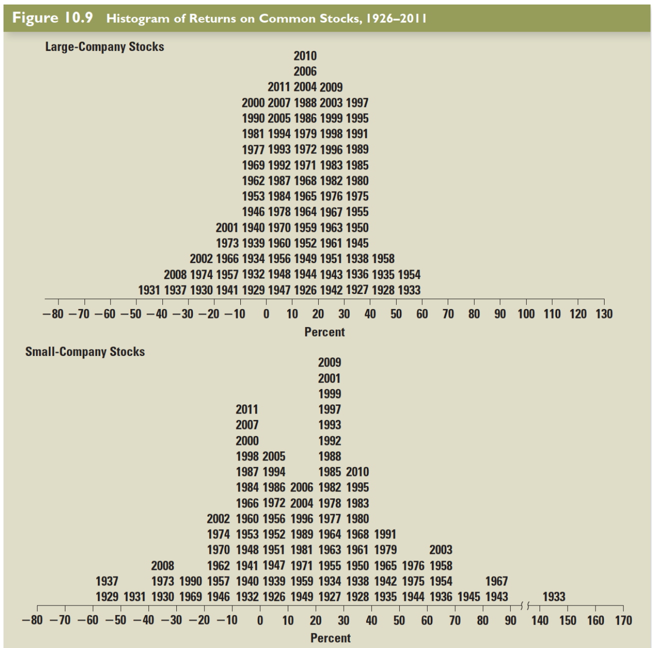
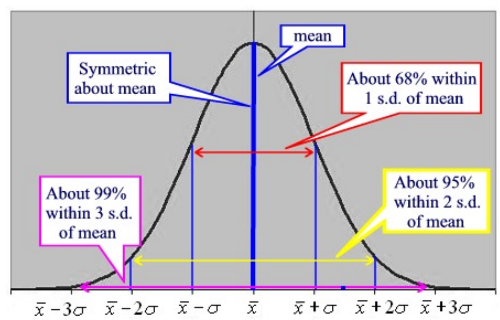
PORTFOLIO ANALYSIS:

Average and Geometric Return: $\overline{R} = \frac{(R_1 + \ldots + R_T)}{T}$ $R = \sqrt[T]{(1+R_1)(1+R_2)\ldots(1+R_T)} - 1$. If we can assume that the returns are normally distributed, than we can make future predictions just by using the average and the standard deviation of a data set on returns.

Distribution of Small and Large Company Stock Returns from 1926 to 2011:



Redrawn from *Stocks, Bonds, Bills and Inflation: 2012 Yearbook,*™ annual updates to the work by Roger G. Ibbotson and Rex A. Sinquefield (Chicago: Morningstar).



Average and Standard Deviation:

$$\overline{R} = \frac{(R_1 + \ldots + R_T)}{T} \qquad \sigma = SD = \sqrt{VAR} = \sqrt{\frac{(R_1 - \overline{R})^2 + (R_2 - \overline{R})^2 + \ldots (R_T - \overline{R})^2}{T-1}}$$

Normal Distribution has the following nice property: By only knowing the mean and the standard deviation of a normally distributed random variable you can predict what its realization will be with a given level of certainty.

Example: A stock has an expected rate of return of 8.3% and a standard deviation of 6.4%. What is the probability that this stock will lose 11% or more in any one given year?

Lower bound of 99% probability range = .083 - (3 × .064) = -.109 = -10.9%;
Probability of losing 11% or more is less than 0.5%.

Example: A stock has returns of 3%, 18%, -24%, and 16% for the past four years. Based on this information, in what range will this stock's return for any one given year be with 95% probability (sometimes this is expressed as "95% certainty", and the resulting range is called the "confidence interval")?

Average return = (.03 + .18 - .24 + .16) ÷ 4 = .0325; Total squared deviation = $(.03 - .0325)^2 + (.18 - .0325)^2 + (-.24 - .0325)^2 + (.16 - .0325)^2$ = .00000625 + .02175625 + .07425625 + .01625625 = .112275; Standard deviation = √(.112275 ÷ (4 - 1) = √.037425 = .19346 = 19.346%; 95% probability range = 3.25% ± (2 × 19.346%) = -35.4 to 41.9%

Covariance of the returns on two stocks: $Cov(A,B) = \sigma_{A,B} = \sum_i r_i^A - \bar{r}_A)(r_i^B - \bar{r}_B)/(T-1)$ where $\bar{r}_A$ is the average (mean) return on stock A, $\bar{r}_B$ is the average (mean) return on stock B, and T is the number of observations and *i* is the index for observations. Note that $Cov(A,B) = Cov(B,A)$. Covariance is a measure that shows the relationship between two variables, in this case, the stock returns of A and B. Note that if both variables move in the same direction then the deviation from the mean would be positive at the same time or negative at the same time for both variables. Hence, each term in the equation is more likely to be positive when two variables more or less move in the same direction. This will produce a positive covariance number. On the other hand, if more often than not, one variable is going up when the other is going down, then the deviations from means will have opposite signs (more often) which will make each term in the equation more likely to be negative and this will produce a negative covariance number. If the covariance number is closer to zero, we can say that the two variables have little to do with each other.

Can I compare the covariance between two variables, say daily temperature in LA and daily humidity in LA, with the covariance of another set of variables, say daily temperature in LA and traffic accidents per day in LA? Does the temperature in LA co-vary more closely with the humidity or with the daily traffic accidents?

For this comparison to make sense, we need to make sure that the covariance is not affected by the units of the variables. If you go back and study the Covariance Equation above, you will note that Cov(A,B) very much depends on the units of A and B. With a little modification, we can obtain a unit-free measure, called “correlation coefficient”:

Corr(A,B) = $\rho_{A,B} = \frac{\sigma_{A,B}}{\sigma_A \sigma_B}$ where $\sigma_{A,B}$ is the covariance between A and B, σ_A is the standard deviation of A and σ_B is the standard deviation of B. Note that Corr(A,B) = Corr(B,A).

Although we are not going to attempt here, it can be easily proven that Correlation Coefficient between any two variables lies between -1 and 1. $-1 \le$ Corr(A,B) ≤ 1

Interpretation: -1 will imply a perfect negative correlation, +1 will imply a perfect positive correlation, 0 will imply no correlation between the variables.

Expected Return and Variance of a Portfolio:

$E(r_P) = x_A E(r_A) + x_B E(r_B)$ $\sigma_P^2 = x_A^2\sigma_A^2 + x_B^2\sigma_B^2 + 2\, x_A x_B\, Cov(A,B)$

Stock A Returns	Stock B Returns	Deviation from mean for A	Deviation from mean for B	Squared Deviation for A	Squared Deviation for B	Product of Deviations
0.03	-0.18	-0.0025	-0.2700	6.25E-06	0.0729	0.0007
0.18	0.03	0.1475	-0.0600	0.0218	0.0036	-0.0089
-0.24	0.32	-0.2725	0.2300	0.0743	0.0529	-0.0627
0.16	0.19	0.1275	0.1000	0.0163	0.0100	0.0128
Mean	Mean			Var A	Var B	Cov(A,B)
0.0325	0.09			0.04	0.05	-0.019
				S.D. A	S.D. B	
				0.19	0.22	

Efficient Set: