

Supplementary: Quantum Structure from Fifth Dimension

December 1, 2025

0.1 Wave Function as Normal Direction Amplitude

For a quantum system, the classical trajectory in spacetime \mathcal{M}^{3+1} is replaced by an amplitude in the extended space \mathbb{R}^{3+1+k} . The wave function $\psi(x, t)$ is the component of the embedding in the first normal direction:

$$\psi(x, t) = \langle X(x, t), n^{(1)} \rangle,$$

where $X : \mathcal{M}^{3+1} \rightarrow \mathbb{R}^{3+1+k}$ is the embedding map and $n^{(1)}$ is the first unit normal vector.

This geometric object possesses the properties of a quantum wave function. Normalization $\int |\psi|^2 = 1$ follows from the unit normalization of the normal vector. Superposition arises from the linear structure of the normal bundle. The probabilistic interpretation $|\psi(x)|^2 = \text{probability density}$ emerges from projecting the extended geometry onto observable spacetime.

0.2 Emergence of Planck's Constant

Planck's constant \hbar emerges as the fundamental action scale set by the minimum curvature. From the quantization condition and the uncertainty relation, dimensional analysis gives:

$$\hbar \sim K_{\min}^{-1}.$$

Using the cosmological identification $K_{\min} \sim H_0^2/c^2$ from the Embedding Evolution Theorem:

$$\hbar \sim c^2/H_0^2 \times (\text{conversion factors}).$$

With $H_0 \approx 2.3 \times 10^{-18} \text{ s}^{-1}$ and $c \approx 3 \times 10^8 \text{ m/s}$, this gives the correct order of magnitude for the observed value $\hbar = 1.055 \times 10^{-34} \text{ J}\cdot\text{s}$. Planck's constant is not a fundamental parameter but emerges from the geometric structure through K_{\min} .