Assignment 3

1. Adding Fog of War to Game

This assignment will deal with a variation of the game from assignment two in which each play only has access to the position of their own pieces. They will not be able to see the location of the opponent's pieces of the location of the pits. Instead each player will receive an observation if one of their pieces is in a cell adjacent to a pit or to one of the opponent's pieces. If the player's units is adjacent to a pit then the unit observes a breeze in the cell that it is in. If the unit is in a cell adjacent to one of the opponent's wumpuses then it observes as stench. If it is in a cell adjacent to one of the opponent's heroes then it observes the noise of the hero moving. If it is in a cell that is adjacent to one of the opponent's mages then it observes hear from the Mage's fire magic.

You should update your interface to include any observations made by the human player's units along with what cell the observation is made in. You should also update your interface to include an option for toggling the fog of war on and off (this will be used during your demo to help us test that your method is working).

Outside of these changes, the mechanics of the game remain the same as described in assignment 2.

2. Probability Distribution of State Variables

We would next like to build a method for tracking the probability distribution of the location of each of your opponent's pieces (as well as the probability distribution of the pits). Each unobserved cell could potentially contain a pit, one of the opponent's heroes, one of the opponent's wumpuses or one of the opponent's mages. For each cell x,y, let $P(P_{x,y})$ be the probability that cell x,y contains a pit, let $P(P_{x,y})$ be the probability that cell x,y contains a wumpus, let $P(H_{x,y})$ be the probability that cell x,y contains a hero and let $P(M_{x,y})$ be the probability that cell x,y contains a mage. We can therefore describe the probability of the states of the board as a set of variables $P(P_{x,y})$, $P(P_{x,y})$, $P(H_{x,y})$ and $P(M_{x,y})$ for each cell in the board.

At the beginning of the game a player knows the position of the opponent's pieces (because you know the setup) and you can compute the probability of each cell containing a pit. You can also assume that you know what pieces the opponent has lost (and thus what pieces he has remaining).

We will now discuss how we would update the probabilities over the duration of a turn. We will make a Markov assumption and assume that the state of the board at each turn depends only on the state of the board during the previous step. If we have the probabilities (i.e. $P(P_{x,y})$, $P(P_{x,y})$, $P(H_{x,y})$ and $P(M_{x,y})$ for each cell x,y) at the end of the previous and we have a method for estimating the probability of each move the opponent will make the you can compute the probability distribution after the opponent makes his move (i.e. $P'(P_{x,y})$, $P'(W_{x,y})$, $P'(H_{x,y})$ and $P'(M_{x,y})$ for each cell x,y).

As a starting point we will assume that the opponent's moves are random and update the probabilities to reflect a random move by the opponent. It the opponent has c pieces left then we will assume that there is a 1/c probability that they will move any of those pieces and that there is an equal probability that they will make each move, so for a cell x,y that is not on the boarder we will assume a 1/8 probability of moving directly up, a 1/8 probability of moving diagonally up and to the right and so forth. For pieces pieces that are on the edge we will assume that there is an equal probability of make each possible move. The probability of the opponent having a wumpus in cell x,y can be estimated using the following equations.

$$\mathbf{P'(W_{x,y}) = (1\text{-}1/c)*P(W_{x,y}) + \Sigma_{(x',y)\in(neighbors(x,y)}P(W_{x',y'}) *P(W_{x,y}|W_{x',y'})}$$

Where neighbors(x,y) are the nodes within one move of (x,y) and $P(W_{x,y}|W_{x',y'})$ is the probability of a piece at cell x', y' moving to x,y.

$$\mathbf{P(W_{x,y}|W_{x',y'}) = 1/(c*neighbors(x',y'))}$$

Similarly

$$\mathbf{P'(M_{x,y}) = (1\text{-}1/c)*P(M_{x,y}) + \Sigma_{(x',y)\in(neighbors(x,y)}P(M_{x',y'}) *P(M_{x,y}|M_{x',y'})}$$

where

$$\mathbf{P(M_{x,y}|M_{x',y'}) = 1/(c*neighbors(x',y'))}$$

and

$$\mathbf{P'(H_{x,y}) = (1\text{-}1/c)*P(H_{x,y}) + \Sigma_{(x',y)\in(neighbors(x,y)}P(H_{x',y'}) *P(H_{x,y}|H_{x',y'})}$$

where

$$\mathbf{P(H_{x,y}|H_{x',y'}) = 1/(c*neighbors(x',y'))}$$

Because the pits do not move we can set the probability distribution of the pits after the opponent's move to be the same as the distribution before the opponent's move.

$$\mathbf{P'(P_{x,y})=P(P_{x,y})}$$

After the player moves you will receive your observations. If any of your pieces is adjacent to one of your opponent's pieces you will get an observation of stench (S), noise (N), heat (HE) based on what piece it is. Also, if you any of your pieces is adjacent to a pit then you will get an observation of breeze (B) for that piece. You will next need to update the probabilities given your observations.

Next you will make your move. After that move, the moved piece may receive a new observation in its new cell. You will also need to update the probability distribution based on that observation. This can be done using Bayes theorem. Let O be the set of observations you receive at the beginning of your turn. You should first P(O), which is the probability of receiving the observation O given the probability distribution at the end of the opposing player's turn. You should then apply the following formula for each variable. Note that I am only showing the computation for $W_{x,y}$ but you should be able to perform similar computations for $H_{x,y}$, $M_{x,y}$ and $P_{x,y}$.

$$\mathbf{P(W_{xy}|O) = P'(W_{xy})*P(O|\ W_{xy})/P(O)}$$

Where $P'(W_{xy})$ is the probability of W_{xy} after the opponent's moved (which we calculated above) and $P(O|\ W_{xy})$ is the probability of getting the observation O given W_{xy}.

Now we will discuss how to compute the probability of an observation, P(O) for a given probability distribution. Intuitivly P(O) is the probability that we are in a state that would give us the observation

O. Let X_O be the set of states that would give us an observation of O and P be a probably distribution. The probability of O is the sum of the probabilities of each of the states in X_O.

$$\mathbf{P(O)= \Sigma_{x \in XO} P(x)}$$

We can compute $P(O|\ W_{xy})$ by first setting W_{xy}=true, M_{xy}=false, H_{xy}=false and P_{xy}=false. We then need to normalize remaining cells, which for those

for all x'y' such that x ≠ x' or y ≠ y'

$\mathbf{P(W_{x'y'}|\ W_{xy}) = P'(W_{x'y'}) * (w-1)/w}$ //where w is the number of wumpuses your opponent has

$\mathbf{P(M_{x'y'}|\ W_{xy}) = P'(M_{x'y'}) * (1- P'(M_{xy}))}$

$\mathbf{P(H_{x'y'}|\ W_{xy}) = P'(H_{x'y'}) * (1- P'(H_{xy}))}$

$\mathbf{P(P_{x'y'}|\ W_{xy}) = P'(P_{x'y'}) * (1- P'(P_{xy}))}$

$\mathbf{P'(W_{xy}) = 1}$
$\mathbf{P'(M_{xy}) = 0}$
$\mathbf{P'(H_{xy}) = 0}$
$\mathbf{P'(P_{xy}) = 0}$

You can then compute $P(O|\ W_{xy})$ by computing the probability of O given this new distribution (in the same manner that you computed P(O) by computing the probability of O given the original probability distribution.

3. Selecting a Good Move

Now that we have a probability distribution indicating the probability that each cell contains a pit or one of the opponents Wumpuses, heroes or mages we can start thinking about what would be a good move. As part of this assignment you will be asked to come up with a policy for selecting a good move given a probability distribution. **Hint: An ideal policy is one that maximized the future expected reward which in this case would be the number of pieces the agent has minus the number of pieces the opponent has.**

4. Deliverable

1. Formulate the new wumpus game in terms of Boolean variables and define the relationship between these variable. Hint: For each cell you should have a variable for each observation and a variable for each item that the cell can contain (i.e. units, pits). (10 points)

2. Update your game interface to display the observations and to include the fog of war option that can be toggled on and off to display and hide the agent's pieces. (10 points)

3. Implement the method for computing probability that you described in section 3. Modify your interface to display the probabilities $P(P_{x,y})$, $P(P_{x,y})$, $P(H_{x,y})$ and $P(M_{x,y})$ for each cell. (20 points)

4. Propose a policy for selecting a good move given a probability distribution. Implement an agent that uses this policy. (30 points)

5. In Section 2 we assume that the opponent will randomly select a piece to move and then move that piece in a random direction. This is not a realistic approximation of the moves the opponent will select. Propose a better alternative for approximating the opponents moves and discuss how you would

compute the effect of the opponent's move on the probability distribution given this approximation method.   (10 points)

6. Implement the transition probability method that your described in question 5. (20 points)