CS555: Numerical Methods for PDEs
Instructor: L. Olson
Due Thursday February 7, 2013

Instructions:

1. Your submitted work must be your own work. You may *discuss* the homework.
2. It is your responsibility to answer the question in detail, to convince the grader that you answered the problem, and to explain your solution.
3. Your submission should be as short as possible.
4. Items #1 and #2 are required; item #3 should be a guide.

To Submit:

- create a directory `sp13-cs555/`*`yournetid`*`/hw`*`N`*
- add your write-up `hw1.pdf` and supporting files `hw1*.py`
- commit your directory and files (svn details are on the web)

1. **[basic implementation]** For $x \in [-1, 3]$ and $t \in [0, 2.4]$, solve

$$u_t + u_x = 0$$

with initial data

$$u(0, x) = \begin{cases} \cos^2 \pi x & |x| \leq 1/2, \\ 0 & \text{otherwise,} \end{cases}$$

and boundary data $u(t, -1) = 0$. Use the following four schemes with $h = 1/10$, $1/20$, and $1/40$:

(a) FTBS with $\lambda = 0.8$;
(b) FTCS with $\lambda = 0.8$;
(c) Lax-Friedrichs with $\lambda = 0.8$ and $\lambda = 1.6$;
(d) Leapfrog with $\lambda = 0.8$.

Use right boundary condition of $u_M^{n+1} = u_{M-1}^{n+1}$ when needed and use FTCS to start Leapfrog.

For each scheme, argue (numerically) that approximation is either convergent or non-convergent. For the convergent schemes, investigate the effect of the different h values (e.g., is the error reduced?).

2. [**consistency**] Show that the leapfrog scheme is consistent with $u_t + au_x = 0$.

3. [**well-posedness**] Show that the initial value problem $u_t = u_{xxx}$ is *well-posed*.

4. [**stability**] Consider the *box scheme*

$$\frac{1}{2k}\left[(u_m^{n+1} + u_{m+1}^{n+1}) - (u_m^n + u_{m+1}^n)\right] + \frac{a}{2h}\left[(u_{m+1}^{n+1} - u_m^{n+1}) + (u_{m+1}^n - u_m^n)\right] = f_m^n.$$

   This scheme is consistent with $u_t + au_x = f$. Is it convergent? It is stable?

5. [**accuracy**] Consider solving $u_t + u_x = 0$ for $(t, x) \in [0, 1.2] \times [-1, 1]$ with $u(0, x) = \sin(2\pi x)$ and periodic boundary conditions. Using the methods

   (a) FTBS with $\lambda = 0.8$, and

   (b) Lax-Wendroff with $\lambda = 0.8$,

   demonstrate numerically the first-order and second-order accuracy of these schemes, respectively. Use $h = 1/10$, $1/20$, $1/40$, and $1/80$. measure the L^2-norm and L^∞-norm (max-norm) of the error (*note:* do not sum both periodic points in the error calculation).