CS 161 Homework 5

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March 5, 2022

Problem 1

Consider the following sentences and decide for each whether it is valid, unsatisfiable, or neither. Justify your answer using truth tables (worlds).

a) $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$

$$P_1 : (Smoke \Rightarrow Fire)$$
$$P_2 : (\neg Smoke \Rightarrow \neg Fire)$$
$$P_3 : P_1 \Rightarrow P_2$$

$Smoke$	$Fire$	P_1	P_2	P_3
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

This sentence is neither valid nor unsatisfiable. This is because the sentence is true in some worlds ($\{Smoke = True, Fire = True\}$) and is false in others ($\{Smoke = False, Fire = True\}$).

b) $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \lor Heat) \Rightarrow Fire)$

$$P_1 : (Smoke \Rightarrow Fire)$$

$$P_2 : (Smoke \lor Heat)$$

$$P_3 : (P_2 \Rightarrow Fire)$$

$$P_4 : P_1 \Rightarrow P_3$$

$Smoke$	$Fire$	$Heat$	P_1	P_2	P_3	P_4
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	F	F
F	F	F	T	F	T	T

This sentence is neither valid nor unsatisfiable. This is because the sentence is true in some worlds ($\{Smoke = True, Fire = True, Heat = True\}$) and is false in others ($\{Smoke = False, Fire = False, Heat = True\}$).

c) $((Smoke \land Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \lor (Heat \Rightarrow Fire))$

$$P_1 : (Smoke \land Heat)$$

$$P_2 : (P_1 \Rightarrow Fire)$$

$$P_3 : (Smoke \Rightarrow Fire)$$

$$P_4 : (Heat \Rightarrow Fire)$$

$$P_5 : (P_3 \lor P_4)$$

$$P_6 : P_2 \Leftrightarrow P_5$$

$Smoke$	$Fire$	$Heat$	P_1	P_2	P_3	P_4	P_5	P_6
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	T	F	F	F	F	T
T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	F	T	T	F	T	T
F	F	F	F	T	T	T	T	T

This sentence is valid, as it holds true for all possible worlds.

Problem 2

Consider the following:

> ***If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.***

Justify your answers using resolution by providing corresponding resolution derivations. Make sure to clearly define all propositional symbols (variables) first, then define your knowledge base, and finally give your derivations.

a) Represent the above information using a propositional logic knowledge base (set of sentences in propositional logic).

The variables used will be $Mythical$, $Immortal$, $Mammal$, $Horned$, and $Magical$. The KB therefore consists of:

$$P_1 : Mythical \Rightarrow Immortal$$
$$P_2 : \neg Mythical \Rightarrow \neg Immortal \wedge Mammal$$
$$P_3 : Immortal \vee Mammal \Rightarrow Horned$$
$$P_4 : Horned \Rightarrow Magical$$

b) Convert the knowledge base into CNF.

$$P_1 : \neg Mythical \vee Immortal$$
$$P_2 : (Mythical \vee \neg Immortal) \wedge (Mythical \vee Mammal)$$
$$P_3 : (\neg Immortal \vee Horned) \wedge (\neg Mammal \vee Horned)$$
$$P_4 : \neg Horned \vee Magical$$

c) Can you use the knowledge base to prove that the unicorn is mythical? How about magical? Horned?

$P_5 : Immortal \vee \neg Immortal$	by resolution of $Mythical$ in P_1 and P_2
$P_6 : \neg Mythical \vee Mythical$	by resolution of $Immortal$ in P_1 and P_2
$P_7 : Immortal \vee Mammal$	by resolution of $Mythical$ in P_1 and P_2
$P_8 : \neg Mythical \vee Horned$	by resolution of $Immortal$ in P_1 and P_3
$P_9 : Mythical \vee Horned$	by resolution of $Mammal$ in P_2 and P_3
$P_{10} : \neg Immortal \vee Magical$	by resolution of $Horned$ in P_3 and P_4
$P_{11} : \neg Mammal \vee Magical$	by resolution of $Horned$ in P_3 and P_4
$P_{12} : Mammal \vee Magical$	by resolution of $Immortal$ in P_7 and P_{10}
$P_{13} : Magical$	by resolution of $Mammal$ in P_{11} and P_{12}
$P_{14} : Horned$	by resolution of $Mythical$ in P_8 and P_9

Mythical:

$$P_{15} : \neg Mythical \quad \text{Begin proof by contradiction}$$

We cannot resolve P_{15} with any clause to create the empty clause, therefore we cannot prove the unicorn is mythical.

Magical:

$$\begin{array}{ll} P_{15} : \neg Magical & \text{Begin proof by contradiction} \\ P_{16} : \{\} & \text{by resolution of } Magical \text{ in } P_{13} \text{ and } P_{15} \end{array}$$

We reached the empty clause by resolution, so we can prove the unicorn is magical.

Horned:

$$\begin{array}{ll} P_{15} : \neg Horned & \text{Begin proof by contradiction} \\ P_{16} : \{\} & \text{by resolution of } Horned \text{ in } P_{14} \text{ and } P_{15} \end{array}$$

We reached the empty clause by resolution, so we can prove the unicorn is horned.

Problem 3

For each pair of atomic sentences, give the most general unifier if it exists:

a) P(A, B, B), P(x, y, z).

$\theta = \{x/A, y/B, z/B\}$

b) Q(y, G(A, B)), Q(G(x, x), y).

No unifier exists

c) Older(Father(y), y), Older(Father(x), John).

$\{y/John, x/John\}$

d) Knows(Father(y), y), Knows(x, x).

No unifier exists

Problem 4

Consider the following sentences:

- **John likes all kinds of food.**
- **Apples are food.**
- **Chicken is food.**
- **Anything anyone eats and isn't killed by is food.**
- **If you are killed by something, you are not alive.**
- **Bill eats peanuts and is still alive. ***
- **Sue eats everything Bill eats.**

For first-order syntax, feel free to use the following text file notation: `|` (for disjunction), `&` (for conjunction), `-` (for negation), `=>` (for implication), `<=>` (for equivalence), `E` (for existential quantification, e.g., `E x, y, Loves(x, y)`), and `A` (for universal quantification, e.g., `A x, y, Loves(x, y)`).

a) Translate these sentences into formulas in first-order logic.

$$
\begin{aligned}
A &: \forall a\, Food(a) \Rightarrow Likes(John, a) \\
B &: Food(Apples) \\
C &: Food(Chicken) \\
D &: \forall b, c\, Eats(b, c) \wedge \neg KilledBy(b, c) \Rightarrow Food(c) \\
E &: \forall d, e\, KilledBy(d, e) \Rightarrow \neg Alive(d) \\
F &: Eats(Bill, Peanuts) \wedge Alive(Bill) \\
G &: \forall f\, Eats(Bill, f) \Rightarrow Eats(Sue, f)
\end{aligned}
$$

b) Convert the formulas of part `(a)` into CNF (also called clausal form).

$$
\begin{aligned}
A &: \neg Food(a) \vee Likes(John, a) \\
B &: Food(Apples) \\
C &: Food(Chicken) \\
D &: \neg Eats(b, c) \vee KilledBy(b, c) \vee Food(c) \\
E &: \neg KilledBy(d, e) \vee \neg Alive(d) \\
F &: Eats(Bill, Peanuts) \\
G &: Alive(Bill) \\
H &: \neg Eats(Bill, f) \vee Eats(Sue, f)
\end{aligned}
$$

c) Prove that John likes peanuts using resolution.

$I : \neg Likes(John, Peanuts)$	Begin proof by contradiction
$J : \neg Food(Peanuts)$	by resolution of A and I
$K : \neg Eats(b, Peanuts) \lor KilledBy(b, Peanuts)$	by resolution of D and J
$L : \neg Eats(d, Peanuts) \lor \neg Alive(d)$	by resolution of E and K
$M : \neg Alive(Bill)$	by resolution of F and L
$N : \{\}$	by resolution of G and M

We reached the empty clause by resolution, so we can prove John likes peanuts.

d) Use resolution to answer the question, "What food does Sue eat?"

$\exists x\ Food(x) \land Eats(Sue, x)$	There exists a food that Sue eats
$Food(x) \land Eats(Sue, x)$	Convert to CNF
$\neg Food(x) \lor \neg Eats(Sue, x)$	Negate

$I : \neg Food(x) \lor \neg Eats(Sue, x)$	Begin proof by contradiction
$J : \neg KilledBy(Bill, e)$	by resolution of E and G
$K : \neg Eats(Bill, e) \lor Food(e)$	by resolution of D and J
$L : Food(Peanuts)$	by resolution of F and K
$M : \neg Eats(Sue, Peanuts)$	by resolution of I and L
$N : \neg Eats(Bill, Peanuts)$	by resolution of H and M
$O : \{\}$	by resolution of F and N

As shown in M, unifying x with $Peanuts$ leads to a contradiction, therefore it can be concluded that the food Sue eats is peanuts.

e) Use resolution to answer the question, “What food does Sue eat?” if, instead of the axiom marked with an asterisk above, we had:

- **If you don’t eat, you die.**
- **If you die, you are not alive.**
- **Bill is alive.**

New KB:

$$\begin{aligned}
&A : \forall a\ Food(a) \Rightarrow Likes(John, a)\\
&B : Food(Apples)\\
&C : Food(Chicken)\\
&D : \forall b, c\ Eats(b, c) \wedge \neg KilledBy(b, c) \Rightarrow Food(c)\\
&E : \forall d, e\ KilledBy(d, e) \Rightarrow \neg Alive(d)\\
&F : \forall f\ Eats(Bill, f) \Rightarrow Eats(Sue, f)\\
&G : \forall g, h\ \neg Eats(g, h) \Rightarrow Dead(g)\\
&H : \forall i\ Dead(i) \Rightarrow \neg Alive(i)\\
&I : Alive(Bill)
\end{aligned}$$

CNF:

$$\begin{aligned}
&A : \neg Food(a) \vee Likes(John, a)\\
&B : Food(Apples)\\
&C : Food(Chicken)\\
&D : \neg Eats(b, c) \vee KilledBy(b, c) \vee Food(c)\\
&E : \neg KilledBy(d, e) \vee \neg Alive(d)\\
&F : \neg Eats(Bill, f) \vee Eats(Sue, f)\\
&G : Eats(g, h) \vee Dead(g)\\
&H : \neg Dead(i) \vee \neg Alive(i)\\
&I : Alive(Bill)
\end{aligned}$$

Resolution:

$J : \neg Food(x) \vee \neg Eats(Sue, x)$	Begin proof by contradiction
$K : \neg Eats(Bill, x) \vee \neg Food(x)$	by resolution of F and J
$L : \neg Food(x) \vee Dead(Bill)$	by resolution of G and K
$M : \neg Food(x) \vee \neg Alive(Bill)$	by resolution of H and L
$N : \neg Food(x)$	by resolution of I and M
$O : \neg Eats(b, x) \vee KilledBy(b, x)$	by resolution of D and N
$P : \neg Eats(b, x) \vee \neg Alive(b)$	by resolution of E and O
$Q : \neg Eats(Bill, x)$	by resolution of I and P
$R : Dead(Bill)$	by resolution of G and Q
$S : \neg Alive(Bill)$	by resolution of H and R
$T : \{\}$	by resolution of I and S

Therefore, since x was never unified during the resolution, we can only conclude the general statement that Sue and Bill eat the same foods and that there exists some food that they eat.