CS 225 Summer 2014 Homework 0

Chen Zhang, NetID : czhang49, Lab Session: 11 AM

---

1. (4 points) Write a few paragraphs (minimum 300 words) in a file called writeup.txt and commit it to your hw0 directory as mentioned in the Instructions. Answer the following prompts:

   Please see writeup.txt

2. (4 points) Post a short synopsis of your favorite movie to the course piazza space under the "HW0 tell me something!" notice, so that your post is visible to everyone in the class, and tagged by #movie. Also, mention someplace interesting you have traveled in a private post to course staff, also with the tag #travel. Finally, please record the two post numbers corresponding to your posts.

   (a) Favorite Movie Post (Public) number: 130

   (b) Summer Travel Post (Private) number: 131

3. (18 points) Simplify the following expressions as much as possible, Do not approximate. Express all rational numbers as improper fractions. **Show your work.**

   (a) $\prod_{k=2}^{n}(1-\frac{1}{k^2})$

   $\prod_{k=2}^{n}(1-\frac{1}{k^2}) = \frac{1+n}{2n}$

   (b) $3^{1000} \bmod 7$

   $3^{1000} \bmod 7 = (7+2)^{500} \bmod 7 = 2^{500} \bmod 7 = (14+2)^{125} \bmod 7 = 2^{125} \bmod 7 = (28+4)^{25} \bmod 7 = 2^{50} \bmod 7 = (28+4)^{10} \bmod 7 = (14+2)^{5} \bmod 7 = 2^{4} \bmod 7 = 32 \bmod 7 = 4$

   (c) $\sum_{r=1}^{\infty}(\frac{1}{2})^r$

   $\sum_{r=1}^{N}(\frac{1}{2})^r = 1-(\frac{1}{2})^N$ If we set $N=\infty$ then we have $\sum_{r=1}^{\infty}(\frac{1}{2})^r = 1-(\frac{1}{2})^\infty = 1$

   (d) $\frac{\log_7 81}{\log_7 9}$

   $\frac{\log_7 81}{\log_7 9} = \log_9 81 = 2$

   (e) $\log_2 4^{2n}$

   $\log_2 4^{2n} = \log_2 2^{4n} = 4n\log_2 2 = 4n$

   (f) $\log_{17} 221 - \log_{17} 13$

   $\log_{17} 221 - \log_{17} 13 = \log_{17}\frac{221}{13} = \log_{17} 17 = 1$

4. (15 points) Find the formula for $1+\sum_{j=1}^{n} j!j$, and show work proving the formula is correct using induction.

   The formula is $(1+n)!$
   It is obvious that when $n=1$, the formula is satisfied. Now if we assume that for $n=N$, the formular is satisfied. Then we have for $n=N+1$, $1+\sum_{j=1}^{N+1} j!j = 1+\sum_{j=1}^{N} j!j+(N+1)!(N+1) = (1+N)!+(1+N)!(1+N) = (1+N)!(2+N) = (2+N)!$
   Thus from induction we can see that the statement is true for any arbitrary n

5. (12 points) Indicate for each of the following pairs of expressions $(f(n), g(n))$, whether $f(n)$ is O, Ω, or Θ of $g(n)$. Prove your answers to the first two items, but just GIVE an answer to the last two.

   (a) $f(n) = 4^{\log_4 n}$ and $g(n) = 2n+1$

   Since $f(n) = n$, Then we hanve $2f(n) <= g(n) <= 3f(n)$. Thus $f(n) = \Theta(g(n))$

   (b) $f(n) = n^2$ and $g(n) = (\sqrt{2})^{\log_2 n}$

   $f(n) = \Omega(g(n))$

   (c) $f(n) = \log_2 n!$ and $g(n) = n \log_2 n$

   $f(n) = \Theta(g(n))$

   (d) $f(n) = n^k$ and $g(n) = c^n$ where k, c are constants and c is >1

   $f(n) = O(g(n))$

6. (15 points) Solve the following recurrence relations for integer n. If no solution exists, please explain the result.

   (a) $T(n) = T(\frac{n}{2}) + 5$, $T(1) = 1$, assume n is a power of 2.

   When $n = 1$, $T(n) = 1$, when $n >= 1$, $T(n) = \frac{5n}{2} + 1$ and n is a power of 2

   (b) $T(n) = T(n-1) + \frac{1}{n}$, $T(0) = 0$.

   The result is when $n = 0$, $T(n) = 0$. When $n >= 1$, $T(n) = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + ...\frac{1}{n}$. No closed form solution exists for this equation.

   (c) Prove that your answer to part (a) is correct using induction.

7. (16 points) Suppose function call parameter passing costs constant time, independent of the size of the structure being passed.

   (a) Give a recurrence for worst case running time of the recursive Binary Search function in terms of n, the size of the search array. Assume n is a power of 2. Solve the recurrence, explicitly noting the recurrence formlua, base case, and recurrence solution.

   (b) Give a recurrence for worst case running time of the recursive Merge Sort function in terms of n, the size of the array being sorted. Solve the recurrence, explicitly noting the recurrence formlua, base case, and recurrence solution.

8. (16 points) Consider the pseudocode function below.

```
darp( x, n )
     if( n == 0 )
         return 1;
     if( n % 2 == 0 )
         return darp( x * x, n/2 );
     return x * darp( x * x, (n - 1) / 2);
```

   (a) What is the output when passed the following parameters: $x = 2$, $n = 12$. Show your work (activation diagram or similar).

   When put in $x = 2, n = 12$, on the first call, the function returns darp$(4,6)$. On the second call it returns darp$(16,3)$. On the third call it returns 16∗darp$(256,1)$. On the fourth call of darp it returns 256∗darp$(256*256,0)$. Thus the final output is $16*256 = 4096$

   (b) Briefly describe what this function is doing.

   This function is basically calculationg x^n

   (c) Write a recurrence that models the running time of this function. Assume checks, returns, and arithmetic are constant time, but be sure to evaluate all function calls. [Hint: what is the *most* n could be at each level of the recurrence?]

   (d) Solve the above recurrence for the running time of this function.