

Algebraic topology

11-Sept-2023

Ref ① Mumbe ② Miller ③ Hatch.

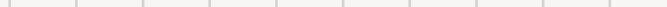
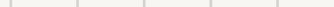
Plan ① Fundamental groups ② Homology Theory

③ Cohomology Theory ⑤ Poincaré duality

Ch1 Fundamental group

2. X, Y Top spaces. Prove $X \stackrel{\sim}{\simeq}_{\text{top}} Y$ (homeomorphic)

Attempt 1 conti with conti inverse?

2.  \neq  ?

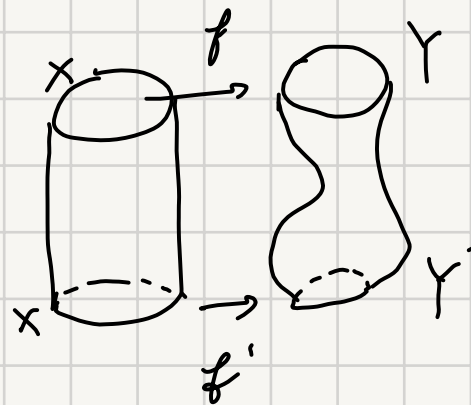
Attempt 1

Attempt 2 Topological invariants

ex. $S' \neq \mathbb{R}$? pf. Using compactness.

3.1 Homotopy of paths.

Def 1.1.1 $f, f' : X \rightarrow Y$ continuous

$$f \sim f' \quad (\text{homotopic}) \text{ iff}$$
$$\exists F: X \times [0, 1] \rightarrow Y \text{ continuous}$$
$$(x, 0) \mapsto f(x)$$
$$(x, 1) \mapsto f'(x)$$


* In part, f is constant. we call it null-homotopic

Def. 1.1.2 f, f' are paths with $f(\omega) = f'(\omega)$, $f(1) = f'(1)$

i.e. $f, f': [0,1] \rightarrow X$, whenever $f \stackrel{F}{\sim} f'$,

f, f' are path homotopic.