CS M51A, Winter 2021, Assignment 1
(Total Mark: 80 points, 8%)

Due: Wed Jan 13, 10:00 AM PT

Student Name: Charles Zhang
Student ID: 305-413-659

Note: You must complete the assignments entirely on your own, without discussing with others.

1. (2 points) Briefly outline the primary differences between digital and analog systems.
   Digital systems have discrete inputs and outputs, like a calculator.
   Analog systems have continuous inputs and outputs, like sound.

2. (2 points) What are the two types of digital systems?
   The 2 types of digital systems are combinational and sequential.

3. (6 points) Given the 8-bit binary number 1101 1010, give its decimal equivalent if these eight bits are interpreted as

   1 1 0 1 1 0 1 0

   (a) an 8-bit unsigned number. (show your steps)
   $2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$
   $2^7+2^6+2^4+2^3+2^1 = \boxed{218}$

   (b) an 8-bit signed magnitude number. (show your steps)
   1 1 0 1 1 0 1 0
   $(-)\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$
   $-(2^6+2^4+2^3+2^1) = \boxed{-90}$

   (c) an 8-bit 2's complement number.
   1 1 0 1 1 0 1 0
   $-2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$
   $-2^7+2^6+2^4+2^3+2^1 = \boxed{-38}$

4. (4 points) Number Representation

   (a) Write the number 125 in binary, extended to 10 bits.
   $\boxed{0001111101}$
   $2^6+2^5+2^4+2^3+2^2+2^0 = 64+32+16+8+4+1 = 125$

   (b) Compute the 2s complement negation of the 10-bit number in (a).
   ↳ complement bits, add 1
   ↳ 1110000010 + 0000000001 = $\boxed{1110000011}$

5. (8 points) Find x and y such that the following conditions are satisfied and show all the steps of your work.

5)

(a) $(817)_9 = (x)_3$ → Every digit in $r=9$ is 2 in $r=3$ → $(8)_9 = (22)_3$, $(1)_9 = (01)_3$, $(7)_9 = (21)_3$ → $\boxed{(220121)_3}$

(b) $(111)_4 = (x)_2$. → Every digit in $r=4$ is 2 in $r=2$ → $(1)_4 = (01)_2$, $(1)_4 = (01)_2$, $(1)_4 = (01)_2$ → $\boxed{(10101)_2}$

(c) $(100)_6 = (x)_9$. $\overset{1}{6^2}\,\overset{0}{6^1}\,\overset{0}{6^0} = (36)_{10}$ → $(9^2)_{10} > (36)_{10} > (9^1)_{10}$ → $4 \times 9^1 = (36)_{10}$ → $\boxed{(40)_9}$

(d) What is the largest number y that can be represented with 3 digit in radix 3. Show y in radix 3 and decimal.. $y = 3^3 - 1 = \boxed{26}$

$\boxed{y = (222)_3} = (2\times3^2)+(2\times3^1)+(2\times3^0) =$
$18 + 6 + 2 = 26$ ✓

6. (8 points) Two's Complement

(a) Write 45 in two's complement representation. $\boxed{0101101}$ → $2^5+2^3+2^2+2^0=45$ ✓

(b) Sign extend the number in part (a) to 8 bits. $\boxed{00101101}$ → Add MSBs

(c) Write -27 in two's complement representation. $\boxed{100101}$ → $-2^5+2^2+2^0 = -27$ ✓

(d) Sign extend the number in part (c) to 8 bits. $\boxed{11100101}$ → Add MSBs

7. (16 points) Add the following pairs of 8-bit two's complement binary numbers, giving a 8-bit result (i.e., throw away the carry-out). Also give the signed decimal value of the the 8-bit result. Note whether or not an overflow occurred for any addition.

```
  1111 1111  (-1)
 +0000 0001  (1)   → 0
```
2's Complement Binary: 0000 0000
Signed Decimal: 0
Overflow? Yes
Is the sum correct? Yes

```
  0110 1010  (106)
 +0010 1101  (45)  → 151
```
2's Complement Binary: 10010111
Signed Decimal: -105
Overflow? Yes
Is the sum correct? No

```
  0010 1100  (44)
 +0011 1101  (61)  → 105
```
2's Complement Binary: 01101001
Signed Decimal: 105
Overflow? No
Is the sum correct? Yes

```
  1011 1001  (185)
 +1011 0001  (177) → 362
```
2's Complement Binary: 01101010
Signed Decimal: 106
Overflow? Yes
Is the sum correct? No

8. (16 points) Draw and fill a truth table for a system which has three inputs (a, b, c) and two outputs (f, g). f and g functions are defined as follow.

- f is a majority function (i.e. it is 1 when more than half of the inputs are 1)
- g is a minority function(i.e. it is 1 when less than half of the inputs are 1.)

(a,b,c)	f(a,b,c)	g(a,b,c)
0 0 0	0	1
0 0 1	0	1
0 1 0	0	1
0 1 1	1	0
1 0 0	0	1
1 0 1	1	0
1 1 1	1	0

9. (8 points) For the following high-level specification, determine the output in both decimal and 4-bits binary. → unsigned

- Input $x \in \{0, 1, 2, 3\}$
- Function $y(x) = x^2 + 2$

$y(0) = 0+2 = 2 = (10)_2$
$y(1) = 1+2 = 3 = (11)_2$
$y(2) = 4+2 = 6 = (110)_2$
$y(3) = 9+2 = 11 = (1011)_2$

$\boxed{y \in (\{2, 3, 6, 11\})_{10}}$

$\boxed{y \in (\{0010, 0011, 0110, 1011\})_2}$

10. (10 points) Find out a high-level specification (input set, output set and input-output function) for a combinational system that compute the distance between two 1's in the input bit-vector $x = (x_{n-1}, \cdots x_0)$. Assume x has exactly two 1's. For example, if $x = (1, 0, 0, 1)$, then the distance is 3.

input: $x = \{(x_{n-1}, \ldots, x_0) \mid n > 0, x_i = 0 \text{ or } 1, \sum_{i=0}^{n-1} x_i = 3\}$
output: $y = \mathbb{N}^+, y \leq n-1$ $f: f(x) = i - j$ if $x_i = 1$ and $x_j = 1$ and $i > j$