# Supplementary: Uncertainty Relations

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## 0.1 Uncertainty Relations from Derivative Hierarchy

The infinite derivative hierarchy constrains fluctuations in the normal direction. Define conjugate variables:

$$q = \text{position coordinate in normal direction}, \quad p = \text{conjugate momentum generating translations in } n^{( \tag{1}$$

From the bounds on spatial derivatives of normal curvature established in Section 4, we have:

$$|K^{(1)}| \sim K_{\min}^2, \quad |\nabla K^{(1)}| \sim K_{\min}^{5/2}. \tag{2}$$

These bounds constrain the precision with which position and momentum in normal space can be simultaneously specified. The curvature bound $|K^{(1)}| \sim K_{\min}^2$ limits spatial localization, giving position uncertainty:

$$\Delta q \sim K_{\min}^{-1/2}. \tag{3}$$

The derivative bound $|\nabla K^{(1)}| \sim K_{\min}^{5/2}$, combined with characteristic velocity $c$ from the Embedding Evolution Theorem, constrains momentum uncertainty:

$$\Delta p \sim c K_{\min}. \tag{4}$$

The uncertainty product yields:

$$\Delta q \cdot \Delta p \sim c K_{\min}^{1/2}. \tag{5}$$

## 0.2 Planck's Constant from Geometry

The fundamental action scale is defined by the characteristic velocity and minimum curvature:

$$\hbar = c K_{\min}^{-1} = \frac{c^2}{H_0}, \tag{6}$$

where we used the cosmological identification $K_{\min} = H_0/c$.

The ratio of uncertainty product to Planck's constant:

$$\frac{\Delta q \cdot \Delta p}{\hbar} \sim \frac{cK_{\min}^{1/2}}{cK_{\min}^{-1}} = K_{\min}^{3/2}. \tag{7}$$

For cosmological $K_{\min} \sim 10^{-26}\ \mathrm{m}^{-1}$, this ratio is extremely small ($\sim 10^{-39}$), ensuring:

$$\Delta q \cdot \Delta p \geq \hbar/2. \tag{8}$$

Uncertainty is not a quantum postulate but a consequence of geometric derivative bounds in the normal direction. Planck's constant emerges from the embedding geometry, not as a free parameter.