

封闭区域的碰问题

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本文主要讨论可以视为质点的刚性球在不同的封闭边界下发生弹性碰撞的运动轨迹,此时质点运动规律为牛顿力学。并通过分析质点的运动轨迹给出混沌出现的条件,利用Vpython 动态展示质点的运动。最后以环形边界为例,比较在不同的物理情境下质点的运动情形,包括粒子带一个单位正电荷的情形。

碰撞原理: 质点与边界碰撞是速度改变:

$$\vec{v}_{i,\perp} = (\vec{v}_i \cdot \hat{n})\hat{n}, \vec{v}_{i,\parallel} = \vec{v}_i - \vec{v}_{i,\perp} \quad (1 \text{ 式})$$

$$\vec{v}_{f,\perp} = -\vec{v}_{i,\perp}, \vec{v}_{f,\parallel} = \vec{v}_{i,\parallel} \quad (2 \text{ 式})$$

设边界在碰撞点的法向量为 $\vec{n} = a\hat{i} + b\hat{j}$

$$\vec{v}_i = v_{i,x}\hat{i} + v_{i,y}\hat{j} \quad (3 \text{ 式})$$

$$\vec{v}_f = v_{f,x}\hat{i} + v_{f,y}\hat{j} \quad (4 \text{ 式})$$

将 4 式 3 式代入 1 式 2 式可得

$$\begin{aligned} v_{f,x} &= (1 - 2a^2)v_{i,x} - 2abv_{i,y} \\ v_{f,y} &= (1 - 2b^2)v_{i,y} - 2abv_{i,x} \end{aligned}$$

在经典情形下质点的运动:

$$\frac{dx}{dt} = v_x, \frac{dy}{dt} = v_y \quad (5 \text{ 式})$$

首先讨论在矩形边界中的碰撞。

整个程序的主要难点在于碰撞的边界如何确定,显然在程序中边界是无法用两个方程求解的出来的,那么我们怎么用手里的两个方程确定呢?我在这里用到的是二分法。我们用 IF, while 语句 if $x[i+1]**2.0+(y[i+1]+alpha/2.0)**2.0>1$:

while $abs(x[i+1]**2+(y[i+1]+alpha/2.0)**2-1)>0.0001$

当然这里的 0.0001 是精度,越小表示越接近边界点,但是太小造成的计算量过大也是个问题。

其次就是话边界问题,对于这个封闭区间我们只需要分成 4 段边界函数即可来写这个碰撞问题。

如下是完整程序:

```
import math
x_0=0
y_0=0
v_x_0=1
v_y_0=0.002
dt=0.01
end_t=7000
x=[]
y=[]
```

```

v_x=[]
v_y=[]
t=[]
x.append(x_0)
y.append(y_0)
v_x.append(v_x_0)
v_y.append(v_y_0)
t.append(0)
alpha=0.001
v_x_1=[]
x_1_1=[]
for i in range(int(end_t/dt)):
    x.append(x[i]+v_x[i]*dt)
    y.append(y[i]+v_y[i]*dt)
    v_x.append(v_x[i])
    v_y.append(v_y[i])
    t.append(t[i]+dt)
    if abs(x[i+1]-1)<0.0001:
        print x[i+1]
        a=-x[i+1]
        b=0
        v_x[i+1]=(1-2*a**2)*v_x[i]-2*a*b*v_y[i]
        v_y[i+1]=(1-2*b**2)*v_y[i]-2*a*b*v_x[i]
    else:
        if y[i+1]<0:
            if x[i+1]**2.0+(y[i+1]+alpha/2.0)**2.0>1:
                while abs(x[i+1]**2+(y[i+1]+alpha/2.0)**2-1)>0.0001:
                    #print abs(x[i+1]**2+y[i+1]**2-1)
                    X=(x[i]+x[i+1])/2
                    Y=(y[i]+y[i+1])/2
                    a=X/((X**2+(Y+alpha/2.0)**2)**0.5)
                    b=(Y+alpha/2.0)/((X**2+(Y+alpha/2.0)**2)**0.5)
                    v_x[i+1]=(1-2*a**2)*v_x[i]-2*a*b*v_y[i]
                    v_y[i+1]=(1-2*b**2)*v_y[i]-2*a*b*v_x[i]
                    #v_y[i+1]=-((2.0*a**3-2.0*a)/b)*v_x[i+1]-(2*a**2-1)*v_y[i+1]
                    if X**2+(Y+alpha/2.0)**2>1:
                        x[i+1]=X
                        y[i+1]=Y
                        continue
                else:
                    x[i]=X
                    y[i]=Y
                    continue
            else:
                else:

```

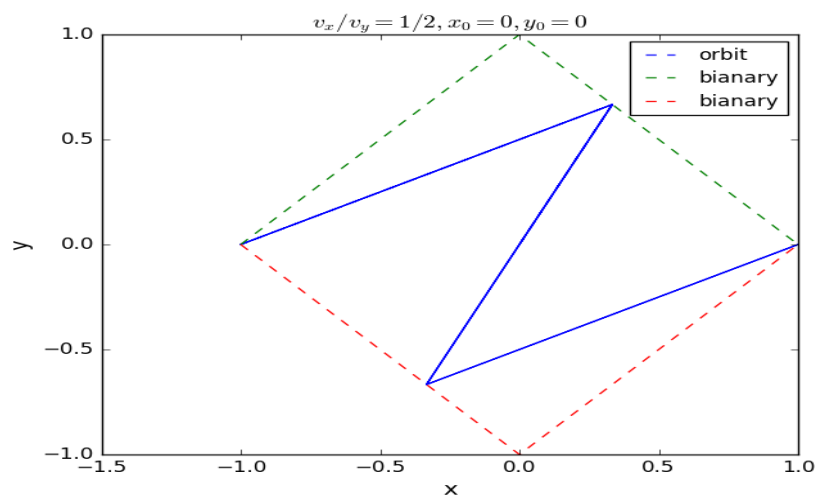
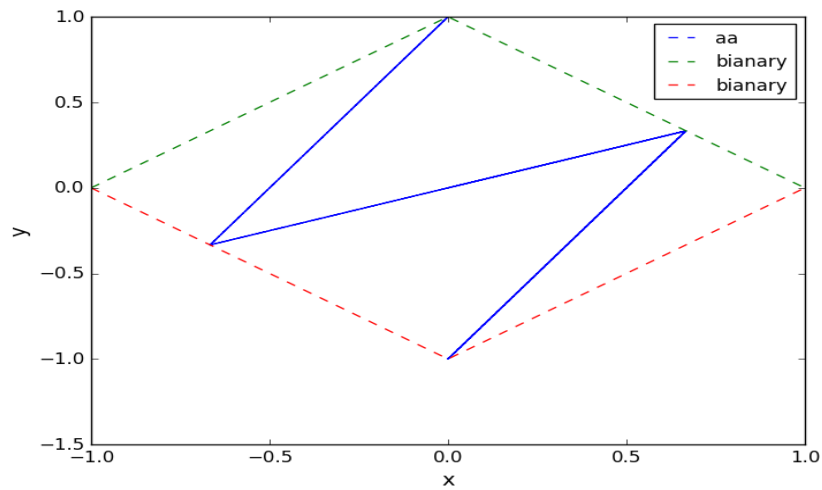
```

if x[i+1]**2.0+(y[i+1]-alpha/2.0)**2.0>1:
    while abs(x[i+1]**2+(y[i+1]-alpha/2.0)**2-1)>0.0001:
        #print abs(x[i+1]**2+y[i+1]**2-1)
        X=(x[i]+x[i+1])/2
        Y=(y[i]+y[i+1])/2
        a=X/((X**2+(Y-alpha/2.0)**2)**0.5)
        b=(Y-alpha/2.0)/((X**2+(Y-alpha/2.0)**2)**0.5)
        v_x[i+1]=(1-2*a**2)*v_x[i]-2*a*b*v_y[i]
        v_y[i+1]=(1-2*b**2)*v_y[i]-2*a*b*v_x[i]
        #v_y[i+1]=-((2.0*a**3-2.0*a)/b)*v_x[i+1]-(2*a**2-1)*v_y[i+1]
        if X**2+(Y-alpha/2.0)**2>1:
            x[i+1]=X
            y[i+1]=Y
            continue
        else:
            x[i]=X
            y[i]=Y
            continue
    if abs(y[i]-0)<0.0001:
        p=x[i]
        q=v_x[i]
        v_x_1.append(q)
        x_1_1.append(p)
print v_x[30]
x_1=[-1]
y_1=[0]
x_2=[-1]
y_2=[0]
dx=0.01
for j in range(199):
    x_1.append(x_1[j]+dx)
    y_1.append(alpha/2.0+(1-x_1[j+1]**2)**0.5)
    x_2.append(x_2[j]+dx)
    y_2.append(-alpha/2.0-(1-x_2[j+1]**2)**0.5)
import matplotlib.pyplot as plt
import numpy as np
import math
fig= plt.figure(figsize=(7,6))
#fig= plt.figure(figsize=(7,6))
plt.scatter(x_1_1,v_x_1,marker='+',color='b',label='phase space plot')
#plt.plot(x,y,'--',label='orbit')
#plt.plot(x_1,y_1,'--',label='bianary')
#plt.plot(x_2,y_2,'--',label='bianary')
plt.title(u'$v_x/v_y=1/0.002,y_0=0,x_0=0,a=0.001,scan-point-y=0$',fontsize=16)

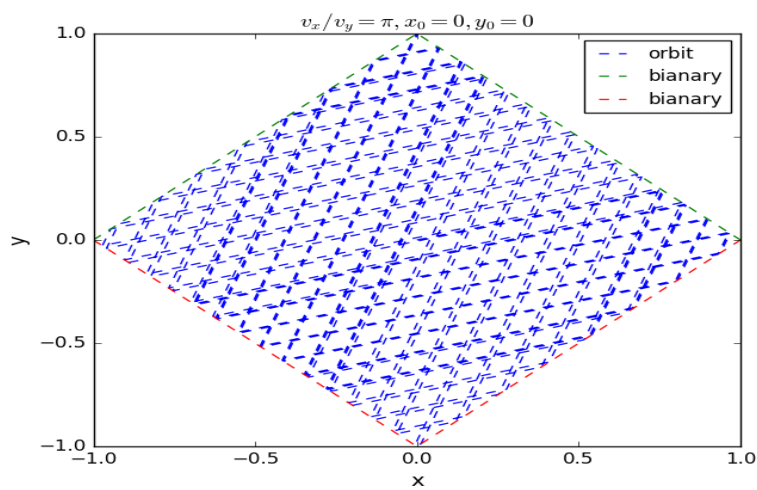
```

```
plt.xlabel(u'x',fontsize=14)
plt.ylabel(u'y',fontsize=14)
plt.legend(fontsize=12,loc='best')
plt.show(fig)
```

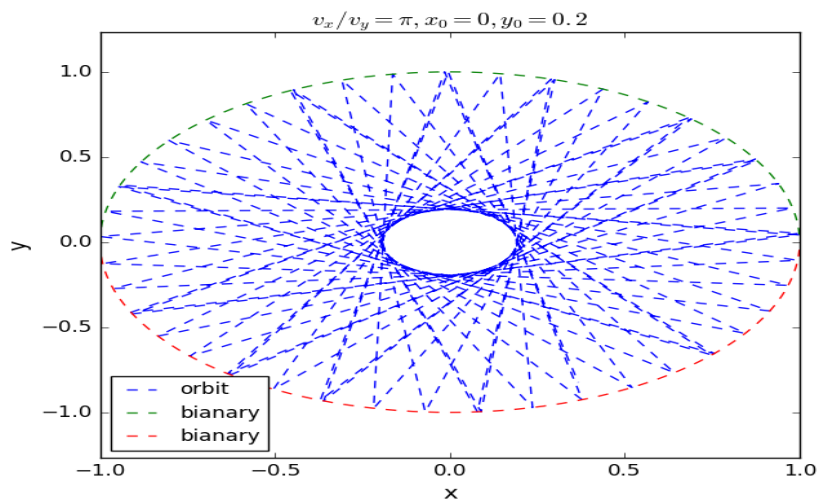
得到的图形



修改取点总时间 endt 得到碰撞多次的图形



修改边界条件得到椭圆边界的情况:



其次我们做出相空间的点的位置:

其程序为

```
import math
x_0=0
y_0=0
v_x_0=1
v_y_0=0.002
dt=0.01
end_t=7000
x=[]
y=[]
v_x=[]
v_y=[]
t=[]
x.append(x_0)
y.append(y_0)
v_x.append(v_x_0)
v_y.append(v_y_0)
t.append(0)
alpha=0.001
v_x_1=[]
x_1_1=[]
for i in range(int(end_t/dt)):
    x.append(x[i]+v_x[i]*dt)
    y.append(y[i]+v_y[i]*dt)
    v_x.append(v_x[i])
    v_y.append(v_y[i])
    t.append(t[i]+dt)
    if abs(x[i+1]-1)<0.0001:
        print x[i+1]
```

```

a=-x[i+1]
b=0
v_x[i+1]=(1-2*a**2)*v_x[i]-2*a*b*v_y[i]
v_y[i+1]=(1-2*b**2)*v_y[i]-2*a*b*v_x[i]
else:
    if y[i+1]<0:
        if x[i+1]**2.0+(y[i+1]+alpha/2.0)**2.0>1:
            while abs(x[i+1]**2+(y[i+1]+alpha/2.0)**2-1)>0.0001:
                #print abs(x[i+1]**2+y[i+1]**2-1)
                X=(x[i]+x[i+1])/2
                Y=(y[i]+y[i+1])/2
                a=X/((X**2+(Y+alpha/2.0)**2)**0.5)
                b=(Y+alpha/2.0)/((X**2+(Y+alpha/2.0)**2)**0.5)
                v_x[i+1]=(1-2*a**2)*v_x[i]-2*a*b*v_y[i]
                v_y[i+1]=(1-2*b**2)*v_y[i]-2*a*b*v_x[i]
                #v_y[i+1]=-((2.0*a**3-2.0*a)/b)*v_x[i+1]-(2*a**2-1)*v_y[i+1]
                if X**2+(Y+alpha/2.0)**2>1:
                    x[i+1]=X
                    y[i+1]=Y
                    continue
            else:
                x[i]=X
                y[i]=Y
                continue
        else:
            if x[i+1]**2.0+(y[i+1]-alpha/2.0)**2.0>1:
                while abs(x[i+1]**2+(y[i+1]-alpha/2.0)**2-1)>0.0001:
                    #print abs(x[i+1]**2+y[i+1]**2-1)
                    X=(x[i]+x[i+1])/2
                    Y=(y[i]+y[i+1])/2
                    a=X/((X**2+(Y-alpha/2.0)**2)**0.5)
                    b=(Y-alpha/2.0)/((X**2+(Y-alpha/2.0)**2)**0.5)
                    v_x[i+1]=(1-2*a**2)*v_x[i]-2*a*b*v_y[i]
                    v_y[i+1]=(1-2*b**2)*v_y[i]-2*a*b*v_x[i]
                    #v_y[i+1]=-((2.0*a**3-2.0*a)/b)*v_x[i+1]-(2*a**2-1)*v_y[i+1]
                    if X**2+(Y-alpha/2.0)**2>1:
                        x[i+1]=X
                        y[i+1]=Y
                        continue
                else:
                    x[i]=X
                    y[i]=Y
                    continue
    if abs(y[i]-0)<0.0001:

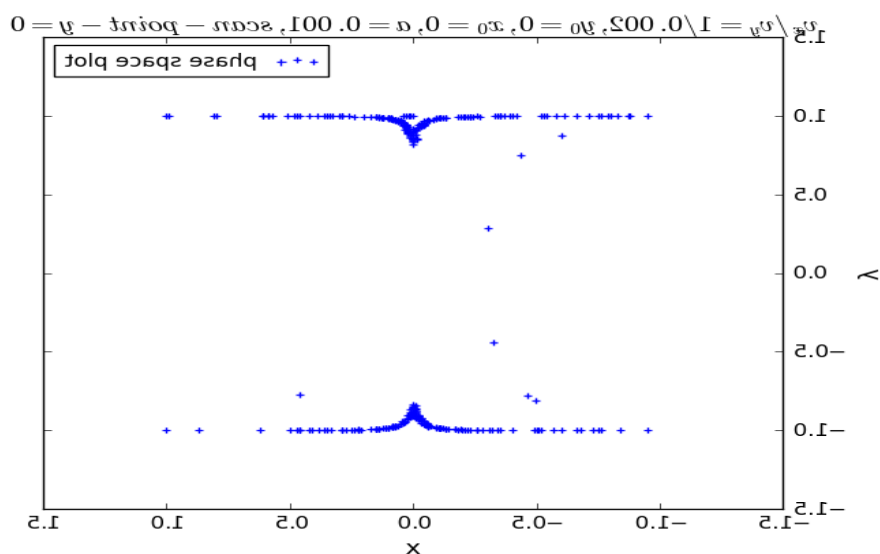
```

```

p=x[i]
q=v_x[i]
v_x_1.append(q)
x_1_1.append(p)
print v_x[30]
x_1=[-1]
y_1=[0]
x_2=[-1]
y_2=[0]
dx=0.01
for j in range(199):
    x_1.append(x_1[j]+dx)
    y_1.append(alpha/2.0+(1-x_1[j+1]**2)**0.5)
    x_2.append(x_2[j]+dx)
    y_2.append(-alpha/2.0-(1-x_2[j+1]**2)**0.5)
import matplotlib.pyplot as plt
import numpy as np
import math
fig= plt.figure(figsize=(7,6))
#fig= plt.figure(figsize=(7,6))
plt.scatter(x_1_1,v_x_1,marker='+',color='b',label='phase space plot')
#plt.plot(x,y,'--',label='orbit')
#plt.plot(x_1,y_1,'--',label='bianary')
#plt.plot(x_2,y_2,'--',label='bianary')
plt.title(u'$v_x/v_y=1/0.002,y_0=0,x_0=0,a=0.001,scan-point-y=0$',fontsize=16)
plt.xlabel(u'x',fontsize=14)
plt.ylabel(u'y',fontsize=14)
plt.legend(fontsize=12,loc='best')
plt.show(fig)

```

做出来的图形如图为：



中间点很稀少具体原因我也不太明确。

我们在中间设置一个带电场的情形，这样我们以前的速度设置也必须改变，我们舍在圆环中存在电场，

具体程序

```
import math
x_0=1.2
y_0=0
v_x_0=1
v_y_0=2
dt=0.01
end_t=25
x=[]
y=[]
v_x=[]
v_y=[]
t=[]
x.append(x_0)
y.append(y_0)
v_x.append(v_x_0)
v_y.append(v_y_0)
t.append(0)
#v_x_1=[]
#x_1_1=[]
q=1.6*10**-19
m=1*10**-19
b=0.9
for i in range(int(end_t/dt)):
    x.append(x[i]+v_x[i]*dt)
    y.append(y[i]+v_y[i]*dt)
    #v_x.append(v_x[i])
    #v_y.append(v_y[i])
    v_x.append(v_x[i]+q*b*v_y[i]*dt/m)
    v_y.append(v_y[i]-q*b*v_x[i]*dt/m)
    t.append(t[i]+dt)
    #print x[i]**2+y[i]**2
    if x[i+1]**2+y[i+1]**2>4:
        print x[i+1]**2+y[i+1]**2
        while abs(x[i+1]**2+y[i+1]**2-4)>0.001:
            #print abs(x[i+1]**2+y[i+1]**2-4)
            X=(x[i]+x[i+1])/2
            Y=(y[i]+y[i+1])/2
            a=X/(X**2+Y**2)**0.5
            b=Y/(X**2+Y**2)**0.5
            v_x[i+1]=(1-2*a**2)*v_x[i]-2*a*b*v_y[i]
```



```

v_y[i+1]=(1-2*b**2)*v_y[i]-2*a*b*v_x[i]
#v_y[i+1]=-((2.0*a**3-2.0*a)/b)*v_x[i+1]-(2*a**2-1)*v_y[i+1]
if X**2+Y**2>4:
    x[i+1]=X
    y[i+1]=Y
    continue
else:
    x[i]=X
    y[i]=Y
    continue
#if abs(abs(X)+abs(Y)-1)<0.1:
# break
if x[i+1]**2+y[i+1]**2<1:
    while abs(x[i+1]**2+y[i+1]**2-1)>0.001:
        X=(x[i]+x[i+1])/2
        Y=(y[i]+y[i+1])/2
        a=X/(X**2+Y**2)**0.5
        b=Y/(X**2+Y**2)**0.5
        v_x[i+1]=(1-2*a**2)*v_x[i]-2*a*b*v_y[i]
        v_y[i+1]=(1-2*b**2)*v_y[i]-2*a*b*v_x[i]
        #v_y[i+1]=-((2.0*a**3-2.0*a)/b)*v_x[i+1]-(2*a**2-1)*v_y[i+1]
        if X**2+Y**2<1:
            x[i+1]=X
            y[i+1]=Y
            continue
        else:
            x[i]=X
            y[i]=Y
            continue
    #if abs(y[i])<0.0001:
    # p=x[i]
    # q=v_x[i]
    # v_x_1.append(q)
    # x_1_1.append(p)
x_1=[-1]
y_1=[0]
x_2=[-1]
y_2=[0]
x_3=[-2]
y_3=[0]
x_4=[-2]
y_4=[0]
dx=0.0001
for j in range(20000):

```

```

x_1.append(x_1[j]+dx)
y_1.append((1-x_1[j+1]**2)**0.5)
x_2.append(x_1[j]+dx)
y_2.append(-(1-x_1[j+1]**2)**0.5)
for k in range(40000):
    x_3.append(x_3[k]+dx)
    y_3.append((4-x_3[k+1]**2)**0.5)
    x_4.append(x_4[k]+dx)
    y_4.append(-(4-x_4[k+1]**2)**0.5)
#x_4=x_4[1:39998]
#y_4=y_4[1:39998]
import matplotlib.pyplot as plt
import numpy as np
import math
fig= plt.figure(figsize=(6,6))
#plt.scatter(x_1_1,v_x_1,marker='+',color='b',label='phase space plot')
plt.plot(x,y,'--',label='orbit')
plt.plot(x_1,y_1,'--',label='binary')
plt.plot(x_2,y_2,'--',label='binary')
plt.plot(x_3,y_3,'--',label='binary')
plt.plot(x_4,y_4,'--',label='binary')
plt.title(u'$v_x/v_y=1/2, x_0=1.2, y_0=0, magnitude=0.9T$', fontsize=14)
plt.xlabel(u'x', fontsize=14)
plt.ylabel(u'y', fontsize=14)
plt.legend(fontsize=12, loc='best')
plt.show(fig)

```

如图

