CS 161 Fundamentals of Artificial Intelligence
Lecture 8

Logical agents

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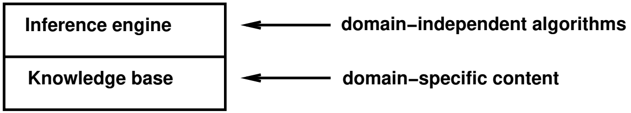
Outline

- Knowledge-based agents
- Wumpus world
- Logic in general—models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - forward chaining
  - resolution

Knowledge bases



- **Knowledge base** = set of **sentences** in a **formal** language
- **Declarative** approach to building an agent (or other system):
  TELL it what it needs to know

Then it can ASK itself what to do—answers should follow from the KB

A simple knowledge-based agent

```
**function** KB-AGENT( *percept*) **returns** an *action*
    **static**: *KB*, a knowledge base
            *t*, a counter, initially 0, indicating time

    TELL(KB, MAKE-PERCEPT-SENTENCE( percept, t))
    action ← ASK(KB, MAKE-ACTION-QUERY(t))
    TELL(KB, MAKE-ACTION-SENTENCE(action, t))
    $t \leftarrow t + 1$
    return action
```

The agent must be able to:

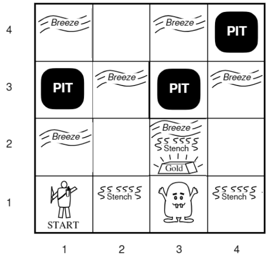
- Represent states, actions, etc.
- Incorporate new percepts
- Update internal representations of the world
- Deduce hidden properties of the world
- Deduce appropriate actions

Wumpus World description

To put it simple

- Grid world
- Pit causes breeze in adjacent cells
- Wumpus causes stench in adjacent cells
- Find the gold

Details are in next slide.



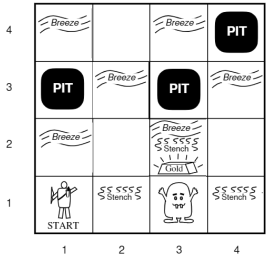
Wumpus World PEAS description

Performance measure gold +1000, death -1000
-1 per step, -10 for using the arrow

Environment
Squares adjacent to wumpus are smelly
Squares adjacent to pit are breezy
Glitter iff gold is in the same square
Shooting kills wumpus if you are facing it
Shooting uses up the only arrow
Grabbing picks up gold if in same square
Releasing drops the gold in same square

Actuators Left turn, Right turn, Forward, Grab, Release, Shoot

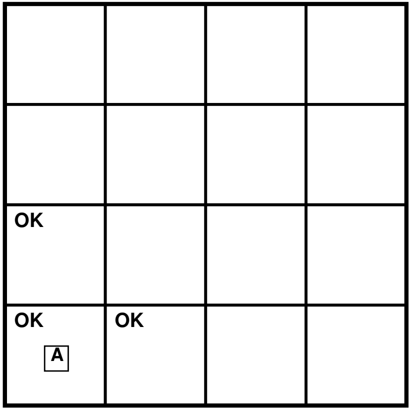
Sensors Breeze, Glitter, Smell



Wumpus world characterization

Observable?? No—only local perception
Deterministic?? Yes—outcomes exactly specified
Episodic?? No—sequential at the level of actions
Static?? Yes—Wumpus and Pits do not move
Discrete?? Yes
Single-agent?? Yes—Wumpus is essentially a natural feature

Exploring a wumpus world



A = Agent; **B** = Breeze; **G** = Glitter, Gold; **OK** = Safe square;
P = Pit; **S** = Stench; **V** = Visited; **W** = Wumpus

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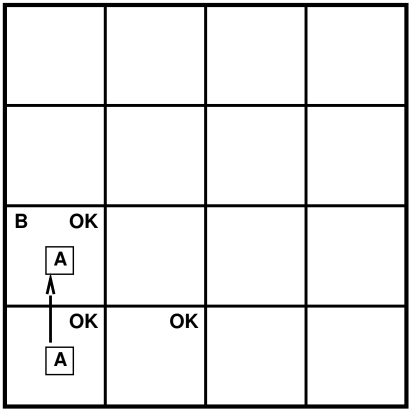

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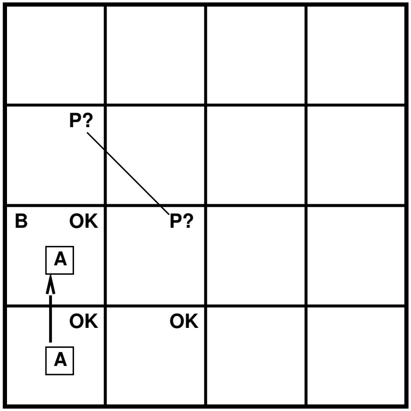

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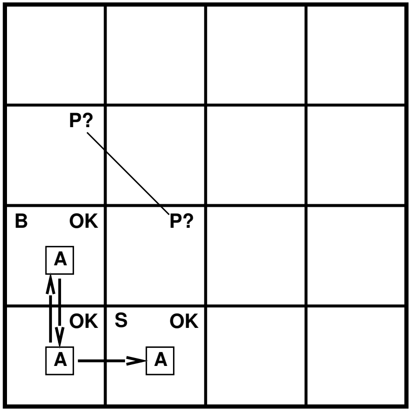

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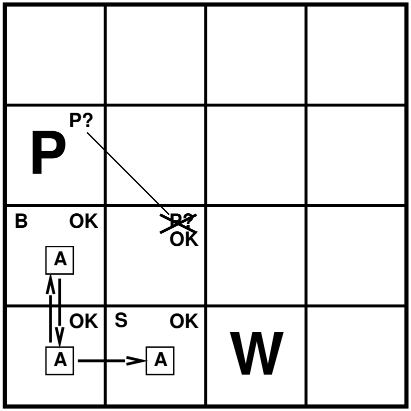

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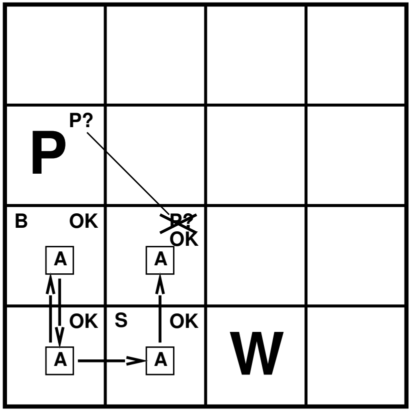

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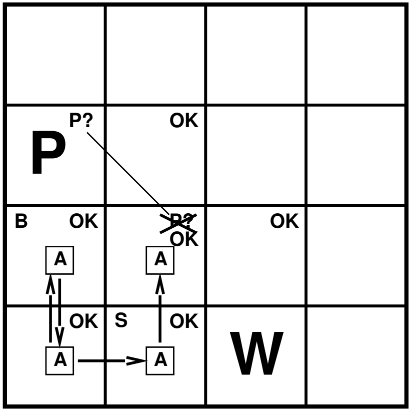

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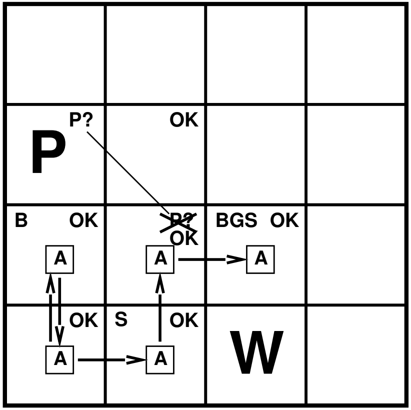

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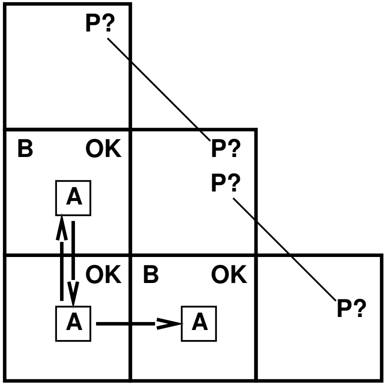
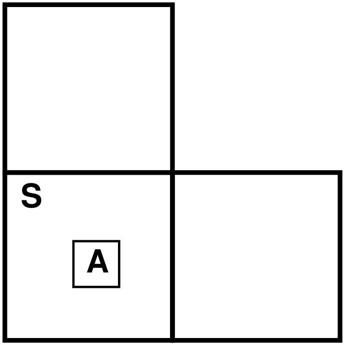
Other tight spots



Breeze in (1,2) and (2,1)
$\Rightarrow$ no safe actions

We can calculate the probability!

Other tight spots



Smell in (1,1)
$\Rightarrow$ cannot move
Can use a strategy of **coercion**:
shoot straight ahead
wumpus was there $\Rightarrow$ dead $\Rightarrow$ safe
wumpus wasn't there $\Rightarrow$ safe

Logic in general

Logics are formal languages for representing information
such that conclusions can be drawn

Syntax defines the sentences in the language

Semantics define the "meaning" of sentences;
i.e., define **truth** of a sentence in a world

E.g., the language of arithmetic

$x + 2 \geq y$ is a sentence; $x2 + y >$ is not a sentence

$x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number y

$x + 2 \geq y$ is true in a world where $x = 7,\ y = 1$

$x + 2 \geq y$ is false in a world where $x = 0,\ y = 6$

Entailment

Entailment means that one thing **follows from** another:

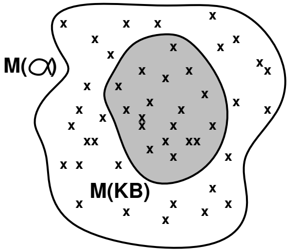
$$KB \models \alpha$$

Knowledge base KB entails sentence α
if and only if
α is true in all worlds where KB is true
E.g., the KB containing "the Giants won" and "the Reds won" entails "Either the Giants won or the Reds won"
E.g., $x + y = 4$ entails $4 = x + y$
Entailment is a relationship between sentences (i.e., **syntax**) that is based on **semantics**
Note: brains process **syntax** (of some sort)

Models

- Logicians typically think in terms of **models**, which are formally structured worlds with respect to which truth can be evaluated
- We say m is a model of a sentence α if α is true in m
- $M(\alpha)$ is the set of all models of α

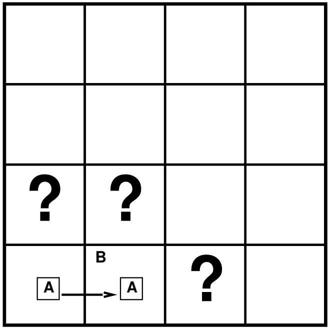
Given sentences α and β, we say α **entails** β ($\alpha \models \beta$), iff $M(\alpha) \subset M(\beta)$. $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$



Entailment in the wumpus world

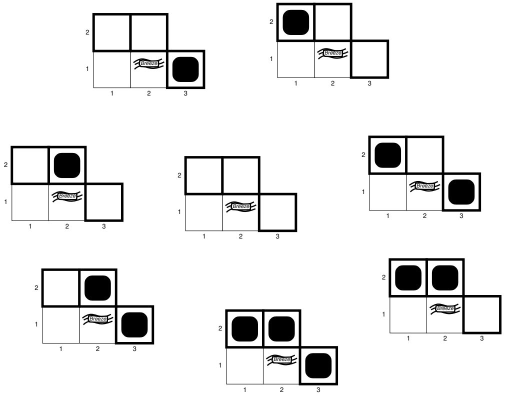
Situation after detecting nothing in [1,1],
moving right, breeze in [2,1]

Consider possible models for ?s
assuming only pits

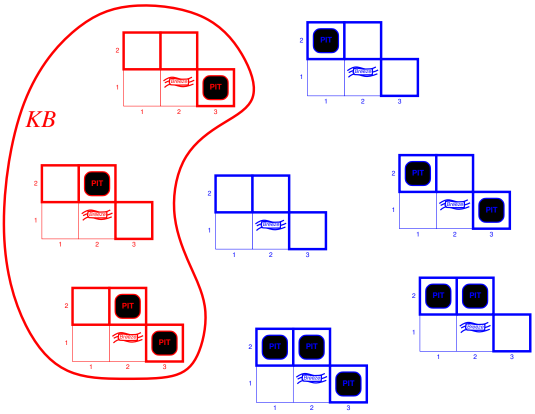


3 Boolean choices $\Rightarrow$ 8 possible models

Wumpus models

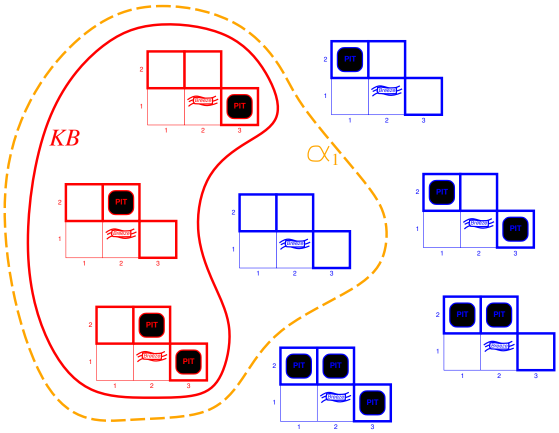


Wumpus models



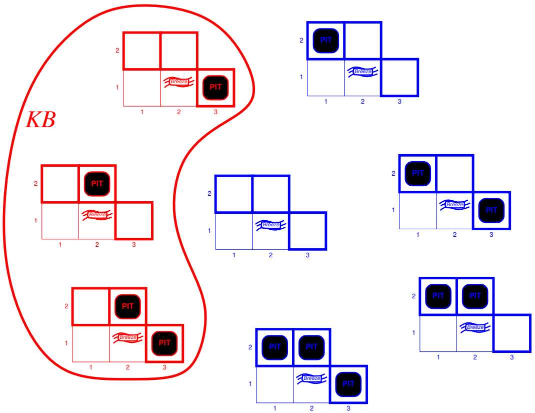
KB = wumpus-world rules + observations

Wumpus models



KB = wumpus-world rules + observations
α_1 = "no pit in [1,2]", $KB \models \alpha_1$, proved by **model checking**: enumerates all possible models to check that α is true in all models in which KB is true.

Wumpus models



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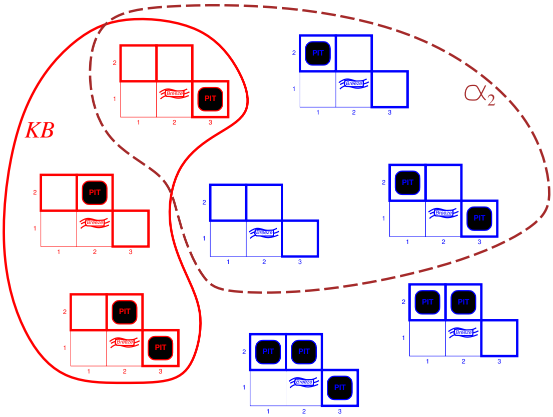
Wumpus models



KB = wumpus-world rules + observations
α_2 = "no pit in [2,2]", $KB \not\models \alpha_2$

Inference

$KB \vdash_i \alpha$ = sentence α can be derived from KB by some inference algorithm i

For instance, model checking is an inference algorithm!

Soundness: i is sound if

whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

Completeness: i is complete if

whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.

That is, the procedure will answer any question whose answer follows from what is known by the KB.

Propositional logic: Syntax

- Propositional logic is the simplest logic, a.k.a., boolean logic
- The proposition symbols P_1, P_2 etc are **atomic sentences**
- We can generate **complex sentences** based on atomic sentences using logical connectives.

If S is a sentence, $\neg S$ is a sentence (**negation**)
If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (**conjunction**)
If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (**disjunction**)
If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (**implication**)
If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (**biconditional**)

Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	$true$	$true$	$false$

(With these symbols, 8 possible models, can be enumerated automatically.)

Rules for evaluating truth with respect to a model m (True: T; False: F)

$\neg S$	is T iff	S	is F		
$S_1 \wedge S_2$	is T iff	S_1	is T **and**	S_2	is T
$S_1 \vee S_2$	is T iff	S_1	is T **or**	S_2	is T
$S_1 \Rightarrow S_2$	is T unless	S_1	is T **and**	S_2	is F
$S_1 \Leftrightarrow S_2$	is T iff	$S_1 \Rightarrow S_2$	is T **and**	$S_2 \Rightarrow S_1$	is T

Simple recursive process evaluates an arbitrary sentence, e.g.,

$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = false \wedge (true \vee false) = false$

Truth tables for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i, j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i, j]$.

$$\neg P_{1,1}$$
$$\neg B_{1,1}$$
$$B_{2,1}$$

"Pits cause breezes in adjacent squares"

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i, j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i, j]$.

$$\neg P_{1,1}$$
$$\neg B_{1,1}$$
$$B_{2,1}$$

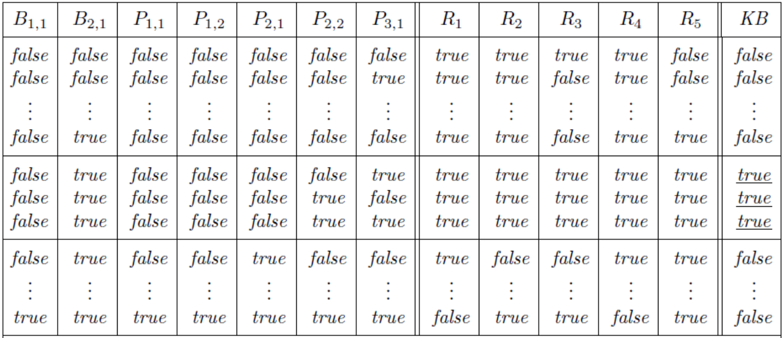
"Pits cause breezes in adjacent squares"

$$\begin{aligned} B_{1,1} &\quad\Leftrightarrow\quad (P_{1,2} \lor P_{2,1}) \\ B_{2,1} &\quad\Leftrightarrow\quad (P_{1,1} \lor P_{2,2} \lor P_{3,1}) \end{aligned}$$

"A square is breezy **if and only if** there is an adjacent pit"

Truth tables for inference

$R_1 : \neg P_{1,1}$ $R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$ $R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
$R_4 : \neg B_{1,1}$ $R_5 : B_{2,1}$
$KB : R_1 \wedge \cdots \wedge R_5$

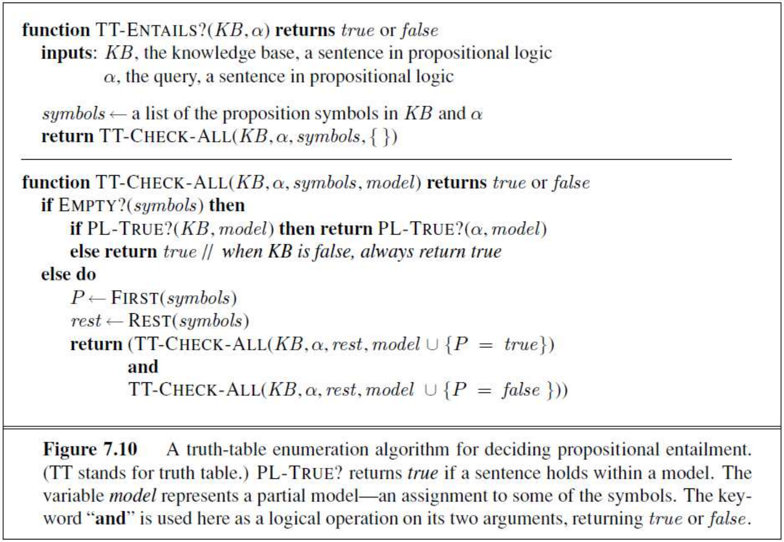


$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Figure 7.9 A truth table constructed for the knowledge base given in the text. *KB* is true if R_1 through R_5 are true, which occurs in just 3 of the 128 rows (the ones underlined in the right-hand column). In all 3 rows, $P_{1,2}$ is false, so there is no pit in [1,2]. On the other hand, there might (or might not) be a pit in [2,2].

Inference by enumeration

Depth-first enumeration of all models is sound and complete



```
function TT-ENTAILS?($KB$, $\alpha$) returns true or false
  inputs: $KB$, the knowledge base, a sentence in propositional logic
          $\alpha$, the query, a sentence in propositional logic

  symbols ← a list of the proposition symbols in $KB$ and $\alpha$
  return TT-CHECK-ALL($KB$, $\alpha$, symbols, { })
```

```
function TT-CHECK-ALL($KB$, $\alpha$, symbols, model) returns true or false
  if EMPTY?(symbols) then
     if PL-TRUE?($KB$, model) then return PL-TRUE?($\alpha$, model)
     else return true // when KB is false, always return true
  else do
     P ← FIRST(symbols)
     rest ← REST(symbols)
     return (TT-CHECK-ALL($KB$, $\alpha$, rest, model $\cup$ {P = true})
             and
             TT-CHECK-ALL($KB$, $\alpha$, rest, model $\cup$ {P = false }))
```

Figure 7.10 A truth-table enumeration algorithm for deciding propositional entailment. (TT stands for truth table.) PL-TRUE? returns *true* if a sentence holds within a model. The variable *model* represents a partial model—an assignment to some of the symbols. The keyword "**and**" is used here as a logical operation on its two arguments, returning *true* or *false*.

$O(2^n)$ for n symbols; problem is **co-NP-complete**

Logical equivalence

Two sentences are **logically equivalent** iff true in same models:
$\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$

$(\alpha \wedge \beta)$	$\equiv$	$(\beta \wedge \alpha)$	commutativity of $\wedge$
$(\alpha \vee \beta)$	$\equiv$	$(\beta \vee \alpha)$	commutativity of $\vee$
$((\alpha \wedge \beta) \wedge \gamma)$	$\equiv$	$(\alpha \wedge (\beta \wedge \gamma))$	associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma)$	$\equiv$	$(\alpha \vee (\beta \vee \gamma))$	associativity of $\vee$
$\neg(\neg\alpha)$	$\equiv$	α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \vee \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \wedge \beta)$	$\equiv$	$(\neg\alpha \vee \neg\beta)$	De Morgan
$\neg(\alpha \vee \beta)$	$\equiv$	$(\neg\alpha \wedge \neg\beta)$	De Morgan
$(\alpha \wedge (\beta \vee \gamma))$	$\equiv$	$((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma))$	$\equiv$	$((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of $\vee$ over $\wedge$

Validity and satisfiability

- A sentence is **valid** if it is true in **all** models,
  e.g., $True$, $A \lor \neg A$, $A \Rightarrow A$, $(A \land (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the **Deduction Theorem**:
  $\alpha \models \beta$ if and only if $(\alpha \Rightarrow \beta)$ is valid

- A sentence is **satisfiable** if it is true in **some** model
  e.g., $A \lor B$, C
- A sentence is **unsatisfiable** if it is true in **no** models
  e.g., $A \land \neg A$

Satisfiability is connected to inference via the following:
  $\alpha \models \beta$ if and only if $(\alpha \land \neg\beta)$ is unsatisfiable

i.e., prove β by contradiction

Proof methods

- Proof methods divide into (roughly) two kinds:

Application of inference rules (detailed)

- Legitimate (sound) generation of new sentences from old
- **Proof** = a sequence of inference rule applications
  Can use inference rules as operators in a standard search algorithm
- Typically require translation of sentences into a **normal form**

Model checking (brief)

truth table enumeration (always exponential in n)
improved backtracking, e.g., Davis–Putnam–Logemann–Loveland
heuristic search in model space (sound but incomplete)
e.g., min-conflicts-like hill-climbing algorithms

- We use $\frac{\alpha}{\beta}$ to imply that α can infer β.

Resolution

Conjunctive Normal Form (CNF—universal)

conjunction of $\underbrace{\textbf{disjunctions} \text{ of } \textbf{literals}}_{\textbf{clauses}}$

E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

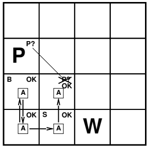
Resolution inference rule (for CNF):

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$

where ℓ_i and m_j are complementary literals.

E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$



Resolution is sound and complete for propositional logic

Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.

$$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$$

3. Move $\neg$ inwards using de Morgan's rules:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$$

4. Apply distributivity law ($\vee$ over $\wedge$) and flatten:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$$

Resolution algorithm

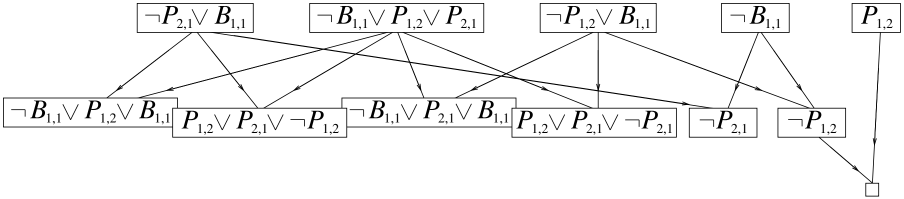
Proof by contradiction, i.e., show $KB \wedge \neg\alpha$ unsatisfiable

```
function PL-RESOLUTION(KB, $\alpha$) returns true or false
    inputs: KB, the knowledge base, a sentence in propositional logic
            $\alpha$, the query, a sentence in propositional logic

    clauses ← the set of clauses in the CNF representation of $KB \wedge \neg\alpha$
    new ← { }
    loop do
        for each $C_i$, $C_j$ in clauses do
            resolvents ← PL-RESOLVE($C_i$, $C_j$)
            if resolvents contains the empty clause then return true
            new ← $new \cup resolvents$
        if $new \subseteq clauses$ then return false
        clauses ← $clauses \cup new$
```

Resolution example

$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})) \land \neg B_{1,1}$ $\alpha = \neg P_{1,2}$
First, transfer $(B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1}))$ into CNF:
$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1})$
Second, apply PL-Resolution until empty clause appears!



$\neg P_{2,1} \lor B_{1,1}$
$\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}$
$\neg P_{1,2} \lor B_{1,1}$
$\neg B_{1,1}$
$P_{1,2}$
$\neg B_{1,1} \lor P_{1,2} \lor B_{1,1}$
$P_{1,2} \lor P_{2,1} \lor \neg P_{1,2}$
$\neg B_{1,1} \lor P_{2,1} \lor B_{1,1}$
$P_{1,2} \lor P_{2,1} \lor \neg P_{2,1}$
$\neg P_{2,1}$
$\neg P_{1,2}$

Forward chaining

- KB = **conjunction** of **Horn clauses**
- **Horn clauses**: disjunction of literals of which at most one is positive
- **Horn clauses** =
  - literal; or
  - (conjunction of symbols) $\Rightarrow$ literal

  E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$
- **Modus Ponens** (for Horn Form): complete for Horn KBs

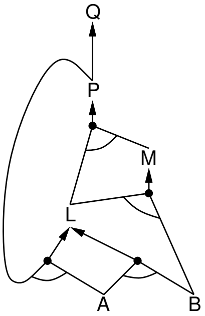
$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \cdots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

- Can be used with **forward chaining**, which is very natural and runs in **linear** time

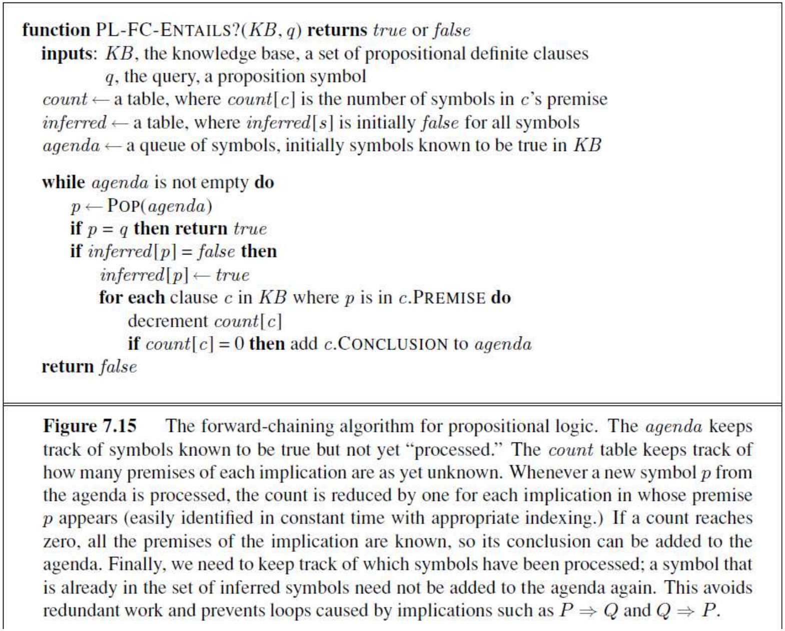
Forward chaining

- Idea: fire any rule whose premises are satisfied in the KB, add its conclusion to the KB, until query is found
- We want to know whether Q is true? How to derive it?

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



Forward chaining algorithm

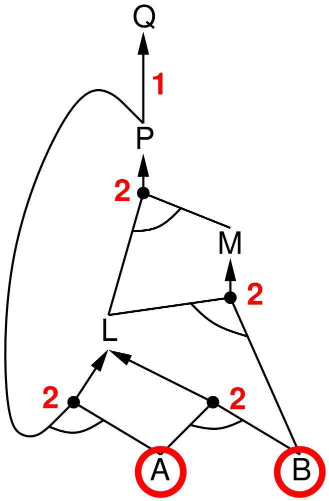


```
function PL-FC-ENTAILS?($KB, q$) returns $true$ or $false$
  inputs: $KB$, the knowledge base, a set of propositional definite clauses
          $q$, the query, a proposition symbol
  $count \leftarrow$ a table, where $count[c]$ is the number of symbols in $c$'s premise
  $inferred \leftarrow$ a table, where $inferred[s]$ is initially $false$ for all symbols
  $agenda \leftarrow$ a queue of symbols, initially symbols known to be true in $KB$

  while $agenda$ is not empty do
     $p \leftarrow$ POP($agenda$)
     if $p = q$ then return $true$
     if $inferred[p] = false$ then
        $inferred[p] \leftarrow true$
        for each clause $c$ in $KB$ where $p$ is in $c$.PREMISE do
           decrement $count[c]$
           if $count[c] = 0$ then add $c$.CONCLUSION to $agenda$
  return $false$
```

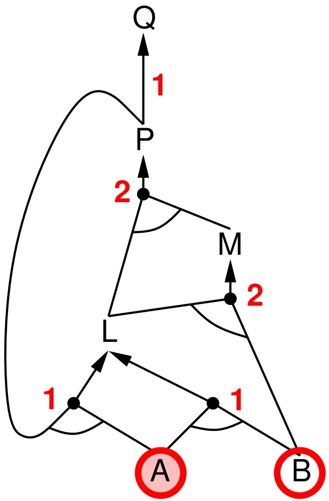
Figure 7.15 The forward-chaining algorithm for propositional logic. The *agenda* keeps track of symbols known to be true but not yet "processed." The *count* table keeps track of how many premises of each implication are as yet unknown. Whenever a new symbol p from the agenda is processed, the count is reduced by one for each implication in whose premise p appears (easily identified in constant time with appropriate indexing.) If a count reaches zero, all the premises of the implication are known, so its conclusion can be added to the agenda. Finally, we need to keep track of which symbols have been processed; a symbol that is already in the set of inferred symbols need not be added to the agenda again. This avoids redundant work and prevents loops caused by implications such as $P \Rightarrow Q$ and $Q \Rightarrow P$.

Forward chaining example



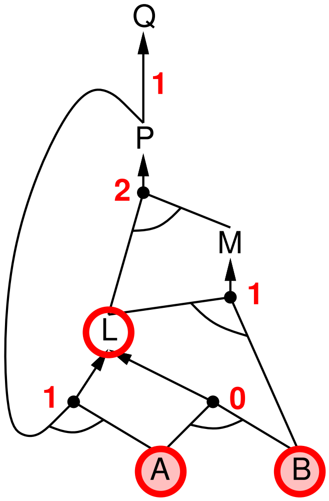
Start from A

Forward chaining example



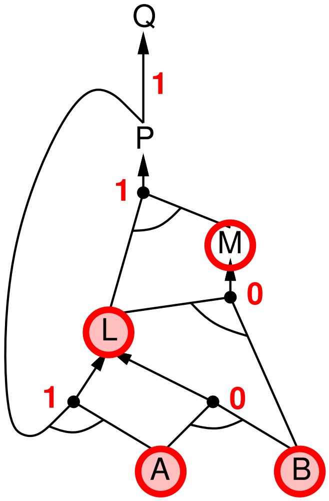
$A \wedge B \Rightarrow L$ decrease 1, $A \wedge P \Rightarrow L$ decrease 1

Forward chaining example



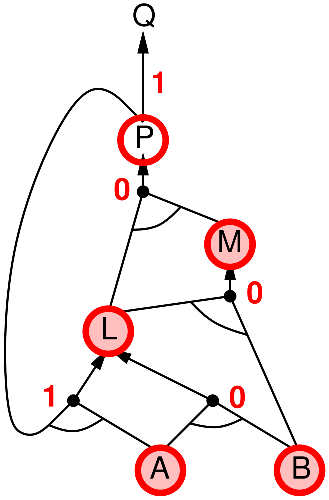
$count(A \land B \Rightarrow L) = 0$, then add L into agenda

Forward chaining example



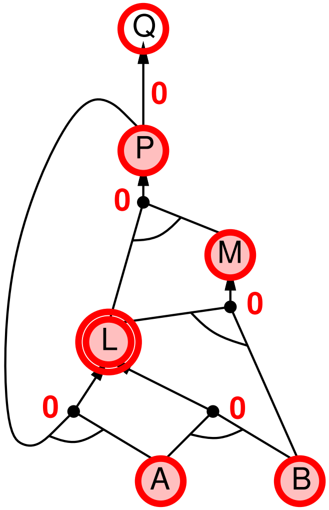
$count(L \wedge B \Rightarrow M) = 0$, then add M into agenda

Forward chaining example



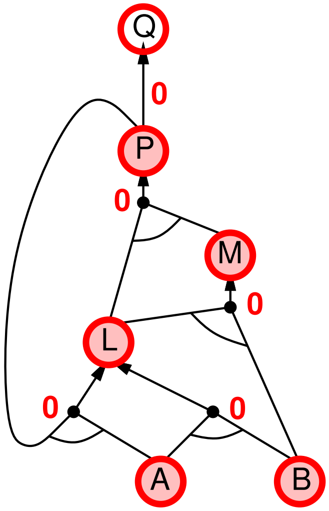
$count(L \wedge M \Rightarrow P) = 0$, then add P into agenda

Forward chaining example



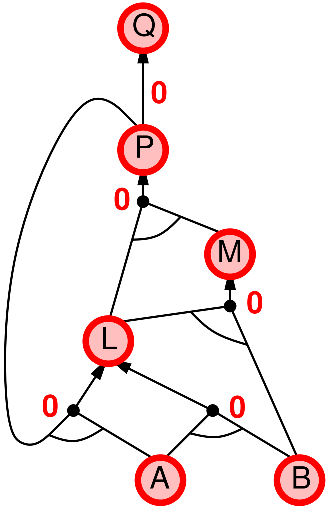
L has already been infered

Forward chaining example



$Count(P \Rightarrow Q) = 0$, Add Q into agenda

Forward chaining example



Forward chaining and Backward chaining

- Forward Chaining (FC) is **data-driven**, which may do lots of work that is irrelevant to the goal
- Another related algorithm is called Backward Chaining (BC), which is **goal-directed**. See the textbook for more details.

Summary

Logical agents apply **inference** to a **knowledge base**
to derive new information and make decisions

Basic concepts of logic:

- **syntax**: formal structure of **sentences**
- **semantics**: **truth** of sentences wrt **models**
- **entailment**: necessary truth of one sentence given another
- **inference**: deriving sentences from other sentences
- **soundness**: derivations produce only entailed sentences
- **completeness**: derivations can produce all entailed sentences

Propositional logic lacks expressive power

Acknowledgment

The slides are adapted from Stuart Russell.