

# Scalable Infomin Learning

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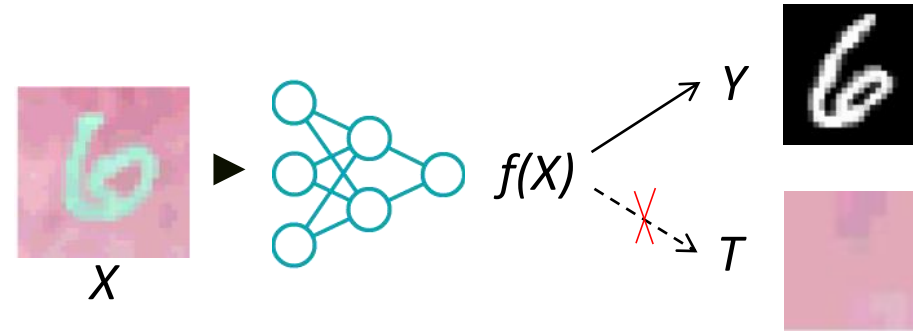
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# Agenda

- **Background**
- **Method**
- **Results**
- **Conclusion**

# Background

## Infomin representation learning



$$\min_f \underbrace{\mathcal{L}(f(X), Y)}_{\text{utility}} + \beta \cdot \underbrace{I(f(X); T)}_{\text{mutual info}}$$

Applications:


(a) domain adaptation; (b) disentanglement; (c) fairness, ...

# Background

## Why infomin learning is hard?

$$\min_f \underbrace{\mathcal{L}(f(X), Y)}_{\text{utility}} + \beta \cdot \underbrace{I(f(X); T)}_{\text{mutual info}}$$

intractable

estimate MI by a neural net  $t^*$  



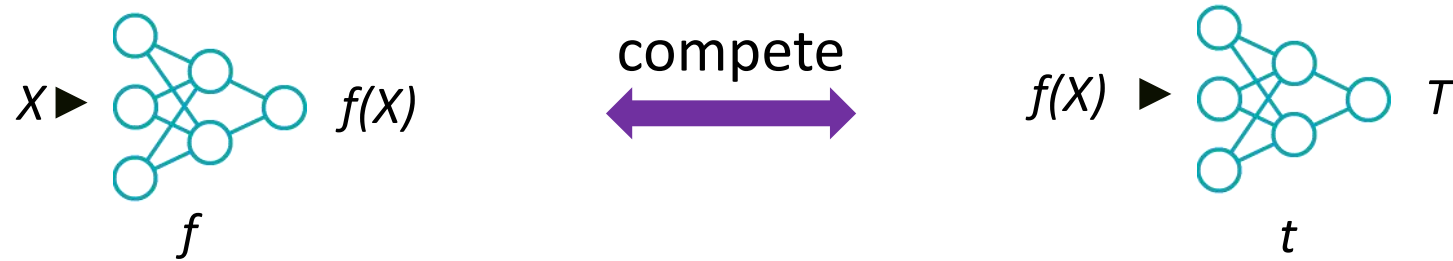
$$\min_f \max_t \mathcal{L}(f(X); Y) + \beta \cdot \hat{I}_t(f(X); T)$$

\*see e.g. DANN, LAFTR, Factor-VAE, CLUB, Learning-not-to-learn...

# Background

## Why infomin learning is hard?

$$\min_f \max_t \mathcal{L}(f(X); Y) + \beta \cdot \hat{I}_t(f(X); T)$$



- time consuming
- hard to optimise

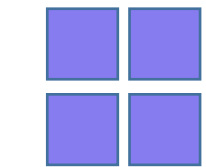
# Method

**Goal:**  ~~$\min_f \max_t \mathcal{L}(f(X); Y) + \beta \cdot \hat{I}_t(f(X); T)$~~

**Idea:** find a proxy to  $I(Z; T)$  whose estimate is easy to compute

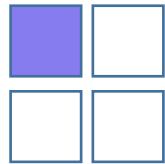


sliced mutual information (SI)



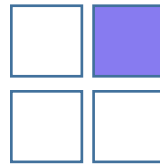
$\min I(Z; T)$

$\approx$



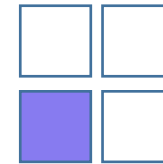
$\min SI_1(Z; T)$

$\&$



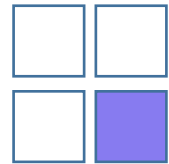
$\min SI_2(Z; T)$

$\&$



$\min SI_3(Z; T)$

$\&$

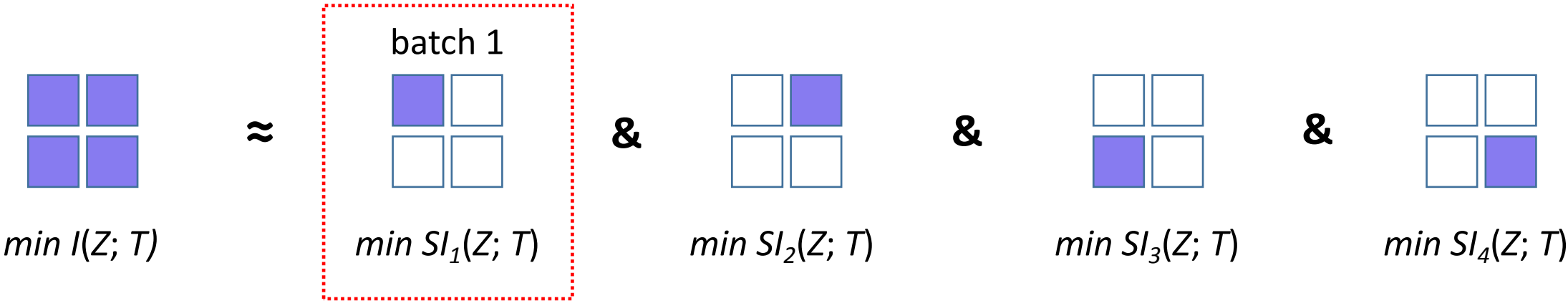


$\min SI_4(Z; T)$

# Method

$$\min_f \mathcal{L}(f(X); Y) + \beta \cdot \underline{SI(f(X); T)},$$

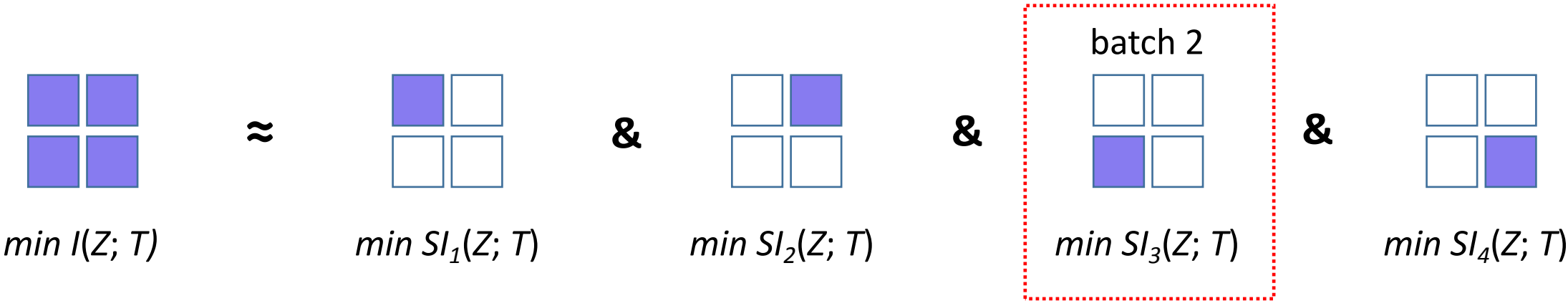
minimise randomly chosen  $SI$  in different mini-batches



# Method

$$\min_f \mathcal{L}(f(X); Y) + \beta \cdot \underline{SI(f(X); T)},$$

minimise randomly chosen  $SI$  in different mini-batches

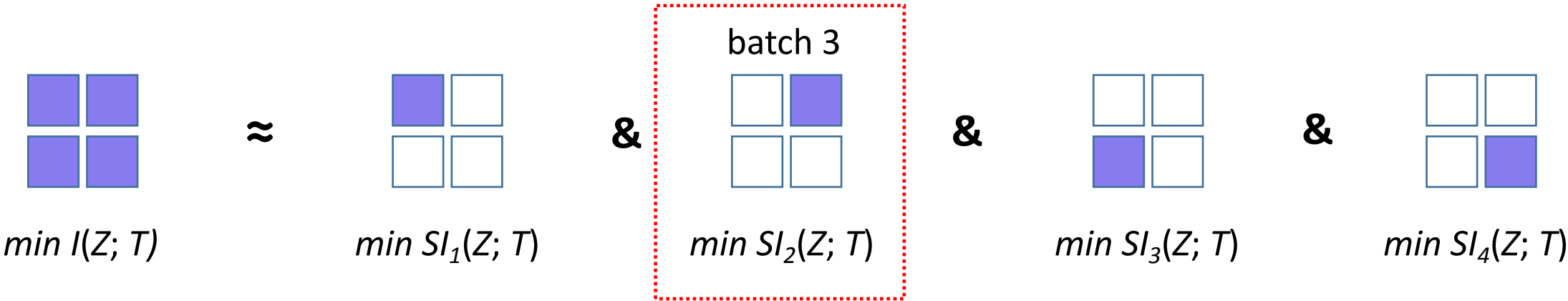




# Method

$$\min_f \mathcal{L}(f(X); Y) + \beta \cdot \underline{SI(f(X); T)},$$

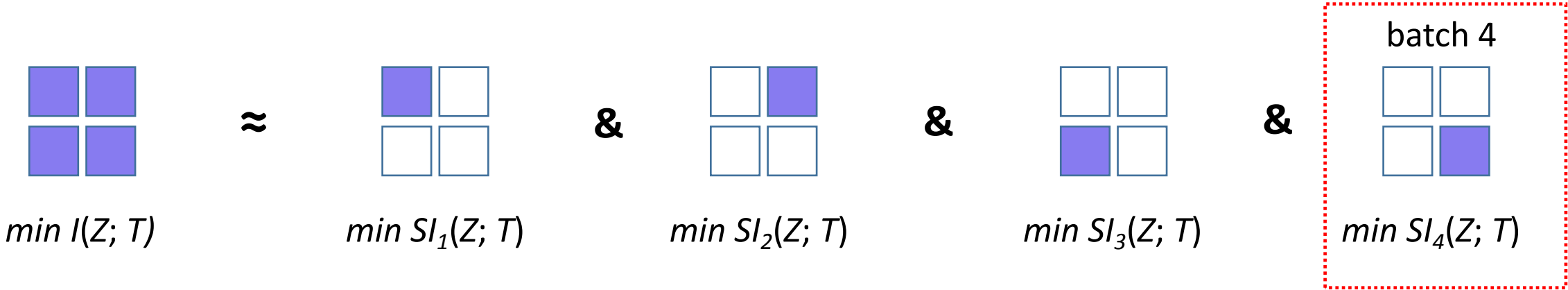
minimise randomly chosen  $SI$  in different mini-batches



# Method

$$\min_f \mathcal{L}(f(X); Y) + \beta \cdot \underline{SI(f(X); T)},$$

minimise randomly chosen  $SI$  in different mini-batches



# Method

## Estimate of $SI$

$$SI(Z; T) \approx \sup_{i,j} \sup_{h_i, g_j} \rho(h_i(\theta_i^\top Z), g_j(\phi_j^\top T)),$$

Diagram illustrating the components of the formula:

- Pearson corr.** points to  $\rho$
- 1D polynomials** points to  $h_i$  and  $g_j$
- uniformly sampled from unit spheres** points to  $\theta_i$  and  $\phi_j$

\*can be solved analytically by eigendecomposition

# Method

## Properties of $SI$

1.  $SI(Z;T) = 0 \Leftrightarrow I(Z;T) = 0$

2.  $SI(Z;T) \in [0, 1]$

3. approximation has analytic solution

# Method

## Adversarial infomin learning

$$\min_f \max_t \mathcal{L}(f(X); Y) + \beta \cdot \hat{I}_t(f(X); T)$$

Estimate  $I$  by SGD

*// Max-step*  
**for**  $l_2$  in 1 to  $L_2$  **do**  
     $t \leftarrow t + \eta \nabla_t \hat{I}_t(f(X); T)$  with data in  $\mathcal{D}'$   
**end for**

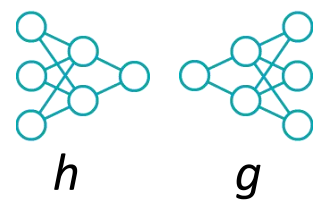
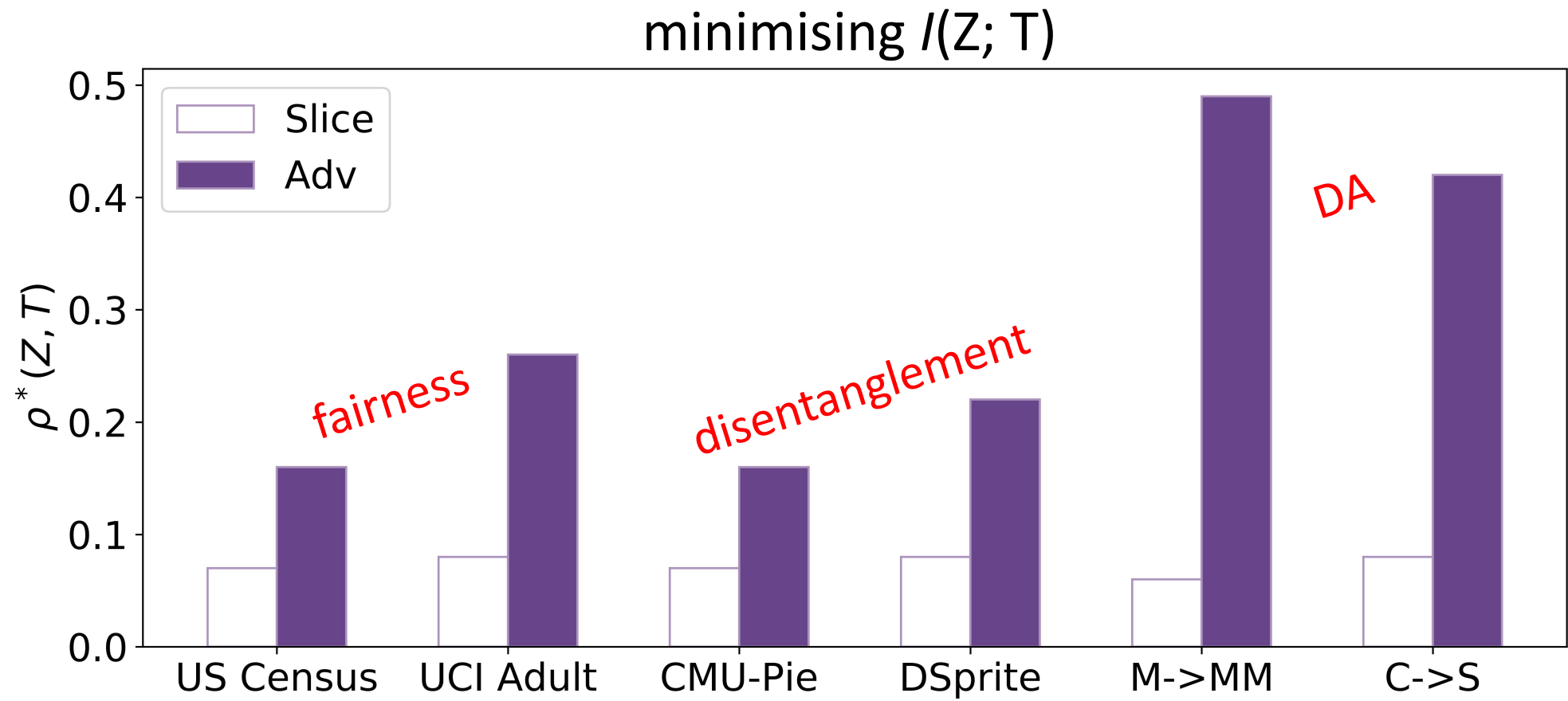
## Slice infomin learning

$$\min_f \mathcal{L}(f(X); Y) + \beta \cdot SI(f(X); T),$$

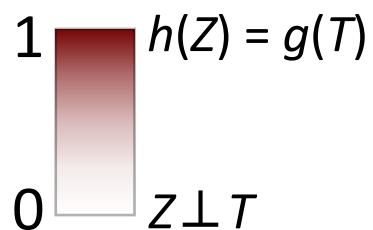
Estimate  $SI$  analytically

*// Max-step*  
sample  $S$  slices  $\Theta = \{\theta_i\}_{i=1}^S, \Phi = \{\phi_j\}_{j=1}^S$   
solve the parameters  $w, v$  in  $\hat{SI}$  analytically  
with  $\Theta, \Phi, \mathcal{D}'$  by eigendecomposition

# Results

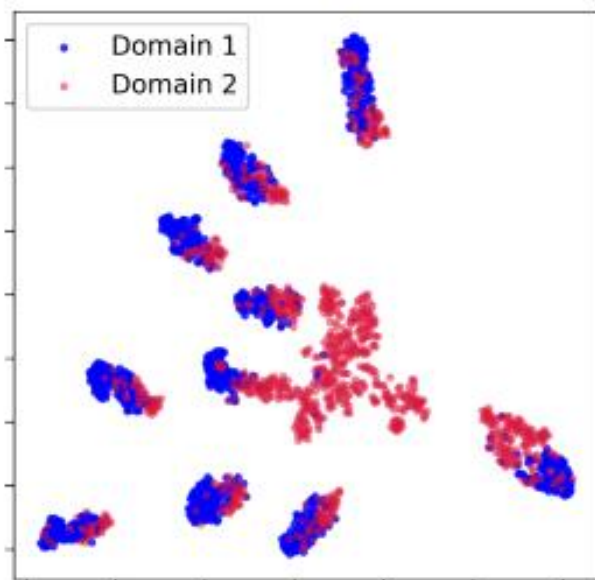


$$\rho^*(Z, T) = \sup_{h, g} \rho(h(Z), g(T))$$

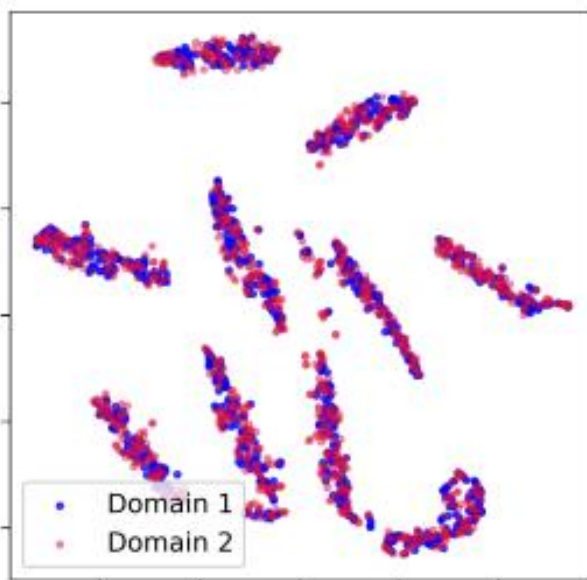


# Results

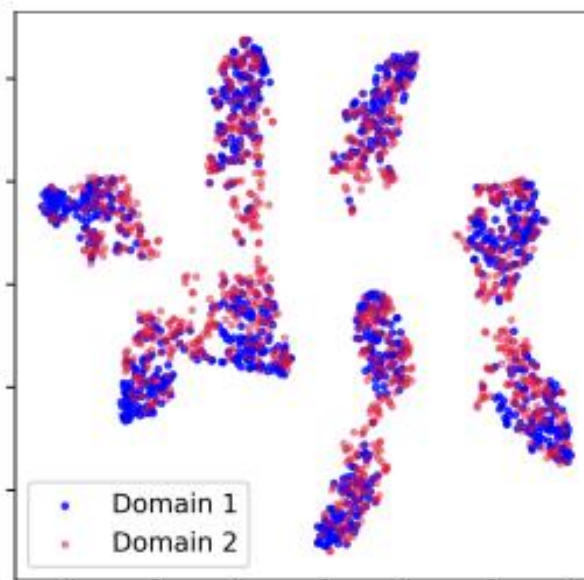
## Domain-invariant representation learning



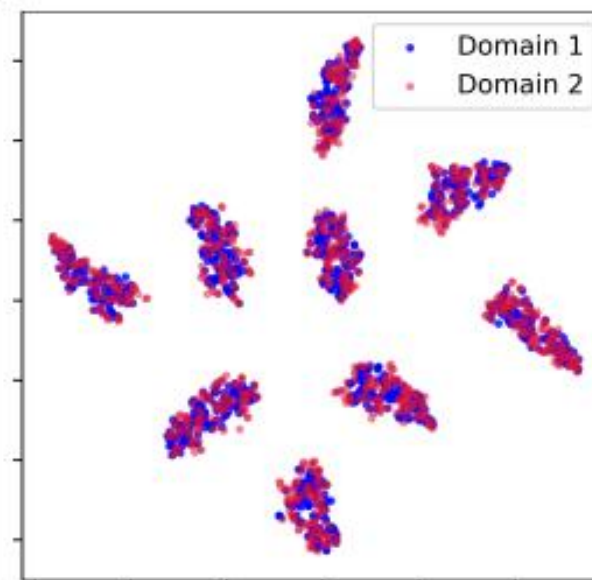
(a)  $M \rightarrow MM$ , adversarial



(b)  $M \rightarrow MM$ , slice



(c)  $C \rightarrow S$ , adversarial

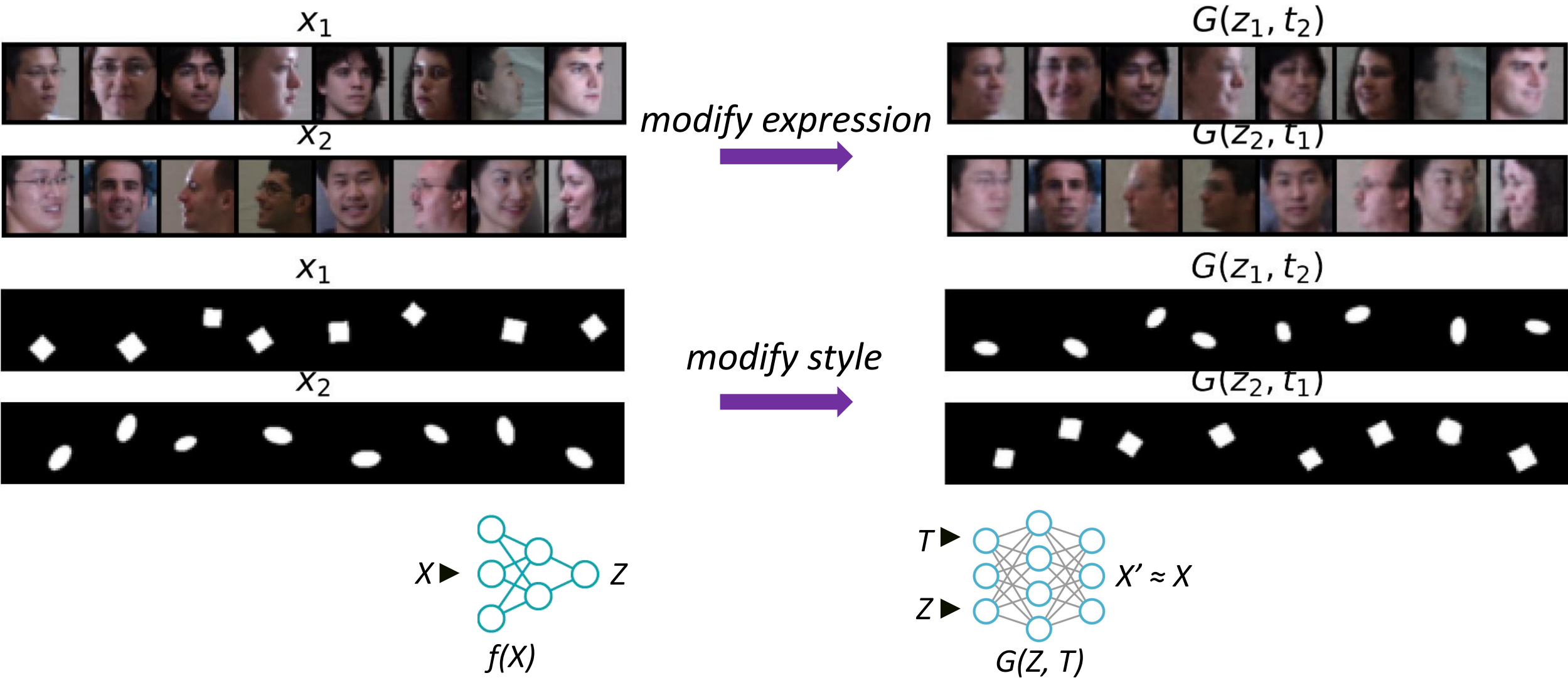


(d)  $C \rightarrow S$ , slice

\*M: MNIST, MM: MNIST-M, C: CIFAR-10, S: STL-10

# Results

## Disentangled representation learning





# Results

## Fair representation learning

Table 4: Algorithmic fairness: US Census

Neural Rényi				Neural TC			
$L_2$	$\rho^*(Z, Y)$	$\rho^*(Z, T)$	time (sec./max step)	$L_2$	$\rho^*(Z, Y)$	$\rho^*(Z, T)$	time (sec./max step)
2	$0.95 \pm 0.02$	$0.23 \pm 0.10$	0.092	3	$0.95 \pm 0.02$	$0.27 \pm 0.03$	0.097
10	$0.95 \pm 0.02$	$0.19 \pm 0.06$	0.642	10	$0.95 \pm 0.02$	$0.21 \pm 0.02$	0.362
50	$0.95 \pm 0.01$	$0.06 \pm 0.02$	2.456	20	$0.95 \pm 0.01$	$0.08 \pm 0.02$	2.146
Slice	$0.95 \pm 0.01$	$0.07 \pm 0.02$	0.102	Slice	$0.95 \pm 0.00$	$0.07 \pm 0.02$	0.102

$T$  = sensitive attribute

$L_2$  = num of gradient steps for training the adversary

## Conclusions

- ***To minimise MI, we do not need to estimate it***  
only consider random facets of MI in each mini-batch
- ***A method widely applicable to many applications***  
fairness, disentanglement, domain adaptation, invariance, ...

Thanks!