$$\xrightarrow{(a_i,\, b_j+\delta_j)} \mu_{J,I}(E) \le \mu_{J,J}(E) + \underbrace{\frac{\varepsilon}{2} + \frac{\varepsilon}{4} + \cdots}_{\varepsilon}$$

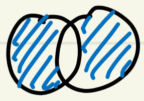
Thm, μ L-S measure, $\mathcal{M}_\mu : \{[C]\}$. $\forall E \in \mathcal{M}_\mu$, we have that

$$\mu(E) = \inf\{\mu(O) \mid O \supset E,\ O \text{ open}\} \quad \text{(by lemma } \uparrow\text{)}$$

outer regular

inner regular $= \sup\{\mu(K) \mid K \subset E,\ K \text{ compact}\} \leftarrow$ Ex

Littlewood's 1 st principle: Borel set Δ good set = null



Good sets includes

$\begin{cases} G_\delta\text{-set} & \text{countable } \cap \text{ of open sets} \\ F_\sigma\text{-set} & \sim\ \cup\ \sim \text{closed} \sim \end{cases}$

Thm $E \subset \mathbb{R}^n$, TFAE

① $E \in \mathcal{M}_\mu$;

② $E = V \setminus N_1$, V is G_δ, $\mu(N_1) = 0$;

③ $E = H \cup N_2$, H is F_σ, $\mu(N_2) = 0$.

pf. ② or ③ $\Rightarrow$ ① μ is complete in $\mathcal{M}_\mu$. $G_\delta, F_\sigma \in \mathcal{B}$.

①, ② $\Rightarrow$ ③ By $\uparrow$ $K_j \subseteq E \subseteq O_j$. $\mu(K_j) \le \mu(E) \le \mu(O_j)$ For $\mu(E) < \infty$.

($\mu(E) = \infty$, use σ-finite)

Convention/Def. $\mu_F =: \mathcal{L}^1$ in (1D case) $\mathcal{M}_{\mu_F} =: \mathcal{M}_{\mathcal{L}^1}$

Eg. (Pathological Ex) Top big $\not\Rightarrow$ Measure big

(i) open dense $E \subset I$, with arbitrary small $\mathcal{L}^1(-)$

(ii) uncountable null set (Cantor)

(iii) $\exists$ non Borel measurable set.