CS 181 HW5 2021 CS181

CHARLES ZHANG

TOTAL POINTS

16 / 21

QUESTION 1

1 PDA 3 / 7

- **0 pts** Correct
- **1 pts** Small Mistake
- **2 pts** Missed ε case

$$a^0b^0c^0$$

- **2 pts** Misses $$i=0$$ or $$j=0$$ cases

Your PDA should accept aaaccc and bbbccc

✓ **- 4 pts Accepts some strings not of the form $$a^i b^j c^k$$**

Your PDA should not accept ababcccc or abccabcc

- **5 pts** Major Mistake

$$i + j \neq k$$

- **7 pts** Incorrect

Accepts a lot of strings not in $$L_{add}$$

Rejects a lot of strings in $$L_{add}$$

- **7 pts** No answer

Mixes a's and b's

QUESTION 2

2 CFL Pumping Lemma 7 / 8

+ **8 pts** Correct or nearly correct

✓ **+ 1 pts Appropriate string**

✓ **+ 3 pts Effective use of constraints on vxy**

+ **2 pts** Mostly effective use of constraints

+ **1 pts** Somewhat effective use of constraints

- **0.5 pts** Does not explicitly state how |vxy| $\leq$ p is used.

✓ **+ 2 pts Clearly shows coverage of all cases**

+ **1 pts** Partially shows coverage of all cases

+ **2 pts** Correct logic in every case

✓ **+ 1 pts Partially correct logic in cases**

+ **0 pts** No answer

+ **0 pts** Your string is not in the language.

+ **0 pts** Your string can be pumped and remain in the language.

+ **7.5 pts** forget one of v and y can be empty (usually bc case)

straddling bc also ahst the case where neither v nor y contain bs (consider v = empty string, x = b y = c)

QUESTION 3

3 DFA Construction 6 / 6

✓ **- 0 pts Correct**

- **4 pts** Attempted
- **2 pts** Partially Correct
- **1 pts** Almost correct
- **6 pts** Not attempted

CS 181 Homework 5

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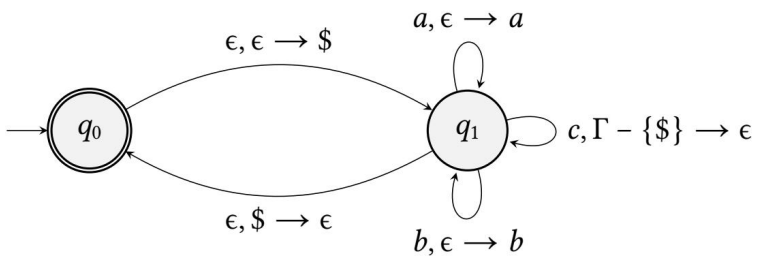
May 2, 2021

Problem 1

$$L_{add} = \{a^i b^j c^k \mid k = i + j\}$$

PDA Model:

$$\Sigma = \{a, b, c\} \quad \Gamma = \{\$, a, b\}$$



Justification:
This PDA starts by pushing \$ onto the stack to indicate an empty stack. It then proceeds to push *a*s and *b*s onto the stack as they are read from the input string, allowing us to "calculate" $i + j$, as defined by L_{add}. Since the language guarantees that all occurrences of a and b occur before any occurrences of c, we know that after calculating $i + j$, we simply need to check if k is equivalent by counting the remaining *c*s in the string. We do this by popping a symbol (either a or b) off the stack for each c that is read from the input string. If there are less *c*s than *a*s and *b*s, the PDA will remain in state q_1 and reject the input. If there are more *a*s and *b*s than *c*s, the PDA will move back into q_0, but won't have reached the end of the input yet, so it will also reject. It will only accept if it moves back into q_0 and there are no more symbols to read, indicating that the number of *c*s was exactly equal to the number of *a*s and *b*s.

1 PDA 3 / 7

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- **1 pts** Small Mistake
- **2 pts** Missed ε case

$$a^0b^0c^0$$

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Mixes a's and b's

Problem 2

$$L_2 = \{a^i b^j c^k \mid k = i + j \text{ and } i > j\}$$

Proof (by contradiction):

- Assume L_2 is context-free.
- Let p be the pumping length given by the pumping lemma for CFLs.
- Let s be the string $a^{2p} b^p c^{3p}$, noting that $s \in L_2$, since $i = 2p$, $j = p$, and $k = 3p$, satisfying the condition that $k = i + j$ and $i > j$.
- With s being a member of L_2 and having a length greater than p, the pumping lemma for CFLs guarantees that s can be split into five substrings of the form $s = uvxyz$ such that $^{(1)}$for each $n \geq 0$, $uv^n xy^n z \in L_2$, $^{(2)}|vy| > 0$, and $^{(3)}|vxy| \leq p$.
- Note that the string s can be split into 3 blocks A, B, and C such that A only contains as and is of length $2p$, B only contains bs and is of length p, and C only contains cs and is of length $3p$.
- By condition (3) of the pumping lemma for CFLs, we know that $|vxy|$ has length of at most p.
- This means that vxy can *at most* cross two of the three blocks A, B, and C, as each block has a length of at least p.
- This leaves us with five possible cases to consider: vxy is entirely within block A, vxy is entirely within block B, vxy is entirely within block C, vxy crosses from block A to block B, and vxy crosses from block B to block C.
- Create the string s' by pumping s using $n = t$ such that $t > p$, noting that $t > 0$ by the specification of the language.
- If vxy is entirely within a single block, only the number of one symbol increases; if vxy crosses from A to B, the number of as and bs increases, while the number of cs remains constant.
- In all four of these cases, only a single side of the expression $k = i + j$ increases, while the other remains constant, indicating that the pumped string s' is not a member of L_2.
- This violates the pumping lemma for CFLs, therefore all four of these cases lead to a contradiction.
- The only remaining case is when vxy crosses from B to C.
- By the specification of the case, there must be at least one b in either the substring v or the substring y.
- This means that the number of bs is pumped by t in this case, leading to there being at least $p + p$ bs, since t is defined as some finite value greater than p.
- This violates the condition for L_2 that $i > j$, as the number of bs is now greater than or equal to the number of as.
- In all cases of vxy, the s' is no longer a member of L_2, violating the pumping lemma of CFLs, contradicting the claim that L_2 is a CFL. $\Rightarrow\Leftarrow$

2 CFL Pumping Lemma 7 / 8

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Problem 3

$$A \text{ PS } B = \{w \mid w = a_1 b_1 ... a_k b_k \text{ where } a_i ... a_k \in A \text{ and } b_1 ... b_k \in B \text{ each } a_i, b_i \in \Sigma\}$$

Proof (by construction):

- If the languages A and B are regular, there must exist some DFAs M_A and M_B that accept them.
- To show that FSLs are closed under perfect shuffle, we must construct a finite automaton M based on the machines M_A and M_B that accepts the perfect shuffle of A and B.
- We define $M_A = \{Q_A, \Sigma, \delta_A, q_{0,A}, F_A\}$, $M_B = \{Q_B, \Sigma, \delta_B, q_{0,B}, F_B\}$, and $M = \{Q, \Sigma, \delta, q_0, F\}$.
- We define $Q = (Q_A \times Q_B) \cup \{q_0\}$.
- We define $\delta(q_0, \epsilon) = (q_{0,A}, q_{0,B})$.
- We define $(\delta_A(q_i, a), q_j) \in \delta((q_i, q_j), a)$ for states $q_i \in Q_A$ and $q_j \in Q_B$.
- We define $(q_i, \delta_B(q_j, a)) \in \delta((q_i, q_j), a)$ for states $q_i \in Q_A$ and $q_j \in Q_B$.
- We define $F = (F_A \times F_B)$.

Justification:
The resultant NFA M models the states of M_A and M_B as ordered pairs. By doing this, we can then use the transitions from M_A and M_B, while also taking into account the state of both machines. This means that transitions that M_A would take can now only be taken if M_B is in the appropriate state, and vice versa. We then create a new initial state that uses an ϵ-edge to allow M to model M_A and M_B's initial states. Finally, we create our set of accepting states from the cartesian product of the accepting states of M_A and M_B, as both machines would have needed to end in an accepting state in order for M to accept the input.

3 DFA Construction 6 / 6

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