

Supplementary: Evolving Manifold Theorems (EMT)

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Evolving Manifold Theorems (EMT) establish the interface between spatial embedding geometry and temporal evolution, transforming geometric structure into time-coupled predictions. Overdetermined Riemannian embeddings predict the speed of light is bounded rather than constant, with $c \sim K_{\min}^{1/2}$ where K_{\min} is the minimum curvature scale.

0.1 Embedding Evolution Theorem

Consider an n -dimensional Riemannian manifold \mathcal{M}^n embedded in \mathbb{R}^{n+k} with $k > n^2 - n - 1$ orthonormal normal directions. The Gauss-Codazzi-Ricci equations overdetermine the geometry, forcing a curvature bound $K_G \geq K_{\min}^2$.

Embedding Evolution Theorem relates spatial geometry to temporal evolution:

$$c_{\text{char}} = \frac{\ell_{\text{spatial}}}{t_{\text{evolution}}} \sim K_{\min}^{1/2}.$$

This characteristic velocity emerges from geometric compatibility—the requirement that spatial slices evolve consistently within the embedding. It is not a postulate but a derived quantity.

0.2 Derivation of Speed of Light Variation

From $c \sim K_{\min}^{(1/2)}$:

$$\frac{\Delta c}{c} = \frac{1}{2} \frac{\Delta K_{\min}}{K_{\min}}.$$

The fine structure constant depends on c :

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}.$$

Holding e , ϵ_0 , and \hbar fixed (as they are set by embedding geometry at different scales):

$$\frac{\Delta\alpha}{\alpha} = -\frac{\Delta c}{c}.$$

Therefore:

$$\frac{\Delta c}{c} = -\frac{\Delta\alpha}{\alpha}.$$

0.3 Predictions from Quasar Spectroscopy Data

Murphy et al. measured $\Delta\alpha/\alpha = (-0.543 \pm 0.116) \times 10^{-5}$ at $z \sim 2$. This implies:

$$\frac{\Delta c}{c} = +0.543 \times 10^{-5}.$$

Numerically:

$$\Delta c = 0.543 \times 10^{-5} \times 3 \times 10^8 \text{ m/s} \approx +1600 \text{ m/s}.$$

Speed of light was larger at $z = 2$ than today by approximately 1600 m/s.

Webb et al. found a spatial dipole with amplitude $|\Delta\alpha/\alpha| = 1.0 \times 10^{-5}$. This implies:

$$|\Delta c| \approx 3000 \text{ m/s}$$

spatial variation across the sky.

0.4 Inverse Embedding Evolution Theorem

The spatial curvature scale K_{\min} is determined by the temporal evolution velocity:

$$K_{\min} = \frac{c}{c_{\text{char}}} \times K_{\text{cosmological}}.$$

This establishes that temporal observations determine the spatial curvature scale.

0.5 EEP Theorem (Einstein Equivalence Principle)

Equivalence principle emerges from embedding geometry when $k \geq 3$ normal directions. For $k = 3$, the embedding $\mathcal{M}^{3+1} \subset \mathbb{R}^{3+1+3}$ has sufficient normal directions to represent different inertial frames. Normal bundle provides the structure needed for locally inertial coordinates. Gauss-Codazzi compatibility ensures that physics is locally equivalent in all frames.

0.6 Velocity Universality

The characteristic velocity c_{char} is independent of the choice of spatial foliation. The embedding geometry is defined in the ambient space \mathbb{R}^{n+1+k} . Spatial foliations correspond to different ways of decomposing the $(n+1)$ -dimensional manifold. Gauss-Codazzi equations are tensor equations independent of coordinate choices.

0.7 Experimental Tests

Modern optical lattice clocks achieve fractional precision $\sim 10^{-18}$. The predicted effect is $\Delta c/c \sim 10^{-5}$, which is 10^{13} times larger than clock precision.

Proposed tests:

- Compare clocks at different gravitational potentials (different local K_{\min})
- Compare clocks separated by large distances (sample spatial dipole)
- Monitor temporal drift over years (secular evolution of K_{\min})

Experimental challenge lies in systematic control: distinguishing c variation from gravitational redshift and other known effects.

0.8 Falsification Condition

If clock comparisons establish $|\Delta c/c| < 10^{-10}$ with no directional dependence, overdetermined embedding is falsified.

0.9 Physical Implications

EMT theorems establish that fundamental constants are environmental variables determined by local embedding geometry. Speed of light varies with K_{\min} , producing observable effects at cosmological scales. This provides a geometric explanation for the fine-tuning problems in physics, where constants emerge from embedding structure rather than being arbitrarily set.