CS M51A

Logic Design of Digital Systems

Winter 2021

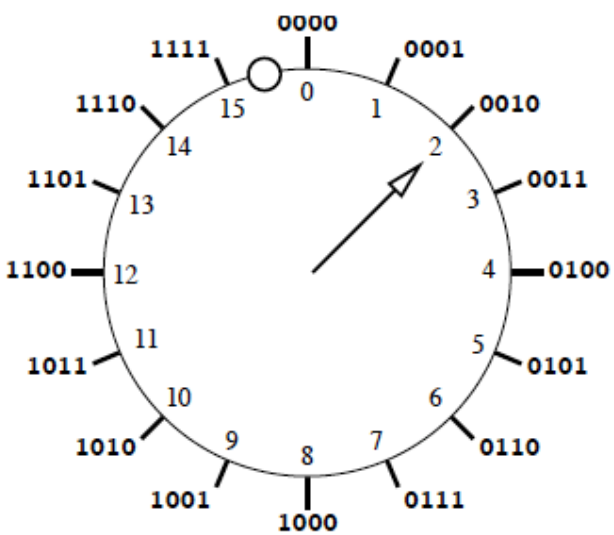
Some slides borrowed and modified from:

M.D. Ercegovac, T. Lang and J. Moreno, Introduction to Digital Systems.

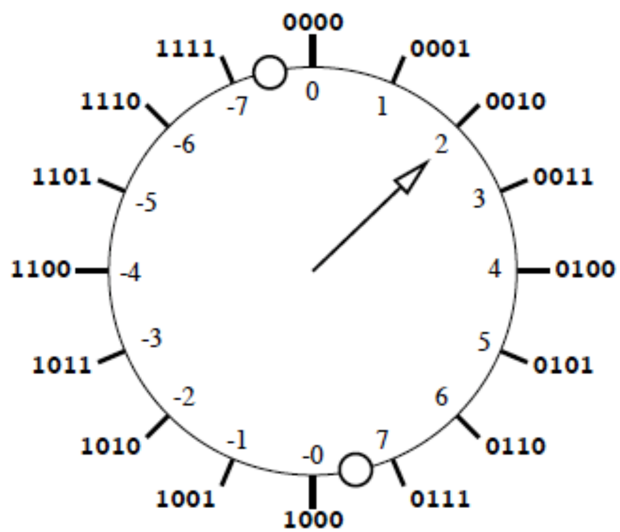
D. Patterson and J. Hennessy, Computer Organization and Design

So Far….

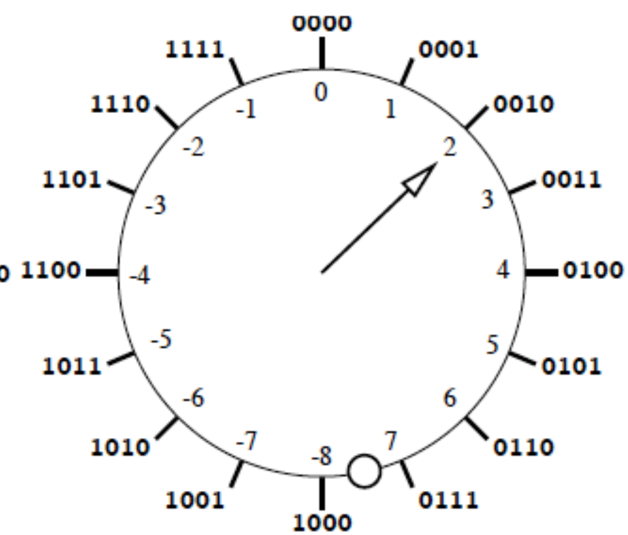
- A system and a digital system
- High level specification of a system
- Data Representation



Unsigned



Signed Magnitude



Two's Complement

Addition

- To add two two's complement numbers, simply use the "elementary school algorithm", throwing away any carry out of the MSB position
- To subtract, simply negate and add
- Problem: what if answer cannot be represented? (called overflow)
- Overflow in addition cannot occur if one number is positive and the other negative
- If both addends have same sign but answer has different sign, overflow has occurred

Examples

```
 0 1 0 1
          +
 0 0 1 1
---------------


 1 1 0 1
          +
 1 1 1 1
---------------


 0 1 0 1
          -
 0 0 1 1
---------------
```

Examples

0 1 0 1 + 0 0 1 1 ---------------	5 + 3 ---------------	Over flow
1 1 0 1 + 1 1 1 1 ---------------	-3 + -1 ---------------	
0 1 0 1 - 0 0 1 1 ---------------	5 - 3 ---------------	

Clicker Question

- In which one, overflow has occurred? (numbers are presented in 2’s compliment)

- A)
```
  0 1 0 1
         +
  0 1 0 1
---------------
```
- B)
```
  0 0 0 1
         +
  0 0 1 1
---------------
```
- C)
```
  0 1 0 1
         +
  1 0 1 1
---------------
```
- D)
```
  1 0 0 0
         +
  1 0 1 1
---------------
```
- E) A and D

Clicker Question

Convert $(121)_4$ to decimal:

a) 481
b) 28
c) 25
d) 100
e) none

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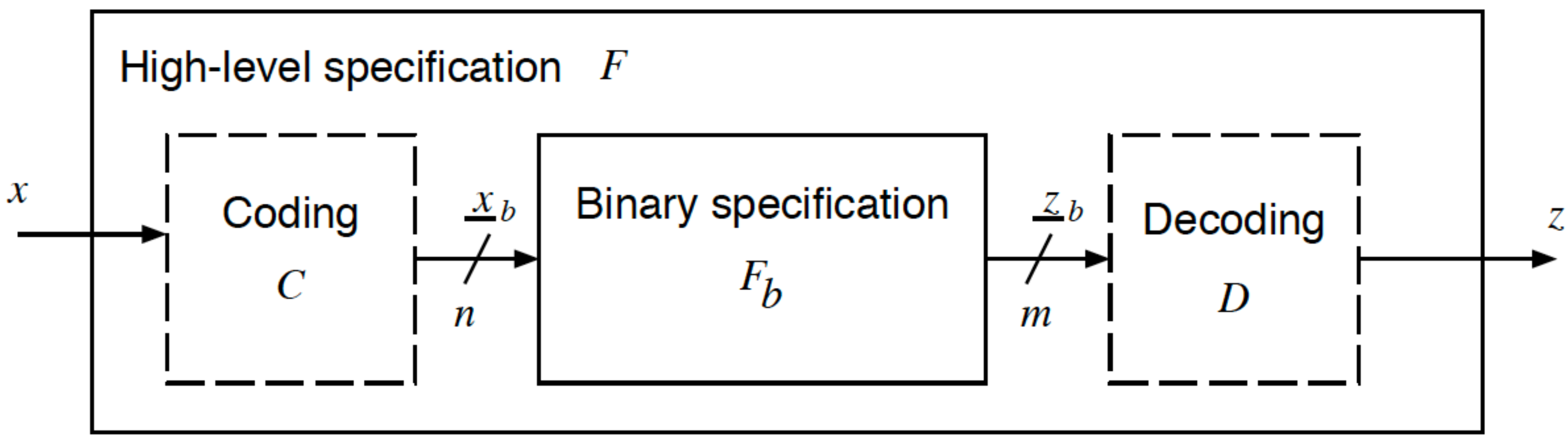

Next….

- Binary Specification and switching functions

SWITCHING FUNCTIONS

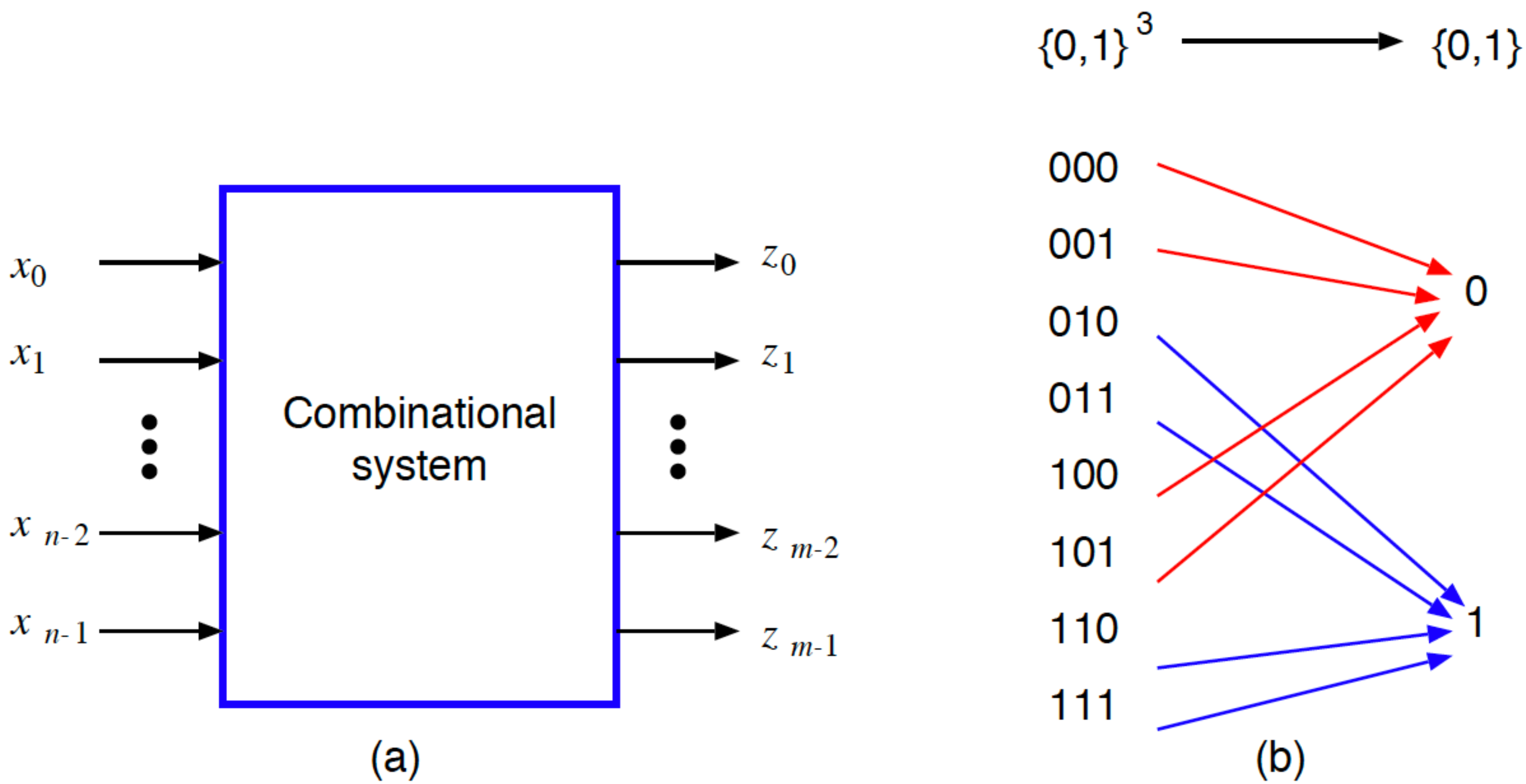

Figure 2.7: a) BINARY COMBINATIONAL SYSTEM; b) A SWITCHING FUNCTION FOR $n = 3$

TABULAR REPRESENTATION OF SWITCHING FUNCTIONS

$x_2 x_1 x_0$	$f(x_2, x_1, x_0)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	0
1 1 0	1
1 1 1	1

2D TABULAR REPRESENTATION

x_4x_3 \ $x_2x_1x_0$	000	001	010	011	100	101	110	111
00	0	0	1	1	0	1	1	1
01	0	1	1	1	1	0	1	1
10	1	1	0	1	1	0	1	1
11	0	1	0	1	1	0	1	0

f

INCOMPLETE SWITCHING FUNCTIONS

$x\ y\ z$	f
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	–
1 0 0	1
1 0 1	0
1 1 0	–
1 1 1	1

SWITCHING EXPRESSIONS

1. Symbols 0 and 1 are SEs.
2. A symbol representing a binary variable is a SE.
3. If A and B are SEs, then
   - $(A)'$ is a SE. This is referred to as "A complement." Sometimes we use $\overline{A}$ to denote complementation.
   - $(A) + (B)$ is a SE. This is referred to as "A OR B"; it is also called "A plus B" or "sum" due to the similarity with the corresponding arithmetic symbol.
   - $(A) \cdot (B)$ is a SE. This is referred to as "A AND B"; it is also called "A times B" or "product" due to the similarity with the corresponding arithmetic symbol.

SWITCHING ALGEBRA AND EXPRESSION EVALUATION

- *Switching algebra*:

  two elements 0 and 1

  operations $+$, $\cdot$, and $'$

$+$	0	1
0	0	1
1	1	1

$\cdot$	0	1
0	0	0
1	0	1

$'$	
0	1
1	0

$$E(x_2, x_1, x_0) = x_2 + x_2' x_1 + x_1 x_0'$$

The value of E for assignment $(1, 0, 1)$ is

$$E(1, 0, 1) =$$

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  operations $+$, $\cdot$, and $'$

$+$	0	1
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1	1	1

$\cdot$	0	1
0	0	0
1	0	1

$'$	
0	1
1	0

$$E(x_2, x_1, x_0) = x_2 + x_2'x_1 + x_1x_0'$$

The value of E for assignment $(1, 0, 1)$ is

$$E(1, 0, 1) = 1 + 1' \cdot 0 + 0 \cdot 1' = 1 + 0 + 0 = 1$$

REPRESENTING SFs BY SWITCHING EXPRESSIONS

$E(x_2, x_1, x_0) = x_2 + x_2' x_1 + x_1 x_0'$ represents f:

$x_2 x_1 x_0$	f
000	0
001	0
010	
011	
100	
101	
110	
111	_

	2 variables	n variables
AND	$x_1 x_0$	$x_{n-1} x_{n-2} \ldots x_0$
OR	$x_1 + x_0$	$x_{n-1} + x_{n-2} + \ldots + x_0$
XOR	$x_1 x_0' + x_1' x_0 = x_1 \oplus x_0$	
EQUIV	$x_1' x_0' + x_1 x_0$	
NAND	$(x_1 x_0)' = x_1' + x_0'$	$(x_{n-1} x_{n-2} \ldots x_0)' = x_{n-1}' + x_{n-2}' + \ldots + x_0'$
NOR	$(x_1 + x_0)' = x_1' x_0'$	$(x_{n-1} + x_{n-2} + \ldots + x_0)' = x_{n-1}' x_{n-2}' \ldots x_0'$

They can also be presented using tables:

ALGEBRAIC METHOD OF OBTAINING EQUIVALENT EXPRESSIONS

- MAIN IDENTITIES OF BOOLEAN ALGEBRA

1.	$a + b$	$= b + a$	ab	$= ba$	Commutativity
2.	$a + (bc)$	$= (a + b)(a + c)$	$a(b + c)$	$= (ab) + (ac)$	Distributivity
3.	$a + (b + c)$	$= (a + b) + c$	$a(bc)$	$= (ab)c$	Associativity
		$= a + b + c$		$= abc$	
4.	$a + a$	$= a$	aa	$= a$	Idempotency
5.	$a + a'$	$= 1$	aa'	$= 0$	Complement
6.	$1 + a$	$= 1$	$0a$	$= 0$	
7.	$0 + a$	$= a$	$1a$	$= a$	Identity
8.	$(a')'$	$= a$			Involution
9.	$a + ab$	$= a$	$a(a + b)$	$= a$	Absorption
10.	$a + a'b$	$= a + b$	$a(a' + b)$	$= ab$	Simplification
11.	$(a + b)'$	$= a'b'$	$(ab)'$	$= a' + b'$	DeMorgan's Law

1.	$a + b = b + a$	$ab = ba$	Commutativity

2.	$a + (bc) = (a + b)(a + c)$	$a(b + c) = (ab) + (ac)$	Distributivity

3. $a + (b + c) = (a + b) + c$ $\quad a(bc) = (ab)c$ $\quad$ Associativity

$ = a + b + c$ $\quad = abc$

4.	$a + a = a$	$aa = a$	Idempotency

5.	$a + a' = 1$	$aa' = 0$	Complement

6. $1 + a = 1$ $0a = 0$

7.	$0 + a = a$	$1a = a$	Identity

8.	$(a')' = a$	Involution

9.	$a + ab = a$	$a(a + b) = a$	Absorption

10.	$a + a'b = a + b$	$a(a' + b) = ab$	Simplification

11. $(a + b)' = a'b'$ $(ab)' = a' + b'$ DeMorgan's Law

EXAMPLE

SHOW THAT E_1 AND E_2 ARE EQUIVALENT:

$$\begin{aligned} E_1(x_2, x_1, x_0) &= x_2x_1 + x_2x_1' + x_2x_0 \\ E_2(x_2, x_1, x_0) &= x_2 \end{aligned}$$

EXAMPLE

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$$\begin{aligned} E_1(x_2, x_1, x_0) &= x_2x_1 + x_2x_1' + x_2x_0 \\ E_2(x_2, x_1, x_0) &= x_2 \end{aligned}$$

$$\begin{aligned} x_2x_1 + x_2x_1' + x_2x_0 &= x_2(x_1 + x_1') + x_2x_0 && \text{using } ab + ac = a(b + c) \\ &= x_2 \cdot 1 + x_2x_0 && \text{using } a + a' = 1 \\ &= x_2(1 + x_0) && \text{using } ab + ac = a(b + c) \\ &= x_2 \cdot 1 && \text{using } 1 + a = 1 \\ &= x_2 && \text{using } a \cdot 1 = a \end{aligned}$$

EXAMPLE

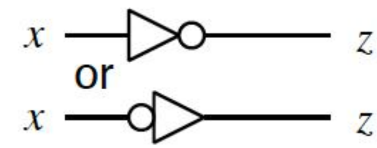
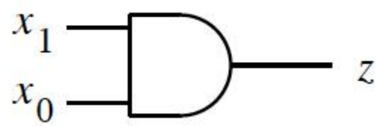
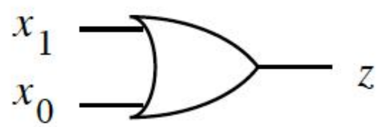
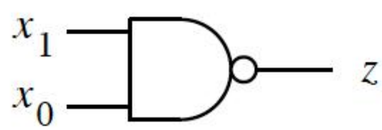
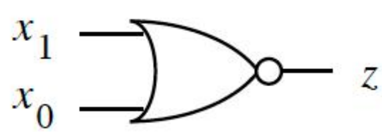
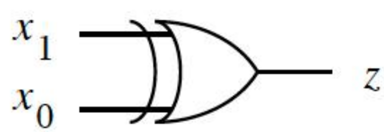
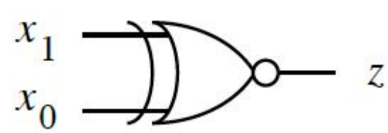
SHOW THAT E_1 AND E_2 ARE EQUIVALENT: (Using a table)

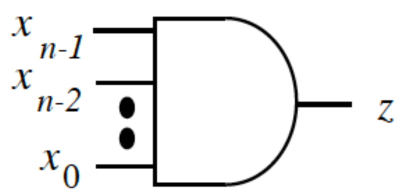
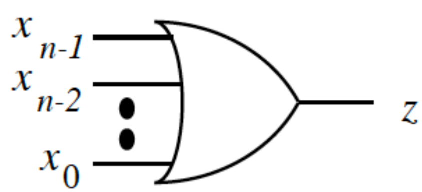
$$
\begin{aligned}
E_1(x_2, x_1, x_0) &= x_2 x_1 + x_2 x_1' + x_2 x_0 \\
E_2(x_2, x_1, x_0) &= x_2
\end{aligned}
$$

Clicker Question

Which one is equal to $x + y$

a) $x' + y'$
b) $(x . y)'$
c) $(x'. y')'$
d) $(xx'+x+y)$
e) c and d

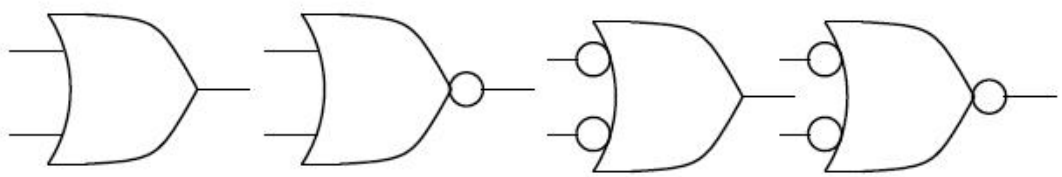
Gate type	Symbol	Switching expression
NOT		$z = x'$
AND		$z = x_1 x_0$
OR		$z = x_1 + x_0$
NAND		$z = (x_1 x_0)'$
NOR		$z = (x_1 + x_0)'$
XOR		$z = x_1 x_0' + x_1' x_0$ $= x_1 \oplus x_0$
XNOR		$z = x_1' x_0' + x_1 x_0$

Gate type	Symbol	Switching expression
AND		$z = x_{n-1}\, x_{n-2} \,....\, x_0$
OR		$z = x_{n-1} + x_{n-2} \,....+ x_0$

Clicker Question

Gates

Which of the following is/are a NOR gate?



a b c d

e b and c