

# Supplementary: Geometric Principles of Quantum Field Theory

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## 0.1 Quantization from Topological Constraints

Consider the connection  $A^{(1)}$  in the normal bundle, which describes parallel transport in the first normal direction. For a closed loop  $\gamma$  in spacetime, the holonomy (accumulated rotation in normal space) is:

$$\Phi[\gamma] = \oint_{\gamma} A^{(1)}.$$

The ambient space  $\mathbb{R}^{3+1+k}$  is simply connected, imposing topological constraints on the holonomy. For consistency of the embedding, the accumulated phase around any closed loop must satisfy:

$$\oint_{\gamma} A^{(1)} = 2\pi n, \quad n \in \mathbb{Z}.$$

This is integer quantization arising from topology. The connection  $A^{(1)}$  relates to the extrinsic curvature  $K^{(1)}$  through the Gauss-Codazzi equations, giving:

$$\oint_{\gamma} A^{(1)} = \int_{\Sigma} K^{(1)} d^2x,$$

where  $\Sigma$  is any surface bounded by  $\gamma$ . The quantization condition is precisely the Bohr-Sommerfeld quantization of action, derived from geometric compatibility rather than postulated.

## 0.2 Observable Consequences

The geometric origin of quantum mechanics produces testable predictions:

**Quantization modifications.** The Bohr-Sommerfeld condition receives corrections at energies approaching the curvature scale  $E \sim K_{\min}$ .

For atomic systems with  $K_{\text{atomic}} \ll K_{\text{min}}$ , standard quantization holds, but for high-energy processes, deviations emerge:

$$\oint p dq = 2\pi\hbar n \left( 1 + \alpha \frac{K_{\text{system}}}{K_{\text{min}}} + \dots \right).$$

**Uncertainty relation bounds.** The uncertainty relation is saturated at the geometric scale  $K_{\text{min}}$ . Ultra-high-precision measurements approaching this limit would reveal departures from standard quantum mechanics:

$$\Delta q \cdot \Delta p \geq \frac{\hbar}{2} \left( 1 + \beta \frac{\Delta E}{K_{\text{min}}} + \dots \right).$$

**Planck constant variation.** If  $K_{\text{min}}$  varies cosmologically,  $\hbar(t) \sim K_{\text{min}}(t)^{-1}$  evolves correspondingly. This induces correlated variations in atomic spectra, detectable through precision spectroscopy of distant quasars.

### 0.3 UV Regulator Mechanism

Resolution does not require QFT vacuum energy to vanish. Vacuum fluctuations exist and carry energy, but embedding geometry bounds effective contribution to  $\Lambda$ .

Quantum fields arise as fluctuations of the embedding map  $X : \mathcal{M}^4 \rightarrow \mathbb{R}^5$  in normal directions. Once quantization emerges from the fifth dimension's topology, quantum fields must satisfy the same derivative hierarchy that constrains classical geometry. Since  $X$  must satisfy the derivative hierarchy:

$$|\nabla^m X| \leq C_m K_{\text{min}}^{2+m/2}, \quad m = 0, 1, 2, \dots$$

any fluctuation  $\delta X$  inherits these bounds, preventing UV divergences from propagating to macroscopic curvature.

Overdetermined embedding establishes that quantum mechanics is the inevitable structure of the fifth dimension. Quantization, uncertainty,  $\hbar$ , wave functions, and the Schrödinger equation all emerge from embedding geometry, not as independent postulates. Quantum principles are geometric necessity.