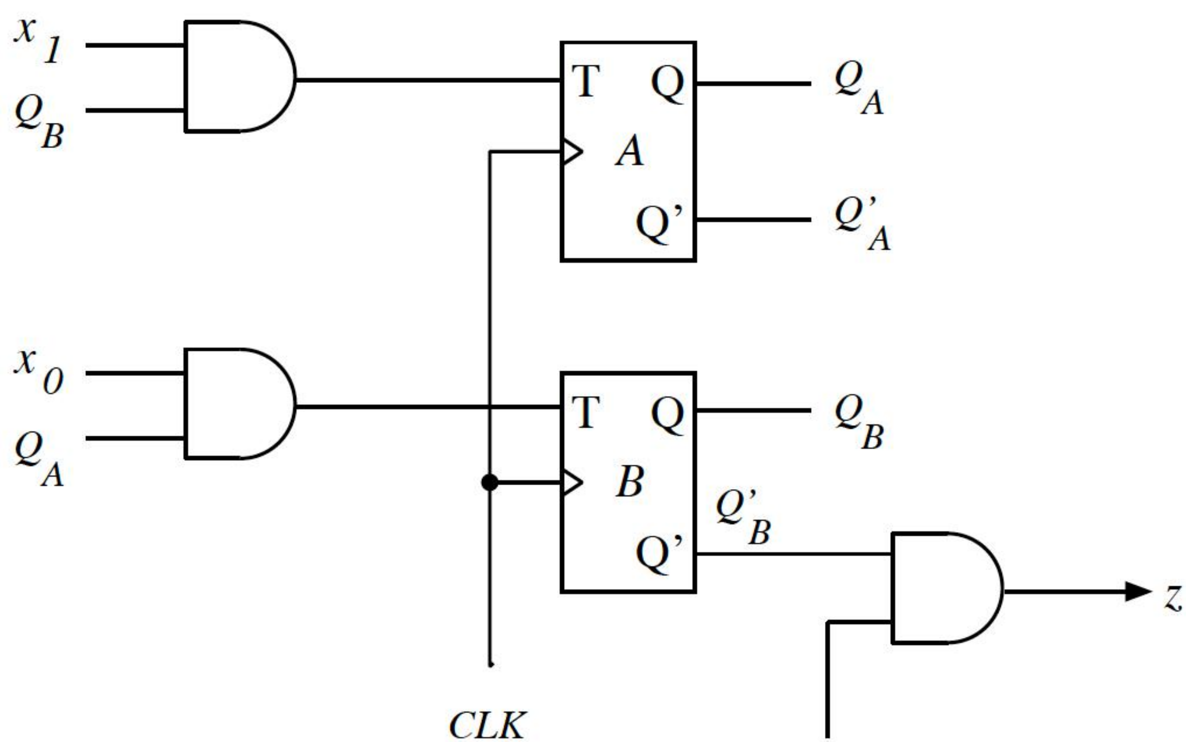
CS M51A

Logic Design of Digital Systems

Winter 2021

Some slides borrowed and modified from:
M.D. Ercegovac, T. Lang and J. Moreno, Introduction to Digital Systems.

EXAMPLE: ANALYSIS



$$
\begin{aligned}
T_A &= x_1 Q_B \\
T_B &= x_0 Q_A \\
z(t) &= x_1(t) Q'_B(t)
\end{aligned}
$$

$$
\begin{aligned}
Q_A(t+1) &= Q_A(t) \oplus x_1 Q_B(t) \\
Q_B(t+1) &= Q_B(t) \oplus x_0 Q_A(t)
\end{aligned}
$$

EXAMPLE (cont.)

- STATE-TRANSITION AND OUTPUT FUNCTIONS

PS	Input							
Q_AQ_B	x_1x_0				x_1x_0			
	00	01	10	11	00	01	10	11
00	00	00	00	00	0	0	1	1
01	01	01	11	11	0	0	0	0
10	10	11	10	11	0	0	1	1
11	11	10	01	00	0	0	0	0
	Q_AQ_B				z			
	NS				Output			

- CODING:

Q_A	Q_B	s
0	0	S_0
0	1	S_1
1	0	S_2
1	1	S_3

x_1	x_0	x
0	0	a
0	1	b
1	0	c
1	1	d

EXAMPLE (cont.)

- HIGH-LEVEL DESCRIPTION:

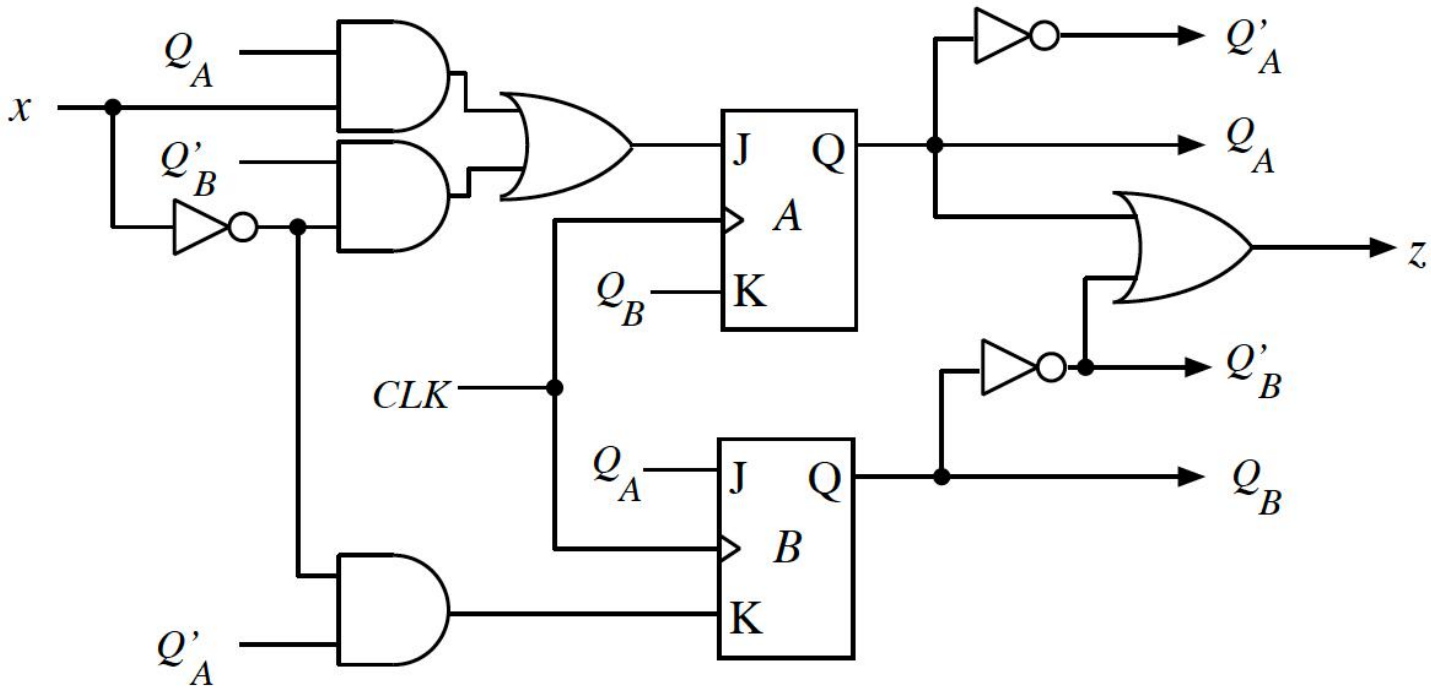
  Input: $x(t) \in \{a, b, c, d\}$
  Output: $z(t) \in \{0, 1\}$
  State: $s(t) \in \{S_0, S_1, S_2, S_3\}$
  Initial state: $s(0) = S_0$

  Functions: The state-transition and output functions

PS	x				x			
	a	b	c	d	a	b	c	d
S_0	S_0	S_0	S_0	S_0	0	0	1	1
S_1	S_1	S_1	S_3	S_3	0	0	0	0
S_2	S_2	S_3	S_2	S_3	0	0	1	1
S_3	S_3	S_2	S_1	S_0	0	0	0	0
	NS				z			

State Diagram

EXAMPLE: ANALYSIS



$$
\begin{aligned}
J_A &= x'Q'_B + xQ_A \\
J_B &= Q_A
\end{aligned}
$$

$$
\begin{aligned}
K_A &= Q_B \\
K_B &= x'Q'_A
\end{aligned}
$$

$$z = Q_A + Q'_B$$

$$
\begin{aligned}
Q_A(t+1) &= Q_A K'_A + Q'_A J_A \\
&= Q_A Q'_B + Q'_A(x'Q'_B + xQ_A) \\
&= Q'_B(Q_A + x')
\end{aligned}
$$

$$Q_B(t+1) = Q_B K'_B + Q'_B J_B$$

EXAMPLE (cont.)

- STATE-TRANSITION AND OUTPUT FUNCTIONS

PS	NS		Output
	$x = 0$	$x = 1$	z
$Q_A Q_B$	$Q_A Q_B$	$Q_A Q_B$	
00	10	00	1
01	00	01	0
10	11	11	1
11	01	01	1

- STATE CODING

Q_A	Q_B	S
0	0	S_0
0	1	S_1
1	0	S_2
1	1	S_3

EXAMPLE (cont.)

- HIGH-LEVEL DESCRIPTION

Input: $x(t) \in \{0, 1\}$
Output: $z(t) \in \{0, 1\}$
State: $s(t) \in \{S_0, S_1, S_2, S_3\}$
Initial state: $s(0) = S_0$

Functions: The state-transition and output functions

PS	Input $x = 0$	Input $x = 1$	
S_0	S_2	S_0	1
S_1	S_0	S_1	0
S_2	S_3	S_3	1
S_3	S_1	S_1	1
	NS		z

State Diagram

EXAMPLE: DESIGN MODULO-5 COUNTER

- USE T FLIP-FLOPS

Input:	$x(t) \in \{0, 1\}$
Output:	$z(t) \in \{0, 1, 2, 3, 4\}$
State:	$s(t) \in \{S_0, S_1, S_2, S_3, S_4\}$
Initial state:	$s(0) = S_0$
Functions:	Counts modulo-5, i.e., (0,1,2,3,4,0,1,2,3,4,0...),

State Diagram:

EXAMPLE (cont.)

z	z_2	z_1	z_0
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0

PS	Input		Input	
$Q_2Q_1Q_0$	$x = 0$	$x = 1$	$x = 0$	$x = 1$
000	000	001	000	001
001	001	010	000	011
010	010	011	000	001
011	011	100	000	111
100	100	000	000	100
	NS		$T_2T_1T_0$	

EXAMPLE (cont.)

T_2:

		x			
	0	0	0	0	
	0	0	1	0	Q_1
Q_2	-	-	-	-	
	0	1	-	-	
			Q_0		

T_1:

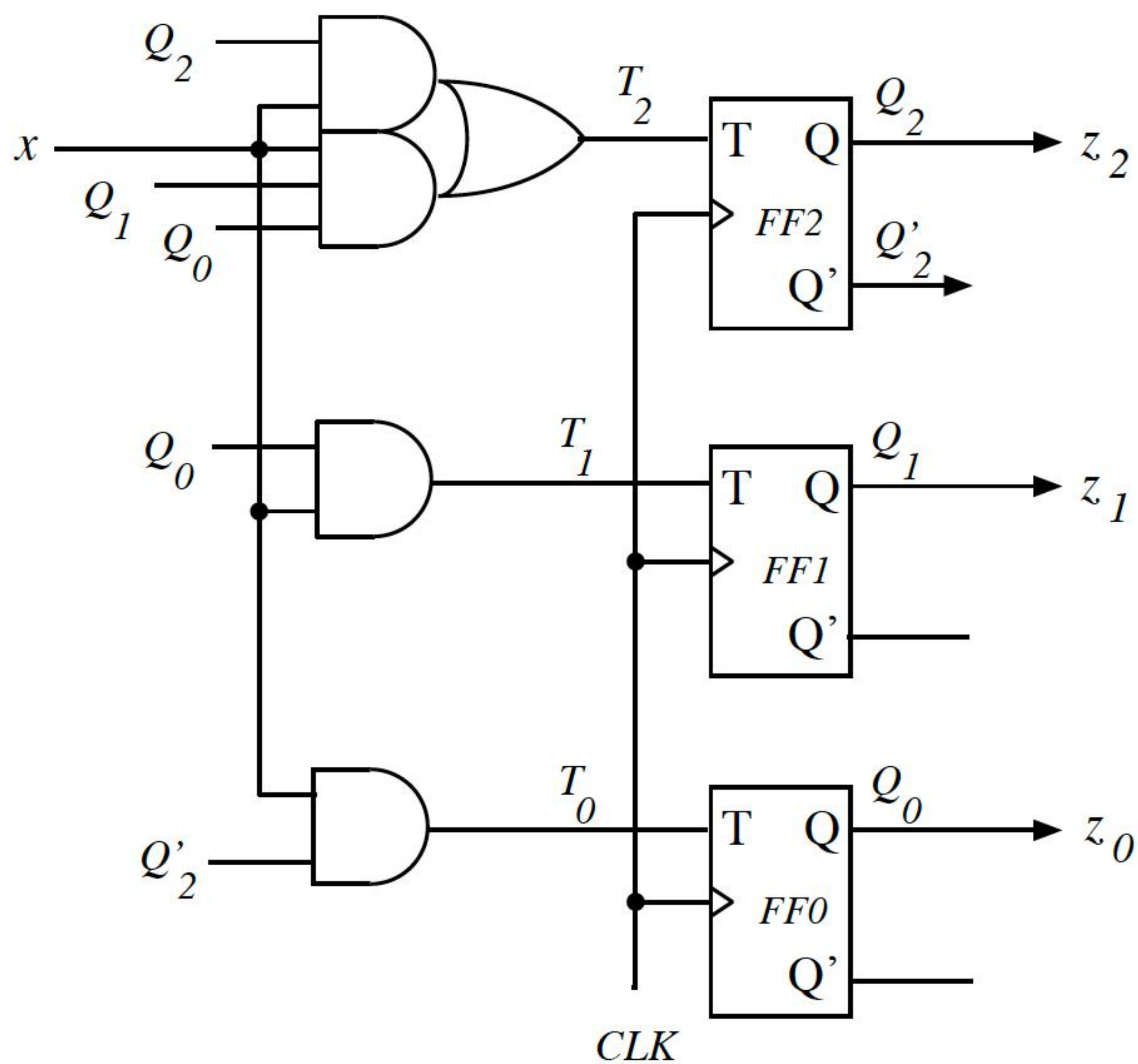
		x			
	0	0	1	0	
	0	0	1	0	Q_1
Q_2	-	-	-	-	
	0	0	-	-	
			Q_0		

T_0:

		x			
	0	1	1	0	
	0	1	1	0	Q_1
Q_2	-	-	-	-	
	0	0	-	-	
			Q_0		

$$T_2 =$$
$$T_1 =$$
$$T_0 =$$

EXAMPLE (cont.)

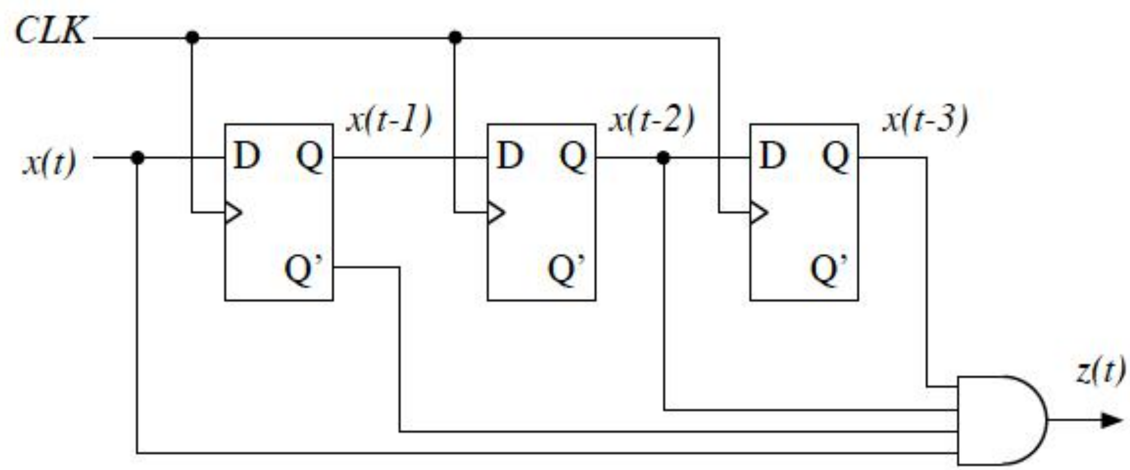


Example: Pattern Detector

Input: $x(t) \in \{0,1\}$

Output: $z(t) \in \{0,1\}$

$$\text{Function: } z(t) = \begin{cases} 1 & \textbf{if } \; x(t-3,t) = 1101 \\ 0 & \textbf{otherwise} \end{cases}$$

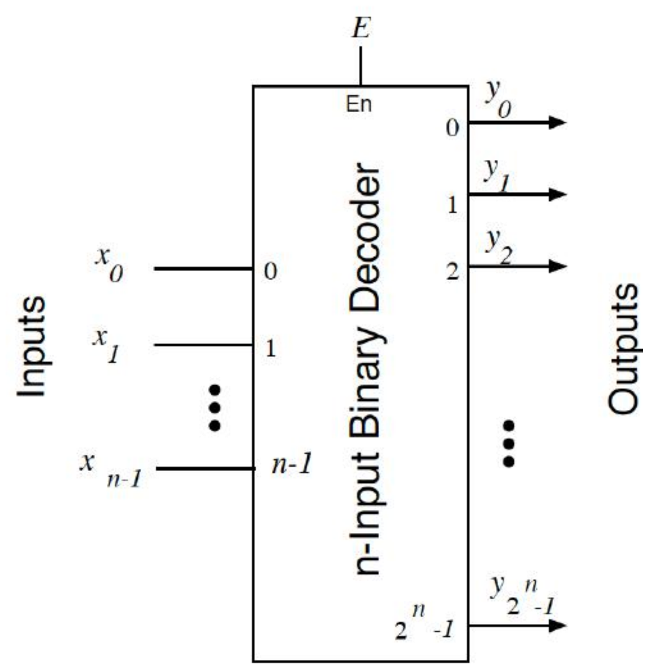


HIGH-LEVEL DESCRIPTION:

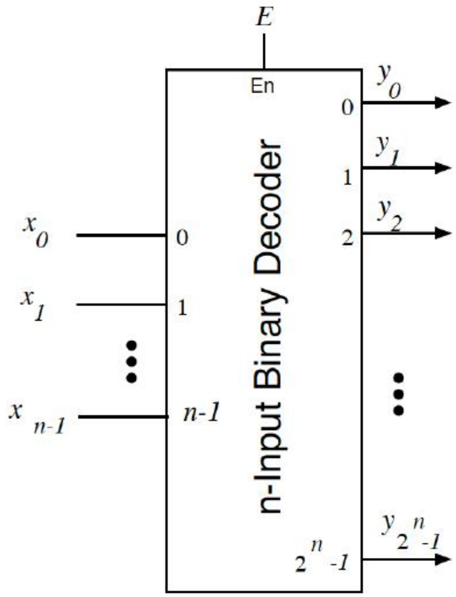
Inputs: $\underline{x} = (x_{n-1}, \ldots, x_0), \quad x_j \in \{0, 1\}$
Enable $E \in \{0, 1\}$

Outputs: $\underline{y} = (y_{2^n-1}, \ldots, y_0), \quad y_i \in \{0, 1\}$

Function: $y_i = \begin{cases} 1 \ \textbf{if} \ \ (x = i) \ \textbf{and} \ (E = 1) \\ 0 \ \textbf{otherwise} \end{cases}$



3-INPUT BINARY DECODER



E	x_2	x_1	x_0	x	y_7	y_6	y_5	y_4	y_3	y_2	y_1	y_0
1	0	0	0	0	0	0	0	0	0	0	0	1
1	0	0	1	1	0	0	0	0	0	0	1	0
1	0	1	0	2	0	0	0	0	0	1	0	0
1	0	1	1	3	0	0	0	0	1	0	0	0
1	1	0	0	4	0	0	0	1	0	0	0	0
1	1	0	1	5	0	0	1	0	0	0	0	0
1	1	1	0	6	0	1	0	0	0	0	0	0
1	1	1	1	7	1	0	0	0	0	0	0	0
0	-	-	-	-	0	0	0	0	0	0	0	0

BINARY SPECIFICATION:

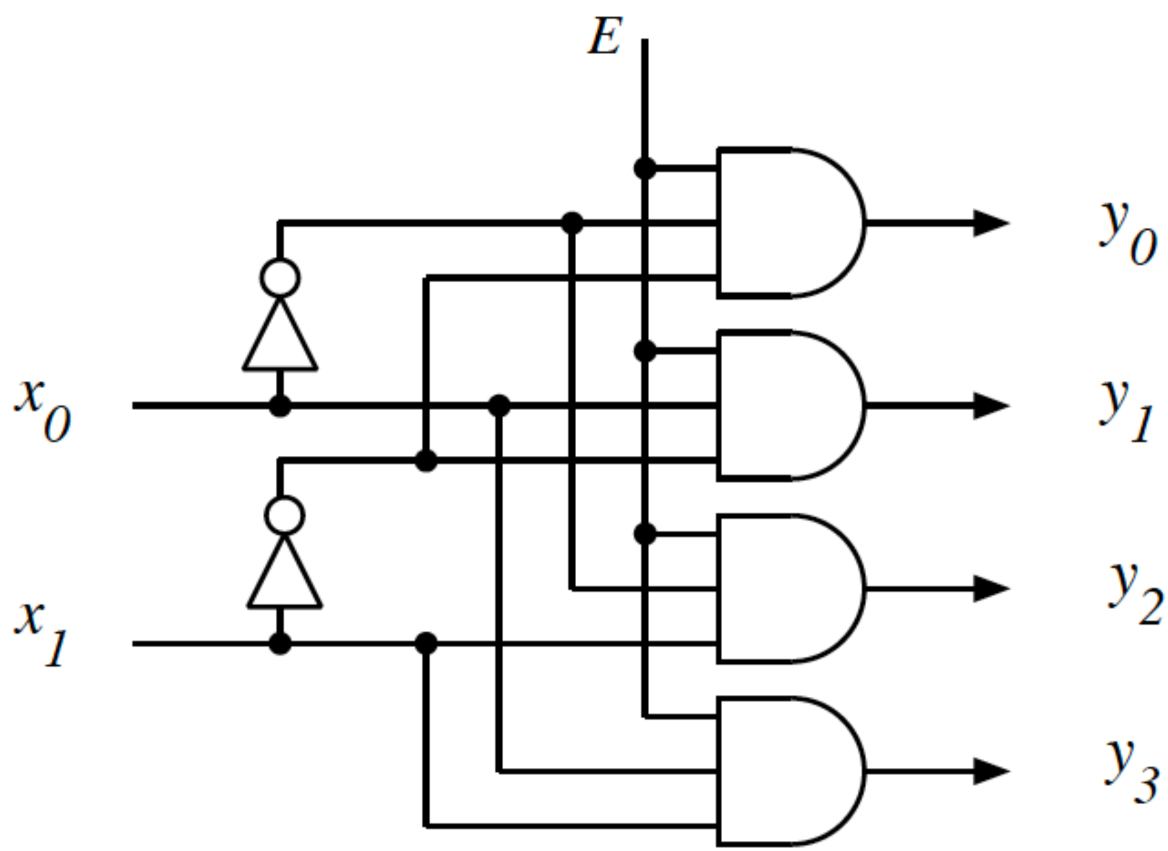
Inputs: $\underline{x} = (x_{n-1}, \ldots, x_0), \quad x_j \in \{0, 1\}$

$E \in \{0, 1\}$

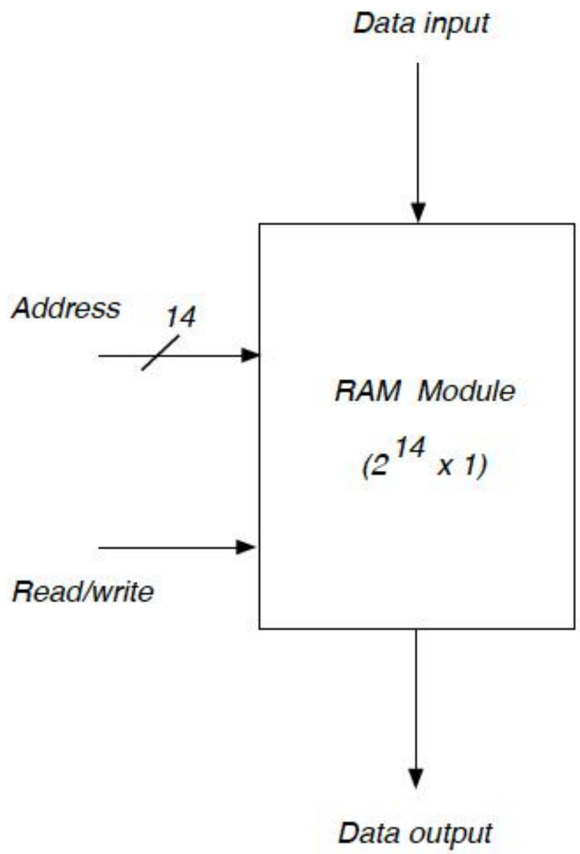
Outputs: $\underline{y} = (y_{2^n-1}, \ldots, y_0), \quad y_i \in \{0, 1\}$

IMPLEMENTATION OF 2-INPUT DECODER

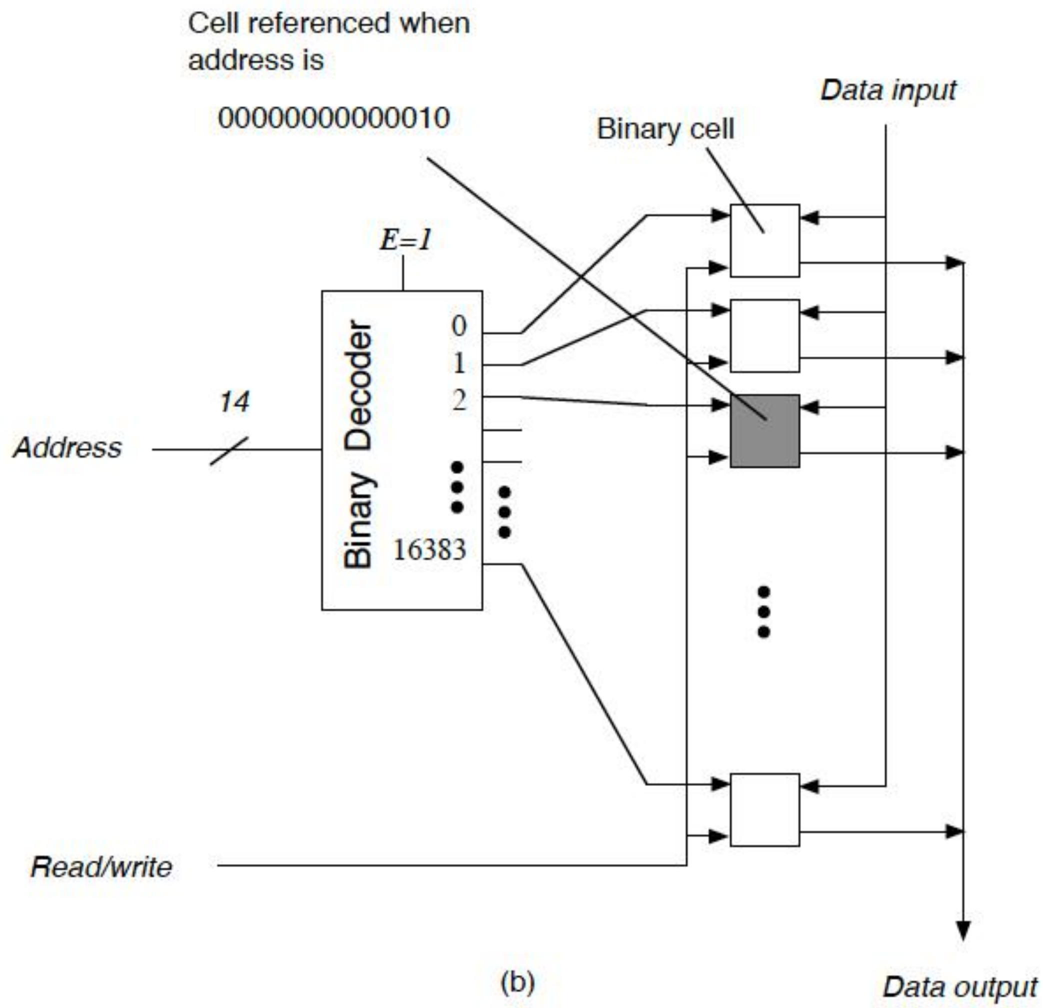
$$y_0 = x_1' x_0' E \quad y_1 = x_1' x_0 E \quad y_2 = x_1 x_0' E \quad y_3 = x_1 x_0 E$$



DECODER USES



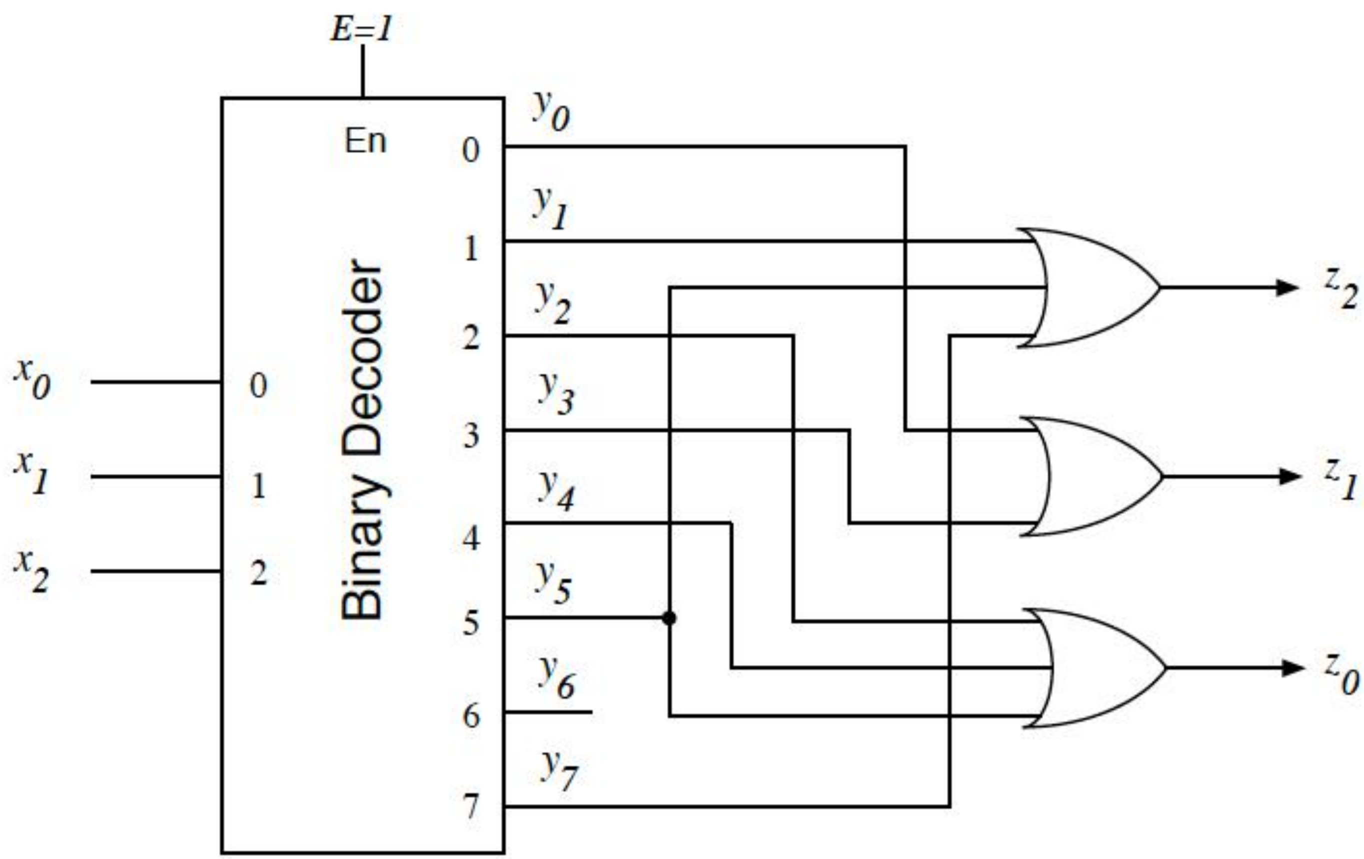
(a)



(b)

Clicker Question

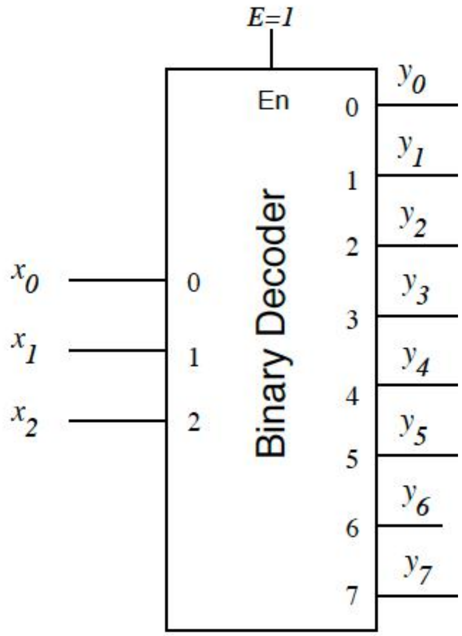
Which one is correct?



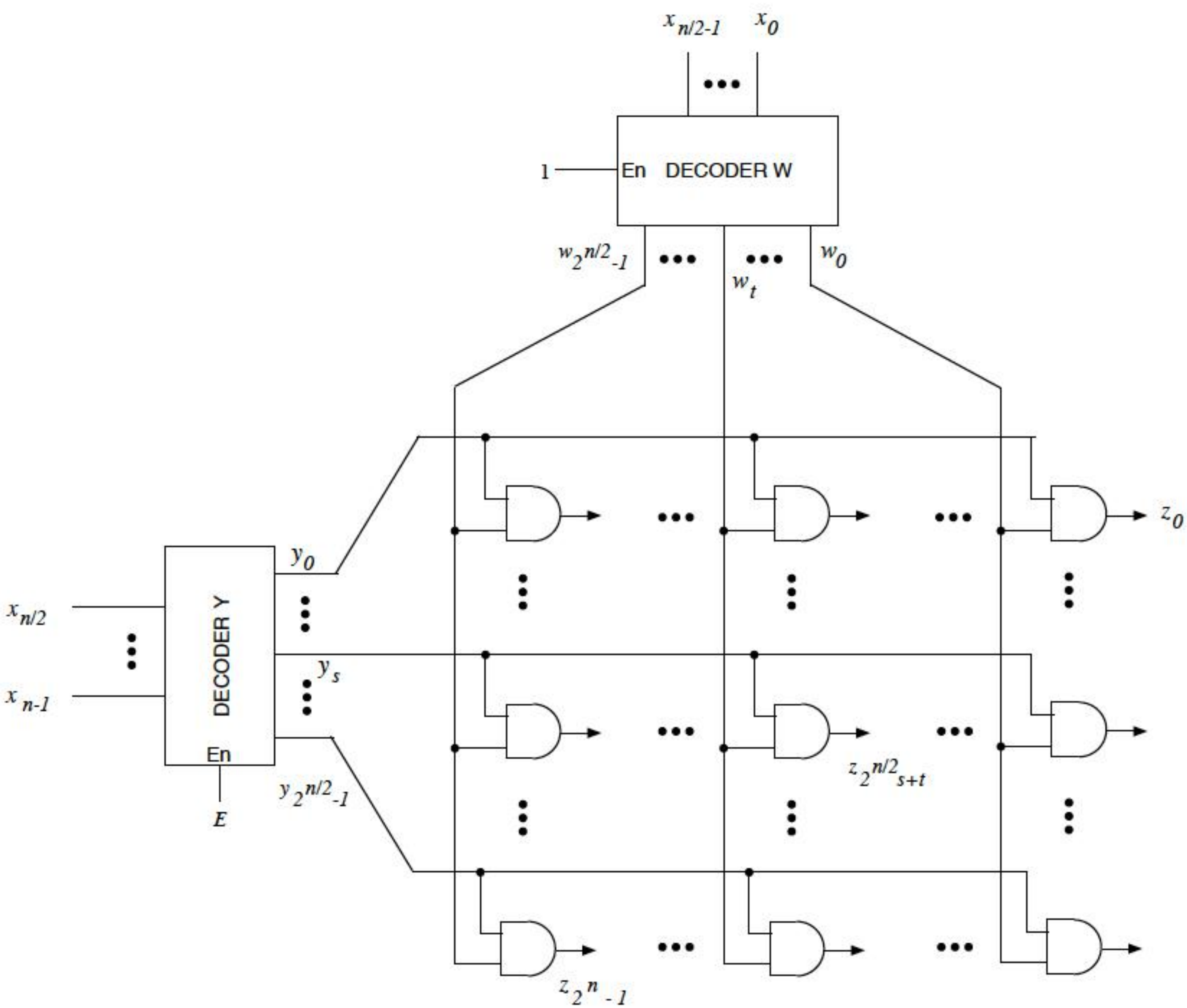
- A) $Z_1 = x_2x'_1x_0 + x_2x_1x_0$
- B) $Z_1 = x'_2x_1x_0 + x'_2x'_1x'_0$
- C) $Z_0 = x_2x_1x_0 + x_2x'_1x'_0$
- D) $Z_2 = x'_2x'_1x_0 + x_2x'_1x_0$
- E) none

Example

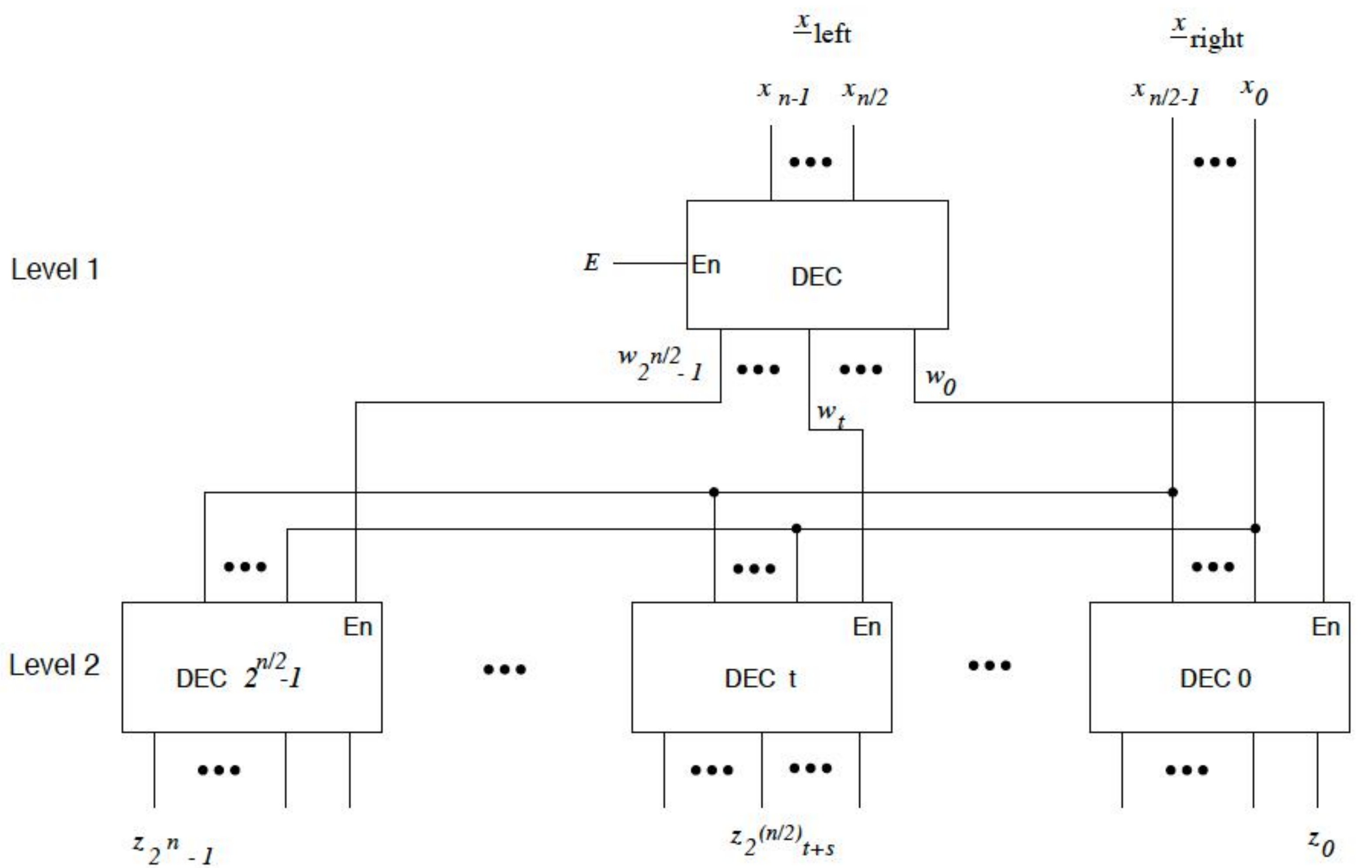
$x_2x_1x_0$	z_2	z_1	z_0
000	0	1	0
001	1	0	0
010	0	0	1
011	0	1	0
100	0	0	1
101	1	0	1
110	0	0	0
111	1	0	0



Coincident Decoder



Tree Decoder



EXAMPLE: 6-INPUT DECODER