# CS174A Lecture 9

# Announcements & Reminders

- *10/26/22 and 10/27/22: Office hours, Noon – 1 PM PST, Zoom*
- *10/27/22: Midterm Exam: 6:00 – 7:30 PM PST, in person, in class*
- *11/08/22: Team project proposals due, initial version*
- *11/09/22: A3 due*
- *11/10/22: Midway demo, online zoom*

# Last Lecture Recap

- ***Rendering Pipeline:***
  - Model space
  - Object/world space
  - Eye/camera space
  - Projection space
  - Screen space

# Next Up

- ***Geometric Calculations***
- ***Midterm: Oct 27, 6:00-7:30 PM PST***
- ***Hidden Surface Removal***
  - Backface Culling
  - Object Space & Image Space Algorithms
- ***Lighting***
- ***Flat and Smooth Shading***

# Transforming Lines & Planes

Transforming Lines

- Given by 2 end points
- Given by line equation $y = mx + b$

Transforming Planes

- Given by 3 non-collinear points
- Given by a plane equation: $Ax + By + Cz + D = 0$
- Given by a normal and a point: $n_x(x - p_x) + n_y(y - p_y) + n_z(z - p_z) = 0$
- If $M_{point}$ is matrix to transform point, then $M_{normal} = (M_{point}^T)^{-1}$
- For orthogonal matrices, $M^T = M^{-1}$ implying $M_{normal} = M_{point}$

# Orthonormal Matrices

Properties of Orthonormal Matrices

- Consider upper-left 3x3 matrix
- Each row is a unit vector
- Each row is orthogonal to the others, i.e., their dot product = 0
- These vectors can be rotated to align with the xyz-axis
- Determinant = 1
- Inverse of orthogonal matrix $M^{-1} = M^T$
- Preserves angles and lengths => rigid-body transformations
- Examples of rigid-body transformations: translations, rotations

$$\mathrm{M} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Geometric Calculations

- **Point in Polygon Test**

i. Convex polys only: if point lies to the left of ALL edges, then inside; if it lies to the right of even one edge, then outside

ii. Semi-infinite ray
($y_1 > y_0$ and $y_2 \le y_0$) or ($y_1 \le y_0$ and $y_2 > y_0$) where $y_0$ is the middle vertex of 3 consecutive vertices
Intersection point (x,y): $x > x_0$ for true intersection with semi-infinite ray to the right

iii. Angle summation
If directed angle sum = 0, then outside, else inside

- **Normal Vector**

i. 3 consecutive vertices (convex vertices): find cross product

ii. Summation method
$(\sum(y_i - y_j)(z_i + z_j), \sum(z_i - z_j)(x_i + x_j), \sum(x_i - x_j)(y_i + y_j))$
where j = (i+1) mod n; n = total number of vertices

# Geometric Calculations (Contd)

- **Plane Equation**

i. Surface normal and distance from origin

$n_x x + n_y y + n_z z = d$

ii. 3 points on plane

$n_x(x – x_i) + n_y(y – y_i) + n_z(z – z_i) = 0$

- **On-Line Test**

P is on $P_1P_2$ means $\frac{x - x_1}{y - y_1} = \frac{x_2 - x_1}{y_2 - y_1}$

If $T_{1,2}(P) = (x – x_1)(y_2 – y_1) – (x_2 – x_1)(y – y_1)$

if +ve, P is on the right; if -ve, P is on the left

- **Edge-Edge Intersection**

$P_1$ and $P_2$ are on opposite sides of line defined by $P_3P_4$ and

$P_3$ and $P_4$ are on opposite sides of line defined by $P_1P_2$

Equivalently, check for intersection:

$(T_{1,2}(P_3) * T_{1,2}(P_4) < 0)$ and $(T_{3,4}(P_1) * T_{3,4}(P_2) < 0)$

# Geometric Calculations (Contd)

- **Collinearity Test**

  $t$ = distance from point P to line $P_1P_2$
  $\theta$ = angle between $P_1P$ and $P_1P_2$

  $$t = |P_1P|\sin\theta = \frac{|P_1P||P_1P_2|sin\theta}{|P_1P_2|} = \frac{|P_1P \text{ X } P_1P_2|}{|P_1P_2|}$$

  if $t < \epsilon$, P is considered to be on $P_1P_2$