CS M51A
Logic Design of Digital Systems
Winter 2021

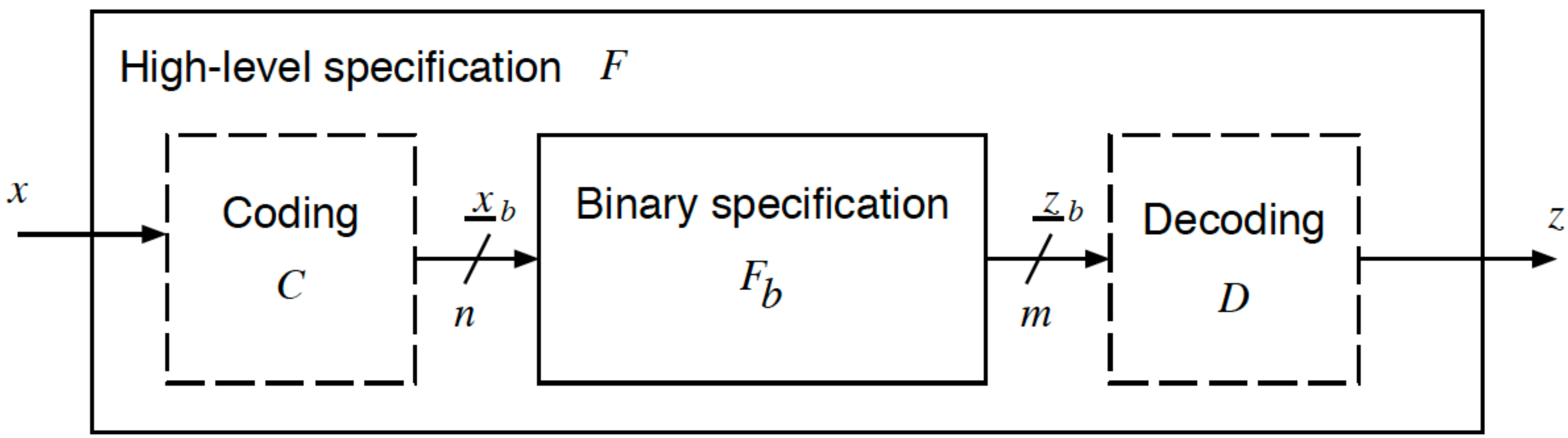
Some slides borrowed and modified from:

M.D. Ercegovac, T. Lang and J. Moreno, Introduction to Digital Systems.

D. Patterson and J. Hennessy, Computer Organization and Design

So Far….

- A system and a digital system
- High level specification of a system
- Data Representation



Next….

- Binary Specification and switching functions

SWITCHING FUNCTIONS

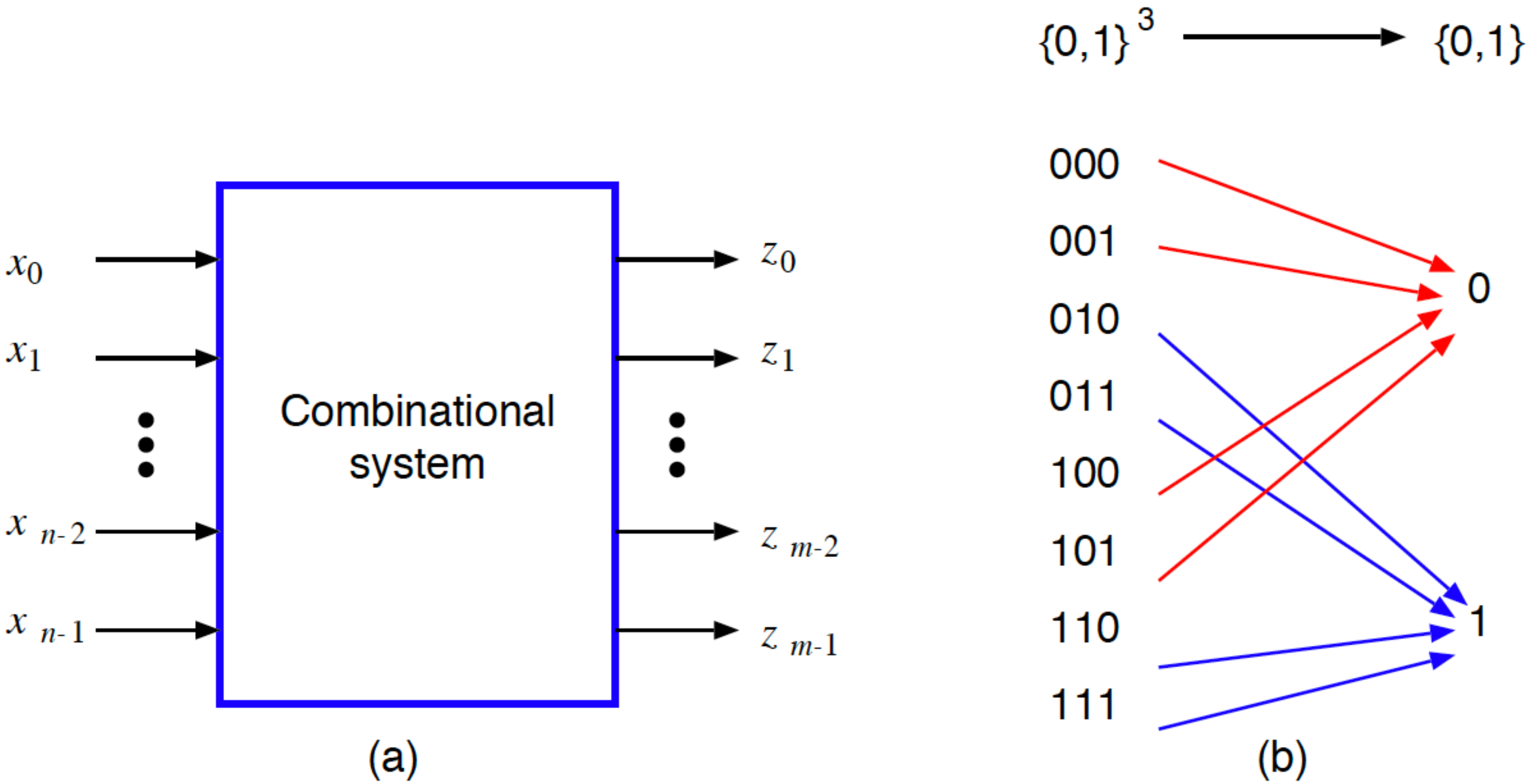


Figure 2.7: a) BINARY COMBINATIONAL SYSTEM; b) A SWITCHING FUNCTION FOR $n = 3$

TABULAR REPRESENTATION OF SWITCHING FUNCTIONS

$x_2 x_1 x_0$	$f(x_2, x_1, x_0)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	0
1 1 0	1
1 1 1	1

2D TABULAR REPRESENTATION

x_4x_3	$x_2x_1x_0$							
	000	001	010	011	100	101	110	111
00	0	0	1	1	0	1	1	1
01	0	1	1	1	1	0	1	1
10	1	1	0	1	1	0	1	1
11	0	1	0	1	1	0	1	0

f

INCOMPLETE SWITCHING FUNCTIONS

$x\ y\ z$	f
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	–
1 0 0	1
1 0 1	0
1 1 0	–
1 1 1	1

SWITCHING EXPRESSIONS

1. Symbols 0 and 1 are SEs.
2. A symbol representing a binary variable is a SE.
3. If A and B are SEs, then
   - $(A)'$ is a SE. This is referred to as "A complement." Sometimes we use $\overline{A}$ to denote complementation.
   - $(A) + (B)$ is a SE. This is referred to as "A OR B"; it is also called "A plus B" or "sum" due to the similarity with the corresponding arithmetic symbol.
   - $(A) \cdot (B)$ is a SE. This is referred to as "A AND B"; it is also called "A times B" or "product" due to the similarity with the corresponding arithmetic symbol.

SWITCHING ALGEBRA AND EXPRESSION EVALUATION

- *Switching algebra*:

  two elements 0 and 1

  operations $+$, $\cdot$, and $'$

$+$	0	1
0	0	1
1	1	1

$\cdot$	0	1
0	0	0
1	0	1

$'$	
0	1
1	0

$$E(x_2, x_1, x_0) = x_2 + x_2'x_1 + x_1x_0'$$

The value of E for assignment $(1, 0, 1)$ is

$$E(1, 0, 1) =$$

SWITCHING ALGEBRA AND EXPRESSION EVALUATION

- *Switching algebra*:

  two elements 0 and 1

  operations $+$, $\cdot$, and $'$

$+$	0	1
0	0	1
1	1	1

$\cdot$	0	1
0	0	0
1	0	1

$'$	
0	1
1	0

$$E(x_2, x_1, x_0) = x_2 + x_2'x_1 + x_1x_0'$$

The value of E for assignment $(1,0,1)$ is

$$E(1,0,1) = 1 + 1' \cdot 0 + 0 \cdot 1' = 1 + 0 + 0 = 1$$

REPRESENTING SFs BY SWITCHING EXPRESSIONS

$E(x_2, x_1, x_0) = x_2 + x_2'x_1 + x_1x_0'$ represents f:

$x_2x_1x_0$	f
000	0
001	0
010	1
011	1
100	1
101	1
110	1
111	1

Example

$$z_2 = \begin{cases} 1 & \textbf{if} \quad x_1 > y_1 \textbf{ or } (x_1 = y_1 \textbf{ and } x_0 > y_0) \\ 0 & \textbf{otherwise} \end{cases}$$

$$z_1 = \begin{cases} 1 & \textbf{if} \quad x_1 = y_1 \textbf{ and } x_0 = y_0 \\ 0 & \textbf{otherwise} \end{cases}$$

$$z_0 = \begin{cases} 1 & \textbf{if} \quad x_1 < y_1 \textbf{ or } (x_1 = y_1 \textbf{ and } x_0 < y_0) \\ 0 & \textbf{otherwise} \end{cases}$$

	y_1y_0			
x_1x_0	00	01	10	11
00				
01				
10				
11				

$z_2z_1z_0$

	2 variables	n variables
AND	$x_1 x_0$	$x_{n-1} x_{n-2} \ldots x_0$
OR	$x_1 + x_0$	$x_{n-1} + x_{n-2} + \ldots + x_0$
XOR	$x_1 x_0' + x_1' x_0 = x_1 \oplus x_0$	
EQUIV	$x_1' x_0' + x_1 x_0$	
NAND	$(x_1 x_0)' = x_1' + x_0'$	$(x_{n-1} x_{n-2} \ldots x_0)' = x_{n-1}' + x_{n-2}' + \ldots + x_0'$
NOR	$(x_1 + x_0)' = x_1' x_0'$	$(x_{n-1} + x_{n-2} + \ldots + x_0)' = x_{n-1}' x_{n-2}' \ldots x_0'$

They can also be presented using tables:

ALGEBRAIC METHOD OF OBTAINING EQUIVALENT EXPRESSIONS

- MAIN IDENTITIES OF BOOLEAN ALGEBRA

1.	$a + b = b + a$	$ab = ba$	Commutativity
2.	$a + (bc) = (a + b)(a + c)$	$a(b + c) = (ab) + (ac)$	Distributivity
3.	$a + (b + c) = (a + b) + c$	$a(bc) = (ab)c$	Associativity
	$= a + b + c$	$= abc$	
4.	$a + a = a$	$aa = a$	Idempotency
5.	$a + a' = 1$	$aa' = 0$	Complement
6.	$1 + a = 1$	$0a = 0$	
7.	$0 + a = a$	$1a = a$	Identity
8.	$(a')' = a$		Involution
9.	$a + ab = a$	$a(a + b) = a$	Absorption
10.	$a + a'b = a + b$	$a(a' + b) = ab$	Simplification
11.	$(a + b)' = a'b'$	$(ab)' = a' + b'$	DeMorgan's Law

9.	$a + ab = a$	$a(a + b) = a$	Absorption

10.	$a + a'b = a + b$	$a(a' + b) = ab$	Simplification

11. $(a + b)' = a'b'$ $(ab)' = a' + b'$ DeMorgan's Law

EXAMPLE

SHOW THAT E_1 AND E_2 ARE EQUIVALENT:

$$
\begin{aligned}
E_1(x_2, x_1, x_0) &= x_2x_1 + x_2x_1' + x_2x_0 \\
E_2(x_2, x_1, x_0) &= x_2
\end{aligned}
$$

EXAMPLE

SHOW THAT E_1 AND E_2 ARE EQUIVALENT:

$$
\begin{aligned}
E_1(x_2, x_1, x_0) &= x_2x_1 + x_2x_1' + x_2x_0 \\
E_2(x_2, x_1, x_0) &= x_2
\end{aligned}
$$

$$
\begin{aligned}
x_2x_1 + x_2x_1' + x_2x_0 &= x_2(x_1 + x_1') + x_2x_0 && \text{using } ab + ac = a(b + c) \\
&= x_2 \cdot 1 + x_2x_0 && \text{using } a + a' = 1 \\
&= x_2(1 + x_0) && \text{using } ab + ac = a(b + c) \\
&= x_2 \cdot 1 && \text{using } 1 + a = 1 \\
&= x_2 && \text{using } a \cdot 1 = a
\end{aligned}
$$

EXAMPLE

SHOW THAT E_1 AND E_2 ARE EQUIVALENT: (Using a table)

$$E_1(x_2, x_1, x_0) = x_2x_1 + x_2x_1' + x_2x_0$$
$$E_2(x_2, x_1, x_0) = x_2$$

Clicker Question

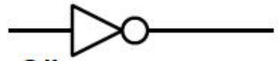
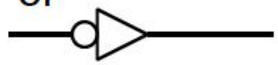
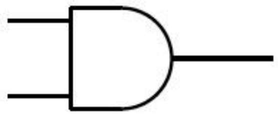
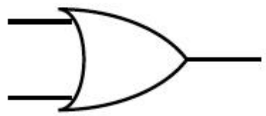
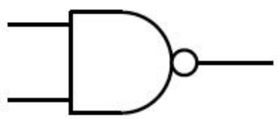
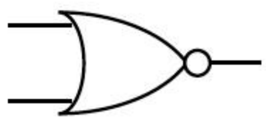
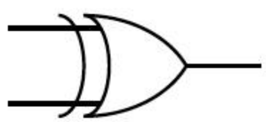
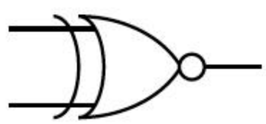
Which one is equal to $x + y$

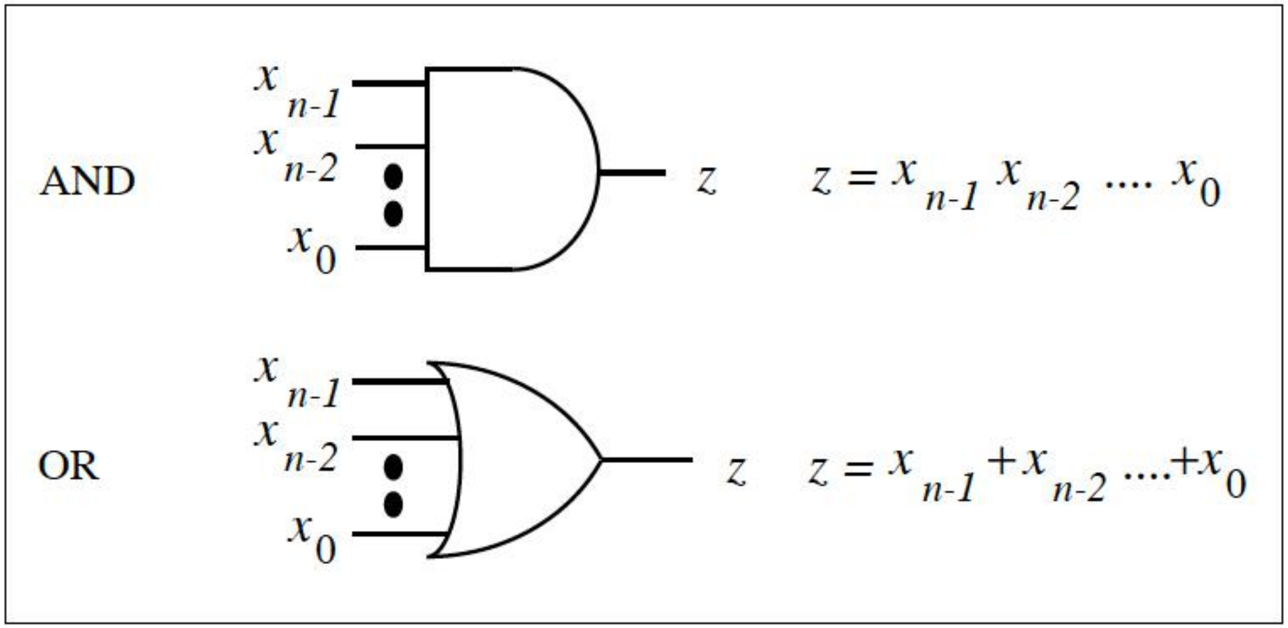
a) $x' + y'$
b) $(x \cdot y)'$
c) $(x' \cdot y')'$
d) $(xx'+x+y)$
e) c and d

Clicker Question

Which one is equal to $ABC + A' + AB'C$

a) A
b) $A + C$
c) $A' + C$
d) $ABC+A$
e) c and d

Gate	Symbol	Output	Expression
NOT	x  or x 	z	$z = x'$
AND	x_1, x_0 	z	$z = x_1 x_0$
OR	x_1, x_0 	z	$z = x_1 + x_0$
NAND	x_1, x_0 	z	$z = (x_1 x_0)'$
NOR	x_1, x_0 	z	$z = (x_1 + x_0)'$
XOR	x_1, x_0 	z	$z = x_1 x_0' + x_1' x_0 = x_1 \oplus x_0$
XNOR	x_1, x_0 	z	$z = x_1' x_0' + x_1 x_0$

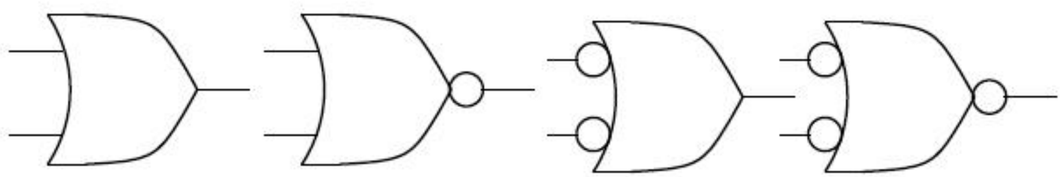


AND
x_{n-1}
x_{n-2}
x_0
z
$z = x_{n-1}\, x_{n-2}\, \ldots.\, x_0$
OR
x_{n-1}
x_{n-2}
x_0
z
$z = x_{n-1} + x_{n-2} \ldots. + x_0$

Clicker Question

Gates

Which of the following is/are a NOR gate?



a b c d

e b and c

Example

Show the truth table and symbol gate design

$$F= x' + xy + xyz$$

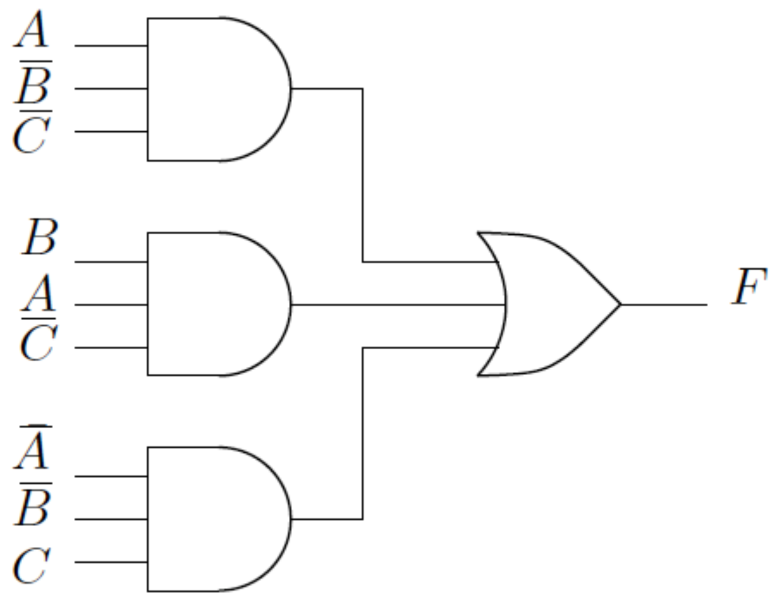
Example

Show the truth table and symbol gate design for simplified version.

$$F= x' + xy + xyz$$

Clicker Question

Digital Design



a $F = ABC + \bar{A}\bar{B}\bar{C}$

b $F = A\bar{B}\bar{C} + AB\bar{C} + A\bar{B}C$

c $F = A\bar{B}C + BAC + \bar{A}\bar{B}C$

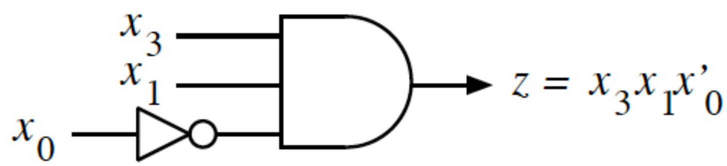
d $F = \bar{A}\bar{B}C + AB\bar{C} + A\bar{B}\bar{C}$

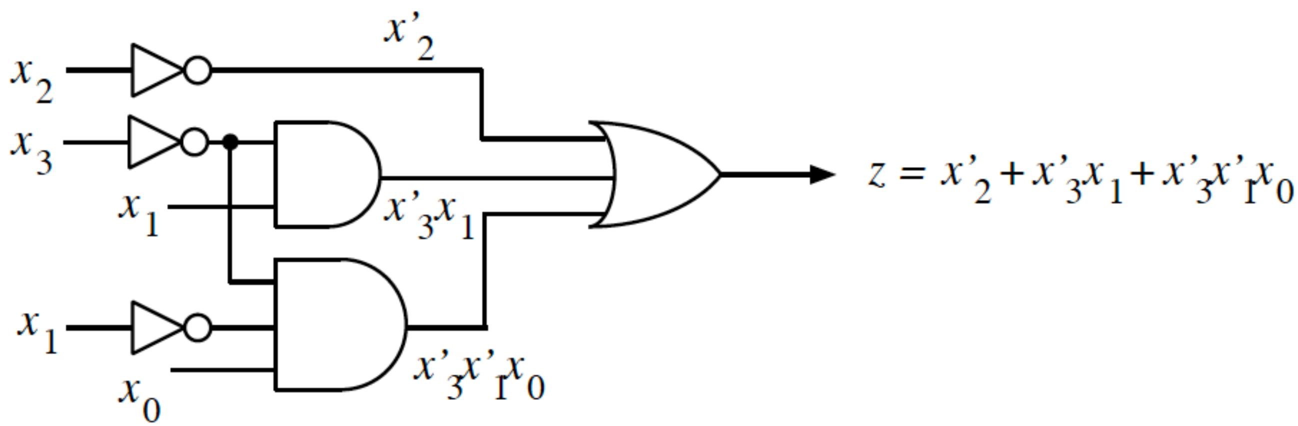
e None of the above

Sum of Products

PRODUCT TERMS	x_0, x_2x_1, $x_3x_1x_0'$
SUM OF PRODUCTS (SP)	$x_2' + x_3x_1' + x_3'x_1'x_0$

Sum of Products

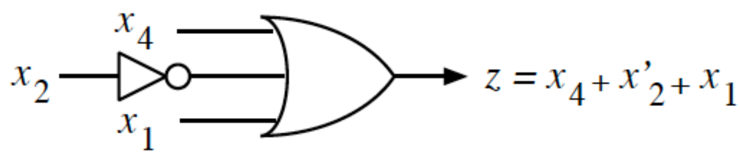


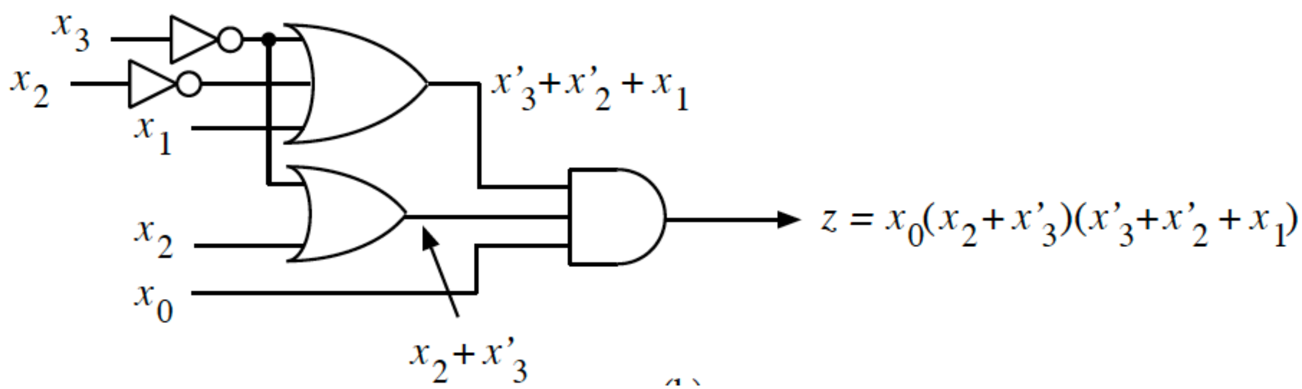


Product of Sums

SUM TERMS	$x_0,\ x_2 + x_1,\ x_3 + x_1 + x_0'$
PRODUCT OF SUMS	$(x_2' + x_3 + x_1')(x_3' + x_1)x_0$

Product of Sums





MINTERM NOTATION

$$x_i \longleftrightarrow 1; \qquad x_i' \longleftrightarrow 0$$

MINTERM m_j, j INTEGER

EXAMPLE: MINTERM $x_3x_2'x_1'x_0$ DENOTED m_9
BECAUSE $1001 = 9$

$$m_j(\underline{a}) = \begin{cases} 1 & \textbf{if} \quad a = j \\ 0 & \textbf{otherwise} \end{cases}$$

$$a = \sum_{i=0}^{n-1} a_i 2^i$$

EXAMPLE: $m_{11} = x_3x_2'x_1x_0$
– HAS VALUE 1 ONLY FOR $\underline{a} = (1, 0, 1, 1)$

MAXTERM NOTATION

$$x_i \longleftrightarrow 0; \qquad x'_i \longleftrightarrow 1$$

MAXTERM M_j , j INTEGER

EXAMPLE: MAXTERM $x_3 + x'_2 + x_1 + x'_0$ DENOTED M_5
BECAUSE 0101 = 5

$$M_j(\underline{a}) = \begin{cases} 0 & \textbf{if} \quad a = j \\ 1 & \textbf{otherwise} \end{cases}$$

EXAMPLE: $M_5 = x_3 + x'_2 + x_1 + x'_0$
– HAS VALUE 0 ONLY FOR ASSIGNMENT 0101