CS M51A
Logic Design of Digital Systems
Winter 2021

Some slides borrowed and modified from:
M.D. Ercegovac, T. Lang and J. Moreno, Introduction to Digital Systems.
D. Patterson and J. Hennessy, Computer Organization and Design

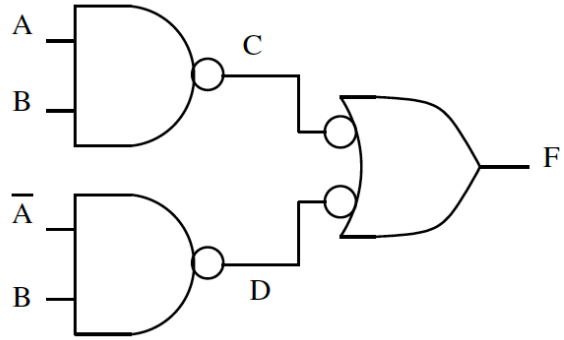
Example

Show the symbol gate design for F

$$F= (x' + y + xz)'$$

Clicker Question

Which truth table is the correct one for the following system.



A	**B**	$\bar{A}$	C	D	F
0	**0**	1	1	0	1
0	**1**	1	1	1	0
1	**0**	0	1	1	0
1	**1**	0	0	1	1

a

A	**B**	$\bar{A}$	C	D	F
0	**0**	1	1	1	1
0	**1**	1	1	0	0
1	**0**	0	1	1	0
1	**1**	0	0	1	1

b

A	**B**	$\bar{A}$	C	D	F
0	**0**	1	1	1	0
0	**1**	1	1	0	1
1	**0**	0	1	1	1
1	**1**	0	0	1	0

c

A	**B**	$\bar{A}$	C	D	F
0	**0**	1	1	1	0
0	**1**	1	1	0	1
1	**0**	0	1	1	0
1	**1**	0	0	1	1

d

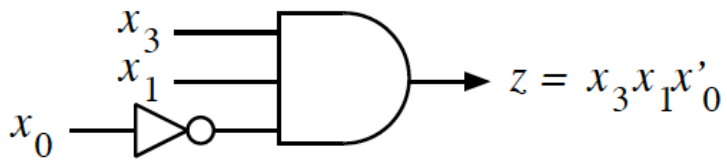
e: none of the above

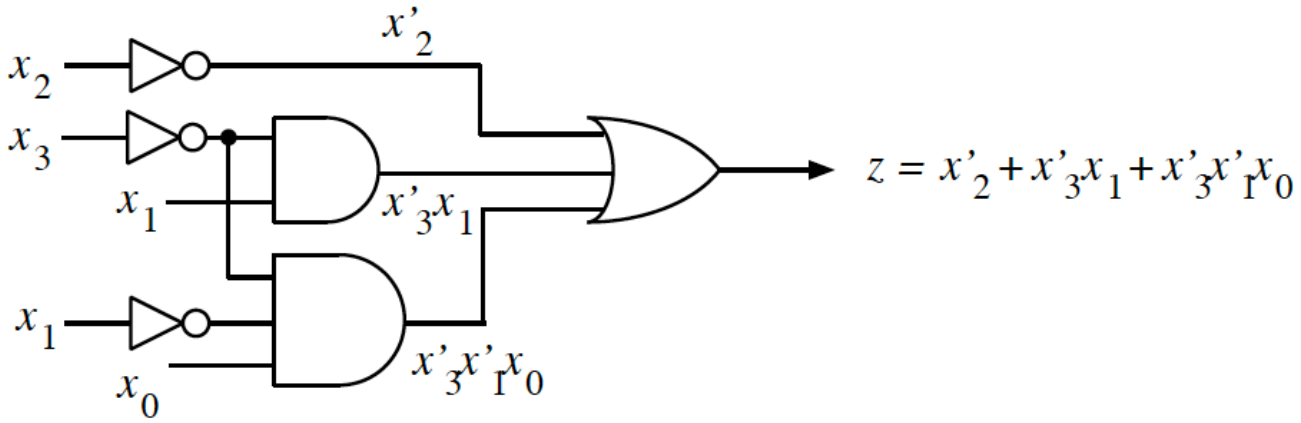
Sum of Products

PRODUCT TERMS $x_0,\ x_2x_1,\ x_3x_1x_0'$

SUM OF PRODUCTS (SP) $x_2' + x_3x_1' + x_3'x_1'x_0$

We can present them using symbols





MINTERM NOTATION

$$x_i \longleftrightarrow 1; \qquad x_i' \longleftrightarrow 0$$

MINTERM m_j, j INTEGER

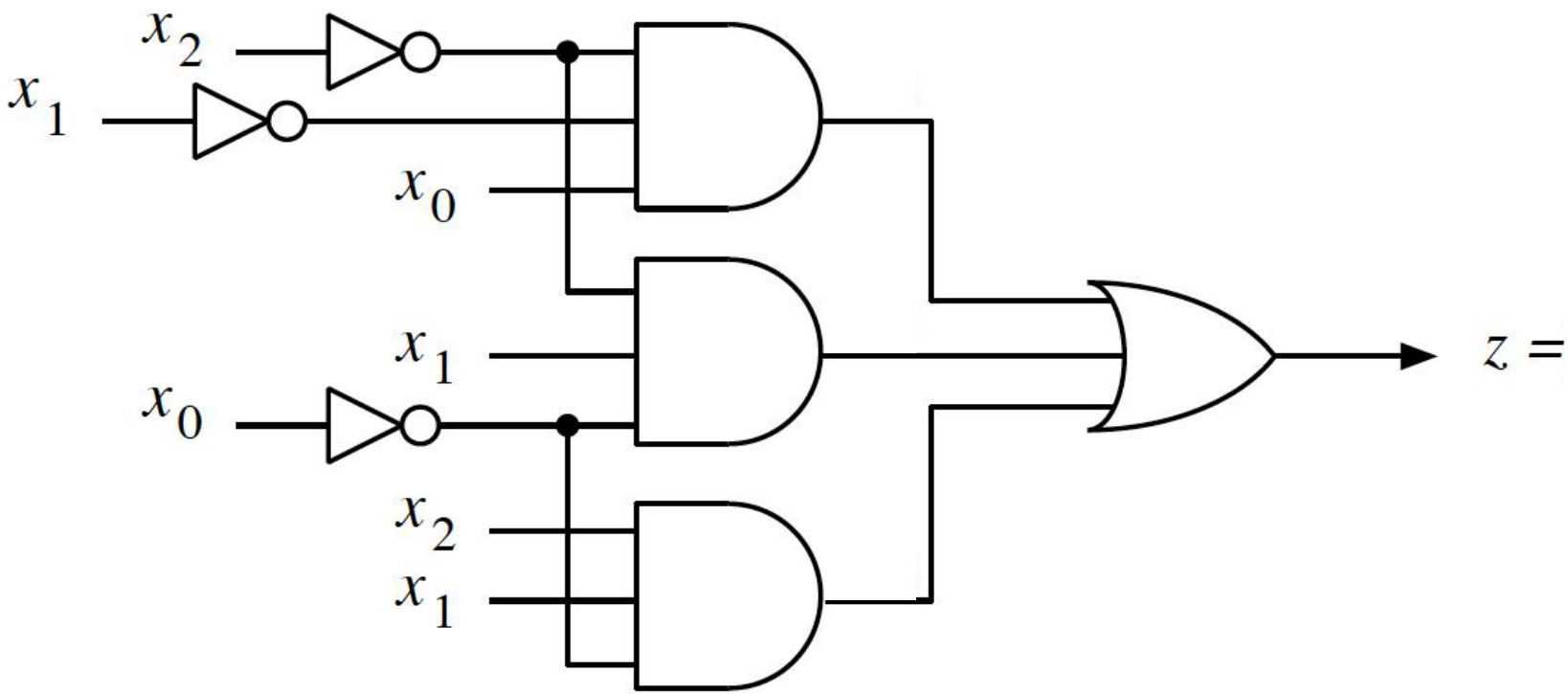
EXAMPLE: MINTERM $x_3x_2'x_1'x_0$ DENOTED m_9
BECAUSE $1001 = 9$

EXAMPLE: $m_{11} = x_3x_2'x_1x_0$
– HAS VALUE 1 ONLY FOR $\underline{a} = (1, 0, 1, 1)$

MINTERM FUNCTIONS

$x_2x_1x_0$	m_0 $x_2'x_1'x_0'$	m_1 $x_2'x_1'x_0$	m_2 $x_2'x_1x_0'$	m_3 $x_2'x_1x_0$	m_4 $x_2x_1'x_0'$	m_5 $x_2x_1'x_0$	m_6 $x_2x_1x_0'$	m_7 $x_2x_1x_0$
000	1	0	0	0	0	0	0	0
001	0	1	0	0	0	0	0	0
010	0	0	1	0	0	0	0	0
011	0	0	0	1	0	0	0	0
100	0	0	0	0	1	0	0	0
101	0	0	0	0	0	1	0	0
110	0	0	0	0	0	0	1	0
111	0	0	0	0	0	0	0	1

EXAMPLE



Example : TABLE $\rightarrow$ SUM OF MINTERMS

j	$x_2\ x_1\ x_0$	f
0	000	0
1	001	0
2	010	1
3	011	1
4	100	0
5	101	1
6	110	0
7	111	0

$E =$

Example

Present F and G in sum of minterms format

X	Y	Z	F	G
0	0	0	0	1
0	0	1	1	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	0

$F =$?

$G =$?

Clicker Question

Digital Design

X	Y	Z	F	X	Y	Z	F
0	0	0	1	1	0	0	0
0	0	1	0	1	0	1	1
0	1	0	0	1	1	0	1
0	1	1	1	1	1	1	0

Which is the correct sum of product form for F? (sum of minterms)

a $F = \bar{X}\bar{Y}Z + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XYZ$

b $F = \bar{X}\bar{Y}\bar{Z} + X\bar{Y}\bar{Z} + X\bar{Y}\bar{Z}$

c $F = \bar{X}\bar{Y}\bar{Z} + \bar{X}YZ + X\bar{Y}Z + XY\bar{Z}$

d $F = XY\bar{Z} + X\bar{Y}Z + \bar{X}\bar{Y}\bar{Z} + X\bar{Y}Z$

e None of the above

CONVERSION TO SUM OF MINTERMS

1. CONVERT TO EQUIVALENT SUM OF PRODUCTS

$$\begin{aligned} E(x_2, x_1, x_0) &= (x_2 x_1)' x_0 \\ &= (x_2' + x_1') x_0 \\ &= x_2' x_0 + x_1' x_0 \end{aligned}$$

2. CONVERT PRODUCT TERMS TO MINTERMS

$$\begin{aligned} E(x_2, x_1, x_0) &= x_2' x_0 + x_1' x_0 \\ &= x_2' x_0 (x_1 + x_1') + x_1' x_0 (x_2 + x_2') \\ &= x_2' x_1 x_0 + x_2' x_1' x_0 + x_2 x_1' x_0 + x_2' x_1' x_0 \end{aligned}$$

3. ELIMINATE REPEATED MINTERMS

$$E(x_2, x_1, x_0) = x_2' x_1' x_0 + x_2' x_1 x_0 + x_2 x_1' x_0$$

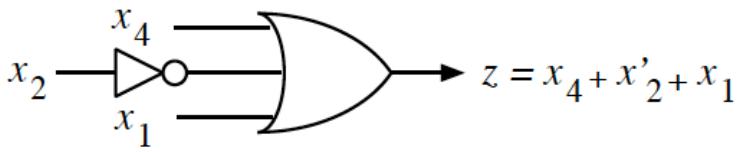
Example : CONVERSION TO SUM OF MINTERMS

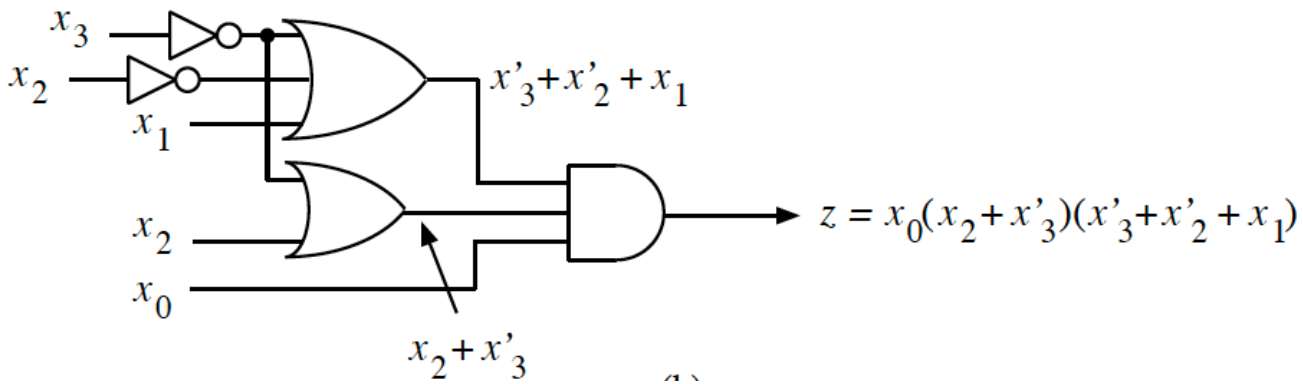
$$E(x_2, x_1, x_0) = x_2x_1' + x_2x_0' + x_1x_0'$$

Product of Sums

SUM TERMS	$x_0,\ x_2 + x_1,\ x_3 + x_1 + x_0'$
PRODUCT OF SUMS	$(x_2' + x_3 + x_1')(x_3' + x_1)x_0$

Product of Sums





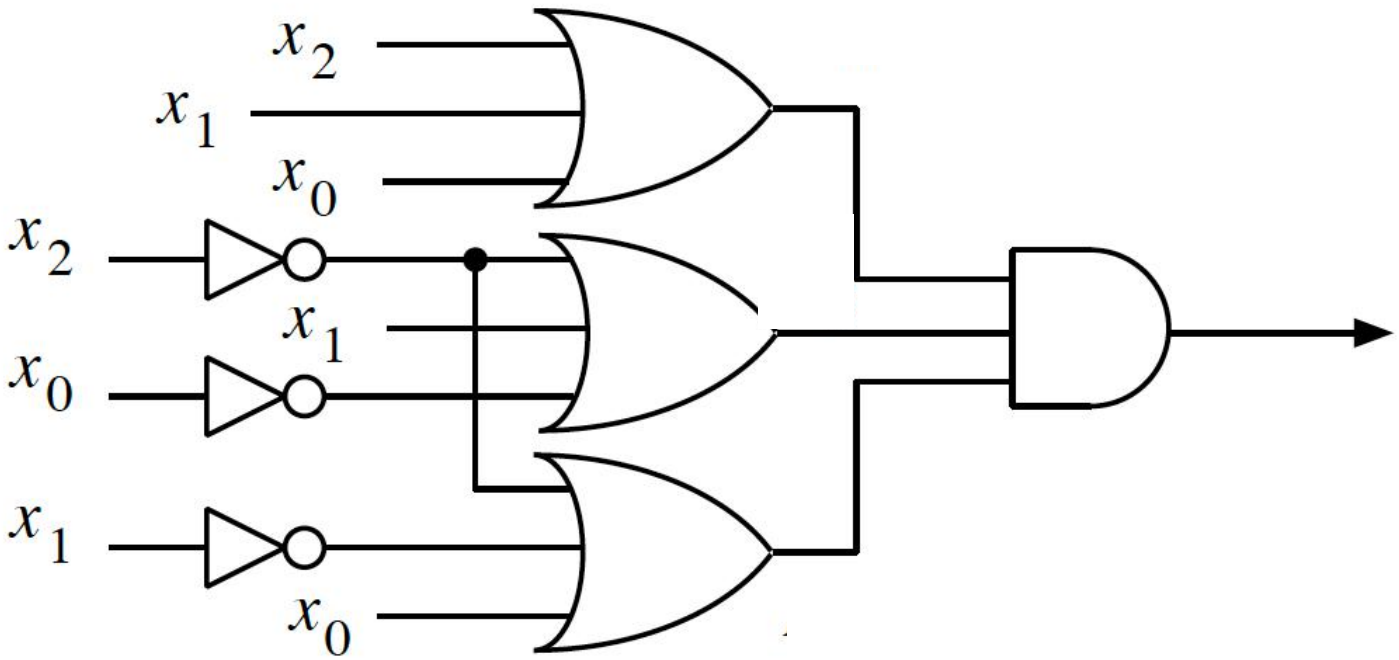
MAXTERM NOTATION

$$x_i \longleftrightarrow 0; \qquad x_i' \longleftrightarrow 1$$

MAXTERM M_j , j INTEGER

EXAMPLE: MAXTERM $x_3 + x_2' + x_1 + x_0'$ DENOTED M_5
BECAUSE $0101 = 5$

EXAMPLE: $M_5 = x_3 + x_2' + x_1 + x_0'$
– HAS VALUE 0 ONLY FOR ASSIGNMENT 0101


x_2
x_1
x_0
x_2
x_1
x_0
x_1
x_0

Example : TABLE → PRODUCT OF SUMS

j	$x_2x_1x_0$	f
0	000	0
1	001	1
2	010	1
3	011	0
4	100	0
5	101	0
6	110	1
7	111	0

$$E(x_2, x_1, x_0) \; =$$

Example

Present F and G in product of maxterms format

X	Y	Z	F	G
0	0	0	0	1
0	0	1	1	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	0

$F =$?

$G =$?

Clicker Question

Product of Maxterms

X	Y	Z	F	X	Y	Z	F
0	0	0	1	1	0	0	0
0	0	1	0	1	0	1	1
0	1	0	0	1	1	0	1
0	1	1	1	1	1	1	0

Which one is correct?

a) F= (x'+y+z). (x+y'+z). (x+y+z'). (x'+y'+z)
b) F= (x'+y+z'). (x+y+z). (x'+y+z). (x'+y'+z)
c) F= (x+y+z'). (x+y'+z). (x'+y+z). (x'+y'+z')
d) F= (x+y'+z). (x+y+z'). (x+y'+z'). (x'+y+z)
e) none

CONVERSION AMONG CANONICAL FORMS

SUM OF MINTERMS $\longleftrightarrow$ *one-set*
PRODUCT OF MAXTERMS $\longleftrightarrow$ *zero-set*
$\Rightarrow$ CONVERSION STRAIGHTFORWARD

$$\sum m(\{j \mid f(j) = 1\}) = \prod M(\{j \mid f(j) = 0\})$$

EXAMPLE:

m-NOTATION:

$$f(x, y, z) = \sum m(0, 4, 7)$$

M-NOTATION:

$$f(x, y, z) = \prod M(1, 2, 3, 5, 6)$$