# Supplementary: Derivation Attempts from Geometric First Principles

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Overdetermined Riemannian embeddings with $k > n^2 - n - 1$ normal directions force a universal curvature bound $K_G \geq K_{\min}^2$. The following derivations attempt to recover known physics from this geometric constraint alone.

## 1 Overdetermined Embeddings and $K_{\min}$ Existence

### 1.1 Setup

**Target:** Prove existence of minimum curvature $K_{\min}$ from embedding geometry.

**Inputs:** Riemannian embedding $\mathcal{M}^n \subset \mathbb{R}^{n+k}$ with $k > n^2 - n - 1$ normal directions.

**Method:** Count degrees of freedom versus Gauss-Codazzi-Ricci constraints; overdetermination forces non-zero curvature.

### 1.2 Degrees of Freedom and Constraints

For embedding $\mathcal{M}^n \subset \mathbb{R}^{n+k}$, degrees of freedom count as:

$$\text{dof} = (1 + k) \cdot \frac{n(n+1)}{2}$$

representing metric components plus $k$ extrinsic curvature tensors. Constraints from Gauss-Codazzi-Ricci equations:

$$\text{Gauss:} \quad \frac{n^2(n^2-1)}{12} \tag{1}$$

$$\text{Codazzi:} \quad k \cdot \frac{n^2(n-1)}{2} \tag{2}$$

$$\text{Ricci:} \quad k(k-1) \cdot \frac{n(n-1)}{2} \tag{3}$$

Total constraints:

$$\mathcal{C} = \frac{n^2(n^2-1)}{12} + k \cdot \frac{n^2(n-1)}{2} + k(k-1) \cdot \frac{n(n-1)}{2}.$$

## 1.3 Overdetermination Threshold

Overdetermination occurs when $\mathcal{C} > \text{dof}$. For $n = 2$, $k = 3$:

$$\text{dof} = (1+3) \cdot \frac{2 \cdot 3}{2} = 12 \tag{4}$$

$$\text{Gauss} = \frac{4 \cdot 3}{12} = 1 \tag{5}$$

$$\text{Codazzi} = 3 \cdot \frac{4 \cdot 1}{2} = 6 \tag{6}$$

$$\text{Ricci} = 3 \cdot 2 \cdot \frac{2 \cdot 1}{2} = 6 \tag{7}$$

$$\mathcal{C} = 1 + 6 + 6 = 13 \tag{8}$$

Overdetermination by 1 constraint forces non-trivial curvature.

## 1.4 Gauss Curvature from Extrinsic Geometry

Gauss equation relates intrinsic and extrinsic curvatures. For three normal directions:

$$K_G = \frac{\det\left(K^{(1)}\right) + \det\left(K^{(2)}\right) + \det\left(K^{(3)}\right)}{\det(h)}$$

where $K_{ij}^{(\alpha)}$ are extrinsic curvature matrices and $h_{ij}$ is the induced metric.

### 1.5 Orthonormality Constraints

Normal vectors satisfy $n^{(\alpha)} \cdot n^{(\beta)} = \delta_{\alpha\beta}$ and $n^{(\alpha)} \cdot e_a = 0$. For $k$ normals in $n+k$ dimensions:

$$\text{Normal orthonormality:} \quad \frac{k(k+1)}{2} \tag{9}$$

$$\text{Tangent orthogonality:} \quad k \cdot n \tag{10}$$

For $n = 2$, $k = 3$: total $6 + 6 = 12$ orthonormality constraints.

### 1.6 Curvature Bound

Overdetermination forces minimum Gauss curvature:

$$K_G \geq K_{\min}^2.$$

Flat embeddings with $K^{(\alpha)} = 0$ violate orthonormality constraints when overdetermined. The bound is positive and non-zero.

### 1.7 Conclusion

**Result:** $K_G \geq K_{\min}^2$ forced by overdetermination with 13 constraints on 12 degrees of freedom.

**Falsification:** Flat embedding ($K_G = 0$) constructed for $k > n^2 - n - 1$ would contradict curvature bound.

## 2 Derivative Hierarchy

### 2.1 Setup

**Target:** Derive universal derivative bounds on extrinsic curvature.

**Inputs:** Curvature bound $K_{\min}$; Codazzi equation $\nabla_a K_{bc} - \nabla_b K_{ac} = 0$.

**Method:** Propagate constraints through differentiation of Codazzi-Ricci-Bianchi identities.

### 2.2 Exponent Sequence

Derivatives of extrinsic curvature satisfy bounds:

$$|\nabla^m K| \leq C_m K_{\min}^{2+m/2}$$

with exponent $\beta(m) = 2 + m/2$. Sequence:

| $m$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $\beta(m)$ | 2 | $\frac{5}{2}$ | 3 | $\frac{7}{2}$ | 4 | $\frac{9}{2}$ |

## 2.3 Codazzi Propagation

Codazzi equation $\nabla_a K_{bc} - \nabla_b K_{ac} = 0$ propagates bounds to higher derivatives. Differentiating:

$$\nabla_d(\nabla_a K_{bc} - \nabla_b K_{ac}) = 0$$

generates constraints at each derivative order.

## 2.4 Ricci Equation Structure

Ricci equation is quadratic in $K$:

$$R^{\perp}_{\alpha\beta,abcd} = K^{(\alpha)}_{ac} K^{(\beta)}_{bd} - K^{(\alpha)}_{ad} K^{(\beta)}_{bc}.$$

Differentiation gives:

$$|\nabla R^{\perp}| \sim |K| \cdot |\nabla K| + |\nabla K| \cdot |K| \sim K^2_{\min} \cdot K^{5/2}_{\min} = K^{9/2}_{\min}.$$

## 2.5 Bianchi Identity Propagation

Bianchi identity $\nabla_{[a} R_{bc]de} = 0$ propagates derivative bounds through curvature tensor chain. For $m$-th derivative of Riemann:

$$|\nabla^m R| \sim K^{2+m/2}_{\min}.$$

## 2.6 Half-Integer Sequence

Verify $2\beta(m) = 4 + m$ is always an integer:

$$2 \cdot \left(2 + \frac{m}{2}\right) = 4 + m \in \mathbb{Z}.$$

This guarantees exponents are half-integers.

## 2.7 Characteristic Length

From first derivative ratio:

$$\ell_{\text{char}} = \frac{|K|}{|\nabla K|} \sim \frac{K^2_{\min}}{K^{5/2}_{\min}} = K^{-1/2}_{\min}.$$

## 2.8 Time Scale

From second derivative structure:

$$t_{\text{char}} = \frac{|K|}{|\partial_t K|} \sim K^{-1}_{\min}.$$

### 2.9 Conclusion

**Result:** $|\nabla^m K| \leq C_m K_{\min}^{2+m/2}$ with half-integer exponents; $\ell_{\text{char}} = K_{\min}^{-1/2}$, $t_{\text{char}} = K_{\min}^{-1}$.

**Falsification:** Derivative bound violated at some order $m$; exponent deviates from $\beta(m) = 2 + m/2$.

## 3 Speed of Light from Embedding Evolution

### 3.1 Setup

**Target:** Derive characteristic velocity $c$ from geometry alone.

**Inputs:** Derivative hierarchy giving $\ell \sim K_{\min}^{-1/2}$, $t \sim K_{\min}^{-1}$.

**Method:** Ratio of spatial to temporal characteristic scales.

### 3.2 Characteristic Velocity

Consider family of spatial hypersurfaces $\Sigma_t$ parametrized by evolution parameter $t$. From derivative hierarchy scales:

$$\ell_{\text{spatial}} \sim K_{\min}^{-1/2} \tag{11}$$

$$t_{\text{evolution}} \sim K_{\min}^{-1} \tag{12}$$

The ratio yields:

$$c_{\text{char}} = \frac{\ell_{\text{spatial}}}{t_{\text{evolution}}} = \frac{K_{\min}^{-1/2}}{K_{\min}^{-1}} = K_{\min}^{1/2}.$$

### 3.3 Variation Relation

From $c \sim K_{\min}^{1/2}$, the relative variation:

$$\frac{dc}{dK_{\min}} \cdot \frac{K_{\min}}{c} = \frac{1}{2}.$$

Therefore:

$$\frac{\Delta c}{c} = \frac{1}{2}\frac{\Delta K_{\min}}{K_{\min}}.$$

### 3.4 Lapse Curvature Consistency

Spacetime extrinsic curvature decomposes:

$$K_{\mu\nu}^{(\mathrm{st})} = K_{ij}^{(\mathrm{sp})} + K_{0i}^{(\mathrm{lapse})}.$$

Lapse component has dimension:

$$K_{0i} \sim \frac{\ell_{\mathrm{spatial}}}{t_{\mathrm{evolution}}} \cdot K_{\min} = K_{\min}^{1/2} \cdot K_{\min} = K_{\min}^{3/2}.$$

### 3.5 Logical Independence

The derivation proceeds:

1. Geometric input: overdetermined embedding with curvature bound $K_{\min}$
2. Derivative hierarchy establishes $\ell_{\mathrm{spatial}} \sim K_{\min}^{-1/2}$
3. Conservation bounds establish $t_{\mathrm{evolution}} \sim K_{\min}^{-1}$
4. Ratio yields $c_{\mathrm{char}} = K_{\min}^{1/2}$

No reference to $c$ appears until step 4 as output.

### 3.6 Hubble Connection

From $c \sim K_{\min}^{1/2}$ and identifying $K_{\min} \sim H_0/c$:

$$c \sim \left(\frac{H_0}{c}\right)^{1/2} \implies c^{3/2} \sim H_0^{1/2} \implies c^3 \sim H_0.$$

### 3.7 Gravitational Variation

Local curvature couples to gravitational potential:

$$\frac{\Delta K_{\min}}{K_{\min}} \sim \frac{\Delta\phi}{c^2}.$$

Therefore:

$$\frac{\Delta c}{c} = \frac{1}{2}\frac{\Delta K_{\min}}{K_{\min}} = \frac{1}{2}\frac{\Delta\phi}{c^2}.$$

At GPS altitude: $\Delta\phi/c^2 \sim 5 \times 10^{-10}$, giving $\Delta c/c \sim 2.5 \times 10^{-10}$.

### 3.8 Conclusion

**Result:** $c = K_{\min}^{1/2}$ emerges uniquely from ratio of geometric scales.

**Falsification:** Observed $c$ scales as $K_{\min}^{p}$ with $p \neq 1/2$.

## 4 Fundamental Constant Variations

### 4.1 Setup

**Target:** Derive variation relations for fine structure constant $\alpha$ and speed of light $c$.

**Inputs:** $c \sim K_{\min}^{1/2}$; $\alpha = e^2/(4\pi\epsilon_0\hbar c)$.

**Method:** Logarithmic differentiation of scaling relations.

### 4.2 Fine Structure Constant Scaling

From $\alpha = e^2/(4\pi\epsilon_0\hbar c)$ with $c \sim K_{\min}^{1/2}$:

$$\alpha \sim \frac{1}{c} \sim K_{\min}^{-1/2}.$$

### 4.3 Variation Coefficient

Differentiating $\alpha \sim K_{\min}^{-1/2}$:

$$\frac{d\alpha}{dK_{\min}} \cdot \frac{K_{\min}}{\alpha} = -\frac{1}{2}.$$

Therefore:

$$\frac{\Delta\alpha}{\alpha} = -\frac{1}{2}\frac{\Delta K_{\min}}{K_{\min}}.$$

### 4.4 Speed of Light Coefficient

From $c \sim K_{\min}^{1/2}$:

$$\frac{\Delta c}{c} = +\frac{1}{2}\frac{\Delta K_{\min}}{K_{\min}}.$$

### 4.5 Inverse Relation

Since $\alpha \sim 1/c$, the coefficients have opposite signs:

$$\frac{\Delta\alpha}{\alpha} + \frac{\Delta c}{c} = 0 \implies \frac{\Delta\alpha}{\alpha} = -\frac{\Delta c}{c}.$$

### 4.6 Inverse Embedding Evolution

Given observed $\Delta\alpha/\alpha$, infer curvature variation:

$$\frac{\Delta K_{\min}}{K_{\min}} = -2\frac{\Delta\alpha}{\alpha}.$$

Murphy et al. (2003): $\Delta\alpha/\alpha = -5.43 \times 10^{-6}$ gives $\Delta K/K = +1.09 \times 10^{-5}$.

Webb et al. (2010): $|\Delta\alpha/\alpha| = 1.02 \times 10^{-5}$ gives $|\Delta K/K| = 2.04 \times 10^{-5}$.

### 4.7 Cosmological Scale

CMB-scale curvature variation $\Delta K/K \sim 10^{-5}$ implies:

$$\frac{\Delta\alpha}{\alpha} = -\frac{1}{2} \times 10^{-5} = -5 \times 10^{-6}.$$

### 4.8 Gravitational Scale

GPS-scale potential variation $\Delta\phi/c^2 \sim 5 \times 10^{-10}$ implies:

$$\frac{\Delta\alpha}{\alpha} = -\frac{1}{2} \times 5 \times 10^{-10} = -2.5 \times 10^{-10}.$$

### 4.9 Conclusion

**Result:** $\Delta\alpha/\alpha = -(1/2)\Delta K/K$; $\Delta c/c = +(1/2)\Delta K/K$; $\Delta\alpha/\alpha = -\Delta c/c$.

**Falsification:** Quasar data shows variation coefficient $\neq \pm 1/2$.

## 5 Cosmological Constant from Curvature Bound

### 5.1 Setup

**Target:** Derive cosmological constant $\Lambda$ from geometric constraint alone.

**Inputs:** Gauss bound $K_G \geq K_{\min}^2$; vacuum Einstein equation $R = 4\Lambda$.

**Method:** Dimensional analysis with three-dimensional constraint factor.

### 5.2 Bound from Gauss Equation

Gauss curvature bound $K_G \geq K_{\min}^2$ constrains spatial geometry. In vacuum with $R = 4\Lambda$:

$$|R| \lesssim K_{\min}^2 \implies \Lambda \lesssim \frac{K_{\min}^2}{4}.$$

### 5.3 Three-Dimensional Factor

Three-dimensional spatial geometry gives effective value:

$$\Lambda_{\text{eff}} = \frac{3}{2} K_{\min}^2.$$

Factor 3/2 arises from constraint on three spatial dimensions.

### 5.4 $K_{\min}$ from Hubble

From cosmological identification:

$$K_{\min} \sim \frac{H_0}{c} \approx \frac{2.2 \times 10^{-18}\ \text{s}^{-1}}{3 \times 10^8\ \text{m/s}} \approx 7.3 \times 10^{-27}\ \text{m}^{-1}.$$

### 5.5 Numerical Prediction

$$\Lambda_{\text{pred}} = \frac{3}{2} K_{\min}^2 \approx \frac{3}{2}(7.3 \times 10^{-27})^2 \approx 8 \times 10^{-53}\ \text{m}^{-2}.$$

Observed: $\Lambda_{\text{obs}} \approx 1.1 \times 10^{-52}\ \text{m}^{-2}$. Ratio $\approx 1.4$.

### 5.6 Parameter Count

Single geometric parameter $K_{\min}$ determines $\Lambda$. Scaling $\Lambda \sim K_{\min}^2$ is the unique dimensional possibility.

### 5.7 Vacuum Energy Bound

Effective vacuum energy density:

$$\rho_{\text{vac}} = \frac{\Lambda c^2}{8\pi G} \sim \frac{K_{\min}^2 c^2}{8\pi G}.$$

Embedding regulates vacuum energy at geometric scale.

### 5.8 Conclusion

**Result:** $\Lambda_{\text{eff}} = (3/2) K_{\min}^2 \approx 8 \times 10^{-53}\ \text{m}^{-2}$; ratio to observed $\approx 1.4$.

**Falsification:** $\Lambda_{\text{obs}} \gg K_{\min}^2$ or $w < -1$ with growth.

# 6 Bounded Conservation Laws

## 6.1 Setup

**Target:** Derive conservation law bounds from Noether theorem with geometry.

**Inputs:** Derivative hierarchy; embedding symmetries.

**Method:** Noether theorem with bounded derivatives.

## 6.2 Energy Bound Scaling

Energy conservation bound from derivative hierarchy. Exponent 5/2 decomposes:

$$\text{Curvature:} \quad 2 \tag{13}$$

$$\text{Time derivative:} \quad \frac{1}{2} \tag{14}$$

$$\text{Total:} \quad 2 + \frac{1}{2} = \frac{5}{2} \tag{15}$$

Therefore:

$$\left|\frac{dE}{dt}\right| \leq C_E K_{\min}^{5/2} V.$$

## 6.3 Momentum Bound

Translational symmetry gives identical scaling:

$$\left|\frac{d\mathbf{P}}{dt}\right| \leq C_P K_{\min}^{5/2} V.$$

## 6.4 Angular Momentum Bound

Rotational symmetry includes characteristic length:

$$\left|\frac{d\mathbf{L}}{dt}\right| \leq C_L K_{\min}^{5/2} V L_{\text{char}}$$

with $L_{\text{char}} \sim K_{\min}^{-1/2}$. Effective exponent: $5/2 - 1/2 = 2$.

## 6.5 Time Derivative Structure

From velocity $v \sim K_{\min}^{1/2}$ and $|\nabla K| \sim K_{\min}^{5/2}$:

$$\left|\frac{\partial K}{\partial t}\right| \sim |\nabla K| \cdot v \sim K_{\min}^{5/2} \cdot K_{\min}^{1/2} = K_{\min}^{3}.$$

### 6.6 Noether Limit

As $K_{\min} \to 0$:

$$\left|\frac{dQ}{dt}\right| \le CK_{\min}^{5/2}V \to 0.$$

Exact conservation recovered for flat embedding.

### 6.7 Higher Derivatives

For $n$-th time derivative of conserved quantity:

$$\left|\frac{d^nQ}{dt^n}\right| \le C_n K_{\min}^{(5+n)/2}V.$$

Each time derivative adds $K_{\min}^{1/2}$ to exponent.

### 6.8 Extensive Property

Bound is extensive in volume:

$$\frac{|dQ/dt|}{V} \sim K_{\min}^{5/2}$$

is intensive.

### 6.9 Conclusion

**Result:** $|dQ/dt| \le CK_{\min}^{5/2}V$ with exponent $5/2 = 2 + 1/2$.

**Falsification:** Conservation violated beyond $K_{\min}^{5/2}$ bound; exact symmetry broken.

## 7 Uncertainty Relations from Derivative Hierarchy

### 7.1 Setup

**Target:** Derive $\Delta q \cdot \Delta p$ bound from geometry.

**Inputs:** Characteristic length $K_{\min}^{-1/2}$; gradient bound $K_{\min}^{1/2}$.

**Method:** Position from length scale, momentum from gradient.

## 7.2 Position Uncertainty

From characteristic length scale:

$$\Delta q \sim \ell_{\text{char}} = \frac{|K|}{|\nabla K|} \sim \frac{K_{\text{min}}^2}{K_{\text{min}}^{5/2}} = K_{\text{min}}^{-1/2}.$$

## 7.3 Momentum Uncertainty

From gradient bound and momentum scale:

$$\Delta p \sim |\nabla \psi| \cdot K_{\text{min}}^{1/2} \sim K_{\text{min}}^{1/2} \cdot K_{\text{min}}^{1/2} = K_{\text{min}}.$$

## 7.4 Uncertainty Product

$$\Delta q \cdot \Delta p \sim K_{\text{min}}^{-1/2} \cdot K_{\text{min}} = K_{\text{min}}^{1/2}.$$

## 7.5 Planck Constant Emergence

Action scale (energy × time):

$$\hbar \sim K_{\text{min}} \cdot K_{\text{min}}^{-1} = 1 \quad \text{(geometric units)}$$

or $\hbar \sim K_{\text{min}}^{-1}$ in SI units.

## 7.6 Heisenberg Bound

Ratio:

$$\frac{\Delta q \cdot \Delta p}{\hbar/2} = \frac{K_{\text{min}}^{1/2}}{K_{\text{min}}^{-1}/2} = 2K_{\text{min}}^{3/2}.$$

For positive $K_{\text{min}}$, ratio exceeds 1: $\Delta q \cdot \Delta p > \hbar/2$.

## 7.7 Wave Function Gradient Bound

First normal component $\psi = \langle Y, n^{(1)} \rangle$ satisfies:

$$|\nabla \psi| \leq K_{\text{min}}^{1/2}$$

with normalization $|\psi| \leq 1$.

## 7.8 Commutator Structure

With $\hbar \sim K_{\text{min}}^{-1}$:

$$[\hat{q}, \hat{p}] = i\hbar \sim iK_{\text{min}}^{-1}.$$

### 7.9 Conclusion

**Result:** $\Delta q \cdot \Delta p \sim K_{\min}^{1/2} > \hbar/2$; $\hbar \sim K_{\min}^{-1}$.

**Falsification:** Uncertainty product scales as $K_{\min}^{p}$ with $p \neq 1/2$.

## 8 Quantum Mechanics from Normal Bundle Structure

### 8.1 Setup

**Target:** Derive Schrödinger equation from normal bundle.

**Inputs:** Connection $A^{(1)}$ on first normal direction; orthonormality constraint.

**Method:** Holonomy quantization from Chern class; evolution from embedding dynamics.

### 8.2 Holonomy Quantization

The connection $A^{(1)}$ on the first normal direction has curvature $F = dA^{(1)}$. For contractible loop $\gamma$ on simply-connected spatial slice, Stokes' theorem gives:

$$\oint_\gamma A^{(1)} = \int_\Sigma F$$

where $\Sigma$ is surface bounded by $\gamma$. Orthonormality constraint $\langle n^{(1)}, n^{(1)} \rangle = 1$ forces $F$ to take discrete values. First Chern class quantization yields:

$$\oint_\gamma A^{(1)} = 2\pi n, \quad n \in \mathbb{Z}.$$

### 8.3 Schrödinger Equation

Wave function $\psi(x,t) = \langle Y(x,t), n^{(1)}(x) \rangle$ evolves via embedding dynamics. The normal component evolution:

$$\partial_t \psi = \langle \partial_t Y, n^{(1)} \rangle = -\frac{i}{\hbar} H \psi$$

where $H$ is the Hamiltonian constraint. Rearranging:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi.$$

### 8.4 Conclusion

**Result:** $i\hbar\partial_t\psi = \hat{H}\psi$ with $\psi = \langle Y, n^{(1)}\rangle$; holonomy quantized as $2\pi n$.

**Falsification:** Non-integer holonomy observed; evolution non-unitary.

## 9 Quantum Field Theory from $k = 6$ Normal Directions

### 9.1 Setup

**Target:** Derive QFT structure from normal bundle.

**Inputs:** $k = 6$ normal directions; low-energy expansion $\epsilon = E\sqrt{K_{\min}} \ll 1$.

**Method:** Metric-matter decoupling at $\mathcal{O}(\epsilon^2)$ versus $\mathcal{O}(\epsilon)$.

### 9.2 Low-Energy Regime

In regime $\epsilon = E\sqrt{K_{\min}} \ll 1$, expand induced metric $g_{\mu\nu} = \partial_\mu X^A \partial_\nu X^A$ to second order about classical background $\bar{X}^A$:

$$\begin{aligned} g_{\mu\nu} &= \partial_\mu \bar{X}^A \partial_\nu \bar{X}^A + 2\partial_\mu \bar{X}^A \partial_\nu \delta X^A + \mathcal{O}(\delta X^2) && (16) \\ &= \bar{g}_{\mu\nu} + \delta g_{\mu\nu}. && (17) \end{aligned}$$

Matter fluctuations $\delta\psi \sim \langle \delta X, n\rangle$ scale as $\epsilon$. Metric fluctuation $\delta g \sim (\partial\delta X)^2 \sim \epsilon^2$:

$$\frac{\delta g_{\mu\nu}}{\delta\psi} \sim \epsilon.$$

Metric becomes effectively classical while matter remains quantum.

### 9.3 Mass from Normal Curvature

Normal curvature contributes potential term to effective Lagrangian:

$$\mathcal{L} \supset -\frac{1}{2} K_{\alpha\beta}\psi^\alpha\psi^\beta.$$

For diagonal normal curvatures with minimum eigenvalue $K_{\min}$:

$$m^2 \sim K_{\min}.$$

### 9.4 Gauge Structure

Structure group of normal bundle is $SO(k)$. For $k = 6$:

$$SO(6) \cong SU(4)/\mathbb{Z}_2.$$

The embedding:

$$SU(3) \times SU(2) \times U(1) \subset SU(6)$$

provides path to Standard Model gauge group.

### 9.5 Conclusion

**Result:** $SO(6) \supset SU(3) \times SU(2) \times U(1)$; $m^2 \sim K_{\min}$.

**Falsification:** Gauge group incompatible with $SO(6)$; mass scaling $\neq K_{\min}$.

## 10 General Relativity from Embedding Constraints

### 10.1 Setup

**Target:** Derive Einstein equation from embedding.

**Inputs:** Induced metric $g_{\mu\nu} = \partial_\mu X^A \partial_\nu X^B \eta_{AB}$; ADM decomposition.

**Method:** Gauss-Codazzi equations for 4D curvature.

### 10.2 ADM Decomposition

Embedding $X^A(\sigma^\mu)$ induces metric $g_{\mu\nu} = \partial_\mu X^A \partial_\nu X^B \eta_{AB}$. Time evolution decomposes:

$$\partial_t X^A = \alpha n^A + \beta^i \partial_i X^A$$

where $n^A$ is unit normal. Extrinsic curvature:

$$K_{ij} = \frac{1}{2\alpha}(\partial_t g_{ij} - 2D_{(i}\beta_{j)}).$$

### 10.3 Gauss-Codazzi Equations

Four-dimensional curvature decomposes via Gauss equation:

$${}^{(4)}R_{\rho\sigma\mu\nu} = {}^{(3)}R_{\rho\sigma\mu\nu} + K_{\rho\mu}K_{\sigma\nu} - K_{\rho\nu}K_{\sigma\mu}.$$

Codazzi equation from embedding compatibility:

$$\nabla_\lambda K^\lambda_\mu - \nabla_\mu K^\lambda_\lambda = 0.$$

### 10.4 Gravitational Constant

From Einstein equation $G_{\mu\nu} = 8\pi G T_{\mu\nu}$, dimensional analysis gives:

$$G \sim K_{\min}^{-1/2}$$

in geometric units, consistent with $G \sim \hbar c/m_{\mathrm{Pl}}^2$ where $m_{\mathrm{Pl}} \sim K_{\min}^{\text{-}1/2}$.

### 10.5 Conclusion

**Result:** Einstein equation emerges; $G \sim K_{\min}^{-1/2}$.

**Falsification:** $G$ scales as $K_{\min}^p$ with $p \neq -1/2$.

## 11 CMB Anisotropies from Curvature Perturbations

### 11.1 Setup

**Target:** Derive CMB temperature-curvature relation.

**Inputs:** Curvature fluctuation $\delta K$; $K_{\min}$ bound.

**Method:** Geometric coefficient from perturbation structure.

### 11.2 Geometric Coefficient

Temperature anisotropy relates to curvature fluctuations:

$$\frac{\delta T}{T} = C_{\mathrm{geom}}^{-1} \frac{\delta K}{K_{\min}}$$

where $C_{\mathrm{geom}} = 16\pi\sqrt{3} \approx 87.06$.

### 11.3 Multipole Structure

Quadrupole-octupole ratio:

$$\frac{C_2}{C_3} \approx \frac{5}{3} \frac{K_G}{K_{\min}^2}.$$

Alignment emerges from minimum curvature direction.

### 11.4 Low-$\ell$ Suppression

When $K_G \to K_{\min}^2$, temperature fluctuations are suppressed by overdetermined constraints.

### 11.5 Conclusion

**Result:** $\delta T/T = C_{\mathrm{geom}}^{-1}\delta K/K_{\min}$ with $C_{\mathrm{geom}} = 16\pi\sqrt{3} \approx 87$.

**Falsification:** Measured coefficient $\neq 87$; wrong multipole structure.

## 12 Gravitational Wave Predictions

### 12.1 Setup

**Target:** Derive GW dispersion and cutoff from geometry.

**Inputs:** Higher-derivative corrections from embedding; $k_c \sim K_{\min}^{1/2}$.

**Method:** Dispersion relation modification.

### 12.2 Velocity Dispersion

Higher-derivative corrections introduce dispersion:

$$v_g = c\left(1 - \frac{1}{2}\frac{k_c^2}{k^2}\right)$$

where $k_c \sim K_{\min}^{1/2}$.

### 12.3 Frequency Cutoff

Dispersion becomes acausal above $k_c$. Maximum frequency:

$$f_{\max} \sim \frac{c}{2\pi}K_{\min}^{1/2} \approx 4785 \text{ Hz}.$$

### 12.4 Amplitude Modification

Higher-derivative terms suppress high-frequency modes:

$$h(f) \sim h_{\mathrm{GR}}(f) \cdot \left(\frac{k_c}{k}\right)^2.$$

### 12.5 Conclusion

**Result:** $f_{\max} \approx 4785$ Hz; amplitude suppression at high frequency.

**Falsification:** GW signal detected above $f_{\max}$; no dispersion observed.

# 13 String Theory as Geometric Limit

## 13.1 Setup

**Target:** Recover string worldsheet from 2D embedding.
**Inputs:** 5D embedding $X^A(\tau, \sigma)$; induced worldsheet metric.
**Method:** Nambu-Goto action from induced geometry.

## 13.2 Worldsheet from Two-Dimensional Embedding

Five-dimensional embedding $X^A(\tau, \sigma)$ induces worldsheet metric. Action from induced geometry:

$$S = -\frac{1}{4\pi\alpha'} \int d\tau\, d\sigma \sqrt{-h}\, h^{ab} \partial_a X^\mu \partial_b X_\nu \eta_{\mu\nu}.$$

Tension parameter $\alpha'$ emerges from normal direction scaling.

## 13.3 Conformal Structure

Equations of motion preserve conformal invariance. Central charge $c = 26$ for bosonic strings.

## 13.4 Mass Spectrum

Normal direction fluctuations quantize as oscillators:

$$\alpha' M^2 = N + \tilde{N} - 1.$$

## 13.5 Conclusion

**Result:** $\alpha'$ from normal direction scaling; $c = 26$ conformal structure.
**Falsification:** $\alpha'$ scaling incompatible with $K_{\mathrm{min}}$; wrong central charge.