ECE 5460 Image Processing AU2018
Project #1
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Linear Camera Calibration Methodology

Given a set of 2D and 3D points, the intrinsic and extrinsic parameters of cameras can been linearly estimated. In the homogenous system, suppose we have 2D point $p = (u, v, 1)^T$ on an image and its corresponding 3D point $P = (X, Y, Z, 1)^T$ in 3D space:

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = M \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

where *M* is a 3x4 projection matrix with

$$x = \frac{u_i}{w_i} = \frac{m_{11}X_i + m_{12}Y_i + m_{13}Z_i + m_{14}}{m_{31}X_i + m_{32}Y_i + m_{33}Z_i + m_{34}} \qquad y = \frac{v_i}{w_i} = \frac{m_{21}X_i + m_{22}Y_i + m_{23}Z_i + m_{24}}{m_{31}X_i + m_{32}Y_i + m_{33}Z_i + m_{34}}$$

Given N pairs of 3D and 2D coordinates, we have

$$A\mathbf{m} = 0$$

where

$$A = \begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 & -x_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -y_1X_1 & -y_1Y_1 & -y_1Z_1 & -y_1 \\ \vdots & \vdots & & & \vdots & & \vdots & & & \vdots & \vdots & \\ & & & & & & & & & & & \\ X_N & Y_N & Z_N & 1 & 0 & 0 & 0 & 0 & -x_NX_N & -x_NY_N & -x_NZ_N & -x_N \\ 0 & 0 & 0 & 0 & X_N & Y_N & Z_N & 1 & -y_NX_N & -y_NY_N & -y_NZ_N & -y_N \end{bmatrix}$$

and

$$m = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{33} & m_{34} \end{bmatrix}$$

Since ***A*** has rank 11, the vector ***m*** can be recovered from SVD related techniques as the column of V corresponding to the zero (in practice the smallest) singular value of ***A***. This algorithm can be illustrated in details in the Matlab code.

As the multiplication of intrinsic camera matrix and extrinsic camera matrix, after divided by a scale factor $\sqrt{m_{31}^2 + m_{32}^2 + m_{33}^2}$ the expression of M can be written as

$$M = \begin{bmatrix} s_x f r_1 + c_0 r_3 & s_x f r_1 + c_0 r_3 \\ s_x f r_1 + c_0 r_3 & s_x f r_1 + c_0 r_3 \\ s_x f r_1 + c_0 r_3 & s_x f r_1 + c_0 r_3 \end{bmatrix}$$

Now let

$$q_1 = [m_{11} \quad m_{12} \quad m_{13}]^T \qquad q_2 = [m_{21} \quad m_{22} \quad m_{23}]^T \qquad q_3 = [m_{31} \quad m_{32} \quad m_{33}]^T$$

Then we have

$$r_3 = q_3^T \qquad t_z = m_{34} \qquad c_0 = q_1^T q_3 \qquad r_0 = q_2^T q_3$$

$$s_x f = \pm\sqrt{q_1^T q_1 - c_0^2} \qquad s_y f = \pm\sqrt{q_2^T q_2 - r_0^2}$$

$$t_x = (m_{14} - c_0 t_z)/(s_x f) \qquad t_x = (m_{14} - c_0 t_z)/(s_x f)$$

$$r_1 = (q_1^T - c_0 r_3)/(s_x f) \qquad r_2 = (q_2^T - r_0 r_3)/(s_y f)$$

The above equations solve all the camera parameters.

Experimental Results

We randomly chose several points in the Fig. 1 and Fig. 2 as 2D references. The visual results are as follows

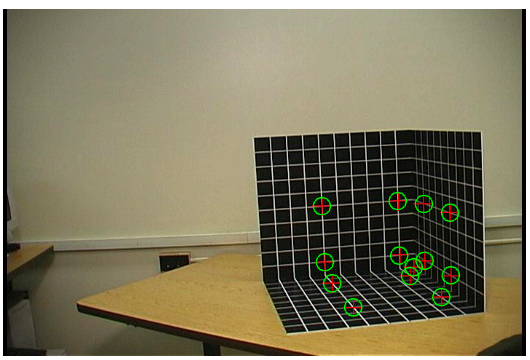

Figure 1 Reference points

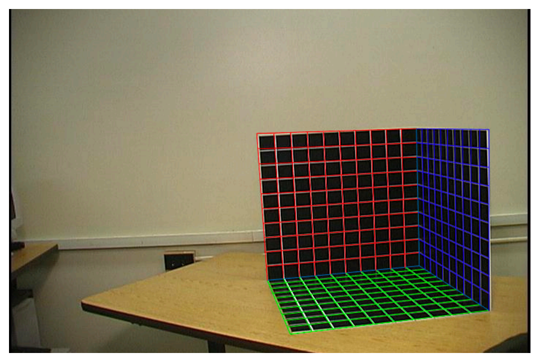

Figure 2 Calibration grid

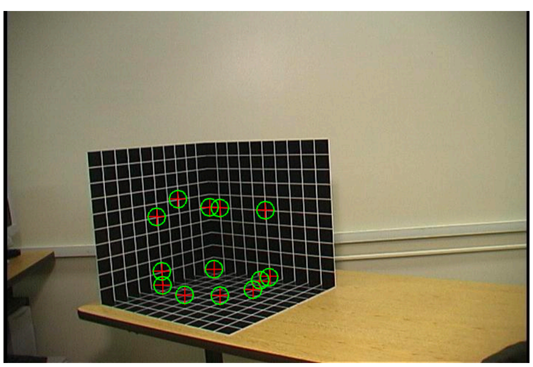

Figure 3 Reference points

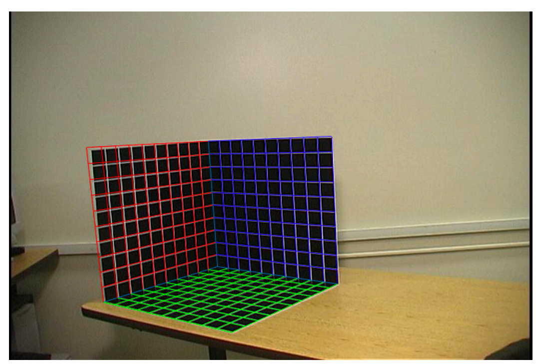

Figure 4 Calibration gird

Reference

[1] https://www.mathworks.com/matlabcentral/fileexchange/67132-camera-calibration-and-calculating-intrinsic-and-extrinsic-parameters

[2] https://github.com/cdbunker/linearCameraCalibration