CS 181 Homework 9

Charles Zhang, 305-413-659

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Problem 1

The language L is not FSL.

Proof (by contradiction):

- Assume L is a finite state language.
- FSLs are closed under complementation, therefore $\bar{L}$ must also be an FSL.
- We can define $\bar{L}$ as the set of languages over Σ such that all runs of 0s must be of the same length.
- Since $\bar{L}$ is an FSL, it must satisfy the conditions of the pumping lemma.
- Let p be the pumping length given by the pumping lemma.
- Let $w = 0^p 1 0^p$, noting that $w \in \bar{L}$.
- By Sipser's condition 1 of the pumping lemma, we know that w can be split into three parts such that $w = abc$ and, for any $i \geq 0$, the string $w' = ab^i c$ is in $\bar{L}$.
- Sipser's condition 3 of the pumping lemma states that, when pumping w, it must be split such that $|ab| \leq p$.
- Since w begins with p 0s, condition 3 guarantees that ab is made up entirely of 0s.
- Sipser's condition 2 of the pumping lemma states that, when pumping w, it must be split such that $|b| \geq 1$.
- Taken together with condition 3, this condition implies that b must contain at least one 0, and be made up entirely of 0s, therefore $b = 0^t$, where $p \geq t \geq 1$.
- We then pump the substring b using $i = 2$.
- $w' = 0^{p+t} 1 0^p$.
- Since we know that $t \geq 1 \geq 0$, we know that $p + t \neq p$.
- This means that w' is not a part of $\bar{L}$, as there exist runs of 0s that have different lengths.
- Thus, the pumping lemma is not satisfied, meaning that $\bar{L}$ is not an FSL.
- This tells us that L doesn't follow the property that FSLs are closed under complementation, as $\bar{L}$ is not an FSL, and a contradiction has been found $\Rightarrow\Leftarrow$.