality of potential splits based on some criterion

**Regression: Purity** Split on node $(x^{(i)}, v)$,
create $\boldsymbol{D}, \boldsymbol{D_L}, \boldsymbol{D_R}$ (parent / left child/ right child dataset)

$$|\boldsymbol{D}| \cdot \boldsymbol{Var}(\boldsymbol{D}) - \left(|\boldsymbol{D_L}| \cdot \boldsymbol{Var}(\boldsymbol{D_L}) + |\boldsymbol{D_R}| \cdot \boldsymbol{Var}(\boldsymbol{D_R})\right)$$

**Classification: Information Gain** **IG(Y|X)**
How much information about Y is contained in X.

$$\boldsymbol{IG(Y|X) = H(Y) - H(Y|X)}$$

Entropy $H(x) = -\sum_{j=1}^{n} p_j \, log \, p_j$

Conditional entropy
$H(Y|X) = \sum_{j=1}^{n} P\left(X = v_j\right) H(Y|X = v_j)$

# When to stop?

- When the leaf is “pure” (variance below threshold)
- When # of examples in a leaf node is too small

# How to predict?

Regression

- predict average $y_i$ of examples in the leaf
- Build linear regression model on the example points

Classification

- Predict most common $y_i$ in the leaf

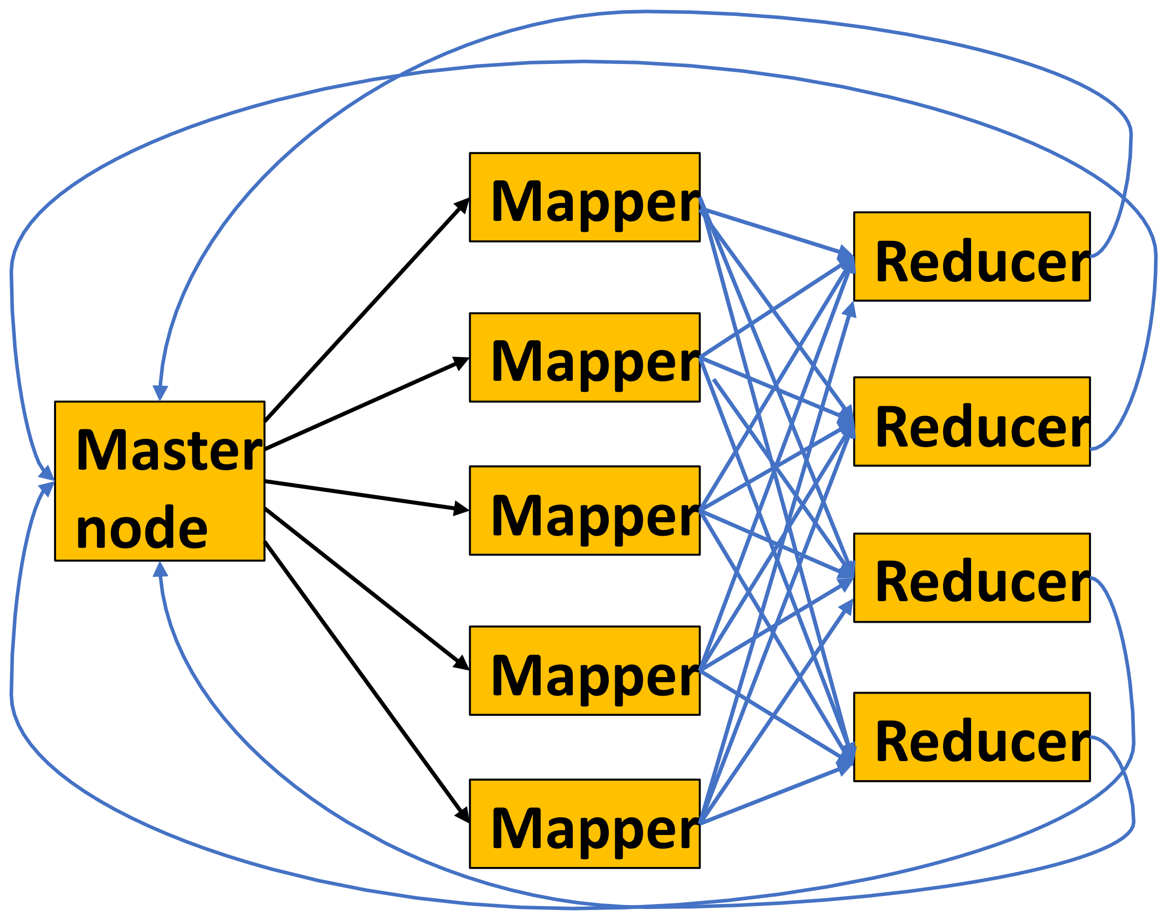
## Building decision trees with MapReduce: PLANET

- Tree small (in memory), data too large to keep in memory
- Hundreds of numerical (discrete or continuous) attributes
- Target variable is numerical (i.e. regression)

Build the decision tree one level at a time

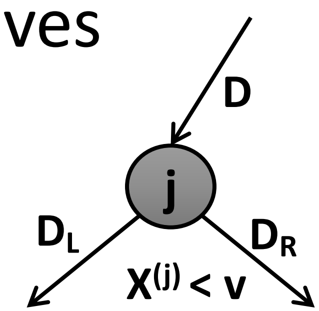
**Master Node**
Keeps track of the model and decides how to grow the tree

**MapReduce**
Do the actual work on data



**3 Types of MapReduce jobs**:

- **Initialization** (run once first)
  - Find candidate splits (node n, attribute $X^{(j)}$, value v)
  - Ideally divide data into similar-sized buckets
- **FindBestSplit** (run multiple times)
  - For a split node j find $X^{(j)}$ and v that **maximizes purity**
- **InMemoryBuild** (run once last)
  - If there is little data entering a tree node, Master runs an InMemoryBuild MapReduce job to grow the entire subtree below that node, including leaves



# Learning Ensembles

## Bagging

- Learn **multiple trees**, each using an independently sampled subset of the training data (sampled with replacement)
- Predictions from all trees are aggregated (e.g. majority vote, average) to compute the final model prediction

## Improvement: Random Forests

- At each candidate split, consider only a random subset of all available features
- Avoids cases where all trees select the same few strong features (Breaks correlation between different decision trees)
- Achieves state-of-the-art results in many classification problems

# SVM

Given training data

$$(x_1, y_1) \dots (x_n, y_n)$$

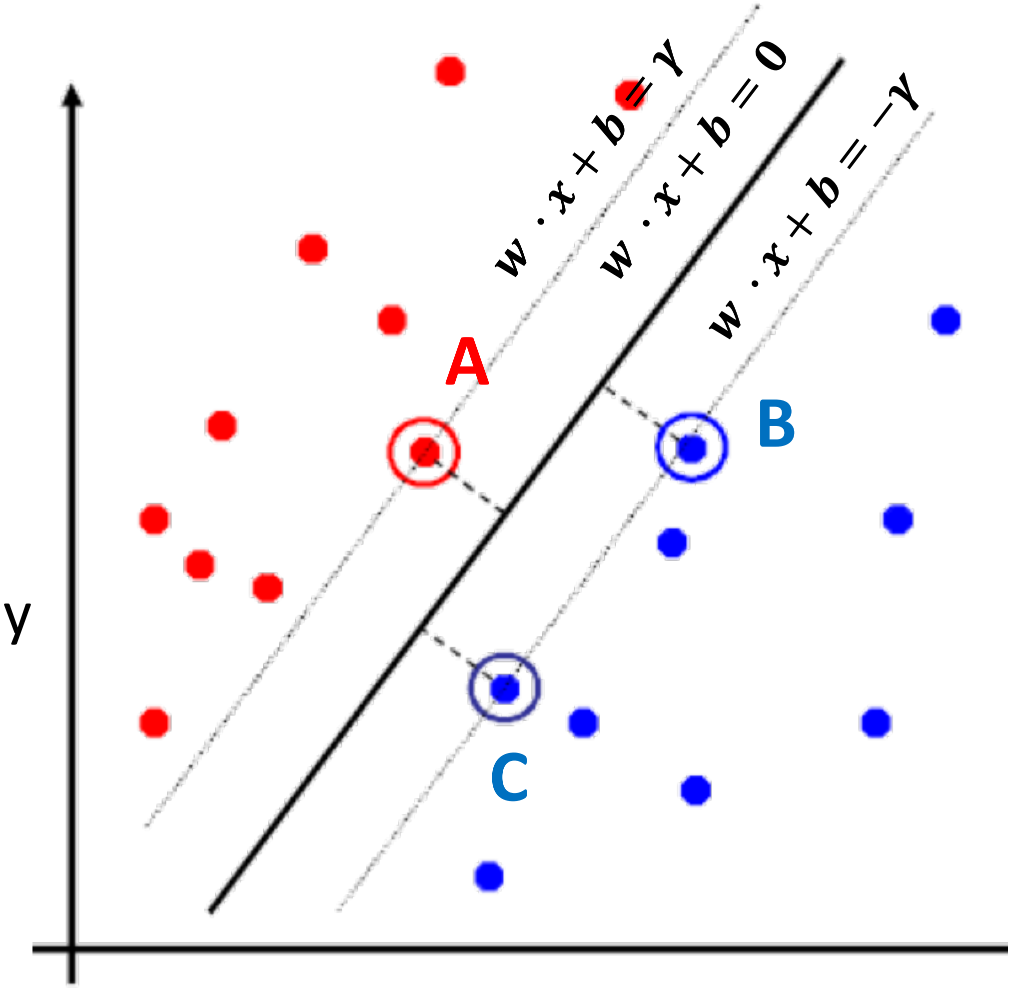
x: d-dimensional, real valued

$$x_i = (x_i^{(1)}, x_i^{(2)}, \dots, , x_i^{(d)})$$
$$y_i = -1 \; or + 1$$

$\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}$: **support vectors**, uniquely define the decision boundary

**Margin $\gamma$**: distance of closest example from the decision line (hyperplane)

Goal: **maximize margin $\gamma$,** find separating hyperplane with the largest distance possible from both positive / negative point



**From maximize $\gamma$ to minimize $\frac{1}{2}||w||^2$**

A lying on support plane

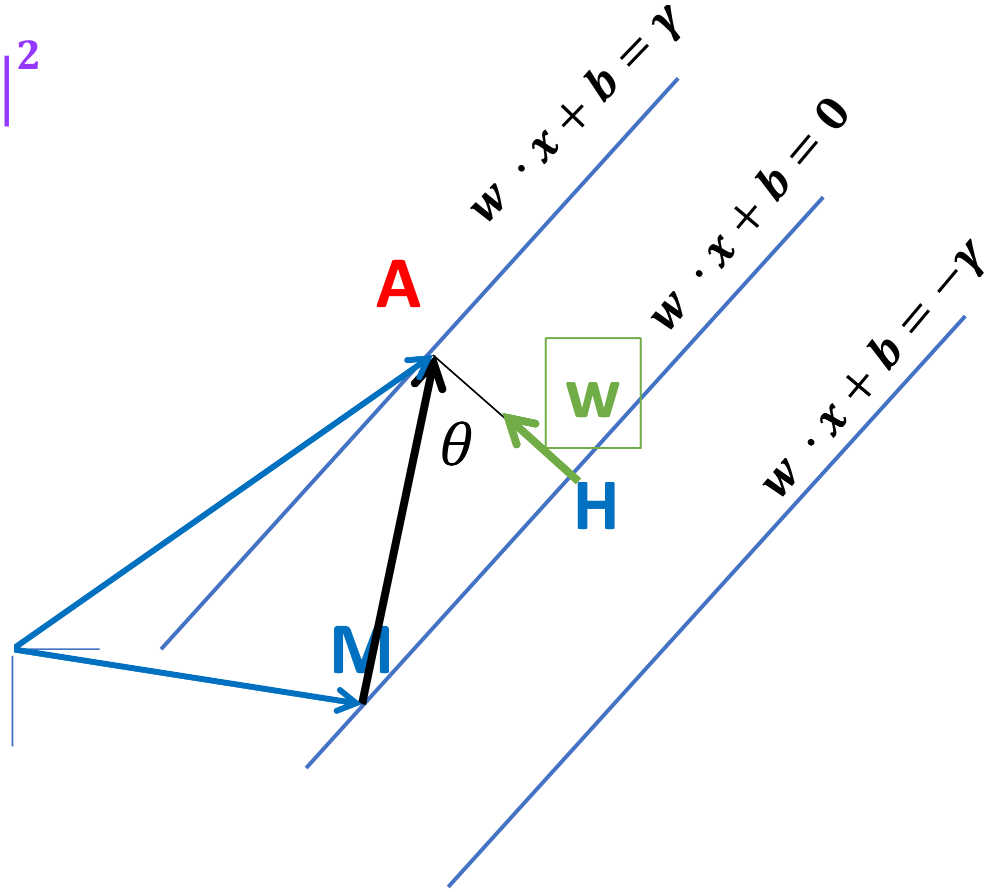
Goal: **Maximize distance $||AH||$**

$$\mathbf{MA}\cdot\mathbf{w} = ||\mathbf{w}|| \times ||\mathbf{AM}|| \times \cos\theta$$

$$\mathbf{w}\cdot\mathbf{A} + b = \gamma$$

$$\mathbf{w}\cdot\mathbf{M} + b = 0$$

$$\begin{aligned} ||\mathbf{AH}|| &= ||\mathbf{AM}|| \times \cos\theta \\ &= \frac{|\mathbf{AM}\cdot\mathbf{w}|}{||\mathbf{w}||} \\ &= \frac{|(\mathbf{A}-\mathbf{M})\cdot\mathbf{w}|}{||\mathbf{w}||} \\ &= \frac{|\mathbf{A}\cdot\mathbf{w}-\mathbf{M}\cdot\mathbf{w}|}{||\mathbf{w}||} \\ &= \frac{|(\gamma-b)-(-b)|}{||\mathbf{w}||} \\ &= \frac{\gamma}{||\mathbf{w}||} \end{aligned}$$




- **Distance from A to H: $\gamma$ measured in $||w||$**
- Scale $\gamma$ and $||\boldsymbol{w}||$ both by 2, nothing changes, thus we can either
- **Normalize w, i.e $||w|| = 1$, maximize $\gamma \Leftrightarrow$**
- **Fix margin $\gamma = 1$, minimize length of w**
- We use the second way

Optimization problem formalized

**fix margin $\gamma = 1$, minimize length of w**

$$\min_w \tfrac{1}{2} \| w \|^2$$
$$s.t. \forall i, y_i (w \cdot x_i + b) \geq 1$$

In real world, data is often not linear separable - Introduce penalty

$$\arg\min_{w,b} \ \tfrac{1}{2} \underbrace{w \cdot w}_{\text{Margin}} + C \cdot \sum_{i=1}^{n} \underbrace{\max\{0, 1 - y_i (w \cdot x_i + b)\}}_{\text{Empirical } \mathbf{loss\ L} \text{ (how well we fit training data)}}$$

Regularization hyperparameter

Penalize mis-predicted points AND correctly predicted points that fall within the margin

# Cost Function

$$J(w,b) = \tfrac{1}{2}\sum_{j=1}^{d}\left(w^{(j)}\right)^2 + C\sum_{i=1}^{n}\max\left\{0, 1 - y_i\left(\sum_{j=1}^{d} w^{(j)} x_i^{(j)} + b\right)\right\}$$

Minimizing cost function J

- Batch Gradient Descent
- Stochastic Gradient Descent
- Mini-batch Gradient Descent

$$\theta = \theta - \eta \cdot \nabla_\theta J(\theta; x^{(i)}; y^{(i)}).$$

# Large-scale machine learning

## Summary

- **Decision Tree**
  - Classification or Regression
  - Numerical or categorical features, usually dense
  - Complicated decision boundaries

- **Support Vector Machine (SVM)**
  - Classification (usually $y = \pm 1$)
  - High-dimensional, sparse feature space
  - Simple, linear decision boundary

# Streaming Algorithms

Wensi Yin

# Bloom filters - Problem

- You have a stream of ads. How to make sure a user **doesn't see the same ad** multiple times?
- Naïve approach: store the ads in a hash table.
  - This takes O(# ads) space!
- What if we want to use at most 100 slots of memory?
  - We can not have a deterministic answer, but we can answer it **with high prob**!

# Bloom filters - Construction

- What if we want to use at most 100 slots of memory?
- Create a bit array B of size 100, initialized to all 0's
- Create a hash function that hashes ads to 100 different possible buckets
- When an ad is seen, hash the ad to a bucket (say, bucket 79), and set B[79] = 1

# Bloom filters - Test

- How to check whether a new incoming ad has been seen?
- Suppose the ad hashes to bucket 89
- If B[89] = 0, you know the ad has NOT been seen
- If B[89] = 1, the ad might have been seen, but we also might have seen a different ad that happened to hash to the same bucket.
- **Prob false positive**: $1 - \left(1 - \frac{1}{100}\right)^m, (m: \#\ distcint\ ads\ seen\ so\ far)$
- K number of hash functions: check if all of the k bits corresponding to the hash functions are set to 1.
- **Reasonable number** of hash functions will help **reduce false positive prob.**

# Flajolet-Martin Algorithm

- Problem: a data stream consists of elements chosen from a set of size n. Maintain **a count of the number of distinct elements** seen so far.
- Pick a hash function h that element in set to $\log_2 n$ bits.
- For each stream element a, let r(a) be the number of trailing 0's in h(a).
  - Record R = the maximum r(a) seen for any a in the stream.
  - Also known as the "tail length"
- Estimate of distinct elements = 2^R .
- Intuitively, seeing r trailing 0s is "unusual" (prob 1/(2^r))
  - More distinct elements leads to a higher chance of seeing this "unusual" event
- If we notice this "unusual" event, our estimate should be correspondingly higher

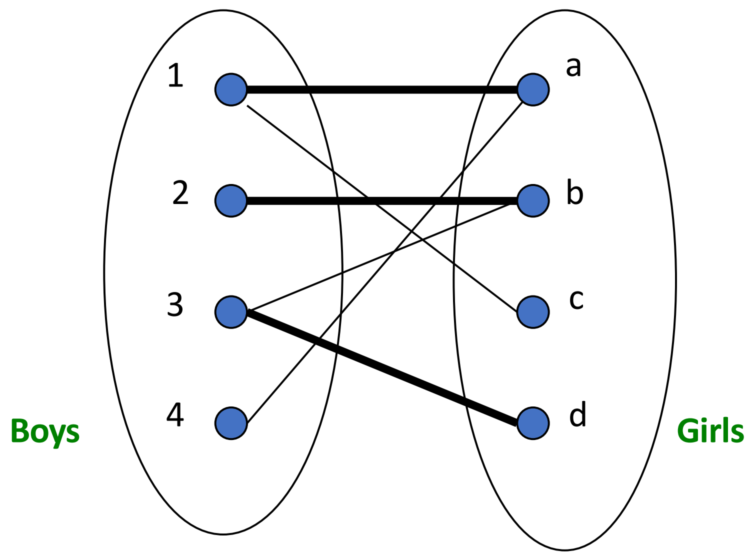
# AMS method

- Problem: Suppose a stream has elements chosen from a set of n values. Let $m_i$ be the number of times value i occurs. **Estimate the k-th moment** which is the sum of $m_i^k$ over all i.
  - $0^{th}$ moment = number of distinct elements in the stream.
  - $1^{st}$ moment = sum of counts of the numbers of elements = length of the stream.
  - $2^{nd}$ moment = measure of how uneven the distribution is.
- Algorithm for $2^{nd}$ moment:
  - Assume stream seen so far has n elements
  - Pick a random starting and let the chosen time have element a in the stream.
  - Let X = # times a is seen in the stream from that point onward
  - Estimate of $2^{nd}$ moment = n(2X -1)
- Application:
  - $2^{nd}$ moment can be used to estimate self-join size in database.

# Computational Advertising

Stefanie

# Advertising: Bipartite Matching




**M = {(1,a),(2,b),(3,d)}** is a **matching**

**Cardinality of matching = |M| = 3**

# Advertising: Online Algorithms and Competitive Ratio

- Question: How to find a maximum matching for a given bipartite graph
- Polynomial offline algorithm exists, but what's the best we can do in online setting?

**Competitive ratio =**

$$\min_{\text{all possible inputs } I} (|M_{greedy}| / |M_{opt}|)$$

**(greedy's worst performance over all possible inputs *I*)**

- In maximization problem, competitive ratio <=1
- In minimization problem, competitive ratio >= 1
- **Greedy bipartite matching algorithm: competitive ratio = 1/2.**
  - Easy to find examples, proofs are more difficult

# Advertising: Adwords and Click Through Rate

- Adwords problem is example of online algorithm

**Instead of sorting advertisers by bid, sort by expected revenue**

| Advertiser | Bid | CTR | Bid * CTR |
|---|---|---|---|
| B | $0.75 | 2% | 1.5 cents |
| C | $0.50 | 2.5% | 1.125 cents |
| A | $1.00 | 1% | 1 cent |

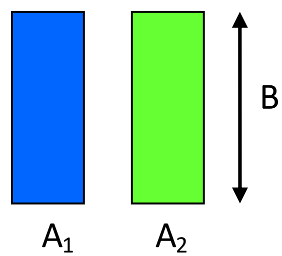
**Challenges:**

- CTR of an ad is unknown
- Advertisers have limited budges and bid on multiple queries

# Advertising: Greedy vs BALANCE Algorithm

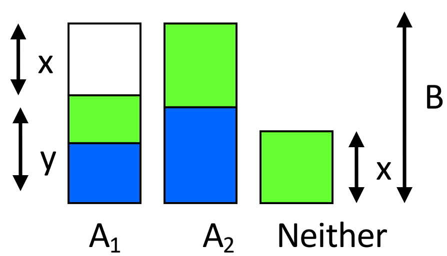
- Simplified setting:
  - There is **1** ad shown for each query
  - All advertisers have the same budget ***B***
  - All ads are equally likely to be clicked
  - Value of each ad is the same (=**1**)
- Greedy Algorithm: Pick any advertiser who has a bid for query
  - Competitive ratio is 1/2.
- **BALANCE Algorithm**: Pick advertiser with largest unspent budget
  - **Competitive ratio is (1-1/e) = 0.63**
  - No online algorithm can do better!

# Advertising: 2 Case Analysis




- Queries allocated to $A_1$ in optimal solution
- Queries allocated to $A_2$ in optimal solution

Opt revenue = 2B
Balance revenue = 2B-x




Balance allocation

We claim $x \leq B/2$
=> Competitive Ratio = 3/4.

# Advertising: Generalized BALANCE Algorithm

- Generalized Scenario:
  - Arbitrary bids and arbitrary budgets
- BALANCE algorithm has arbitrary bad competitive ratio
  - competitive ratio -> 0
- **Generalized BALANCE:** consider query $q$, bidder $i$
  - Bid = $x_i$
  - Budget = $b_i$
  - Amount spent so far = $m_i$
  - Fraction of budget left over $f_i = 1 - m_i/b_i$
  - Define $\psi_i(q) = x_i(1 - e^{-f_i})$
- Allocate query $q$ to bidder $i$ with largest value of $\psi_i(q)$
- **Same competitive ratio (1-1/$e$) = 0.63**

# Learning Through Experimentation

Baige(Alice) Liu

# Learning Through Experimentation

- Take action, get reward, learn from that reward.
- Approach: formalize as a Multiarmed Bandits. Take action = pull an arm.

# Multiarmed Bandits

- K-armed bandit: $|action| = K$.
- Each arm $a$ wins (reward = 1) with fixed (unknown) probability $\mu_a$, and loses (reward = 0) with fixed (unknown) probability $1 - \mu_a$.
- Want to maximize total reward, but need information about (unknown) $\mu_a$.
- Every time we pull $a$, we learn a bit about $a$, so we can estimate $\mu_a$ (denoted as $\widehat{\mu_a}$).

# Bandit Algorithm: Greedy and Epsilon-Greedy

- Tradeoff between exploration and exploitation.
- Exploration: pull arm haven't tried before. Exploitation: pull arm with current highest $\widehat{\mu_a}$.
- Greedy algorithm takes action with highest average reward based on samples seen so far ($\widehat{\mu_a}$). But it does not explore sufficiently.
- Epsilon-Greedy takes a random $a$ with a decaying probability $\varepsilon_t$ ($O(\frac{1}{t})$), and it takes the same action that Greedy would take with probability $1 - \varepsilon_t$. During exploration time, it selects random $a$ equally likely, which is suboptimal.

# Bandit Algorithm: $UCB_1$

- Balances exploration and exploitation by taking confidence into consideration.
- A confidence interval is a range of values within which we are sure the mean lies with a certain probability.
- Let $m_a$= number of times $a$ is pulled, $\delta$ = given confidence level.
- Then, confidence interval $b = \max(\mu_a|\delta) - \min(\mu_a|\delta) = 2\sqrt{\frac{2 \ln T}{m_a}}$.
- $UCB(a) = \widehat{\mu_a} + \alpha\sqrt{\frac{2 \ln T}{m_a}}$.

# Bandit Algorithm: $UCB_1$

- The accuracy of $\widehat{\mu_a}$ is dependent on how many times we have tried $a$: trying $a$ too few times means our estimate of $\mu_a$ could be very off from the true value $\mu_a$, which means it has a large confidence interval. This interval shrinks as we try $a$ more often.
- Strategy: try arm $a$ with the highest upper bound on its confidence interval, i.e., action as good as possible given the available evidence. It is called an optimistic policy.
- $UCB(a) = \widehat{\mu_a} + \alpha\sqrt{\frac{2\ln T}{m_a}}$.