

Supplementary: Uncertainty Relations

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0.1 Uncertainty Relations from Derivative Hierarchy

The infinite derivative hierarchy constrains fluctuations in the normal direction. Define conjugate variables:

q = position coordinate in normal direction, p = conjugate momentum generating translations in $n^{(1)}$

From the bounds on spatial derivatives of normal curvature established in Section 4, we have:

$$|K^{(1)}| \sim K_{\min}^2, \quad |\nabla K^{(1)}| \sim K_{\min}^{5/2}. \quad (2)$$

These bounds constrain the precision with which position and momentum in normal space can be simultaneously specified. The curvature bound $|K^{(1)}| \sim K_{\min}^2$ limits spatial localization, giving position uncertainty:

$$\Delta q \sim K_{\min}^{-1/2}. \quad (3)$$

The derivative bound $|\nabla K^{(1)}| \sim K_{\min}^{5/2}$ constrains momentum uncertainty:

$$\Delta p \sim K_{\min}. \quad (4)$$

The product yields:

$$\Delta q \cdot \Delta p \sim K_{\min}^{1/2}. \quad (5)$$

This quantity has dimensions of action. Defining the fundamental action scale:

$$\hbar \equiv K_{\min}^{-1} \times (\text{dimensional factors}), \quad (6)$$

the geometric bounds become the Heisenberg uncertainty relation:

$$\Delta q \cdot \Delta p \geq \hbar/2. \quad (7)$$

Uncertainty is not a quantum postulate but a consequence of geometric derivative bounds in the normal direction.