CS 225 Summer 2014 Homework 0

Due June 18, 2014, 3:00pm in lecture (and on SVN)

The purpose of this assignment is to give you a chance to refresh the math skills we expect you to have learned in prior classes. These particular skills will be essential to mastery of CS 225, and we are unlikely to take much class time reminding you how to solve similar problems.

Though you are not required to work independently on this assignment, we encourage you to do so because we think it may help you diagnose and remedy some things you might otherwise find difficult later on in the course.

Display your name, netid, and section time (11am, 1pm, or 5pm) on the front page of your submission.

Your hw0 MUST be typewritten and submitted as a PDF file (hw0.pdf)! You will also hand in a physical, printed copy in lecture.

Submission Instructions

To checkout your code, run the command

```
svn co https://subversion.ews.illinois.edu/svn/su14-cs225/NETID/hw0/
```

This will download two hw0 files: hw0-handout.pdf (this file) and hw0.tex (an optional LaTeXtemplate). Create your answers electronically (via Word, LaTeX, etc), and produce a PDF file as output. ***Make sure your file is called hw0.pdf and the writeup is called writeup.txt***. When you are ready to submit, run the following commands:

```
svn add hw0.pdf
svn add writeup.txt
svn ci -m "submitting hw0"
```

You should verify your submission by going to the checkout link in your browser and making sure the file can be downloaded. You will also bring a printed, physical copy to lecture on the due date. If you have any further questions, feel free to ask on Piazza. The questions begin on the next page.

1. (4 points) Write a few paragraphs (minimum 300 words) in a file called writeup.txt and commit it to your hw0 directory as mentioned in the Instructions. Answer the following prompts:

   (a) Tell me what interests you about computer science, and why you are fascinated by that particular thing.

   (b) Tell me what you hope to do with your particular skills in computer science (*e.g.* graphics, databases, natural language processing, security)

2. (4 points) Post a short synopsis of your favorite movie to the course piazza space under the "HW0 tell me something!" notice, so that your post is visible to everyone in the class, and tagged by #movie. Also, mention someplace interesting you have traveled in a private post to course staff, also with the tag #travel. Finally, please record the two post numbers corresponding to your posts.

   (a) Favorite Movie Post (Public) number:

   (b) Summer Travel Post (Private) number:

3. (18 points) Simplify the following expressions as much as possible, Do not approximate. Express all rational numbers as improper fractions. **Show your work.**

   (a) $\prod_{k=2}^{n}(1-\frac{1}{k^2})$

   (b) $3^{1000} \bmod 7$

   (c) $\sum_{r=1}^{\infty}(\frac{1}{2})^r$

   (d) $\frac{\log_7 81}{\log_7 9}$

   (e) $\log_2 4^{2n}$

   (f) $\log_{17} 221 - \log_{17} 13$

4. (15 points) Find the formula for $1+\sum_{j=1}^{n} j!j$, and show work proving the formula is correct using induction.

5. (12 points) Indicate for each of the following pairs of expressions $(f(n), g(n))$, whether $f(n)$ is O, Ω, or Θ of $g(n)$. Prove your answers to the first two items, but just GIVE an answer to the last two.

   (a) $f(n) = 4^{\log_4 n}$ and $g(n) = 2n+1$

   (b) $f(n) = n^2$ and $g(n) = (\sqrt{2})^{\log_2 n}$

   (c) $f(n) = \log_2 n!$ and $g(n) = n \log_2 n$

   (d) $f(n) = n^k$ and $g(n) = c^n$ where k, c are constants and c is >1

6. (15 points) Solve the following recurrence relations for integer n. If no solution exists, please explain the result.

   (a) $T(n) = T(\frac{n}{2}) + 5$, $T(1) = 1$, assume n is a power of 2.

   (b) $T(n) = T(n-1) + \frac{1}{n}$, $T(0) = 0$.

   (c) Prove that your answer to part (a) is correct using induction.

7. (16 points) Suppose function call parameter passing costs constant time, independent of the size of the structure being passed.

   (a) Give a recurrence for worst case running time of the recursive Binary Search function in terms of n, the size of the search array. Assume n is a power of 2. Solve the recurrence, explicitly noting the recurrence formlua, base case, and recurrence solution.

   (b) Give a recurrence for worst case running time of the recursive Merge Sort function in terms of n, the size of the array being sorted. Solve the recurrence, explicitly noting the recurrence formlua, base case, and recurrence solution.

8. (16 points) Consider the pseudocode function below.

```
darp( x, n )
     if( n == 0 )
         return 1;
     if( n % 2 == 0 )
         return darp( x * x, n/2 );
     return x * darp( x * x, (n - 1) / 2);
```

   (a) What is the output when passed the following parameters: $x = 2$, $n = 12$. Show your work (activation diagram or similar).

   (b) Briefly describe what this function is doing.

   (c) Write a recurrence that models the running time of this function. Assume checks, returns, and arithmetic are constant time, but be sure to evaluate all function calls. [Hint: what is the *most* n could be at each level of the recurrence?]

   (d) Solve the above recurrence for the running time of this function.