CS 118 Assignment 1

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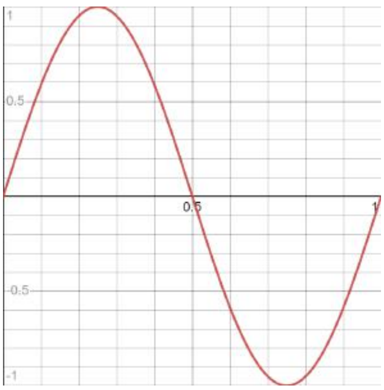
Problem 1

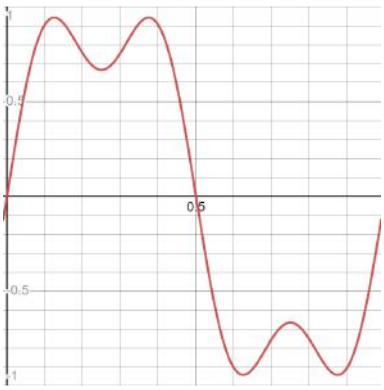
A) By Chain Rule we know that, when integrating $\sin(2\pi nt)$, we arrive at a coefficient of $\frac{1}{2\pi n}$ which is inversely proportional to n, since 2π is a constant.

B1)

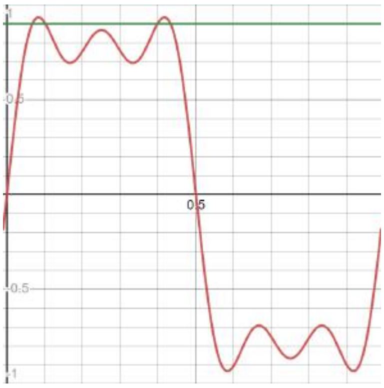
$\sin(2\pi t)$:



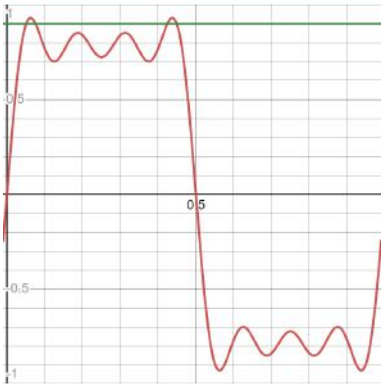
$\sin(2\pi t) + \frac{\sin(6\pi t)}{3}$:



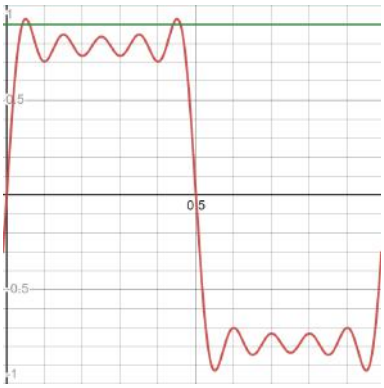
$\sin(2\pi t) + \frac{\sin(6\pi t)}{3} + \frac{\sin(10\pi t)}{5}$:



$\sin(2\pi t) + \frac{\sin(6\pi t)}{3} + \frac{\sin(10\pi t)}{5} + \frac{\sin(14\pi t)}{7}$:



$\sin(2\pi t) + \frac{\sin(6\pi t)}{3} + \frac{\sin(10\pi t)}{5} + \frac{\sin(14\pi t)}{7} + \frac{\sin(18\pi t)}{9}$:



B2)

$\sin(2\pi t)$:

$$\boxed{1\text{Hz}}$$

$\sin(2\pi t) + \frac{\sin(6\pi t)}{3}$:

$$\boxed{3\text{Hz}}$$

$\sin(2\pi t) + \frac{\sin(6\pi t)}{3} + \frac{\sin(10\pi t)}{5}$:

$$\boxed{5\text{Hz}}$$

$\sin(2\pi t) + \frac{\sin(6\pi t)}{3} + \frac{\sin(10\pi t)}{5} + \frac{\sin(14\pi t)}{7}$:

$$\boxed{7\text{Hz}}$$

$\sin(2\pi t) + \frac{\sin(6\pi t)}{3} + \frac{\sin(10\pi t)}{5} + \frac{\sin(14\pi t)}{7} + \frac{\sin(18\pi t)}{9}$:

$$\boxed{9\text{Hz}}$$

B3)

$\sin(2\pi t)$:

$$\frac{|1 - 1.2732|}{1} \times 100 = \boxed{27.32\%}$$

$\sin(2\pi t) + \frac{\sin(6\pi t)}{3}$:

$$\frac{|1 - 1.2004|}{1} \times 100 = \boxed{20.04\%}$$

$\sin(2\pi t) + \frac{\sin(6\pi t)}{3} + \frac{\sin(10\pi t)}{5}$:

$$\frac{|1 - 0.8821|}{1} \times 100 = \boxed{11.79\%}$$

$\sin(2\pi t) + \frac{\sin(6\pi t)}{3} + \frac{\sin(10\pi t)}{5} + \frac{\sin(14\pi t)}{7}$:

$$\frac{|1 - 0.8917|}{1} \times 100 = \boxed{10.83\%}$$

$\sin(2\pi t) + \frac{\sin(6\pi t)}{3} + \frac{\sin(10\pi t)}{5} + \frac{\sin(14\pi t)}{7} + \frac{\sin(18\pi t)}{9}$:

$$\frac{|1 - 0.8959|}{1} \times 100 = \boxed{10.41\%}$$

B4)

$\sin(2\pi t)$:

$$\boxed{0.18\text{s}}$$

$\sin(2\pi t) + \frac{\sin(6\pi t)}{3}$:

$$\boxed{0.0985\text{s}}$$

$\sin(2\pi t) + \frac{\sin(6\pi t)}{3} + \frac{\sin(10\pi t)}{5}$:

$$\boxed{0.0674\text{s}}$$

$\sin(2\pi t) + \frac{\sin(6\pi t)}{3} + \frac{\sin(10\pi t)}{5} + \frac{\sin(14\pi t)}{7}$:

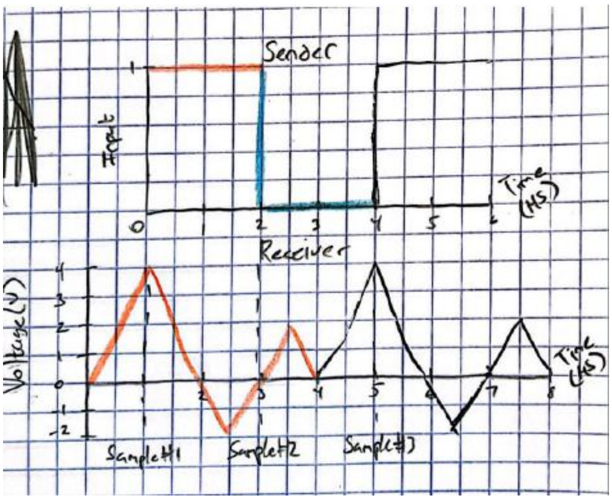
$$\boxed{0.051\text{s}}$$

$\sin(2\pi t) + \frac{\sin(6\pi t)}{3} + \frac{\sin(10\pi t)}{5} + \frac{\sin(14\pi t)}{7} + \frac{\sin(18\pi t)}{9}$:

$$\boxed{0.041\text{s}}$$

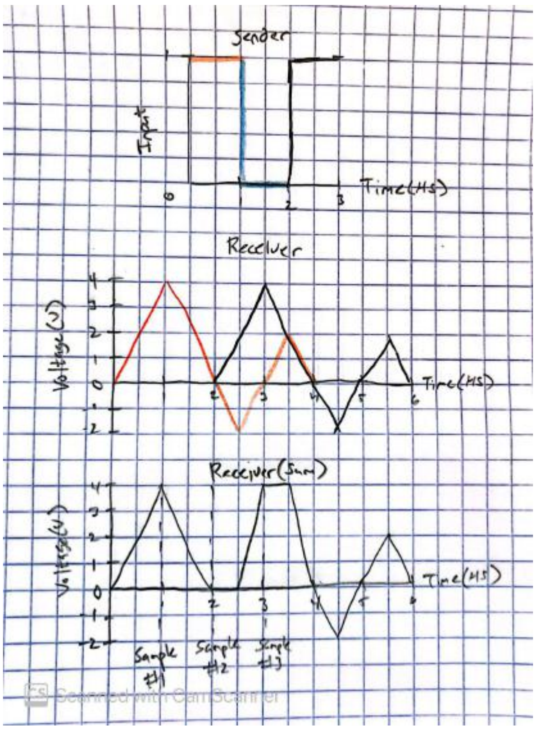
Problem 2

2.1)



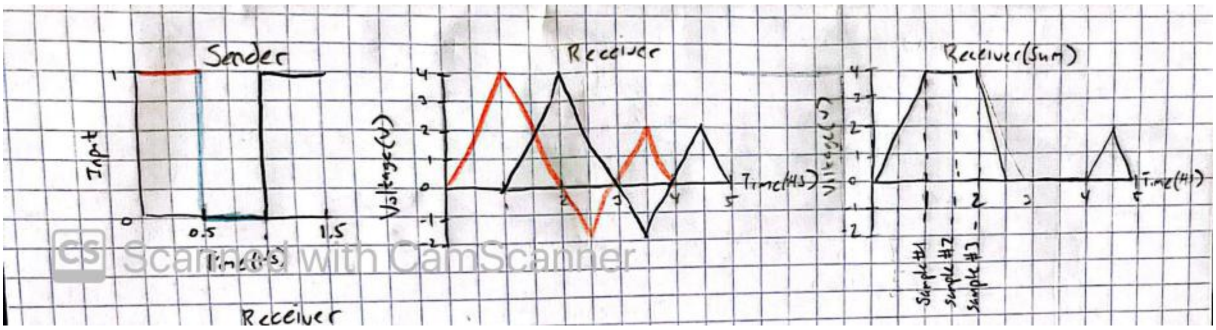
The measured outputs are 4V at $1\mu s$, 0V at $3\mu s$, and 4V at $5\mu s$. These translate to output bits of `1`, `0`, and `1`.

2.2)



The measured outputs are 4V at $1\mu s$, 0V at $2\mu s$, and 4V at $3\mu s$. These translate to output bits of `1`, `0`, and `1`.

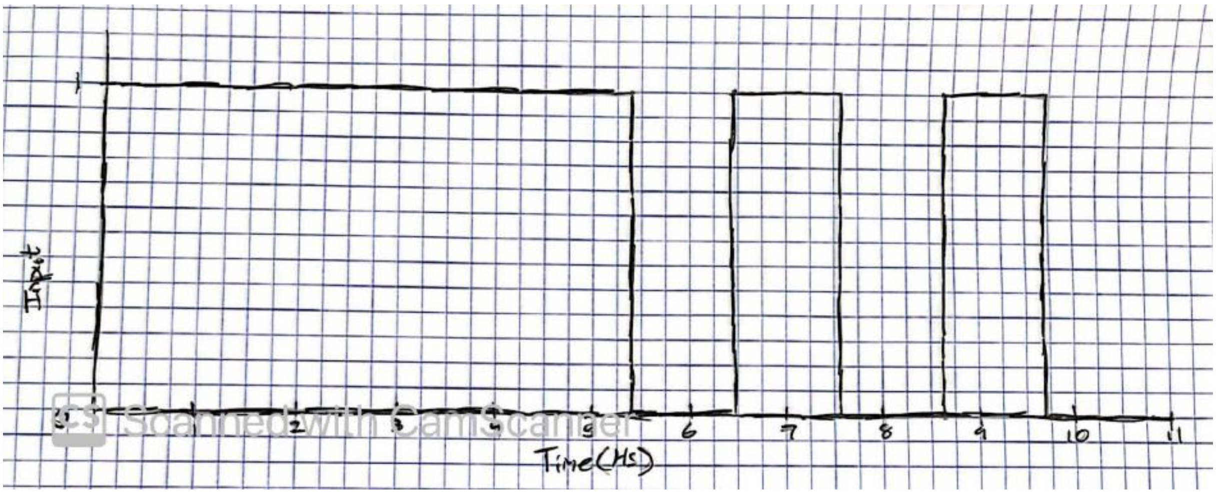
2.3)



The measured outputs are 4V at 1μs, 4V at 1.5μs, and 4V at 2μs. These translate to output bits of 1, 1, and 1, showing clear ISI.

Problem 3

3.1)

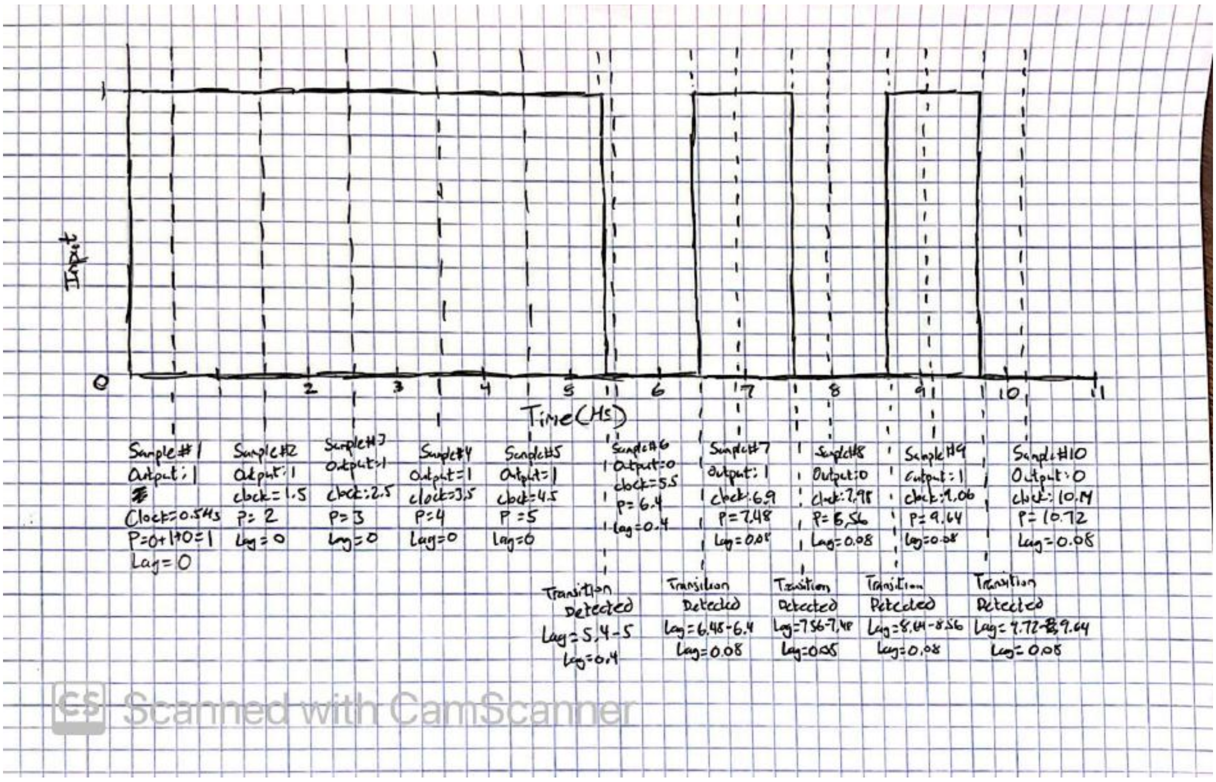


3.2)

Knowing that the sender is sending 8% slower, we know that the actual middle of bits occur at 0.54μs, 1.62μs, 2.7μs... We also know that, without clock recovery, the receiver samples at 0.5μs, 1.5μs, 2.5μs... Following this pattern, we can calculate that the middle of the 10th bit occurs at 10.26μs and is sampled at 9.5μs, which tells us the sampling is off by:

$$|10.26\mu\text{s} - 9.5\mu\text{s}| = \boxed{0.76\mu s}$$

3.3)



3.4)

A sharp noise spike at 0.3μs wouldn't have any affect on sampling times or output since the clock recovery algorithm we use doesn't begin sampling or searching for transitions until 0.5μs. At 0.3μs, we would still be waiting for the initial $\frac{T}{2}$ timer to expire.

3.5)

At the time the spike occurs, we have $P = 3$ and $A = 2.7$, resulting in $lag = -0.3$. In the context of this algorithm, a negative lag means that the actual transition occurred before the predicted transition, or that the sender's clock is faster than the receiver's. As a result, the receiver will begin to sample earlier after detecting this noise, when it should be trying to sample later, since the sender's clock is actually slower than the receiver's. The new samples are as follows:

Sample #	Output	Clock	P	Lag
1	1	0.5	1	0
2	1	1.5	2	0
3	1	2.5	3	0
4	1	3.5	3.7	-0.3
5	1	4.2	4.4	-0.3
6	1	4.9	5.1	-0.3
7	0	5.6	6.4	0.3
8	1	6.9	7.48	0.08
9	0	7.98	8.56	0.08
10	1	9.06	9.64	0.08

As seen by the values in the table, samples #4-6 are performed too early, resulting in an extra `1` being sampled in this time frame, and the following `0101` being shifted 1 bit to the right, while the final `0` bit is dropped due to the extra sampling. The receiver manages to recover using the actual transitions from the input bits, but not before this error is introduced.