NUMERIC PORTFOLIO EXAMPLE:

You have $100 in your pocket. You would like to invest on a portfolio of stock A and B.

You collected data on stock A and B returns and found the expected returns and standard deviations as follows:

$E(r_A)$ = 5%, $E(r_B)$ = 10%, σ_A= 10%, σ_B= 20%.

Let's assume that you invest $80 on A and $20 on B. Then you formed a portfolio with 80% of your money invested on A (weight of A, X_A = 0.8) and 20% on B (weight of B, X_B=0.2).

In order to find the expected return and the risk level of the portfolio, $E(r_P)$ and σ_P, we need one more piece of information: the covariance of A and B.

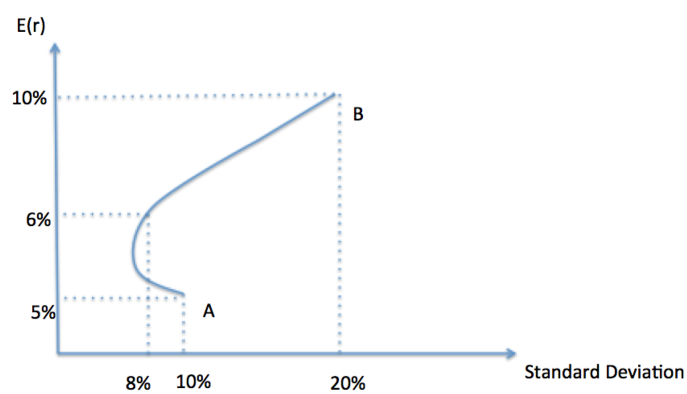
Assume Cov(A,B) = $\sigma_{A,B}$ = -0.005. Note that this implies the correlation between A and B as $\rho_{A,B} = \frac{\sigma_{A,B}}{\sigma_A \sigma_B}$ = -0.005/(0.1*0.2) = -0.25

$E(r_P)$ = 0.8*5%+0.2*10% = 6%

$\sigma_P = (0.8^2*0.1^2+0.2^2*0.2^2+2*0.2*0.8*(-0.005))^{(1/2)}$ = 8%

Note that by combining two risky assets (whose risk levels are 10% and 20%), we obtain a portfolio whose risk level is less then either at 8%!!! This is due to diversification!

Consider all the portfolios we could form by investing a portion of our money on A and the rest on B. We could calculate the expected return and the risk levels. We would obtain a graph as follows when we put all such portfolios we can find on a return-risk plane:



Given the above graph, there exists a portfolio we can form by combining A and B that has a minimum standard deviation (or minimum variance). Let's call this portfolio the minimum variance portfolio to be denoted by MV. Can we find the expected return and the risk level of MV? What would be the weight of A and B in the minimum variance portfolio?

We can find out by minimizing the following expression over X_A (Note that $X_B = 1-X_A$):

$\sigma_P = (X_A{}^2*0.1^2+ (1-X_A)^2*0.2^2+2* X_A*(1- X_A)*(-0.005))^{(1/2)}$, then this is the same as minimizing the variance σ_P^2:

Minimize $(X_A{}^2*0.1^2+ (1-X_A)^2*0.2^2+2* X_A*(1- X_A)*(-0.005))$ with respect to $X_{A.}$

First order condition will give us:

$2*0.1^2X_A-2*(1-X_A)*0.2^2+(2*(-0.005)(1-2*X_A))=0$

Now we can solve for $X_{A.}$