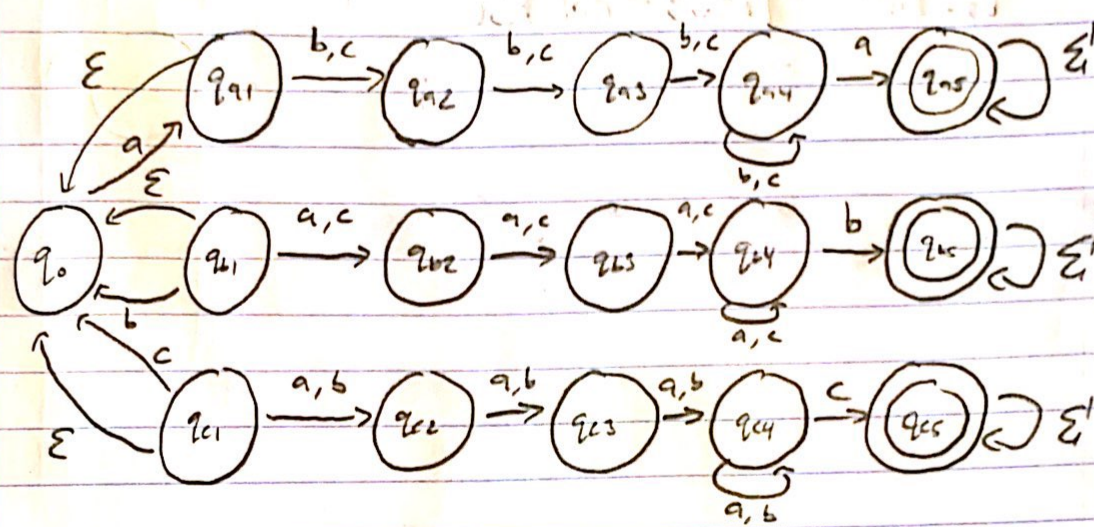
Charles Zhang
305413659

CS181 HW#2

1) $\Sigma' = \{a, b, c\}$

$L_1 = \{w \in \Sigma'^{+} \mid$ w contains at least one substring of two of the same symbol separated by at least three occurrances of the other two symbols$\}$



This design splits the language into 3 parts: substrings that begin with a, those that begin with b, and those that begin with c. It uses 3 intermediary states for each case to fulfill the condition that the start/end symbol are separated by 3+ occurrences of the other 2 symbols. At the 3rd state (q_{a4}, q_{b4}, q_{c4}), the design loops into the same state if it receives anything other than the symbol the substring starts with (accounting for 3+) and enters an accepting state if the target symbol is read. The accepting states loop into themselves, since, once a valid substring is found, the string is in the language. This makes use of nondeterminism by using the blocking convention and ε-arrows to simplify the design.

2) $\boxed{\text{Cancelled}}$

3) $L_3 = \{ w \in \Sigma^* \mid \#(a,w) = 2\#(b,w) \}$

$\Sigma = \{a, b\}$

$L_2 = \{ a^{(2n)} b^n \mid n \geq 0 \} \rightarrow$ not FSL

Union, ~~Concatenation~~, Kleene

Complementation, Intersection ✓

~~Reversal~~

Proof:

- Assume L_3 is regular
- By closure properties of FSL, L_3 must be closed under intersection
- By the notation of regular expressions, $L_3 \cap a^*b^* = \{ a^{(2n)} b^n \mid n \geq 0 \}$
- In other words $L_3 \cap a^*b^* = L_2$
- a^*b^* is regular by closure under Kleene star and concatenation
- Therefore, if L_3 is regular, $L_3 \cap a^*b^*$ is regular by closure under intersection
- However, $L_3 \cap a^*b^* = L_2$, and L_2 is known to be nonregular, leading to a contradiction $\rightarrow\leftarrow$
- L_3 cannot be regular by contradiction

4) $\Sigma_1' = \{0, 1, \#\}$

$L_4 = \{w \in \Sigma_1'^* \mid$ in w, to the right of any 0s, there is at least one 1 before any #'s $\}$

~~$(1 \cup \#)^* (0 (0 \cup 1)^* (1\#(1 \cup \#)^*)^*)^*$~~

$$\boxed{(1 \cup \#)^* (0 (0^* (1\# (1 \cup \#)^* \cup 1)^*)^*)^*}$$

This reg. exp. first checks for any prefixes of w that don't contain a 0, which should all be accepted. If it finds a 0, it then accepts any 0s that follow it. Afterwards, it looks for a 1# or a 1. If a # is found instead, the string will not be accepted. If a 1# is found, it will then accept everything past it until it hits another 0, where it once again must check the language's condition. If a 1 is found, it will look for a 1# or 1 again.

5) Let T be a k-ary directed rooted tree. Show by induction on h that for any degree $k > 1$ If T has $k^h + 1$ leaf nodes, then T has height of $h+1$

Basis: $h = 2$

- T has k^2+1 leaf nodes at a height of 3.
- Since each node has at most k children, the most leaf nodes possible at height 2 is $k \cdot k$, or k^2.
- Since the number of leaf nodes is k^2+1, the minimum height possible is 3.
- Basis solved ✓

Induction hypothesis: Assume that for a tree with k^x+1 leaf nodes, where $x>1$, the tree must have a height of at least $x+1$

Inductive step: Prove that for a tree with $k^{x+1}+1$ leaf nodes, the minimum height of that tree is $x+2$

- Since height is h+1 in all cases, an induction on the height acts as an induction on h
- By the induction hypothesis, we know a tree of height x can have <u>at most</u> k^x leaf nodes, since adding any more leaf nodes would increase the height of the tree to $x+1$
- Let T' be a k-ary tree of height $x+1$
- By the definition of a tree, T's subtrees have a height $\leq x$
- Using the induction hypothesis, we know these subtrees have at most k^x leaf nodes each
- By the definition of a k-ary tree, at most k such subtrees exist in T'
- By the nature of trees, the total number of leaf nodes in T' is equal to the number of leaf nodes in its subtrees
- Having at most k subtrees with k^x leaf nodes in each subtree tells us that T' has <u>at most</u> $k(k^x)$, or k^{x+1} leaf nodes
- Therefore, a tree with a height of $x+1$ can have at most k^{x+1} leaf nodes
- Therefore, for a tree to have $k^{x+1}+1$ leaf nodes, it must have a height of $x+2$
- The property holds for trees of height $x+2$
- Induction solved ✓

Scanned with CamScanner