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Homework 10 Problem 2	Chen Zhang (czhang49)

Solution: (a) Let V and E denote the set of vertices and edges in graph [1]. Define a variable x_e for each edge $e \in E$. Then, the maximum-cardinality bipartite matching can be formulated as:

Maximize $\sum_{e \in E} x_e$

Subject to $\sum_{e \sim v} x_e \leq 1$ for all $v \in V$

$x_e \in \{0, 1\}$ for all $e \in E$

where $e \sim v$ denotes the set of edges connected to vertex v. Note that the Linear Program can be written as

Maximize cx

Subject to $Ax \leq b$

$x \geq 0$

with c a $1 \times |E|$, where all elements are equal to 1 and $|E|$ denotes the total number of edges. Moreover, b is a $|V| \times 1$ vector with all elements equal to 1, where $|V|$ denotes the total number of vertices. The matrix A has a 1 on entry (i, j) if vertex i is connected to edge j and 0 otherwise.

(b) The dual problem is given by

Minimize $\sum_{v \in V} y_v$

Subject to $\sum_{v \sim e} y_v \geq 1$ for all $e \in E$

$y_v \in \{0, 1\}$ for all $v \in V$

where $v \sim e$ denotes the vertices v connected to edge e. The dual variable y_i is equal to 1 if vertex i is matched and 0 otherwise. The objective function minimizes the sum of matched vertices subject to the given constraints. ■

1 References

[1] http://www.imsc.res.in/ meena/matching/lecture5.pdf