**STOCK VALUATION:**

Another way to raise capital for a corporation is to offer a part of the ownership of the corporation to the public, that is, to issue a stock. Unlike a bond, a common stock does not promise to pay a fixed payment at any time. The cash flow that is expected from investing on a stock is the expected possible dividend payments (a portion of the profit the company distributes) and the capital gains acquired if the price that can be obtained for the stock when it is sold is higher than the price at the time of the purchase. What determines the price of a stock?

*Technical Analysis*: Looks into past data and the price movements of the stock in relation to the market movements and makes predictions. No attention is paid to what the 'value' of a stock is. Price predictions are only extrapolations from the historical price patterns.

*Fundamental Analysis*: Looks into the underlying fundamentals of a company like, balance sheet, income statement, cash flow, growth opportunities etc…. and tries to determine the true value of a stock. A model that is widely used towards this goal is called the Dividend Discount Model (Gordon Growth Model):

**Dividend Discount Model (DDM):**

The value of an asset is the present value of the total of current and future cash flows. A stock provides two kinds of cash flows: 1. Dividends, 2. Sale price when the stock is sold. Suppose I am considering buying a stock today. In one year I will collect the dividend payment and sell the stock. Then the value of the stock today, $P_0$, is:

$$P_0 = \frac{Div_1}{1+R} + \frac{P_1}{1+R}$$

The person who purchases the stock at time 1 will make the same kind of calculation (without loss of generality, let's assume that he will also keep the stock for 1 year and sell it the following year):

$$P_1 = \frac{Div_2}{1+R} + \frac{P_2}{1+R}$$

If we substitute this $P_1$ in the previous equation and follow the same logic for the subsequent buyers we will arrive at:

$$P_0 = \frac{Div_1}{1+R} + \frac{Div_2}{(1+R)^2} + \frac{Div_3}{(1+R)^3} + \cdots = \sum_{t=1}^{\infty} Div_t/(1+R)^t$$

Thus the price of a share of common stock to the investor is equal to the present value of all the expected future dividends.

Constant Dividend: $P_0 = \frac{Div}{1+R} + \frac{Div}{(1+R)^2} + \frac{Div}{(1+R)^3} + \cdots = \frac{Div}{R}$

Growing Dividend: $P_0 = \frac{Div}{1+R} + \frac{Div(1+g)}{(1+R)^2} + \frac{Div(1+g)^2}{(1+R)^3} + \cdots = \frac{Div}{R-g}$ when R>g

What if g>R? You would expect a firm to grow at a bigger rate than the market rate for a finite horizon then settle at a rate that is below R. It is a natural pattern that businesses go through.

Differential Growth: Dividends will grow at rate $g_1$ for T years and grow at $g_2$thereafter:

$$P = \frac{C}{R - g_1}\left[1 - \frac{(1+g_1)^T}{(1+R)^T}\right] + \frac{\left(\frac{\text{Div}_{\text{T+1}}}{R - g_2}\right)}{(1+R)^T} \quad \text{R} > g_1, g_2$$

**Example:** A company will distribute $1.15 dividends per share a year from today. Dividens grow at 15% for four years after that and then the growth rate will settle at 10% forever. If the market rate is 15%, what is the stock price today?

$$P_0 = \frac{Div_1}{1+R} + \frac{Div_2}{(1+R)^2} + \frac{Div_3}{(1+R)^3} + \frac{Div_4}{(1+R)^4} + \frac{Div_5}{(1+R)^5} + \frac{Div_5(1+0.10)}{(1+R)^6} + \cdots$$

$$P_0 = \frac{1.15}{1.15} + \frac{1.15^2}{(1.15)^2} + \frac{1.15^3}{(1.15)^3} + \frac{1.15^4}{(1.15)^4} + \frac{1.15^5}{(1.15)^5} + \frac{1.15^5(1+0.10)}{(1.15)^6} + \cdots$$

$$P_0 = 1 + 1 + 1 + 1 + 1 + \frac{1.15^5(1+0.10)}{(1.15)^6} + \cdots$$

$$P_0 = 5 + \frac{2.2125}{(1.15)^6} + \frac{2.2125(1.10)}{(1.15)^6} + \frac{2.2125(1.10)^2}{(1.15)^6} + \frac{2.2125(1.10)^3}{(1.15)^6} \ldots$$

$$P_0 = 5 + \frac{1}{(1.15)^5}\left[\frac{2.2125}{(0.15-0.10)}\right] = \$27$$

Note that at the last step the "growing dividend formula" is used!

**Where does g come from?**

Earnings Next Year = Earnings This Year + Retained Earnings this year x Return on Retained Earnings

Dividing either side of this equation by "Earnings This Year" we get:

(1+g) = 1+Retention Ratio x Return on Retained Earnings.

Where g is the growth rate of the earnings. For a company with a constant dividend payout ratio policy, g would also be equal to the growth rate of the dividends.

If we can approximate the Return on Retained Earnings by ROE then,

g = Retention Ratio x ROE

**Where does R come from?**

P = $\frac{Div}{R-g}$ , then, $R = \frac{Div}{\text{P}} + g$ = dividend yield + capital gains

**Growth Opportunities:**

If a company distributes all of its earnings, such a company is called a 'cash cow'. In this case Dividends = EPS, retention ratio (b) will be zero and company is not investing in any future projects (assuming no other financing is raised). If the company decides to retain some earnings and invest on a project, the value of the company will increase by the same amount as the Net Present Value of the project (NPVGO, Net Present Value of Growth Opportunities). Then, P = EPS/R + NPVGO.

Does a Higher Retention Ratio Benefit the Stock Holder?

P = $\frac{Div}{R-g}$ by plugging g=b * ROE and Div = (1-b) * EPS, P = (1-b)*EPS/[R-(b*ROE)]

Note that b appears both in the numerator and the denominator. The effect of an increase in b on P can be positive or negative. Taking the derivative of P with respect to b suggests that when ROE>R, P will increase as b increases. This also makes economic sense.

What if the company offers no dividends? How to value its stock?

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Two companies each having the same EPS, do not necessarily have the same stock price. Everything else being equal, we can attribute the difference in stock price to the differences in growth opportunities. P = EPS/R + NPVGO, dividing both sides by EPS will give us

P/E ratio = 1/R + NPVGO / EPS

Therefore, the P/E ratio of a company depends on R, NPVGO, and EPS.