

# Cheng Zhang

## Research Statement

✉ [czhang03@bu.edu](mailto:czhang03@bu.edu)  
🌐 [cs-people.bu.edu/czhang03/](https://cs-people.bu.edu/czhang03/)  
🔗 [czhang03](#)  
📄 Curriculum vitae

*"Simplicity is a great virtue." - Edsger W. Dijkstra (1984)*

Computer science as a field is facing many complex problems in today's ever-developing world. Instead of directly attacking these convoluted questions, I believe in building a wealth of mathematical knowledge on simple foundational problems, and compose their solutions to conquer real-world challenges in a clean and efficient manner.

Guided by these goals, my PhD study focuses on three fundamental theories in computer science: Kleene algebra (KA), coalgebra, and automata theory. I was fascinated not only by their elegant mathematical structures and simple formulations, but also by their vast applications across seemingly unrelated domains. Hence, theory and practice always go hand-in-hand in my research: whenever we discover an application, we seek to perfect its theory; and similarly, when a new theory is developed, we will thoroughly explore its use cases in the real-world.

### TopKAT: A Unified View Of Program Logic

Incorrectness logic [14], although simple, has shown great potential for bug detection across various semantical domains [17, 12, 24]. To unify the theory of incorrectness logic in these semantical domains, we tried to use Kleene algebra with tests (KAT) as a simple and abstract semantical foundations. This task was soon proven to be impossible [23]: we showed that the theory of KAT is insufficient to encode incorrectness logic, mainly because KAT lacks the relational domain operation. Surprisingly, instead of adding a fully-fledged domain operator, we only need to add a top element. We named our framework Kleene algebra with top and tests (TopKAT), and unlike KAT extensions with domain operators [5, 19], TopKAT preserves the complexity class of KAT [23].

However, upon diving into the theory of TopKAT, we discovered its unexpected limitation: despite its power to subsume both propositional Hoare and incorrectness logic, TopKAT is incomplete with respect to its relational model. Our followup work [22] resolves this weakness by focusing on the inequalities used to encode incorrectness and Hoare logic, which we named "domain-comparison inequalities". In this work, we used techniques in universal algebra to streamline the definition of reduction [16, 10]. This new perspective greatly simplified previous completeness proofs, and also allowed us to prove the relational completeness with regard domain-comparison inequalities. This result has not only demonstrated the effectiveness of reasoning about incorrectness and Hoare logics using TopKAT, but also other logics like reachability logic [13] as well.

In the future, we plan to extend the universal algebra techniques in this work to prove more complicated completeness results. We hope this will yield another compositional framework for completeness proof in Kleene Algebra.

## Kleene Algebra With Commutativity Hypothesis

Commutativity hypothesis has long been recognized for its importance in control-flow analysis [7], yet recent work [1] has also established its vital role in relational verification. Contrary to its broad applications, the theory of KA with commutativity hypothesis remains stale; specifically, the decidability of the theory has made no progress since the question was raised by Kozen [7].

Independently, Kuznetsov [11] has shown that Kleene Algebra with commutativity is indeed undecidable. We, on the other hand, has shown the same result without using the induction or right unfolding rule [2]. Our result exhibits a large class of equational theories that are all undecidable when extended commutativity hypothesis, generalizing the result of Kuznetsov.

This work settles a long-standing open problem in Kleene algebra, and also demonstrates the limitation of relational reasoning with algebra of alignment [1]. In fact, we envision a decidable, yet less robust, extension of Kleene Algebra useful for reasoning about alignment problems in relational verifications.

## Ongoing Works

### CF-GKAT, control flow verification in nearly linear time

Control-flow manipulation is a prevalent task in software engineering, thus verifying its correctness is crucial to ensure software reliability. Our work, building upon foundational researches on guarded Kleene algebra with tests (GKAT) [20] and the theory of non-local control flow in Kleene algebra [8], greatly simplifies process of control-flow verification.

Specifically, We extended GKAT with common control structures, including break, return, goto, and indicator variables; while preserving its efficiency, soundness, and completeness. These extensions enable us to verify a large class of control-flow restructuring algorithms [21, 3, 6, 9]. And its efficiency, soundness, and completeness allow our works to be invoked automatically on-the-fly, or be used as a framework in a proof assistant.

### Theory and practice of symbolic GKAT

Although GKAT is extremely efficient in some use cases, its efficiency can be improved in many other scenarios. For example, when there is a large amount of primitive test (primitive conditional statements used in if-statement and while-loops), the memory usage and runtime of the original algorithm [20] will blowup exponentially. The large memory usage is typically resolved using derivatives to produce the automaton on-the-fly [4, 18], whereas the long runtime can be optimized using symbolic automaton [15].

Our latest work marries these two ideas, and built a theory of symbolic guarded Kleene coalgebra with tests (sGKCT), where we use category theory to streamline some languages in previous works of symbolic automata, and designed an efficient derivative-based symbolic decision procedure for GKAT. Unlike similar works on KAT [15], the structure of GKAT enables us to export the complex boolean logic into a fast and reliable solvers like z3; further improving the efficiency of our implementation.

This work also characterized the exact complexity of GKAT.

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