

CS118: Lecture 2, Limits to Transmission

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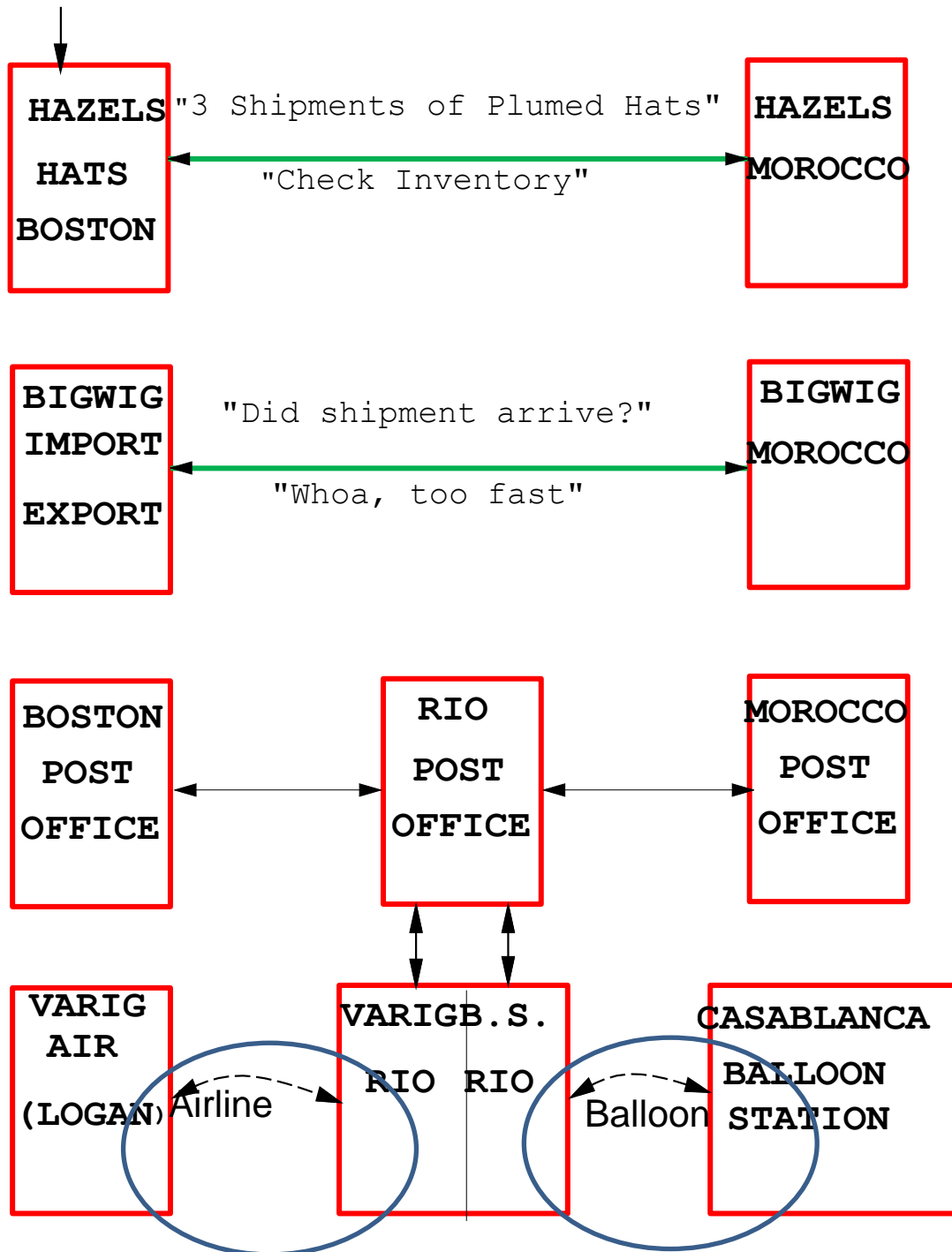
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Main Idea: There are limits to the speed at which bits can be transmitted between a sender and a receiver over a channel based on how fast the channel can react (bandwidth) and the noise.

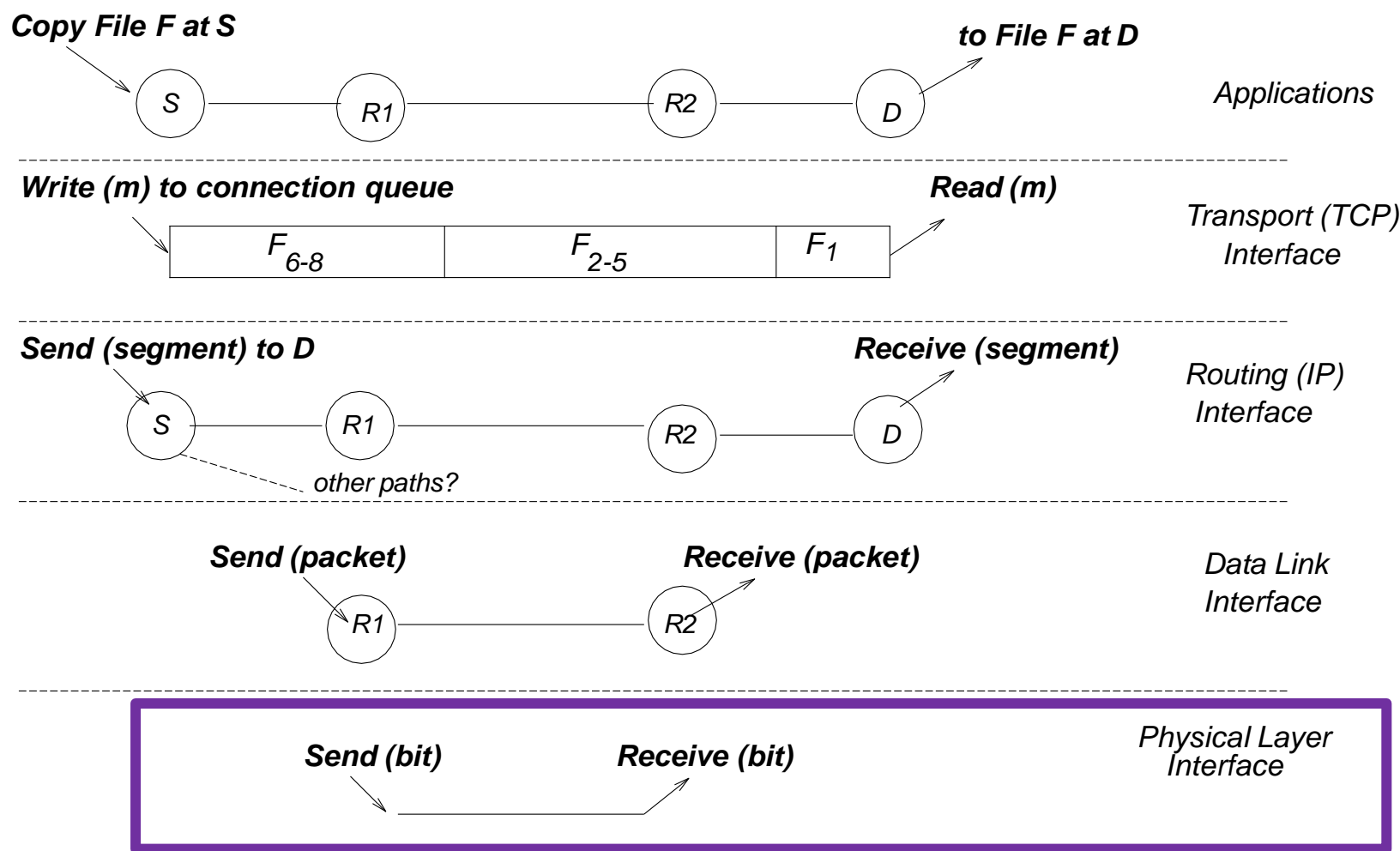
REMEMBER THE HAT TRANSFER

A Plumed Hat, Please"



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We will start with the raw links (physical layer) and work our way up



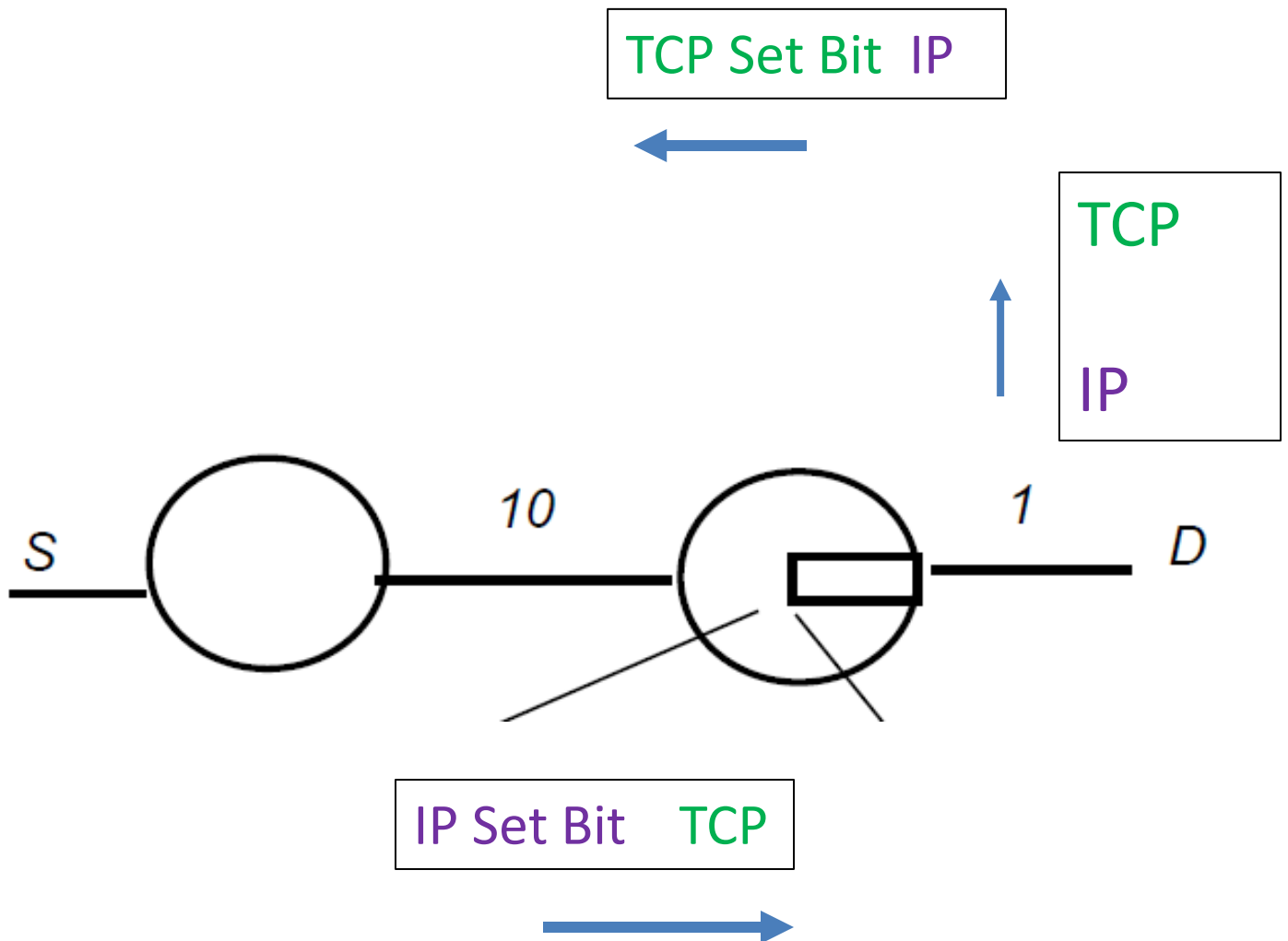
PHYSICAL LAYER ABSTRACTION:

Sending a bit over a “channel”. This lecture: how fast can we transmit?

Watch those headers!

- Communication between layer entities shares physical medium by using a layer header for each layer in each message. Think of data in envelope with transport header, stuffed in envelope with routing header, stuffed in envelope with DL header.
- Sharing headers saves postage and also trivially coordinates headers with corresponding data (compared to out-of-band transmission between layers).
- Strict Layering: Each layer only looks at its header and interface data to do its job. Software engineering: changes to one layer do not cause other layers to be reimplemented. Information can be passed between layers via interface.
- As data moves down the layers, each layer adds its header. As data moves up, each layer strips off its header.

Sample Midterm Question on Layering



The Foundation: Sending Bits

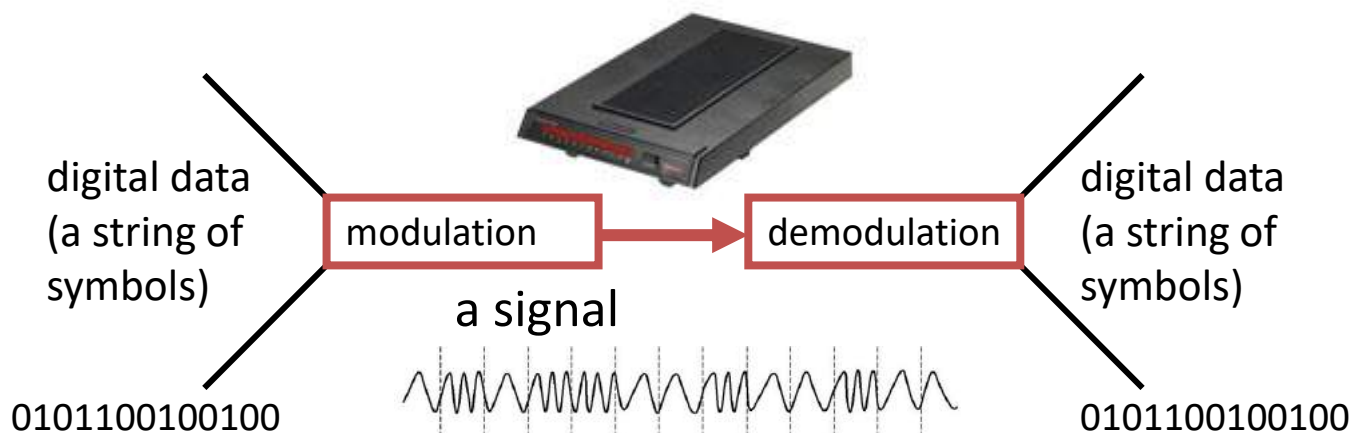
A three-step process

Take an input stream of bits (digital data)

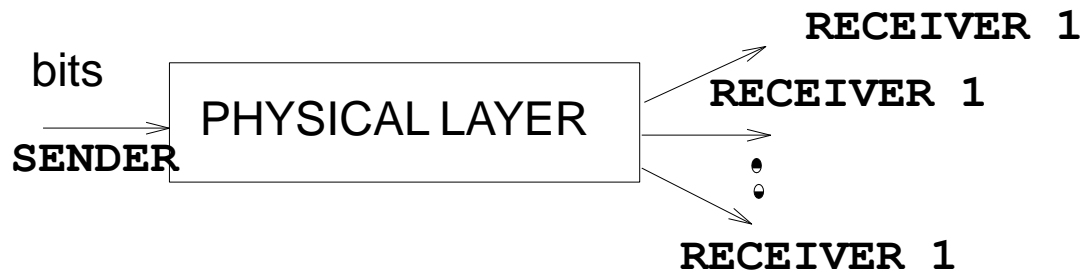
Modulate some physical media to send data (analog)

Demodulate the signal to retrieve bits (digital again)

Anybody heard of a **modem** (Modulator-demodulator)?



What does the Physical Layer Do?



- A possibly faulty, single-hop, bit pipe that connects a sender to possibly multiple receivers



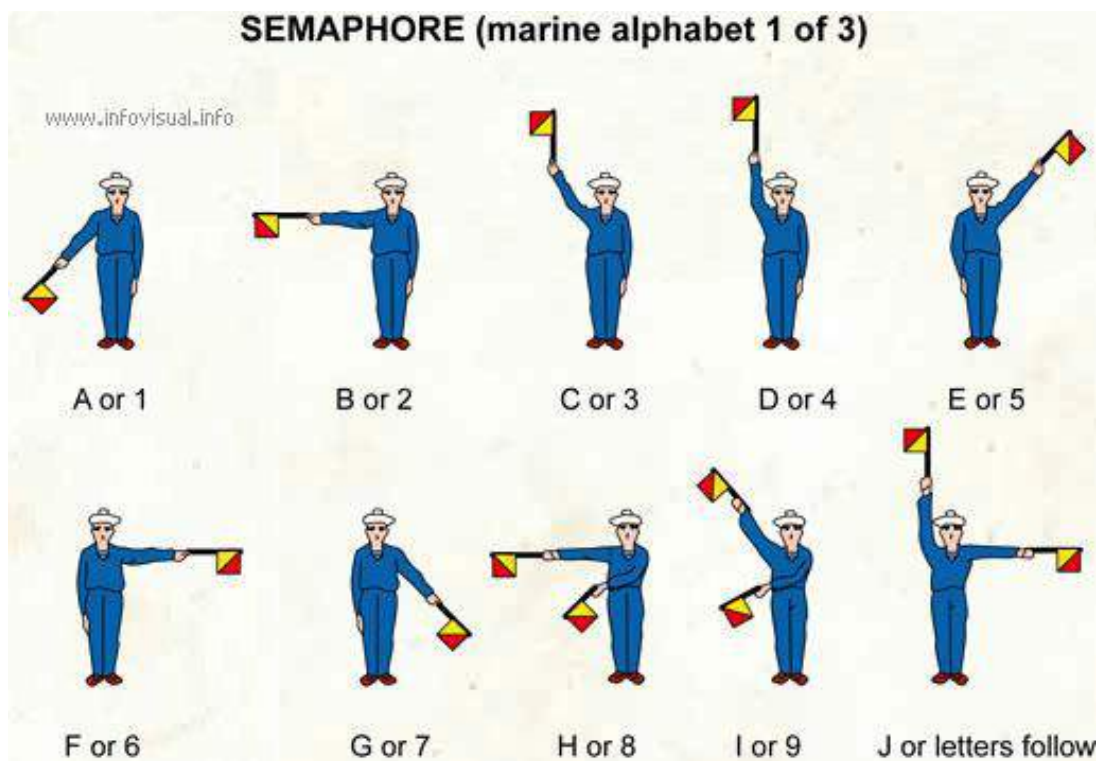
Imagine your radio has failed and you wish to communicate with another ship across a few miles. You use some colored flags and semaphores. You are in hurry to get your message across? What limits your transmission speed?

Morse Code Transmission

A	• —	V	• • • —
B	— • • •	W	• — —
C	— • — •	X	— • • —
D	— • •	Y	— • — —
E	•	Z	— — • •
F	• • — •	.	• — • — • —
G	— — •	,	— — • • — —
H	• • • •	?	• • — — • •
I	• •	/	— • • — •
J	• — — —	@	• — — • • •
K	— • —	1	• — — — —
L	• — • •	2	• • — — —
M	— —	3	• • • — —
N	— •	4	• • • • —
O	— — —	5	• • • • •
P	• — — •	6	— • • • •
Q	— — • —	7	— — • • •
R	• — •	8	— — — • •
S	• • •	9	— — — — •
T	—	0	— — — — —
U	• • —		

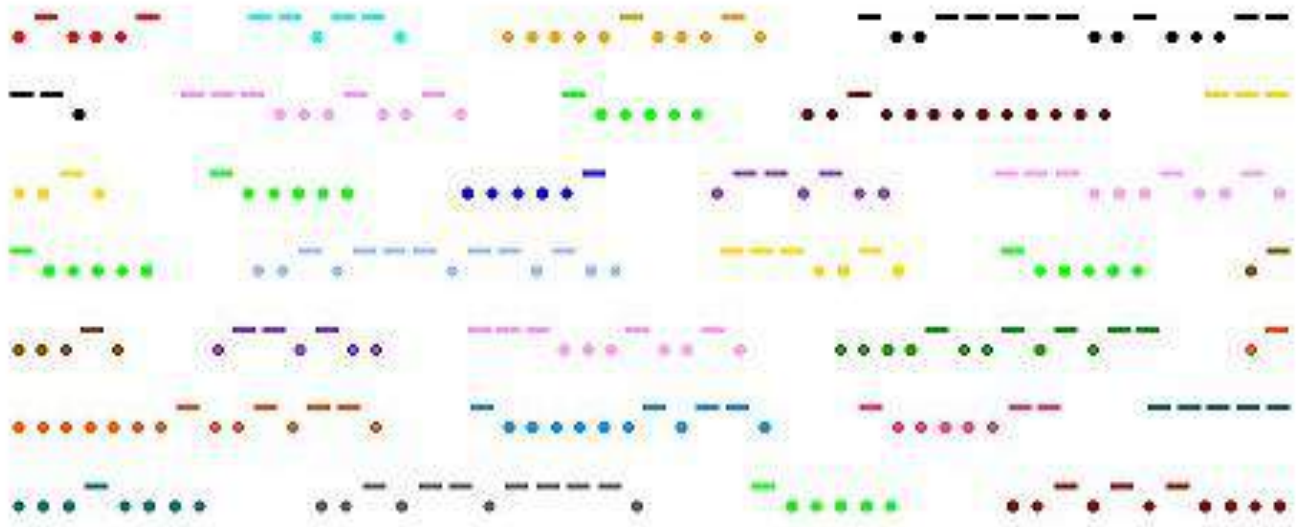


Signaling with Semaphores



Morse Code Message

Let man have dominion over the fish of the sea and over the fowl of the air and over every living thing that moves upon the earth



Think: Deeper Issues with the Analogy

- Equivalents of semaphore signaling and your cable link?
- Equivalents of fog
- How to increase information rate sent in a semaphore signal per second? And in a cable
- **Think about these differences before class!**

Morse Code Analogy

Example bit pipe: sending Morse Code to receivers using a flashlight. Issues:

- **Fundamental Limits:** Brain-eye system processing limits leads to Inter Symbol Interference
- **Media Issues:** Flashlight, semaphore
- **Coding:** Morse code, getting in synch, knowing receiver rate.

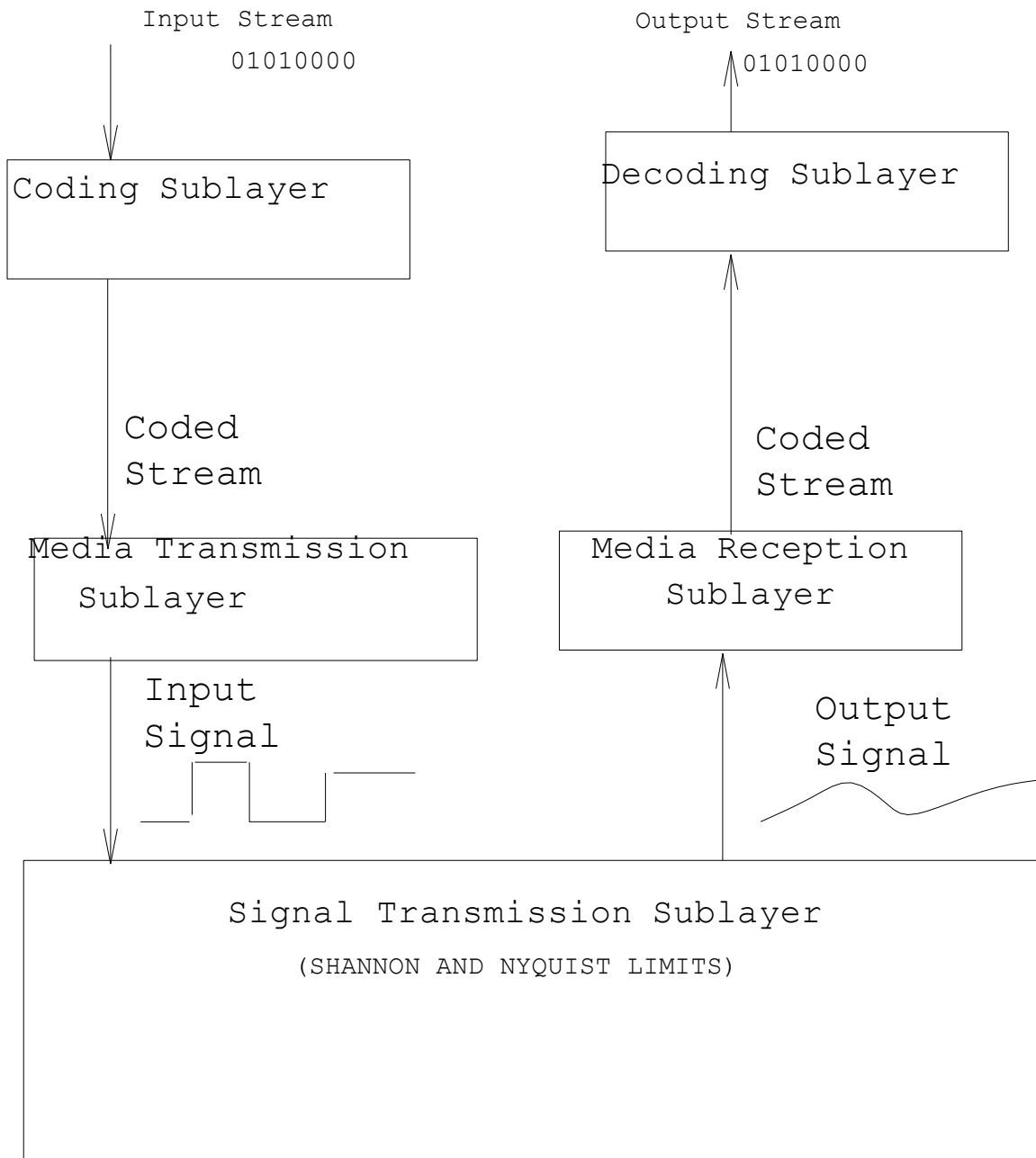
Think: Deeper Issues with the Analogy

- Equivalents of semaphore signaling and your cable link?
- Equivalents of fog
- How to increase information rate sent in a semaphore signal per second? And in a cable
- **Discuss the answers in your Breakout Group and Report your answers**

Now in more detail

- We will divide the physical layer into sublayers, starting with a coding layer (semaphores have codes as well)
- To understand the Shannon Limit we have to understand what bandwidth means in Hz (EE idea) and why its related to bandwidth in bits/sec (CS)
- To do so we need to take a small painless detour into the world of signals and systems and Fourier Analysis. Don't get scared. Chill, we'll go easy on the math

PHYSICAL LAYER: SUBLAYERS



Why understand the Physical Layer in Sublayers?

- The bottom sublayer is really describing the essential properties of the media (frequency response, bit error rate). These influence data transmission rates (Nyquist, Shannon limits). **This lecture.**
- The middle sublayer describes properties of particular media — e.g.. satellites, coaxial cable, fibre.
- The top sublayer is about things like clock recovery, synchronization etc.

Can study Sublayers independently. Separate concerns. Each sublayer exacts its price!

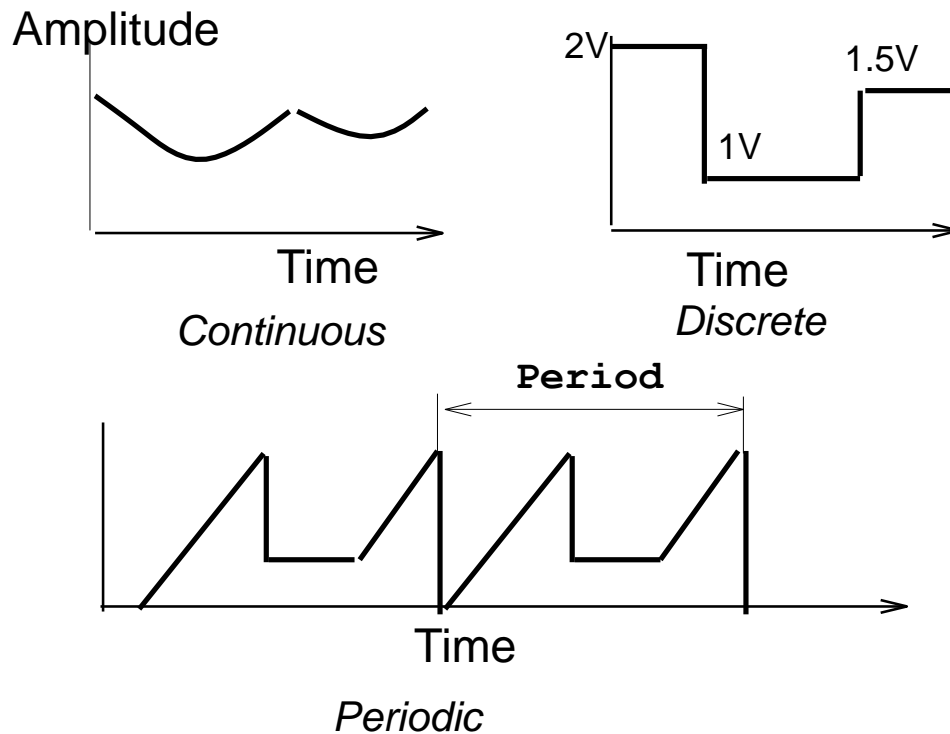
Bottom Sublayer: Signal Transmission and Limits

How fast can you send and what prevents you from sending faster?

Sending bits to a receiver

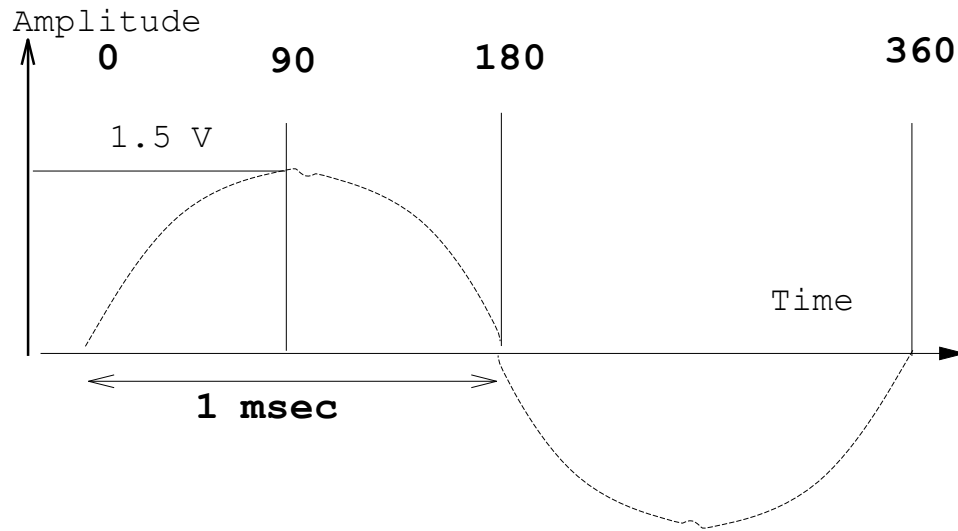
- **Goal:** to send a sequence of 0's and 1's from a sender to a receiver by sending energy (e.g., light, electricity) over a channel (e.g., fiber, cable). One coding: 0 = no energy, 1 = energy.
- **Problem:** Real channels distort input energy signals. Leads to two questions.
 - **Q1:** How can we predict what a given channel will do to an input signal given some properties of the channel. Answer: Fourier Analysis.
 - **Q2:** How does distortion affect maximum bit rate? Answer: Nyquist (sluggishness) and Shannon (noise) limits.

Signals, and channels



- **Signal:** energy (e.g., voltage, light) that varies with time. Continuous and Discrete. Periodic. Period and frequency.
- **Channel:** physical medium that conveys energy from a sender to a receiver (e.g., a fiber link) with possible distortion.

Sine Waves

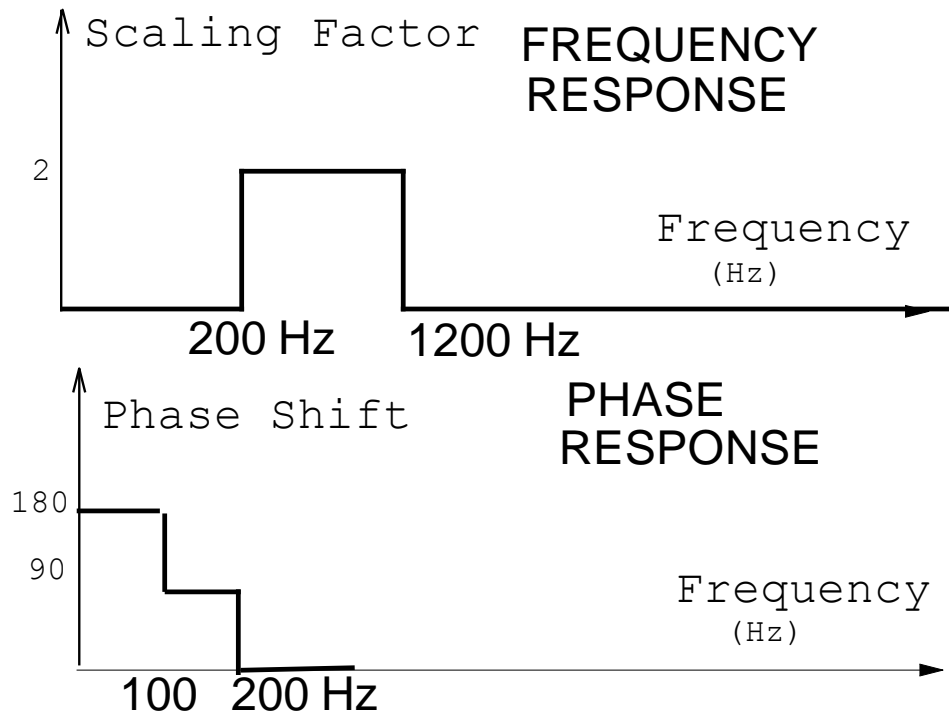


- Sine waves are special because all signals can be rewritten in terms of sine waves.
- Mathematically: $A \sin(2\pi f t + \theta)$, A is max value, f is frequency, θ is initial phase shift
- Example: Frequency 1 Hz, $\theta = 0$. Values at $t = 0$ and $t = 1/4$. Use calculator but express angle in radians!

Fourier Analysis: the big picture

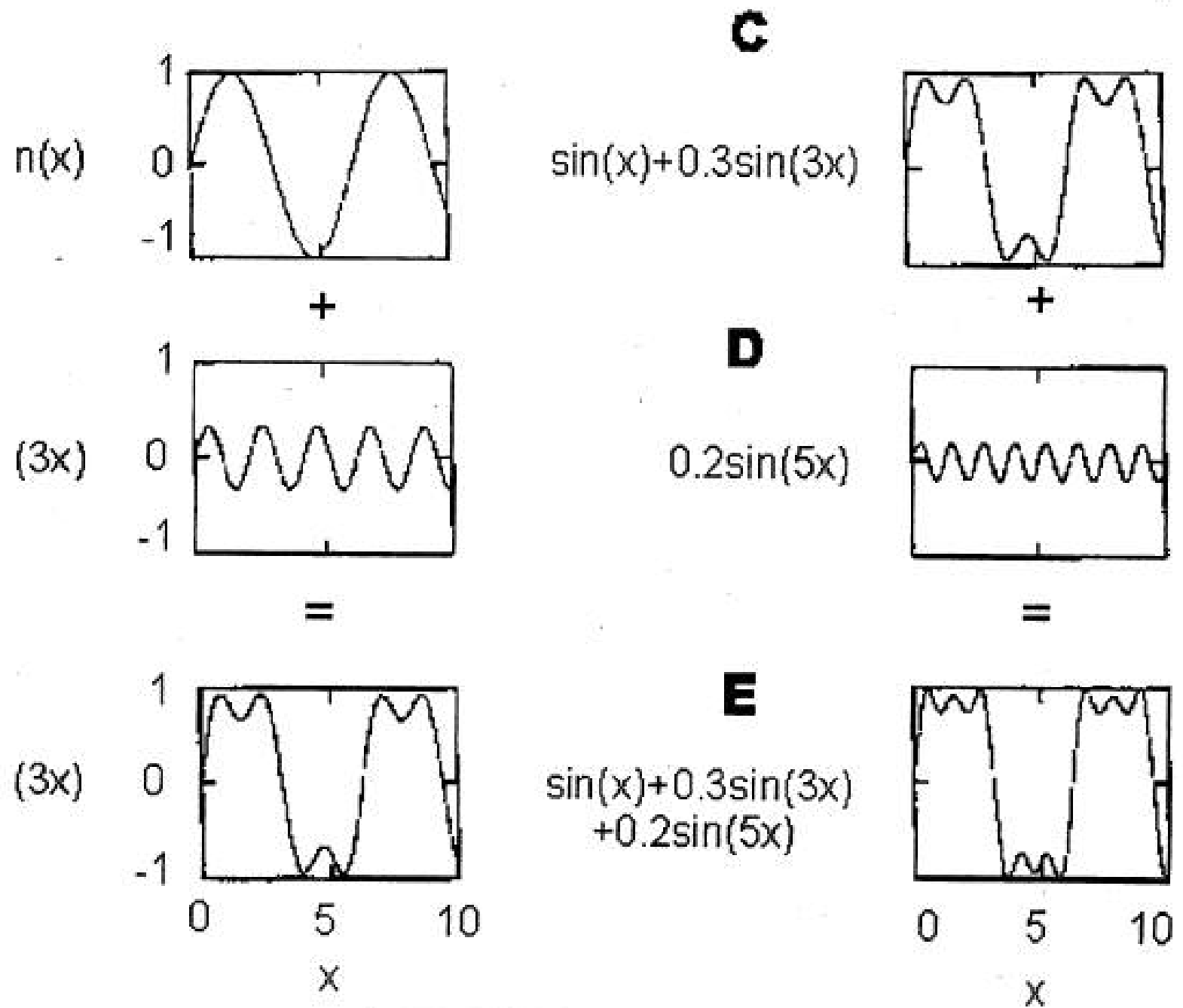
- If we forget about noise, most channels are “nice” to sine waves. A sine wave of frequency f is always scaled by a fixed factor $S(f)$ and phase shifted by a fixed amount $p(f)$ regardless of amplitude.
- Thus we can completely describe a channel by plotting the values of $S(f)$ (frequency response) and $p(f)$ (phase response) for all values of frequency f .
- To find what happens to arbitrary signal S , we i) Use Fourier Analysis to rewrite S as a sum of sine waves of diff frequencies ii) Use frequency and phase response to see effect of each sine wave iii) Add scaled sine waves to find output signal.

Frequency and Phase Response Examples

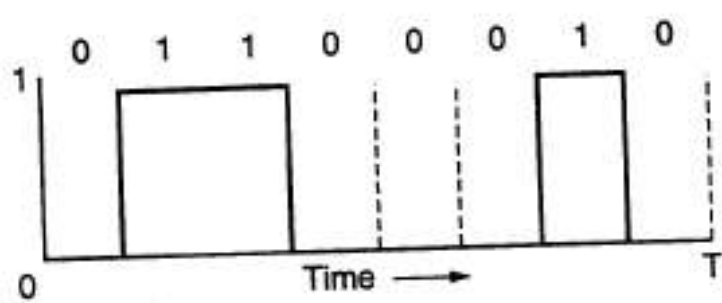


- Bandwidth: range of frequencies for which channel passes signal through. Not very precise.

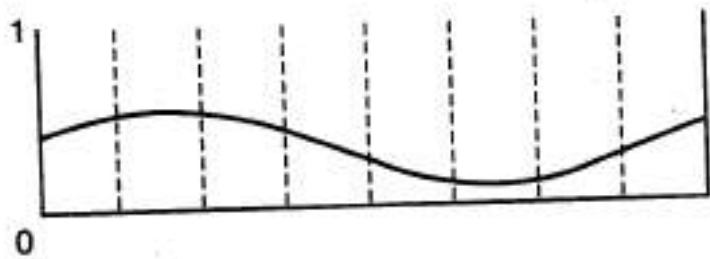
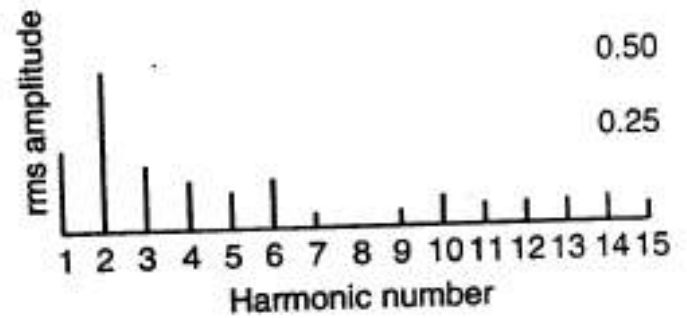
More bandwidth, more fidelity



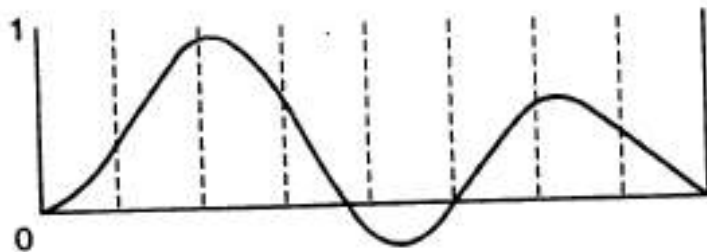
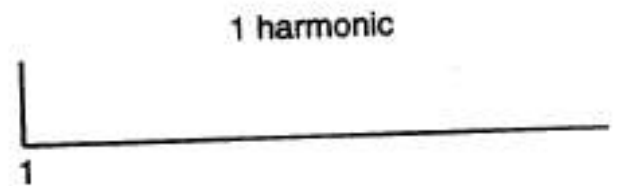
More fidelity, better recovery of bits



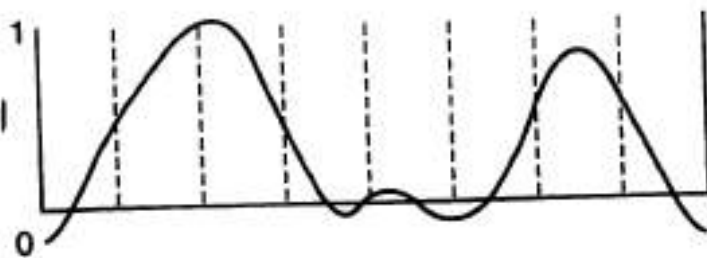
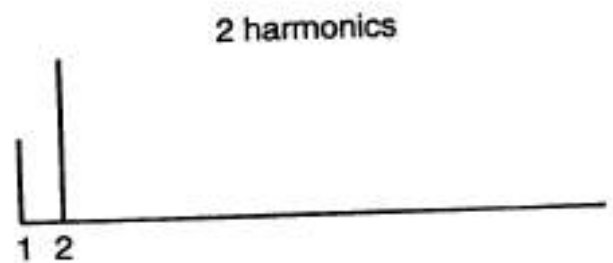
(a)



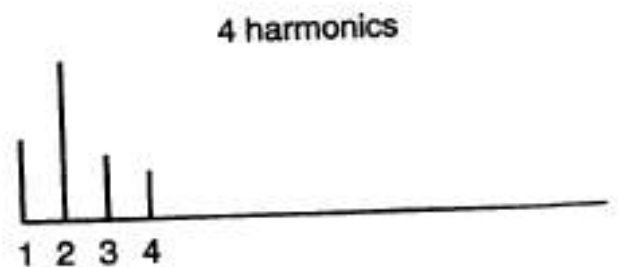
(b)



(c)



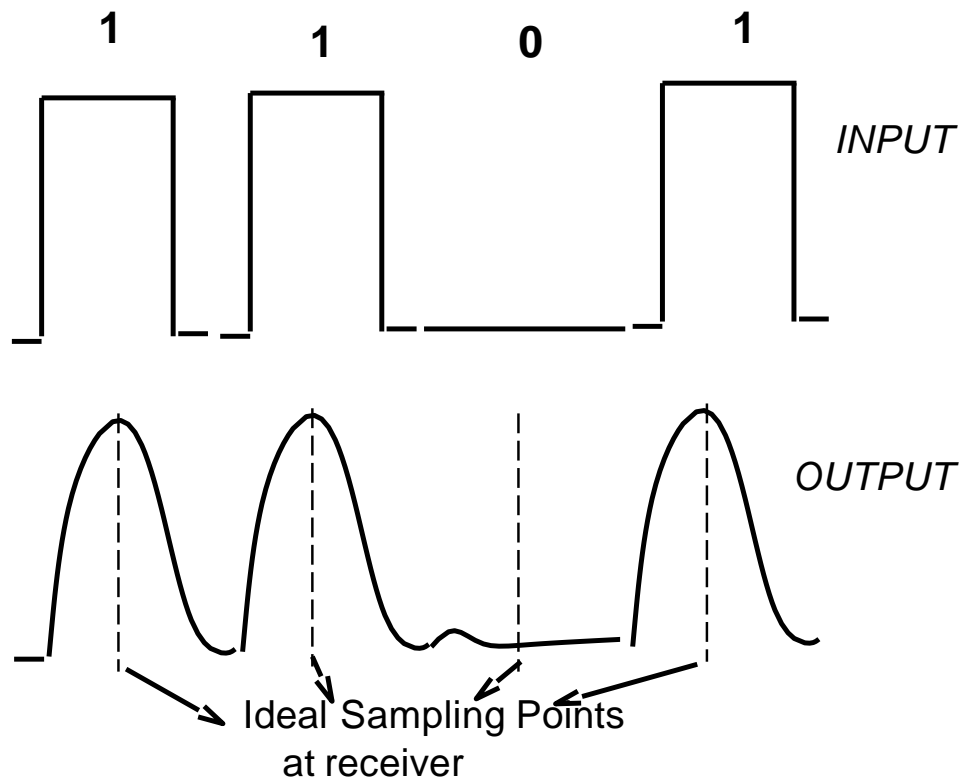
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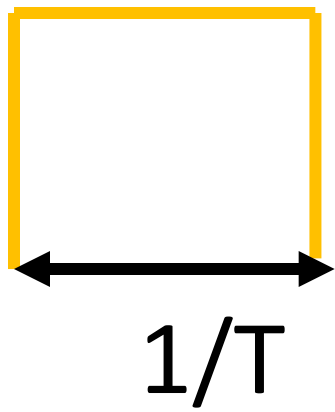
Sluggishness and Noise

- Most channels are sluggish (they take time to respond) because they turn a deaf ear to higher frequencies in the input signal. Thus lower bandwidth channels are more sluggish.
- What about noise? Different models for different channels. Simplest and common model: white noise (uniformly distributed at all frequencies and normally distributed within a frequency)

Sampling Bits



- Receivers recover the bits in the input signal by *sampling* output signal close to middle of bit period.
- Two limits to bit rate: channel bandwidth (Nyquist) and noise (Shannon).



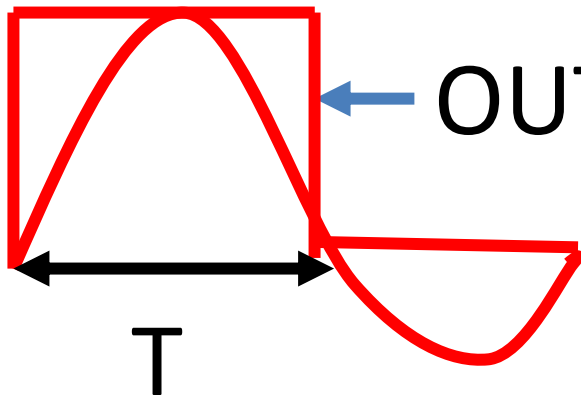
Frequency \rightarrow

Sender  Receiver



By Fourier Analysis

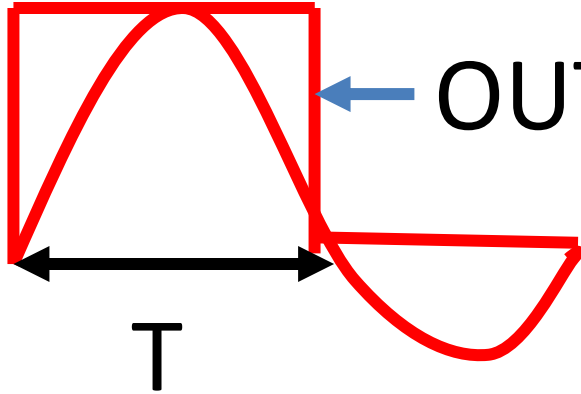
INPUT BIT



OUTPUT BIT (SINC)

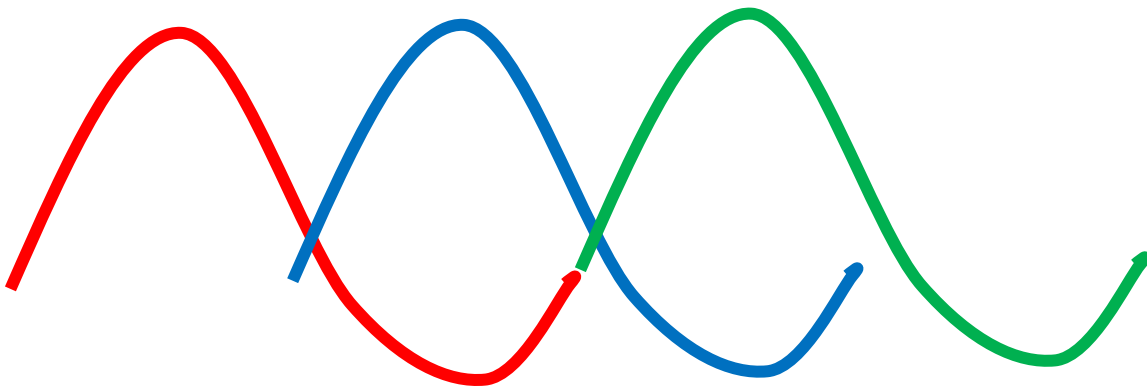
Time \rightarrow

INPUT BIT



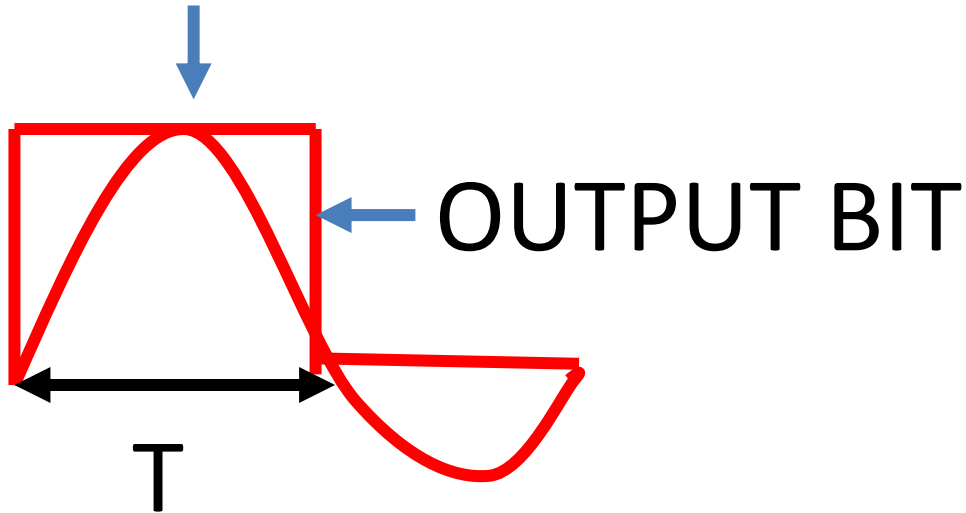
OUTPUT BIT

**So how fast can we send next
(blue, green) bit without
Intersymbol-interference?**

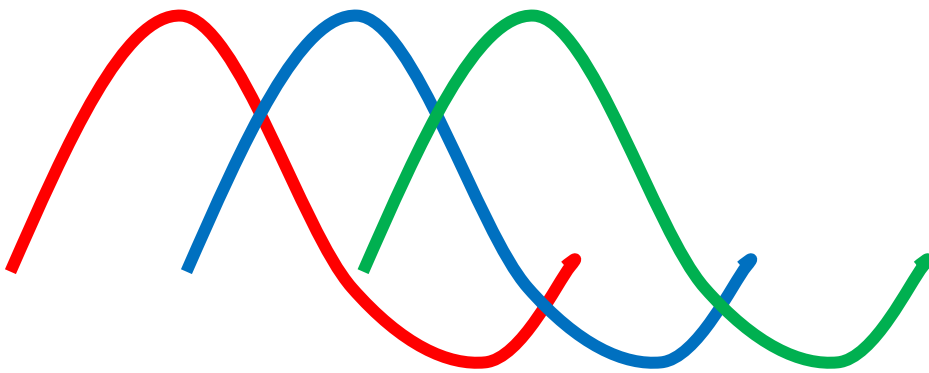


Sending every T clearly works

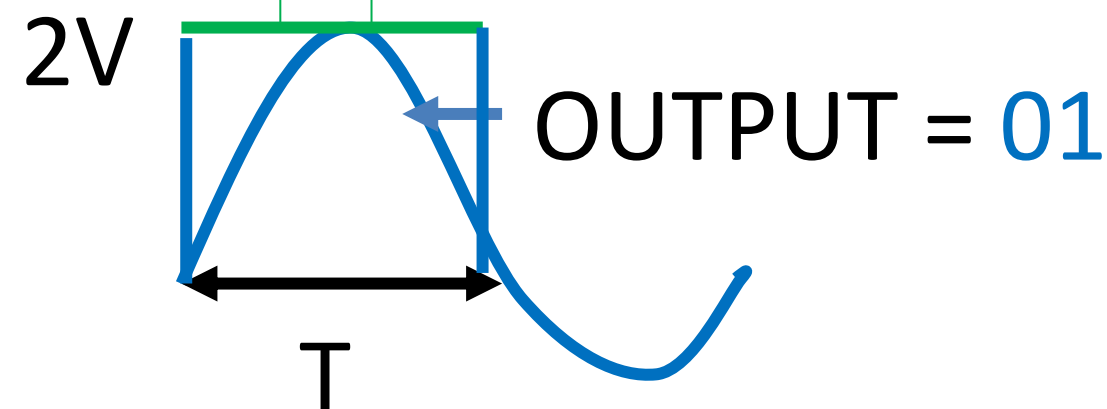
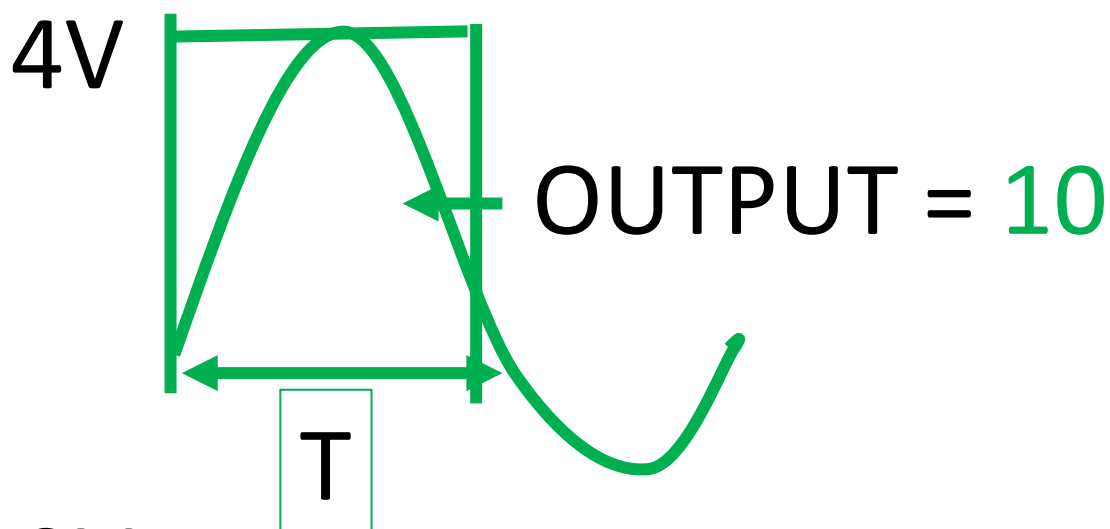
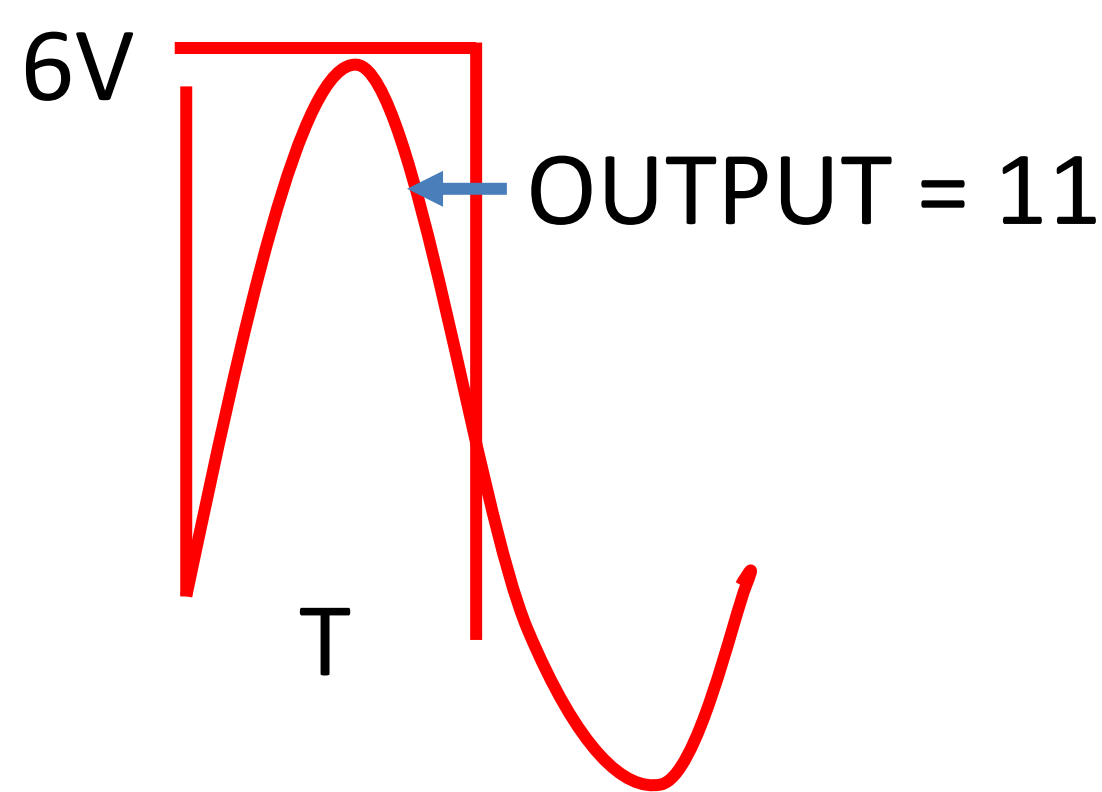
INPUT BIT



But Nyquist noticed that sending Every $T/2$ also works because peak of current lines up with zeroes of past bits for sinc



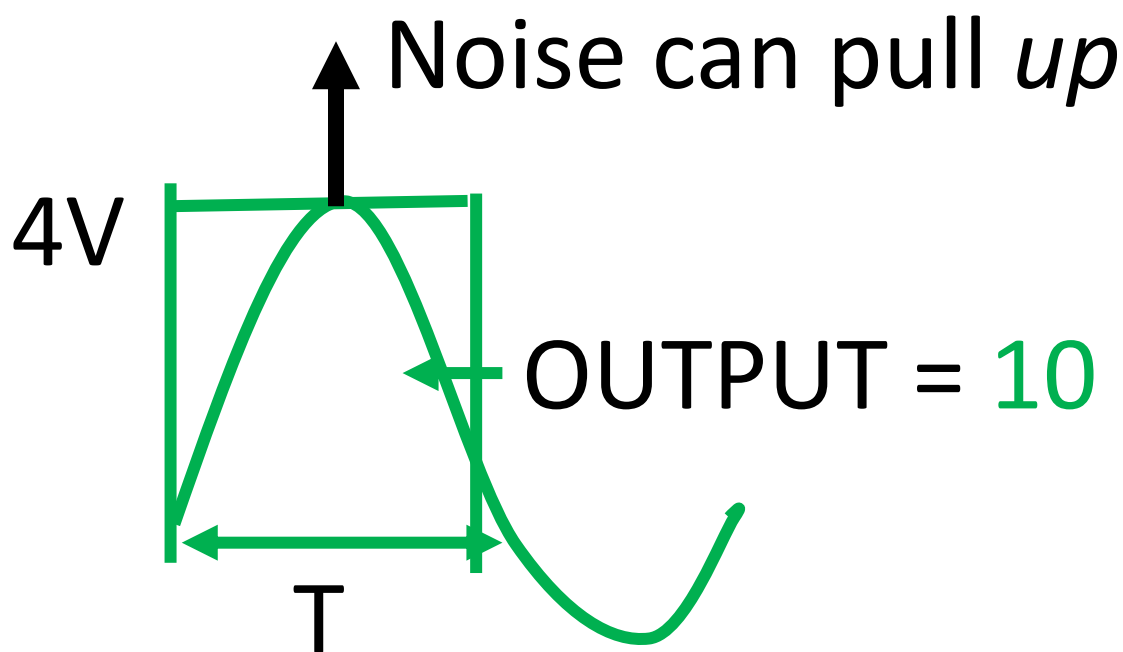
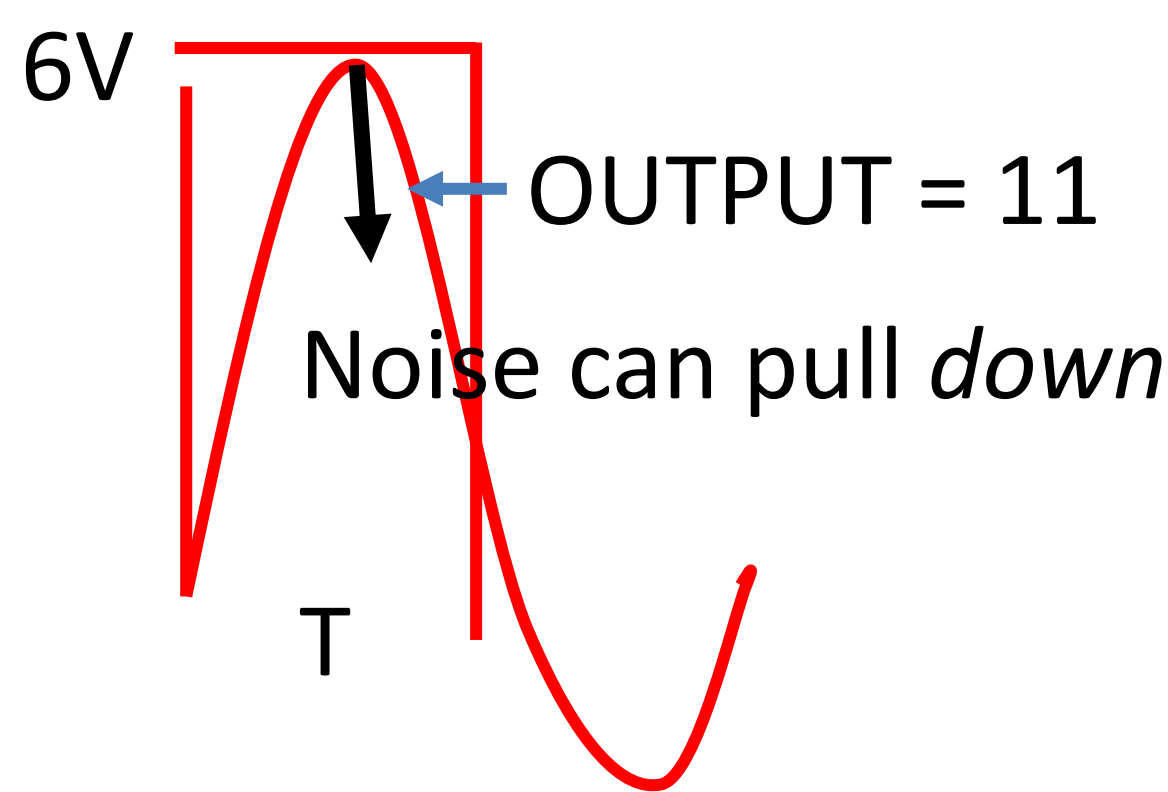
Since bandwidth = $1/T$, max bit Rate = $2/T = 2 * \text{bandwidth}$



But we can play with the y-axis (amplitude) to send more bits

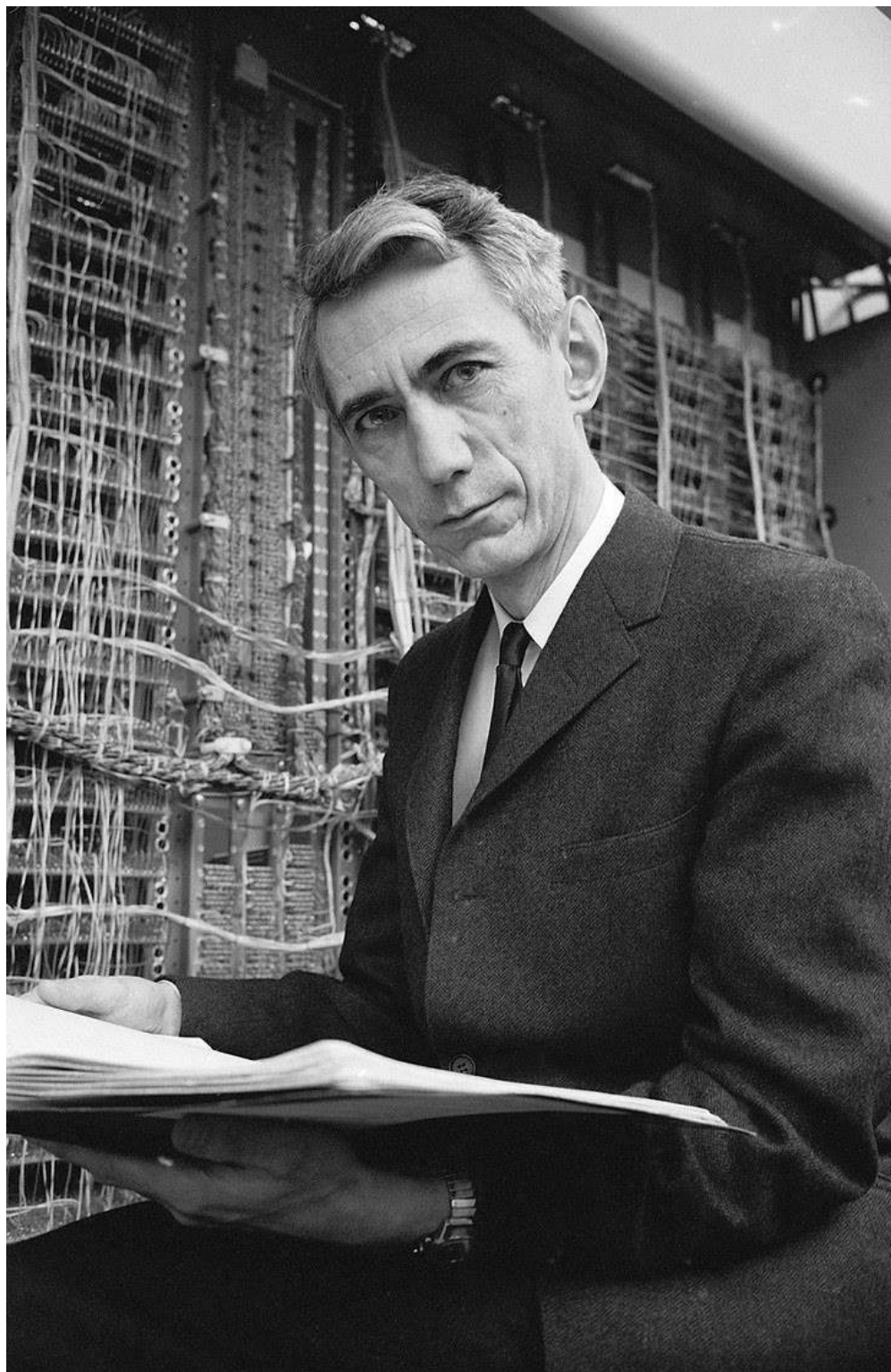
Baud Rate and Bit Rate

- To prevent ISI, we cannot send “symbols” faster than $2B$ times per second. Nyquist rate is max rate of sending *symbols* not *bits* (*baud rate*)
- But as we saw each symbol in a signal can carry multiple bits. For example: 0, 2, 4 and 6 V . 4 possible values and 2 bits per symbol.
- With L signal levels, bit rate is $\log L$ times baud rate.
- So why can't we transmit at terabits over a phone line? Noise will make one output level look like a nearby one.

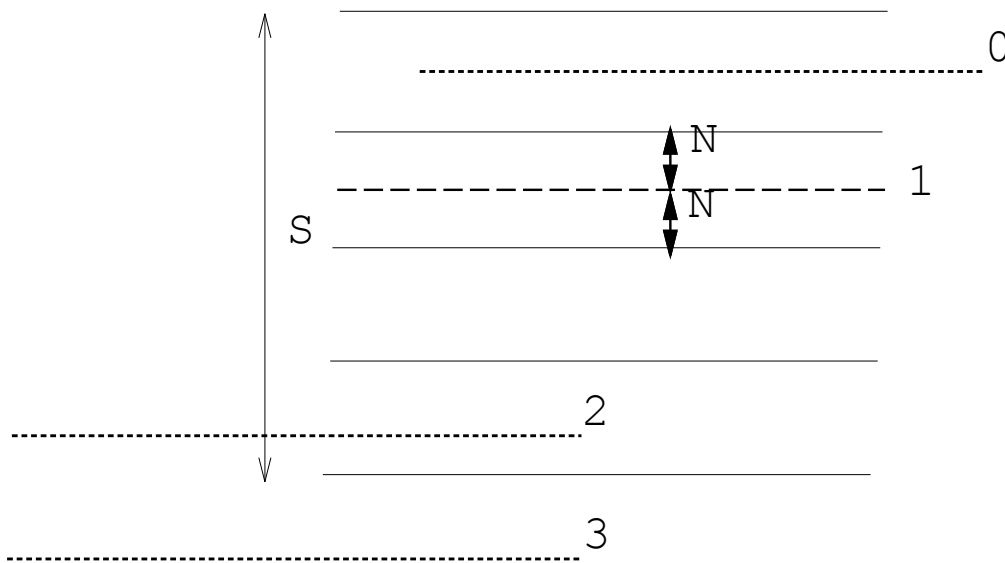


How much noise can we tolerate?

The Man: Claude Shannon



THE SHANNON BOUND



S = Maximum Signal
Amplitude

N = Maximum Noise Amplitude

$\log(S/2N)$ bits per signal

$2 B$ signals/sec (Nyquist)

Naive Bound = $2 B \log(S/2N)$

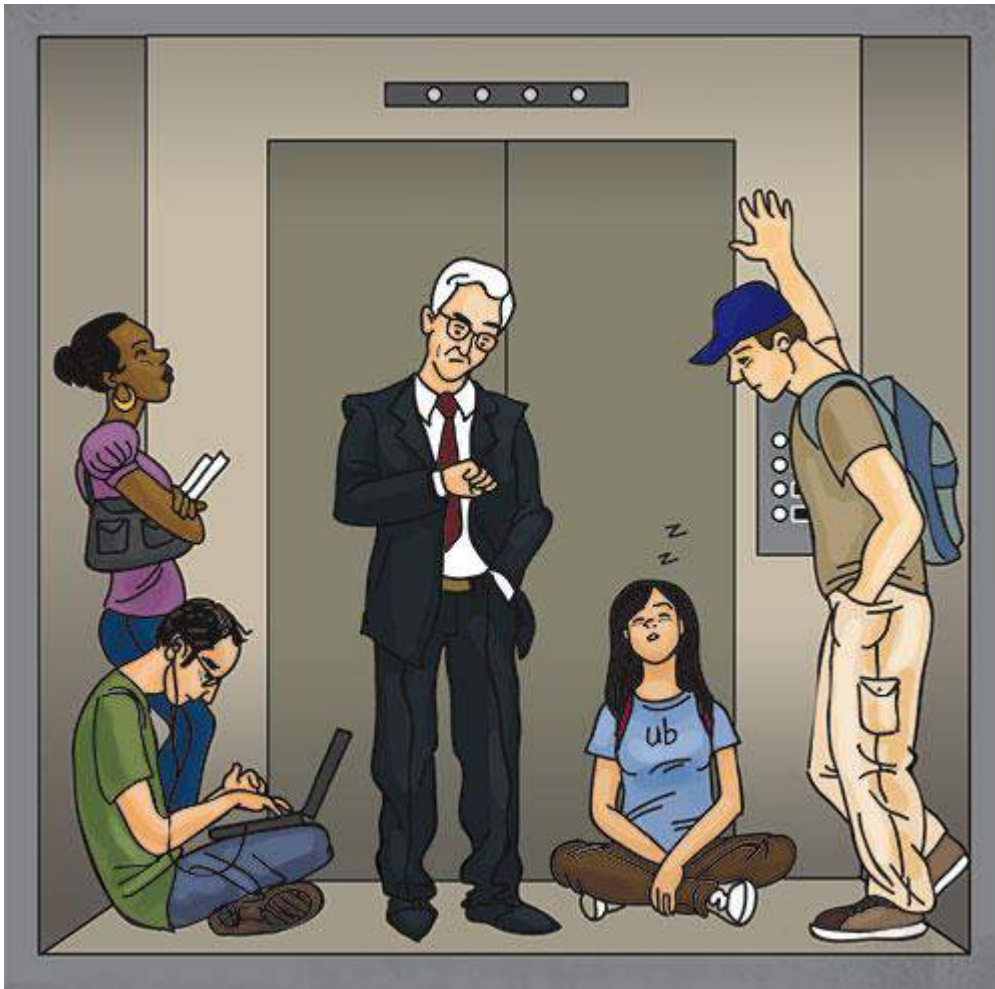
Shannon Bound = $B \log(1 + S/2N)$

More on Shannon Bound

- The real Shannon bound does not have the factor of 2 and has an extra 1 added
- This is because our simple model was only for a simple coding and for fixed deterministic noise
- Shannon bound works for any coding scheme (frequency, phase modulation) and for Gaussian additive noise. Needs a deep probabilistic argument
- . Telephone line (not DSL) with SNR of 30dB bandwidth 3kHz, we get a maximum data ¹⁵rate of 30 kbps.

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FOOD FOR THOUGHT



SLOW ELEVATOR PROBLEM: HOW
TO DEAL WITH PROBLEM MORE
CHEAPLY THAN BUYING A NEW
ELEVATOR