LEI / LCC
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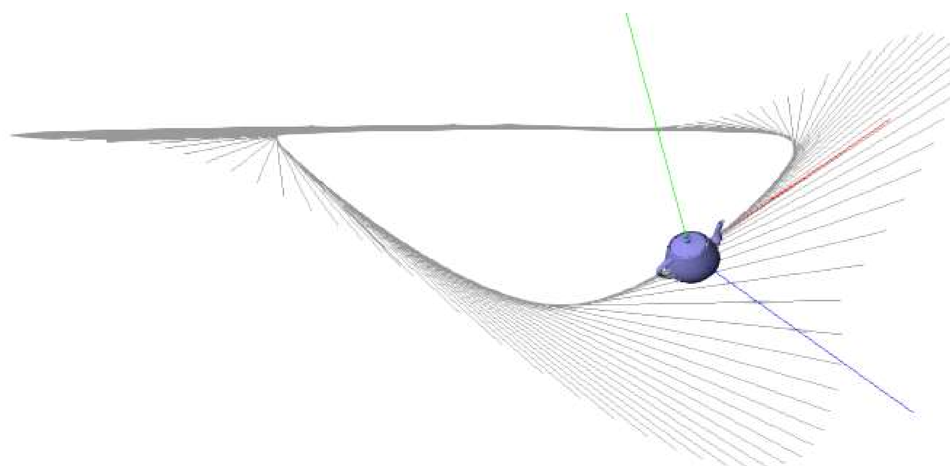
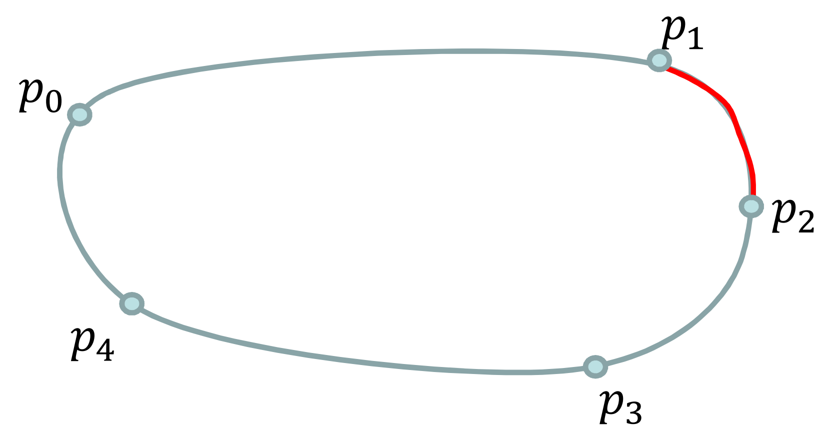
Animation with Catmull-Rom Curves



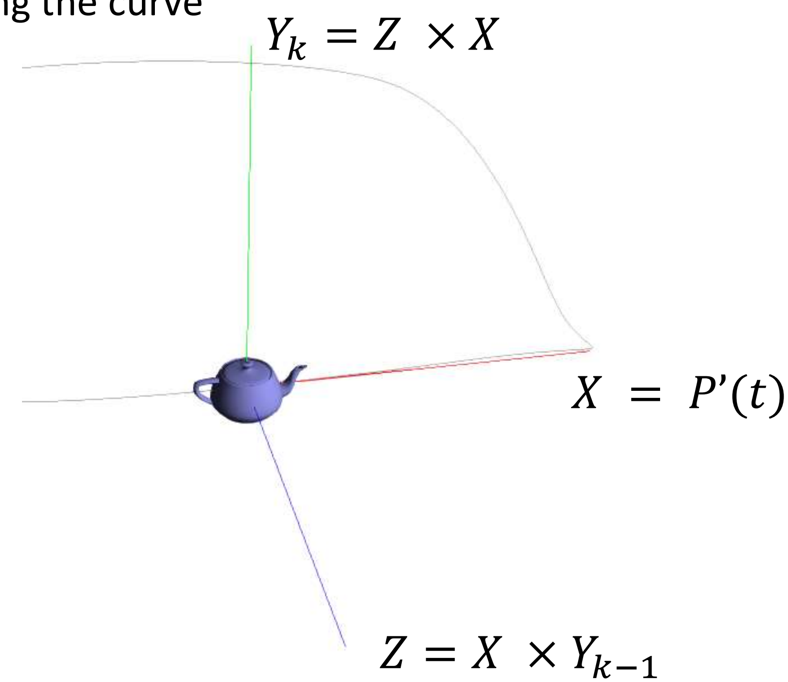
Cubic Curves – Catmull-Rom

- Matrix formulation
  - $P(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} -0.5 & 1.5 & -1.5 & 0.5 \\ 1 & -2.5 & 2 & -0.5 \\ -0.5 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_o \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$
  - $P'(t) = \begin{bmatrix} 3t^2 & 2t & 1 & 0 \end{bmatrix} \begin{bmatrix} -0.5 & 1.5 & -1.5 & 0.5 \\ 1 & -2.5 & 2 & -0.5 \\ -0.5 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_o \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$



Cubic Curves – Catmull-Rom

- Axis for Rotation Matrix
    - Available data at instant t
        - $P(t)$ - position of an object “walking” along the curve
        - $P’(t)$ - vector tangent to the curve
    - Transform for teapot
        - Translation to place teapot
        - Rotation to align with curve
        - $Y_0 = (0,1,0)$



Cubic Curves – Catmull-Rom

- Assuming an initial specification of an $\overrightarrow{up_0}$ vector, to align the object with the curve, we need to build a rotation matrix for the object:

$$\begin{aligned}\vec{X} &= P'(t)\\ \vec{Z} &= X \times \vec{Y}_{i-1}\\ \vec{Y_i} &= \vec{Z} \times \vec{X}\end{aligned} \qquad M = \begin{bmatrix} X_x & Y_x & Z_x & 0\\ X_y & Y_y & Z_y & 0\\ X_z & Y_z & Z_z & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

```
glMultMatrixf(float *m)
```

- Current matrix gets multiplied by `m`

Note: OpenGL matrices are column major => compute the transpose instead

Assignment

- Complete the function

```
void getCatmullRomPoint(float t,
                        float *p0, float *p1, float *p2, float *p3,
                        float *pos, float *deriv) {

     // catmull-rom matrix
     float m[4][4] = {   {-0.5f,  1.5f, -1.5f,  0.5f},
                         { 1.0f, -2.5f,  2.0f, -0.5f},
                         {-0.5f,  0.0f,  0.5f,  0.0f},
                         { 0.0f,  1.0f,  0.0f,  0.0f}};

     // Compute A = M * P
     // Compute pos = T * A
     // compute deriv = T' * A
     // ...
}
```

Assignment

- Write the function

```
void renderCatmullRomCurve() {

// draw the curve using line segments - GL_LINE_LOOP
}
```

To get the points for the full curve call

```
void getGlobalCatmullRomPoint(float gt, float *pos, float *deriv)
```

with `gt` in [0,1[.

- Apply the required transformations to have the teapot travelling along the curve oriented accordingly to the derivative.
  - Use `buildRotMatrix` provided in the source code