

CS M51A, Winter 2021, Assignment 1

(Total Mark: 80 points, 8%)

Due: Wed Jan 13, 10:00 AM PT

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Note: You must complete the assignments entirely on your own, without discussing with others.

- ✓ 1. (2 points) Briefly outline the primary differences between digital and analog systems.
Digital systems have discrete inputs and outputs, like a calculator.
Analog systems have continuous inputs and outputs, like sound.

- ✓ 2. (2 points) What are the two types of digital systems?
The 2 types of digital systems are combinational and sequential.

- ✓ 3. (6 points) Given the 8-bit binary number 1101 1010, give its decimal equivalent if these eight bits are interpreted as

(a) an 8-bit unsigned number. (show your steps)

$$\begin{array}{cccccccc} 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ 1 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 & & & & & & & \\ = 2^7 + 2^6 + 2^4 + 2^3 + 2^1 & & & & & & & \\ = 218 & & & & & & & \end{array}$$

(b) an 8-bit signed magnitude number. (show your steps)

$$\begin{array}{cccccccc} 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ \rightarrow 2^6 + 2^5 + 2^4 + 2^3 + 2^1 & & & & & & & \\ = 2^6 + 2^4 + 2^3 + 2^1 & & & & & & & \\ = -90 & & & & & & & \end{array}$$

(c) an 8-bit 2's complement number.

$$\begin{array}{cccccccc} 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ -2^7 + 2^6 + 2^4 + 2^3 + 2^1 & & & & & & & \\ = -38 & & & & & & & \end{array}$$

4. (4 points) Number Representation

(a) Write the number 125 in binary, extended to 10 bits.

$$2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^0 = 64 + 32 + 16 + 8 + 4 + 1 = 125 \checkmark$$

(b) Compute the 2s complement negation of the 10-bit number in (a).

↳ complement bits, add 1

$$\rightarrow 1110000010 + 0000000001 =$$

$$1110000011$$

5. (8 points) Find x and y such that the following conditions are satisfied and show all the steps of your work.

5)

(a) $(817)_9 = (x)_3 \rightarrow$ Every digit in $r=9$ is 2 in $r=3 \rightarrow$ $(8)_9 = (22)_3$, $(1)_9 = (01)_3$, $(7)_9 = (21)_3 \rightarrow \boxed{(220121)_3}$

(b) $(111)_4 = (x)_2 \rightarrow$ Every digit in $r=4$ is 2 in $r=2 \rightarrow$ $(1)_4 = (01)_2$, $(1)_4 = (01)_2$, $(1)_4 = (01)_2 \rightarrow \boxed{(10101)_2}$

(c) $(100)_6 = (x)_9$. $\overset{1}{6^2} \overset{0}{6^1} \overset{0}{6^0} = (76)_{10} \rightarrow (9^2)_{10} > (76)_{10} > (9^1)_{10} \rightarrow 4 \times 9^1 = (36)_{10} \rightarrow \boxed{(40)_9}$

(d) What is the largest number y that can be represented with 3 digit in radix 3.
Show y in radix 3 and decimal.. $y = 3^3 - 1 = \boxed{26}$

$y = (222)_3 = (2 \times 3^2) + (2 \times 3^1) + (2 \times 3^0) = 18 + 6 + 2 = 26 \checkmark$

6. (8 points) Two's Complement

(a) Write 45 in two's complement representation. $\boxed{0101101} \rightarrow 2^5 + 2^3 + 2^2 + 2^0 = 45 \checkmark$

(b) Sign extend the number in part (a) to 8 bits. $\boxed{00101101} \rightarrow$ Add MSBs

(c) Write -27 in two's complement representation. $\boxed{100101} \rightarrow -2^5 + 2^2 + 2^0 = -27 \checkmark$

(d) Sign extend the number in part (c) to 8 bits. $\boxed{11100101} \rightarrow$ Add MSBs

7. (16 points) Add the following pairs of 8-bit two's complement binary numbers, giving a 8-bit result (i.e., throw away the carry-out). Also give the signed decimal value of the the 8-bit result. Note whether or not an overflow occurred for any addition.

	1111 1111 (-1)	$\} \rightarrow 0$
2's Complement Binary:	+0000 0001 (1)	
Signed Decimal:	0	
Overflow?	Yes	
Is the sum correct?	Yes	

	0110 1010 (106)	$\} \rightarrow 151$
2's Complement Binary:	+0010 1101 (45)	
Signed Decimal:	-105	
Overflow?	Yes	
Is the sum correct?	No	

	0010 1100 (44)	$\} \rightarrow 105$
2's Complement Binary:	+0011 1101 (61)	
Signed Decimal:	105	
Overflow?	No	
Is the sum correct?	Yes	

	1011 1001 (185)	$\} \rightarrow 362$
2's Complement Binary:	+1011 0001 (177)	
Signed Decimal:	106	
Overflow?	Yes	
Is the sum correct?	No	

8. (16 points) Draw and fill a truth table for a system which has three inputs (a, b, c) and two outputs (f, g). f and g functions are defined as follow.

- f is a majority function (i.e. it is 1 when more than half of the inputs are 1)
- g is a minority function (i.e. it is 1 when less than half of the inputs are 1.)

(a, b, c)	f(a, b, c)	g(a, b, c)
0 0 0	0	1
0 0 1	0	1
0 1 0	0	1
0 1 1	1	0
1 0 0	0	1
1 0 1	1	0
1 1 0	1	0
1 1 1	1	0

9. (8 points) For the following high-level specification, determine the output in both decimal and 4-bits binary. \rightarrow unsigned

- Input $x \in \{0, 1, 2, 3\}$
- Function $y(x) = x^2 + 2$

$$y \in (\{2, 3, 6, 11\})_{10}$$

$$y \in (\{0010, 0011, 0110, 1011\})_2$$

$$y(0) = 0 + 2 = 2 = (10)_2$$

$$y(1) = 1 + 2 = 3 = (11)_2$$

$$y(2) = 4 + 2 = 6 = (110)_2$$

$$y(3) = 9 + 2 = 11 = (1011)_2$$

10. (10 points) Find out a high-level specification (input set, output set and input-output function) for a combinational system that compute the distance between two 1's in the input bit-vector $x = (x_{n-1}, \dots, x_0)$. Assume x has exactly two 1's. For example, if $x = (1, 0, 0, 1)$, then the distance is 3.

$$\text{input: } x = \{(x_{n-1}, \dots, x_0) \mid n > 0, x_i = 0 \text{ or } 1, \sum_{i=0}^{n-1} x_i = 2\}$$

$$\text{output: } y = \mathbb{N}^+, y \leq n-1 \quad f = f(x) = i - j \text{ if } x_i = 1 \text{ and } x_j = 1 \text{ and } i > j$$