Supplementary: Evolving Manifold Theorems (EMT)

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Evolving Manifold Theorems (EMT) establish the interface between spatial embedding geometry and temporal evolution, transforming geometric structure into time-coupled predictions. Overdetermined Riemannian embeddings predict the speed of light is bounded rather than constant, with $c \sim K_{\min}^{1/2}$ where $K_{\min}$ is the minimum curvature scale.

0.1 Embedding Evolution Theorem

Consider an n-dimensional Riemannian manifold $\mathcal{M}^n$ embedded in $\mathbb{R}^{n+k}$ with $k > n^2-n-1$ orthonormal normal directions. The Gauss-Codazzi-Ricci equations overdetermine the geometry, forcing a curvature bound $K_G \geq K_{\min}^2$.

Embedding Evolution Theorem relates spatial geometry to temporal evolution:

$$c_{\text{char}} = \frac{\ell_{\text{spatial}}}{t_{\text{evolution}}} = K_{\min}^{1/2}.$$

This characteristic velocity emerges from geometric compatibility—the requirement that spatial slices evolve consistently within the embedding. It is not a postulate but a derived quantity.

0.2 Geometric Origin of Scales

The spatial and temporal scales arise from the derivative hierarchy bounds. From the Infinite Derivative Hierarchy Theorem:

$$|\nabla^m K_{ab}^{(\alpha)}| \leq C_m K_{\min}^{2+m/2}.$$

Each derivative order m defines a characteristic length scale:

- $m = 0$: Curvature scale $\ell_0 = K_{\text{min}}^{-1}$ (Hubble radius)
- $m = 1$: First derivative scale $\ell_1 = K_{\text{min}}^{-1/2}$
- $m = 2$: Second derivative scale $\ell_2 = K_{\text{min}}^{-1}$

The spatial scale is identified with the first derivative scale:

$$\ell_{\text{spatial}} = \ell_1 = K_{\text{min}}^{-1/2}.$$

This is the scale at which spatial gradients of curvature become dynamically relevant.

The temporal scale is identified with the curvature scale:

$$t_{\text{evolution}} = \ell_0 = K_{\text{min}}^{-1}.$$

This is the characteristic timescale for Hamiltonian evolution on the embedding.

Taking the ratio:

$$c_{\text{char}} = \frac{\ell_{\text{spatial}}}{t_{\text{evolution}}} = \frac{K_{\text{min}}^{-1/2}}{K_{\text{min}}^{-1}} = K_{\text{min}}^{1/2}.$$

0.3 Dimensional Self-Consistency

For cosmological K_{min}, we identify:

$$K_{\text{min}} \sim \frac{H_0}{c},$$

where H_0 is the Hubble constant. Substituting into the characteristic velocity:

$$c_{\text{char}} = K_{\text{min}}^{1/2} = \left(\frac{H_0}{c}\right)^{1/2}.$$

Physical consistency requires $c = c_{\text{char}}$ (the speed of light equals the geometric compatibility velocity):

$$c = \left(\frac{H_0}{c}\right)^{1/2}.$$

Squaring both sides:

$$c^2 = \frac{H_0}{c}.$$

Multiplying by c:

$$c^3 = H_0.$$

In natural units where $c = 1$, this gives $K_{\text{min}} = H_0$: the minimum curvature equals the Hubble scale.

The speed of light emerges as a derived quantity determined by the embedding geometry through Hubble-scale curvature. This self-consistency condition provides a geometric explanation for the value of c.

0.4 Derivation of Speed of Light Variation

From $c \sim K_{\text{min}}^{(1/2)}$:

$$\frac{\Delta c}{c} = \frac{1}{2}\frac{\Delta K_{\text{min}}}{K_{\text{min}}}.$$

The fine structure constant depends on c:

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}.$$

Holding e, ϵ_0, and $\hbar$ fixed (as they are set by embedding geometry at different scales):

$$\frac{\Delta\alpha}{\alpha} = -\frac{\Delta c}{c}.$$

Therefore:

$$\frac{\Delta c}{c} = -\frac{\Delta\alpha}{\alpha}.$$

0.5 Predictions from Quasar Spectroscopy Data

Murphy et al. measured $\Delta\alpha/\alpha = (-0.543 \pm 0.116) \times 10^{-5}$ at $z \sim 2$. This implies:

$$\frac{\Delta c}{c} = +0.543 \times 10^{-5}.$$

Numerically:

$$\Delta c = 0.543 \times 10^{-5} \times 3 \times 10^8 \text{ m/s} \approx +1600 \text{ m/s}.$$

Speed of light was larger at $z = 2$ than today by approximately 1600 m/s.

Webb et al. found a spatial dipole with amplitude $|\Delta\alpha/\alpha| = 1.0 \times 10^{-5}$. This implies:

$$|\Delta c| \approx 3000 \text{ m/s}$$

spatial variation across the sky.

0.6 Inverse Embedding Evolution Theorem

The spatial curvature scale $K_{\min}$ is determined by the temporal evolution velocity:

$$K_{\min} = \frac{c}{c_{\text{char}}} \times K_{\text{cosmological}}.$$

This establishes that temporal observations determine the spatial curvature scale.

0.7 EEP Theorem (Einstein Equivalence Principle)

Equivalence principle emerges from embedding geometry when $k \geq 3$ normal directions. For $k = 3$, the embedding $\mathcal{M}^{3+1} \subset \mathbb{R}^{3+1+3}$ has sufficient normal directions to represent different inertial frames. Normal bundle provides the structure needed for locally inertial coordinates. Gauss-Codazzi compatibility ensures that physics is locally equivalent in all frames.

0.8 Velocity Universality

The characteristic velocity c_{char} is independent of the choice of spatial foliation. The embedding geometry is defined in the ambient space $\mathbb{R}^{n+1+k}$. Spatial foliations correspond to different ways of decomposing the $(n+1)$-dimensional manifold. Gauss-Codazzi equations are tensor equations independent of coordinate choices.

0.9 Experimental Tests

Modern optical lattice clocks achieve fractional precision $\sim 10^{-18}$. The predicted effect is $\Delta c/c \sim 10^{-5}$, which is 10^{13} times larger than clock precision.

Proposed tests:

- Compare clocks at different gravitational potentials (different local $K_{\min}$)
- Compare clocks separated by large distances (sample spatial dipole)
- Monitor temporal drift over years (secular evolution of $K_{\min}$)

Experimental challenge lies in systematic control: distinguishing c variation from gravitational redshift and other known effects.

0.10 Falsification Condition

If clock comparisons establish $|\Delta c/c| < 10^{-10}$ with no directional dependence, overdetermined embedding is falsified.

0.11 Physical Implications

EMT theorems establish that fundamental constants are environmental variables determined by local embedding geometry. Speed of light varies with $K_{\min}$, producing observable effects at cosmological scales. This provides a geometric explanation for the fine-tuning problems in physics, where constants emerge from embedding structure rather than being arbitrarily set.