CS 161 Fundamentals of Artificial Intelligence
Lecture 9

First-order Logic: Representation

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Outline

- Why First-order logic (FOL)?
- Syntax and semantics of First-order logic
- Kinship Example
- Wumpus world in First-order logic

Pros and cons of propositional logic

- Propositional logic is **declarative**: pieces of syntax correspond to facts
- Propositional logic allows partial/disjunctive/negated information
  (unlike most data structures and databases)
- Propositional logic is **compositional**:
  meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- Meaning in propositional logic is **context-independent**
  (unlike natural language, where meaning depends on context)
- Propositional logic has very limited expressive power

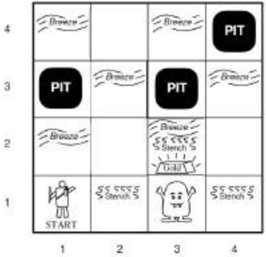
Review: Wumpus World

Performance measure gold +1000, death -1000
-1 per step, -10 for using the arrow

Environment
Squares adjacent to wumpus are smelly
Squares adjacent to pit are breezy
Glitter iff gold is in the same square
Shooting kills wumpus if you are facing it
Shooting uses up the only arrow
Grabbing picks up gold if in same square
Releasing drops the gold in same square

Actuators Left turn, Right turn,
Forward, Grab, Release, Shoot

Sensors Breeze, Glitter, Smell



Limitation of Propositional Logic

How to express 'pits cause breezes in adjacent squares'?

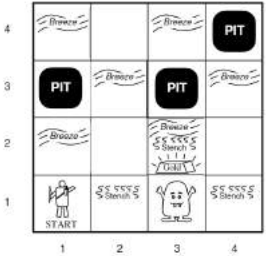
Propositional logic: write down rule for each square

- $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
- $B_{1,2} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{1,3})$
- ...
- Very complicated!

Natural Language: 'pits cause breezes in adjacent squares'

- Simple and powerful

First-order logic: similar to natural language



First-order logic

Whereas propositional logic assumes world contains **facts**,
first-order logic (like natural language) assumes the world contains

- **Objects**: people, houses, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries ...
- **Relations** (can be unary relations i.e., **Properties**): red, round, bogus, prime, multistoried ...,
  brother of, bigger than, inside, part of, has color, occurred after, owns, comes between, ...
- **Functions**: father of, best friend, third inning of, one more than, end of ...

Syntax of First-order Logic: Basic elements

- Constants $KingJohn,\ 2,\ UCLA, \ldots$
- Predicates $Brother,\ >, \ldots$
  - Return True of False
- Functions $Sqrt,\ LeftLegOf, \ldots$
- Variables $x,\ y,\ a,\ b, \ldots$
- Connectives $\wedge \quad \vee \quad \neg \quad \Rightarrow \quad \Leftrightarrow$
- Equality $=$
- Quantifiers $\forall\ \exists$

Atomic sentences

Term: a logical expression that refers to an object

- $constant$
- $variable$
- $function(term_1, \ldots, term_n)$

Atomic sentence: $predicate(term_1, \ldots, term_n)$

- $Brother(KingJohn, RichardTheLionheart)$
- $>$ $(Length(LeftLeg(Richard)), Length(LeftLeg(KingJohn)))$

• An atomic sentence is true in a given model if the relation referred to by the predicate symbol holds among the objects referred to by the arguments.

Complex sentences

- **Complex sentences** are made from atomic sentences using connectives

$$\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \Rightarrow S_2, \quad S_1 \Leftrightarrow S_2$$

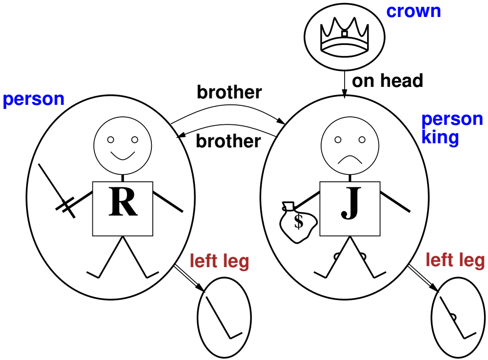
Examples:

$Sibling(KingJohn, Richard) \Rightarrow Sibling(Richard, KingJohn)$
$>(1,2) \vee \leq(1,2)$
$>(1,2) \wedge \neg>(1,2)$

Truth in first-order logic

- Sentences are true with respect to a **model** and an **interpretation**
- Model contains ≥ 1 objects (**domain elements**) and relations among them
- Interpretation specifies referents for
  constant symbols $\rightarrow$ objects
  predicate symbols $\rightarrow$ relations
  function symbols $\rightarrow$ functional relations
- An atomic sentence $predicate(term_1, \ldots, term_n)$ is true iff the objects referred to by $term_1, \ldots, term_n$ are in the relation referred to by $predicate$

Models for FOL: Example



- Five objects: Richard, John, Richard's left leg, John's left leg, crown
- Two binary relations: brother(,), on head (,)
- Three unary relations: person(), king(), crown()
- Unary function: left leg()

Models for FOL: Lots!

- Entailment in propositional logic can be computed by enumerating models
- We **can** enumerate the FOL models for a given KB vocabulary:

For each number of domain elements n from 1 to ∞
  For each k-ary predicate P_k in the vocabulary
    For each possible k-ary relation on n objects
      For each constant symbol C in the vocabulary
        For each choice of referent for C from n objects ...

- Computing entailment by enumerating FOL models is not easy!
  - In FOL, we use universal quantification $\forall$ and existential quantification $\exists$ to entail!

Universal quantification

$\forall \langle variables \rangle \ \langle sentence \rangle$

Everyone at UCLA is smart:

$\forall x \ At(x, UCLA) \Rightarrow Smart(x)$

$\forall x \ P$ is true in a model m iff P is true with x being **each** possible object in the model

- Roughly speaking, equivalent to the conjunction of instantiations of P

$$
\begin{array}{ll}
 & (At(KingJohn, UCLA) \Rightarrow Smart(KingJohn)) \\
\wedge & (At(Richard, UCLA) \Rightarrow Smart(Richard)) \\
\wedge & \ldots
\end{array}
$$

A common mistake to avoid

Typically, $\Rightarrow$ is the main connective with $\forall$
Common mistake: using $\wedge$ as the main connective with $\forall$:

$$\forall x \;\; At(x, UCLA) \wedge Smart(x)$$

means "Everyone is at UCLA and everyone is smart"

- Not everyone is at UCLA!
- Not true!

Existential quantification

$\exists \langle variables \rangle \ \langle sentence \rangle$

Someone at USC is smart:

$\exists x \ \ At(x, USC) \wedge Smart(x)$

$\exists x \ \ P$ is true in a model m iff P is true with x being **some** possible object in the model

- Roughly speaking, equivalent to the disjunction of instantiations of P

$$
\begin{array}{ll}
 & (At(KingJohn, USC) \wedge Smart(KingJohn)) \\
\vee & (At(Richard, USC) \wedge Smart(Richard)) \\
\vee & \ldots
\end{array}
$$

Another common mistake to avoid

Typically, $\wedge$ is the main connective with $\exists$
Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

$$\exists x \; At(x, USC) \Rightarrow Smart(x)$$

is true iff

- Someone at USC is smart
  or
- There exists anyone who is not at USC!
- Always true! Meaningless

Properties of quantifiers

$\forall x\ \forall y$ is the same as $\forall y\ \forall x$

$\exists x\ \exists y$ is the same as $\exists y\ \exists x$

$\exists x\ \forall y$ is **not** the same as $\forall y\ \exists x$

$\exists x\ \forall y\ Loves(x, y)$

"There is a person who loves everyone in the world"

$\forall y\ \exists x\ Loves(x, y)$

"Everyone in the world is loved by at least one person"

Quantifier duality, i.e., De Morgan rules: each can be expressed using the other (add $model$)

$\forall x\ Likes(x, IceCream) \qquad \neg\exists x\ \neg Likes(x, IceCream)$

$\exists x\ Likes(x, Broccoli) \qquad \neg\forall x\ \neg Likes(x, Broccoli)$

Example: Kinship

- Brothers are siblings

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- "Sibling" is symmetric

$\forall\, x, y \;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x)$.

Example: Kinship

- Brothers are siblings

$\forall\, x, y\;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y)$.

- "Sibling" is symmetric

$\forall\, x, y\;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x)$.

- A first cousin is a child of a parent's sibling

Example: Kinship

- Brothers are siblings

$\forall x, y \;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y)$.

- "Sibling" is symmetric

$\forall x, y \;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x)$.

- A first cousin is a child of a parent's sibling

$\forall x, y \;\; FirstCousin(x, y) \;\Leftrightarrow\; \exists p, ps \;\; Parent(p, x) \wedge Sibling(ps, p) \wedge Parent(ps, y)$

Example: Wumpus World

We transfer some rules in Wumpus world into FOL!

Perception of agent at time t

- $Percept([Breeze, Glitter, Smell], t)$

Percept data implies certain facts about the current state

- $\forall t, s, g, Percept([Breeze, g, s], t) \Rightarrow Breeze(t)$
- $\forall s, b, t, Percept([b, Glitter, s], t) \Rightarrow Glitter(t)$

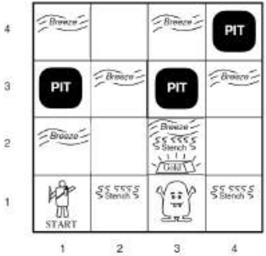
Whether agent is at square s at time t

- $At(agent, s, t)$

Agent is at s and perceives a breeze, then s is breezy

- $\forall s, t, At(agent, s, t) \wedge Breeze(t) \Rightarrow Breeze(s)$

...



Summary

- First-order logic:
  - objects and relations are semantic primitives
  - syntax: constants, functions, predicates, equality, quantifiers
- Increased expressive power: sufficient to define Wumpus world

Acknowledgment

The slides are adapted from Stuart Russell et al.