

$$\log P(x_1 \cdots x_n | u) P(u) \\ = \log \lambda - \lambda u + \sum_{i=1}^n x_i \log u - u \log e - \log x_i!$$

$$\frac{\partial \log P(x_1 \cdots x_n | u) P(u)}{\partial u} = -\lambda + \sum_{i=1}^n \left(\frac{x_i}{u} - 1 \right)$$

set $\frac{\partial \log P(x_1 \cdots x_n | u) P(u)}{\partial u} = 0$, we have

$$-\lambda + \frac{1}{u} \sum_{i=1}^n x_i - n = 0 \Rightarrow u = \frac{\sum_{i=1}^n x_i}{n + \lambda}$$