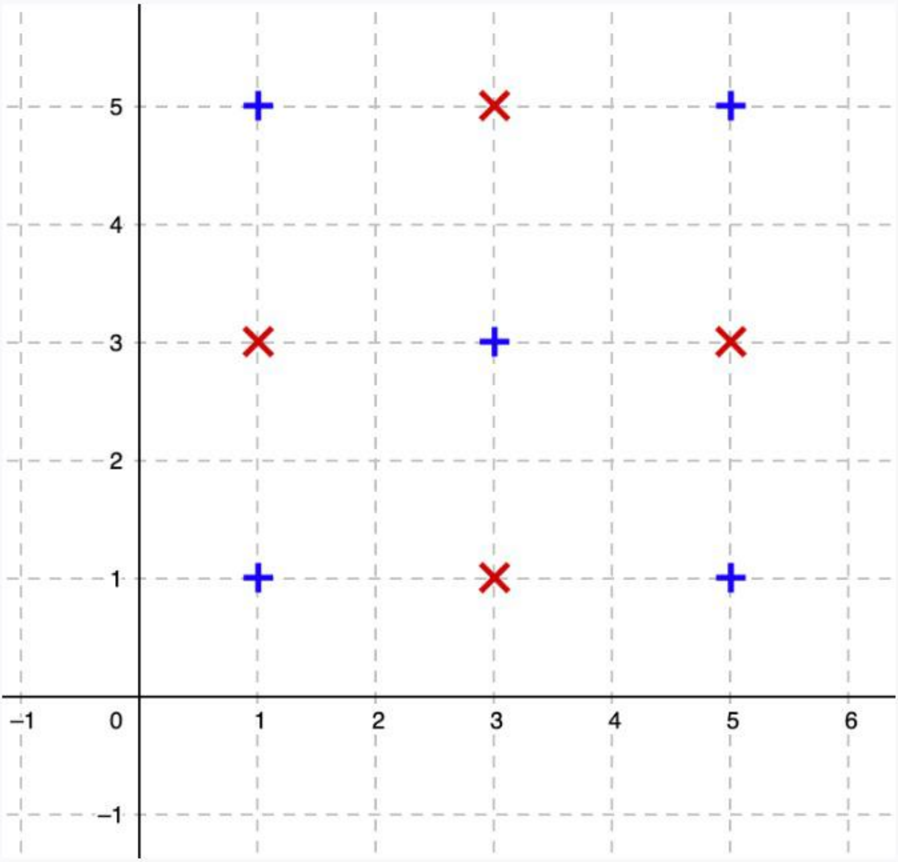
## Q2 KNN
12 Points

Consider the following training dataset, what is the leave-one-out validation accuracy (i.e., accuracy computed using 9-fold cross-validation) for the following classifier?




### Q2.1
4 Points

1-NN with Euclidean distance.
(Round your answer to two decimal places)

Enter your answer here

Save Answer

### Q2.2
4 Points

3-NN with Euclidean distance
(Round your answer to two decimal places)

Enter your answer here

## Q2.3
4 Points

5-NN with Euclidean distance
(Round your answer to two decimal places)

Enter your answer here

Save Answer

## Q3 Decision tree
22 Points

When training deep neural networks, out-of-memory errors often happen, depending on factors such as model size, batch size, and the quality of implementations. We will use the following dataset to learn a decision tree that predicts if an out-of-memory error will happen, based on 3 attributes (batch size, network depth, and the implementation version).

| Batch size | Depth | Implementation | Out-of-memory? |
|---|---|---|---|
| small | Deep | A | No |
| small | Shallow | B | No |
| small | Medium | B | Yes |
| large | Shallow | A | No |
| large | Medium | A | Yes |
| large | Shallow | B | Yes |
| large | Deep | B | Yes |

In case that more than one attribute has equal information gain, the priority of choosing the attributes is ordered as Batch size > Depth > Implementation.

You may use the following formula.
Entropy: $H(p) = -(p \log_2 p + (1-p)\log_2(1-p))$

$H(0) = 0; H(1/2) = 1; H(1/3) = 0.933; H(1/4) = 0.8;$
$H(1/5) = 0.7; H(1/7) = 0.571; H(3/7) = 0.971$
You may also use $\log_2 3 = 1.6$, $\log_2 5 = 2.3$, and $\log_2 7 = 2.8$

Please round your answer to 2 decimal places.

## Q3.1 Decision Tree

6 Points

What is the entropy of $\mathcal{H}$(out-of-memory)?

$\mathcal{H}$(out-of-memory)=

Enter your answer here

Save Answer

## Q3.2

6 Points

What is the information gain if we partition the data on the attribute **Implementation**?
Information Gain (Implementation) =

Enter your answer here

Save Answer

## Q3.3

6 Points

Suppose we learn a decision tree by the ID3 algorithm. What is the attribute used for the first split?

- ○ Batch Size
- ○ Depth
- ○ Implementation

Save Answer

## Q3.4

4 Points

Based on the learned decision tree learned from ID3, what is the prediction for an input with: Batch size=small, Depth=Deep, Implementation=B?

- ○ Yes
- ○ No

Save Answer

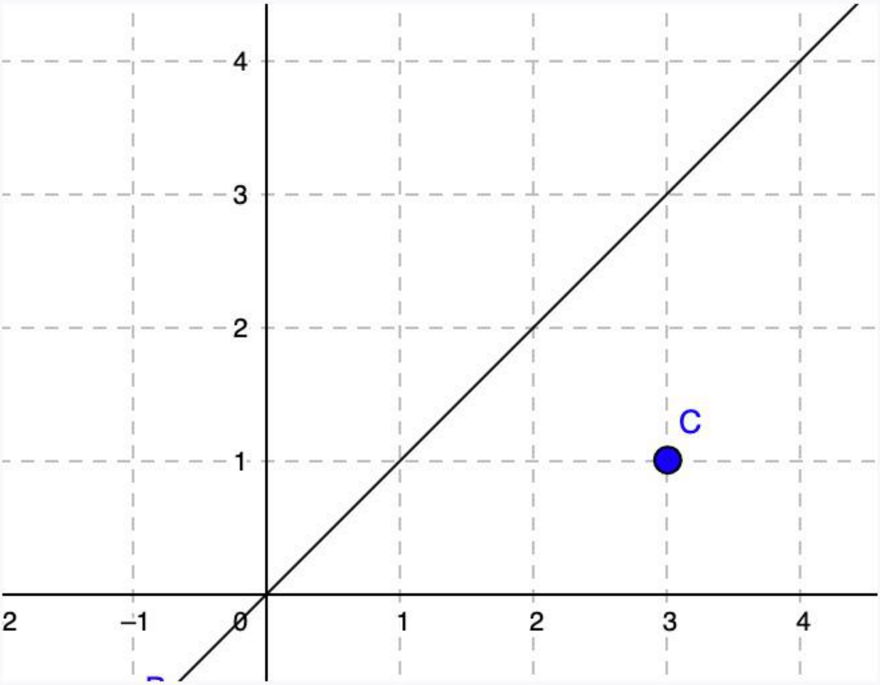
## Q4 Margin

19 Points

### Q4.1

5 Points

Consider the following data point and the hyperplane.




What is the distance between the point C to the line in L2 (euclidean) distance? Round your answer to 2 decimal places.

1.41

Save Answer *Unsaved Changes

### Q4.2

5 Points

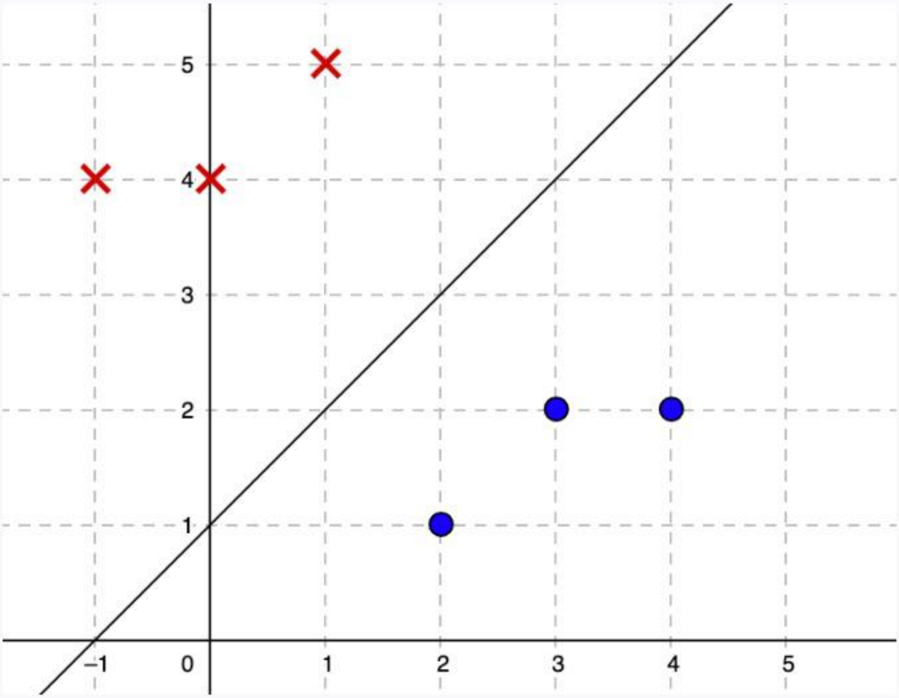
Follow the previous question. What is the distance between the point C to the line in L1 (manhattan) distance? Round your answer to 2 decimal places.

2.00

Save Answer *Unsaved Changes

## Q4.3

5 Points




What is the margin of the hyperplane in L2 (euclidean) distance? Round your answer to 2 decimal places.

Enter your answer here

Save Answer

## Q4.4

4 Points

Follow the previous question. Is there another hyperplane with a larger margin in L2 (euclidean)?

- Yes
- No

Save Answer

## Q5 Maximum Likelihood Estimation

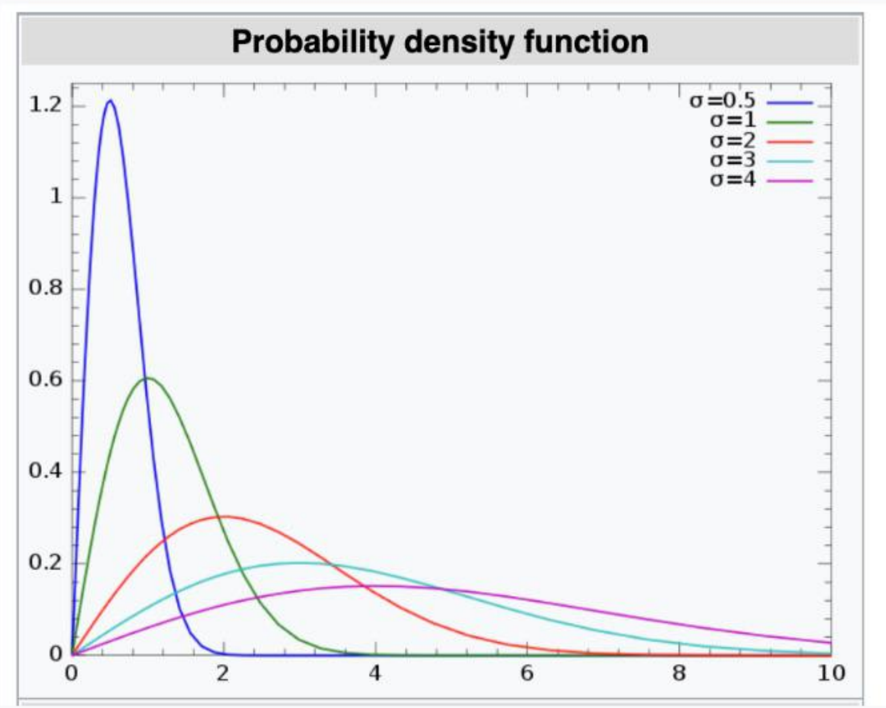
22 Points

In the following questions, we will explore how to estimate the lifetime of a capacitor. The age of a capacitor depends on various factors and we have encoded them into numerical values: $\boldsymbol{x}$. Given a bunch of capacitors drawn i.i.d. from the underlying distribution, their age and the factors, we create a training data set $\{(\boldsymbol{x}_i, y_i)\}_{i=1}^{N}$ where $\boldsymbol{x}_i \in \mathbb{R}^d$ and $y_i \in \mathbb{R}$. We ask some of our friends from the Statistics department and they suggest that we model the capacitor age using a Rayleigh distribution with parameter $\sigma = \boldsymbol{w}^T\boldsymbol{x}$ as follows:

$$P(Y = y|\sigma) = \frac{y}{\sigma^2} \cdot \exp\left(-\frac{y^2}{2\sigma^2}\right) \quad \textbf{where} \quad \exp(z) = e^z$$

That is the probability the lifetime $y$ of a capacitor $\boldsymbol{x}$ follows the Rayleigh distribution with $\sigma = \boldsymbol{w}^T\boldsymbol{x}$, $\boldsymbol{w} \in \mathbb{R}^d$ is the weight vector we can learn.

A visualization of the probability density function of Rayleigh distribution is shown in the following



## Q5.1

4 Points

We first consider giving a learned $w$. How we predict the most likely lifetime of a capacitor.

Rayleigh distribution with parameter $\sigma$ have the following property:

$$\arg\max_y \frac{y}{\sigma^2} \cdot \exp\left(-\frac{y^2}{2\sigma^2}\right) = \sigma$$

That is the peak of the Rayleigh distribution $P(Y = y|\sigma)$ is when $y = \sigma$.

Based on this property, consider the feature vector extracted for a capacitor is $x = [1, 3, 1]$ and the learned weight vector $w = [1, 2, 1]$. What is the most likely lifetime $y$ of the capacitor?

most likely lifetime =

Enter your answer here

Save Answer

## Q5.2

4 Points

Now, let's discuss how to learn $w$ based on the training data set $\{(x_i, y_i)\}_{i=1}^N$. We start with considering one data point $(x_i, y_i)$.

Based on our assumption, the distribution of the lifetime of a capacitor follow a Rayleigh distribution with parameter $\sigma = w^T x$. What is the probability $P(Y = y_i|x_i, w)$ of that the lifetime of a capacitor $x_i$ is $y_i$ based on the assumption.

- ○ $P(y_i|x_i, w) = \frac{y_i}{x_i^2} \cdot \exp\left(-\frac{y_i^2}{2x_i^2}\right)$
- ○ $P(y_i|x_i, w) = \frac{y_i}{w^T x_i} \cdot \exp\left(-\frac{y_i^2}{2w^T x_i}\right)$
- ○ $P(y_i|x_i, w) = \frac{y_i}{(w^T x_i)^2} \cdot \exp\left(-\frac{y_i^2}{2(w^T x_i)^2}\right)$
- ○ $P(y_i|x_i, w) = \log(y_i) - \log(w^T x_i) - \frac{y_i^2}{2w^T x_i}$

Save Answer

## Q5.3

4 Points

Given the dataset $\{(x_i, y_i)\}_{i=1}^N$, what is the log-likelihood of the model $w$?

- ○ $\mathcal{LL}(w) = constant + \sum_{i=1}^N \left(-\log(w^T x_i) - \frac{y_i^2}{2w^T x_i}\right)$
- ○ $\mathcal{LL}(w) = constant + \sum_{i=1}^N \left(-2\log(w^T x_i) - \frac{y_i^2}{2(w^T x_i)^2}\right)$
- ○ $\mathcal{LL}(w) = constant + \prod_{i=1}^N \frac{y_i}{(w^T x_i)^2} \cdot \exp\left(-\frac{y_i^2}{2(w^T x_i)^2}\right)$
- ○ $\mathcal{LL}(w) = constant + \prod_{i=1}^N \left(-\log(w^T x_i) - \frac{y_i^2}{2w^T x_i}\right)$

### Q5.4
4 Points

How we can obtain $w$?

○ We can maximize the log-likelihood in previous question

○ We can minimize the log-likelihood in previous question

Save Answer

### Q5.5
6 Points

How we can solve the optimization problem? (selected all correct options)

☐ The optimization problem can be solved by SGD

☐ The optimization problem can be solved by GD

Save Answer

## Q6 Perceptron
12 Points

Consider the following binary classification dataset with 3 features:

| $x_1$ | $x_2$ | $x_3$ | y |
|---|---|---|---|
| 1 | 1 | 2 | 1 |
| 1 | 2 | 0 | 1 |
| 4 | 0 | 0 | -1 |
| 2 | 0 | 0 | -1 |
| 0 | 0 | 1 | 1 |

Consider training a Perceptron model $y = sgn(w^T x + b)$, with $w$ and $b$ are initialized with 0.

Note that you can augment $b$ into $w$ using the trick we discussed in class.

## Q6.1

4 Points

After running the Perceptron model over these five data points **once**, what is $w$ and $b$

$w_1$=

Enter your answer here

$w_2$=

Enter your answer here

$w_3$=

Enter your answer here

$b$=

Enter your answer here

Save Answer

## Q6.2

4 Points

Given a test data point [-1, -3, 1], what is the model prediction.

y =

Enter your answer here

Save Answer

## Q6.3

4 Points

If we allow the Perceptron model to run over these five data points **until it converges**, what is the final model

$w_1$=

Enter your answer here

$w_2$=

Enter your answer here

$w_3$=

Enter your answer here

$b$=

Enter your answer here

Save Answer

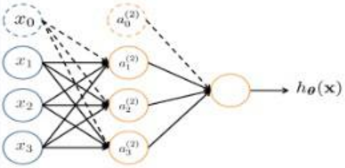
# **Q7** Multiple Choices and short answer

13 Points

## **Q7.1** Neural Network

6 Points

Consider the following neural network we discussed in the class. For a binary classification problem, the model make prediction on $x$ based on the sign of $h_\Theta(x)$. That is the model predicts positive if $h_\Theta(x) > 0$; otherwise it predicts negative.




$a_i^{(j)}$ = "activation" of unit $i$ in layer $j$

$\Theta^{(j)}$ = weight matrix controlling function mapping from layer $j$ to layer $j+1$

$$
\begin{aligned}
a_1^{(2)} &= g(\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3) \\
a_2^{(2)} &= g(\Theta_{20}^{(1)}x_0 + \Theta_{21}^{(1)}x_1 + \Theta_{22}^{(1)}x_2 + \Theta_{23}^{(1)}x_3) \\
a_3^{(2)} &= g(\Theta_{30}^{(1)}x_0 + \Theta_{31}^{(1)}x_1 + \Theta_{32}^{(1)}x_2 + \Theta_{33}^{(1)}x_3) \\
h_\Theta(x) = a_1^{(3)} &= g(\Theta_{10}^{(2)}a_0^{(2)} + \Theta_{11}^{(2)}a_1^{(2)} + \Theta_{12}^{(2)}a_2^{(2)} + \Theta_{13}^{(2)}a_3^{(2)})
\end{aligned}
$$

If the activation function $g(z) = \beta z$, where $\beta$ is a non-zero constant.

Which of the following is True?

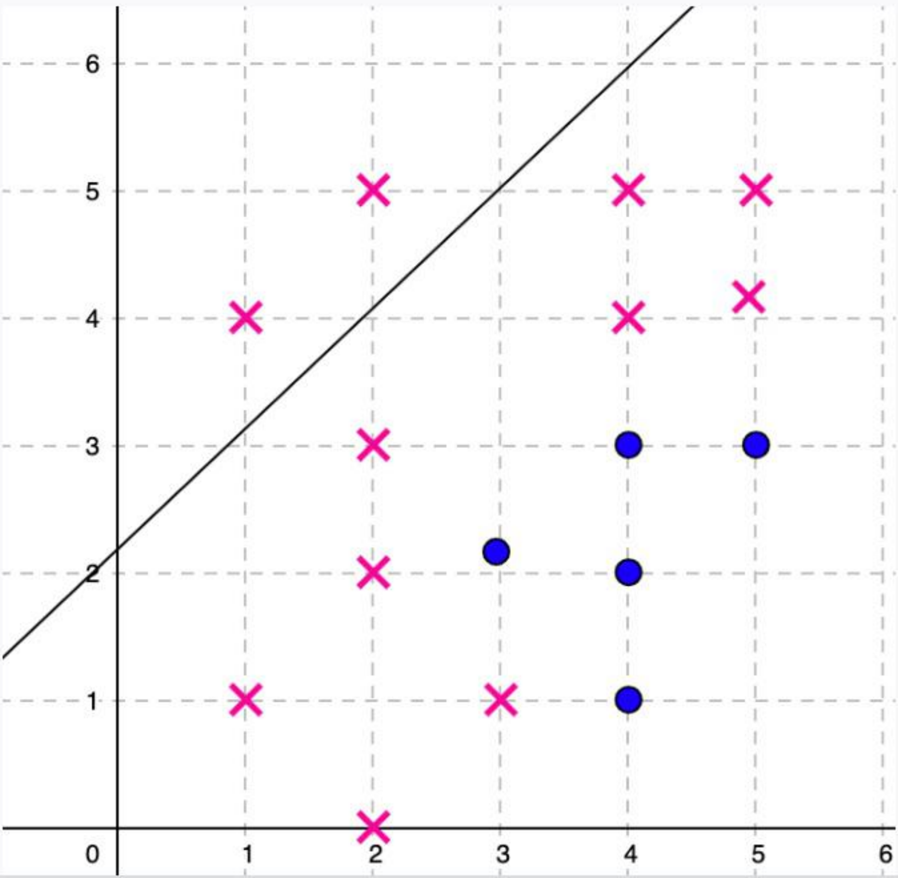
- [ ] If data are linearly separable, we can find a set of parameter $\Theta$ such that $sgn(h_\Theta(x))$ can separate all positive and negative data.
- [ ] If data are linearly separable, we **cannot** find a set of parameter $\Theta$ such that $sgn(h_\Theta(x))$ can separate all positive and negative data.
- [ ] If data are non-linearly separable, we can find a set of parameter $\Theta$ such that $sgn(h_\Theta(x))$ can separate all positive and negative data.
- [ ] If data are non-linearly separable, we **cannot** find a set of parameter $\Theta$ such that $sgn(h_\Theta(x))$ can separate all positive and negative data.

Save Answer

## Q7.2

3 Points

Given a set of positive points (blue circle) and negative points (red cross) as training data. We train a logistic regression model using SGD with 100 iterations. The results are shown in the following figure



## Q7.2

3 Points

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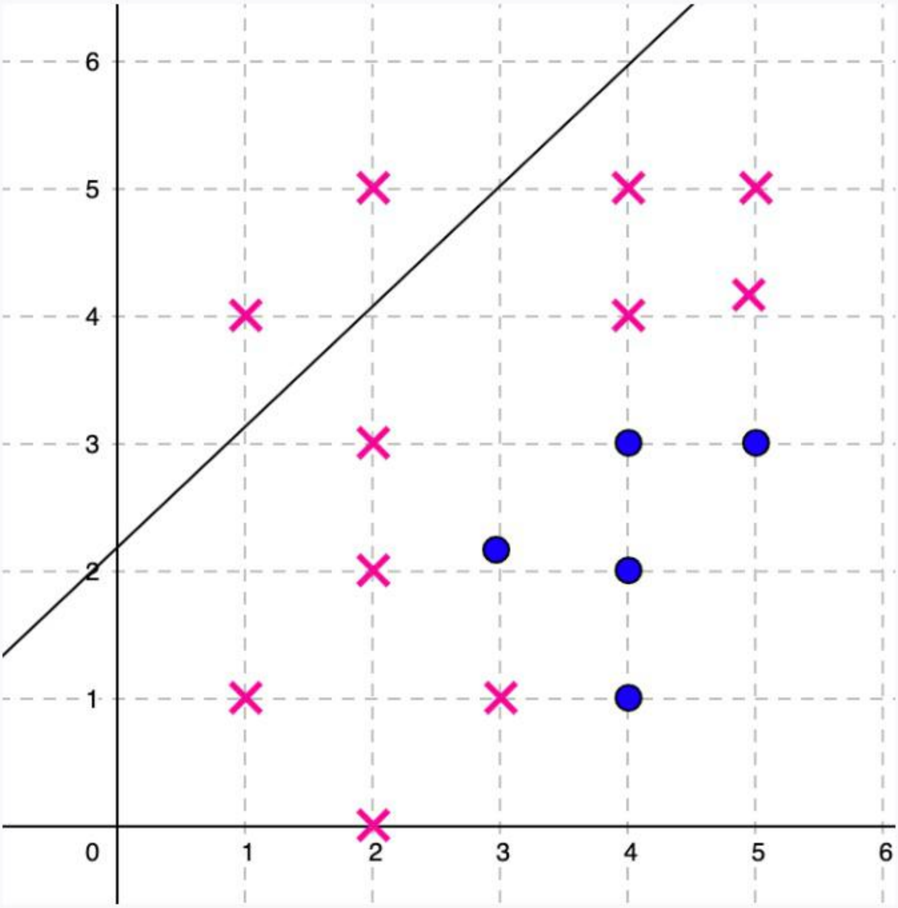


Which of the following statement is true:

- ○ The model is overfitting.
- ○ The model is underfitting.
- ○ The model has 0 training error

Save Answer

## Q7.3

4 Points

Provide one possible reason why the model does not do well in one sentence.

Enter your answer here