Attributed Probabilistic P Systems and Their Application to the Modelling of Social Interactions in Primates

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Abstract. We propose a variant of probabilistic P Systems, Attributed Probabilistic P systems (APP systems), in which objects are annotated with attributes. We use APP systems for modelling social behaviours of some species of primates. In this context attributes can represent position of the animals in the environment, age of the animal, dominance level, aggressiveness, etc. As in standard P systems, the dynamics of the system is described by multiset rewrite rules that are applied in a maximally parallel way. Probabilities of rule application, in a maximal step, are computed according to weight functions associated to rules. As an application, we develop models to compare despotic and egalitarian behaviours on different species of primates.

1 Introduction

P systems [29] were introduced as distributed parallel computing devices inspired by the structure and the functioning of a living cell. A P system consists of a hierarchy of membranes, each of them containing a multiset of objects, representing molecules, a set of evolution rules, representing chemical reactions, and possibly other membranes. For each evolution rule there are two multisets of objects, describing the reactants and the products of the chemical reaction. Evolution rules can be applied more than once to different objects, with maximal parallelism, namely it cannot happen that some evolution rule is not applied when the objects needed for its triggering are available and not consumed by the application of any other rule.

Many variants of P systems exist that include features to increase their expressiveness or which are based on different evolution strategies [30]. In this paper we define a variant of P systems (Attributed Probabilistic P systems, APP systems) which includes features for the description of population dynamics. APP systems are actually the extension of Minimal Probabilistic P Systems [2] with the possibility of enriching objects with attributes. In the modelling of populations, such attributes can be used for describing characteristics of the individuals (position, age, etc.). APP systems include also the probabilistic choice

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of the rules to be applied in each maximally parallel step, and the possibility to include rule promoters to enable/disable rules in different phases of evolution.

We show how APP systems can be used for describing real ecological systems, such as self-organizing populations of animals. Such models have been successfully applied for understanding the behaviour of schools of fishes or flocks of birds [14,19]. Modelling social interactions in primates is an interesting research field which has been usually tackled by means of agent-based models [17,18,20,21,25,26,34]. Such models have been often criticized because, in many cases, they were so poorly documented that the models could not be evaluated. For these reasons protocols have been defined for creating a standard structure by which all the agent-based models could be documented [15,16]. Such protocols document a model by providing a check list of questions which must be answered by the authors of the model. For example, questions of that check list include: "Who (i.e. what entity) does what, and in what order?", "When are state variables updated?", "What kind of entities are in the model?", "By what state variables, or attributes, are those entities characterized?".

We present an APP system model of social interactions in primates. The model is composed of a few unambiguous rules. Such rules can be seen as an implementation of the rules used in agent-based models which, however, are usually programmed "ad hoc" and their effect needs to be documented separately. On the contrary, the rules of our APP-system-based model are almost self-explanatory, showing the ease of use of the formalisms for modelling real-world systems. Simulations of our model are also presented. We show that the obtained results are compatible with the results of agent-based models described in the literature.

From a theoretical point of view, the computational expressiveness of APP systems is the same as that of P systems, since in principle each attributed object could be replaced with a new symbol by embedding the values of the attributes in the symbol itself (thus having one symbol for each combination of attribute values). Nevertheless, the use of APP systems for modelling real systems provides important advantages, mainly with respect to readability of the models, and the compact and unambiguous descriptions which can be obtained.

As related work we mention studies in which formal notations are used to model and simulate population dynamics and ecosystems. In [32,33] a process algebra is proposed and used to model population dynamics by taking spatial distribution of the individuals into account. In [11] the Bio-PEPA process algebra is used to describe epidemiological problems, again by including a notion of spatiality. In [9] a variant of P systems is used to model the dynamics of some endangered species in the Pyrenees. In [23,24] the BlenX and LIME languages are used to compositionally construct models of ecosystems. In [5,6] Spatial P Systems are proposed and used to model the schooling behaviour of fish. In [12,31] formal notations are used to model and simulate the population dynamics of Solea solea in the Adriatic Sea by taking the effect of fishing into account.

The feature that mainly makes a difference between APP systems and other proposals is the use of maximal parallelism for the application of rules. This feature is particularly suitable for the modelling of populations that evolve by stages (e.g. reproductive stages or stages related with seasons). The usefulness of maximal parallelism in the context of ecosystems modelling is confirmed by the recent proposal of S-PALPS [36], an extension of PALPS that incorporates maximal parallelism by means of a synchronous parallel composition operator. The difference between S-PALPS and APP systems is in the modelling approaches, which are process algebraic and rewrite-based, respectively.

As regards the use of attributes, related works are [8,13,22]. The first proposes an extension of the π-calculus process algebra with attributed processes and attribute-dependent synchronization. The second defines Stochastic Concurrent Constraint Programming, in which the constraint programming constructs can be used to perform operations similar to those that can be done on attributes. The third proposes an extension of the Kappa language with annotations representing geometric information.

As regards spatial formalisms we mention also Spatial CLS [4] and Grid Systems [3,35]. In particular, the latter is a rich extension of P systems aimed at the modelling of ecosystems with a focus on population dynamics driven by properties of the environment and environmental events.

The paper is organized as follows: Sect. 2 introduces the Attributed Probabilistic P systems; Sect. 3 presents a model of social behaviour in primates; Sect. 4 shows the results of the experiments; Sect. 5 concludes the paper.

2 Attributed Probabilistic P Systems

We denote with $\{a_1, \ldots, a_n\}$ the set of objects $a_1, \ldots, a_n$, and with $\{\!|a_1, \ldots, a_n|\!\}$ the multiset of objects $a_1, \ldots, a_n$. Moreover, we denote with $|w|$ the size (number of elements) of the multiset w, and with $-$ and $+$ the difference and the union of multisets, respectively.

Definition 1 (**APP System**). *An* Attributed Probabilistic P system, *${}^aP$, is a tuple $\langle A, arity, D_{a_1}, \ldots, D_{a_n}, w_0, R\rangle$ where:*

- *A is an ordered finite alphabet of symbols, $\{a_1, \ldots, a_n\}$;*
- *$arity : A \to \mathbb{N}$ is a function which for each $a_i \in A$ gives the arity of D_{a_i};*
- *each D_{a_i} is a set of tuples, $D_{a_i} = I_1 \times \ldots \times I_{arity(a_i)}$, where each I_j is a (possibly infinite) set of unstructured values; the set D_{a_i} is called the set of attributes of a_i;*
- *w_0 is a multiset of values in $\Sigma = \{\langle a_i, d_i\rangle \mid a_i \in A,\ d_i \in D_{a_i}\}$ describing the initial state of the system, where Σ is called the set of* objects *of P. In the following we will write $w_0 \in \Sigma^*$.*
- *given a set of variables V, R is a finite set of evolution rules having the form*

$$u_{\mathrm{V}} \xrightarrow{f} v_{\mathrm{V}} \mid_{pr_{\mathrm{V}}}$$

where $u_{\mathrm{V}}, pr_{\mathrm{V}} \in \Sigma^*_{\mathrm{V}}$ are multisets (often denoted without brackets) of objects and variables denoting reactants *and* promoters, *respectively; $v_{\mathrm{V}} \in \Sigma^*_{\mathrm{EV}}$ is* *a multiset of objects and* expressions *with variables denoting* products*; and* $f : \Sigma^* \mapsto \mathbb{R}^{\geq 0}$ *is a* weight function*. Precisely:*

$$\Sigma_{\mathrm{V}} = \{(a_i, d_i) \mid a_i \in A, d_i \in D^{\mathrm{V}}_{a_i}\} \qquad \Sigma_{\mathrm{EV}} = \{(a_i, e_i) \mid a_i \in A, e_i \in E^{\mathrm{V}}_{a_i}\}$$

where $D^{\mathrm{V}}_{a_i} = (V \cup I_1) \times \ldots \times (V \cup I_{arity(a_i)})$*; and* $E^{\mathrm{V}}_{a_i} = Exp(V, I_1) \times \ldots \times Exp(V, I_{arity(a_i)})$*, with* $Exp(V, I)$ *denoting the set of well-typed expressions built from operators, variables* V*, and values of* I*. Moreover, we have* $Vars(v_{\mathrm{V}}) \subseteq Vars(u_{\mathrm{V}}) \cup Vars(pr_{\mathrm{V}})$*, where* $Vars(t)$ *denotes the set of variables occurring in* t*. Rules without variables are called* ground rules.

In what follows we will denote an (attributed) object $\langle a, d\rangle$ as $a_{(d)}$. A state (or configuration) of an APP system is a multiset of objects in Σ^*. By definition, the initial state is w_0, and we denote a generic state as w.

The evolution of an APP system is a sequence of probabilistic maximally parallel steps. We formally define the semantics of APP systems as a transition relation in the style of [7]. In each step a maximal multiset of evolution rule instances is selected and applied as described by the following semantic rules:

$$\text{(rule application)} \quad \frac{r_i = \ u \xrightarrow{k} v \in R \qquad u \subseteq w' \qquad K = \{\!| k' | u' \xrightarrow{k'} v' \in R,\ u' \subseteq w' |\!\} \qquad p = k / \sum_{k' \in K} k'}{(w', \overline{w}') \xrightarrow{r_i,\ p}_R (w' - u, \overline{w}' + v)}$$

$$\text{(single rule sequence)} \quad \frac{(w', \overline{w}') \xrightarrow{r_i,\ p}_R (w'', \overline{w}'')}{(w', \overline{w}') \xrightarrow{[r_i],\ p}{}^{+}_R (w'', \overline{w}'')}$$

$$\text{(multiple rules sequence)} \quad \frac{(w', \overline{w}') \xrightarrow{r_i,\ p_i}_R (w'', \overline{w}'') \qquad (w'', \overline{w}'') \xrightarrow{\overline{r},\ \overline{p}}{}^{+}_R (w''', \overline{w}''')}{(w', \overline{w}') \xrightarrow{\overline{r}@[r_i],\ p_i \cdot \overline{p}}{}^{+}_R (w''', \overline{w}''')}$$

$$\text{(step rule)} \quad \frac{(w, \emptyset) \xrightarrow{\overline{r},\ \overline{p}}{}^{+}_{R(w)} (w', \overline{w}') \qquad (w', \overline{w}') \not\rightarrow_{R(w)}}{w \xRightarrow{\overline{r},\ \overline{p}}_R w' + \overline{w}'}$$

where $[r_i]$ denotes the sequence composed of the single element r_i, and @ denotes the concatenation of sequences.

Given a system state, w, the (step rule) describes the evolution in a new state by the $\xRightarrow{\overline{r},\ \overline{p}}_R$ relation, where $\overline{p}$ is the probability of the transition, and $\overline{r}$ is the sequence of applied ground rules. (step rule) invokes $(w, \emptyset) \xrightarrow{\overline{r},\ \overline{p}}{}^{+}_{R(w)} (w', \overline{w}')$ where $R(w)$ is the set of applicable ground rules in the state w, with their weights, namely:

$$R(w) = \left\{ u_{\mathrm{V}}\sigma \xrightarrow{f(w)} v_{\mathrm{V}}\sigma \;\middle|\; u_{\mathrm{V}} \xrightarrow{f} v_{\mathrm{V}} \mid_{pr_{\mathrm{V}}} \in R,\ \exists \sigma.\, u_{\mathrm{V}}\sigma \subseteq w \wedge pr_{\mathrm{V}}\sigma \subseteq w \right\}$$

where (i) $\sigma : V \to flat(D_{a_1}) \cup \ldots \cup flat(D_{a_n})$, with $flat(D_{a_i}) = I_1 \cup \ldots \cup I_{arity(a_i)}$, for all $a_i \in A$; (ii) $u_{\mathrm{V}}\sigma$ ($u_{\mathrm{V}} \in \Sigma^*_{\mathrm{V}}$) is the well-typed multiset obtained by substituting values for variables in u_{V} according to σ; and (iii) $v_{\mathrm{V}}\sigma$ ($v_{\mathrm{V}} \in \Sigma^*_{\mathrm{EV}}$)

$^a\mathcal{P}_{\text{prot}} = (A, arity, D_p, D_{move}, D_{repr}, D_{aging}, w_0, R)$ where:

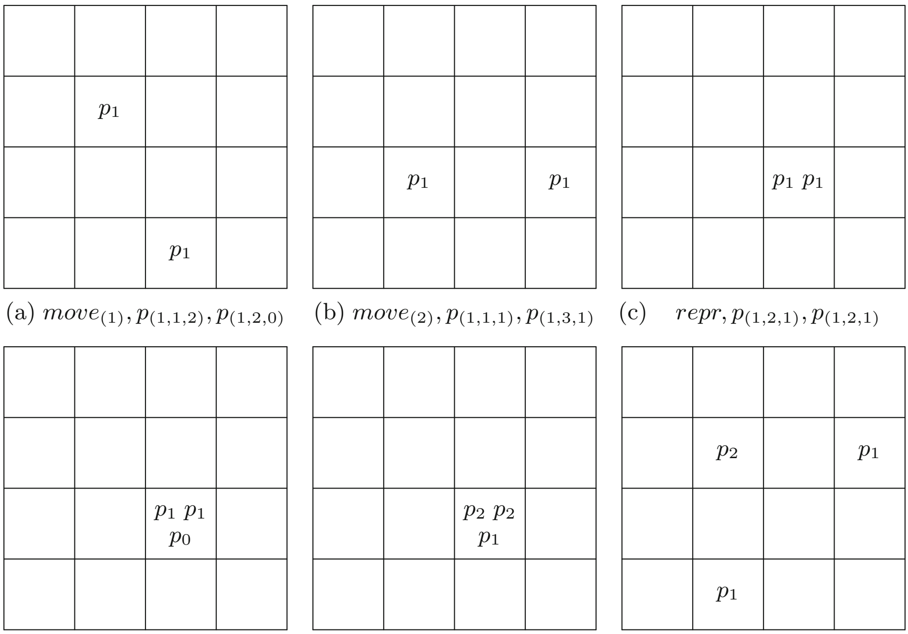
$A = \{p, move, repr, aging\}$ $\qquad arity = \{p \mapsto 3, move \mapsto 1, repr \mapsto 0, aging \mapsto 0\}$

$D_p = \{0,1,2\} \times \{0,1,2,3\} \times \{0,1,2,3\};$

$D_{move} = \{1,2\};\ D_{move2} = D_{repr} = D_{aging} = \emptyset \qquad w_0 = \{\!|move_{(1)},\ p_{(1,1,2)},\ p_{(1,2,0)}|\!\}$

$$R = \left\{ \begin{array}{ll}
r_1 :\ p_{(a,x,y)} \xrightarrow{1} p_{(a,x+1,y)} \ |_{move_{(n)}} & \text{if } x < 3 \\
r_2 :\ p_{(a,x,y)} \xrightarrow{1} p_{(a,x,y+1)} \ |_{move_{(n)}} & \text{if } y < 3 \\
r_3 :\ p_{(a,x,y)} \xrightarrow{1} p_{(a,x-1,y)} \ |_{move_{(n)}} & \text{if } x > 0 \\
\vdots & \\
r_8 :\ p_{(a,x,y)} \xrightarrow{1} p_{(a,x-1,y-1)} \ |_{move_{(n)}} & \text{if } x > 0 \ \wedge y > 0 \\
r_9 :\ p_{(a_1,x,y)}, p_{(a_2,x,y)} \xrightarrow{1} p_{(a_1,x,y)}, p_{(a_2,x,y)}, p_{(0,x,y)} \ |_{repr} & \\
r_{10} : p_{(a,x,y)} \xrightarrow{1} p_{(a+1,x,y)} \ |_{aging} & \text{if } a < 2 \\
r_{11} : p_{(2,x,y)} \xrightarrow{1} |_{aging} & \\
r_{12} : move_{(1)} \xrightarrow{1} move_{(2)} & r_{13} : move_{(2)} \xrightarrow{1} repr \\
r_{14} : repr \xrightarrow{1} aging & r_{15} : aging \xrightarrow{1} move_{(1)}
\end{array} \right\}$$

Fig. 1. APP system modelling protozoans.



(a) $move_{(1)}, p_{(1,1,2)}, p_{(1,2,0)}$ (b) $move_{(2)}, p_{(1,1,1)}, p_{(1,3,1)}$ (c) $repr, p_{(1,2,1)}, p_{(1,2,1)}$

(d) $aging, p_{(1,2,1)}, p_{(1,2,1)}, p_{(0,2,1)}$ (e) $move_{(1)}, p_{(2,2,1)}, p_{(2,2,1)}, p_{(1,2,1)}$ (f) $move_{(2)}, p_{(1,1,0)}, p_{(1,3,2)}, p_{(2,1,2)}$

Fig. 2. Example of the evolution of the protozoans model, with (a)–(f) representing a possible sequence of states reached by the system. The caption of each figure contains the complete multiset of objects present in the system. (For compactness, only the age attributes are shown for the objects in the figures, that is, $p_{(a,x,y)}$ is depicted as p_a.)

is the well-typed multiset obtained by evaluating the expressions in v_{V} under the substitution σ. Transition relation $\xrightarrow{\overline{r},\ \overline{p}}{}^{+}_{R}$ is the transitive closure of $\xrightarrow{r,\ p}_{R}$.

A transition $(w', \overline{w}') \xrightarrow{r_i,\ p}_{R} (w' - u, \overline{w}' + v)$ corresponds to the application of a single rule. When a rule is selected, its application consists in removing its reactants from w' and adding its products to $\overline{w}'$. The $\overline{w}'$ multiset will collect all products of all applied rules. Note that $R(w)$ takes into account that each rule is applied with respect to the weights of the rules computed in the initial state w. Moreover, $R(w)$ contains only the ground rules the promoters of which are present in the initial state w ($pr_{\mathrm{V}}\sigma \subseteq w$). Once objects in w' are such that no further rule in $R(w)$ can be applied to them, by (step rule) the new system state is $w' + \overline{w}'$ (where w' are the unused objects and $\overline{w}'$ are the new products).

Intuitively, the semantic definition states that all the rules to be applied are selected in a probabilistic way from the set of applicable rules, their reactant are removed for the available reactants, w', and their product are added to a suspended multiset $\overline{w}'$. When no further rule can be applied to w' the new state, which is composed be the unused objects in w' plus the suspended products in $\overline{w}'$, is produced. Finally we give the probability of a transition between two states by means of the following rule:

$$\text{(state transition prob.)} \quad \frac{PR = \{(\overline{r}, \overline{p}) \mid w \overset{\overline{r},\ \overline{p}}{\Longrightarrow}_{R} w'\} \qquad p \ = \sum_{(\overline{r}, \overline{p}) \in PR} \overline{p}}{w \overset{p}{\Longrightarrow}_{R} w'}$$

Example 1. We consider a population of sexually reproducing protozoans in which two individuals are necessary for producing an offspring. Protozoans are free ranging on a laboratory Petri dish. The Petri dish is abstracted by a $n \times n$ grid and protozoans can move one step at a time on the grid. They can reproduce if they meet in the same entry of the grid. They have a finite lifespan, at the end of which they die. Each individual is represented by an attributed object $p_{(a,x,y)}$ in which p stands for “protozoan”, attribute a is an integer representing the age of the individual, and attributes x and y are the coordinates of the position of the individual on the Petri dish. The evolution cycles among three phases: a *movement* phase, a *reproduction* phase, and an *aging* phase. Each phase is represented by a different symbol, namely *move*, *repr*, and *aging*, respectively, which are used as promoters to enable different sets of rules for each phase. The movement phase has a duration of two maximally-parallel steps, hence the *move* symbol has an attribute taking values from the set $\{1, 2\}$ to allow modelling it.

For the sake of simplicity we assume a 4×4 grid and a lifespan of 3 age units. We consider an initial configuration in $move_{(1)}$ phase with two individuals, of age 1, in positions $(1, 2)$ and $(2, 0)$, respectively. The APP system is shown in Fig. 1. Rules r_1–r_8 model the movement phase, with a different rule for each possible direction of movement: east for rule r_1, north for rule r_2, and so on, also allowing diagonal movement as exemplified by rule r_8. Rule r_9 handles the reproduction phase, while rules r_{10}, r_{11} model the aging phase. Finally, rules r_{12}–r_{15} are used to switch phases. Note that all the weights associated with the rules are constant and equal to 1, thus for each phase all (and only) the rules specific for that phase can be applied, and such rules are equiprobable. An example of evolution of the system is shown in Fig. 2.

3 Modelling Social Interactions in Primates

In this model, we describe the behaviour of male monkeys and how it changes when a female monkey enters the oestrum. The model is inspired by the social behaviours of species of prosimians as described in [10,27,28]. The population is dispersed in an environment, which is modelled as a continuous 2D space, hence each individual is associated with coordinates (x, y). Male and female monkeys are represented by symbols MMonkey and FMonkey, respectively, and both have attributes in the domain $\mathbb{R}^2 \times \mathbb{N}$. Beyond the actual position, we also keep track of the *dominance* level of each individual, which is used to derive the likeliness of a individual to win (or just engage in) a fight against another individual.

At the beginning, the population is composed only of male monkeys having a dominance level of 1500. All the male monkeys alternate between two phases: a *movement* phase, represented by the special symbol MOV, in which they wander around slowly and move towards other individuals in order to keep the population compact; and a *fight* phase, represented by FGT, in which they chase other individuals to fight, yielding to variations in their levels of dominance. Females alternate between a *normal* phase, denoted by the symbol NORMAL, and an *oestrum* phase, denoted by OEST. In this model, for simplicity, there is only one female monkey, which is explicitly represented only during the oestrum phase. In other words, the individual FMonkey appears only at the beginning of the oestrum phase, and is removed from the model at the end of the phase.

Formally, the model is composed of 6 symbols $A = \{\mathrm{MMonkey}, \mathrm{FMonkey}, \mathrm{MOV}, \mathrm{FGT}, \mathrm{NORMAL}, \mathrm{OEST}\}$, having a corresponding set of attributes defined as $D = \{(\mathbb{R}^2 \times \mathbb{N}), (\mathbb{R}^2 \times \mathbb{N}), \mathbb{N}, \mathbb{N}, \mathbb{N}, \mathbb{N}\}$. As regards symbols $\mathrm{MOV}, \mathrm{FGT}, \mathrm{NORMAL}, \mathrm{OEST}$, an attribute from $\mathbb{N}$ is associated with each of them, denoting the length of those phases in terms of the steps taken by the Attributed P system. The initial state of the system is the following:

$$\begin{aligned} w_0 = \{&\mathrm{MMonkey}_{(x_1,y_1,1500)}, \mathrm{MMonkey}_{(x_2,y_2,1500)}, \mathrm{MMonkey}_{(x_3,y_3,1500)},\\ &\mathrm{MMonkey}_{(x_4,y_4,1500)}, \mathrm{MMonkey}_{(x_5,y_5,1500)}, \mathrm{MMonkey}_{(x_6,y_6,1500)},\\ &\mathrm{MMonkey}_{(x_7,y_7,1500)}, \mathrm{MMonkey}_{(x_8,y_8,1500)}, \mathrm{MOV}_{(1)}, \mathrm{NORMAL}_{(Snl)}\} \end{aligned}$$

where the positions of the individuals $(x_i, y_i) \in \mathbb{R}^2$ are randomly generated within a square area of $cage \in \mathbb{R}$ side length. Parameter Snl denotes the duration of the “normal” phase for females.

The evolution rules of the model are as follows.

$$r_1 : \mathrm{MOV}_{(n)} \xrightarrow{1} \mathrm{MOV}_{(n-1)} \qquad \forall n > 1$$
$$r_2 : \mathrm{MOV}_{(1)} \xrightarrow{1} \mathrm{FGT}_{(Nfl)}|_{\mathrm{NORMAL}_{(n)}}$$
$$r_3 : \mathrm{MOV}_{(1)} \xrightarrow{1} \mathrm{FGT}_{(Ofl)}|_{\mathrm{OEST}_{(n)}}$$
$$r_4 : \mathrm{FGT}_{(n)} \xrightarrow{1} \mathrm{FGT}_{(n-1)} \qquad \forall n > 1$$
$$r_5 : \mathrm{FGT}_{(1)} \xrightarrow{1} \mathrm{MOV}_{(Msn)}$$

Rules r_1–r_5 model the alternation between "movement" and "fight" phases for males. For both MOV and FGT, their attributes decrease to keep track of the number of steps passed, until they reach 1 and a phase switch occurs. In particular, in the switch from MOV and FGT, the number of steps that the fight phase lasts depend on the phase of the female; namely it is either Nfl or Ofl, if the female is currently either in the normal phase (NORMAL) or the oestrum phase (OEST), respectively. The movement phase lasts Msn steps.

$$r_6 : \mathrm{NORMAL}_{(n)} \xrightarrow{1} \mathrm{NORMAL}_{(n-1)} \qquad \forall n > 1$$
$$r_7 : \mathrm{NORMAL}_{(1)} \xrightarrow{1}$$
$$\mathrm{OEST}_{(Sol)}, \mathrm{FMonkey}_{(FemaleInitX, FemaleInitY, FemaleInitDom)}$$
$$r_8 : \mathrm{OEST}_{(n)} \xrightarrow{1} \mathrm{OEST}_{(n-1)} \qquad \forall n > 1$$
$$r_9 : \mathrm{OEST}_{(1)}, \mathrm{FMonkey}_{(x,y,dom)} \xrightarrow{1} \mathrm{NORMAL}_{(Snl)}$$

Rules r_6–r_9 model the alternation between "normal" and "oestrum" phases for the female monkey. The durations of the oestrum phase is modelled by the parameter Sol. The initial coordinates of the female monkey are denoted by the parameters $FemaleInitX$ and $FemaleInitY$, while $FemaleInitDom$ denotes its initial dominance level.

$$r_{10} : \mathrm{MMonkey}_{(x',y',dom')} \xrightarrow{f_{10}} \mathrm{MMonkey}_{(\mathrm{move}(x',x'',SMMA),\mathrm{move}(y',y'',SMMA),dom')}$$
$$|_{\mathrm{MOV}_{(n)},\mathrm{NORMAL}_{(m)},\mathrm{MMonkey}_{(x'',y'',dom'')}}$$
$$r_{11} : \mathrm{MMonkey}_{(x',y',dom')} \xrightarrow{f_{11}} \mathrm{MMonkey}_{(\mathrm{move}(x',x'',SMMA),\mathrm{move}(y',y'',SMMA),dom')}$$
$$|_{\mathrm{MOV}_{(n)},\mathrm{OEST}_{(m)},\mathrm{MMonkey}_{(x'',y'',dom'')}}$$

Rules r_{10}–r_{11} handle the movement of males during either the "normal" or "oestrum" phase for the female. In both cases, a male (in a position x', y') is allowed to move towards any other male (in position x'', y''). The resulting position of the male which moves is computed as $(\mathrm{move}(x', x'', SMMA), \mathrm{move}(y', y'', SMMA))$, where move is a function to move the coordinates (x', y') towards coordinates (x'', y'') with a given speed factor described by the parameter $SMMA$ (Speed Male-Male approach). Formally, this function is defined as:

$$\mathrm{move}(a, b, \gamma) = a + (b - a)/\gamma$$

The most important difference between rules r_{10} and r_{11} lies in the weight functions, which are defined as:

$$f_{10} = \begin{cases} 1 & \text{if } SD/NT < dist((x',y'),(x'',y'')) < SD; \\ 0 & \text{otherwise;} \end{cases}$$

$$f_{11} = \begin{cases} 1 & \text{if } SD/OT < dist((x',y'),(x'',y'')) < SD; \\ 0 & \text{otherwise.} \end{cases}$$

where $dist((x',y'),(x'',y''))$ is a function giving the euclidean distance between two points, parameter SD (Spot Distance) denotes the maximum visibility distance of a monkey, and parameters NT (Normal Tolerance) and OT (Oestrum Tolerance) are used to derive the minimum distance allowed between two monkeys to enable the relative movement of one towards the other. In particular, such a minimum distance depends on the phase of the female, and it is either SD/NT during the NORMAL phase, and SD/OT during the OEST phase. In this manner, during the oestrum phase, males are allowed to come closer one another, hence increasing the possibility to engage in a fight.

$$r_{12} : \text{MMonkey}_{(x',y',dom')} \xrightarrow{f_{12}} \text{MMonkey}_{(\text{move}(x',x'',SMFA),\text{move}(y',y'',SMFA),dom')} \\ |_{\text{MOV}_{(n)},\text{FMonkey}_{(x'',y'',dom'')}}$$

Rule r_{12} models the movement of a male monkey towards the female. In this case, the speed of the male is denoted by the parameter $SMFA$. The corresponding weight function is:

$$f_{12} = \begin{cases} PfF & \text{if } dist((x',y'),(x'',y'')) < SD \text{ and } dom' + FT > dom''; \\ 0 & \text{otherwise;} \end{cases}$$

which enables the movement only if both (i) their relative distance is less than SD, and (ii) the dominance level of the male, plus a tolerance value FT (Female Tolerance), is greater than that of the female. The actual weight used is denoted by the parameter PfF (Preference for Female).

$$r_{13} : \text{MMonkey}_{(x',y',dom')}, \text{MMonkey}_{(x'',y'',dom'')} \xrightarrow{elo\_rating(dom',dom'')} \\ \text{MMonkey}_{(\text{chase}(x',x'',CSN),\text{chase}(y',y'',CSN),elow(dom',dom''))}, \\ \text{MMonkey}_{(\text{flee}(x',x'',FSN),\text{flee}(y',y'',FSN),elol(dom'',dom'))} |_{\text{FGT}_{(n)},\text{NORMAL}_{(m)}}$$

with $dom' \geq dom''$, $dist((x',y'),(x'',y'')) \leq NAD$, and $dom' - dom'' \leq AN$, where NAD (Normal Aggression Distance) and AN (Avoidance Normal) and model parameters representing the minimum distance and the maximum difference in dominance that enable an aggression when the female is in normal condition. In this rule the monkey in position (x',y') has a dominance that is higher or equal to that of the other monkey. The probability that the first monkey wins the fight is given by the standard Elo rating method, originally defined for applications to games, and then used for the modelling of social interactions. Such a method is based on a table that gives the probability of success in a fight depending on the difference of rating (or dominance) of the involved individual. The Elo rating table we consider for this model is in [1]. The function $elo\_rating(\Delta dom)$ looks in the table and gives as result the probability of the victory of the stronger monkey over the weaker one.

Function chase gives the new position of the winner of the fight; function flee gives the new position of the looser of the fight; *elow* gives the new dominance of the winner of the fight following the Elo rating table and method; *elol* the new dominance of the looser of the fight. These functions are defined as follows:

$$\text{chase}(a, b, \rho) = a + \rho \cdot (b - a) \qquad \text{flee}(a, b, \rho) = b + \rho \cdot (b - a)$$

$$elow(d', d'') = d' + \begin{cases} (1 - elo\_rating(\Delta dom)) \cdot stepness & \text{if } d' > d'' \\ elo\_rating(\Delta dom) \cdot stepness & \text{if } d' < d'' \end{cases}$$

$$elol(d', d'') = d' - \begin{cases} elo\_rating(\Delta dom) \cdot stepness & \text{if } d' > d'' \\ 1 - (elo\_rating(\Delta dom)) \cdot stepness & \text{if } d' < d'' \end{cases}$$

where $\Delta dom = |d' - d''|$, and *stepness* is a parameter representing the maximum increase/decrease of dominance. The parameters CSN(Chase Speed Normal) and FSN (Flee Speed Normal) used in rule r_{13} describe how fast the monkeys move.

$$r_{14} : \text{MMonkey}_{(x',y',dom')}, \text{MMonkey}_{(x'',y'',dom'')} \xrightarrow{1-elo\_rating(dom',dom'')}$$
$$\text{MMonkey}_{(\text{flee}(x',x'',FSN),\text{flee}(y',y'',FSN),elol(dom',dom''))},$$
$$\text{MMonkey}_{(\text{chase}(x',x'',CSN),\text{chase}(y',y'',CSN),elow(dom'',dom'))}|\text{FGT}_{(n)}\ \text{NORMAL}_{(m)}$$

with $dom' > dom''$, $dist((x', y'), (x'', y'')) \leq NAD$, and $|dom' - dom''| \leq AN$.

Rule r_{14} is analogous to rule r_{13}, but describes the case in which the winner is the weaker monkey.

$$r_{15} : \text{MMonkey}_{(x',y',dom')}, \text{MMonkey}_{(x'',y'',dom'')} \xrightarrow{elo\_rating(dom',dom'')}$$
$$\text{MMonkey}_{(\text{chase}(x',x'',CSO),\text{chase}(y',y'',CSO),elow(dom',dom''))},$$
$$\text{MMonkey}_{(\text{flee}(x',x'',FSO),\text{flee}(y',y'',FSO),elol(dom'',dom'))}|\text{FGT}_{(n)},\text{OEST}_{(m)}$$

with $dom' \geq dom''$, $dist((x', y'), (x'', y'')) \leq OAD$, and $dom' - dom'' \leq AO$.

$$r_{16} : \text{MMonkey}_{(x',y',dom')}, \text{MMonkey}_{(x'',y'',dom'')} \xrightarrow{1-elo\_rating(dom',dom'')}$$
$$\text{MMonkey}_{(\text{flee}(x',x'',FSO),\text{flee}(y',y'',FSO),elol(dom',dom''))},$$
$$\text{MMonkey}_{(\text{chase}(x',x'',CSO),\text{chase}(y',y'',CSO),elow(dom'',dom'))}|\text{FGT}_{(n)},\text{OEST}_{(m)}$$

with $dom' > dom''$, $dist((x', y'), (x'', y'')) \leq OAD$, and $|dom' - dom''| \leq AO$.

Rules r_{15} and r_{16} is analogous to r_{13} and r_{14}, respectively, but describe the case in which the female is in oestrum state. Parameters OAD (Oestrum

Aggression Distance), AO (Avoidance Oestrum), CSO (Chase Speed Oestrum) and FSO (Flee Speed Oestrum) of these rules are analogous to the corresponding ones of rules r_{13} and r_{14}, but with values that depend on the fact that the female is in oestrum state.

$$\begin{aligned} r_{17} : \ & \mathrm{MMonkey}_{(x',y',dom')}, \mathrm{MMonkey}_{(x'',y'',dom'')} \xrightarrow{1} \\ & \quad \mathrm{MMonkey}_{(\mathrm{chase}(x',x'',CSN),\mathrm{chase}(y',y'',CSN),dom')}, \\ & \quad \mathrm{MMonkey}_{(\mathrm{flee}(x',x'',FSN),\mathrm{flee}(y',y'',FSN),dom'')} | \mathrm{FGT}_{(n)}, \mathrm{NORMAL}_{(m)} \end{aligned}$$

with $dom' > dom''$, $dist((x', y'), (x'', y'')) \leq NAD$ and $|dom' - dom''| > AN$.

$$\begin{aligned} r_{18} : \ & \mathrm{MMonkey}_{(x',y',dom')}, \mathrm{MMonkey}_{(x'',y'',dom'')} \xrightarrow{1} \\ & \quad \mathrm{MMonkey}_{(\mathrm{chase}(x',x'',CSO),\mathrm{chase}(y',y'',CSO),dom')} \\ & \quad \mathrm{MMonkey}_{(\mathrm{flee}(x',x'',FSO),\mathrm{flee}(y',y'',FSO),dom'')} | \mathrm{FGT}_{(n)}, \mathrm{OEST}_{(m)} \end{aligned}$$

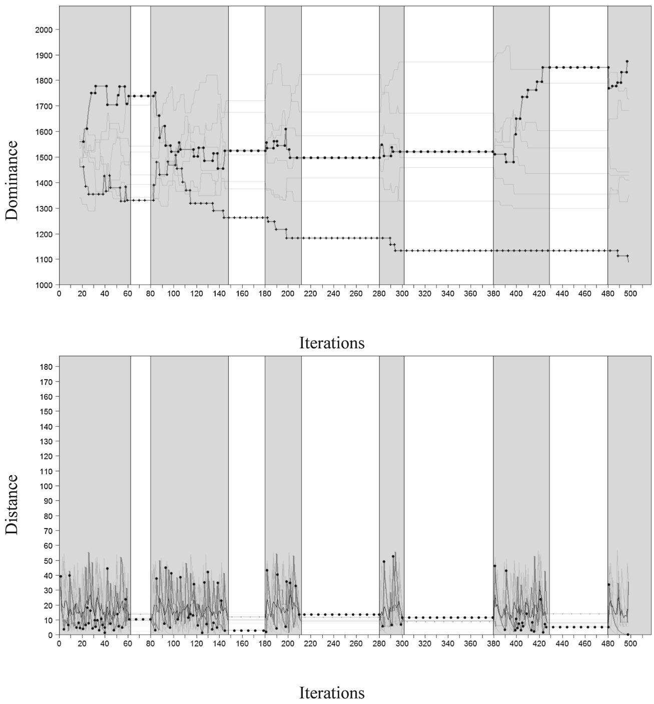
with $dom' > dom''$, $dist((x', y'), (x'', y'')) \leq OAD$ and $|dom' - dom''| > AO$.

Rules r_{17} and r_{18} describe the interaction between two individuals when the difference in dominance is too high to motivate a fight (greater than parameters AN and AO for the normal and oestrum cases, respectively). In these cases the monkeys move but do not fight, so there is no change in dominance levels.

4 Experimental Results

We studied the dynamics of the APP model described in the previous section by running simulations. In particular, we implemented an APP systems interpreter in C# that allows attributed objects and evolution rules to be represented as instantiations of specific C# classes. Once an APP systems model is specified, the interpreter simulates it by performing a number of iterations to be given as a parameter. In each iteration, a maximally parallel step is performed according to the APP systems semantics. The result of a simulation is the sequence of configurations reached by the interpreter at each iteration. In order for the interesting measurements to be easily readable, we processed the simulation results and produced graphical representations by using the statistical framework R.

In Fig. 3 and in Fig. 4 we show the dynamics of two groups of monkeys. In particular, Fig. 3 refers to a group with a low level of aggressiveness (egalitarian), while Fig. 4 describes a group with a higher level of aggressiveness (despotic). The upper part of both figures shows the dominance level of each male in the group during the simulation, while the lower part shows the distance of each male either from the center of the group or from the female (when present). In the figures we put in evidence the lines corresponding to both the monkey with highest dominance level at the end of the simulation (line marked with •) and the one with the lowest one (marked with +). The main model parameters (that are different in the two cases) are reported in the figures. The other parameters have, in both cases, the following values: iteration = 498, Sol = 20, Snl = 80,



$NAD = 5$ $OAD = 5$ $FT = 600$

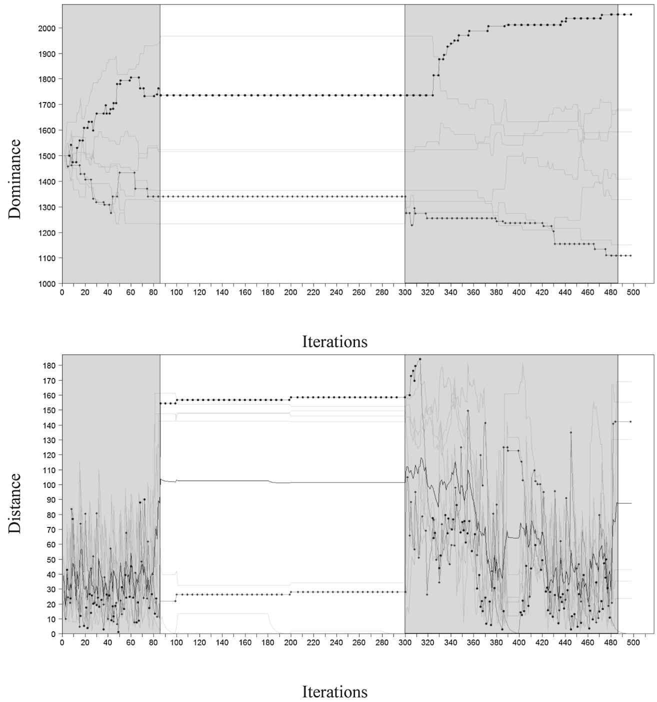
$AN = 200$ $AO = 400$ $FSO = 12$ $FSN = 8$

$CSN = 0$ $CSO = 0$ $Nfl = 1$ $Ofl = 2$

Fig. 3. Simulation of an egalitarian group.

SMMA = 0.25, SD = 100, NT = 3, OT = 2, SMFA = 0.25, PfF = 8, Msn = 1 and Stepness = 100. Note that we ran the simulations for 498 iterations. The time corresponding to an iteration is, in the real world, about a few hours. Actually, such a time could vary among the different phases described by the model. However, this is not a problem since we are not interested in precise description of timing aspects. As regards the other model parameters they have been estimated from the descriptions in [10,27,28]. The results we obtain are compatible with the behaviour of prosimians, as described in the above studies.

As regards Fig. 3, initially all the males have the same dominance level. From the beginning and up to about 210 iterations, the group of monkeys struggles to define a clear dominance among individuals. Between iteration 210 and 390, the dotted line is not the dominant of the group, thus its position is not the closest neither to the group center nor to the female. At the end of the simulation, when the monkey with dotted line becomes the most dominant (alpha male), we can observe that it gains a central position in the group.



$NAD = 10$	$OAD = 10$	$FT = 400$	
$AN = 350$	$AO = 700$	$FSO = 15$	$FSN = 10$
$CSN = 1$	$CSO = 0$	$Nfl = 2$	$Ofl = 4$

Fig. 4. Simulation of a despotic group.

Figure 4 shows the dynamics of a group with a more rigid hierarchy, due to the higher level of aggressiveness. The first phase, up to iteration 90, in which the males establish a first hierarchy, is followed by a phase (up to iteration 320) in which the hierarchy becomes very stable. From iteration 320 onwards, the monkey with the dotted line becomes the alpha male, it gains the position which is closer to the center of the group and to the female, and it does not allow any other monkey to come close.

Normal and oestrum periods last respectively 80 and 20 iterations. In normal phase monkeys are less willing to fight and they keep a distance from each other in order to avoid unnecessary conflicts; this corresponds to the more linear parts of the graphs. In oestrum periods males have the female as the pole of attraction and they are more willing to fight. Fights can change the dominance levels of males, thus oestrum periods correspond to more "hectic" parts of the graphs, where, often, the ranking of dominance changes. When the dominance levels change, the topology of the group changes accordingly. The results of the simulations agree with the behaviour of different species of prosimians as described in [10,27,28]

5 Conclusions

We proposed an extension of probabilistic P Systems, called Attributed Probabilistic P systems (APP systems), in which objects are annotated with attributes. APP systems are intended to be used to model the dynamics of populations and ecosystems. In this context, attributes can be used to represent characteristics of the population individuals such as age, position, and so on. Apart from attributes, the feature that mainly makes a difference between APP systems and other proposals is the use of maximal parallelism for the application of rules. This feature is particularly suitable for the modelling of populations that evolve by stages (e.g. reproductive stages or stages related with seasons).

We used APP systems for modelling social behaviours of some species of primates. In particular, as an application we developed a model to compare despotic and egalitarian behaviours of different species of primates. Such kinds of social systems are usually approached by means of agent-based models that are often poorly documented and ambiguous. On the contrary, since both the syntax and the semantics of APP systems are formally defined, the model based on APP systems is unambiguous.

The model has been inspired by the behaviour of species of prosimians. We plan to adapt our general model to the modelling of the behaviour of particular species of primates by changing the values of the parameters.

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