Def X, Y. $f: X \to Y$, $g: Y \to X$. We call X, Y are homotopy equivalent if $g \circ f \simeq id_X$, $f \circ g \simeq id_Y$ $\quad X \simeq Y$

Eg. $X \times \mathbb{R} \simeq X$ $\quad f: X \times \mathbb{R} \to X$, $(x, s) \mapsto s$ $\quad g: X \to X \times \mathbb{R}$, $x \mapsto (x, 0)$

$g \circ f: (x, s) \mapsto (x, 0)$ $\quad H: (X \times \mathbb{R}) \times I \to (X \times \mathbb{R})$, $((x, s), t) \mapsto (x, ts)$

$f \circ g: x \mapsto x$ is id_X

Def. If $X \simeq \{pt\}$. X is called contractible.

Eg. $E^n \simeq \{pt\}$

Def. If $f: X \to Y \simeq$ constant, f is called null-homotopy

Def X, $A \subset X$. $i: A \to X$, $a \mapsto a$; $r: X \to A$, $x \mapsto r(A)$.

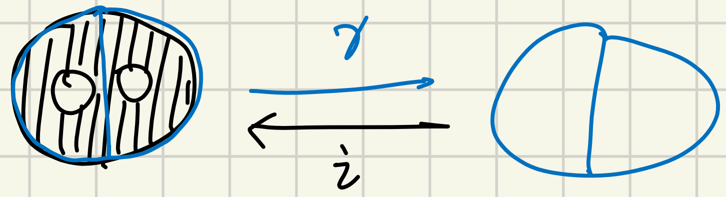
If $\begin{cases} r \circ i = id_A \\ i \circ r \simeq id_X \text{ rel } A \end{cases}$, A is called deformation retract of X

Rmk. Equivalently $H: X \times [0,1] \to X$. s.t. $\begin{cases} H(x, 0) = x \\ H(x, 1) = i \circ r(x) = r(x) \\ H(a, t) = a \quad a \in A, t \in [0,1] \end{cases}$

Eg. $X \times \mathbb{R}$ def retract of $X \times \{0\}$.

$r: X \times \mathbb{R} \to X \times \{0\}$, $(x, s) \mapsto (x, 0)$. i trivial.

Eg.



Def CW complex (cell complex)

$\mathbb{E}^n := \{(x_1, \dots, x_n) \mid x_i \in \mathbb{R}\}$

$D^n := \{(x_1, \dots, x_n) \in \mathbb{E}^n \mid x_1^2 + \dots + x_n^2 \le 1\}$

$S^{n-1} := \partial D^n$ $\quad \{\cdot \ \cdot\} \subset \longmapsto \subset$ ——

Def. $e^n := \mathring{D}^n$ (interior)