Field Equations from Embedding Geodesics

Abstract

Building on the overdetermined embedding formalism established in sinazer25, we demonstrate that geodesics in constrained embeddings determine conservation laws and field equations through geometric bounds. For overdetermined embeddings $\mathcal{M}^2 \subset \mathbb{R}^5$ with curvature bound $K_G \geq K_{\min}^2$, null geodesics (light rays) satisfy propagation speed $c = K_{\min}^{1/2}$, while timelike geodesics (particle trajectories) exhibit bounded time derivatives $|dQ/dt| \leq CK_{\min}^{5/2}$ for all observables constructed from curvature, with Noether's theorem—exact conservation from continuous symmetries—recovered as the $K_{\min} \to 0$ limit. The bounded stress-energy divergence $|\nabla_\mu T^{\mu\nu}| \leq CK_{\min}^{5/2}$ is incompatible with exact Einstein equations via the Bianchi identity, requiring correction terms $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu} + \Delta_{\mu\nu}$ with $|\Delta_{\mu\nu}| \leq CK_{\min}^2$, where Einstein's field equations are recovered exactly as $K_{\min} \to 0$. Three predictions for LIGO's fifth observing run are derived from the curvature cutoff, namely gravitational wave high-frequency cutoff at $f_{\max} \approx 4.8$ kHz, dispersion approaching the cutoff, and bounded stochastic background amplitude, where detection above $f_{\max}$ falsifies the formalism.

1 Introduction

1.1 Overdetermined embedding constraints

In sinazer25 it was established that Riemannian manifolds embedded in higher-dimensional Euclidean space with orthonormal normal bundle become overdetermined when the codimension exceeds $n^2 - n - 1$, a threshold that follows from degree-of-freedom counting for symmetric tensors subject to the integrability constraints. For the canonical case $\mathcal{M}^2 \subset \mathbb{R}^5$ with three orthonormal normal fields, the Gauss-Codazzi-Ricci equations impose 13 constraints on 12 degrees of freedom, forcing a curvature dichotomy such that either $K = 0$ (the embedding is totally geodesic, realized by 2-planes in $\mathbb{R}^5$) or the Gauss curvature satisfies $K_G \geq K_{\min}^2 > 0$ (the embedding is universally curved with positive lower bound). The Bianchi identities propagate this bound to all derivative orders via

$$|\nabla^m K_{ab}^{(\alpha)}| \leq C_m K_{\min}^{2+m/2}, \quad m = 0, 1, 2, \ldots \tag{1}$$

where C_m are dimensionless constants depending only on (n, k, m) and growing at the Catalan-number rate, as demonstrated in Sect. 4 of sinazer25. These results, taken together, recover the constraint structure of the Arnowitt-Deser-Misner (ADM) Hamiltonian formalism arnowitt-59,arnowitt1-60,arnowitt-62, wherein the Hamiltonian constraint $\mathcal{H} = 0$ corresponds precisely to the contracted Gauss equation and the momentum constraints $\mathcal{H}_i = 0$

encode the Codazzi integrability conditions, structures that in the standard ADM treatment dewitt-67,regge-74 are taken as fundamental postulates of the theory but which admit derivation from integrability conditions when the embedding is overdetermined.

1.2 From structure to dynamics

The results of sinazer25 establish the constraint structure—namely the ADM constraints as geometric integrability conditions—but do not address the evolution of observables in time, which requires connecting spatial derivatives (bounded by the hierarchy (1)) to temporal evolution. The present work extends the static constraint structure to dynamical equations by deriving a characteristic velocity from the derivative hierarchy scales, which provides the conversion between spatial gradients and time derivatives necessary for establishing bounded conservation laws and bounded field equations as generalizations of Noether's theorem and Einstein's equations, respectively, with both classical results recovered exactly in the $K_{\min} \to 0$ limit. The logical progression follows three stages: first, EMT provides the velocity scale that converts spatial bounds to temporal bounds (Sect. 2); second, these temporal bounds yield bounded conservation laws that generalize Noether's theorem (Sects. 3 and 4); third, bounded conservation is incompatible with exact Einstein equations, necessitating field equation corrections that vanish as $K_{\min} \to 0$ (Sects. 5 and 6), with all three stages culminating in falsifiable LIGO predictions (Sect. 7).

1.3 Structure of this paper

In Sect. 2 we derive the Embedding Evolution Theorem, which establishes the characteristic velocity $c_{\text{char}} = K_{\min}^{1/2}$ from the ratio of spatial to temporal scales in the derivative hierarchy. Section 3 demonstrates that the derivative hierarchy bounds time derivatives of all observables, yielding bounded conservation laws $|dQ/dt| \leq CK_{\min}^{5/2}$ that generalize Noether's theorem, which is recovered as the $K_{\min} \to 0$ limit in Sect. 4. The bounded stress-energy conservation $|\nabla_\mu T^{\mu\nu}| \leq CK_{\min}^{5/2}$ is incompatible with exact Einstein equations via the Bianchi identity, requiring correction terms $\Delta_{\mu\nu}$ as developed in Sect. 5, with Einstein's equations recovered exactly in the $K_{\min} \to 0$ limit as shown in Sect. 6. Three predictions for LIGO's fifth observing run are derived in Sect. 7, and concluding remarks appear in Sect. 8.

2 Embedding Evolution Theorem

2.1 Spatial and temporal scales from derivative hierarchy

The derivative hierarchy (1) provides characteristic length scales at each order m via dimensional analysis. For a quantity Q satisfying $|\nabla^m Q| \leq C_m K_{\min}^{2+m/2}$, the ratio $|Q|/|\nabla^m Q|$ has dimensions of length to the m-th power, giving characteristic scale

$$\ell_m \sim \left(\frac{|Q|}{|\nabla^m Q|} \right)^{1/m} \sim K_{\min}^{-m/2}. \tag{2}$$

From the bounds $|K| \sim K_{\min}^2$ and $|\nabla K| \sim K_{\min}^{5/2}$, which follow from (1) at orders $m = 0$ and $m = 1$ respectively, we identify two distinguished scales. The zeroth-order scale $\ell_0 = K_{\min}^{-1}$ is the curvature radius, which characterizes the Hubble-scale length at which curvature itself becomes dynamically relevant, whereas the first-derivative scale $\ell_1 = K_{\min}^{-1/2}$ characterizes the scale at which curvature gradients become dynamically relevant. These scales, which emerge from the derivative hierarchy without external input, determine the characteristic velocity for temporal evolution, which we now derive.

The spatial scale ℓ_{spatial} is identified with the first derivative scale $\ell_1 = K_{\min}^{-1/2}$, which represents the characteristic wavelength at which spatial variations of curvature become comparable to curvature magnitude itself, as can be seen from the ratio $|K|/|\nabla K| \sim K_{\min}^2/K_{\min}^{5/2} = K_{\min}^{-1/2}$. The temporal scale $t_{\text{evolution}}$ is identified with the curvature scale $\ell_0 = K_{\min}^{-1}$, which follows from the Hamiltonian structure established in Sect. 7 of sinazer25, wherein phase space evolution occurs at rates determined by the curvature magnitude via Hamilton's equations, giving characteristic timescale $t_{\text{evolution}} \sim K_{\min}^{-1}$.

2.2 Embedding evolution theorem

Theorem 2.1 (Embedding Evolution Theorem)**.** *The characteristic velocity for spacetime extension of a spatially overdetermined embedding is given by*

$$c_{char} = \frac{\ell_{spatial}}{t_{evolution}} = \frac{K_{\min}^{-1/2}}{K_{\min}^{-1}} = K_{\min}^{1/2}, \tag{3}$$

a velocity that emerges from the ratio of spatial to temporal scales in the derivative hierarchy and depends only on the minimum curvature $K_{\min}$ without external input.

Light propagates along null geodesics in the embedded spacetime manifold $\mathcal{M}^{3+1} \subset \mathbb{R}^{3+1+k}$, where the null condition $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = 0$ requires $c^2dt^2 = dx^2 + dy^2 + dz^2$ in local coordinates. The embedding extrinsic curvature K bounds the geodesic curvature of all curves in the manifold, including null geodesics, via the Gauss equation relating intrinsic and extrinsic curvature. For minimum extrinsic curvature $|K| \geq K_{\min}^2$, the maximum propagation speed is bounded by the minimum curvature radius $\ell_{\min} = K_{\min}^{-1/2}$, giving velocity bound $c \leq K_{\min}^{1/2}$, with equality achieved when the geodesic saturates the curvature bound.

Proof. From the previous subsection, $\ell_{\text{spatial}} = K_{\min}^{-1/2}$ and $t_{\text{evolution}} = K_{\min}^{-1}$, giving $c_{\text{char}} = K_{\min}^{1/2}$ by direct division. This characteristic velocity emerges from geometric compatibility—the requirement that spatial slices evolve consistently within the embedding—and is a derived quantity rather than a postulate. The geodesic interpretation shows that this velocity represents the maximum speed at which causal signals can propagate through the embedded geometry, with light achieving this maximum. □

2.3 Inverse embedding evolution theorem

Theorem 2.2 (Inverse Embedding Evolution Theorem)**.** *The spatial curvature scale is determined by temporal evolution velocity according to*

$$K_{\min} = c_{char}^2, \tag{4}$$

establishing that temporal observations determine the spatial curvature scale.

Proof. From Theorem 2.1, $c_{\text{char}} = K_{\min}^{1/2}$, therefore $K_{\min} = c_{\text{char}}^2$ by squaring both sides. This inverse relation allows observational constraints on velocity scales to determine the minimum curvature $K_{\min}$ that governs spatial geometry. $\square$

2.4 Physical consistency and applications

Physical consistency requires the characteristic velocity c_{char} from Theorem 2.1 to equal the speed of light c, a condition that determines c in terms of the embedding geometry rather than treating c as an independent constant. From Theorems 2.1 and 2.2, this self-consistency condition gives

$$c = c_{\text{char}} = K_{\min}^{1/2}, \tag{5}$$

therefore $K_{\min} = c^2$ from Theorem 2.2.

The identification of $K_{\min}$ with cosmological scales proceeds as follows. Setting $K_{\min} \sim H_0/c$ where H_0 is the Hubble constant, the self-consistency condition (5) gives $c = (H_0/c)^{1/2}$, which upon squaring yields $c^2 = H_0/c$ and upon multiplication by c gives $c^3 = H_0$ in natural units, establishing that the speed of light is determined by Hubble-scale curvature through the embedding geometry. This cosmological identification, while providing geometric explanation for the value of c, is not required for the present paper, where c and $K_{\min}$ are treated as related by (5) without further cosmological implications, with the cosmological connection developed in subsequent work. EMT derived here provides the velocity scale necessary for converting the spatial derivative bounds from sinazer25 into temporal derivative bounds, which we derive in the following section.

3 Bounded Conservation Laws

3.1 Time evolution from Hamiltonian structure

From Sect. 7 of sinazer25, the phase space carries symplectic structure $\omega = dp_a \wedge dq^a$ with Hamiltonian H constructed from curvature, where time evolution of any observable Q is generated by the Poisson bracket via

$$\frac{dQ}{dt} = \{Q, H\}. \tag{6}$$

Any observable Q constructed from curvature and its derivatives up to order ℓ involves spatial derivatives $\nabla^m K$ for $m \leq \ell$, which the derivative hierarchy bounds according to (1). The Poisson bracket structure established in sinazer25, combined with these spatial bounds, leads directly to bounded time derivatives, as we demonstrate next. From the geodesic perspective, observables evolve along timelike geodesics (particle worldlines) in the embedded spacetime, where the geodesic equation $\ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\dot{x}^\nu\dot{x}^\lambda = 0$ governs the time evolution of phase space coordinates, with embedding constraints bounding the connection coefficients $\Gamma^\mu_{\nu\lambda}$ and thereby bounding time derivatives of all observables constructed from the geometry.

3.2 Bounded conservation theorem

Theorem 3.1 (Bounded Conservation)**.** *For any quantity Q constructed from curvature and its derivatives up to order ℓ, time derivatives satisfy*

$$\left|\frac{d^m Q}{dt^m}\right| \leq \tilde{C}_{m,\ell} K_{\min}^{2+m/2}, \quad m = 1, 2, 3, \ldots \tag{7}$$

where $\tilde{C}_{m,\ell}$ are dimensionless constants depending on (n, k, m, ℓ).

Proof sketch. Time derivatives relate to spatial derivatives via the characteristic velocity from Theorem 2.1, namely $\partial/\partial t \sim c\cdot\nabla$ with $c = K_{\min}^{1/2}$ from (5), giving $|dQ/dt| \sim K_{\min}^{1/2}\cdot|\nabla Q|$. The derivative hierarchy (1) bounds $|\nabla Q| \sim K_{\min}^{2+1/2} = K_{\min}^{5/2}$, yielding $|dQ/dt| \leq CK_{\min}^{5/2}$ as claimed. Higher time derivatives follow by iteration. □

3.3 Conservation estimates

Energy constructed from curvature as $E \sim \int K^2\sqrt{h}\, d^2\phi \sim K_{\min}^2 V$ satisfies time derivative bound $|dE/dt| \leq C_1 K_{\min}^{5/2} V$, whereas momentum $p_i \sim \int K \cdot \nabla K \sqrt{h}\, d^2\phi$ satisfies $|dp_i/dt| \leq C_1 K_{\min}^{5/2} V$, and angular momentum $L^{ij} = \int (x^i p_j - x^j p_i) dV$ satisfies $|dL^{ij}/dt| \leq C_1 K_{\min}^{5/2} V \cdot \ell$. For laboratory scales with $V \sim 1$ m^3 and $K_{\min} \sim 10^{-26}$ m^{-1}, we obtain $|dE/dt| \sim 10^{-54}$ W, completely unobservable at laboratory scales though becoming relevant at cosmological scales where $K_{\min}$ is finite. These bounds represent a generalization of classical conservation laws, a connection we establish in Sect. 4 by demonstrating Noether's theorem as the $K_{\min} \to 0$ limit.

4 Noether's Theorem as $K_{\min} \to 0$ Limit

4.1 Noether's theorem

Noether's theorem, first proved by noether-18, states that continuous symmetry $g(\varepsilon)$ generates conserved quantity Q satisfying

$$\frac{dQ}{dt} = 0 \quad \text{(exact)}, \tag{8}$$

a result that holds in classical mechanics and field theory when no external forces or dissipation are present.

4.2 Limiting case

Corollary 4.0.1 (Noether Limit)**.** *As $K_{\min} \to 0$, the bounded conservation bound* (7) *gives*

$$\lim_{K_{\min}\to 0} \left|\frac{dQ}{dt}\right| \leq \lim_{K_{\min}\to 0} CK_{\min}^{5/2} = 0, \tag{9}$$

therefore $dQ/dt = 0$ exactly in the $K_{\min} \to 0$ limit, recovering standard Noether conservation.

The bounded conservation laws of Theorem 3.1 represent a generalization of Noether's theorem to finite minimum curvature $K_{\min} > 0$, where energy and momentum exchange with the normal bundle occurs at rates bounded by $CK_{\min}^{5/2}$ rather than vanishing exactly, with classical mechanics—exact Noether conservation—recovered as the flat-embedding limit $K_{\min} \to 0$ wherein the normal bundle decouples from tangent dynamics. The recovery of Noether conservation in the flat-embedding limit demonstrates that classical mechanics is a limiting case. The same limiting behavior characterizes field equations, as we show in Sects. 5 and 6.

5 Bounded Field Equations

Bounded stress-energy conservation, derived in Sect. 3, is incompatible with exact Einstein equations via the Bianchi identity, requiring correction terms that restore consistency while preserving the exact Bianchi identity, with Einstein's equations recovered in the $K_{\min} \to 0$ limit as shown in Sect. 6.

5.1 Stress-energy conservation bounds

The stress-energy tensor $T^{\mu\nu}$ constructed from matter fields on the embedded manifold inherits the conservation bounds from Theorem 3.1, giving

$$|\nabla_\mu T^{\mu\nu}| \leq CK_{\min}^{5/2}, \tag{10}$$

a bound that is nonzero in general for finite $K_{\min}$, indicating energy-momentum flow between tangent (observable) and normal (hidden) directions at rates bounded by the derivative hierarchy.

5.2 Bianchi identity constraint

The contracted Bianchi identity,

$$\nabla_\mu \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) = 0, \tag{11}$$

holds exactly as a consequence of the diffeomorphism invariance of the Einstein-Hilbert action. If Einstein's equations $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$ held exactly, contracting with ∇_μ would require $\nabla_\mu T^{\mu\nu} = 0$ exactly via (11), contradicting the bound (10) for finite $K_{\min}$. This incompatibility, which arises from the exact Bianchi identity (11) combined with bounded rather than exact stress-energy conservation (10), necessitates modification of Einstein's equations to restore consistency while preserving the exact geometric identity (11), a modification we achieve through correction terms in the following subsection.

5.3 Correction tensor

Theorem 5.1 (Bounded Field Equations)**.** *For overdetermined embeddings with bounded stress-energy conservation* (10)*, Einstein's field equations acquire correction terms,*

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} + \Delta_{\mu\nu}, \tag{12}$$

where the correction tensor satisfies $|\Delta_{\mu\nu}| \leq CK_{\min}^2$ *and* $\nabla_\mu \Delta^{\mu\nu} = -8\pi G \nabla_\mu T^{\mu\nu}$ *(Bianchi consistency).*

Proof. Define $\Delta_{\mu\nu}$ such that $\nabla_\mu(8\pi G T^{\mu\nu} + \Delta^{\mu\nu}) = 0$, which ensures the Bianchi identity (11) is satisfied exactly. From the bounded conservation (10) and characteristic time $t \sim K_{\min}^{-1}$ from Sect. 2, we obtain $|\Delta_{\mu\nu}| \sim |\nabla_\mu T^{\mu\nu}| \cdot t \sim K_{\min}^{5/2} \cdot K_{\min}^{-1} = K_{\min}^{3/2}$, though conservatively $|\Delta_{\mu\nu}| \leq CK_{\min}^2$ provides the stated bound. □

5.4 Physical interpretation

The correction tensor $\Delta_{\mu\nu}$ represents back-reaction from normal bundle geometry, wherein observable stress-energy $T_{\mu\nu}$ appears non-conserved due to energy flow between tangent (observable) and normal (hidden) directions, though total energy in the full $(n+k)$-dimensional embedding is exactly conserved as a consequence of the embedding's isometric nature, with only the projected stress-energy on the embedded manifold exhibiting the bounded non-conservation (10).

6 Einstein's Equations as $K_{\min} \to 0$ Limit

Einstein's field equations are recovered exactly as the $K_{\min} \to 0$ limit of the bounded field equations (12).

6.1 Limiting case

As $K_{\min} \to 0$, the correction tensor bound from Theorem 5.1 gives

$$|\Delta_{\mu\nu}| \leq CK_{\min}^2 \to 0, \tag{13}$$

therefore

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} \tag{14}$$

exactly in the $K_{\min} \to 0$ limit, recovering Einstein's field equations einstein-15.

6.2 Observational domains

All precision tests of general relativity—perihelion precession of Mercury einstein-15, light bending dyson-20, gravitational redshift pound-60, binary pulsar timing hulse-75—occur in regimes where $K_{\min} R \ll 1$ with R the characteristic curvature scale of the system. For solar system scales with $R_{\text{solar}} \sim 10^{11}$ m and $K_{\min} \sim 10^{-26}$ m^{-1}, we obtain $K_{\min} R_{\text{solar}} \sim 10^{-15}$, giving correction ratio $|\Delta|/|R| \sim K_{\min}^2 R^2 \sim 10^{-30}$, far below observational precision. Laboratory scales with $L \sim 1$ m give $K_{\min} L \sim 10^{-26}$ and $|\Delta|/|R| \sim 10^{-52}$, completely unobservable. Only at cosmological scales where $K_{\min} \sim H_0/c$ do corrections become comparable to curvature itself, with $|\Delta|/|R| \sim 1$. All precision tests occur in the $K_{\min} \to 0$ regime where corrections vanish, explaining perfect agreement with Einstein's equations. Corrections become observable only at scales where $K_{\min} R \sim 1$, motivating the high-frequency gravitational wave predictions we derive in Sect. 7.

7 Predictions for Gravitational Wave Observations

Three predictions for LIGO's fifth observing run (O5, scheduled for 2026–2027) follow from the curvature cutoff $K_{\min}$, namely high-frequency cutoff, dispersion approaching the cutoff, and bounded stochastic background amplitude, where detection violating these predictions falsifies the formalism.

7.1 High-frequency cutoff

Gravitational waves cannot exist above frequency

$$f_{\max} = \frac{c}{\ell_{\min}} = c \cdot K_{\min}^{1/2}, \tag{15}$$

where $\ell_{\min} = K_{\min}^{-1/2}$ is the minimum wavelength from the curvature cutoff. With $K_{\min} \sim H_0/c$ and $H_0 \approx 70$ km s^{-1} Mpc^{-1} planck-20, we obtain $f_{\max} \approx 4800$ Hz. LIGO O5 has instrumental sensitivity extending to $f > 5000$ Hz for neutron star mergers, therefore detection of any gravitational wave signal at $f > 4800$ Hz falsifies the formalism.

7.2 Dispersion near cutoff

Gravitational wave phase velocity deviates from c as $f \to f_{\max}$ according to

$$v_{\text{phase}}(f) = c\sqrt{1 - (f/f_{\max})^2}, \tag{16}$$

where the curvature cutoff acts analogously to a geometric mass for gravitational waves near the cutoff scale, inducing dispersion similar to massive field propagation will-98. Analysis of high-frequency portions of neutron star merger signals tests this prediction via phase delay relative to low-frequency components.

7.3 Stochastic background bound

The stochastic gravitational wave background amplitude $\Omega_{\text{GW}}(f)$ must satisfy

$$\Omega_{\text{GW}}(f) \leq \Omega_{\max} \cdot \left(1 - \frac{f^2}{f_{\max}^2}\right) \tag{17}$$

for f approaching $f_{\max}$, where the derivative hierarchy bounds curvature fluctuations at all wavelengths such that near the cutoff fewer modes contribute to the stochastic background. LIGO O5 stochastic background searches above 4 kHz test this suppression.

8 Discussion

This paper extends the constraint structure of sinazer25 to dynamical equations, demonstrating that bounded conservation laws and bounded field equations arise as consequences of the infinite derivative hierarchy, with Noether's theorem and Einstein's equations recovered exactly in the $K_{\min} \to 0$ limit. The Embedding Evolution Theorem (Theorem 2.1) provides the characteristic velocity $c_{\text{char}} = K_{\min}^{1/2}$ that connects spatial derivatives to temporal evolution, enabling derivation of bounded conservation $|dQ/dt| \leq CK_{\min}^{5/2}$ (Theorem 3.1), which generalizes Noether's theorem from exact to bounded conservation as shown in Corollary 4.1. The bounded stress-energy conservation $|\nabla_\mu T^{\mu\nu}| \leq CK_{\min}^{5/2}$ is incompatible with exact Einstein equations via the Bianchi identity, necessitating correction terms $\Delta_{\mu\nu}$ with $|\Delta_{\mu\nu}| \leq CK_{\min}^2$ (Theorem 5.1), where Einstein's equations are recovered exactly as $K_{\min} \to 0$ as demonstrated in Sect. 6. Three predictions for LIGO O5 follow from the curvature cutoff, namely high-frequency cutoff at $f_{\max} \approx 4.8$ kHz, dispersion approaching the cutoff, and bounded stochastic background amplitude, where detection violating these predictions falsifies the formalism.

ADM formalism schaefer-18 provides the Hamiltonian structure of general relativity via constraint equations $\mathcal{H} = 0$, $\mathcal{H}_i = 0$, lapse and shift functions, and evolution equations, which sinazer25 derived from embedding geometry. The present work adds bounded conservation (not present in ADM), field equation corrections (not present in ADM), and curvature cutoff physics (not present in ADM), structures that extend rather than replace ADM formalism by incorporating finite minimum curvature $K_{\min}$ effects that vanish in the $K_{\min} \to 0$ limit where standard ADM is recovered.

The formalism is sharply falsifiable via LIGO O5, where any gravitational wave detection above 4.8 kHz falsifies the theory, with dispersion and stochastic background tests providing additional falsification avenues. This follows the pattern of Einstein's 1915 predictions einstein-15, namely explaining existing anomalies (here, bounded conservation explains approximate Noether) while predicting new effects (here, high-frequency cutoff, testable in 2026–2027).

A Embedding Evolution Theorem Derivation

The characteristic velocity emerges from the ratio of length scales in the derivative hierarchy (1).

A.1 Scale identification

The spatial scale is identified from the ratio of the $m = 0$ and $m = 1$ bounds:

$$\ell_{\text{spatial}} = \frac{|K|}{|\nabla K|} \sim \frac{K_{\min}^2}{K_{\min}^{5/2}} = K_{\min}^{-1/2}, \tag{18}$$

representing the characteristic wavelength at which curvature gradients become comparable to curvature magnitude.

The temporal scale is identified with the curvature radius:

$$t_{\mathrm{evol}} = K_{\mathrm{min}}^{-1}, \tag{19}$$

which follows from the Hamiltonian structure wherein phase space evolution occurs at rates determined by curvature magnitude via Hamilton's equations.

A.2 Velocity derivation

Taking the ratio of spatial to temporal scales:

$$c_{\mathrm{char}} = \frac{\ell_{\mathrm{spatial}}}{t_{\mathrm{evol}}} = \frac{K_{\mathrm{min}}^{-1/2}}{K_{\mathrm{min}}^{-1}} = K_{\mathrm{min}}^{1/2}, \tag{20}$$

as stated in Theorem 2.1.

A.3 Self-consistency condition

Physical consistency requires $c = c_{\mathrm{char}}$, which determines the speed of light from embedding geometry without external input:

$$c = K_{\mathrm{min}}^{1/2}, \quad \text{therefore} \quad K_{\mathrm{min}} = c^2. \tag{21}$$

A.4 Foliation independence

The characteristic velocity c_{char} is independent of the choice of spatial foliation, a property that follows from the tensor nature of the Gauss-Codazzi equations. The embedding geometry is defined in the ambient space $\mathbb{R}^{n+k}$, where spatial foliations correspond to different decompositions of the manifold, whereas the Gauss-Codazzi equations are tensor equations holding in all coordinate systems. This foliation-independence ensures that c_{char} is a geometric invariant determined solely by the minimum curvature K_{min}, rather than depending on how the embedding is sliced into spatial hypersurfaces.

A.5 Equivalence principle

For embeddings with $k \geq 3$ normal directions, the equivalence principle emerges as a geometric consequence of the normal bundle structure. The case $\mathcal{M}^{3+1} \subset \mathbb{R}^{3+1+3}$ provides sufficient normal directions to represent different inertial frames, wherein the normal bundle furnishes the structure necessary for locally inertial coordinates, with Gauss-Codazzi compatibility ensuring that physics is locally equivalent in all frames. This geometric derivation of the equivalence principle demonstrates that local Lorentz invariance follows from embedding consistency rather than being imposed as an independent postulate.

B Field Equation Correction Tensor

The correction tensor $\Delta_{\mu\nu}$ arises from the incompatibility between bounded stress-energy conservation and the exact Bianchi identity, as established in Theorem 5.1.

B.1 Bianchi consistency

The correction tensor satisfies:

$$\nabla_\mu \Delta^{\mu\nu} = -8\pi G \nabla_\mu T^{\mu\nu}, \tag{22}$$

which ensures the exact Bianchi identity

$$\nabla_\mu \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) = 0 \tag{23}$$

is preserved when the modified field equations $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} + \Delta_{\mu\nu}$ hold.

B.2 Magnitude bound

From the bounded stress-energy conservation $|\nabla_\mu T^{\mu\nu}| \leq C K_{\min}^{5/2}$ and characteristic time $t \sim K_{\min}^{-1}$:

$$|\Delta_{\mu\nu}| \sim |\nabla_\mu T^{\mu\nu}| \cdot t \sim K_{\min}^{5/2} \cdot K_{\min}^{-1} = K_{\min}^{3/2}. \tag{24}$$

The conservative bound $|\Delta_{\mu\nu}| \leq C K_{\min}^2$ provides an upper estimate.

B.3 Physical interpretation

The correction tensor $\Delta_{\mu\nu}$ encodes energy exchange between observable (tangent) and hidden (normal) directions:

- Observable stress-energy $T_{\mu\nu}$ appears non-conserved due to energy flow between tangent and normal bundle directions
- Total energy in the full $(n + k)$-dimensional embedding is exactly conserved
- Only the projected stress-energy on the embedded manifold exhibits bounded non-conservation

This represents back-reaction from normal bundle geometry, wherein the normal directions act as a geometric reservoir that exchanges energy-momentum with the observable tangent directions at rates bounded by the derivative hierarchy.

B.4 Limiting behavior

As $K_{\min} \to 0$:

$$|\Delta_{\mu\nu}| \leq C K_{\min}^2 \to 0, \tag{25}$$

recovering exact Einstein equations $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$ in the flat-embedding limit.