BOND VALUATION:

When a corporation or a government has a project, the necessary funds are usually acquired by issuing a bond. A bond is a legally binding agreement between a borrower and a lender that specifies the loan amount, the duration and the amount to be paid back over time.

Face (par) Value: The amount to be paid at the end of the loan. This is a lump sum amount and does not change over the life of the bond.

Time to maturity: The number of years until the face value is paid.

Coupon Rate: (Annual coupon payment)/(Face Value)

After the bond is issued and sold, it becomes an asset to the holder. The bond holder can keep it until the maturity date or sell it to another investor before that. Once the bond is issued, the issuer has nothing to do with these subsequent transactions. In this way, bonds are reminiscent of stocks. Once they are issued, they can be bought and sold in the secondary market. Most bonds are issued with a promise of coupon payment. Hence most bonds produce a cash payment periodically, and promise to pay a lump sum at the end of the maturity. The value of such an asset is determined (like any other asset) by properly discounting the cash flow it will supply during its lifetime. Note that the par(face) value is NOT the price of the bond. Par stays constant over the life of the bond whereas the price changes as the market conditions change!!!

There are also bonds, called "pure discount bonds", that have no coupon payments but only the face value payment at the maturity date.

Example: You are offered a 10-year bond with $1,000 face-value and 8% coupon rate today. The first coupon payment will occur a year from today. If in the market, you would get 8% for another asset with comparable risk, would you buy this bond at $1,000 today?

If the price ($1,000) is less than or equal to the present value (PV) of the cash payments, you would be willing to invest your money (buy the bond). Let's calculate the PV of this bond, as of today (time 0):

$$PV_0 = \frac{80}{1.08} + \frac{80}{(1.08)^2} + \ldots + \frac{1080}{(1.08)^{10}} = 80\ A_{.08}^{10} + \frac{1000}{(1.08)^{10}} = \$1{,}000$$

where $A_{.08}^{10}$ is the annuity factor. The present value exactly coincides with the price of the bond. Then, you are indifferent between putting money on this asset or on an alternative investment opportunity. In other words, the alternative opportunities in the market offer as good a return as this bond.

Let's assume that a year passed after the issuance of the bond. Let's also assume that interest rate in the market is still the same!!! What is the value of the bond now? The remaining cash flow from the bond is 9 coupon payments of $80 each and the face value of $1,000 to be paid on the maturity date. Then PV is:

$$PV_1 = \frac{80}{1.08} + \frac{80}{(1.08)^2} + \ldots + \frac{1080}{(1.08)^9} = 80\ A_{.08}^{9} + \frac{1000}{(1.08)^9} = \$1{,}000$$

Therefore, the price of the bond will not change as long as the market conditions do not change. In other words, regardless of the time left until maturity, the price of the bond will be the same as long as the alternatives continue to offer the same interest rate. You can check this for yourself by calculating the present value of the cash stream the bond offers as it gets closer and closer to the maturity.

If instead, the market rate after a year were to change from 8% to, say, 10%, you would expect to have a change in the price of the bond so as to offer the same return as the market. Remember, the face value, the coupon rate, and the maturity date are fixed. Hence, if you wanted to sell this bond as an investment opportunity to another investor, the only way you can offer the now-higher 10% market rate would be to offer to sell it at a lower price. So intuitively, we know that the price of the bond should go down if the interest rate in the market goes up. But, exactly how much should the price be? We can calculate the present value of the cash stream that is offered by the bond using the new interest rate in the market:

$$PV_1 = \frac{80}{1.10} + \frac{80}{(1.10)^2} + \ldots + \frac{1080}{(1.10)^9} = 80\ A^9_{.10} + \frac{1000}{(1.10)^9} = \$884.82$$

As the market opportunities get better, an investor would be willing to pay $884.82 (or less) on this bond to be able to get the same (or better) return as the market.

Now!!!!! If one can buy this bond today (time 1) at $884.82, then the YIELD she would get will be 10%!!! Therefore, even though the coupon rate stays the same; as the price of the bond changes, the YIELD (or YTM, yield to maturity) it offers will change as well.

Note that, if I wished to sell a brand new bond in this market, with $1,000 face value and $1,000 price, and 9 years to maturity, I would need to offer $100 coupon payments, that is, $20, more cash flow than the original bond to match the market rate.

If you calculate the present value of this $20 extra cash stream:

$$PV_1 = \frac{20}{1.10} + \frac{20}{(1.10)^2} + \ldots + \frac{20}{(1.10)^9} = 20\ A^9_{.10} = \$115.18$$

Note that this is exactly by how much you need to reduce the price of the original bond to make it as attractive as the alternatives: $1,000 - $115.18 = $884.82.

Conversely, if the market rates go down, from the original 8%, say, to 6%, then the cash stream that is offered by the original bond is more attractive than the alternatives. This means that you can sell this bond at a higher price than its original $1,000 price. By how much higher? You can again calculate the present value of the cash stream that is offered by the bond using the now-lower interest rate:

$$PV_1 = \frac{80}{1.06} + \frac{80}{\left(1.06\right)^2} + \ldots + \frac{1080}{\left(1.06\right)^9} = A^9_{.10} + \frac{1000}{\left(1.06\right)^9} = \$1{,}136.03$$

There is an inverse relationship between the price of a bond and the yield it offers!

If a bond sells at the same or lower or higher price than its face value, it is called a par, or a discount, or a premium bond respectively.

Also note that, for a discount bond, YTM>coupon rate. For a premium bond YTM<coupon rate. For a bond trading at par, the yield and the coupon rate are the same.

The general formula for calculating the value(price) of a bond is:

$$\text{Bond Value} = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right] + \frac{F}{(1+r)^T}$$

Where C is the coupon payment, r is the discount rate (the market rate), F is the face value and T is the time to maturity.

Example: What does a bond that sells for $1,136.03 now yields if the annual coupon payment is $80, face value is $1,000, and time to maturity is 9 years?

$$1{,}136.03 = \frac{80}{r}\left[1 - \frac{1}{(1+r)^9}\right] + \frac{1{,}000}{(1+r)^9}$$

Solving for r, we get the yield as 6%.

In the US, the coupon payments are usually made semi-annually. Hence, the value of the bond needs to be adjusted to take this into account.

Example: What is the value(price) of a bond that offers 14% coupon rate, $1,000 face value, and maturity time of 7 years if the market rate is 16% and coupon payments are made semi-annually?

$$\frac{[\frac{140}{2}]}{[\frac{0.16}{2}]}\left[1 - \frac{1}{\left(1+[\frac{0.16}{2}]\right)^{7x2}}\right] + \frac{1{,}000}{(1+[\frac{0.16}{2}])^{7x2}} = \$917.56$$

Suppose I have two different bonds issued by two different corporations. The riskiness are the same as far as the timely payments are concerned, that is, the default risks are the same. Do these bonds otherwise bear the same risk? Not necessarily.

If both offer 10% coupon rate and $1,000 face value but they differ in their time to maturity, one being 1 year, and the other being 30 years, the price of the long-term bond will be more sensitive to any market rate changes then the short term bond. If the market rate is 10%, the value of either bond is $1,000. If the market rate goes down to 5%, the 1-year bond's price will increase to $1,047.62, while the 30-year's price will shoot up to $1,768.62. You can expect to see the same kind of discrepancy if the market rate were to increase. This is called the interest rate risk. This is one of the reasons why longer term bonds offer higher yields. Investors demand a compensation for bearing this risk.

On the other hand, two bonds offering different coupon rates, with otherwise equivalent properties, will also have different interest rate risk. The one with higher coupon payments will have a lower interest rate risk than the one with a lower coupon payment. With a lower coupon bond, the payments are relatively pushed to farther in the future. The intuition behind this is: Any time you get a payment later than sooner, you are more exposed to interest rate risk. Remember that the discounting occurs at a higher degree for payments that are going to be made at a later date!

On the other side of the coin, you have what is called a reinvestment risk. With a shorter maturity bond, you will get your money back earlier than with a longer maturity bond. If you cannot find an investment opportunity as attractive as before then you maybe reinvesting at a lower rate. The opposite of what happens with interest rate risk occurs with reinvestment risk. For otherwise equivalent bonds, longer maturity bears less reinvestment risk; and for otherwise equivalent bonds lower coupon bears less reinvestment risk.

Example:

A century bond was offered by Bell-South in 1995 with a maturity date of 2095. The interest rate risk manifests itself in the price change of the bond over the years:

Issue Date	Maturity Date	Coupon Rate	Price in 1995	Price in 1996	Price in 2007
1995	2095	7%	$1,000	$800	$1020.29

For any bond the yield to maturity (YTM) can be divided into two components: current yield and capital gains yield.

Current Yield = Annual coupon payment / Current price of the bond

Capital Gains Yield = YTM – Current Yield.

For a discount bond Current Yield < YTM, and for a premium bond the opposite is correct.

Example: Consider a 6-year bond with 8% coupon rate with par $1,000. If the market rate is 9% the price is $955.14. Then the current yield is 80/955.14 = 8.38%, and the capital gains yield is 9%-8.38%=.62%

Not only corporations but also the federal and various local governments can issue bonds to finance all sorts of projects. The bonds that are issued by the federal government are considered to be free-of-default-risk. When we look for a risk-free rate to use as a benchmark, we look at the treasury rates. Treasury (federal government) offers three kinds of bonds that can be directly bought at treasurydirect.gov :

1. Treasury Bills: maturity less than a year

2. Treasury Notes: maturity between 1 and 10 years

3. Treasury bonds: maturity between 10 and 30 years.

Unlike Treasuries, corporate bonds carry some default risk. Therefore the yield that a corporate bond offers will be the promised yield. The expected yield will be lower depending on the risk level of the bond.

Example: Consider a 1-year bond with $80 coupon and $1,000 face value both of which will be paid one year from today. The promised YTM is 11.9%. The expected average return you get in the market is 9%. What is the default rate (the probability with which the issuer of the bond will default) if you are expected to be paid only $800 if there's a default.

There are two ways to think about this problem.

- If the company does not default, you will be paid $1080 in 1 year. The present value of that value is 1080/1.09=990.83. In other words, if you had $990.83 you could get $1080 at the market in one year. If the company defaults you will get $800 in one year, present value of which being 800/1.09=733.94. If the probability of default is p, the expected value of this bond is p*733.94+(1-p)*990.83. The price you pay for the bond today is 1080/1.119=965.14 which is the expected value of the bond. When you equate these two values, you get p=10%.

- Another way to think about this problem is in terms of future values rather than present values. Under no default, you get $1080 in one year and under default you get $800. What is the expected value you expect to get in the future? The money you put down now plus what you can get at the market rate. The bond is priced at 1080/0.119=965.14. This is the money you pay today. On average you expect to get as much as market pays, which is 965.14*(1.09)=1052. Then, p*800 +(1-p)*1080 = 1052, which gives p=10%.

Since default rate is quite important for an investor, corporations pay to independent companies which are trusted by the market in general, to evaluate and rate their bonds. The biggest rating agencies are Standard&Poors, Moody's and Fitch.

Determinants of Bond Yields:

1. Term Structure of Interest: The longer the maturity, the higher the interest risk and higher the extra yield (interest rate risk premium) that is demanded to bear this risk.

2. Rate of Inflation: On average there would be an expectation for the prices to rise and the purchasing power of a dollar to go down in the future. The extra yield that is demanded is called inflation risk premium.

3. Default Risk Premium: extra return that is demanded for the possibility of default.

4. Liquidity Premium: Not all bonds are as liquid as treasuries. For example, it may be hard to sell the bond of a small, unknown company. To compensate for that, the company may offer extra return.

5. Taxability Premium: If a particular bond is taxed at a higher rate, the issuer would offer a premium for that.

The Yield Curve:

The yield curve represents the relationship between the maturity of treasuries and their respective interest rates. You can see the current yield curve here:

http://www.bloomberg.com/markets/rates-bonds/government-bonds/us/

This curve has a positive slope in a normal economy.

If the yield curve inverts, it is widely regarded as a signal for a coming recession.

Here is a link to an older article that is talking exactly about this:

http://money.cnn.com/2011/08/23/markets/bondcenter/treasuries_yield_curve/index.htm

Example: Define what is meant by interest rate risk. Assume you are the manager of a $100 million portfolio of corporate bonds and you believe interest rates will fall. What adjustments should you make to your portfolio based on your beliefs?

Interest rate risk is the risk that arises for bond owners from fluctuating interest rates. All else the same, if interest rates are expected to fall you should purchase long-term bonds and/or low coupon bonds, and sell shorter-term, higher-coupon bonds.