

# Stochastic Modelling of Seasonal Migration Using Rewriting Systems with Spatiality

Suryana Setiawan<sup>1,2</sup>(✉) and Antonio Cerone<sup>2</sup>(✉)

<sup>1</sup> Dipartimento di Informatica, Università di Pisa, Pisa, Italy  
setiawan@di.unipi.it

<sup>2</sup> UNU-IIST — International Institute for Software Technology, United Nations  
University, Macau SAR, China  
ceroneantonio@gmail.com, setiawan@iist.unu.edu

**Abstract.** Seasonal migration is the long-distance movement of a large number of animals belonging to one or more species that occurs on a seasonal basis. It is an important phenomenon that often has a major impact on one or more ecosystem(s). It is not fully understood how this population dynamics phenomenon emerges from the behaviours and interactions of a large number of animals. We propose an approach to the modelling of seasonal migration in which dynamics is stochastically modelled using rewriting systems, and spatiality is approximated by a grid of cells. We apply our approach to the migration of a wildebeest species in the Serengeti National Park, Tanzania. Our model relies on the observations that wildebeest migration is driven by the search for grazing areas and water resources, and animals tend to follow movements of other animals. Moreover, we assume the existence of dynamic guiding paths. These paths could either be representations of the individual or communal memory of wildebeests, or physical tracks marking the land. Movement is modelled by rewritings between adjacent cells, driven by the conditions in the origin and destination cells. As conditions we consider number of animals, grass availability, and dynamic paths. Paths are initialised with the patterns of movements observed in reality, but dynamically change depending on variation of movement caused by other conditions. This methodology has been implemented in a simulator that visualises grass availability as well as population movement.

## 1 Introduction

Computer scientists have taken inspiration from natural processes to build new computing paradigms/formalisms. Their motivation is either for solving general computing problems or helping natural scientists in modelling and analysing natural phenomena [1, 3, 10, 13, 15, 17–19, 21, 22]. In our work a formalism is being developed, the Grid Systems [2]. It is intended for modelling the dynamics of populations and their interactions with ecosystems. This formalism has taken inspiration from several existing formalisms, especially Cellular Automata [1] and Membrane P Systems [4–6, 16, 18].

In our previous work [2] the syntax and the semantics of the Grid Systems is elaborated and our approach for modelling the population growth of *Aedes albopictus* sp using the formalism is presented. In the model population dynamics were affected by external events: temperature fluctuation and rainfall. Behaviour was also modelled to vary spatially. The model was analysed by using a simulator that was developed based on the semantics of the Grid Systems and results were compared to real data.

In this paper a new feature for expressing the dynamic movement of the population around its habitat is introduced. This feature is expected to enable further analysis of the movement patterns, resulting from the changes in the ecosystem. To examine this feature the migration of wildebeests in Serengeti National Park, Tanzania, was used as our case study. The migration is massive since it involves about 1.2 million wildebeests together with hundreds of thousands of zebras, gazelles, impalas, and other herbivores. The route typically ranges about 1400 km from Ngorongoro crater in the south to Grumeti reserve in the west, then to Masai Mara reserve in the north and finally back to Ngorongoro, covering an area over 30,000 km<sup>2</sup>. The main variables affecting the movement of the wildebeest are grass availability and the dynamic pathways that are formed by geographical boundaries (rivers and hills) and the communal memorisation of the route.

In the following sections the Grid Systems, and *links*, the new feature, will be discussed. They will be followed by the case study, its model, its simulation results and a short discussion of the results.

## 2 Grid Systems

Grid Systems are a formalism for modelling the dynamics of ecosystems [2]. They consist of biotic and abiotic components defined as the objects of the system.

### 2.1 Reaction Rules

The behaviour of the system is defined by reaction rules that rewrite a given multiset of objects into a new multiset of objects. In reaction rule

$$\alpha \rightarrow \beta$$

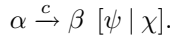
multiset  $\alpha$  represents the objects that are consumed, called *reactants*, and multiset  $\beta$  represents the objects that are produced, called *products*. Some objects, called *promoters*, may be required for the reaction to occur although they are not consumed when the reaction occurs; other objects, called *inhibitors*, may instead inhibit the reaction.

Reaction rule includes promoters  $\psi$  and inhibitors  $\chi$ :

$$\alpha \rightarrow \beta \ [\psi \mid \chi]$$

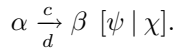
### 2.1.1 Reaction Rate and Duration

The frequency with which a reaction occurs is specified by a rate  $c$  as follows.



In this case the duration of the reaction is implicitly given by  $1/c$ . This supports the high-level modelling of natural processes whose duration is inversely proportional to the frequency of their termination. For example, at population level life expectancy is inversely proportional to natural death rate.

When modelling at lower-level, for instance at individual level, there may be short duration processes that overlap longer duration processes and cause their early termination. For example, death due to predation concludes a very fast predation process that causes the early termination of the life of an individual. In this case the duration of the predation process is independent of its frequency. It is therefore necessary to explicitly specify duration  $d$  of such processes as follows.



### 2.1.2 Object States

Once a reaction starts, reactants can no longer be used by any other reactions but can still act as promoters or inhibitors of other reactions. Therefore, a reactant cannot be removed until the reaction is completed; instead it changes its state from “available” to “committed” when the reaction starts.

## 2.2 Principles in Conducting the Reactions

The Grid System evolves through reactions. During the reactions the system obeys three principles that are also typical in nature: parallelism, stochasticity and spatiality.

### 2.2.1 Maximal Parallelism

Reactions must be applied immediately and maximally whenever the required reactants are available and the conditions related to the promoters and the inhibitors are satisfied. Reactions must also be performed in a parallel (simultaneous) manner.

### 2.2.2 Stochasticity

In Grid Systems distinct rules may require the same reactants at the same time. This non-determinism is resolved stochastically based on the propensity of each rule using Gillespie’s SSA [9].

Stochasticity is also manifested by varying the duration of the reactions. The duration is exponentially distributed with mean  $1/c$  when duration  $d$  is not specified. Otherwise, the duration is exponentially distributed with mean  $d$ . To

specify a rule having this stochastic property, mark  $M$  is placed after  $c$  for the former case,

$$\alpha \xrightarrow{c,M} \beta [\psi \mid \chi]$$

or after  $d$  for the later case, as written as

$$\alpha \xrightarrow[d,M]{c} \beta [\psi \mid \chi]$$

### 2.2.3 Spatiality

Grid Systems also consider the spatial dynamics of the objects that are distributed in the space. For instance, their behaviours (rules or parameters) might vary in different locations. A Grid system divides the space discretely into cells. Each cell is “associated” with a set of rules. Also, they define those rules which can only take the objects in the cell as the reactants. However, the objects from some other cells could be referenced as promoters or inhibitors in a reaction in that cell and the products can be placed in other cells. This referencing to other cells can be defined by any of two methods: relative addressing or absolute addressing. Relative addressing specifies the referenced cell’s address as the column-row distance from the cell which the rule is associated with. Absolute addressing specifies the referenced cell’s address as the referenced cell’s actual row-column numbers. The address of object  $a$  in a cell will be expressed as the subscript to  $a$ . To differentiate their notations the absolute ones are placed within squared brackets “[.,.]” as in  $a_{[3,4]}$ , and relative ones within curved brackets “(.,.)”, as in  $a_{(-1,1)}$ .

Grid Systems provide a method to address objects located in the “environment”, by subscripting respective objects with “[ $E$ ]”, as in  $a_{[E]}$ . As to the cells the rules can be associated with the environment.

## 2.3 Formal Definition of Grid Systems

**Definition 1.** A Grid System  $G(N, M, \Sigma, R, A, C^{(0)})$  is defined as follows:

- $G$  is the grid name;
- $N$  and  $M$  are two integers indicating that  $G$  has  $N \times M$  cells, also called local membranes;
- $\Sigma$  is the alphabet of object types;
- $R$  is a set of transition rules;
- $A$  is the set of associations of the rules with the membranes, i.e.

$$A = \{(\rho, \gamma) \mid \rho \in R, \gamma \in \{G_{i,j} \mid 0 \leq i < N, 0 \leq j < M\} \cup \{G_E\}\};$$

where

- $G_{i,j}$ , with  $0 \leq i < N$  and  $0 \leq j < M$ , denotes the cell in position  $(i, j)$ ;
- $G_E$  is the global membrane surrounding the cells;
- $C^{(0)}$  is the initial configuration of the grid.

**Definition 2.** A transition rule  $\rho : \alpha \xrightarrow[c]{d} \beta [\psi \mid \chi]$  is defined as follows:

- $\rho$  is the unique identifier of the rule;
- $\alpha$  is a non-empty multiset of reactants;
- $\beta$  is a multiset of products;
- $\psi$  is a multiset of promoters;
- $\chi$  is a multiset of inhibitors;
- $c \in \mathbb{R}^+$  is the rate with which the rule may be applied to perform a reaction,  $d \in \mathbb{R}^+$  is the duration of the reaction. When  $d$  is not specified, the duration will take  $1/c$ . When either  $c$  or  $d$  is marked by a ‘M’, it indicates that the duration time is an exponentially distributed random variable with parameter  $1/d$  (or with parameter  $c$ , when  $d$  is not specified).

### 3 Links

Living species have been given by nature the ability to sense and to follow the pathways for movements. Namely, wood ants can memorize snapshot views and landmarks [8], salmon fishes can sense geomagnetic fields [14], and sperm cells can sense chemotaxes to locate the ovum [12]. The pathways are either given by nature (as for salmon and sperm), created by themselves dynamically (as by ant), or a combination of both. Therefore, the model of the pathways might be more complex than just random walks (Brownian motion) as for chemical particles.

Pardini in his thesis reports the use of Spatial P Systems to imitate the movement of schooling fishes [16] which is based on the Boid flock model. This model determines the movement direction of each individual fish by a weighted-averaging computation from other fishes’ directions inside its viewing space. Such heavy computations are originally intended for computer graphics animation rather than for biological simulation [20].

By using the basic definition of the Grid Systems movement may need to be expressed as a large number of rules. Instead, an extended definition of objects, termed ‘links’, is introduced which enables the pathways being modelled to function as pointers.

#### 3.1 Basic Idea

A link is defined as a ‘special object’ that carries pointers, information which provides the address of a destination cell. The pointers can be used by rules in referring the objects in another cell. This is a third addressing method in addition to the relative addressing and the absolute addressing. Being used as an addressing method, different pointers carried by a link introduce another form of non-determinism into the system. In order to resolve this further non-determinism decision will be made stochastically based on the weighing of each pointer. Weights are real numbers between 0 and 1.0, and the total weights in the same cell is 1.0. Like objects, the number of links in a cell can be increased or decreased by applying its related rule.

### 3.2 Formal Definition of Links

#### 3.2.1 Links as Special Objects

Let  $G$  be a Grid System whose object set is  $\Sigma$ . Moreover,  $P$  is the set of links and  $P \subseteq \Sigma$ . An object  $p$  is a link, if  $p$  carries one pointer  $\eta$  or more. The notation  $p : \eta$  indicates that link  $p$  is carrying pointer  $\eta$ . When  $p$  carries more than one pointers in a cell, the pointers are weighted by  $w_1, w_2, \dots, w_k$  respectively, with  $0 \leq w_i \leq 1.0$  and  $\sum w_i = 1.0$ .

#### 3.2.2 Addressing by the Links

Pointer  $\eta$  can be expressed as either  $(dr, dc)$  or  $[r, c]$ . When  $p$  is the link existing in cell  $G_{r_0, c_0}$ , pointer  $[r_1, c_1]$  carried by  $p$  points to the cell  $G_{r_1, c_1}$ , and pointer  $(dr, dc)$  carried by  $p$  points to the cell  $G_{r_0+dr, c_0+dc}$ . Also, regarding link  $p$  in  $G_{r_0, c_0}$ , pointer  $[r_1, c_1]$  equals pointer  $(dr, dc)$  if and only if  $r_0 + dr = r_1$ , and,  $c_0 + dc = c_1$ .

Given that  $p : \eta$ , specifying  $a_p$  indicates object  $a$  in the cell pointed by  $\eta$ . For instance, if in  $G_{5,6}$  there exists link  $p : (2, -1)$  (a link with only one pointer), a rule containing  $a_p^m$  indicates that the number of  $a$  objects is  $m$  and they are located in  $G_{7,5}$ .

If link  $p$  has pointers  $\eta_1, \eta_2, \dots, \eta_k$  whose weights are  $w_1, w_2, \dots, w_k$  respectively with rule  $r$  specifies object  $A$  subscripted by link  $p$  as  $A_p$ , then in applying  $r$ ,  $\eta_i$  will be chosen randomly weighted by  $w_i$ .

#### 3.2.3 Rules for Changing the Weights

If in a cell link  $p^n$  has  $k$  distinct pointers labelled  $\eta_1, \eta_2, \dots, \eta_k$ , and weighings  $w_1, w_2, \dots, w_k$  respectively, then,

- adding  $p^m : \eta_i$  into that cell changes its overall weighing:
 
$$w'_j = \frac{nw_j}{(n+m)}, \text{ for } \eta_i \neq \eta_j, \text{ or}$$

$$w'_j = \frac{(nw_j+m)}{(n+m)}, \text{ for } \eta_i = \eta_j, \text{ or}$$

$$w'_i = \frac{m}{(n+m)}, \text{ for } \eta_i \notin \{\eta_1, \eta_2, \dots, \eta_k\} \text{ and } k > 0, \text{ or}$$

$$w'_i = 1, \text{ for } k = 0.$$
- adding  $p^m$  (without a pointer) into that cell will not affect the weights as they behave as ordinary objects.
- removing  $p^m : \eta_i$ , from that cell, where  $m \leq nw_i$ , changes its overall weighing:
 
$$w'_j = \frac{nw_j}{(n-m)}, \text{ for } \eta_i \neq \eta_j, \text{ or}$$

$$w'_j = \frac{(nw_j-m)}{(n-m)}, \text{ for } \eta_i = \eta_j.$$
- removing  $p^m$  (without a pointer) will not affect the weighing, as they behave as ordinary objects, except in the case of  $m = n$ , all pointers will be removed.
- When a rule performs both adding and removing, adding will be done before removing to maintain the weighing properly.

### 3.3 Links as Objects in the Rules

Let  $p^n$  be a link  $p$  having quantity  $n$  in the cell and  $r$  be a rule applied to that cell. When link  $p$  appears in  $r$  without a pointer, as in  $p^m$ , it will be handled as

an ordinary object. On the other hand, when it appears in a rule with a pointer  $\eta_i$ , as in  $p^m : \eta_i$ , it will be handled according to its role in the rule.

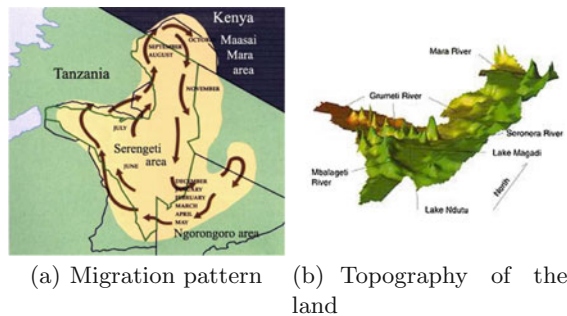
- As reactants, the rule can be applied when  $mw_i \leq n$  and the changes will follow Sect. 3.2.3.
- As promoters, the rule can be applied when  $mw_i \leq n$ .
- As inhibitors, the rule can not be applied when  $mw_i \geq n$ .
- As products, the rule will be applied and the number of link  $p$  will change accordingly and the changes will follow Sect. 3.2.3.

## 4 Experimental Works on Seasonal Migration

Many hypotheses have been proposed to describe the migration phenomenon. Boone et al. report that at least 16 explanations have been given for the cause or timing of the migration in Serengeti [7]. Furthermore, they have observed that the main reason driving the direction of the wildebeest migration is the search for a grazing rather than following the rainfall. By using evolutionary programming they approximated a proportion of 75 % to 25 % for the above reasons. By using dynamic model fitting, Holdo *et al.* report a different result, namely, an opposing rainfall and fertility gradient as the main reason for the migration [11]. They go on to conclude that the rainfall affects the availability of the grass. However, the conclusions of both studies focussed on grass availability and except in the latter, rainfall also played an additional role as the external factor affecting grass availability. The route of migration is likely related by the topographics of the area as shown in Fig. 1.

### 4.1 Pathways of Migration

In our work, the migration is modelled as the result of the animals finding a place for grazing and following existing pathways. The pathways are formed by their memorisation of the animals' previous movement and the initial pathways given from the beginning. The movement to the nearby area to locate grass is simply



**Fig. 1.** Serengeti National Park

performed after the quantity of the grass in the current area is reduced by their consumption. When there is insufficient grass or if there is a strong pathway, the animals move to follow the pathway. To protect themselves from predators they tend to group and therefore, they avoid to be alone in a quiet location. On the other hand, they avoid crowded locations to maximise their chances to access the grass. On each movement they leave more marks (augmenting the pathway) which others will follow. Grass root will continue to produce blades of grass until the quantity reaches the maximum that can be produced in that area. The strength of a pathway will decay according to a decay factor in each interval of time due to being destroyed by natural events or being forgotten by the animals.

## 4.2 Life Cycle

Ideally, there should be at least three dimensions for their state space in animal life cycle: age, health, and physical periods. For this experiment, we simplified them to be one dimension with 10 stages of strength/wellness from  $A_0, A_1, \dots, A_9$ . In every different stage animals will have its own behaviour parameters: death rates, feeding rates, birth rates. Pregnancy is limited only on stages  $A_6$  to  $A_9$ . New born baby will be at  $A_0$ . They will upgrade to one higher stage except at stage  $A_9$  when they gain food. Reversely, lacking the food will degrade to one lower stage except at the most left stage. The death rate will be higher to lower stages and the birth rate will be higher to higher stage. Their feeding rate will peak in  $A_6$  and  $A_7$ . After giving birth they drop their condition 3 stages.

## 4.3 Objects

- Wildebeests: The animals of each stage will be represented as objects  $A_0, A_1, \dots$ , and  $A_9$ , movable animals, or  $B_0, B_1, \dots$ , and  $B_9$ , in-digestion animals. As grass is being consumed,  $A\langle S \rangle$  will immediately become  $B\langle S \rangle$ , then revert back to  $A$  with higher stage,  $A\langle S + 1 \rangle$ , after digestion is complete. This differentiation is intended to avoid other rule applicability being affected by the grass which is already being consumed.
- Grass: Grass will be represented as a root  $R$  in a cell and its quantity produced in a cell as a number of  $G$ . To create a delayed effect, that forces the animal to move, the root will produce  $H$  first then becomes  $G$  after the delay. Moreover,  $R$  can still continuously produce  $H$ .
- Counters: To ease some rules in considering the number of animals in some cells, object counter  $C$  whose number represents the number of animals in the same cell. One  $C$  is created when a birth rule is applied. In the death rule,  $A$  will become  $Ax$  first avoiding object  $C$  as a reactant followed by the rule that removes a pair of  $Ax$  and  $C$ . Moreover, the number of  $C$  changes due to the movement. Such a mechanism is performed by creating  $Ax$  in its origin and creating  $C$  in its destination.
- Pathways: A pathway is represented by object *path* which is a link. It will decay geometrically of a certain rate as the time passes. Links are refreshed

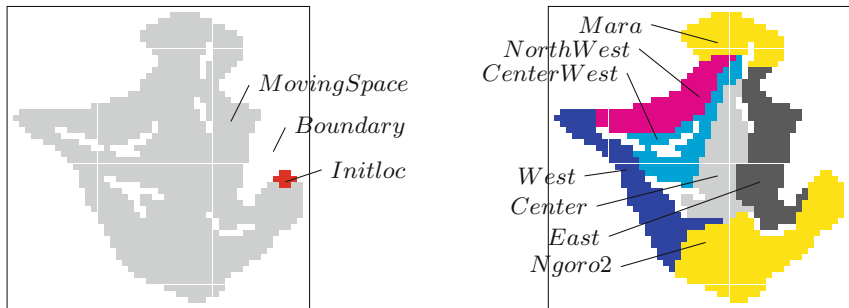


by the animal movements that create objects *path* in the cell. The number of objects *path* created by the movements varies depending on each movement's importance.

- Boundary objects: To limit the movement within an area, in each boundary cell a dummy object *Z* will be placed. Each movement rule will consider this object as the inhibitor in the destination cell.

#### 4.4 Regions

A region is defined as a set of cells identifying an area. Associating a region with a rule implies associating its cells with the rule. Serengeti and its surrounding area are defined as a  $50 \times 50$  grid of cells divided into *MovingSpace* region and *Boundary* region. *MovingSpace* is the grazing area. It is further divided into seven regions whose grass characteristics are different: *Ngoro2*, *West*, *Center*, *East*, *Mara*, *Northwest*, and *Centerwest*. They are characterized by the grass growth duration and the maximum grass quantity that can be produced by the land. *Initloc* is the region where the animals are initially placed. The regions are shown in Fig. 2.



(a) *Boundary* and *MovingSpace* are complementary; *Initloc* is a subset of *MovingSpace* where wildebeests were initially located in the simulation.

(b) The regions are defined as the subsets of *MovingSpace* whose different characteristics of their vegetation (grass).

**Fig. 2.** Regions defined over the cells.

#### 4.5 Reaction Rules

Firstly the rules were written and their parameters and initial objects were just roughly given. After running several combinations some adjustments were made. Insignificant rules were removed, whereas the ones representing important behaviours were modified by increasing their rates or lengthening/shortening their durations. Death/birth rates were also adjusted to approximate reasonable actual death/birth rates. The initial pathways were as little as possible to prevent animals from being trapped at the corners. Grass' growth parameters were

**Table 1.** Constants table for rules of life cycle

Stages	$\langle S \rangle$	0	1	2	3	4	5	6	7	8	9
Upstages	$ug\langle S \rangle$	A1	A2	A3	A4	A5	A6	A7	A8	A9	A9
Downstages	$dg\langle S \rangle$	A0	A0	A1	A2	A3	A4	A5	A6	A7	A8
Birthstages	$bs\langle S \rangle$	-	-	-	-	-	-	A3	A4	A6	A7
Birth rates	$br\langle S \rangle$	-	-	-	-	-	-	0.0017	0.0033	0.005	0.008
Death rates (nat.)	$dn\langle S \rangle$	0.05	0.035	0.021	0.015	0.011	0.008	0.006	0.004	0.003	0.002
Death rates (prey)	$dp\langle S \rangle$	0.5	0.35	0.21	0.15	0.11	0.07	0.005	0.003	0.03	0.015
Feeding rates	$fr\langle S \rangle$	1	2	4	6	8	10	15	20	16	0.1

**Table 2.** Constants table for grass growth

Region	$\langle R \rangle$	Ngorongoro	West	Center	East	Mara	Northwest	Centerwest
Max p. cell	$mg\langle R \rangle$	100	100	30	100	100	100	20
Duration est.	$gr\langle S \rangle$	1	1	15	1	1	1	15

set to assure right direction of migration. Ideally the time frame should be set up before defining parameters. However, the definition of “one year” was eventually justified based on the result instead. Then the migration cycle time length resulted from the simulation was taken as “one year” and finally the parameters were re-adjusted based on it. This method was repeatedly worked through many simulation runs until a reasonable behaviour was produced. The rules and parameters shown below illustrate one example that resulted in a stable migration cycles.

In specifying the rules the following constants are listed in Tables 1 and 2. Also, the rules refer to the following assignments.

$stages = \{0, 1, 2, \dots, 9\}$ ,  $directions = \{1, 2, \dots, 8\}$ ,  $declev = \{1, \dots, 10\}$ ,  $Mnp = 5$ ,  $Mng = 5$ , and  $\{(dr\langle x \rangle, dc\langle x \rangle) | x = 1..8\} = \{(-1, -1), (-1, 0), (-1, 1), (0, -1), (0, 1), (1, -1), (1, 0), (1, 1)\}$ .

Consuming the grass,  $A$  becomes  $B$ ,

$$\forall \langle S \rangle \in stages. \textbf{Feeding}\langle S \rangle : G \ A\langle S \rangle \xrightarrow[1/4, M]{fr\langle S \rangle} B\langle S \rangle$$

if  $Feeding\langle S \rangle \in assoc(MovingSpace)$ .

Food digesting is needed to create delay after the grass is removed and wellness level increases one level except at A9,

$$\forall \langle S \rangle \in stages. \textbf{Digesting}\langle S \rangle : B\langle S \rangle \xrightarrow{1, M} ug\langle S \rangle$$

if  $Digesting\langle S \rangle \in assoc(MovingSpace)$ .

Being unable to feed because of grass shortage, wellness level decreases one stage except at A0,

$$\forall \langle S \rangle \in stages. \textbf{Starving}\langle S \rangle : A\langle S \rangle \xrightarrow{1/2, M} dg\langle S \rangle \quad [ \ \lambda \quad | \quad G^{Mng/2} \ ]$$

if  $Starving\langle S \rangle \in assoc(MovingSpace)$

Time needed to balance the death propensities,

$$\forall \langle S \rangle \in \text{stages. } \mathbf{Resting}(\mathbf{S}) : A\langle S \rangle \xrightarrow[1/2, M]{1} A\langle S \rangle$$

if  $\text{Resting}\langle S \rangle \in \text{assoc}(\text{MovingSpace})$ .

Giving birth only at stages A6, A7, A8, A9 with different birth rates; after delivering a baby (at state A0) its wellness level decreases three levels,

$$\forall \langle S \rangle \in \text{stages. } \mathbf{Birth}(\mathbf{S}) : A\langle S \rangle \xrightarrow[5, M]{br\langle S \rangle} bs\langle S \rangle \text{ A0 } C$$

if  $\text{Birth}\langle S \rangle \in \text{assoc}(\text{MovingSpace})$ .

Mortality because of natural factors; more healthy having lower death rate,

$$\forall \langle S \rangle \in \text{stages. } \mathbf{DeathNat}(\mathbf{S}) : A\langle S \rangle \xrightarrow[0.5, M]{dn\langle S \rangle} Ax \quad [ \quad C^{Mnp} \quad | \quad \lambda \quad ]$$

if  $\text{DeathNat}\langle S \rangle \in \text{assoc}(\text{MovingSpace})$ .

Mortality because of predators and natural causes due to grazing in a quiet place; the rate is higher than the normal,

$$\forall \langle S \rangle \in \text{stages. } \mathbf{DeathPred}(\mathbf{S}) : A\langle S \rangle \xrightarrow[0.1, M]{dp\langle S \rangle} Ax \quad [ \quad \lambda \quad | \quad C^{Mnp} \quad ]$$

if  $\text{DeathPred}\langle S \rangle \in \text{assoc}(\text{MovingSpace})$ .

Decreasing counter  $C$  after being mortality,

$$\mathbf{DecCount} : Ax \text{ } C \xrightarrow{\infty} \lambda$$

if  $\text{DecCount} \in \text{assoc}(\text{MovingSpace})$ .

Initial growth of grass; root  $R$  produces one unit of grass  $H$  until maximum capacity that the land can produce is reached,

$$\forall \langle R \rangle \in \text{regions. } \mathbf{Grass}(\mathbf{R}) : R \xrightarrow[gr\langle R \rangle, M]{1} R \text{ } H \quad [ \quad \lambda \quad | \quad G^{mg\langle R \rangle} \quad ]$$

if  $\text{Grass}\langle R \rangle \in \text{assoc}(\text{reg}\langle R \rangle)$ .

Grass growing to be available for future grazing,

$$\mathbf{GrassReady} : H \xrightarrow[25, M]{1} G \text{ } \lambda$$

if  $\text{GrassReady} \in \text{assoc}(\text{MovingSpace})$ .

Movement along the path unless there is enough grass for grazing,

$$\forall \langle S \rangle \in \text{stages. } \mathbf{MoveByPath}(\mathbf{S}) : A\langle S \rangle \xrightarrow{10, M} Ax \text{ } A\langle S \rangle_{path} \text{ } C_{path} \text{ } path^{10} : path \quad [ \quad \lambda \quad | \quad Z_{path} \text{ } G^{7Mng} \quad ]$$

if  $\text{MoveByPath}\langle S \rangle \in \text{assoc}(\text{MovingSpace})$ .

Random movement to a place having a plenty of grass (3, 6, and 9 times minimum quantity),

$$\forall \langle G \rangle \in \{3, 6, 9\}, \forall \langle X \rangle \in \text{directions}, \forall \langle S \rangle \in \text{stages. } \mathbf{MoveToGrass}(\mathbf{G})\langle \mathbf{X} \rangle \langle \mathbf{S} \rangle : A\langle S \rangle \xrightarrow[1/2, M]{mr\langle G \rangle} Ax \text{ } A\langle S \rangle_{(dr\langle X \rangle, dc\langle X \rangle)} \text{ } C_{(dr\langle X \rangle, dc\langle X \rangle)} \text{ } path^{pn\langle G \rangle} : (dr\langle X \rangle, dc\langle X \rangle)$$

$$[ \quad G^{Mng.\langle G \rangle}_{(dr\langle X \rangle, dc\langle X \rangle)} \quad | \quad G^{Mng} \text{ } Z_{(dr\langle X \rangle, dc\langle X \rangle)} \quad ]$$

if  $\text{MoveToGrass}\langle G \rangle \langle X \rangle \langle S \rangle \in \text{assoc}(\text{MovingSpace})$ , and  $\{mr\langle x \rangle | x = 3, 6, 9\} = \{20, 25, 35\}$ , and  $\{pn\langle x \rangle | x = 3, 6, 9\} = \{10, 25, 50\}$ .

**Table 3.** Initial numbers of animals according to their levels

Level	A0	A1	A2	A3	A4	A5	A6	A7	A8	A9	Total
Number in a cell	10	10	15	25	30	50	60	75	50	25	350
Number in all cells	80	80	120	200	240	400	480	600	400	200	2800

Movement to a cell with a less dense grouping of animals due to overcrowding,  
 $\forall \langle X \rangle \in \text{directions}, \forall \langle S \rangle \in \text{stages}$ . **MoveToLessDens** $\langle X \rangle \langle S \rangle$ :

$$A \langle S \rangle \xrightarrow[1/2, M]{4} A \langle S \rangle_{(dr \langle X \rangle, dc \langle X \rangle)} \quad C_{(dr \langle X \rangle, dc \langle X \rangle)} \quad Ax \quad path^7 : (dr \langle X \rangle, dc \langle X \rangle)$$

$$[ \quad C_{(dr \langle X \rangle, dc \langle X \rangle)}^{10Mnp} \quad C_{(dr \langle X \rangle, dc \langle X \rangle)}^{Mnp} \quad | \quad C_{(dr \langle X \rangle, dc \langle X \rangle)}^{5Mnp} \quad Z_{(dr \langle X \rangle, dc \langle X \rangle)} \quad path \quad ]$$

if  $MoveToLessDens \langle X \rangle \langle S \rangle \in \text{assoc}(\text{MovingSpace})$ .

Movement to a cell with a more dense grouping of animals due to quietness,  
 $\forall \langle X \rangle \in \text{directions}, \forall \langle S \rangle \in \text{stages}$ . **MoveToMoreDense** $\langle X \rangle \langle S \rangle$ :

$$A \langle S \rangle \xrightarrow[1/2, M]{8} A \langle S \rangle_{(dr \langle X \rangle, dc \langle X \rangle)} \quad C_{(dr \langle X \rangle, dc \langle X \rangle)} \quad Ax \quad path^7 : (dr \langle X \rangle, dc \langle X \rangle)$$

$$[ \quad C_{(dr \langle X \rangle, dc \langle X \rangle)}^{2Mnp} \quad | \quad C_{(dr \langle X \rangle, dc \langle X \rangle)}^{Mnp} \quad Z_{(dr \langle X \rangle, dc \langle X \rangle)} \quad C_{(dr \langle X \rangle, dc \langle X \rangle)}^{10Mnp} \quad path \quad ]$$

if  $MoveToMoreDense \langle X \rangle \langle S \rangle \in \text{assoc}(\text{MovingSpace})$ .

Decaying of 10 % per an interval of time (7 time units),  
 $\forall \langle P \rangle \in \text{declev}$ . **Decay** $\langle P \rangle$ :

$$path^{\langle P \rangle} \xrightarrow[7, M]{1} path^{\langle P \rangle - 1} \quad [ \quad \lambda \quad | \quad path^{ip \langle P \rangle} \quad ]$$

if  $Decay \langle P \rangle \in \text{assoc}(\text{MovingSpace})$ , and  $\{ip \langle x \rangle | x = 1, \dots, 10\} = \{2, \dots, 9, 10, 0\}$ .

## 4.6 Initial Configuration

### 4.6.1 Initial Population Size and Location

A total population of 2800 wildebeests is initially placed evenly in the cells of *Initloc* region. They were distributed in proportion to their wellness/strength stage, is shown in Table 3.

### 4.6.2 Grass and Grass Root

The numbers were set according to the figures from previous migration. Each region has a different duration in producing grass available from grazing. Grass growth is bounded to a maximum quantity per cell which is assumed as being caused by the different condition of the soil and water in each region. The following are the initial quantities of grass *G* per region: *Ngorongoro* 50 (50 %), *West* 40 (40 %), *Northwest* 20 (20 %), *Mara* 20 (20 %), *East* 2 (20 %), *Center12* (40 %) and *Centerwest* 2 (10 %).

### 4.6.3 Initial Pathways

Initially some pathways were placed in some regions especially at the corners (*West*, *Mara* and *East*, to avoid being isolated at the corners), at turning

positions (*East* to the tail of *Ngoro2*), and in *Center* (to force them going downward). Visually the initial pathways are shown in Fig. 4(a).

4.7 Results: Migration Movement

A simulation was run producing several stable cycles of migration. Figure 3 shows each “month” in the second cycle. In the fourth cycle a small group of animals was trapped in the tail part of Ngorongoro. They were misled since the initial pathways were already overwritten by the new pathways created later. As more groups were being trapped, the size of the overall population was declined. By increasing the decay interval from 7 time units to 20 (to sustain the paths) there were no longer trapped groups until 7 cycles later.

The movement of frontiers were affected by the grass and movement of the followers were affected by the pathways. As the followers moved forward they become the next frontiers. Figure 4 shows the pathways in the beginning ( $t = 0$ ) and at about the end of second cycle ( $t = 17$ ).

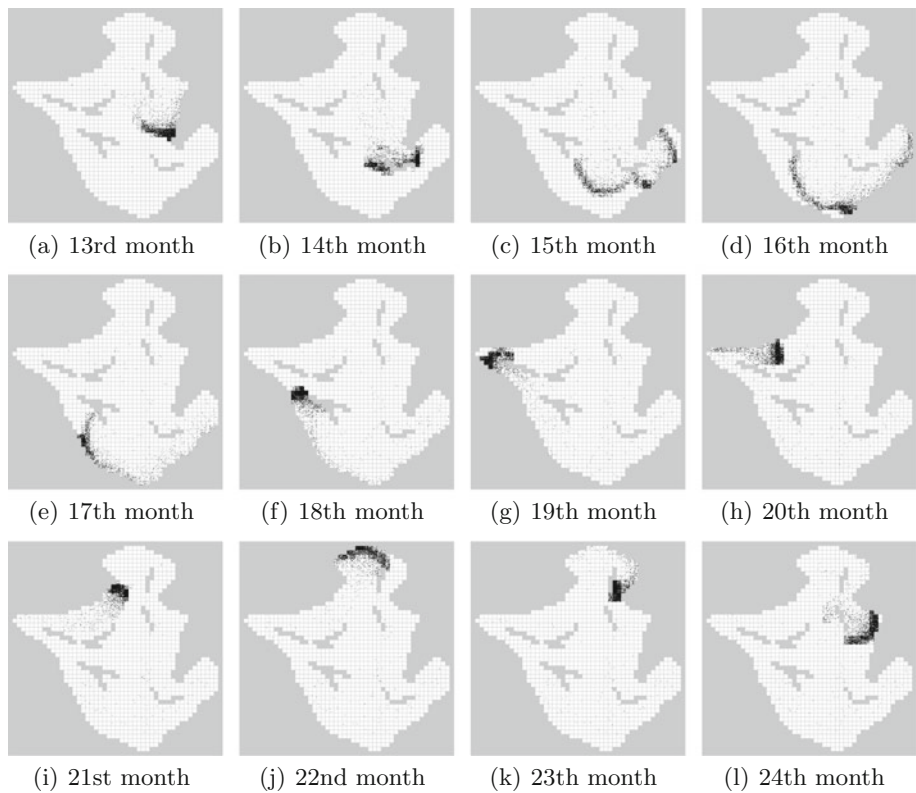
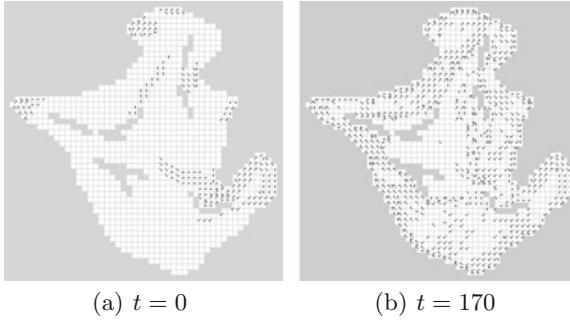


Fig. 3. “Monthly” sequence of migration at the second cycle since being started.



**Fig. 4.** Initial Pathways (at  $t = 0$ ) and pathways at  $t = 17$  (about the end of second cycle).

There are some interesting observations from the experiments by changing the rule parameters or adding/removing some other rules.

- When there was no initial pathway, animals were blocked by each other at the corner creating isolated groups. They stayed there until the grass regrew or they just died. Then, the population size declined faster. since in cells where isolated groups were located there was no more grass.
- When the initial pathways were given in a small quantity, similar isolations occurred after those initial pathways decayed. The population size also declined faster.
- When the grass characteristics were made equal for all regions, the animals divided into small groups moving without pattern in the area. Later some groups blocked each others when they met from opposite directions. As a result the overall population size declined.
- When the rate of rule MoveToPath was made exceeding the rate of rule MoveToGrass, migration sped up, the population spread along the migration route and the average of their wellness dropped, thus increasing death rate.
- When initial grass quantities were made much smaller and the migration had not been realized, the animals spread over the entire grid and the overall population size quickly declined.
- When the rate of random movement was set higher the animals were evenly spread throughout an area and there was no significant migration. Since the death rate for isolated animal was high the population size immediately declined.

#### 4.8 Results: Migration Pathways

The pathways created during the migration (at the end of second cycle) are shown in Fig. 4(b). They can be compared to the initial paths in Fig. 4(a). In each cell the animals passed through pathways were created in almost every direction. However, Fig. 4 shows only the pathway whose weight is greater than

0.30. Figure 4(b) shows that most initial paths were still there. After the fourth cycle most original pathways were overwritten by the new pathways.

## 5 Discussion

As described previously the model was simplified in some aspects compared to the real situation. The state space for life-cycle could be more complex than our ten-stage life cycle. The topology of the area was not fully represented. The parameters were not based on real-world measurements. The external events (rainfalls, temperature) and water availability which may affect the animal movements were not included. The resulting migration pattern has not been compared with actual migration patterns. However, this modelling effort was carried out with an experimental purpose rather than as a theoretical work of its biological domain. All parameters and behaviours expressed as the rules need to be further validated by the biologists. The main motivation of working on this case was to explore a new feature of the Grid Systems. As seen in the case its modelling required only expressing the behaviours in terms of rules and parameters. A working prototype simulator has been developed and used in our work. It was developed based on the semantics of the Grid Systems.

## 6 Conclusion and Future Work

A new feature of the Grid Systems is introduced. The new feature is the ‘link’ which is a special object that can carry pointers. The pointers carried by the links enable modelling the pathways for population movement in a more dynamic way. To illustrate the use of links a case was modelled and the model run using a simulator. The case study refers to a wildebeest population in Serengeti National Park, which performs seasonal migrations around the park area. The simulation of the model imitated the migration.

In our future work we aim at using real data for the same case or for others having similar problem characteristics.

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