a)

③ since $P(X=x) = \frac{u^x e^{-u}}{x!}$, $u>0$,

$$\Rightarrow P(x_1 \cdots x_n | u) = \prod_{i=1}^{n} \frac{u^{x_i} e^{-u}}{x_i!}$$

$$\Rightarrow \log P(x_1 \cdots x_n | u) = \sum_{i=1}^{n} \log\left(\frac{u^{x_i} e^{-u}}{x_i!}\right)$$

$$= \sum_{i=1}^{n} \log u^{x_i} + \log e^{-u} - \log x_i!$$

$$= \sum_{i=1}^{n} x_i \log u - u \log e - \log x_i!$$

$$\frac{\partial \log P(x_1 \cdots x_n | u)}{\partial u} = \sum_{i=1}^{n} \left(\frac{x_i}{u} - \log e\right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} \frac{x_i}{u} = \sum_{i=1}^{n} \log e \Rightarrow \frac{1}{u} \sum_{i=1}^{n} x_i = n \log e$$

$$\Rightarrow u = \frac{\sum_{i=1}^{n} x_i}{n \log e} = \frac{\bar{x}}{\log e} = \bar{x} \quad (\log = \ln)$$

b) now $P(x_1 \cdots x_n | u) \cdot P(u)$

$$= \lambda e^{-\lambda u} \cdot \prod_{i=1}^{n} \frac{u^{x_i} e^{-u}}{x_i!}$$