

CS174A Lecture 8

Announcements & Reminders

- *10/26/22 and 10/27/22: Office hours, Noon – 1 PM PST, Zoom*
- *10/27/22: Midterm Exam: 6:00 – 7:30 PM PST, in person, in class*
- *11/08/22: Team project proposals due, initial version*
- *11/09/22: A3 due*
- *11/10/22: Midway demo, online zoom*

TA Session This Friday

- ***Team project***
 - First draft of proposal due: 11/08/22
 - What's expected in the proposal
 - Still looking for teammates? Resolve this Friday
- ***Project assignment #3, due 11/09/22***
- ***Midterm review***

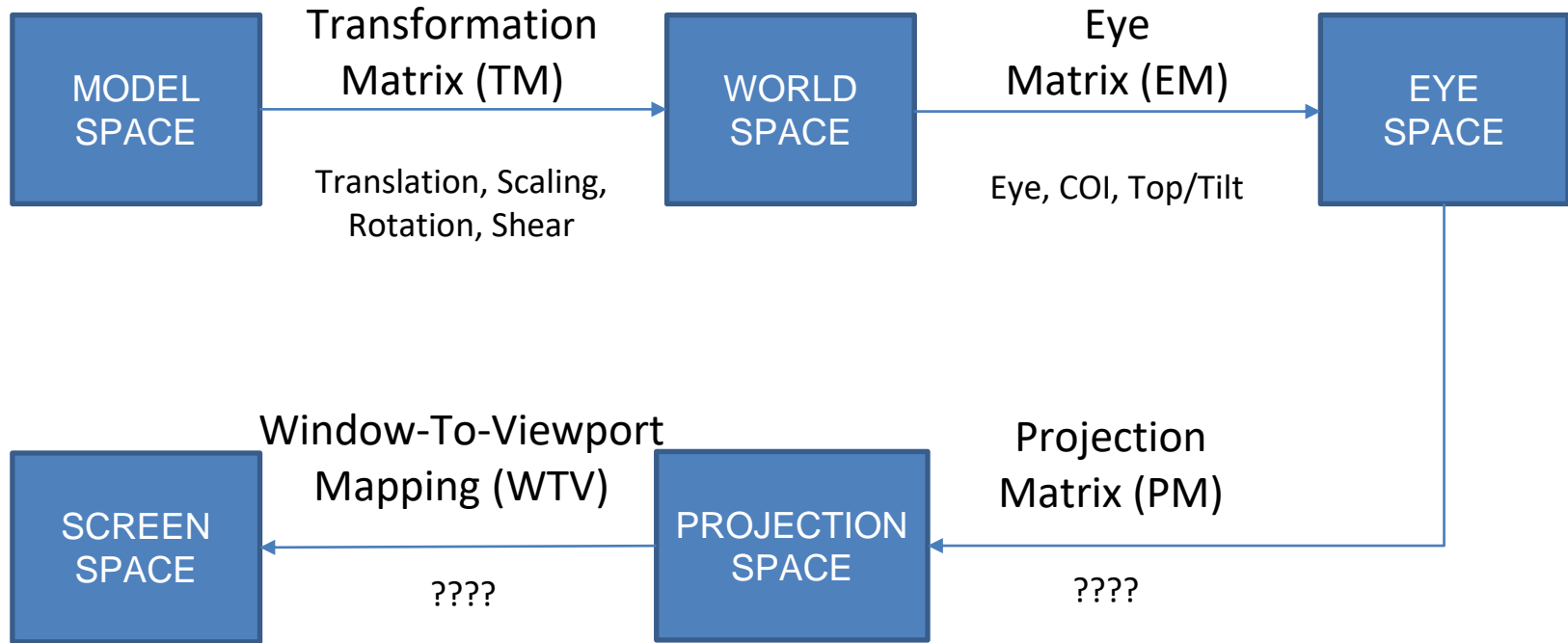
Last Lecture Recap

- ***Spaces:***
 - Model space
 - Object/world space
 - Eye/camera space
 - Projection Space
 - Screen space

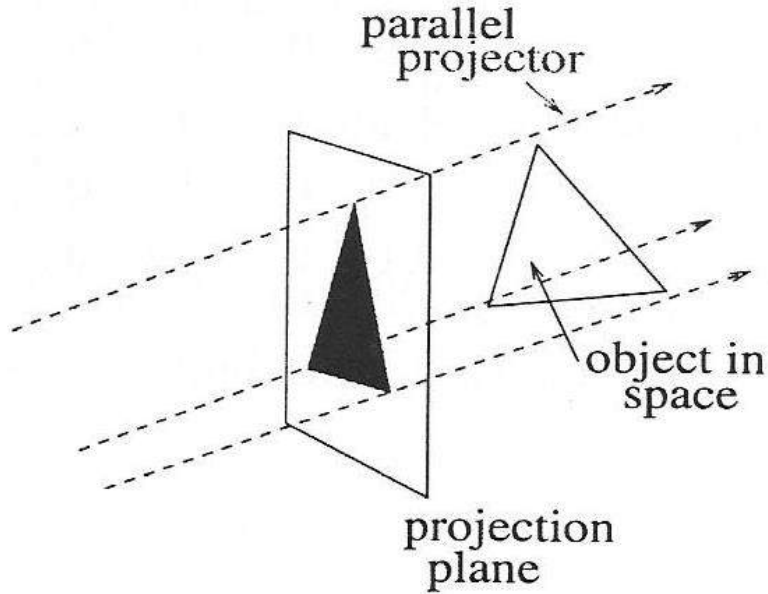
Next Up

- ***Projections: parallel and perspective***
 - Parallel and perspective view volumes
 - Canonical (normalized) view volume
- ***Geometric Calculations***
- ***Midterm***
- ***Hidden Surface Removal***
 - Backface Culling
- ***Lighting***
- ***Flat and Smooth Shading: introduction***

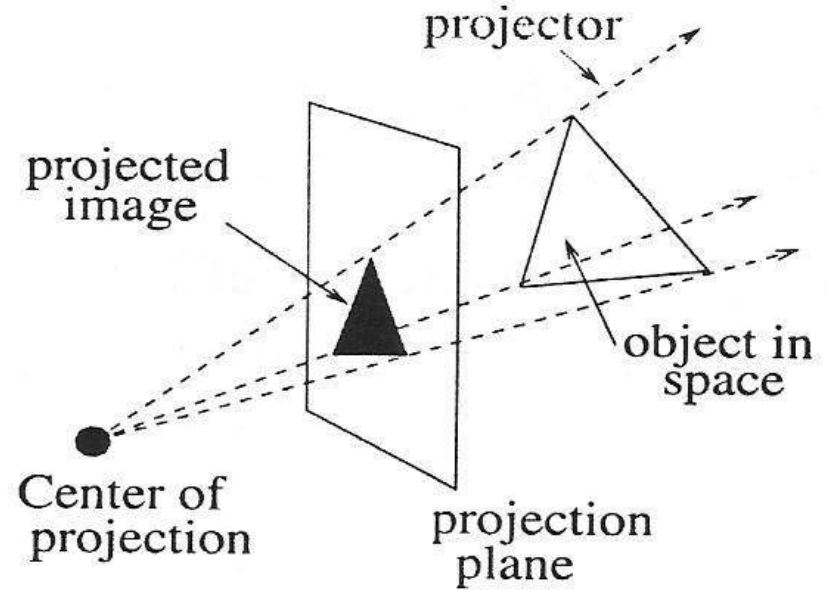
Rendering Pipeline



Orthographic Projections



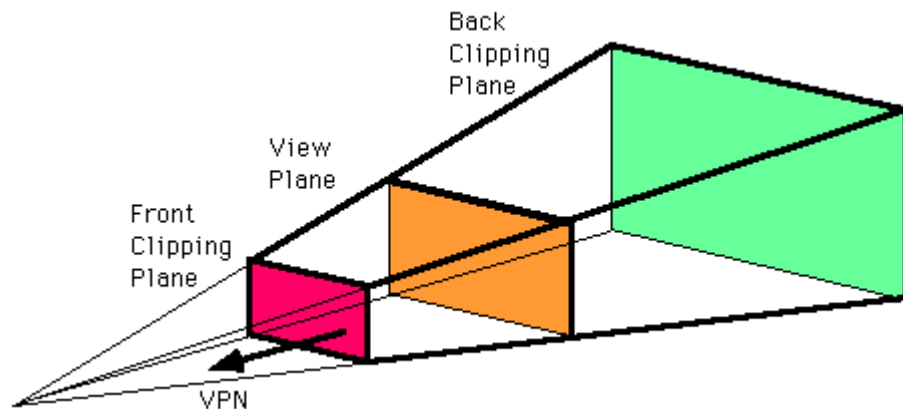
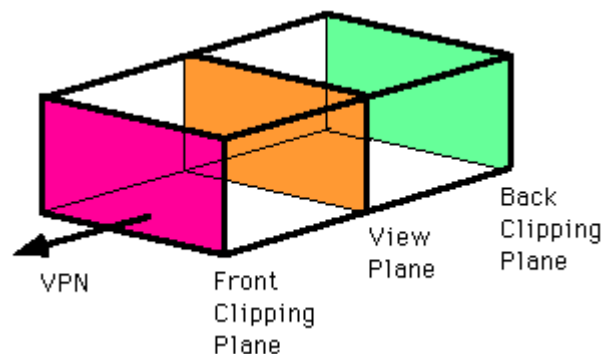
Parallel projection



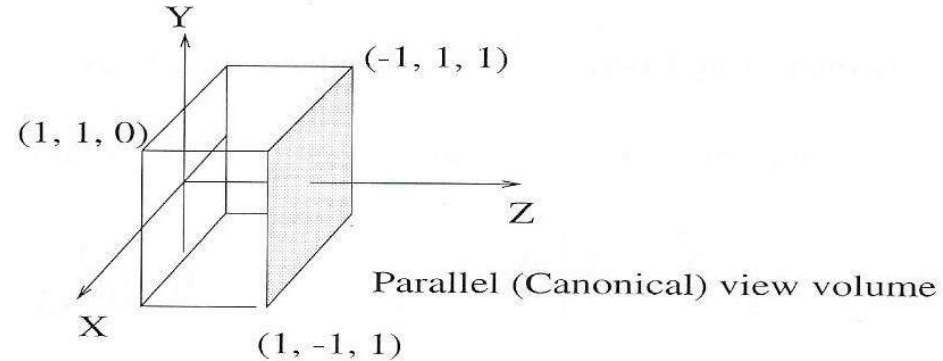
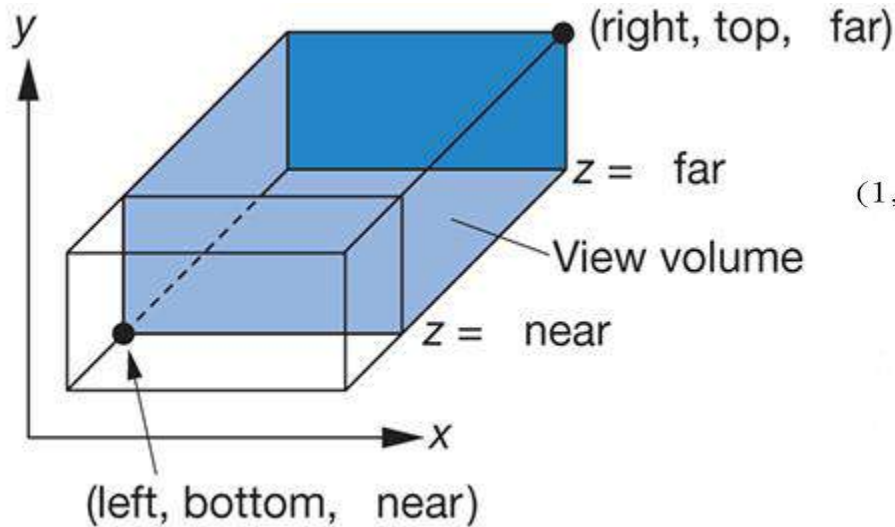
Perspective projection

Orthographic Projections

View Volumes



Parallel Projection



$$\text{Parallel PM} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

View Volume

$$-\frac{W}{2} \leq X \leq \frac{W}{2}$$

$$-\frac{H}{2} \leq Y \leq \frac{H}{2}$$

$$N \leq Z \leq F$$

Parallel Projection

$$\text{Parallel PM} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

View Volume

$$-\frac{W}{2} \leq X \leq \frac{W}{2}$$

$$-\frac{H}{2} \leq Y \leq \frac{H}{2}$$

$$N \leq Z \leq F$$

$$\text{Normalized Parallel PM} = \begin{bmatrix} \frac{2}{W} & 0 & 0 & 0 \\ 0 & \frac{2}{H} & 0 & 0 \\ 0 & 0 & \frac{1}{F-N} & -\frac{N}{F-N} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

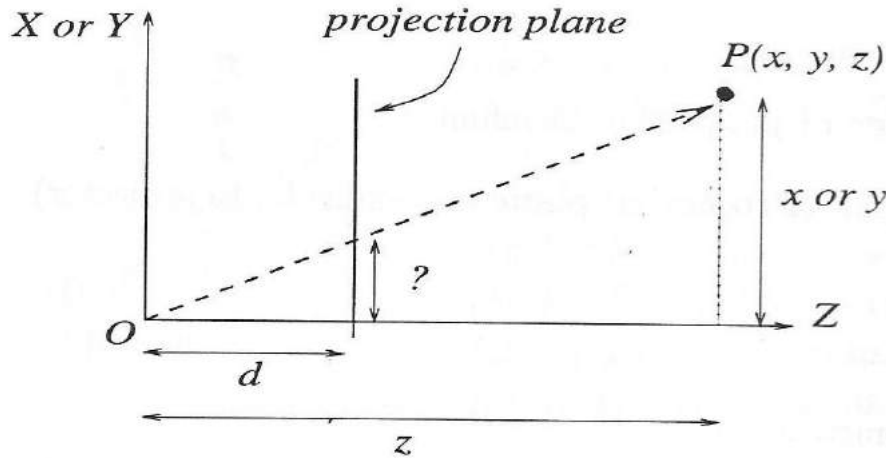
Canonical View Volume

$$-1 \leq X' \leq 1$$

$$-1 \leq Y' \leq 1$$

$$0 \leq Z' \leq 1$$

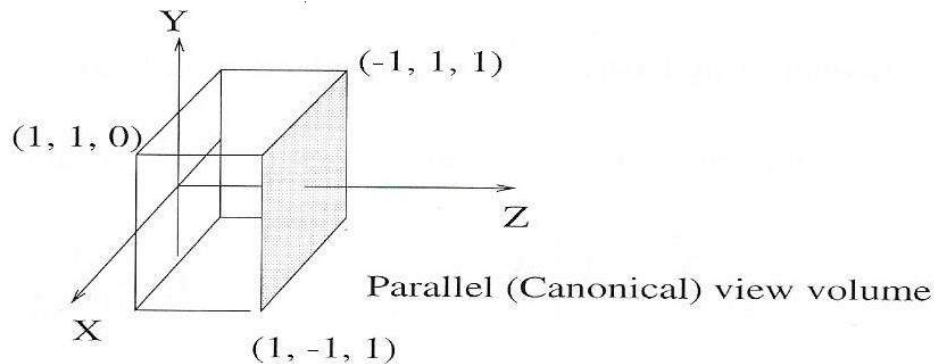
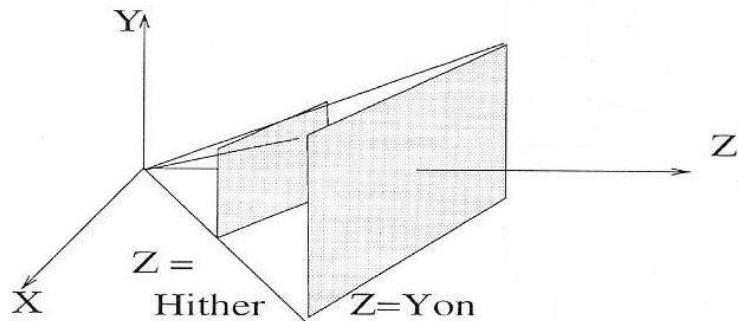
Perspective Projection



$$x \leftarrow \frac{x}{z}d; \quad y \leftarrow \frac{y}{z}d$$

Perspective PM (square viewport) =
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d} & 0 \end{bmatrix}$$

Perspective Projection



Aspect Ratio (A_r) = $\frac{W}{H}$

Half Angle of View = θ

θ is defined wrt to x-axis

$$\text{Normalized PPM} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & A_r & 0 & 0 \\ 0 & 0 & A \tan(\theta) & B \tan(\theta) \\ 0 & 0 & \tan(\theta) & 0 \end{bmatrix}$$

Perspective Projection

$$\text{Normalized PPM} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & A_r & 0 & 0 \\ 0 & 0 & A \tan(\theta) & B \tan(\theta) \\ 0 & 0 & \tan(\theta) & 0 \end{bmatrix} \quad \begin{aligned} A &= \frac{F}{F - N} \\ B &= -\frac{N * F}{F - N} \end{aligned}$$

Apply Perspective Division

How to handle -ve values of w? What does it mean?

As examples,

- Lower-left-near vertex of view volume (in eye space) with coordinates: $(-N * \tan(\theta), -N * \tan(\theta) / A_r, N)$ will map to $(-1, -1, 0)$ after pers div
- Upper-right-far vertex of view volume with coordinates: $(F * \tan(\theta), F * \tan(\theta) / A_r, F)$ will map to $(1, 1, 1)$ after pers div

Window-to-Viewport Mapping

Change from normalized volume (xyz) to screen coordinates (XY)

xyz: normalized point after perspective division

XY: screen coordinates

v_l, v_b : lower-left corner of viewport

v_r, v_t : upper-right corner of viewport

$$\text{WTV Matrix} = T((v_l+v_r)/2, (v_b+v_t)/2) \cdot S((v_r-v_l)/2, (v_t-v_b)/2) \cdot T(0,0)$$

$$X = x \frac{v_r - v_l}{2} + \frac{v_r + v_l}{2}$$

$$Y = y \frac{v_t - v_b}{2} + \frac{v_t + v_b}{2}$$

In general (window= $x_{\min}, y_{\min}, x_{\max}, y_{\max}$; viewport= $u_{\min}, v_{\min}, u_{\max}, v_{\max}$)

$$\text{WTV Matrix} = T(u_{\min}, v_{\min}) \cdot S\left(\frac{u_{\max} - u_{\min}}{x_{\max} - x_{\min}}, \frac{v_{\max} - v_{\min}}{y_{\max} - y_{\min}}\right) \cdot T(-x_{\min}, -y_{\min})$$

Rendering Pipeline

