Lifecycle Financial Portfolios in General Equilibrium

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Abstract

This paper quantifies the general equilibrium effects of financial innovation that increases access to equity markets. I study an overlapping generations model with idiosyncratic and aggregate risk. The benchmark economy with frictions on asset allocation of saving flows matches lifecycle financial portfolio holdings; the joint distribution of financial wealth and portfolios across age and wealth; macroeconomic aggregates; equity and bond returns; and recent estimates of stock demand elasticity and risk transfers. Counterfactual experiments show widespread adoption of target date funds decreases equity premium from 6.7% to 2.3%, reduces annual return standard deviation from 21.1% to 15%, and shrinks Sharpe ratio from 0.318 to 0.153. Top 10% financial wealth concentration drops from 70% to 63%, as top 10% equity market share declines from 80% to 60%. These outcomes are comparable to an economy without any participation costs or rebalancing frictions. Following transition to target date investing, rich young households lose in welfare by up to 3% consumption equivalents, while the rest of young households gain almost 6%; and retirees benefit by 0-3%.

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1 Introduction

Households' ability to access equity markets shapes their lifecycle portfolio dynamics and generates large general equilibrium effects on asset prices, inequality, and welfare. Traditionally, access to equity markets is limited and unequal: stock market participation is low, particularly among young workers and low net worth individuals, and portfolio rebalancing is infrequent. Recent financial innovation like target date funds increases access to equity markets by using an age based asset allocation rule that determines holdings of risky and riskfree assets according to investor's age. How effective is age-based asset allocation rule in improving households' access to equity markets? What are the impacts on asset prices, inequality, and welfare?

This paper answers these questions with a general equilibrium model that is consistent with salient features of lifecycle consumption, saving, and portfolio behavior; the joint distribution of financial wealth and portfolios across age and wealth; macroeconomic aggregates; and asset pricing dynamics. In addition, the model produces a low stock demand elasticity and limited risk transfers which traditional models struggle to do. A new type of rebalancing constraint on asset flows imply high equity return volatility, high Sharpe ratio, and large equity premium. I show this result using a model with heterogeneous equity access that I discipline with household portfolio data by age and wealth, and the model is solved with machine learning techniques to overcome the curse of dimensionality. The adoption of age-based asset allocation rule, as is commonly used by target date funds, reduces equity premia by stabilizing equity returns and improving risk sharing, almost to the extent that these frictions are absent. The equity premium drops from 6.7% to 2.3%, while the annual standard deviation of equity returns stabilizes from 21.1% to 15.0%, and the Sharpe ration shrinks from 0.318 to 0.153. Following transition to target date investing, young households in the bottom 90% of wealth distribution gain 6% consumption equivalents, with welfare gains declining gradually to 0-2% into retirement. Among the top 10%, young households lose by 3%, but older households gain 2-3% consumption equivalents.

The new rebalancing friction studied in this paper fixes the asset allocation of saving flows into portfolios, bridging the gap between empirically observed asset demand and traditional asset pricing models. Recent empirical studies point out that traditional models of asset prices and portfolio choice imply impractically high stock demand elasticity and risk transfers, two measures gauging the sensitivity of stock demand at the aggregate and across individuals respectively. This inconsistency between data and theory can potentially lead to misinformed policy evaluations due to unrealistic asset demand behavior embedded in such models. To address this concern, this paper studies a new type of friction on saving flows. In particular, agents allocate the flows of savings between equities and bonds using a fixed rule and do not rebalance portfolios. This setup captures three prominent features in household portfolios documented in previous and recent empirical literature. The first feature is that asset flows to the stock market across the wealth distribution are insensitive to market returns, even among the ultra-rich households, resulting in limited risk transfers and inelastic stock demand (Hartzmark and Solomon, 2021, Gabaix and Koijen, 2021, Gabaix, Koijen, Mainardi, Oh and Yogo, 2023, 2025). The second feature is inertia in household financial portfolios.(Agnew, Balduzzi and Sundén, 2003, Ameriks and Zeldes, 2004, Brunnermeier and Nagel, 2008). The third feature is that a substantial fraction of U.S. households do not participate in the stock market (Mankiw and Zeldes, 1991); hence, these households effectively have a flow allocation rule that is 0% in stocks. The rebalancing friction in this paper differs from the existing literature which constrains portfolio weights and requires rebalancing to the targeted allocation or frictions that impose periodic access to equity markets through intermittent rebalancing.

Stock market non-participation and infrequent rebalancing generate inelastic stock demand and limit risk transfers which impede risk sharing in the economy, implying high stock return volatility, Sharpe ratio, and equity premia. Due to these rebalancing frictions, stockowners grow increasingly more exposed to aggregate risk over time because stock returns tend to be higher than bond returns. Their portfolio share in stocks grows as stockowners age, which is a feature confirmed by empirical studies on household portfolios in both proprietary data and in survey data. Moreover, risk transfers, which measure flows in and out of households financial portfolios, are low across the wealth distribution, prohibiting older cohorts from swiftly unloading aggregate risk bearing to younger cohorts. The result is that stockowners demand high compensation for holding disproportionately large amount of risk. The combination of high stock volatility and high price of risk leads to high risk premia in the economy.

After quantification of the benchmark model that features rebalancing frictions on asset flows, I show that age-based asset allocation rule used by target date funds reduces these frictions almost completely. The benchmark economy matches macro aggregates, equity and bond prices, portfolio distributions by age and wealth, as well as stock demand elasticity and risk transfers. I then change portfolio choice constraints in the benchmark model to capture financial innovation that reduces these frictions. In the first counterfactual exercise, households use target date funds which have an age-dependent rebalancing strategy. The second counterfactual exercise removes both participation and rebalancing frictions, and households freely optimize portfolios. Outcomes are similar under the two alternative asset market arrangements.

The overlapping generations model in this paper connects lifecycle portfolio dynamics to asset prices in general equilibrium. The aggregate state of the economy switches between expansions and recessions. Households have time-separable CRRA preferences over consumption, and derive utility from bequests. While working, agents receive labor income, which features an age profile and idiosyncratic risk that is higher in recessions. After retirement, retirees receive social security payments. Households can save in stocks and riskfree bonds, subject to frictions on asset allocation of flows and short-selling constraints. Competitive firms implement capital structure and payout rules which can be micro-founded with maximizing firm value and to smooth out payouts. The government balances its budget by collecting taxes, financing spending, and supplying government bonds.

The benchmark economy introduces rebalancing constraints in the form of frictions on asset flows to capture stock market non-participation and asset flow insensitivity before the introduction of target date funds. Specifically, households receive stock market participation shocks that are correlated with income. Before receiving a participation shock, households save in bonds only. When a participation shock arrives, a household sets up a flow allocation rule that fixes the stock-bond ratio for future flows into the portfolio. Households do not actively rebalance portfolios afterwards.

Using parameter values that match household portfolio data, the benchmark model generates realistic lifecycle wealth and portfolio age profiles, matches macroeconomic, asset pricing dynamics, stock demand elasticity, and risk transfers. The distributions of financial wealth and equity market shares are concentrated among the richest 10% of households. The quantification process of the benchmark model involves two stages. In the first stage, I take parameters either from the literature or from data. This set of parameters include firm parameters, government parameters, and most household parameters. Then, in the second stage, I use the CRRA parameter to target the average equity portfolio share in stocks.

Infrequent rebalancing of the asset allocation of asset flows implies that consumption processes observed in the data are compatible with high equity premia, high stock return volatility, and low riskfree rate, which traditional consumption based asset pricing models fail to explain. In particular, participation and rebalancing frictions separate the pricing of risky and riskfree rates. The benchmark model in this paper deviates from a standard consumption based asset pricing model because rebalancing frictions prevent participating households from freely optimizing their portfolio weights. As a result, the usual Euler equations for optimal portfolio weights do not hold. Instead, households make consumption and saving decisions, taking portfolio weights as fixed. Therefore, their Euler conditions hold for returns on their portfolios, which are mixtures of the risky and riskfree rates. Nonparticipants, who tend to be low-wealth agents and who have strong precautionary savings motives, price the riskfree rate.

Rebalancing frictions help the model produce realistic lifecycle consumption, saving, and portfolio dynamics; and hence, successfully matching wealth and portfolio distributions by age and by wealth. Due to consumption smoothing incentives, agents save while they are working and dissave in retirement, which leads to a hump-shaped wealth age profile as seen in the data. Portfolio share in equity at any age, conditional on participation, is a consequence of the initial allocation rule and subsequent market outcomes. Since equity returns tend to be higher than bond returns, the equity portfolio share increases with age, which is consistent with empirical observations among people born between 1920 and 1970, who are the participants in asset markets around 2000. These lifecycle portfolio dynamics lead to realistic distribution of financial wealth and equity market shares.

I show the adoption of a simple financial product, target date funds, can mitigate participation and rebalancing frictions, almost to the extent that the frictions are absent. To show this, I consider two alternative asset market arrangements. In the target date economy, households follow the age based asset allocation rule. The free access economy drops rebalancing frictions, allowing free choices of portfolio allocation at all times, subject to the same short selling constraints.

In the target date economy, where everyone invests in target date funds, the average annual equity premium is 2.3%, as opposed to 6.7% in the benchmark economy. The steep drop in the equity premium is a result of stabilized equity returns and improved risk sharing. The annual standard deviation of equity returns drops from 21.1% in the benchmark economy to 15.0%, and the aggregate Sharpe ratio declines from 0.318 in the benchmark economy to 0.153. These outcomes are very close to those of an economy where agents freely optimize portfolios, which I call the "free access economy." In the free access economy, the equity premium declines further to 1.5%, equity returns are stable with a 15.5% annual standard deviation, risk sharing further improves, and the Sharpe ratio is 0.099.

The reduction in the aggregate Sharpe ratio in the target date economy comes from redistributing equity shares towards the young and towards the bottom 90% of wealth distribution. Workers are rich in relatively safe human capital compared to retirees, which is why their demanded compensation for bearing risk is lower than retirees. Redistributing equity shares to the young, therefore, lowers the aggregate Sharpe ratio. Similarly, redistributing to the bottom 90% of wealth distribution alters the aggregate Sharpe ratio because rich households have significantly higher exposures to risk than the rest of the agents. As inequality attenuates, risk sharing in the economy improves.

Equity return stabilization is a result of increased equity holdings in household financial portfolios. Compared to the benchmark economy, households in the target date world participate in the stock market per the age based asset allocation rule. In addition, conditional on participation, the asset allocation rule features higher portfolio weights on stocks than households in the benchmark economy, whose equity portfolio weights are low because frictions on portfolio rebalancing hinders the ability to minimize risk exposure as people approach retirement. As a consequence, the target date economy sees increased equity holdings by the young, the old, the rich, and the poor. Equity returns stabilize as the whole economy deleverages.

Welfare consequences of target date funds are heterogeneous and sizable. During the transition from benchmark economy to target date economy, following an unanticipated and immediate adoption of target date funds, households in the bottom 90% mostly gain in welfare. Such welfare gains are concentrated among younger households, enjoying up to 6% gains in consumption equivalents. Such welfare gains are much attenuated for elderly households who have much shorter investment horizons and who do not have the adequate financial resources to buffer against volatility in the stock market. Therefore, retirees in the bottom 90% benefit only moderately by 0-2% consumption equivalents. Households in the top 10% of wealth distribution also witness differential welfare consequences. Young rich households lose up to 3% consumption equivalents because of suppressed equity returns coming from general equilibrium forces, while elderly rich households still enjoy 2-3% gains in consumption equivalents because of stabilized equity returns on their financial assets which they have accumulated by the time of target date funds adoption.

The analysis above are facilitated by recent advancements in machine learning tools that address the technical challenge of solving high dimensional heterogeneous agent models with aggregate risk. The computational challenge comes from the distribution of individual state variables, which is infinite dimensional by definition, being a state variable. I follow the recent literature and approximate the distribution functions with sufficient statistics that are learned by neural networks (Kahou, Fernández-Villaverde, Perla and Sood, 2021, Han, Yang and E, 2021). I then adapt a reinforcement learning algorithm based on DeepHAM proposed in Han, Yang and E (2021) and apply the algorithm to finite horizon settings, with hyperparameters configured specifically for the model set up in this paper. The algorithm can be thought of as automated version of Krusell and Smith (1998) where AI learns to summarize sufficient statistics from the cross sectional distribution rather than using any pre-specified moments. On the ergodic set of the economy, the average relative consumption error, which measures residual in Euler equation from full information rational expectations equilibrium, is 0.6% which is in line with previous literature that solves lifecycle models (Maliar, Maliar and Winant, 2021) or OLG economies but without heterogeneity within an age cohort (Krueger and Kubler, 2004, Hasanhodzic and Kotlikoff, 2013).

Literature This paper contributes to the existing literature on three fronts. Firstly, this paper proposes and studies a new rebalancing friction on the asset allocation of flows, bridging the literature on access frictions in financial markets with past and recent empirical facts on household financial portfolios. Secondly, this paper studies these frictions in an OLG environment, emphasizing on the interaction between these frictions with two strong dimensions of heterogeneity, wealth and age, in understanding asset returns, inequality, and welfare in the economy. Thirdly, this paper advances welfare analysis of target date funds from choice problem frameworks to general equilibrium.

Non-participation in the stock market and infrequent rebalancing are well-known patterns in micro data on U.S. household portfolios. Before the introduction of target-date funds, many households did not participate in the stock market (Blume, Crockett and Friend, 1974, Blume and Friend, 1978, King and Leape, 1985, Mankiw and Zeldes, 1991, Poterba and Samwick, 1995, Vissing-Jorgensen, 1998, 2002a,b, Agnew, Balduzzi and Sundén, 2003, Ameriks and Zeldes, 2004).[1] Generally, households rarely rebalance their portfolios (Samuelson and Zeckhauser, 1988, Madrian and Shea, 2001, Choi, Laibson, Madrian and Metrick, 2002a,b, Agnew, Balduzzi and Sundén, 2003, Ameriks and Zeldes, 2004, Beshears, Choi, Laibson and Madrian, 2009, Brunnermeier and Nagel, 2008, Bilias, Georgarakos and Haliassos, 2009, Calvet, Campbell and

[1] Ameriks and Zeldes (2004) analyze Surveys of Consumer Finances between 1962 and 2001. They estimate the upper bound for stock market participation during this period to be 29.6% (1962), 43.7% (1983), 47.5% (1989), 49.6% (1992), 54.0% (1995), 57.0% (1998), and 59.7% (2001).

Sodini, 2009, Mitchell, Mottola, Utkus and Yamaguchi, 2009, Bianchi, 2018).[2]

This paper confirms that non-participation in the stock market and infrequent portfolio rebalancing are quantitatively important for understanding risk premia and volatility in asset markets. These ideas go back to early work by Mankiw and Zeldes (1991) who document that data on consumption growth by stockholders is more volatile than consumption growth by non-participants. Early theoretical work assumes that non-stockholders save in bonds which are in zero net supply (Saito, 1995, Basak and Cuoco, 1998). For the bond market to clear, stockowners must therefore hold leveraged positions in stocks, which imply high risk exposures by few investors and therefore higher risk premia. Allen and Gale (1994) endogenize the participation decision with a fixed cost for participation. Vissing-Jorgensen (2002a) estimates these participation costs to be large. Heaton and Lucas (1996) study how transaction costs increase the equity premium in equilibrium. Guvenen (2009) adds heterogeneity in preferences as well as stochastic labor income of non-stockholders, which further concentrates risk exposures among stockholders.

Infrequent rebalancing goes back to Grossman and Laroque (1990) who introduce adjustment costs in consumption, which implies that assets are illiquid. As a consequence, households want compensation for holding illiquid assets in equilibrium, in addition to demanding the standard compensation for aggregate risk taking that is familiar from frictionless consumption-based models. Lynch (1996) analyzes the quantitative importance of these liquidity premia in discrete time, while Gabaix and Laibson (2002) derive analytical solutions in continuous time. Chien, Cole and Lustig (2012) demonstrate that infrequent rebalancing is also quantitatively important for understanding the high volatility in the Sharpe ratio of stock market and its countercyclicality. They study an economy with households that differ in their adjustment costs: some continuously rebalance, while others do so infrequently. This paper analyzes rebalancing frictions on flows in a model with strong age heterogeneity. Moreover, I study whether target date funds can help address these frictions.

[2] For example, Ameriks and Zeldes (2004) study a 10-year panel dataset of retirement accounts in the U.S. They find that 73% of plan participants made no change to portfolio asset allocation during the ten years, and an additional 14% made only one change in ten years.

More recently, Hartzmark and Solomon (2021), Gabaix and Koijen (2021), Gabaix, Koijen, Mainardi, Oh and Yogo (2023, 2025) have provided key empirical estimates for moments that discipline asset pricing models. These new moments focus on the inelastic demand of stock and limited risk transfers between heterogeneous agents. The authors found traditional models to perform poorly on these new moments, potentially hinting at false mechanisms that are incompatible with reality. This paper studies a new rebalancing friction that fixes the asset allocation of flows, which leads to inelastic stock demand and limited risk transfers that are consistent with recent empirical counterparts and produces realistic macroeconomic aggregates, asset prices, and distributions of financial wealth and portfolios.

In addition, this paper adds to a literature that shows age is an important source of heterogeneity to understand equity valuations. Abel (2003) connects the baby boom with stock prices and shows that social security can potentially affect national saving and investment. Geanakoplos, Magill and Quinzii (2004) argue that population booms and busts cause bull and bear stock markets. Gomes and Michaelides (2008) studies asset returns and stock market participation with heterogeneous preferences and incomes in an OLG economy with capital depreciation shocks. Gârleanu and Panageas (2015) highlight the potential in preference heterogeneity across age cohorts to resolve some key asset pricing puzzles in an endowment economy.

Quantitative papers that study equity valuation in OLG economies with aggregate risk have mostly used a version of the Krusell and Smith (1998) approach that finds a self-confirming equilibrium where agents form beliefs about a set of moments selected by the modelers. Storesletten, Telmer and Yaron (2007) study idiosyncratic risk and risk premia in an OLG economy with production and incomplete markets. Favilukis (2013) jointly considers the rise in wage inequality, decrease in stock market participation costs, and relaxation of borrowing constraints. He finds that these observations have led to the sharp rise in wealth inequality, declines in interest rate and in equity premium. One exception in the quantitative strand of the literature is Leombroni, Piazzesi, Schneider and Rogers (2020) who solve for the temporary equilibrium of a model with exogenous expectations to study the entry of baby boomers into asset markets and inflation disagreement across age cohorts. The paper takes the joint distribution of income and initial endowments by age directly from the data, and feeds in survey forecasts to study equilibrium asset prices, wealth, and portfolios. In this paper, I use machine learning tools designed to approximate the rational expectations equilibrium.

Finally, this paper studies the general equilibrium implications of target date investing in a world in which many households do not participate in the stock market and rarely rebalance their portfolios. The existing literature on the introduction of target-date funds studies consumption-portfolio choice problems with exogenous asset returns. Moreover, the literature compares the welfare of target date investing with optimal portfolio choice in the absence of any frictions in stock market participation and portfolio rebalancing (Gomes, Kotlikoff and Viceira, 2008, An and Sachdeva, 2021, Duarte, Fonseca, Goodman and Parker, 2021, Gomes, Michaelides and Zhang, 2022). In contrast, this paper studies welfare effects of target date funds in general equilibrium with frictions that are consistent with salient features of household financial portfolios.

2 OLG Model with Idiosyncratic and Aggregate Risk

I consider a dynamic continuous time economy, with time indexed by $t \in [0, \infty)$. The state of the economy $Z_t \in \{0, 1\}$, follows a two-state continuous time persistent Markov chain with switching intensity $\lambda^Z(Z_{t-})$ between recessions ($Z_t = 0$) and expansions ($Z_t = 1$).

2.1 Household Sector

To model consumption, saving, and portfolio decisions through the life cycle, this section introduces a continuum of overlapping generations households that populate the economy. In the benchmark economy, households face portfolio frictions on asset allocation of saving flows.

Birth, aging, and death A household starts working at age a^{entry}, retires at age a^{retire}, and lives at most until age a^{exit}. The household dies with an age-dependent probability $\eta(a_t)\Delta$

during the time Δ, where a_t is current age of the household at time t. New households enter to replace dying and exiting households, and the population distribution is stationary over time.

Preferences A household has time-separable CRRA utility $u(\cdot)$ over consumption c_t at time t, discounts the future at a constant rate ρ, and derives utility from bequests $u^B(\cdot)$. Therefore, for a consumption process c, the discounted utility of a household at time t can be expressed as

$$U_t = E_t \left[\int_t^{t+a^{exit}-a_t} e^{-\rho(v-t)-\int_t^v \eta(a_s)ds} \Big(u(c_v) + \eta(a_v) u^B(q_v) \Big) \, dv \right], \tag{2.1}$$

where

$$\begin{aligned} u(c) &= \frac{c^{1-\gamma}}{1-\gamma} \\ u^B(q) &= \underline{b} \frac{(\bar{b}+q)^{1-\gamma}}{1-\gamma}. \end{aligned}$$

Income dynamics Before retiring, each household inelastically supplies labor and earns labor income. Household productivity $l(a_t, y_t)$ evolves according to a deterministic age profile and an idiosyncratic component y_t. Households are heterogeneous in idiosyncratic labor productivity, $y_t \in \{y_l, y_h, y_s\}$, where the "star" state y_s captures top earners in the economy. Labor productivity y_t switches between the three states according to a Poisson jump process. The probability of switching depends on both the pre-switch productivity state y_{t-} and the aggregate state Z_{t-}. By allowing idiosyncratic and aggregate risk to be correlated, this setup accommodates cyclical movements in labor income risk (Constantinides and Duffie, 1996, Guvenen, Ozkan and Song, 2014). Following Huggett (1996), retired households receive constant social security payment $\bar{s}$. This assumption is a rough approximation of the progressive replacement rate schedule of the U.S. Social Security program. Thus, at time t, a household receives income $m_t(a_t, y_t)$ which is either labor income $w_t l(a_t, y_t)$ or social security income $\bar{s}$. The wage rate w_t is the compensation for one efficient unit of labor supply.

A household consumes c_t and saves (or dissaves) s_t from income $m_t(a_t, y_t)$

$$c_t + s_t = m_t(a_t, y_t). \tag{2.2}$$

2.1.1 Asset Market Arrangements in Benchmark Economy

Households save in equity and in riskfree bond. The benchmark economy features frictions on asset flows. Firstly, participation shocks determine whether a household can participate in the stock market. The arrival rate of participation shocks can depend on the productivity of the household. Additionally, households set up a flow allocation rule which they choose at the arrival of a participation shock and do not actively rebalance portfolio afterwards.

Assets Household net worth n_t composes of savings in bonds b_t and in stocks e_t

$$n_t = e_t + b_t.$$

The riskfree rate is r_t^f. Equity payouts stay invested in equity (Duffie and Sun, 1990, Chien, Cole and Lustig, 2012). The cum-dividend return rate on equity follows

$$dr_t^e = \mu_t^e dt + \sigma_t^e dW_t,$$

where W is a standard Brownian motion, and the drift μ_t^e and the volatility σ_t^e are determined in equilibrium.[3]

The savings flow s_t into the portfolio splits between bonds and stocks according to the allocation rule $f_t \in [0, 1]$, which indicates the fraction of the savings flow s_t that goes into equity holdings e_t. The remaining $(1 - f_t)$ fraction of savings flow s_t goes into bond holdings b_t. Households start with an allocation rule that has zero weight on equity, $f_t = 0$ at time t when $a_t = a^{entry}$, thus households initially only save in bonds.

[3]The equity return follows a diffusion process rather than a jump diffusion process because the switch between recession and expansion induces change in capital depreciation rate rather than in level, as Section 2.2 explains.

Households whose allocation rule is 0 on stocks ($f_t = 0$) may receive a participation shock, which is a counting process N_t^f with associated intensity λ_t^f. Once the household receives a participation shock, the flow allocation rule switches to $F_t \in [0, 1]$ which is endogenously chosen by the household. Thereafter, households do not actively rebalance existing assets. Thus, the flow allocation rule of a household that receives a participation shock at time t switches from $f_t = 0$ to $f_s = F_t$ for $s > t$.

The jump intensity λ_t^f is dependent on pre-jump idiosyncratic productivity y_{t-} to capture that high-income individuals are more likely to participate in the stock market. A fraction of households, whose labor productivity is y_h, immediately receive a participation shock when they start working at age a^{entry}. Additionally, "star" earners, those with productivity is y_s, always receive participation shock if they have an allocation rule that is 0 ($f_t = 0$). Households with the *low* productivity state, however, do not receive participation shocks,

$$\lambda_t^f = \begin{cases} \bar{\lambda}^f & a_t = a^{entry} \text{ and } y_t = y_h \\ +\infty & y_t = y_s \text{ and } f_t = 0 \\ 0 & otherwise. \end{cases}$$

Withdrawals are proportional to current portfolio shares (Chien, Cole and Lustig, 2012, Choukhmane and de Silva, 2022). Therefore, the effective flow allocation to equity $\widetilde{f}_t$, depending on if the flow s_t is saving or withdrawal, is

$$\widetilde{f}_t = \begin{cases} f_t & s_t \geqslant 0 \\ \frac{e_t}{e_t + b_t} & s_t < 0. \end{cases}$$

Equity and bond holdings evolve according to

$$\begin{aligned} de_t &= (\mu_t^e e_t + \widetilde{f}_t s_t)dt + \sigma_t^e e_t dW_t \\ db_t &= [r_t^f b_t + (1 - \widetilde{f}_t)s_t]dt, \end{aligned} \tag{2.3}$$

and the household net worth n_t evolves by

$$dn_t = de_t + db_t = (\mu_t^e e_t + r_t^f b_t + s_t)dt + \sigma_t^e e_t dW_t.$$

In addition, households cannot short stocks or bonds

$$0 \leqslant e_t, b_t, 0 \leqslant F_t \leqslant 1. \tag{2.4}$$

Appendix B formulates the household problem.

Bequest distribution A small fraction of households receive bequests at age a^{entry} in the form of the average household portfolio. The probability of receiving bequests depends on idiosyncratic labor productivity y_t (Hendricks, 2007, De Nardi, 2004, Wolff and Gittleman, 2014). To capture the fact that a substantial fraction of estates does not pass on as inheritances but rather goes to expenses/charities that are not for production purposes, a certain amount of terminal wealth flows out of the economy (Joulfaian, 1994, Hurd and Smith, 1999, Hendricks, 2001).

2.2 Production Sector

This section describes production, capital structure and payout rules. Following Chien, Cole and Lustig (2012), I model equity as a leveraged claim to risky capital held through firms who implement a set of capital structure and payout rules to maximize value and to smooth payouts.[4] Additionally, I allow leverage to fluctuate to capture time-varying endogenous aggregate risk (Fernández-Villaverde, Hurtado and Nuno, 2023). The benefit of deriving capital structure and payout rules in analytical form is alleviating pressure on model solution, allowing rich heterogeneity and complex frictions in the household sector.

[4]The derivation of capital structure and payout rules can be found in Appendix C.

Technology The firms produce consumption goods with capital and labor

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}.$$

Firms own capital and hire labor at the competitive wage rate w_t.

Capital evolves according to

$$dK_t = [\iota_t - \Phi(\iota_t) - \delta(Z_t)] K_t dt + \sigma K_t dW_t,$$

where $\iota_t = I_t/K_t$ is the investment rate, and investment is subject to adjustment cost

$$\Phi(\iota_t) = \frac{1}{2}\phi[\iota_t - \delta(Z_t)]^2.$$

The depreciation rate $\delta(Z_t)$ is correlated with the aggregate state Z_t, while W_t is a standard Brownian motion that captures quality shocks to capital (Merton, 1973, Gertler and Kiyotaki, 2010, He and Krishnamurthy, 2019, Brunnermeier and Sannikov, 2014, Fernández-Villaverde, Hurtado and Nuno, 2023). Expected excess return on capital is the marginal product of capital, MPK_t, minus the riskfree rate r_t^f, adjustment cost $\Phi(\iota_t)$, and depreciation $\delta(Z_t)$

$$ER_t = MPK_t - r_t^f - \Phi(\iota_t) - \delta(Z_t).$$

Payout and capital structure Firms issue riskfree bonds B_t^f to finance investments in risky capital. Their balance sheets have assets

$$K_t = N_t + B_t^f,$$

where N_t is net worth of firms. Firms' leverage is the ratio of debt to their assets, B_t^f/K_t, and the capital-to-net-worth ratio is

$$\omega_t = \frac{K_t}{N_t} = \frac{1}{1 - leverage_t},$$

which maps one-to-one to the leverage ratio B_t^f/K_t, and the two ratios move in the same direction. Firms can rent capital to each other through a competitive rental market and collect rent. A representative firm exists in aggregation.

Net worth of the firms evolves according to

$$dN_t = \left(\left[r_t^f + \omega_t ER_t\right] N_t - D_t\right) dt + \sigma\omega_t N_t dW_t, \tag{2.5}$$

where D_t is payout to share holders at time t. The evolution of firm net worth indicates that increasing firm leverage and thus its ratio of capital-to-net-worth ω_t by issuing more debt (for example, for tax benefits) can amplify earnings from excess return on capital. However, more leverage also involves more risk that could be costly (for example, bankruptcy cost).

I assume firms follow a set of payout and capital structure rules (C.13) and (C.12) which are consistent with empirical observations on firm payout behavior. These rules can be micro-founded by modeling firms maximizing their value and smoothing their payouts (Brav, Graham, Harvey and Michaely, 2005, Farre-Mensa, Michaely and Schmalz, 2014, Bianchi, Ilut and Schneider, 2018),

$$\max_{D_t,\omega_t} E_t\left[\int_t^{+\infty} e^{-\bar{\rho}s}\log D_s ds\right], \tag{2.6}$$

subject to the evolution of net worth (2.5) and budget constraint

$$D_t = MPK_t K_t - I_t. \tag{2.7}$$

The curvature of logarithm function leads to intertemporal smoothing of payouts. The trade-off between earning amplified excess return on capital and more risk associated with higher leverage leads to an interior solution for the capital-to-net-worth ratio ω_t, which is increasing in the expected excess return on capital but decreasing in quality shock volatility σ. The optimal payout yield $\rho_t = D_t/N_t$ equals to the firm's discount rate $\bar{\rho}$ on average but fluctuates over time due to the adjustment costs.

This setup is agnostic on the open theoretical question of how to model firm objective function in an heterogeneous agent environment with incomplete markets where different agents disagree.[5] Given that the financial innovation I focus on, target date funds, happen exclusively on the demand side, these rules capture capital structure and payout dynamics that are invariant under such innovation. This particular objective function can be interpreted through the lens of either a financial sector (Brunnermeier and Sannikov, 2014, Fernández-Villaverde, Hurtado and Nuno, 2023) or through financial intermediaries (He and Krishnamurthy, 2019).

Finally, inflows/outflows from the household sector into the production sector open/close such identical firms. Section 2.4 explains the aggregation in details.

2.3 Government

The government collects income taxes at a constant tax rate τ. In addition, the government borrows by issuing riskfree bonds B_t^g that make up a constant share g of the total bond market.[6] Fiscal spending has three components: social security payments, debt payments, and discretionary spending G_t. Discretionary spending G_t adjusts to balance budget

$$\left((1-\tau)\int_{\mathcal{I}_t} \bar{s}\mathbf{1}_{\{a_{it}>a^{retire}\}}di + r_t^f B_t^g + G_t - \tau w_t L_t\right) dt = dB_t^g, \tag{2.8}$$

where $i \in \mathcal{I}_t$ indexes households alive in the economy at time t.[7]

2.4 Market Clearing and Aggregation

To prepare for the equilibrium definition, this section first describes the market clearing conditions and the evolution of aggregate variables.

Labor market clears by equating labor demand with labor supply

$$\int_{\mathcal{I}_t} y_{it}\mathbf{1}_{\{a_{it}\leqslant a^{retire}\}}di = L_t. \tag{2.9}$$

[5] Please see Gottardi, Bisin and Clementi (2022) for some recent progress on this question.

[6] The Treasury market relative to the fixed-income market share is stable around 20% to 30% between 1995 and 2005 (SIFMA, 2021).

[7] The government spending G_t is not transferred to households. Government does not solve an optimization problem.

Bond market clears by equating household bond holdings with corporate and government bond supplies

$$B_t = \int_{\mathcal{I}_t} b_{it} di = B_t^f + B_t^g. \tag{2.10}$$

Equity market clears by equating household equity holdings with net worth of the firm

$$N_t = \int_{\mathcal{I}_t} e_{it} di. \tag{2.11}$$

The numeraire good market clears by Walras's Law.

In the benchmark economy, the aggregate inflow of equity from the household sector to the production sector is

$$F_t^e = \int_{\mathcal{I}_t} \widetilde{f}_{it} s_{it} di + D_t,$$

which includes new purchases of stocks and reinvested payouts. The aggregate inflow of bonds is

$$F_t^b = \int_{\mathcal{I}_t} (1 - \widetilde{f}_{it}) s_{it} di.$$

The market clearing conditions plus aggregated resource constraint

$$D_t + w_t L_t = C_t + F_t^e + F_t^b + G_t$$

imply that aggregate capital evolves according to

$$\begin{aligned} dK_t &= \Big[\iota_t - \Phi(\iota_t) - \delta(Z_t) - O_t\Big] K_t dt + (F_t^e + F_t^b) dt + \sigma K_t dW_t \\ &= \left[\frac{Y_t - C_t - G_t}{K_t} - \Phi(\iota_t) - \delta(Z_t) - O_t\right] K_t dt + \sigma K_t dW_t, \end{aligned}$$

where O_t is the rate at which resources flow out of the economy because some estates do not pass on as bequests, as described in Section 2.1.

2.5 Equilibrium

In the benchmark economy, the household-specific individual state variables are age, bond holdings, equity holdings, flow allocation rule, and idiosyncratic labor productivity. Denote individual state variables of households as $x_t = (a_t, b_t, e_t, f_t, y_t)$, and the associated distribution is $\varphi_t(\cdot)$. Aggregate state variables consist of $X_t = (Z_t, W_t, \varphi_t)$. The entire collection of household state variables is then (x_t, X_t).

For a given initial aggregate state variables X_0, the equilibrium consists of pricing functions $\{r_t, r_t^f, MPK_t, w_t\}_{t\geqslant 0}$, household policy functions $\{c_t, s_t, F_t\}_{t\geqslant 0}$, and firm policy functions $\{D_t, \omega_t, K_t, L_t\}_{t\geqslant 0}$, such that

- given prices r_t, r_t^f, and w_t, policy functions c_t, s_t and F_t are the solution to maximizing household discounted utility (2.1), subject to budget constraint (2.2), the evolution of equity and bond holdings (2.3), and short selling constraints (2.4);
- given prices MPK_t, w_t, r_t, and r_t^f, policy functions K_t, and L_t are the solution to maximizing firm production profit, and policy functions D_t, ω_t implement capital structure and payout rules (C.13) and (C.12) to maximize discounted payouts (2.6), subject to the evolution of net worth (2.5) and firm budget constraint (2.7);
- the government collects income taxes, issues government bonds, and pays for social security benefits and government spending to balance budget (2.8);
- given prices r_t, r_t^f, and w_t, policy functions c_t, s_t and F_t, distribution function $\varphi_t(\cdot)$ is the solution to Kolmogorov forward equation (A.1);
- given prices MPK_t, w_t, r_t, and r_t^f, labor, capital, bond, equity, and numeraire good markets clear, as described in equations (2.9), (2.10), and (2.11).[8]

[8]Market clearing condition for equity implies that the cum-dividend equity return drift is $\mu_t^e = r_t^f + \omega_t ER_t$, and the cum-dividend return local volatility is $\sigma_t^e = \sigma\omega_t$, where r_t^f is the riskfree rate, ω_t is the capital-to-net-worth ratio, and ER_t is the expected excess return on risky capital.

I then solve the model using machine learning techniques by adapting the DeepHAM algorithm introduced in Han, Yang and E (2021). Appendix E summarizes the implementation of the computational strategy. The average relative consumption error is around 0.6%, which is in line with previous literature that solves lifecycle models (Maliar, Maliar and Winant, 2021) or OLG economies but without heterogeneity within an age cohort (Krueger and Kubler, 2004, Hasanhodzic and Kotlikoff, 2013).

3 Quantification of the Model

Table 1 shows the parameter values used for quantification of the model, most of which are either taken from previous literature or from data before 2001 (before the rise of target date funds). The CRRA parameter is set to match the average portfolio share in stocks.

Aggregate State The switching intensity between expansions and recessions is from Krusell and Smith (1998). On average, expansions and recessions last 8 quarters. The switching probabilities are symmetric between the two states.

Households The starting age of working life, retirement age, and death age are 30, 65, and 80. The age distribution and mortality risk by age are from the 1998 U.S. Mortality Database. Discount rate is from Huggett (1996), which reflects pure time preference that is separate from discounting induced by mortality risk. The risk aversion coefficient is calibrated to match the average portfolio share on stocks among stock market participants (44.03% in the data vs. 43.27% in the model). The CRRA of 5 is common in the macro-finance literature (Landvoigt, Piazzesi and Schneider, 2015, Favilukis and Van Nieuwerburgh, 2021). The income age profile comes from estimates by İmrohoroğlu, İmrohoroğlu and Joines (1995). Three idiosyncratic labor productivity states and their transition matrix come from Den Haan (2010) and Dávila, Hong, Krusell and Ríos-Rull (2012). Table B.6 shows the (quarterly) transition matrices, states, stationary distributions, and normalization. Social security benefit is set at 0.3, which is on average 30% of average wage (U.S. Social Security Administration, 2013, 2024).[9] The arrival in-

[9]According to U.S. Social Security Administration (2013, 2024), the average monthly retirement is $1,217.60 in January 2013, and the national average wage index is $ 44,321.67 for 2012, implying a ratio of 32%.

Parameter	Notation	Value	Source
Switching intensity	$\lambda^Z(\cdot)$	0.125	Krusell and Smith (1998)
Entry age	a^{entry}	30	
Retirement age	a^{retire}	65	
Death age	a^{exit}	80	
Age distribution			1998 US Mortality Database
Mortality risk	$\eta(a)$		1998 US Mortality Database
Discount rate	β	1.011	Huggett (1996)
CRRA	γ	5	Average equity portfolio share 44.03%
Income age profile			İmrohoroğlu, İmrohoroğlu and Joines (1995)
Labor productivity			Den Haan (2010) and Dávila, Hong, Krusell and Ríos-Rull (2012)
Social security	$\bar{s}$	0.3	U.S. Social Security Administration (2013)
Access shock at a^{entry}	$\bar{\lambda}^f$	0.5	Survey of Consumer Finances, 1995-2001
Bequest intensity	$\underline{b}$	23.6	Nardi, French and Jones (2010)
Bequest intercept	$\bar{b}$	20	Nardi, French and Jones (2010)
Bequest arrival by type		0, 0.05, 0.1	Wolff and Gittleman (2014)
Capital share	α	0.36	Kydland and Prescott (1982)
Adjustment cost	ϕ	1	Hall (2002)
Capital volatility	σ	0.1	Brunnermeier and Sannikov (2014)
Depreciation	$\delta(Z)$	0.09, 0.11	Krusell and Smith (1998)
Average payout yield	$\bar{\rho}$	0.049	Fernández-Villaverde, Hurtado and Nuno (2023)
Income tax rate	τ	0.2	De Nardi and Yang (2014)
Government bond	$\bar{g}$	1/3	SIFMA (2021)

Table 1: Parameters from Literature and Data

Notes: This table shows the parameter values used for quantification. They either are from data or previous estimates, with the exception of CRRA parameter which is calivrated targeting the average equity portfolio share among stock market participants. The four divisions correspond to parameters for the aggregate state of the economy, the household sector, the production sector, and the government sector.

tensity of participation shocks for the high type matches the share of stock market participants aged 30 years old. Parameters governing bequest motives, including intensity and intercept, are taken from Nardi, French and Jones (2010). The probability of receiving bequests depends on idiosyncratic labor productivity y_t (Hendricks, 2007, De Nardi, 2004, Wolff and Gittleman, 2014).

Production Firms The capital share is 0.36 as in Kydland and Prescott (1982). The capital volatility is from Brunnermeier and Sannikov (2014) who also study quality shocks to capital. Previous literature has estimated adjustment cost to range from 0.5 to 2 using disaggregated data (Shapiro, 1986, Hall, 2002, Cooper and Haltiwanger, 2006). Adjustment cost 1 is approximately the average across industries as estimated in Hall (2002).[10] The depreciation rate is equal to 10% on average, which is standard in the business cycle literature. Expansions (recessions) increase (lower) the depreciation rate by 1%, which amounts to the same size of the aggregate shock in Krusell and Smith (1998). The average payout yield is 4.9% as in Fernández-Villaverde, Hurtado and Nuno (2023), which is approximately the average payout rate of non-financial corporate businesses in the U.S. according to the Financial Accounts of the United States between 1970 Q1 and 2021 Q4.

Government Income tax rate τ is 20%, which is in line with the literature (De Nardi and Yang, 2014). Government bonds make up stably around one third of the entire U.S. fixed-income asset market between 1995 and 2005, according to the Securities Industry and Financial Markets Association (SIFMA, 2021). So, the fraction of government bond supply as a fraction of the total bond market g equals to 1/3.

3.1 Aggregates

The model is able to generate dynamics for macro aggregates and asset prices that match the data. Moreover, the model does not feature standard asset pricing puzzles. The implied aggregate stock demand elasticity and risk transfers are also consistent with recent empirical estimates.

3.1.1 Dynamics of Macroeconomic Aggregates and Asset Prices

The benchmark model delivers realistic macroeconomic and financial aggregates. The model is able to overcome the well known tension among low and stable riskfree rate, high and volatile risky rate, and stable consumption growth.

Table 2 shows that the benchmark model does well in matching the dynamics of macroeconomic aggregates and financial variables. Specifically, consumption growth rate is stable,

[10] In Hall (2002), he estimates the average adjustment cost, using quadratic specification, to be 0.91, which is in line with other estimates using micro data (Shapiro, 1986, Cooper and Haltiwanger, 2006). Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2019) use similar values for adjustment cost.

	Quarterly Standard Deviations of Growth Rates			
	Output	Cconsumption	Investment	Labor Supply
Benchmark	0.016	0.013	0.023	0.010
Data	0.012	0.012	0.041	0.014
	Annualized Asset Returns			
	$E[r_t - r_t^f]$	$\sigma(r_t - r_t^f)$	Sharpe Ratio	Leverage
Benchmark	0.067	0.211	0.320	0.512
Data	0.066	0.178	0.371	0.560

Table 2: Macroeconomic Aggregates and Financial Moments

Notes: This table compares model generated financial and macroeconomic aggregates with empirical estimates. The data sample contains 1970Q1 to 2022Q2 (210 quarters). Macroeconomic variables are from the Federal Reserve Bank of St. Louis. All data series are real and seasonally adjusted. Output is the gross domestic product. Consumption is the personal consumption expenditures. Investment is the gross private domestic investment. Labor supply is the hours worked for all employed persons (nonfarm business sector). For the model, simulation period is also 210 quarters. Asset prices come from CRSP value weighted index and the 1 month T-bill rate. Leverage is from estimate in Graham, Leary and Roberts (2015) for U.S. public firms in 2010.

but equity premium is high and volatile. These moments are common criteria in gauging performance of asset pricing models. The upper panel displays the quarterly standard deviations of the growth rates in output, consumption, investments, and labor supply for the benchmark model and the data. The bottom panel shows the equity premium and its volatility, Sharpe ratio, and leverage.

The benchmark model is able to simultaneously match asset prices and macro aggregates because participation and rebalancing frictions separate the pricing of the risky and the riskfree rates. This segmentation avoids puzzles seen in conventional consumption based asset pricing models which price both the risky and the riskfree rates with the same consumption process.

Households can adjust their savings at any time, implying that standard Euler equations hold for the return on savings, which are portfolios of stocks and bonds. For non-stockholders, the portfolio consists of bonds only, so that only the Euler equation for the riskfree rate holds. These households tend to be less wealthy, because lower productivity households have a lower arrival rate of participation shocks and because these households earn a lower average return on savings than stock-owners. There are many states of the world in which these underprivileged households may like to borrow but they face a borrowing constraint. To avoid these states of the world, less wealthy households will save and thereby depress the riskfree rate in equilibrium relative to an economy without participation frictions.

The portfolio of stockholders contains both bonds and stocks, but their Euler equation only holds for the return on the entire portfolio, not for the return on each asset individually. The reason is that rebalancing frictions prevent stockholder-households from adjusting their portfolios. Instead, household choose how much to save and sell their portfolios, with fixed portfolio weights.

A two-agent economy with participation and rebalancing frictions can illustrate the intuition for these asset pricing dynamics at the extreme. Suppose one agent can only hold stocks, while the other agent only holds bonds. Stocks and bonds are in non-zero net supply. In equilibrium, the stockowner will price stocks, and the bond holder will price the riskfree rate. The stockowner has massive exposure to risk and demands a high compensation, pushing up the equity return rate. The intertemporal smoothing motives of the (less wealthy) non-stockowner will determine the riskfree rate. The segmentation of stock and bond pricing will avoid the well-known asset pricing puzzles seen in a standard consumption-based asset pricing model.

It remains to clarify how the stock-owner ends up with the high exposure to risk, which the stylized two-agent model abstracts away from. Stock returns are on average much higher than the riskfree rate. As a consequence, mechanically, a stock market participant's portfolio share in equity increases with time, due to rebalancing frictions. Empirical studies on household portfolios confirm this pattern in both proprietary data and in survey data (Ameriks and Zeldes, 2004). This mechanism concentrates equity holdings even more, in addition to the concentration that participation frictions induce. Moreover, rebalancing frictions prevent stockowners from choosing portfolio shares, so these agents still do not price the riskfree rate. Thus, the segmentation intuition remains intact, even if stockowners in the full model hold both stocks and bonds.

3.1.2 New Benchmarks for Macro-Finance Models

Recent empirical literature has uncovered inconsistencies between conventional Macro-Finance models and empirical features of the stock market. The two prominent new statistics are stock demand elasticity and risk transfers. These new moments suggest that models consistent with macroeconomic and financial aggregates might still miss the correct mechanisms that generate these results, potentially leading to wrong conclusions for policy evaluations. The benchmark model studied in this paper generates inelastic stock demand and limited risk transfers, closely matching recent empirical estimates.

Conventional Models	
Rep. Agent Endowment Economy	20
Disaster Model	8
Endowment Model w/ corr. labor inc.	17.5
Long Run Risk Model	30
Empirical Estimates	
Gabaix–Koijen (2023)	0.2
Hartzmark–Solomon (2021)	0.4–0.7
Benchmark Economy	0.9

Table 3: Stock Demand Elasticities

Notes: Empirical estimates for the aggregate demand elasticity come from Hartzmark and Solomon (2021) and Gabaix and Koijen (2021). Gabaix and Koijen (2021) also provides the demand elasticities that are implied by the various models mentioned in the table. Benchmark economy stock demand elasticity is estimated from the agent policy function, on the ergodic distribution of the model economy.

The benchmark economy features a realistic low stock demand elasticity. Hartzmark and Solomon (2021) and Gabaix and Koijen (2021) both deliver empirical demand elasticity estimates that are several orders of magnitude smaller than implied in typical asset pricing models. As Table 3 demonstrates, the benchmark economy studied in this paper generates inelastic stock demand because frictions on asset flows dampen agents' response to movements in asset prices. The model implies that the aggregate demand for stocks drop by 0.9% for an 1% increase in stock price on average, substantially closing the gap between previous asset pricing models that generate demand elasticities around 10-30 and the empirical estimates ranging from 0.2-0.7.

In addition to generating inelastic stock demand, the benchmark economy also exhibits limited risk transfers between agents, suggesting realistic risk sharing arrangements which conventional heterogeneous-agent models struggle to produce. Gabaix, Koijen, Mainardi, Oh and Yogo (2025) use proprietary data on financial accounts managed by financial advisors to investigate the average change in risk exposure among financial market participants. The authors find that risk transfer, which measures the average flow change with respect to current risk held in an investor's portfolio, is very limited across the wealth distribution. Their estimates suggest that the risk transfer is on average 0.65%. Leading asset pricing models with heterogeneous agents produce risk transfers that are one or two orders of magnitude larger. Table 4 compares risk transfers between empirical estimates and the benchmark economy for two groups sorted by their portfolio share on stocks. The low group has an average portfolio share of 42.8% and an average risk transfer of 0.010, while the high exposure group holds 81.4% of their portfolio in stocks and has an average risk transfer of 0.0031. Both groups have similar risk transfers in the benchmark economy. As a comparison, Gabaix, Koijen, Mainardi, Oh and Yogo (2025) report that the leading asset pricing model with heterogeneous agents features risk transfers of 3.5% and 78% for the two groups.

	Empirical Estimates		Benchmark Economy	
	Low	High	Low	High
Portfolio Share in Stocks	0.428	0.814	0.457	0.782
Average Risk Transfer	0.010	0.0031	0.0076	0.0064

Table 4: Risk Transfers

Notes: The empirical estimates of risk tranfer are from Gabaix, Koijen, Mainardi, Oh and Yogo (2025). The model implied risk transfers come from siulation using 800 quarters on the ergodic distribution of the economy. To align the benchmark model with the sample used for estimation, I apply the same set of filters listed. These filters include being active in at least three asset classes (I used stocks, bonds, and cashs) be having at least $100,000 of financial assets. These criteria filter out 40% of individuals in 2019 SCF, to which the benchmark economy is calibrated. The divide between the high and low groups is then calibrated to match the average portoflio shares in the data sample used by Gabaix, Koijen, Mainardi, Oh and Yogo (2025).

3.2 Lifecycle Financial Wealth and Stock Holdings

The benchmark model not only generates realistic aggregate moments but replicates salient features of lifecycle household financial portfolios. The model's ability to replicate these features testifies to the quantitative success of the benchmark model in capturing significant forces shaping consumption, saving, and portfolio holdings over the life cycle.

Figure 1 compares the wealth age profile in the model and in the data. Both the model and the data show a hump-shaped wealth age profile. Due to consumption-smoothing incentives, workers save while income is high and draw down savings in retirement. As a result, household wealth peaks around retirement age. During late retirement, the bequest motive becomes strong, and households do not consume all of their financial wealth even near the end of their lifecycle.

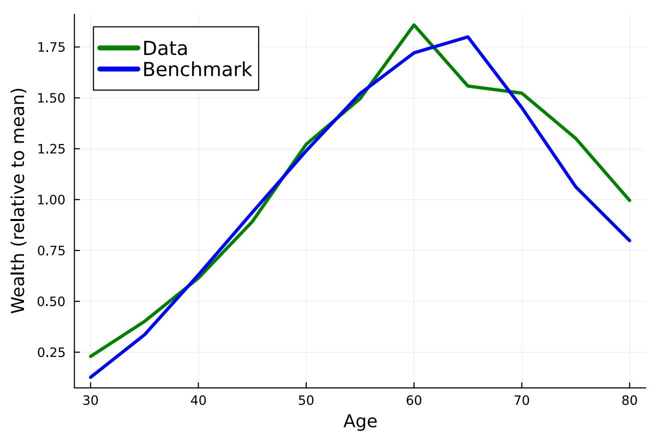

Figure 1: Age Profile of Wealth and Stock Holdings

Notes: The left panel plots the average wealth by age, and the right panel illustrates equity market shares by age. Data is from the Survey of Consumer Finances 1995, 1998, and 2001. Household wealth is calculated as non-housing, non-business net worth. Wealth, in both the model and data, has been normalized by the mean household wealth. The graph excludes households with negative net worth.

Figure 2 then examines lifecycle portfolio holdings among stock market participants in the benchmark economy against the data. In the pooled cross section from the Surveys of Consumer Finances, stock market participants hold roughly 35% of their portfolios in stocks at age 30, compared to 30% in the model. The equity portfolio share then rises to around

50% at age 63 before declining to 40% at age 80. The model successfully replicates the rise of equity portfolio share to age 63 but slightly misses on the decline thereafter. This slight mismatch is a result of cohort effect, as the right panel demonstrates. Following the approach in (Ameriks and Zeldes, 2004), The right panel plots the lifecycle portfolio holdings separately for each age cohort for people born between 1920 to 1960s. Echoing the benchmark model, the equity portfolio share climbs with age across all cohorts. Particularly, households born between 1930 to 1950, who hold a lion's share of the stock market, have age profiles nearly identical to the benchmark model. The cohort view of the cross sectional equity portfolio holdings by age demonstrates that the missed decline post retirement in the left panel is due to cohort effect shown among people born in 1920s in the data. As Section 3.3 shows, missing the particular cohort effect exhibited among the 1920 cohort does not lead to serious concerns with the distribution of financial portfolios, as this cohort of agents hold minimal financial wealth, bearing limited impact on pricing risk in the economy.

The main driver in the model that leads to such a realistic age profile of equity holdings is rebalancing frictions. Recall that the only portfolio-related target used in quantification is the average portfolio share in stocks. Conditional on participation, rebalancing frictions in the model imply that portfolio equity share at any age depends only on the initial allocation, savings according to flow allocation rule, and the subsequent market outcomes. In fact, Section 4.2 re-visits this plot when asset market arrangements change, the age profile of equity holdings is drastically different. This finding confirms earlier conclusion in Ameriks and Zeldes (2004) that household lifecycle financial portfolio holdings are influenced by flows and market returns, highlighting the importance of rebalancing frictions in understanding household lifecycle portfolio dynamics.[11] The fact that the benchmark economy is able to replicate salient facts on lifecycle consumption, saving, and portfolio dynamics is a strong indication about the quantitative importance of embedded frictions in the model.

[11] Ameriks and Zeldes (2004) decompose the change in household portfolio allocation using a balanced panel data and find that market returns drove up equity allocation by 9.2 percentage points during a 52-quarter period, while non-market effects, including active rebalancing, only offset 1.1 percentage points in the increase in equity portfolio share among the households they study.

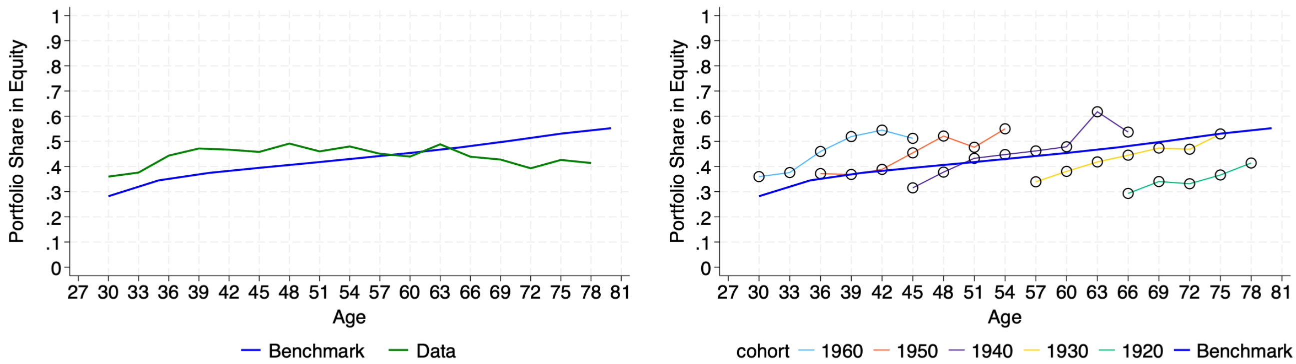

Figure 2: Age Profile of Wealth and Stock Holdings

Notes: The left panel plots the average wealth by age, and the right panel illustrates equity market shares by age. Data is from the Survey of Consumer Finances 1989-2001. Household wealth is calculated as non-housing, non-business net worth. Wealth, in both the model and data, has been normalized by the mean household wealth. The graph excludes households with negative net worth.

3.3 The Distribution of Wealth and Equity Holdings

This section compares distributions of financial wealth and portfolios by age and by wealth to demonstrate the power of benchmark frictions in replicating empirical observations on financial wealth, equity holdings concentrations in the economy. Success of generation realistic asset holdings distributions is key to producing realistic price of risk in the economy and to understanding forces that shape the evolution of inequality.

As Figure 3 illustrates, the benchmark model generates hump shaped distributions by age for financial wealth and for equity market shares. The left panel in Figure 3 plots the share of financial wealth by age. Households between age 40 and age 70 hold disproportionately more financial wealth and equity market share than other households. The right panel of Figure 3 illustrates equity market share by age, defined as the ratio of total equity holdings by an age cohort to the total equities outstanding in the economy. The success of replicating these empirical distributions comes from the realistic age profiles of financial wealth and of equity holdings, as Figure 1 and Figure 2 have shown. Intertemporal consumption smoothing incentives encourage financial assets accumulation before retirement and dis-saving post-

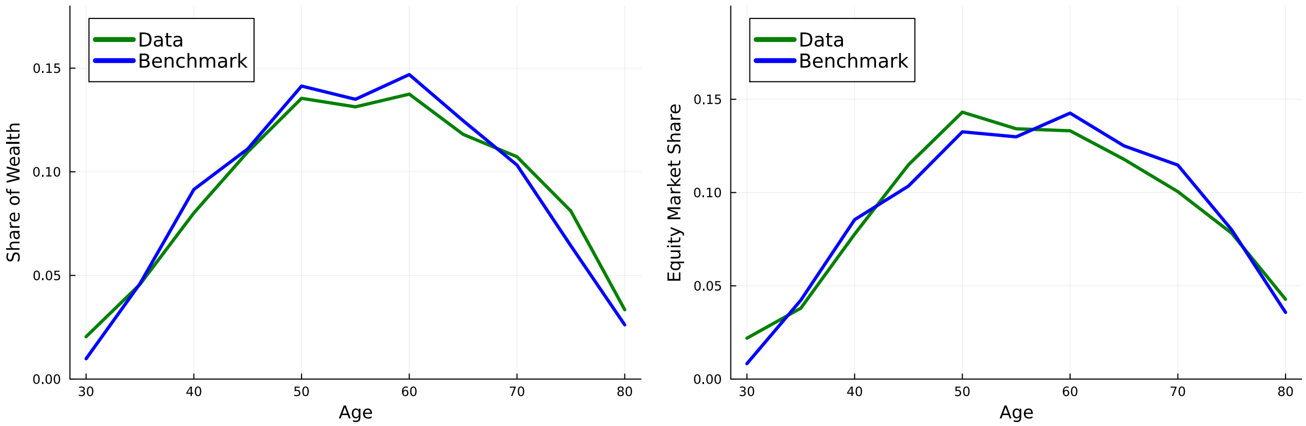

Figure 3: Financial Wealth Distribution

Notes: Model moments are calculated with simulation for the ergodic distribution of the benchmark economy, which features participation frictions and rebalancing frictions on flows. Data is from the Survey of Consumer Finances 1995, 1998, and 2001. Household wealth is calculated as non-housing, non-business net worth. The graph excludes households with negative net worth.

retirement, leading to middle aged households being wealthier than their younger and older counterparts. This realistic distribution of financial wealth by age, along with matching the lifecycle profile of portfolio holdings, leads to a close fit for equity market shares by age.

The success of the model in delivering realistic distributions of wealth and equity holdings by age is a part of the reason why the aggregate Sharpe ratio is close to its empirical estimate, as Table 2 shows. Given the heterogeneous risk bearing abilities by households of different ages for having varying levels of human capital, it is crucial for the model to match the distribution of equity market shares by age in order to produce realistic risk sharing arrangements by households at all stages of their life cycle.

A second factor that leads to a high Sharpe ratio in the model is inequality in financial wealth and the concentration of equity shares distribution. A direct consequence of a small group of households owning a lion's share of the equity market is that these people are highly exposed to aggregate risk, demanding high compensation for holding risk.

As Figure 4 demonstrates, the top 10% richest households account for 70% of financial wealth in the economy and 80% of the stock market. The model produces the same level of

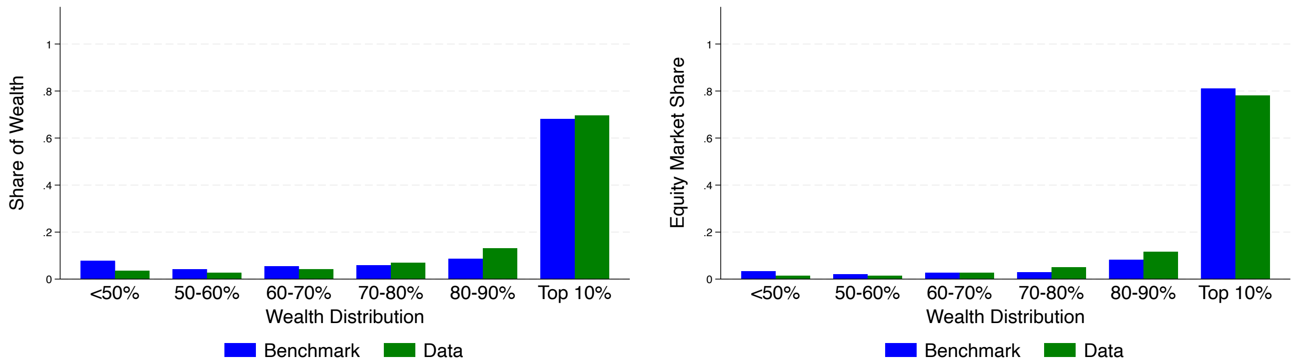

Figure 4: Financial Wealth Distribution

Notes: Model moments are calculated with simulation for the ergodic distribution of the benchmark economy, which features participation frictions and rebalancing frictions on flows. Data is from the Survey of Consumer Finances 1995, 1998, and 2001. Household wealth is calculated as non-housing, non-business net worth. The graph excludes households with negative net worth.

concentration in equity holdings as observed in the data due to frictions in participation and in rebalancing. For equity holdings to be even more concentrated than the wealth distribution, richer households must have higher portfolio shares in equity.

In the model, households who end up at the top of the wealth distribution are equity market participants who have been enjoying the equity premium for a long time. The positive correlation between equity participation and idiosyncratic productivity means that equity participants tend to have lucky histories of idiosyncratic productivity draws. In addition, rebalancing frictions imply that, conditional on participation, the portfolio share in equity results from the initial allocation and subsequent market outcomes. Given that equities tend to outperform bonds, the portfolio share in equities trends upwards as agents age. Agents around 50 to 60, just before they start drawing down savings, tend to be the richest households whose portfolios are also high in equities. Therefore, the distribution of equity holdings is more concentrated than the distribution of wealth.

4 Counterfactuals: Target Date and Free Access Economy

This section considers two alternative asset market arrangements to gauge the general equilibrium effects of recent financial innovation on asset prices, inequality and welfare. In the first alternative, households use an age based asset allocation rule that resembles the target date glide path. In the second alternative, free access economy, households choose their portfolios optimally without any participation and rebalancing restrictions. Widespread adoption of target date funds reduces equity premium and inequality. The outcomes are very close to those of an economy without any participation costs and rebalancing frictions. On the transition path following an unanticipated, immediate adoption of target date funds, young and rich households suffer welfare losses, while other gain to varying degrees.

Target date economy Households use an age based asset allocation rule that splits their portfolios between stocks and bonds. A household still chooses consumption c_t and savings s_t to maximize (2.1), subject to budget constraint (2.2), but portfolio frictions in the benchmark economy are replaced with an age based asset allocation rule

$$\frac{e_t}{e_t + b_t} = T(a_t) \quad \forall t,$$

where $T(a_t)$ is the portfolio share invested in equity at age a_t, as the target date glide path dictates (See Appendix D an illustration of the glide path).

The household net worth $n_t = e_t + b_t$ evolves according to

$$dn_t = \{[r_t^f + T(a_t)(\mu_t^e - r_t^f)]n_t + s_t\}dt + \sigma_t^e T(a_t) n_t dW_t.$$

Short selling constraints (2.4) stay the same.

Free access economy There are no participation and rebalancing frictions like in the benchmark economy. Households can choose their portfolio share in stocks and rebalance their portfolio anytime. As a result, households choose consumption c_t, savings s_t, and portfolio equity share E_t to maximize (2.1), subject to budget constraint (2.2). Household net worth $n_t = e_t + b_t$ evolves according to

$$dn_t = \{[r_t^f + E_t(\mu_t^e - r_t^f)]n_t + s_t\}dt + E_t n_t \sigma_t^e dW_t,$$

where the drift is the expected return on the portfolio invested in bonds and stocks plus any additional savings (or minus any withdrawals). Any share E_t invested in stocks contributes to the volatility of net worth because of the volatility σ^N of stock returns. Short selling constraints (2.4) stay the same.

4.1 Asset Prices

This section studies how counterfactual asset market arrangements change equilibrium asset prices. Compared to the benchmark economy, target date investing lowers equity premium, stabilizes equity returns, and decreases the aggregate Sharpe ratio. Results are similar for the free access economy.

Table 5 compares moments on asset prices for the benchmark economy, the target date economy, and the free access economy, including annualized average return rates and standard deviations of, equity, bond, and risk premium, the aggregate Sharpe ratio, and the average capital-to-net worth ratio ω.

Compared to the benchmark economy, the two counterfactual worlds have drastically different asset pricing dynamics. In the target date economy, equity returns are lower and more stabilized: the average equity return rate is 2.3% with standard deviation 15.0%, compared to 6.7% and 21.1% respectively in the benchmark economy. The riskfree rate does not show noticeable differences between the benchmark and the target date economies. Consequently, lower and more stabilized equity returns translate into the smaller and less volatile equity premium in the target date economy. In addition, the aggregate Sharpe ratio diminishes from 0.318 in the benchmark economy to 0.153 in the target date economy. Accompanying all these changes in asset prices is a sharp decline in the firm leverage, with capital-to-net worth ratio cut to 1.523 from 2.049. Outcomes for the free access economy are very much comparable with those from the target date economy. The average equity return rate, its standard deviation, the equity premium and its volatility, the Sharpe ratio, and the capital-to-net-worth ratio ω drop further, but only to a limited extent.

	$E[r_t]$	$\sigma(r_t)$	$E[r_t^f]$	$\sigma(r_t^f)$	$E[r_t - r_t^f]$	$\sigma(r_t - r_t^f)$	Sharpe Ratio	ω_t
Benchmark	0.052	0.211	-0.015	0.006	0.067	0.211	0.318	2.049
Target Date	0.011	0.150	-0.011	0.008	0.023	0.150	0.153	1.523
Free Access	0.008	0.155	-0.007	0.008	0.015	0.155	0.097	1.705

Table 5: Counterfactuals: Annualized Asset Returns

Notes: This table demonstrates the asset pricing moments in counterfactual exercises. The benchmark economy features participation frictions and rebalancing frictions on flows. The target date economy imposes an age based allocation for households. The free access economy allows for free portfolio optimization.

The reduction in equity premium, under alternative asset market arrangements, shown in Table 5 is driven by stabilized equity returns and improved risk sharing. As Section 4.2 and Section 4.3 demonstrate, the stabilization of equity returns takes place because of increased equity holdings across all households. The two alternative financial market arrangements redistribute financial wealth to young households who hold a large share of their portfolios in stocks and to the less wealth who do not participate in the stock market under benchmark economy. Consequently, the economy deleverages. In addition, as the young and the underprivileged hold more shares of the equity markets, demanded price of risk declines substantially.

4.2 Higher Stock Holdings at Younger Ages

To understand these movements in asset prices, I examine lifecycle portfolio holdings and the distributions of wealth and equity shares by age and wealth groups under the two alternative asset market arrangements. The changes in asset prices shown in Table 5 are driven by increased stock holdings, particularly by the young and by the less wealthy.

Figure 5 illustrates average portfolio age profiles under the three asset market arrangements. In the benchmark economy, the average portfolio share, including participants and non-participants, in equity starts off around 20% at age 30 and goes up with age, reaching 35% at age 80. The equity premium mechanically drives most of this pattern. Given that equities tend to outperform bonds, the equity portfolio share goes up as stockowners age due to rebalancing frictions.

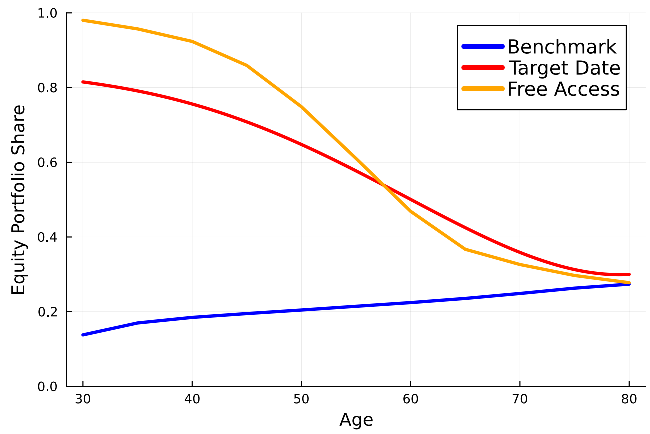

Figure 5: Counterfactuals: Portfolio Age Profiles

Notes: This figure illustrates the average portfolio share in stocks by age. The benchmark economy features participation frictions and rebalancing frictions on flows. The target date economy imposes an age based allocation for households. The free access economy allows for free portfolio optimization.

Compared to the benchmark economy, the target date economy shows substantially more equity holdings across almost all ages, especially for the young. The target date glide path slides from around 80% for 30-year-old agents to about 40% at retirement and continues declining to 25% at age 80. The higher average portfolio share in equities among the young reflects two margins of agents' portfolios: participation rate is lower in the benchmark economy (around 55% across all working ages); conditional on participation, the glide path sets the portfolio equity share higher than a stock market participant in the benchmark economy.

The average portfolio share in equities in the free access economy is similar to the glide path throughout the life cycle. Initially at close to 100%, the portfolio share in equities declines with age in a similar fashion to the target date glide path, as non-tradable, relatively safe human capital decreases with age.

4.3 Redistribution of Financial Wealth and Stock Holdings

In the two counterfactual worlds, equity ownership is no longer as concentrated at the late stage of life cycle and among the wealthiest households. Younger agents and people across the wealth distribution experience widely improved access to the equity markets. Stock ownership equalization along dimensions of age and wealth is a direct result of both higher stock holdings at early ages and general equilibrium forces that reduce the equity premium. At the aggregate, risk sharing improves, and financial inequality dwindles.

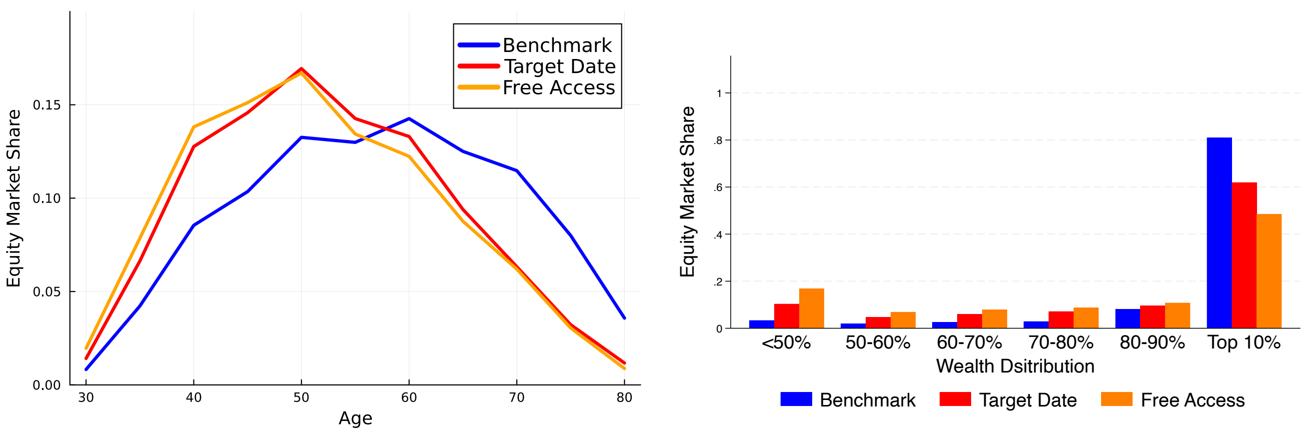

Figure 6: Counterfactuals: Distribution of Equity Market Shares

Notes: The left panel plots the equity market share by age, and the right panel illustrates the equity market share by wealth groups. In the two counterfactual worlds, equity holdings are redistributed towards the young and towards the bottom 90%. The extent of redistribution is higher in the free access economy than in the target date economy.

Figure 6 demonstrates the dramatic redistribution of equity holdings to the young and to the less wealthy. The left panel of Figure 6 plots the equity market share by age for three different asset market arrangements. Compared to the benchmark economy, working agents hold more equity shares in the counterfactual economies. Specifically, households below age 60 hold about three to five percentage points more of the stock market at every age. In contrast, retirees see a steep cut in their equity market shares. In other words, both the target date and the free access economies redistribute equity shares towards young workers, to a more aggressive extent in the free access economy. The right panel of Figure 6 illustrates a equally spectacular redistribution of equity ownership from the rich to everyone else. As we move from the benchmark economy to target date and free access economies, the top 10% richest households see their equity market share shrink from 80% to 60% and 50%, respectively.

Redistribution of financial wealth is also substantial, although not as pronounced as the redistribution of equity shares. The left panel of Figure 7 suggests that households below age 60 hold between 1 to 2 percentage points more of financial wealth in the economy at all ages. IN contrast, retirees who lose financial wealth share by one to two percentage points. The right panel of Figure 7 shows 7% and 10% reductions in the top 10% financial wealth share, as we move from benchmark economy to target date and free access economies.

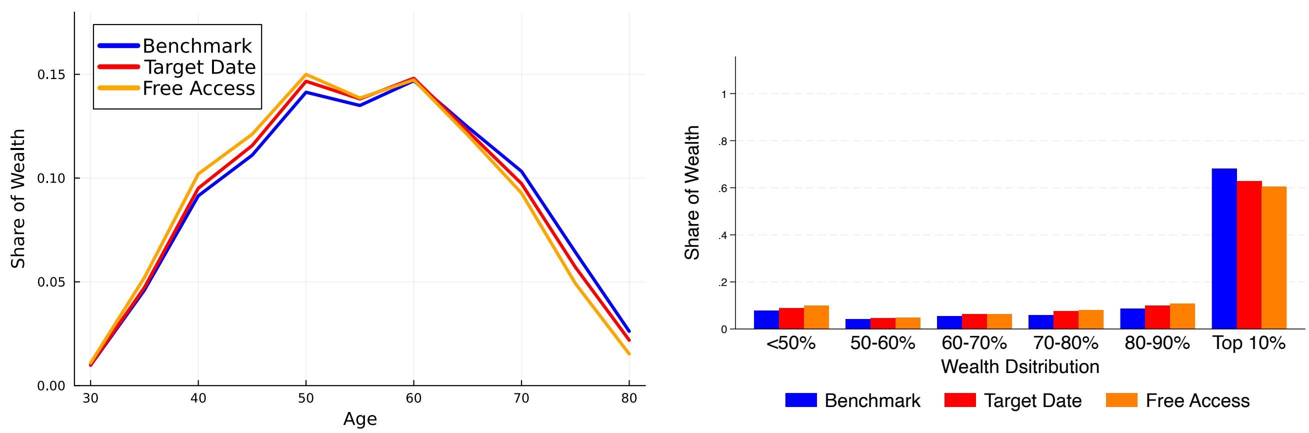

Figure 7: Counterfactuals: Distribution of Equity Market Shares

Notes: The left panel plots the share of financial wealth by age, and the right panel illustrates the share of financial wealth by wealth groups. In the two counterfactual worlds, financial wealth is redistributed towards the young and towards the bottom 90%. The extent of redistribution is higher in the free access economy than in the target date economy.

Age patterns of wealth and stock holdings depicted in Figure 6 and Figure 7 are a consequence of increased equity holdings among the young. In fact, Figure 5 hints at these age patterns in the three economies. In the target date economy, both stock market participation and, conditional on participation, the portfolio share in equities are higher than those in the benchmark economy. This is especially true for workers for whom the target date glide path sets a high portfolio share in equities. As a result, equity shares and financial wealth redistribute towards the young in the target date economy. This redistribution is more dramatic in the free access economy. Compared to the target date glide path, households dial up even more their equity portfolio shares at early life cycle.

Increased access to equity markets and reduction in equity premium both drive the wealth and stock holdings distributions along wealth portrayed in Figure 6 and in Figure 7. In the benchmark economy, participation in equity markets is restricted to rich agents who are either bequest receivers or who have enjoyed lucky draws of labor productivity. In contrast, everyone participates in the stock market in the two counterfactual economies. In addition, the concentration of equity market shares is substantially mitigated, because risk premium drops from over 6.7% to around 2.3%. Stock owners accumulate stock wealth much slower than they do in the benchmark economy, reducing the concentration of equity ownership.

4.4 Welfare

To investigate the welfare consequences from target date investing, this section studies on the transition path following an unanticipated, immediate adoption of target date funds. This experiment resembles the passage of the 2006 Pension Protection Act which significantly contributed to the subsequent rise of target date funds.

Figure 8 demonstrates the heterogeneous welfare consequences following the adoption of target date funds. Households in the bottom 90% of the wealth distribution generally gain in welfare. Such welfare gains are more pronounced among working age households who enjoy 4-6% gains in consumption equivalents. Retirees in the bottom 90% of wealth distribution witness much smaller welfare improvements, with gains decreasing with age to around 0-2% in retirement. Wealthy households who are in the top 10% of wealth distribution can lose 0-3% consumption equivalents during young ages before 40. Rich households above age 50, however, see sizable welfare gains at around 2-4% consumption equivalents. These welfare changes are driven by the trade-off of higher returns achieved with stocks versus volatility of savings. As target date funds increase household access to equity markets using an age based asset allocation rule, young households in the bottom 90% are able to accumulate wealth with

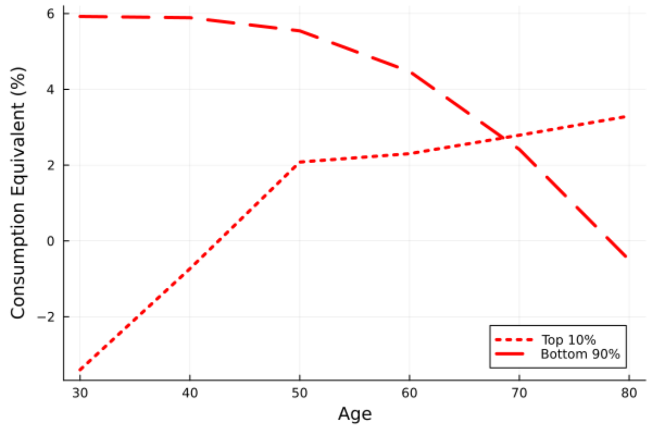

Figure 8: Counterfactuals: Welfare

Notes: Following an unanticipated, immediate adoption of target date funds, households experience heterogeneous welfare consequences. Households in the bottom 90% of wealth distribution mostly gain in welfare. Young households enjoy up to 6% consumption equivalents, while retirees gain 0-2%. Rich households in the top 10% can lose up to 3% among the young. Older households still gain in welfare by 2-3%.

higher returns, leading to large welfare gains. As we move to the older households, however, such gains from faster asset accumulation diminishes because of shorter investment horizon. In addition, particularly among the retirees, a lot of households in the bottom 90% do not wish to hold large amount of equity as they lack the resources to buffer against volatility coming from stock returns. Both forces chip away the welfare gains observed for young households. This mechanism works in the opposite direction for households in the top 10% of wealth distribution. Rich and young households are generally stock market participants under the benchmark asset market arrangement. As general equilibrium forces slash equity premium, these households can no longer accumulate assets to the scale they would have if equity returns were dramatically higher, which explains their welfare losses of up to 3%. Older rich households, however, have accumulated large financial resources and have shorter investment horizons. The more relevant margin for these people who are approaching retirement is the stability of their wealth. As general equilibrium forces stabilize equity returns, the welfare losses witnessed for young rich households are reverse to 2-5% welfare gains for rich retirees.

5 Conclusion

To conclude, this paper investigates the general equilibrium effects of financial innovation that increases access to equity markets for asset prices, inequality, and welfare. I set up an overlapping generations model with idiosyncratic and aggregate risk to study lifecycle portfolio choices in general equilibrium. I then solve the model by applying machine learning techniques to overcome the curse of dimensionality. The benchmark economy features frictions in the asset allocation of flows and replicates lifecycle portfolio patterns, macro aggregates dynamics, equity and bond returns, and distributions of wealth and portfolios by age and wealth, as well as recent estimates for stock demand elasticity and risk transfers.

Target date investing stabilizes equity returns, improves risk sharing, and reduces inequality. The equity premium plunges from 6.7% to 2.3%, and the annualized standard deviation of equity returns falls from 21.1% to 15.0%. In addition, the aggregate Sharpe ratio plummets from 0.318 to 0.153. These results are a consequence of increased access to equity markets, particularly among the young and the less wealthy. Overall, outcomes are comparable between the target date economy and the free access economy. Free access economy removes frictions in participation and in rebalancing altogether, leading to further improvements in risk sharing, and a bigger reduction in inequality.

Following an unanticipated, immediate adoption of target date funds, households in the bottom 90% of wealth distribution gain in welfare measured by consumption equivalents by 6% among the young and around 0-2% among retirees. Households in the top 10% suffer losses of up to 3% for the young, but retirees gain 2-3% in welfare.

Findings in the paper suggest that age based asset allocation rule dramatically increases households' access to equity markets and generates quantitatively significant general equilibrium effects on asset prices, inequality, and welfare. Evaluations of policies that encourage the adoption of financial innovations, such as the 2006 Pension Protection Act and the 2022 Secure Act 2.0, should consider these general equilibrium implications.

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Appendices

A Kolmogorov Forward Equation

The dynamics of the density function $\varphi_t(\cdot)$ are described by the Kolmogorov forward equation

$$\begin{aligned}
&\frac{\partial \varphi_t(a,b,e,f,y_i)}{\partial t} = \\
&-\frac{\partial}{\partial a}\varphi_t(a,b,e,f,y_i) \\
&-\frac{\partial}{\partial b}\{[r_t^f b_t + (1-\tilde{f}(a,b,e,f,y_i,X_t))s(a,b,e,f,y_i,X_t)]\varphi_t(a,b,e,f,y_i)\} \\
&-\frac{\partial}{\partial e}[(\mu_t^e e_t + \tilde{f}(a,b,e,f,y_i,X_t)s(a,b,e,f,y_i,X_t))\varphi_t(a,b,e,f,y_i)] \\
&+\frac{1}{2}\frac{\partial^2}{\partial e \partial e}[(\sigma_t^e e)^2 \varphi_t(a,b,e,f,y_i)] \\
&+\sum_{j\neq i}\lambda_{ji}^y(Z_t)\varphi_t(a,b,e,f,y_j) - \sum_{j\neq i}\lambda_{ij}^y(Z_t)\varphi_t(a,b,e,f,y_i) \\
&+\sum_{j\neq i}\int \mathbf{1}(F(a,b,e,f,y_i,X_t)=f)\lambda^f(a,b,e,f,y_j)\varphi_t(a,b,e,f,y_j)d(a,b,e,f) \\
&-\int \mathbf{1}(F(a,b,e,f,y_i,X_t)\neq f)\lambda^f(a,b,e,f,y_i)\varphi_t(a,b,e,f,y_i)d(a,b,e,f) \\
&+_t(a,b,e,f,y_i) - \eta(a)\varphi_t(a,b,e,f,y_i),
\end{aligned} \tag{A.1}$$

where $y_i, y_j \in \{y_l, y_h, y_s\}$. The first four lines describe the dynamics of state variables that follow a diffusion process. Line five is associated with inflows and outflows coming from labor productivity transitions, with transition intensities in Table B.6. Line six and line seven come from inflows and outflows that are induced by shocks to access the stock market. The last line describes entry and exit of agents. The distribution of newborn agents is described by distribution function $\psi_t(\cdot)$. The number of households entering and exiting are equal at all time, $\int_{\mathcal{X}} \psi_t(x)dx = \int_{\mathcal{X}} \eta(a)\varphi_t(x)dx$, where η is the age dependent mortality rate, and $\mathcal{X}$ is the state space of age, bond holdings, equity holdings, flow allocation rule, and idiosyncratic labor productivity, which is consistent with a stationary age distribution. Distribution function $\varphi_t(\cdot)$ describes the distribution of individual states over state space $\mathcal{X}$ among living agents.

B Household Problem

$$
\begin{aligned}
V(x_t, X_t) = \sup_{c\geq 0, F\geq 0} \quad & E_t \quad \left[\int_t^{t+a^{exit}-a_t} e^{-\rho(u-t)-\int_t^u \eta(a_s)ds}\left(u(c_u)+\eta(a_u)u^B(q_u)\right)du\right] \\
s.t. \qquad & da_t \ = dt \\
& db_t \ = [r_t^f b_t + (1-\widetilde{f}_t)s_t]dt \\
& de_t \ = (\mu_t^e e_t + \widetilde{f}_t s_t)dt + \sigma_t^e e_t dW_t \\
& df_t \ = d\left(\sum_{i=1}^{N_t^f} F_t\right) \\
& dy_t \ = d\left(\sum_{i=1}^{N_t^y} \xi_i^y\right),
\end{aligned}
$$

where

$$
\widetilde{f}_t \ = \begin{cases} f_t & s_t \geqslant 0 \\ \frac{e_t}{e_t+b_t} & s_t < 0 \end{cases}
$$

and budget constraint and short selling constraint

$$
\begin{aligned}
c_t + s_t \ &= \begin{cases} w_t l(a_t, y_t) & a_t \leqslant a^{retire} \\ \bar{s} & a_t > a^{retire} \end{cases} \\
e_t, b_t \ &\geqslant 0.
\end{aligned}
$$

N_t^f is a Poisson counting process with intensity λ_t^f that is dependent on age a_t, labor productivity y_t

$$
\lambda_t^f = \begin{cases} \bar{\lambda}^f & a_t = a^{entry},\, y_t = y_h \\ +\infty & y_t = y_s,\, f_t = 0 \\ 0 & otherwise. \end{cases}
$$

N_t^y is a Poisson counting process with intensity λ_t^y that is dependent on pre-jump labor productivity y_{t-} and the aggregate state of the economy Z_t. The quarterly transition probabilities are shown below in Table B.6.

		$Z=0$		
i,j	y_l	y_h	y_s	stationary distribution
y_l	0.6	0.4	0	0.104930
y_h	0.05	0.948625	0.001375	0.839444
y_s	0	0.02075	0.97925	0.055626
y	0.107914	0.719424	5.805755	0.938189
		$Z=1$		
i,j	y_l	y_h	y_s	stationary distribution
y_l	0.3	0.7	0	0.031369
y_h	0.025	0.973625	0.001375	0.879255
y_s	0	0.02075	0.97925	0.089376
y	0.107914	0.719424	5.805755	1.005461

Table B.6: Transition Probabilities for Idiosyncratic Labor Productivity

Notes: This table demonstrates the transition matrices for idiosyncratic labor productivity. The top panel shows the transition probabilities during recession periods, and the bottom panel is for boom periods. In addition, the last column shows the stationary distributions among the three labor productivity states. All numbers are in quarterly frequency.

C Firm Problem

The capital structure and payout rules implemented by firms can be micro-founded by maximizing discounted log payouts.

Each individual firm solves

$$\max_{D_t, I_t, \omega_t} E_t \left[\int_t^{t+s} e^{-\bar{\rho}s} log(D_s) ds \right]$$

s.t.

$$dN_t = \left(\left[\frac{I_t}{K_t} - \Phi\left(\frac{I_t}{K_t}\right) - \delta(Z_t) \right] K_t - r_t^f B_t^f \right) dt + \sigma K_t dW_t \tag{C.1}$$

$$D_t = MPK_t K_t - I_t \tag{C.2}$$

$$\omega_t = \frac{K_t}{N_t} \tag{C.3}$$

$$K_t = N_t + B_t^f \tag{C.4}$$

$$\Phi(\frac{I_t}{K_t}) = \frac{1}{2}\phi(\frac{I_t}{K_t} - \delta(Z_t))^2. \tag{C.5}$$

Rewrite equation (C.1) with (C.2)-(C.5)

$$\begin{aligned} dN_t &= \Bigg((MPK_t - \delta(Z_t))[1 - \frac{1}{2}(MPK_t - \delta(Z_t))]\omega_t N_t - r_t^f \omega_t N_t \\ &+ [\phi(MPK_t - \delta(Z_t)) - 1]D_t - \frac{1}{2}\phi \frac{D_t^2}{\omega_t N_t} + r_t^f N_t \Bigg) dt + \sigma \omega_t N_t dW_t, \end{aligned}$$

or, for simplicity,

$$dN_t = \mu_t^e dt + \sigma_t^e dW_t.$$

Let X_t^{agg} be the collection of aggregate state variables (distribution replaced by generalized moments) except jump Z_t, and

$$dX_t^{agg} = \mu_t^{agg} dt + \sigma_t^{agg} dW_t$$

Conjecture the firm value function as

$$V(X_t^{agg}, Z_t, N_t) = \chi_0(X_t^{agg}, Z_t) + \chi_1 log(N_t) \tag{C.6}$$

The firm HJB is

$$\sup_{\omega_t, D_t} \mu_t^{agg} V_X + \mu_t^e V_N + \frac{1}{2} trace\left[\begin{bmatrix} \sigma_t^{agg} \\ \sigma_t^e \end{bmatrix} \begin{bmatrix} \sigma_t^{agg} & \sigma_t^e \end{bmatrix} Hess_{X,N} V \right]$$
$$-\bar{\rho} V + log(D_t) + \lambda_Z \left[V(X_t^{agg}, Z_t + \Delta Z, N_t) - V \right]$$

Notice that conjecture (C.6) implies that the last row and the last column of $Hess_{X,N}V$ are populated by 0's except the bottom right corner element V_{NN}. Therefore, HJB can be re-written as

$$\begin{aligned} \sup_{\omega_t, D_t} \mu_t^{agg} V_X + \mu_t^e V_N + \frac{1}{2} trace\left[\sigma_t^{agg} (\sigma_t^{agg})^\top Hess_X V \right] + \frac{1}{2} (\sigma_t^e)^2 Hess_N V \\ -\bar{\rho} V + log(D_t) + \lambda_Z \left[V(X_t^{agg}, Z_t + \Delta Z, N_t) - V \right] = 0 \end{aligned} \tag{C.7}$$

The implied first order conditions with respect to ω_t is

$$(MPK_t - \delta(Z_t))[1 - \frac{1}{2}\phi(MPK_t - \delta(Z_t))] - r_t^f + \frac{1}{2}\phi \frac{D_t^2}{\omega_t^2 N_t^2} - \sigma^2 \omega_t = 0$$

which means that payout is proportional to net worth:

$$D_t = \omega_t N_t \underbrace{\sqrt{\frac{2}{\phi}\left\{ r_t^f + \sigma^2 \omega_t - (MPK_t - \delta(Z_t))[1 - \frac{1}{2}\phi(MPK_t - \delta(Z_t))] \right\}}}_{x} \tag{C.8}$$

The first order condition with respect to D_t is

$$\chi_1[\phi(MPK_t - \delta(Z_t)) - 1]x - \chi_1 \phi x^2 + \frac{1}{\omega_t} = 0 \tag{C.9}$$

The roots to (C.9) are

$$\frac{[\phi(MPK_t - \delta(Z_t)) - 1] \pm \sqrt{[\phi(MPK_t - \delta(Z_t)) - 1]^2 + \frac{4\phi}{\chi_1 \omega_t}}}{2\phi}$$

Given that $x > 0$,

$$x = \frac{[\phi(MPK_t - \delta(Z_t)) - 1] + \sqrt{[\phi(MPK_t - \delta(Z_t)) - 1]^2 + \frac{4\phi}{\chi_1 \omega_t}}}{2\phi}. \tag{C.10}$$

Equating x implied by equations (C.8) and (C.10) yields

$$\begin{aligned} r_t^f =& \frac{1}{8\phi}\left\{[\phi(MPK_t - \delta(Z_t)) - 1] + \sqrt{[\phi(MPK_t - \delta(Z_t)) - 1]^2 + \frac{4\phi}{\chi_1 \omega_t}}\right\}^2 \\ &+ (MPK_t - \delta(Z_t))[1 - \frac{1}{2}\phi(MPK_t - \delta(Z_t))] - \sigma^2 \omega_t. \end{aligned} \tag{C.11}$$

Equation (C.11) also implies that ω_t does not depend on N_t. This, combined with D_t being proportional to N_t, mean that

$$\chi_1 = \frac{1}{\bar{\rho}}$$

in order for HJB condition (C.7) to be true for all N_t. Therefore,

$$\begin{aligned} r_t^f = & \frac{1}{8\phi}\left\{[\phi(MPK_t - \delta(Z_t)) - 1] + \sqrt{[\phi(MPK_t - \delta(Z_t)) - 1]^2 + \frac{4\phi\bar{\rho}}{\omega_t}}\right\}^2 \\ & + (MPK_t - \delta(Z_t))[1 - \frac{1}{2}\phi(MPK_t - \delta(Z_t))] - \sigma^2 \omega_t \end{aligned} \tag{C.12}$$

Substituting (C.12) into (C.8) gives

$$D_t = \frac{1}{2\phi}\omega_t N_t \left\{[\phi(MPK_t - \delta(Z_t)) - 1] + \sqrt{[\phi(MPK_t - \delta(Z_t)) - 1]^2 + \frac{4\phi\bar{\rho}}{\omega_t}}\right\} \tag{C.13}$$

Since a firm is a price taker, choice variables do not show up in μ_t^{agg} and the trace term in equation (C.7). Therefore, first order conditions for choice variables do not involve aggregate state variables (myopia). Plugging in $\chi_0(X_t^{agg}, Z_t)$ into HJB (C.7) along with optimal choices and condition (C.13) yields a system of two PDEs that do not involve N_t. Solving the system of PDEs pins down $\chi_0(X_t^{agg}, Z_t)$ but does not change the optimal decisions. One can verify sufficiency, following the same steps as in a Merton's problem with logarithmic utility.

D Target Date Glidepath

The target date glide path used in the counterfactual exercise is estimated using data from CRSP. Figure D.9 illustrates the estimated portfolio allocation based on remaining time to retirement.

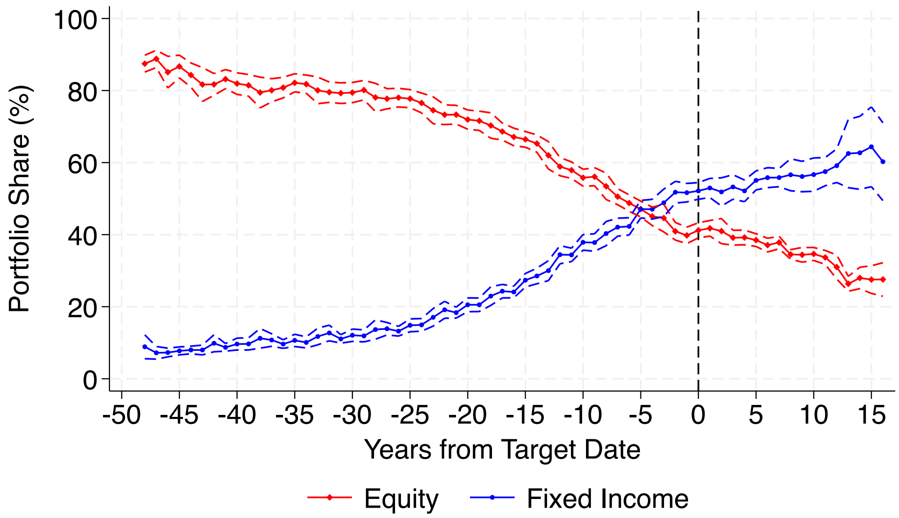

Figure D.9: Target Date Funds Glide Path

Notes: Center for Research in Security Prices (CRSP) Mutual Fund Database (CRSP, 2022). Annual fund summary file, 2006-2021. Target date funds are identified using Lipper class labels that lead with MAT.

E Computational Strategy

To overcome the curse of dimensionality associated with the overlapping generations model with aggregate risk and multiple assets, this section elaborates on the machine learning algorithm used in the paper.

The model solution has two components. In the first component, I reduce the dimensionality of the problem and approximate the distribution φ with sufficient statistics. The idea of replacing the distribution φ with some sufficient statistics of the distribution is familiar from Krusell and Smith (1998). Their paper uses the first moment of the distribution, its mean, to compute an equilibrium model in which approximate aggregation holds. In their setting, the first moment alone is enough to well approximate the rational expectations equilibrium. However, the OLG structure prevents an approximate aggregation (Krueger and Kubler, 2004). In the setting of this paper, the first moment is not sufficient to approximate the full information rational expectations equilibrium.

To find sufficient statistics of the distribution φ, I use a machine learning algorithm. The algorithm instructs the computer to summarize the distribution using

$$\widetilde{\varphi} = \int_{\mathcal{X}} \mathcal{G}(x)\varphi_t(x)dx, \tag{E.1}$$

where $\mathcal{G}$ is a basis function, and $\mathcal{X}$ is the state space of individual states, including age, bond holdings, equity holdings, flow allocation rule, and idiosyncratic labor productivity. In the case that $\mathcal{G}$ is a polynomial function, $\widetilde{\varphi}$ consists of standard moments (first, second, third moments, etc). $\mathcal{G}$ can also be more general than polynomials, hence the outputs are sometimes called generalized moments (Han, Yang and E, 2021). Several recent papers have demonstrated the power of finding sufficient statistics of distribution using neural networks (Kahou, Fernández-Villaverde, Perla and Sood, 2021, Han, Yang and E, 2021). I use a similar strategy but on an overlapping generations economy with heterogeneity within an age cohort, aggregate risk, and multiple assets.

The intuition for why replacing φ by generalized moments is sufficient to approximate the rational expectations equilibrium is that agents do not interact with each other but rather interact through the market. Thus, instead of focusing on how each individual matters for one another, keeping track of how each individual matters to the aggregate dynamics is sufficient. The interaction form in equation (E.1) is common in the mean-field literature. In the typical application in this literature, one generalized moment is sufficient. For the computation of the OLG model in this paper, the algorithm was given the capacity to choose two generalized moments. However, the two chosen generalized moments are identical to each other, indicating one generalized moment is sufficient.

In the second component of the model solution, reinforcement learning fits neural networks that parameterize the basis function $\mathcal{G}$ and the policy functions (Han, Yang and E, 2021). In this component, the computer simulates the model environment with a cross section of agents. An artificial intelligence (AI) lives in the simulated environment and maximizes realized lifetime utility along simulated life paths. In attempts to maximize utility, the AI learns the utility-maximizing policy functions and the correct generalized moments.

Algorithm Table 1 shows the pseudo code of the computational strategy. There are a total of two sets of two neural nets involved, with each set containing a policy neural net $\mathcal{C}$ and a basis function neural net $\mathcal{G}$. Depths, widths, and activation functions of the two sets of nets are identical. The first set will go through reinforcement training, whereas the second set is for storage purposes.

Algorithm 1: DeepHAM (Han, Yang and E, 2021) Adapted to an OLG Economy

Input : 1) initialized neural nets $\mathcal{C}^0$ and $\mathcal{G}^0$ for policy functions and basis function; 2) duplicates of the two neural nets $\mathcal{C}^{dup}$ and $\mathcal{G}^{dup}$

for $k = 1, 2, ..., N^k$ **do**
- simulate a panel of OLG agents (with replacement) for $T^B + T^E$ periods, using $\mathcal{C}^{k-1}$ and $\mathcal{G}^{k-1}$ (distributions of agents from $T^B + 1$ to $T^B + T^E$ represent the ergodic distribution of the economy, whereas the first T^B periods are burnouts)
- **for** $m = 1, 2, ..., N^m$ **do**
  - set $\mathcal{C}^{dup} = \mathcal{C}^{(k-1)N^m+m-1}$ and $\mathcal{G}^{dup} = \mathcal{G}^{(k-1)N^m+m-1}$
  - draw initial state variables of OLG agents $X_{\mathcal{I},0}$ from the ergodic distribution
  - initialize state variables of a single agent $X_{i,0}$ at age a^{entry}
  - **for** $t = a^{entry}, ..., a^{exit}$ **do**
    - update state variables $X_{\mathcal{I},t+1}$ using $\mathcal{C}^{dup}$ and $\mathcal{G}^{dup}$
    - update state variables $X_{i,t+1}$ using $\mathcal{C}^{(k-1)N^m+m-1}$ and $\mathcal{G}^{(k-1)N^m+m-1}$
    - collect realized utility for the single agent $u_{i,t}$
  - **end**
  - update neural nets to obtain $\mathcal{C}^{(k-1)N^m+m}$ and $\mathcal{G}^{(k-1)N^m+m}$, based on collected and discounted utility u_i for the single agent
- **end**

end

Output: trained policy functions and basis function $\mathcal{C}^{N^k \times N^m}$ and $\mathcal{G}^{N^k \times N^m}$

The training process involves two loops. The outer loop prepares the ergodic distribution of the economy, by simulating for a long enough period of time which includes burnouts (Judd, Maliar and Maliar, 2011). In the inner loop, the AI and the cross section of OLG agents update their separate sets of neural nets iteratively, in the spirit of fictitious play (Brown, 1951, Han and Hu, 2020, Hu, 2021, Han, Yang and E, 2021). In each play, the OLG agents use neural nets from the previous iteration, whereas the AI tries to figure out the best response to the OLG agents. Specifically, the inner loop initializes by drawing from the ergodic set, copies parameters from the first set of neural nets to the second set, and adjusts the first set of neural nets based on realized utility via stochastic gradient descent. The loss function is the realized discounted lifetime utility.

The purpose of the cross section of OLG agents is to provide the AI with the model environment from which the AI tries to learn. For this reason, the cross section of OLG agents and the individual AI use separate sets of neural nets to obtain a well defined loss function. Without doing so, general equilibrium prices would become manipulable to the AI who really should take prices as given instead. After training for N^m lifetimes, with each lifetime lasting from a^{entry} to a^{exit}, the algorithm falls back to the outer loop for a new ergodic set of individual states and repeats.

This adaptation deviates from the original DeepHAM algorithm by dropping the value function training. This decision is for theoretical and practical reasons. On the theoretical side, agents in the Han, Yang and E (2021) setup solve infinite-horizon problems, whereas the setup of this paper has a life cycle component. To obtain the value function at every age means the training must alternate between ages. Moreover, while training the value function recursively makes sense in an infinite-horizon setting, it is not obvious that their recursive definition can easily apply in the finite horizon case. Last but not least, analysis in this paper does not require the value function. On the practical side, training the value function slows down the algorithm and takes up memory. The OLG plus heterogeneity structure in this paper is heavily demanding in memory. Given the hardware constraints, training additional neural nets for the value function would come at the cost of less accurate simulations by decreasing the size of the cross section. For these theoretical and practical reasons, the adaptation removes the value function training.

A second deviation of this adaptation is that the inner loop takes place multiple times before re-simulating for a new ergodic set. In the original algorithm, Han, Yang and E (2021) obtain a new ergodic set after every neural net update. When the cross section is large, as in the setting of this paper which involves OLG and heterogeneity within each age, simulation becomes costly. This training scheme in the adaptation shortens the time spent on simulations.

E.1 Implementation and Accuracy Check

This section discusses the implementation details of the training process, which includes the convergence speed and accuracy.

The solve the continuous time model numerically, I use the standard Euler-Maruyama discretization method. Following Krusell and Smith (1998), I use a large number of agents to replace the continuum. The difference is that given there is a strong age dimension in the model, I simulate a large number of agents per age cohort, with a total of 20100 agents in one cross section. Every 80 episodes make up an epoch. Model is trained for 2400 epochs to reach convergence on an NVIDIA H100 graphic card with 80G memory. Learning rate is set at $1 \times e^{-4}$ and $5 \times e^{-5}$ respectively for the consumption neural net and for the basis function neural net.

The consumption neural net is configured with 256 and 128 neurons for the two hidden layers, both activated by sigmoid function. The output layer is also activated by sigmoid to constrain output between 0 and 1 (fraction of cash on hand consumed). The basis function is configured with 32 and 16 neurons, each activated with sigmoid function. The output contains two generalized moments. The two neural nets are initialized with Glorot Normal initialization.

Results are robust to increasing neural network sizes on both consumption neural net and basis function neural nets, alternative activation functions (in the sigmoid family), increasing or decreasing the number of generalized moments. Deeper nets and activation functions outside of the sigmoid function tend to destabilize the training process, often resulting in NaN traps.

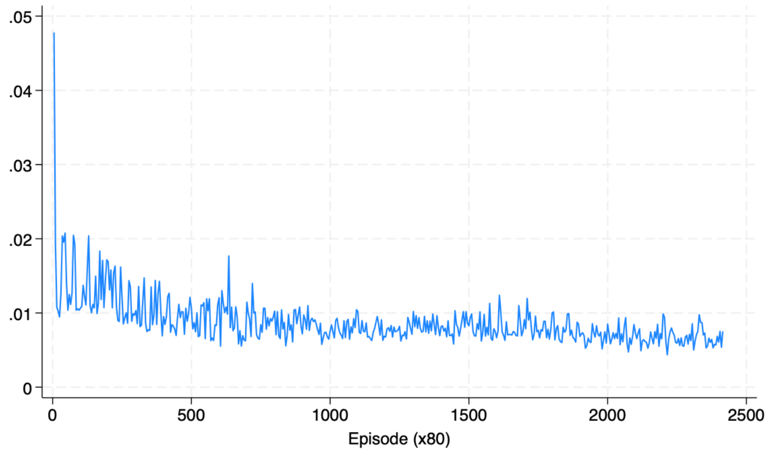

Figure E.10: Benchmark Economy - Average Relative Consumption Errors

Notes: Each epoch contains 80 episodes. The upper figure checks the relative consumption errors across all individual agents who are unconstrained. Unconstrained agents are those whose assets are 0.2 away from borrowing constraint. Results are similar if I define unconstrained agents as those who consume less than 96% of cash on hand.

After convergence, the neural nets start oscillating around the solution. To check for accuracy, I look at both the average relative consumption errors in the cross section. The advantage of using the relative Euler equation error is that this measure is invariant to the magnitude of consumption. The average relative consumption error across individuals is about 0.6% on the ergodic set of the economy, which means that the neural network determined consumption is on average 0.6% different from the Euler equation implied consumption, in line with previous literature that solves lifecycle models (Maliar, Maliar and Winant, 2021) or OLG economies but without heterogeneity within an age cohort (Krueger and Kubler, 2004, Hasanhodzic and Kotlikoff, 2013).