

# CS174A Lecture 14

# Announcements & Reminders

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- *11/20/22: A4 due*
- *11/22/22: Team project proposals due, final version*
- *11/24/22-12/03/22: Student evaluations of course/instructors/TAs*
- *11/29/22: Prof Demetri's talk*
- *12/02/22 (Discussion Sessions): Team project presentations*
- *12/05/22-12/06/22: Office hours for final exam, see Canvas*
- *12/06/22: Final Exam, 6:30-8:30 PM PST, in class, in person*

# TA Session This Friday

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- *Team project demo logistics*
- *Assignment #4*

# Last Lecture Recap

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- *Non-Photorealistic Rendering*
- *Global Illumination: Radiosity*
- *Mappings: Texture, Bump, Displacement, Environment*
- *Shadows*

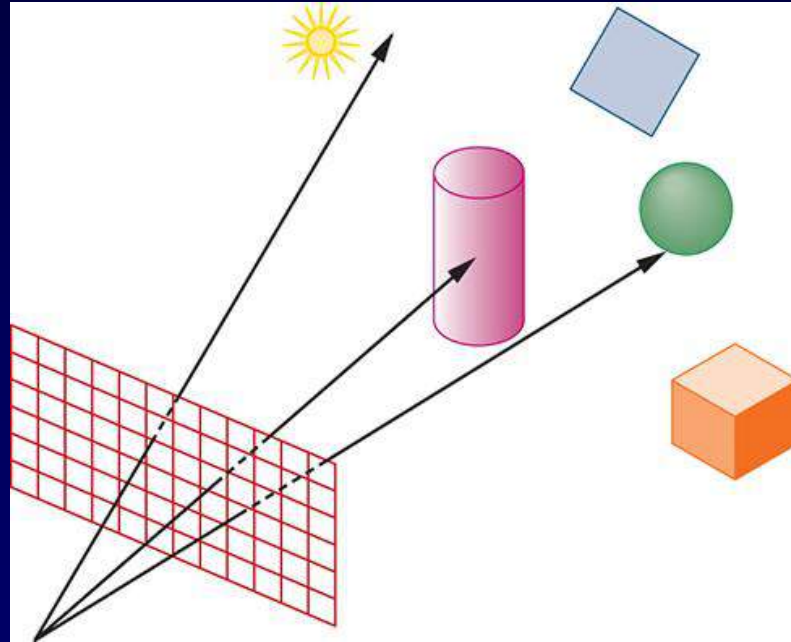
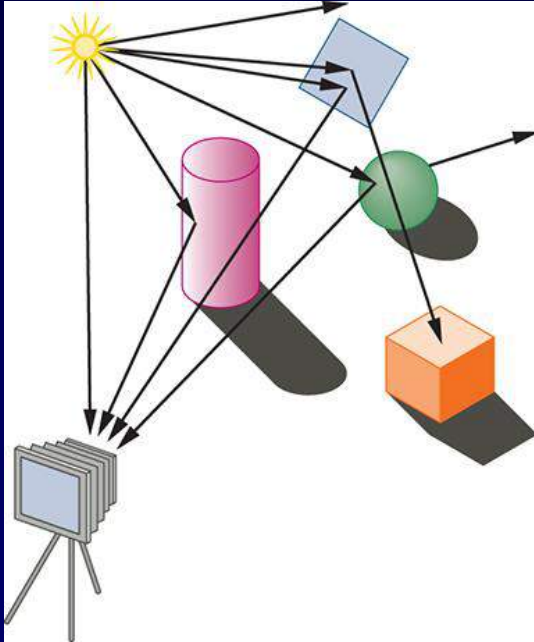
# Next Up

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- ***Shadows***
  - Shadow volumes
  - 2-pass z-buffer algorithm
- ***Hidden Surface Removal***
  - Ray casting
- ***Ray Tracing***

# HSR: Ray Casting Algorithm

- *Forward vs. backward ray casting*

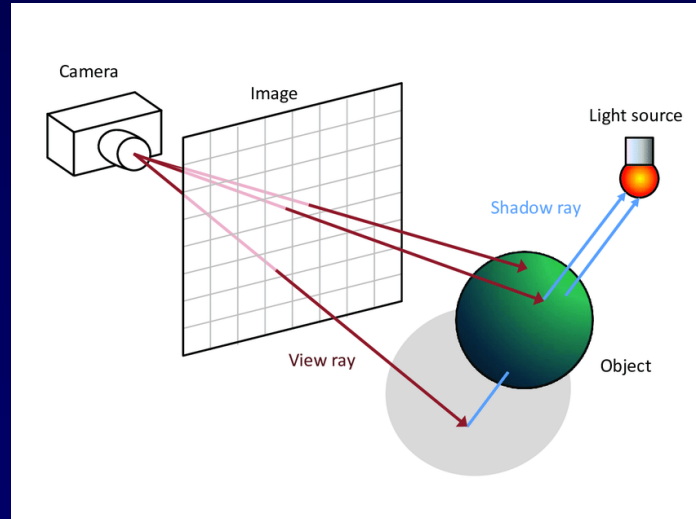


# HSR: Ray Casting Algorithm

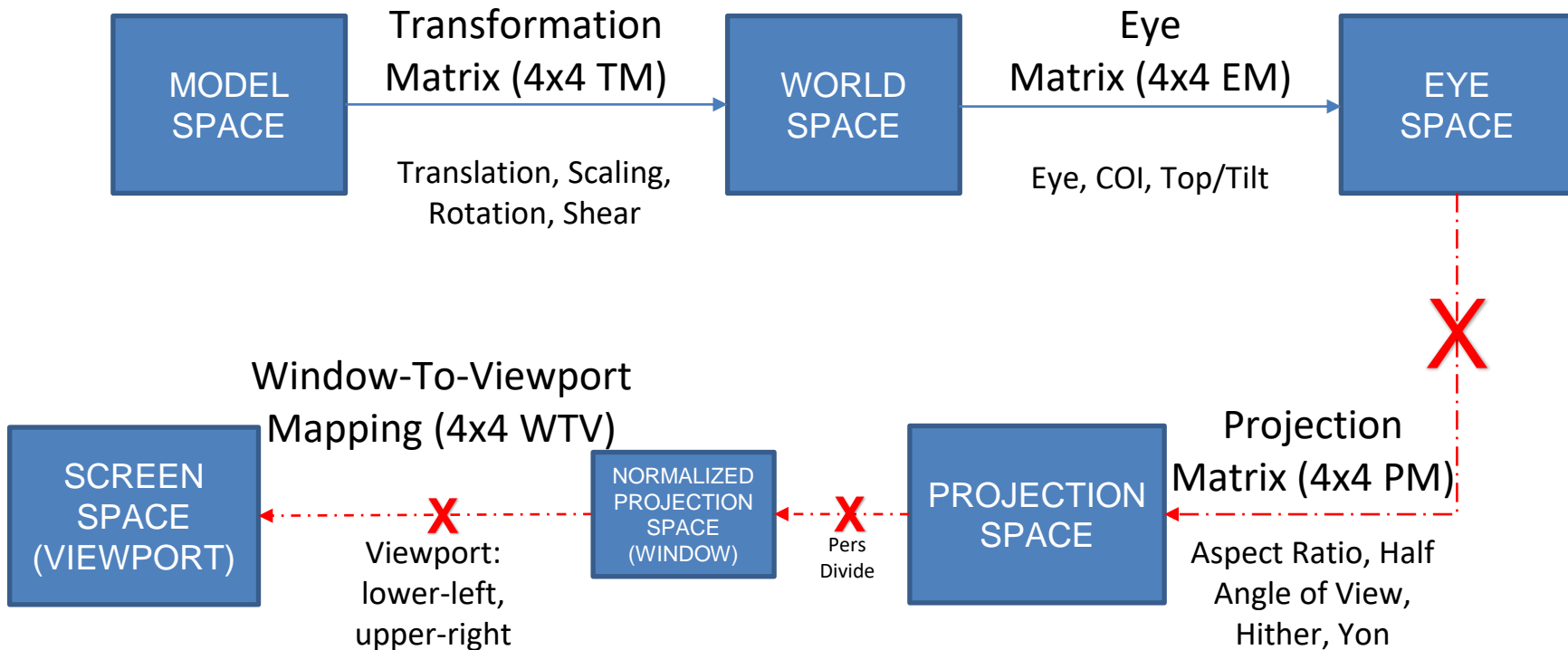
- *Ray Casting is a WS or ES visible surface algorithm*
- *Window is divided into a rectangular array of pixels*
- *Algorithm*

For each pixel in viewport

1. Generate ray emanating from Eye (O) through pixel
2. Find intersection between ray and objects
3. Pick intersection point closest to Eye (O)
4. Illuminate the closest intersection point
5. Plot pixel with illuminated color of closest object



# Ray Casting Pipeline





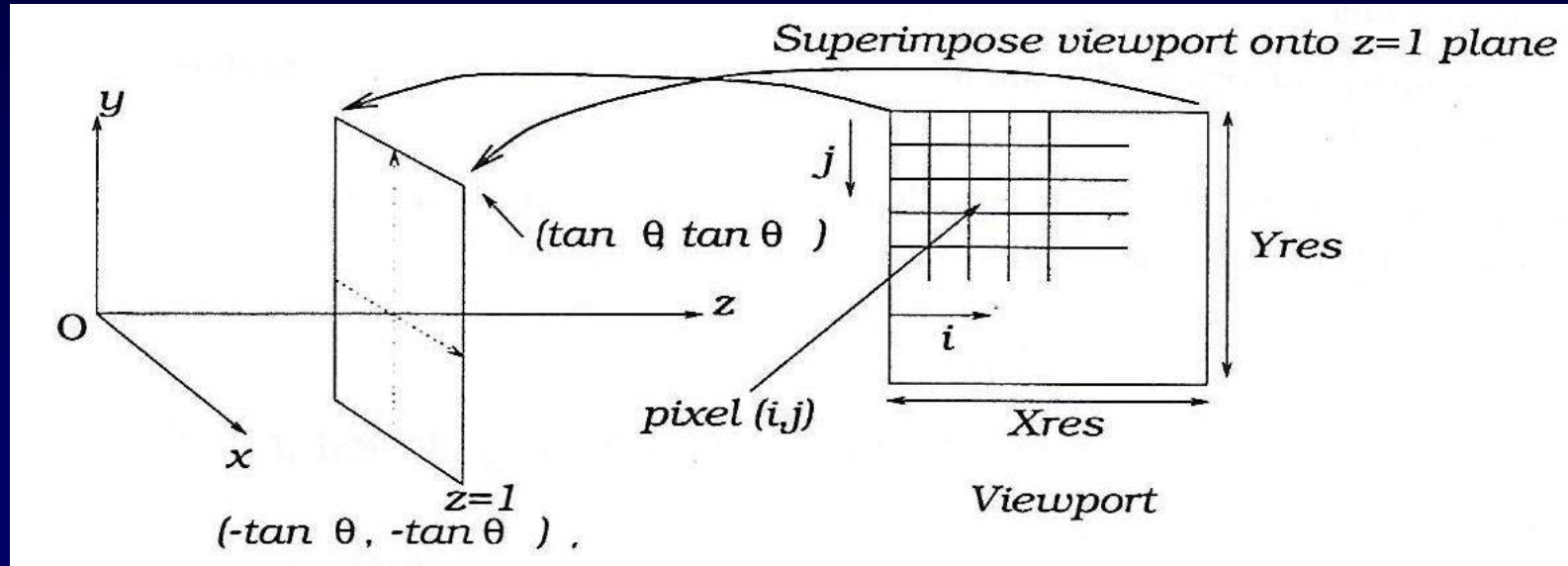
# Ray Casting Algorithm

**Step 1: Generate ray emanating from Eye (O) through pixel**

- **Superimpose a grid (representing viewport) onto the area defined by  $(-\tan\theta, -\tan\theta/A_r)$ ,  $(\tan\theta, \tan\theta/A_r)$  on  $z = 1$  plane**
- **Width of a pixel on the  $z = 1$  plane will be  $\frac{2\tan\theta}{X_{res}}$**
- **Point corresponding to center of pixel  $(i,j)$  is  $P = (x_p, y_p, 1)$**   
$$x_p = -\tan\theta + (i + 0.5) \frac{2\tan\theta}{X_{res}} \quad y_p = \frac{1}{A_r}(\tan\theta - (j + 0.5) \frac{2\tan\theta}{Y_{res}})$$
- **Parametric ray emanating from eye (origin) through pixel  $P$ :**
  - $x = x_e + t(x_p - x_e)$ ,  $y = y_e + t(y_p - y_e)$ ,  $z = z_e + t(z_p - z_e)$
  - For ES ( $x_e = y_e = z_e = 0$ ):  $x = tx_p$ ,  $y = ty_p$ ,  $z = t$

# Ray Casting Algorithm

*Step 1: Generate ray emanating from Eye (O) through pixel*



# Ray Casting Algorithm

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## *Step 2: Find intersection between ray and objects*

- a. Intersection test: if no intersection, skip part b
- b. Intersection calculation: for spheres and for polygons

# Ray Casting Spheres

- **For spheres**
  - a. Transform center of sphere to ES (radius remains same)
  - b. Equation of sphere:  $(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 - R^2 = 0$
  - c. Parametric equation of ray (ES):  $x = tx_p, y = ty_p, z = t$
  - d. Find intersection of ray with sphere: plug in eqn of ray in eqn of sphere  
 $At^2 + Bt + C = 0$ 
    - i. *No real solution  $\Rightarrow$  ray does not intersect sphere*
    - ii. *1 real solution  $\Rightarrow$  ray grazes sphere*
    - iii. *2 real solutions  $\Rightarrow$  entering & exiting points, pick lower +ve  $t$ , and calculate  $P$*
  - e. Find normal at intersection point  $P$ :  $N = P - P_c$
  - f. Illuminate using  $P, N, E$  of closest +ve  $t$

# Ray Casting Polygons

- **For polygons**
  - a. Transform poly to ES
  - b. Equation of the plane containing poly:  $Ax + By + Cz + D = 0$
  - c. Parametric equation of ray (ES):  $x = tx_p, y = ty_p, z = t$
  - d. Find intersection of ray with plane: plug in eqn of ray in eqn of plane
    - i. *If denom = 0, ray is parallel to plane  $\Rightarrow$  no intersection*
  - e. Check if intersection point P is contained inside the poly
    - i. *Project poly and point onto one of the primary planes (use max of N components)*
    - ii. *Use one of the “point inside poly” tests*
  - f. Find normal N at intersection point using bilinear interpolation
  - g. Illuminate using P, N, E of closest +ve t

# Ray Casting in World Space

- **Vectors**

- $V_z = \text{normalize}(\text{COI} - \text{OBS})$
- $V_x = V_z \times (0, 1, 0)$
- $V_y = V_x \times V_z$

- **Ray**

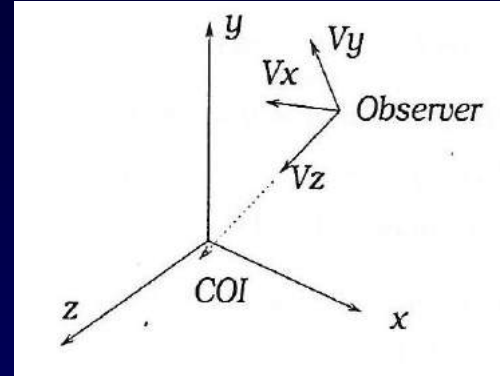
- Parametric equation:  $\text{OBS} + t(\text{P} - \text{OBS})$ , where P is given by

$$x_p = -\tan\theta + (i + 0.5) \frac{2\tan\theta}{X_{res}} V_x$$

$$y_p = \frac{1}{A_r} (\tan\theta - (j + 0.5) \frac{2\tan\theta}{Y_{res}} V_y)$$

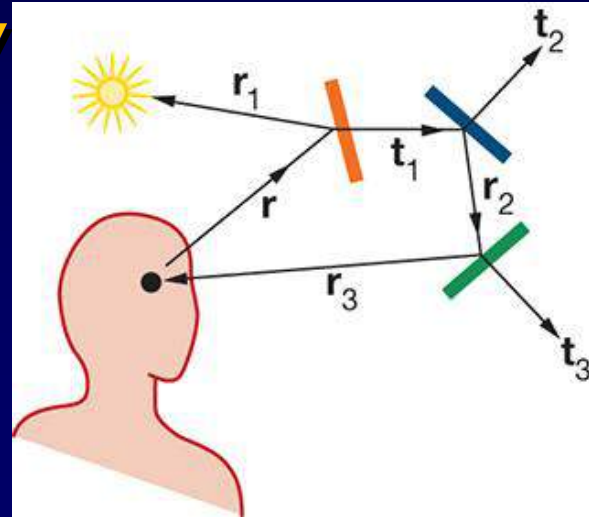
$$z_p = V_z$$

- Rest of calculations are similar to ray casting in ES



# Ray Tracing

- Also referred to as *Recursive Ray Tracing*
- Handles reflections, refractions, shadows
- Primary ray: ray from eye into the world
- Secondary rays: reflected, refracted, shadow
- Illumination for ray tree:  
 $I = \text{ambient} + \text{diffuse} + \text{specular} + k_r I_r + k_t I_t$   
 $k_r$ : coefficient of reflection  
 $k_t$ : coefficient of transmission
- Attenuate  $I_r$  and  $I_t$  by distance the ray travels



# Ray Tracing

- Reflected Ray Direction**

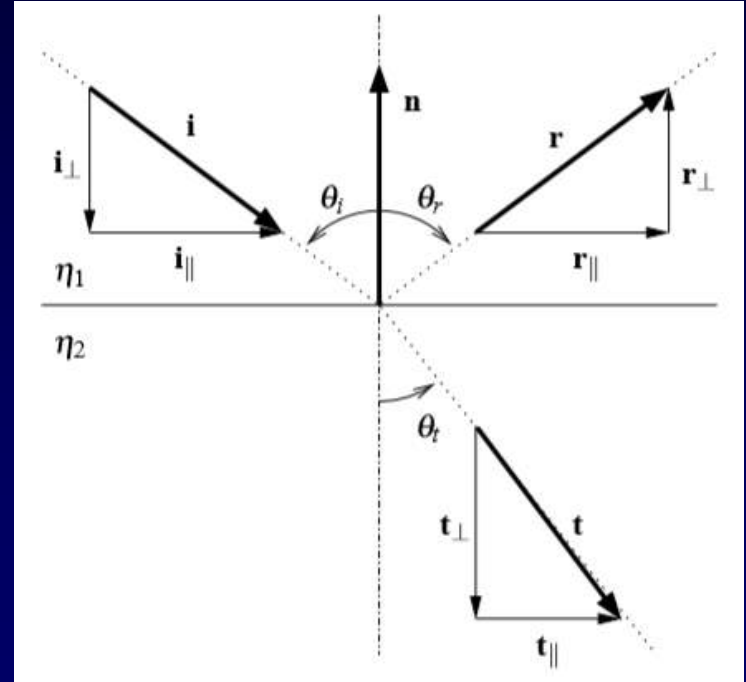
$$r = i - 2(i \cdot n)n$$

- Refracted Ray Direction**

$$\text{Snell's Law: } \eta_1 \sin \theta_i = \eta_2 \sin \theta_t$$

$$t = \frac{\eta_1}{\eta_2} i + \left( \frac{\eta_1}{\eta_2} \cos \theta_i - \sqrt{1 - \sin^2 \theta_t} \right) n$$

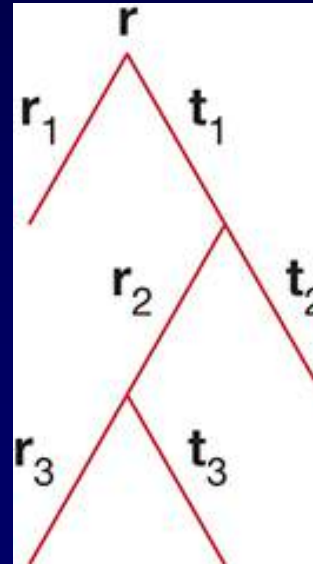
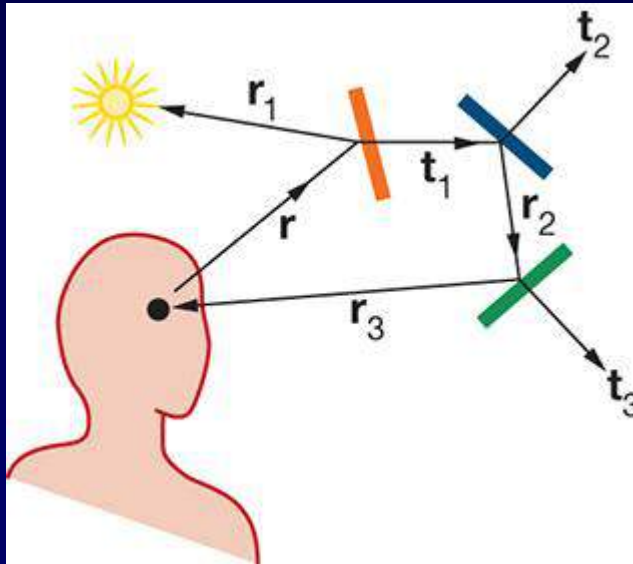
$$\sin^2 \theta_t = \left( \frac{\eta_1}{\eta_2} \right)^2 \sin^2 \theta_i = \left( \frac{\eta_1}{\eta_2} \right)^2 (1 - \cos^2 \theta_i)$$





# Ray Tracing: Illumination

- *Ray Tree*
  - Formed of primary, secondary, shadow rays
  - Evaluated bottom up



# Ray Tracing: Illumination

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- *Tree terminates if*
  - No intersection for a ray
  - Tree depth has reached a specified level
  - Intensity of  $I_r$  and  $I_t$  becomes very low
  - Ray has traveled a max distance

# Ray Tracing: Speed

- ***Efficiency Considerations***

- Total # of shadow rays spawned =  $m(2^n - 1)$   
 $m$  = # light sources;  $n$  = depth of ray-tree
- Total # of rays =  $(m + 1)(2^n - 1)$
- Back faces cannot be culled
- Clipping cannot be done for view volume or behind eye
- 75%-95% of time is spent in intersection calculations
- Use bounding box/sphere testing or hierarchies (octrees)