Week 5

Discounted Cash Flow Valuation

Net Present Value

In the one-period case, the formula for *NPV* can be written as:

- $NPV = -Cost + PV$

If we had *not* undertaken the positive *NPV* project considered on the last slide, and instead invested our \$9,500 elsewhere at 5 percent, our *FV* would be less than the \$10,000 the investment promised, and we would be worse off in *FV* terms :

$$\$9{,}500 \times (1.05) = \$9{,}975 < \$10{,}000$$

4.2 The Multi-period Case

- The general formula for the future value of an investment over many periods can be written as:

$$FV = C_0 \times (1 + r)^T$$

Where

C_0 is cash flow at date 0,

r is the appropriate interest rate, and

T is the number of periods over which the cash is invested.

Future Value

- Suppose a stock currently pays a dividend of \$1.10, which is expected to grow at 40% per year for the next five years.
- What will the dividend be in five years?

$$FV = C_0 \times (1 + r)^T$$

$$\$5.92 = \$1.10 \times (1.40)^5$$

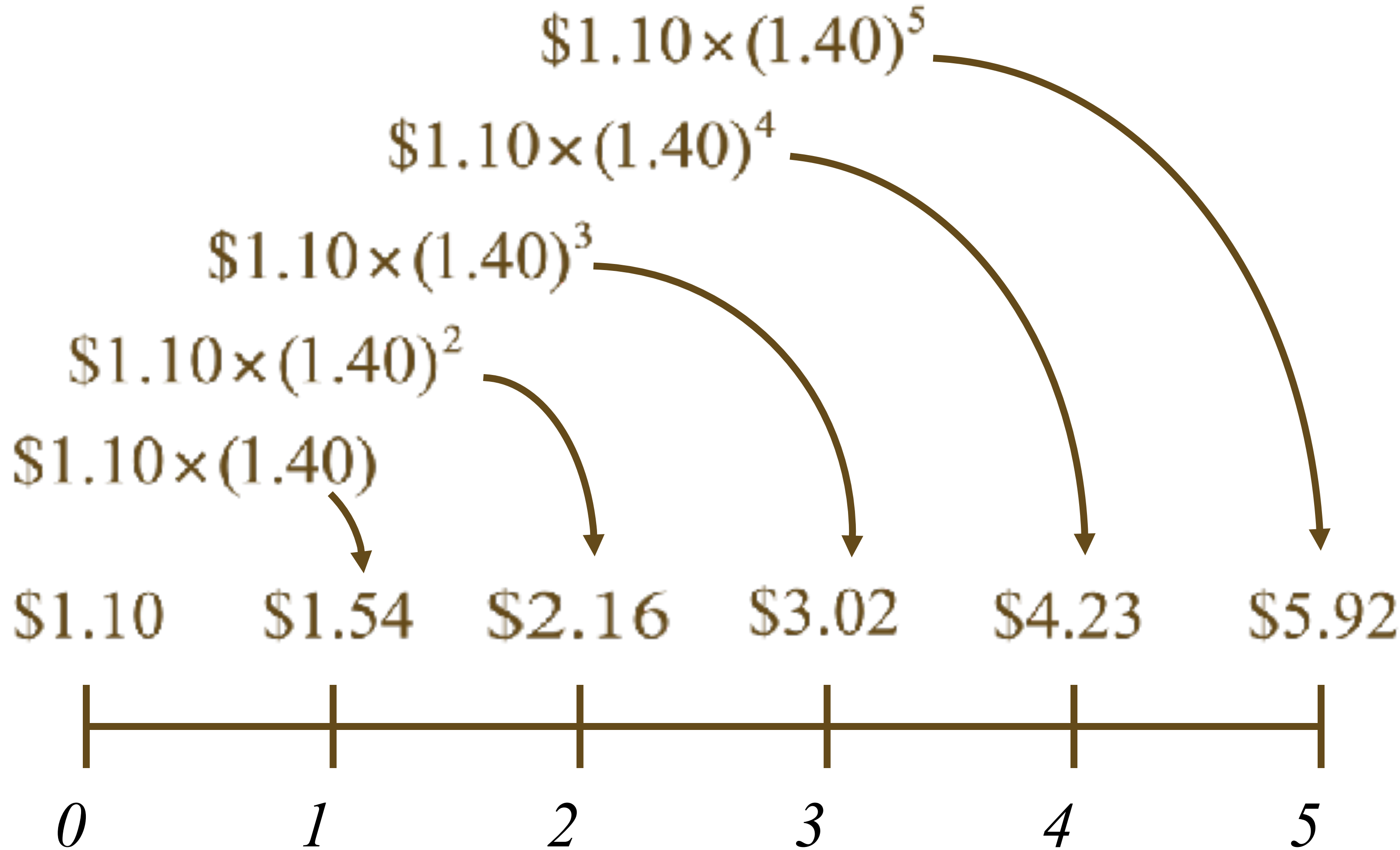
Future Value and Compounding

- Notice that the dividend in year five, \$5.92, is considerably higher than the sum of the original dividend plus five increases of 40-percent on the original \$1.10 dividend:

$$\$5.92 > \$1.10 + 5\times[\$1.10\times.40] = \$3.30$$

This is due to *compounding*.

Future Value and Compounding



Present Value and Discounting

$$PV = C_{\mathrm{T}} / (1 + r)^{T}$$

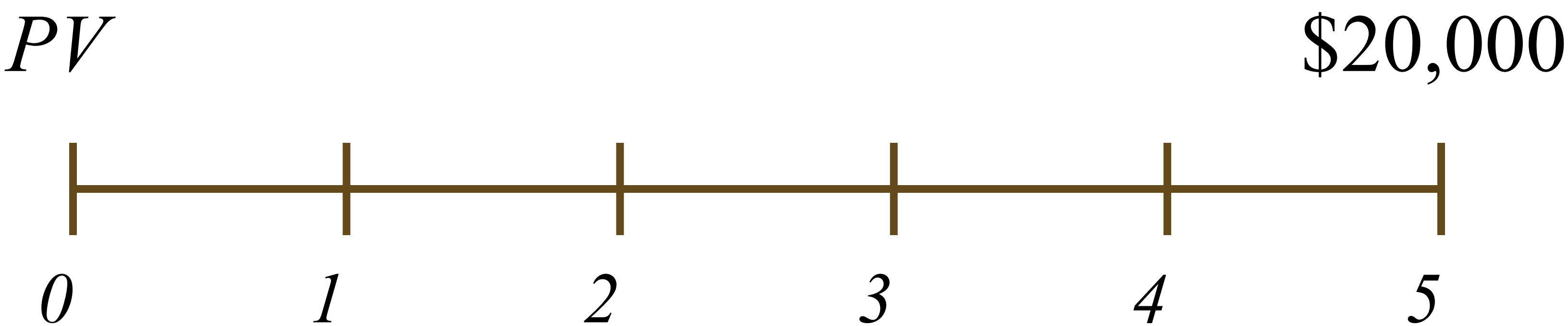
C_{T} is cash flow at date T,

r is the appropriate interest rate, and

T is the number of periods over which the cash is invested.

Present Value and Discounting

- How much would an investor have to set aside today in order to have $20,000 five years from now if the current rate is 15%?



$$\$9{,}943.53 = \frac{\$20{,}000}{(1.15)^5}$$

4.5 Finding the Number of Periods

If we deposit \$5,000 today in an account paying 10%, how long does it take to grow to \$10,000?

$$FV = C_0 \times (1+r)^T \qquad \$10{,}000 = \$5{,}000 \times (1.10)^T$$

$$(1.10)^T = \frac{\$10{,}000}{\$5{,}000} = 2$$

$$\ln(1.10)^T = \ln(2)$$

$$T = \frac{\ln(2)}{\ln(1.10)} = \frac{0.6931}{0.0953} = 7.27 \text{ years}$$

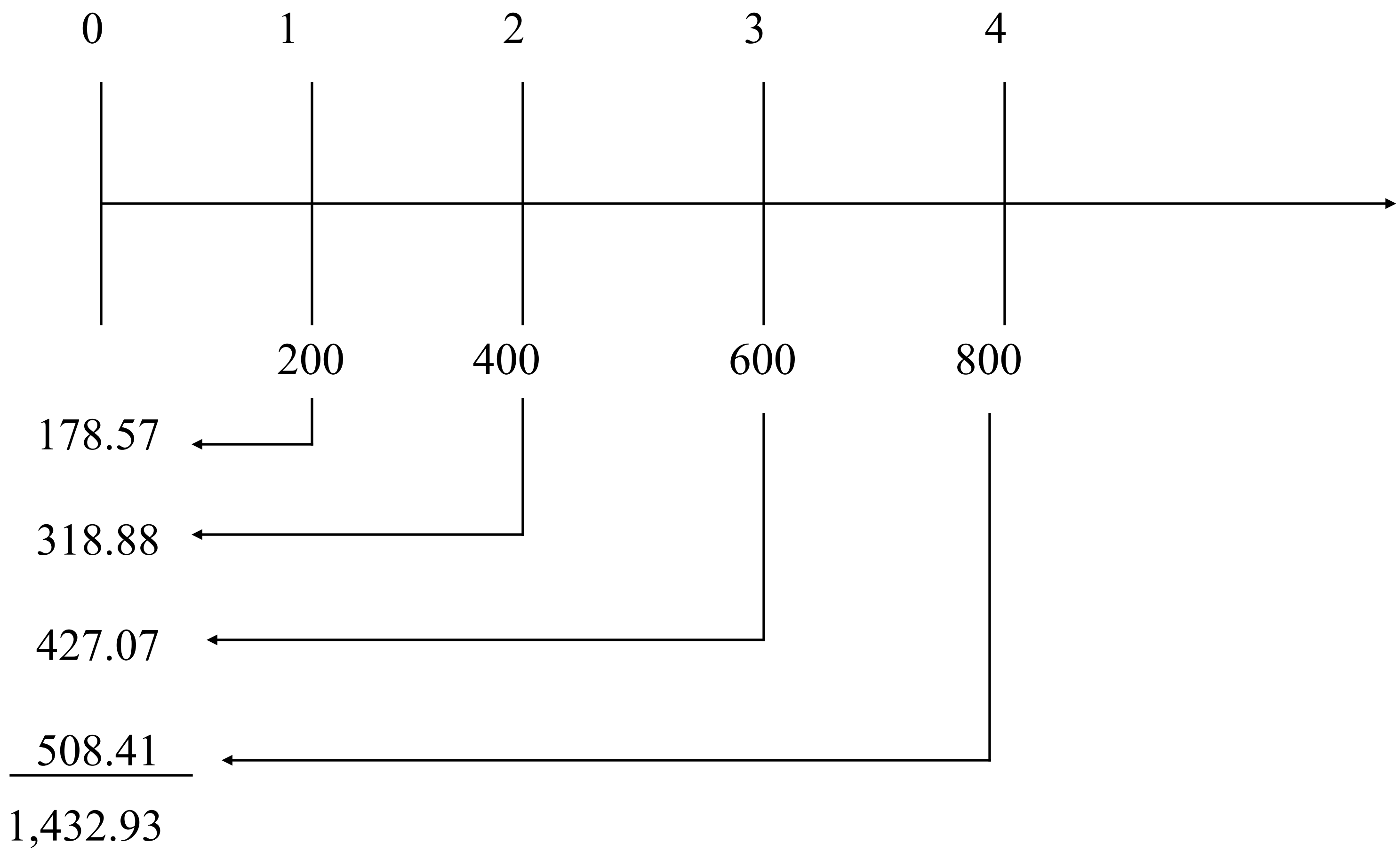
Effect of Interest Rate on Tail Heavy Cash Flow

Year	0	1	2	3	4	5	6	7	8	9	10
Cash Flow I	-10,000	3,000	3,000	3,000	3,000	3,000	4,000	4,000	4,000	4,000	4,000
Cash Flow II	-10,000	4,000	4,000	4,000	4,000	4,000	3,000	3,000	3,000	3,000	3,000

Multiple Cash Flows

- Consider an investment that pays \$200 one year from now, with cash flows increasing by \$200 per year through year 4. If the interest rate is 12%, what is the present value of this stream of cash flows?
- If the issuer offers this investment for \$1,500, should you purchase it?

Multiple Cash Flows



Present Value < Cost → Do Not Purchase

LET'S THINK ABOUT NPV ONE MORE TIME

An Investment Opportunity

Year 0	Year 1	Year 2	Year 3
-1000	200	200	1200

NPV = 249

Market

-1000	100	100	1100

Difference

100	100	100

249

An Investment Opportunity

Year 0	Year 1	Year 2	Year 3
-1000	200	200	1200

NPV = 249

Market

-1000			1331

Difference

200	200	-131

249

WAIT: we can invest the money we get each year, then what?

-1000	200	200	1200
	-200	220	462
		-420	
-1000	0	0	1662

NPV = 249

-1000	200	200	1200		…		Inv. Opp.
-1000	100	100	100	100	…	1100	Market
	100	100	1100	-100	…	-1100	Opp-Market

249 ?

4.3 Compounding Periods

Compounding an investment m times a year for T years provides for future value of wealth:

$$FV = C_0 \times \left(1 + \frac{r}{m}\right)^{m \times T}$$

12% annual rate

compounded monthly

then, monthly rate is 1%

$(1+0.01)^{12} - 1 = 12.68\%$

Compounding Periods

- For example, if you invest \$50 for 3 years at 12% compounded semi-annually, your investment will grow to

$$FV = \$50 \times \left(1 + \frac{.12}{2}\right)^{2\times 3} = \$50 \times (1.06)^6 = \$70.93$$

Effective Annual Rates of Interest

A reasonable question to ask in the above example is "what is the effective *annual* rate of interest on that investment?"

$$FV = \$50 \times (1 + \frac{.12}{2})^{2\times3} = \$50 \times (1.06)^6 = \$70.93$$

The Effective Annual Rate (EAR) of interest is the annual rate that would give us the same end-of-investment wealth after 3 years:

$$\$50 \times (1 + EAR)^3 = \$70.93$$

Effective Annual Rates of Interest

$$FV = \$50 \times (1 + EAR)^3 = \$70.93$$

$$(1 + EAR)^3 = \frac{\$70.93}{\$50}$$

$$EAR = \left(\frac{\$70.93}{\$50}\right)^{1/3} - 1 = .1236$$

So, investing at 12.36% compounded annually is the same as investing at 12% compounded semi-annually.

Effective Annual Rates of Interest

- Find the Effective Annual Rate (EAR) of an 18% APR loan that is compounded monthly.
- What we have is a loan with a monthly interest rate rate of 1½%.
- This is equivalent to a loan with an annual interest rate of 19.56%.

$$\left(1+\frac{r}{m}\right)^{m}=\left(1+\frac{.18}{12}\right)^{12}=(1.015)^{12}=1.1956$$

Continuous Compounding

- The general formula for the future value of an investment compounded continuously over many periods can be written as:

$FV = C_0 \times e^{rT}$

Where

C_0 is cash flow at date 0,

r is the stated annual interest rate,

T is the number of years, and

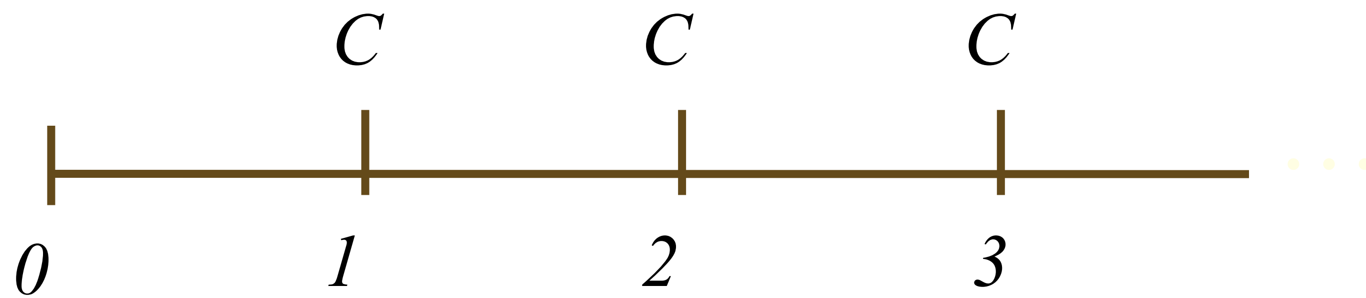
e is a transcendental number approximately equal to 2.718. e^x is a key on your calculator.

4.4 Simplifications

- Perpetuity
  - A constant stream of cash flows that lasts forever
- Growing perpetuity
  - A stream of cash flows that grows at a constant rate forever
- Annuity
  - A stream of constant cash flows that lasts for a fixed number of periods
- Growing annuity
  - A stream of cash flows that grows at a constant rate for a fixed number of periods

Perpetuity

A constant stream of cash flows that lasts forever



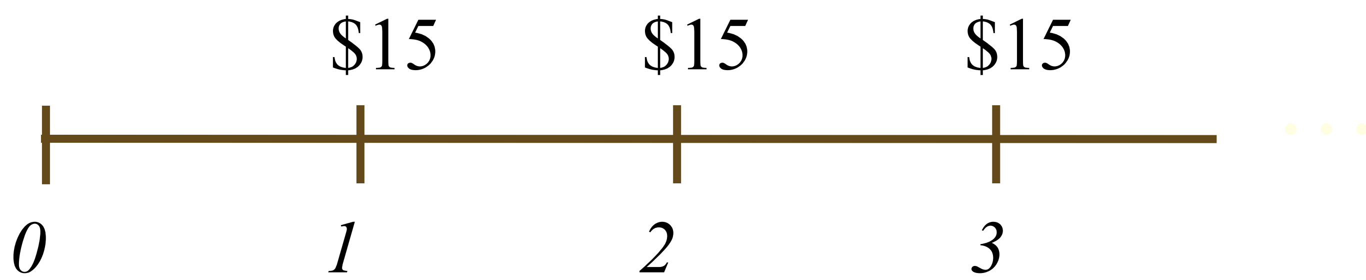
$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots$$

$$PV = \frac{C}{r}$$

Perpetuity: Example

What is the value of an asset that promises to pay $15 every year for ever?

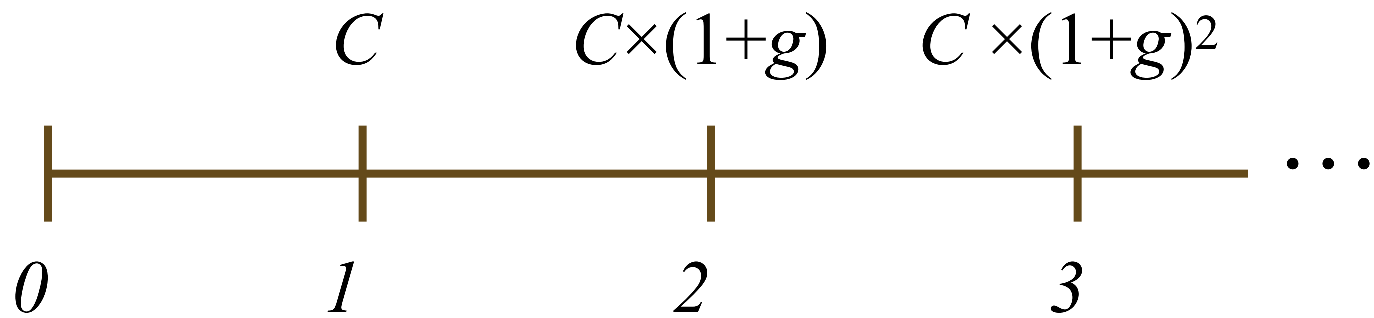
The interest rate is 10-percent.



$$PV = \frac{\$15}{.10} = \$150$$

Growing Perpetuity

A growing stream of cash flows that lasts forever



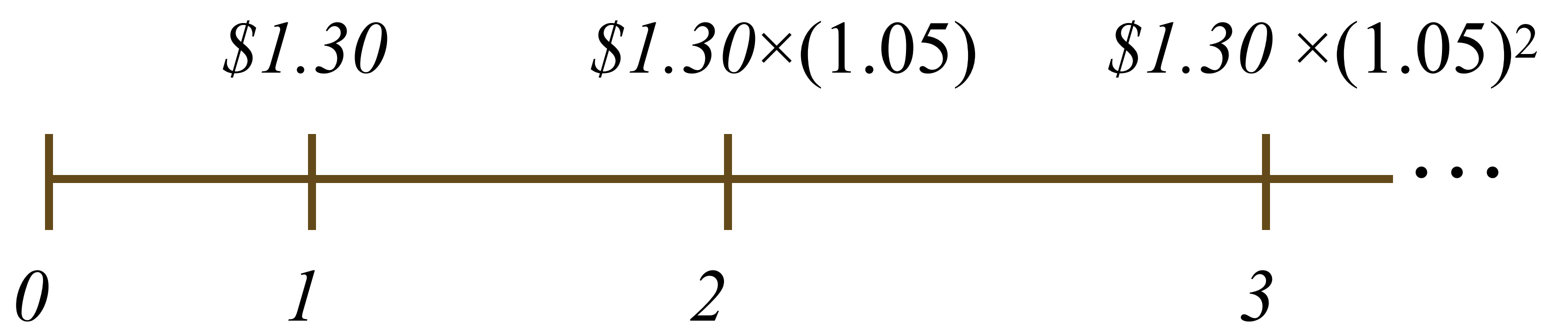
$$PV = \frac{C}{(1+r)} + \frac{C\times(1+g)}{(1+r)^2} + \frac{C\times(1+g)^2}{(1+r)^3} + \cdots$$

$$PV = \frac{C}{r-g}$$

Growing Perpetuity: Example

The expected dividend next year is \$1.30, and dividends are expected to grow at 5% forever.

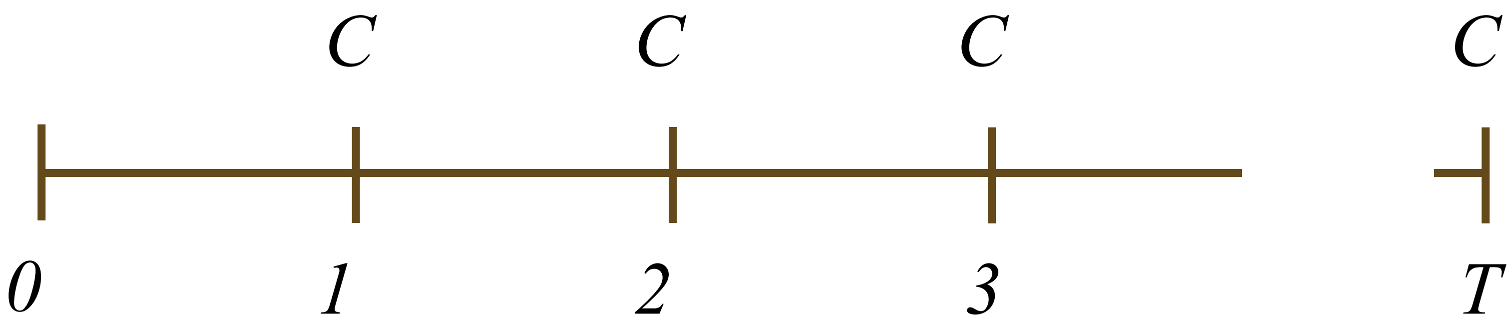
If the discount rate is 10%, what is the value of this promised dividend stream?



$$PV = \frac{\$1.30}{.10 - .05} = \$26.00$$

Annuity

A constant stream of cash flows with a fixed maturity

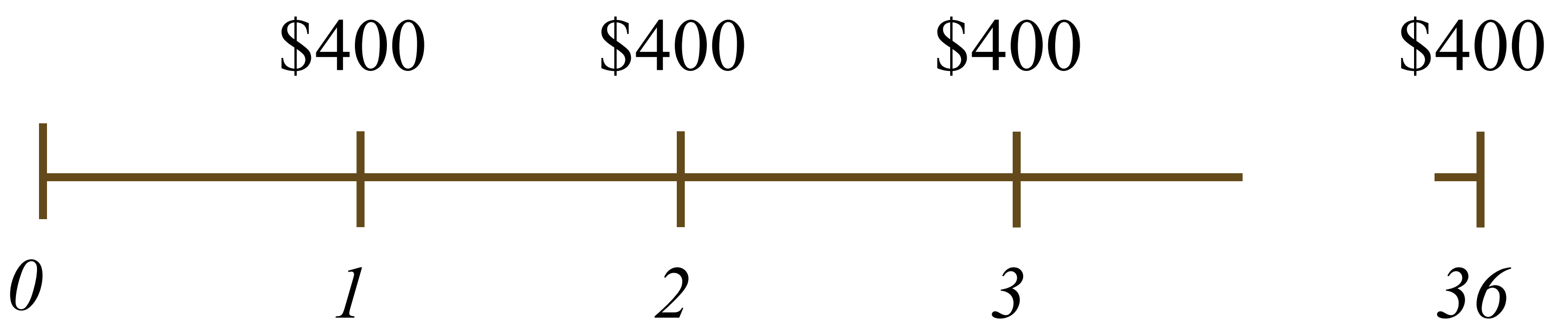


$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \quad \frac{C}{(1+r)^T}$$

$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

Annuity: Example

If you can afford a $400 monthly car payment, how much car can you afford if interest rates are 7% on 36-month loans?

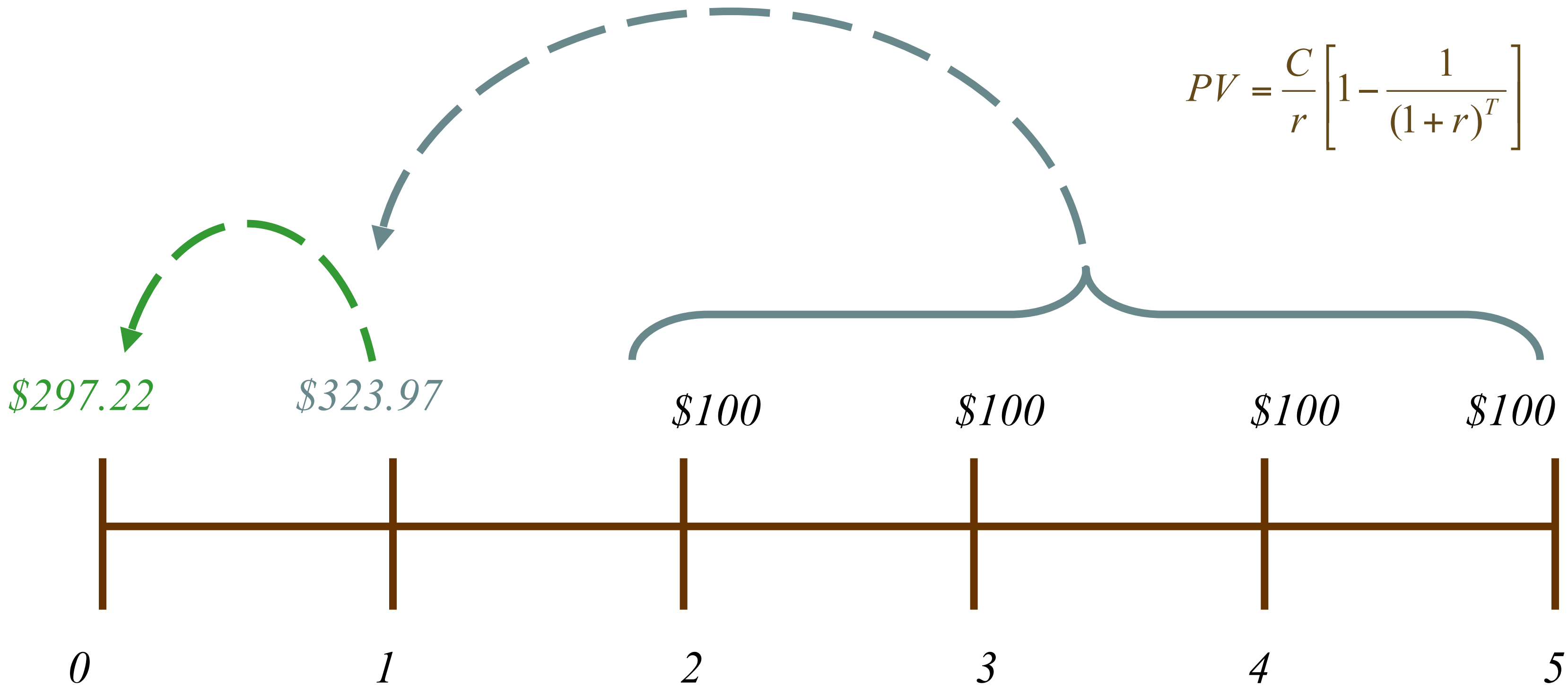


$$PV = \frac{\$400}{.07/12}\left[1 - \frac{1}{(1+.07/12)^{36}}\right] = \$12,954.59$$

What is the present value of a four-year annuity of $100 per year that makes its first payment two years from today if the discount rate is 9%?

$$PV_1 = \sum_{t=1}^{4} \frac{\$100}{(1.09)^t} = \frac{\$100}{(1.09)^1} + \frac{\$100}{(1.09)^2} + \frac{\$100}{(1.09)^3} + \frac{\$100}{(1.09)^4} = \$323.97$$

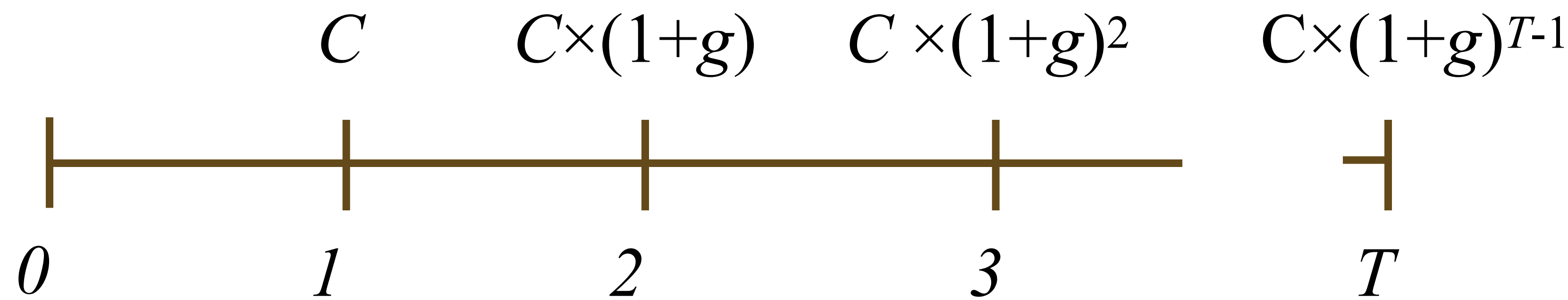
$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$



$$PV_0 = \frac{\$323.97}{1.09} = \$297.22$$

Growing Annuity

A growing stream of cash flows with a fixed maturity

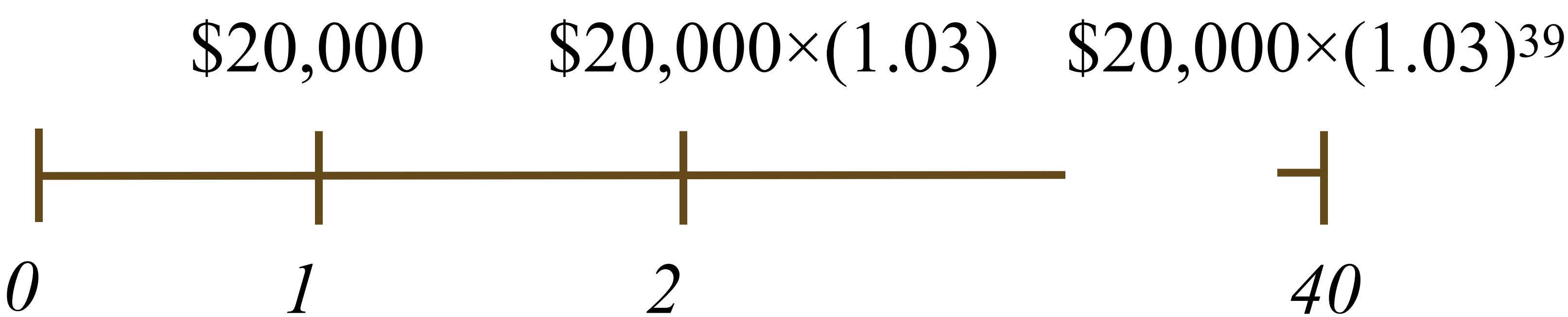


$$PV = \frac{C}{(1+r)} + \frac{C\times(1+g)}{(1+r)^2} + \quad + \frac{C\times(1+g)^{T-1}}{(1+r)^T}$$

$$PV = \frac{C}{r-g}\left[1-\left(\frac{1+g}{(1+r)}\right)^T\right]$$

Growing Annuity: Example

An asset offers to pay \$20,000 per year for 40 years and increase the annual payment by 3% each year beginning a year from today. What is the present of this asset today if the discount rate is 10%?



$$PV = \frac{\$20{,}000}{.10 - .03}\left[1 - \left(\frac{1.03}{1.10}\right)^{40}\right] = \$265{,}121.57$$