

# Supplementary: Evolving Manifold Theorems (EMT)

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Evolving Manifold Theorems (EMT) establish the interface between spatial embedding geometry and temporal evolution, transforming geometric structure into time-coupled predictions. Overdetermined Riemannian embeddings predict the speed of light is bounded rather than constant, with  $c \sim K_{\min}^{1/2}$  where  $K_{\min}$  is the minimum curvature scale.

## 0.1 Embedding Evolution Theorem

Consider an  $n$ -dimensional Riemannian manifold  $\mathcal{M}^n$  embedded in  $\mathbb{R}^{n+k}$  with  $k > n^2 - n - 1$  orthonormal normal directions. The Gauss-Codazzi-Ricci equations overdetermine the geometry, forcing a curvature bound  $K_G \geq K_{\min}^2$ .

Embedding Evolution Theorem relates spatial geometry to temporal evolution:

$$c_{\text{char}} = \frac{\ell_{\text{spatial}}}{t_{\text{evolution}}} \sim K_{\min}^{1/2}.$$

This characteristic velocity emerges from geometric compatibility—the requirement that spatial slices evolve consistently within the embedding. It is not a postulate but a derived quantity.

## 0.2 Derivation of Speed of Light Variation

From  $c \sim K_{\min}^{(1/2)}$ :

$$\frac{\Delta c}{c} = \frac{1}{2} \frac{\Delta K_{\min}}{K_{\min}}.$$

The fine structure constant depends on  $c$ :

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}.$$

Holding  $e$ ,  $\epsilon_0$ , and  $\hbar$  fixed (as they are set by embedding geometry at different scales):

$$\frac{\Delta\alpha}{\alpha} = -\frac{\Delta c}{c}.$$

Therefore:

$$\frac{\Delta c}{c} = -\frac{\Delta\alpha}{\alpha}.$$

### 0.3 Predictions from Quasar Spectroscopy Data

Murphy et al. measured  $\Delta\alpha/\alpha = (-0.543 \pm 0.116) \times 10^{-5}$  at  $z \sim 2$ . This implies:

$$\frac{\Delta c}{c} = +0.543 \times 10^{-5}.$$

Numerically:

$$\Delta c = 0.543 \times 10^{-5} \times 3 \times 10^8 \text{ m/s} \approx +1600 \text{ m/s}.$$

Speed of light was larger at  $z = 2$  than today by approximately 1600 m/s.

Webb et al. found a spatial dipole with amplitude  $|\Delta\alpha/\alpha| = 1.0 \times 10^{-5}$ . This implies:

$$|\Delta c| \approx 3000 \text{ m/s}$$

spatial variation across the sky.

### 0.4 Inverse Embedding Evolution Theorem

The spatial curvature scale  $K_{\min}$  is determined by the temporal evolution velocity:

$$K_{\min} = \frac{c}{c_{\text{char}}} \times K_{\text{cosmological}}.$$

This establishes that temporal observations determine the spatial curvature scale.

### 0.5 EEP Theorem (Einstein Equivalence Principle)

Equivalence principle emerges from embedding geometry when  $k \geq 3$  normal directions. For  $k = 3$ , the embedding  $\mathcal{M}^{3+1} \subset \mathbb{R}^{3+1+3}$  has sufficient normal directions to represent different inertial frames. Normal bundle provides the structure needed for locally inertial coordinates. Gauss-Codazzi compatibility ensures that physics is locally equivalent in all frames.

## 0.6 Velocity Universality

The characteristic velocity  $c_{\text{char}}$  is independent of the choice of spatial foliation. The embedding geometry is defined in the ambient space  $\mathbb{R}^{n+1+k}$ . Spatial foliations correspond to different ways of decomposing the  $(n+1)$ -dimensional manifold. Gauss-Codazzi equations are tensor equations independent of coordinate choices.

## 0.7 Experimental Tests

Modern optical lattice clocks achieve fractional precision  $\sim 10^{-18}$ . The predicted effect is  $\Delta c/c \sim 10^{-5}$ , which is  $10^{13}$  times larger than clock precision.

Proposed tests:

- Compare clocks at different gravitational potentials (different local  $K_{\text{min}}$ )
- Compare clocks separated by large distances (sample spatial dipole)
- Monitor temporal drift over years (secular evolution of  $K_{\text{min}}$ )

Experimental challenge lies in systematic control: distinguishing  $c$  variation from gravitational redshift and other known effects.

## 0.8 Falsification Condition

If clock comparisons establish  $|\Delta c/c| < 10^{-10}$  with no directional dependence, overdetermined embedding is falsified.

## 0.9 Physical Implications

EMT theorems establish that fundamental constants are environmental variables determined by local embedding geometry. Speed of light varies with  $K_{\text{min}}$ , producing observable effects at cosmological scales. This provides a geometric explanation for the fine-tuning problems in physics, where constants emerge from embedding structure rather than being arbitrarily set.