

# STDGLM

```
library(STDGLM)
```

## Introduction

The **STDGLM** package provides a framework for fitting spatio-temporal dynamic generalized linear models. These models are useful for analyzing data that varies over both space and time, allowing for the incorporation of spatial and temporal dependencies in the modeling process. The package provides functions for fitting these models, as well as tools for visualizing and interpreting the results.

## Installation

You can install the package from GitHub using the following command:

```
if (!requireNamespace("devtools", quietly = TRUE)) {  
  install.packages("devtools")  
}  
devtools::install_github("czaccard/STDGLM")
```

## Detailed Explanation on supported STDGLMs

As for the current version of the package (0.0.0.9000), only Gaussian outcomes are supported. Specifically only dynamic linear models (DLMs) with the following specification can be handled:

$$y_t = x_t' \beta_t + z_t' \gamma + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2)$$
$$\beta_t = F_t \beta_{t-1} + \eta_t, \quad \eta_t \sim N(0, \Sigma_\eta)$$

where:

- $y_t$  is the response variable at time  $t$ ,
- $x_t$  is a  $p$ -dimensional vector of covariates at time  $t$  (an intercept may or may not be included here),
- $\beta_t$  is the state vector at time  $t$ ,
- $z_t$  is a  $q$ -dimensional vector of covariates whose effects are constant (an intercept may or may not be included here),
- $\gamma$  is a vector of non-varying coefficients,
- $\epsilon_t$  is the observation error at time  $t$ ,
- $F_t = \phi I_p$  is a transition matrix,
- $\Sigma_\eta$  is the covariance matrix of the state evolution error  $\eta_t$ .

The state equation is adapted to account for spatial correlations in the state vector. Let  $p$  denote the number of *spatial units* (either georeferenced locations or areal units) where data is collected, then  $\beta_t = (\beta_{1,t}, \dots, \beta_{p,t})'$ , where  $\beta_{i,t}$  is the effect of the covariate  $x_{i,t}$  on the outcome at spatial unit  $i = 1, \dots, p$  and time  $t = 1, \dots, T$ .

The state evolution covariance matrix  $\Sigma_\eta$  can be structured to reflect *spatial relationships*, e.g. by assuming an **exponential covariance function** if the data are point-referenced:

$$(\Sigma_\eta)_{ij} = \rho_1 \exp\left(-\frac{d_{ij}}{\rho_2}\right), \quad d_{ij} = \|s_i - s_j\|$$

where  $\rho_1$  is the partial sill,  $\rho_2$  is the range parameter, and  $d_{ij}$  is the Euclidean distance between locations  $s_i$  and  $s_j$ . At the moment, this is the only supported covariance structure for point-referenced data.

In this case, the evolution error  $\eta_t$  is assumed to be a zero-mean Gaussian process with exponential covariance matrix parameterized by  $\rho_1$  and  $\rho_2$ , which we will denote as  $\eta_t \sim GP(0, \rho_1, \rho_2; exp)$ .

If the data are areal, a **proper conditional autoregressive (PCAR) covariance structure** is assumed:

$$\Sigma_\eta = \rho_1 (D_w - \rho_2 W)^{-1}$$

where  $W$  is a binary adjacency matrix,  $D_w$  is a diagonal matrix with row sums of  $W$  on the diagonal, and  $\rho_1$  and  $\rho_2$  are the conditional variance and autocorrelation parameters, respectively.

In this case, the evolution error  $\eta_t$  follows a zero-mean PCAR process, and we will denote this as  $\eta_t \sim PCAR(0, \rho_1, \rho_2)$ .

## ANOVA Decomposition of the State Vector

The function `stdglm` allows for the decomposition of the state vector into components that can be interpreted as contributions from different sources of variability. The state vector  $\beta_t$  is decomposed as follows:

$$\beta_{i,t} = \bar{\beta} + \beta_i^{(S)} + \beta_t^{(T)} + \beta_{i,t}^{(ST)}$$

where:

- $\bar{\beta}$  is the overall mean effect,
- $\beta_i^{(S)}$  is the spatial effect at location  $i$ ,
- $\beta_t^{(T)}$  is the temporal effect at time  $t$ ,
- $\beta_{i,t}^{(ST)}$  is the interaction effect between space and time at location  $i$  and time  $t$ .

## Bayesian Hierarchical Structure

The Bayesian model is as follows, for  $t = 1, \dots, T$ :

$$\begin{aligned}
y_t &\sim N(x'_t \beta_t + z'_t \gamma, \sigma_\epsilon^2) \\
\beta_t &= 1_p \bar{\beta} + \beta^{(S)} + 1_p \beta_t^{(T)} + \beta_t^{(ST)} \\
\bar{\beta} &\sim N(0, V_\gamma) \\
\beta^{(S)} &= (\beta_1^{(S)}, \dots, \beta_p^{(S)})' \sim GP(0, \rho_1^{(S)}, \rho_2^{(S)}; exp) \text{ or } PCAR(0, \rho_1^{(S)}, \rho_2^{(S)}) \\
\beta_t^{(T)} &\sim N(\phi^{(T)} \beta_{t-1}^{(T)}, V_\beta^{(T)}) \\
\beta_t^{(ST)} &= (\beta_{1,t}^{(ST)}, \dots, \beta_{p,t}^{(ST)})' \sim GP(\phi^{(ST)} \beta_{t-1}^{(ST)}, \rho_1^{(ST)}, \rho_2^{(ST)}; exp) \text{ or } PCAR(\phi^{(ST)} \beta_{t-1}^{(ST)}, \rho_1^{(ST)}, \rho_2^{(ST)}) \\
\gamma &\sim N(0, V_\gamma) \\
\sigma_\epsilon^2 &\sim IG(a_\epsilon, b_\epsilon) \\
\rho_1^{(S)} &\sim IG(0.01, 0.01) \\
\rho_2^{(S)} &\sim U(a_\rho, b_\rho) \\
\rho_1^{(ST)} &\sim IG(0.01, 0.01) \\
\rho_2^{(ST)} &\sim U(a_\rho, b_\rho) \\
\phi^{(T)} &\sim TN_{(-1,1)}(0, 1) \\
\phi^{(ST)} &\sim TN_{(-1,1)}(0, 1) \\
V_\beta^{(T)} &\sim IG(a^{(T)}, b^{(T)})
\end{aligned}$$

where  $TN_{(q,r)}(\mu, \sigma^2)$  denotes a normal distribution with mean  $\mu$  and variance  $\sigma^2$  truncated to the interval  $(q, r)$ . The hyperparameters  $a_\rho$  and  $b_\rho$  depend on the type of spatial data. If the data are point-referenced, they are set based on the minimum and maximum distances between points, respectively. If the data are areal,  $a_\rho = 0.1$  and  $b_\rho \rightarrow 1$ .