Lecture 8: Neural Network & Deep Learning

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Announcements

- Quiz 2 is due today
- Hw 1 is due next Tue
  - The definition of F1 score will be covered today
- Midterm postpones to 11/1?
  - The practice exam will be posted

What you will learn today

- Optimization
  - Gradient descent
  - Stochastic gradient descent (SGD)
- Evaluation Metrics
- Neural network / Deep learning
  - Non-linear classifier
  - Feed-forward neural network
  - Deep learning architecture

Gradient Descent

Example $\min f(\boldsymbol{\theta}) = 0.5(\theta_1^2 - \theta_2)^2 + 0.5(\theta_1 - 1)^2$

- Use the following iterative procedure for gradient descent

$$\nabla f(\theta) = \begin{bmatrix} 2(\theta_1^2 - \theta_2)\theta_1 + \theta_1 - 1 \\ -(\theta_1^2 - \theta_2) \end{bmatrix}$$

1. Initialize $\theta_1^{(0)}$ and $\theta_2^{(0)}$, and $t = 0$
2. do

Type equation here.

$$\theta_1^{(t+1)} \leftarrow \theta_1^{(t)} - \eta \left[2({\theta_1^{(t)}}^2 - \theta_2^{(t)})\theta_1^{(t)} + \theta_1^{(t)} - 1 \right]$$

$$\theta_2^{(t+1)} \leftarrow \theta_2^{(t)} - \eta \left[-({\theta_1^{(t)}}^2 - \theta_2^{(t)}) \right]$$

$$t \leftarrow t + 1$$

3. until $f(\boldsymbol{\theta}^{(t)})$ *does not change much*

Remarks

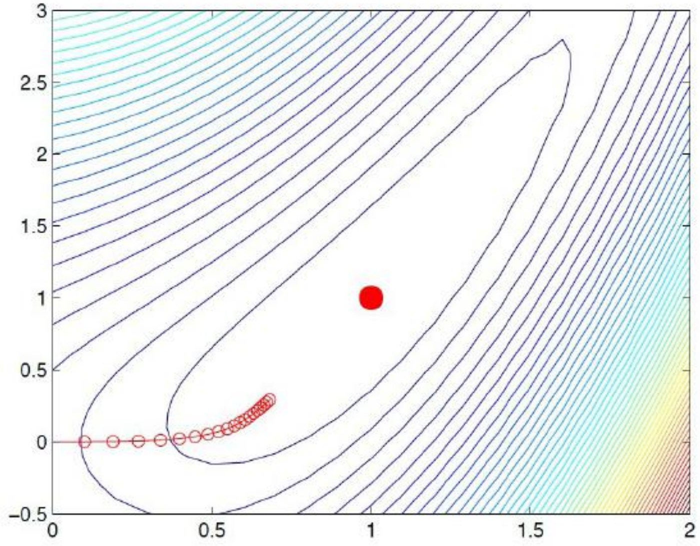
- η is often called step size or learning rate -- how far our update will go along the the direction of the negative gradient
- With a suitable choice of η, the iterative procedure converges to a stationary point where

$$\frac{\partial f}{\partial \boldsymbol{\theta}} = 0$$

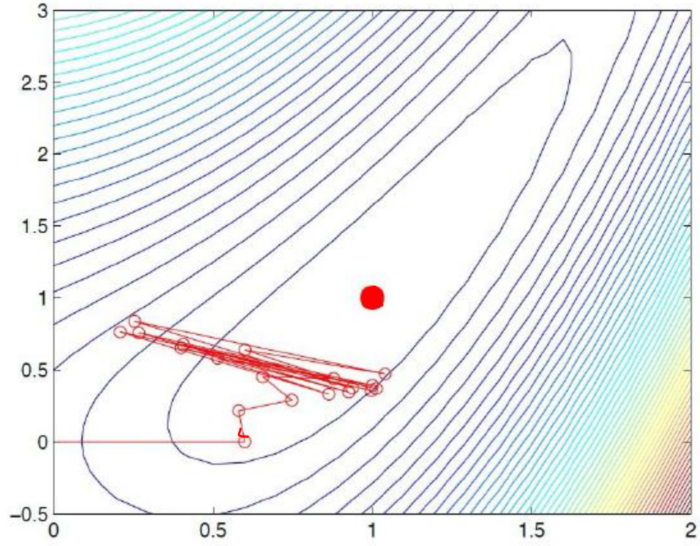
- A stationary point is only necessary for being the minimum

Choosing the right learning rate (η) is important

small η is too slow?



large η is too unstable?



Recap: Logistic Regression

- Training data: $S = \{(x_i, y_i)\}$, m examples
- Hypothesis space:

$$H = \{\ \mathrm{h} \mid h : X \rightarrow P(Y \mid X), h(x) = \sigma\,(\,w^T x + b)\,\}$$

$$\sigma(z) = \frac{1}{1 + \exp(-z)}$$

i.e., model $P(Y|X)$ by $\sigma(w^T x + b)$

- How to find the best $h \in H$: maximum log-likelihood

$$\arg\max - \sum_{i=1}^{m} \log(1 + \exp(-y_i(w^T x_i + b)))$$

Gradient Descent for Logistic Regression

- Maximum log-likelihood

$$\arg\max -\sum_{i=1}^{m} \log(1 + \exp(-y_i(w^T x_i + b)))$$

- Equivalent to the following minimization problem

$$\arg\min \underbrace{\sum_{i=1}^{m} \log(1 + \exp(-y_i(w^T x_i + b)))}_{L(w,b)}$$

- Gradient of $L(w, b)$

$$\nabla L(w, b) = \sum_{i=1}^{m} \nabla \log(1 + \exp(-y_i(w^T x_i + b)))$$

Recap: Gradient

- Let z to be a n-dimensional vector of variables, $f(z)$ is a function of z

$$\nabla f(z) = \begin{bmatrix} \partial f(z)/\partial z_1 \\ \partial f(z)/\partial z_2 \\ \vdots \\ \partial f(z)/\partial z_{n-1} \\ \partial f(z)/\partial z_n \end{bmatrix}$$

Exercise

- Let $z = [z_1, z_2, z_3]^T$ to be a 3-dimensional vector of variables, $a = [3, 2, 4]^T$

  $f(z) = \log(a^T z)$

  $= \log(3z_1 + 2z_2 + 4z_3)$

$$\nabla f(z) = \begin{bmatrix} \partial f(z)/\partial z_1 \\ \partial f(z)/\partial z_2 \\ \vdots \\ \partial f(z)/\partial z_{n-1} \\ \partial f(z)/\partial z_n \end{bmatrix}$$

- $\nabla f(z) = \begin{bmatrix} \partial f(z)/\partial z_1 \\ \partial f(z)/\partial z_2 \\ \partial f(z)/\partial z_3 \end{bmatrix} = ?$

Exercise

- Let $z = [z_1, z_2, z_3]^T$ to be a 3-dimensional vector of variables, $a = [3, 2, 4]^T$
  $f(z) = \log(a^T z)$
  $= \log(3z_1 + 2z_2 + 4z_3)$

$$\nabla f(z) = \begin{bmatrix} \partial f(z)/\partial z_1 \\ \partial f(z)/\partial z_2 \\ \vdots \\ \partial f(z)/\partial z_{n-1} \\ \partial f(z)/\partial z_n \end{bmatrix}$$

- $\nabla f(z) = \begin{bmatrix} \partial f(z)/\partial z_1 \\ \partial f(z)/\partial z_2 \\ \partial f(z)/\partial z_3 \end{bmatrix} = \begin{bmatrix} \frac{3}{3z_1+2z_2+4z_3} \\ \frac{2}{3z_1+2z_2+4z_3} \\ \frac{4}{3z_1+2z_2+4z_3} \end{bmatrix}$

$$= \frac{1}{3z_1+2z_2+4z_3} \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix} = \frac{1}{a^T z} a$$

Gradient of $L(w, b)$

$$\nabla L(w,b) = \Sigma_{i=1}^{m} \ \nabla \underbrace{\log(1 + \exp(-y_i(w^T x_i + b)))}_{\nabla \log \frac{1}{\sigma(y_i(w^T x_i + b))}}$$

$$\sigma(z) = \frac{1}{1 + \exp(-z)}$$

- $\nabla \log \frac{1}{\sigma(z)} = \nabla \log\big(1 + \exp(-z)\big) = -\frac{\exp(-z)}{1+\exp(-z)}$

$$= -\frac{1 + \exp(-z) - 1}{1 + \exp(-z)} = -1 + \frac{1}{1 + \exp(-z)}$$

$$= \sigma(z) - 1$$

Gradient of $L(w, b)$

$$\nabla L(w,b) = \Sigma_{i=1}^{m} \nabla \underbrace{\log(1 + \exp(-y_i(w^T x_i + b)))}_{\nabla \log \frac{1}{\sigma(y_i(w^T x_i + b))}}$$

$$\sigma(z) = \frac{1}{1 + \exp(-z)}$$

- Using $\nabla \log \frac{1}{\sigma(z)} = \sigma(z) - 1$

Partial gradient w.r.t w

$$\nabla_w L(w,b) = \Sigma_{i=1}^{m} \nabla_{\mathrm{w}} \log \frac{1}{\sigma(y_i(w^T x_i + b))}$$

$$= \Sigma_{i=1}^{m} (\sigma(y_i(w^T x_i + b)) - 1) y_i x_i$$

$$\nabla_b L(w,b) = \Sigma_{i=1}^{m} (\sigma(y_i(w^T x_i + b)) - 1) y_i$$

Gradient descent for logistic regression

Given a training data set $S = \{(x_i, y_i)\}, i = 1 \ldots m$

1. Initialize w (e.g., $w \leftarrow 0 \in R^n$)
2. For epoch $1 \ldots T$:
3. Compute $\nabla_w L(w, b)$ and $\nabla_b L(w, b)$

Loop over instance to compute the summation

$$\nabla_w L(w,b) = \Sigma_{i=1}^{m} (\sigma(y_i(w^T x_i + b)) - 1) y_i x_i$$
$$\nabla_b L(w,b) = \Sigma_{i=1}^{m} (\sigma(y_i(w^T x_i + b)) - 1) y_i$$

4. Update w and b

$$w \leftarrow w - \eta \nabla_w L(w, b)$$
$$\mathrm{b} \leftarrow \mathrm{b} - \eta \nabla_b\, L(w, b)$$

5. Return w and b

Remark

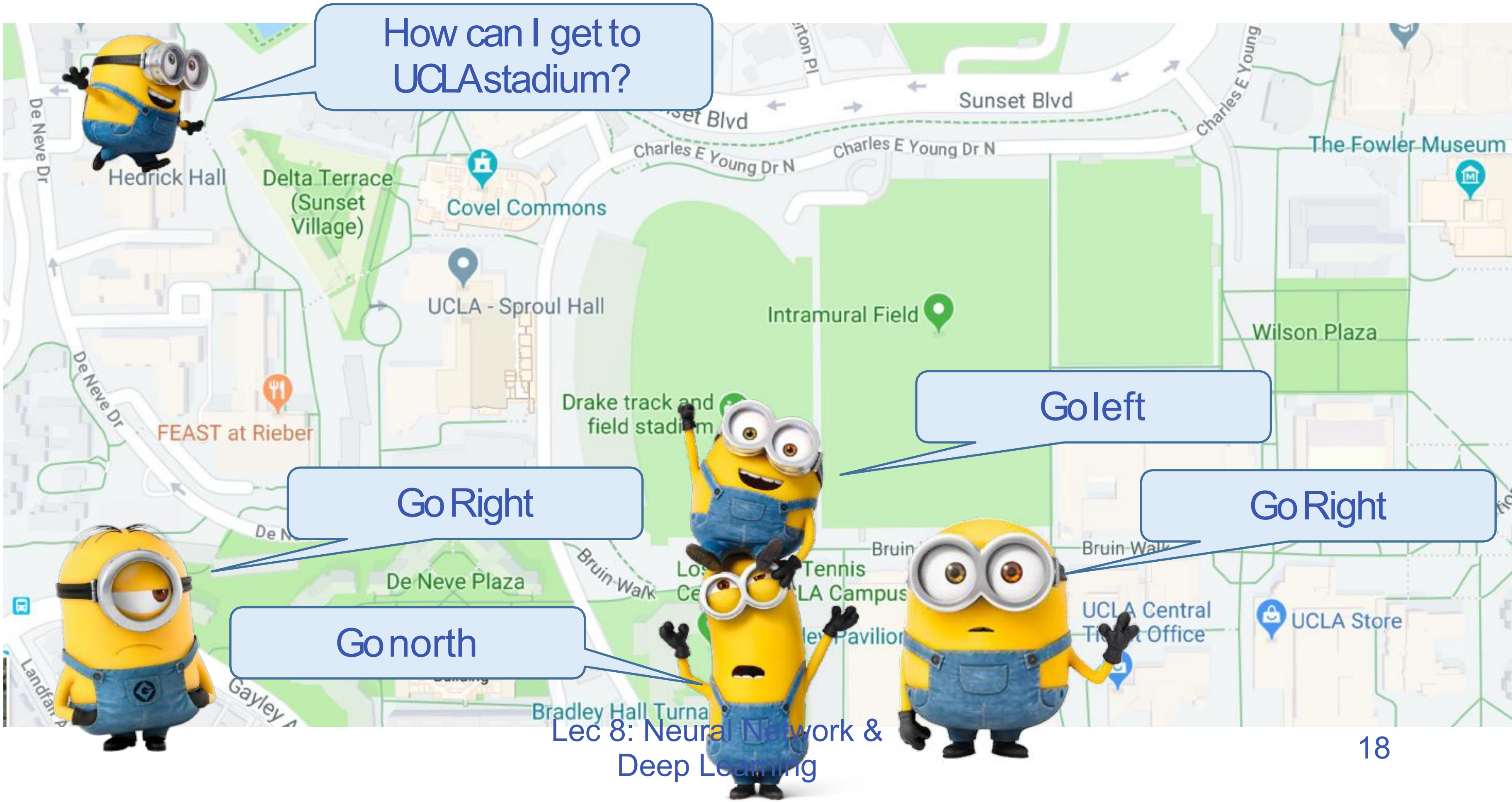
$$\nabla_w L(w,b) = \Sigma_{i=1}^{m}(\sigma(y_i(w^T x_i + b)) - 1)y_i x_i$$
$$\nabla_b L(w,b) = \Sigma_{i=1}^{m}(\sigma(y_i(w^T x_i + b)) - 1)y_i$$

- Need to compute $(\sigma(y_i(w^T x_i + b)) - 1)$ for every data point (x_i, y_i)
- Gradient descent usually needs many iterations to converge
- When size of data (m) is large, computing $\nabla L(w,b)$ is expensive

Stochastic Gradient Descent

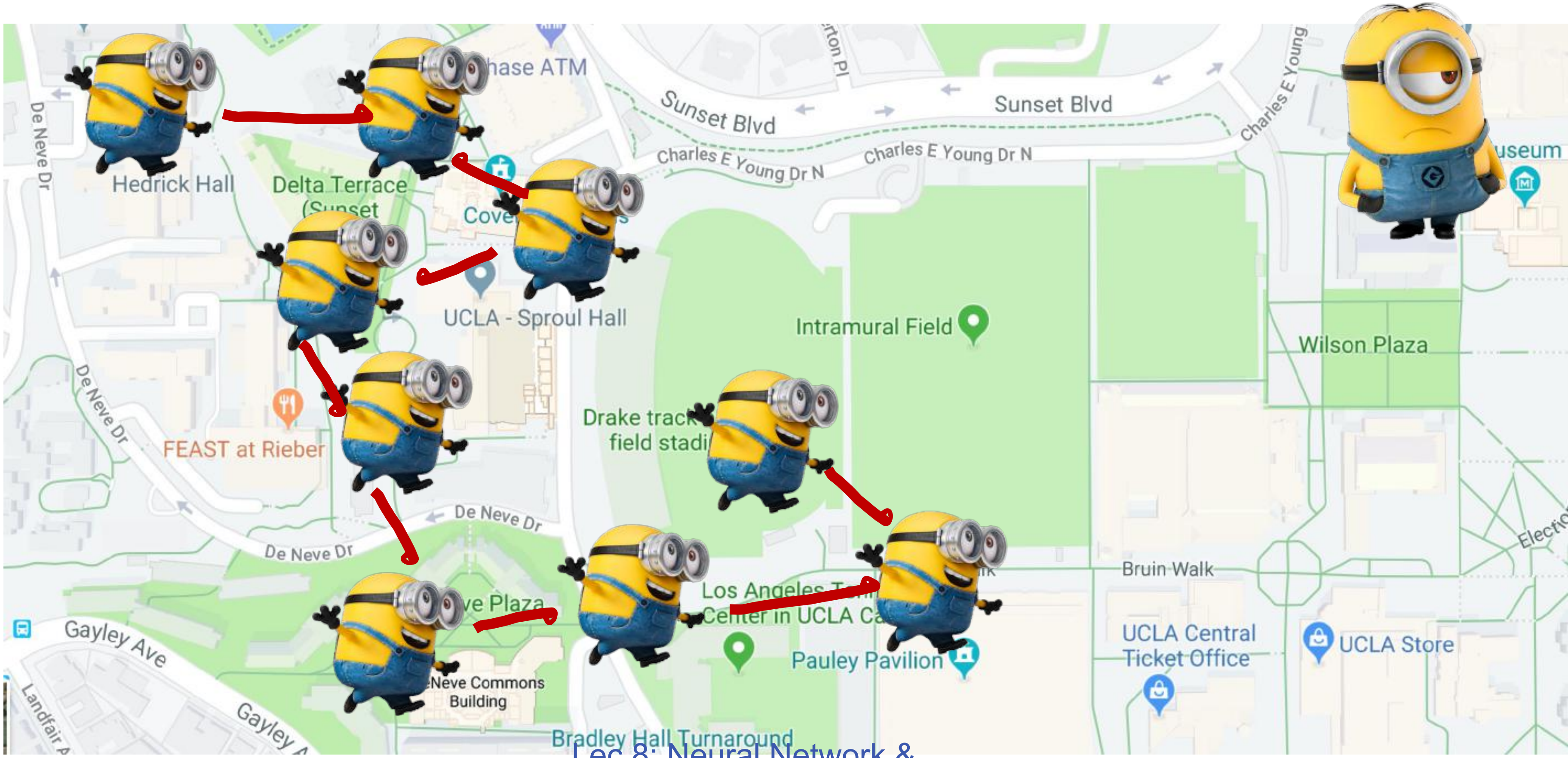
Intuition

Asking direction. Gradient descent: compute gradient of all instances.



Intuition

Asking direction. Stochastic Gradient descent:
compute approximate gradient by one instance



Incremental/Stochastic gradient descent

Repeat for each example ($\mathbf{x}_i$, y_i)

- Use this example to calculate approximate the gradient and update the model

Contrast with *batch gradient descent* which makes one update to the weight vector for every pass over the data

Recap: Gradient Descent

- $\nabla_w L(w,b) = \sum_{i=1}^{m} \underbrace{(\sigma(y_i(w^T x_i + b)) - 1) y_i x_i}_{\nabla L_i(w,b)}$
- Gradient descent update:
  $w \leftarrow w - \eta \sum_{i=1}^{m} \nabla_{\mathrm{w}} L_i(w,b)$
- Alternative way of gradient update
  For i = 1 … m
  $$w \leftarrow w - \eta \, \nabla_{\mathrm{w}} L_i(w,b)$$

Stochastic Gradient Descent

- $\nabla_w L(w,b) = \sum_{i=1}^{m} \nabla_w L_i(w,b) = m \frac{\sum_{i=1}^{m} \nabla L_i(w,b)}{m} = m\, \underbrace{\text{avg}\left(\nabla_{\mathrm{w}} L_i(w,b)\right)}_{\text{Average } L_i(w,b) \text{ over instance}}$
- $\text{avg}\left(\nabla_{\mathrm{w}} L_i(w,b)\right) = E_{(x_i,y_i)\sim S}\left[\nabla_{\mathrm{w}} L_i(w,b)\right]$
- Gradient descent update:
  $w \leftarrow w - \eta \sum_{i=1}^{m} \nabla_{\mathrm{w}} L_i(w,b)$
- Stochastic gradient descent
  - Repeat until converge
    Sample a data point (x_i, y_i) from S
    $w \leftarrow w - \eta' \nabla_{\mathrm{w}} L_i(w,b)$

Expectation of gradient $L_i(w,b)$ over dataset S

Stochastic Gradient descent for logistic regression

Given a training data set $S = \{(x_i, y_i)\}, i = 1 \ldots m$

1. Initialize w (e.g., $w \leftarrow 0 \in R^n$)
2. For epoch $1 \ldots T$:
3. Sample a data point (x_i, y_i) from S
4. Compute $\nabla_w L_i(w, b)$ and $\nabla_b L_i(w, b)$

$$\nabla_w L_i(w, b) = (\sigma(y_i(w^T x_i + b)) - 1) y_i x_i$$

$$\nabla_b L_i(w, b) = (\sigma(y_i(w^T x_i + b)) - 1) y_i$$

5. Update w and b

$$w \leftarrow w - \eta \nabla_w L_i(w, b)$$

$$\mathrm{b} \leftarrow \mathrm{b} - \eta \nabla_b L_i(w, b)$$

6. Return w and b

The Perceptron Algorithm [Rosenblatt 1958]

Given a training set $\mathcal{D} = \{(\boldsymbol{x}, y)\}$

1. Initialize $\boldsymbol{w} \leftarrow \mathbf{0} \in \mathbb{R}^n$
2. For epoch $1 \ldots T$:
3. For $(\boldsymbol{x}, y)$ in $\mathcal{D}$:
4. if $y(\boldsymbol{w}^\top \boldsymbol{x}) < 0$
5. $\boldsymbol{w} \leftarrow \boldsymbol{w} + \eta y \boldsymbol{x}$
6. Return $\boldsymbol{w}$

Prediction: $y^{\text{test}} \leftarrow sgn(\boldsymbol{w}^\top \boldsymbol{x}^{\text{test}})$

The Perceptron Algorithm [Rosenblatt 1958]

Given a training set $\mathcal{D} = \{(\boldsymbol{x}, y)\}$

1. Initialize $\boldsymbol{w} \leftarrow \mathbf{0} \in \mathbb{R}^n$
2. For epoch $1 \ldots T$:
3. For $(\boldsymbol{x}, y)$ in $\mathcal{D}$:
4. if $y(\boldsymbol{w}^\top \boldsymbol{x}) < \mathbf{0}$
5. $\boldsymbol{w} \leftarrow \boldsymbol{w} + \eta y \boldsymbol{x}$
6. Return $\boldsymbol{w}$

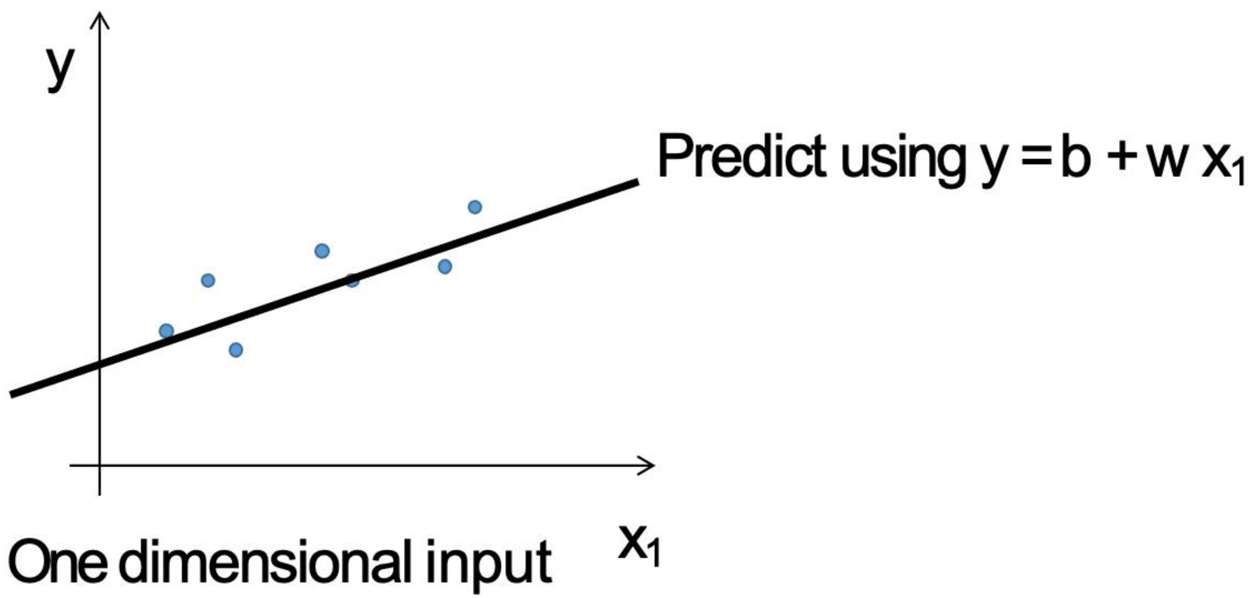
Prediction: y^{test}

Perceptron effectively minimizing:

$$\sum_i \max(0,\ 1 - \ y_i(\mathbf{w}^T \mathbf{x}_i))$$

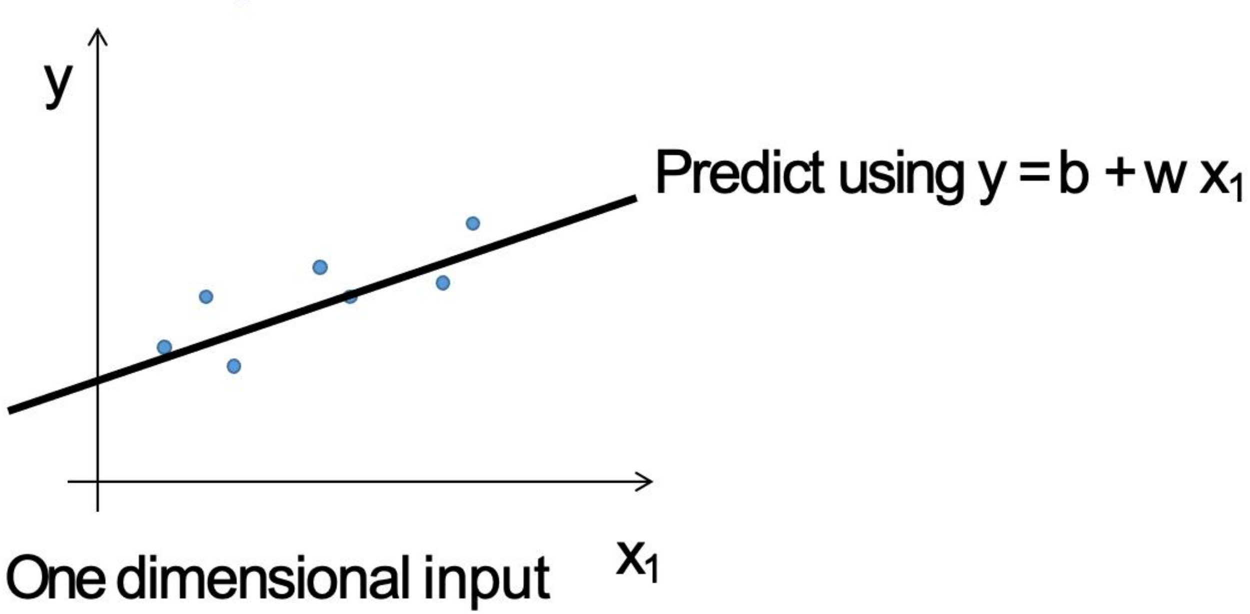
Linear Regression

- Find a line $w^T x + b$ to approximate real-value output y based on input x
  e.g., predict house price next year



Linear Regression

- Find a line to approximate real-value output
  e.g., predict house price next year

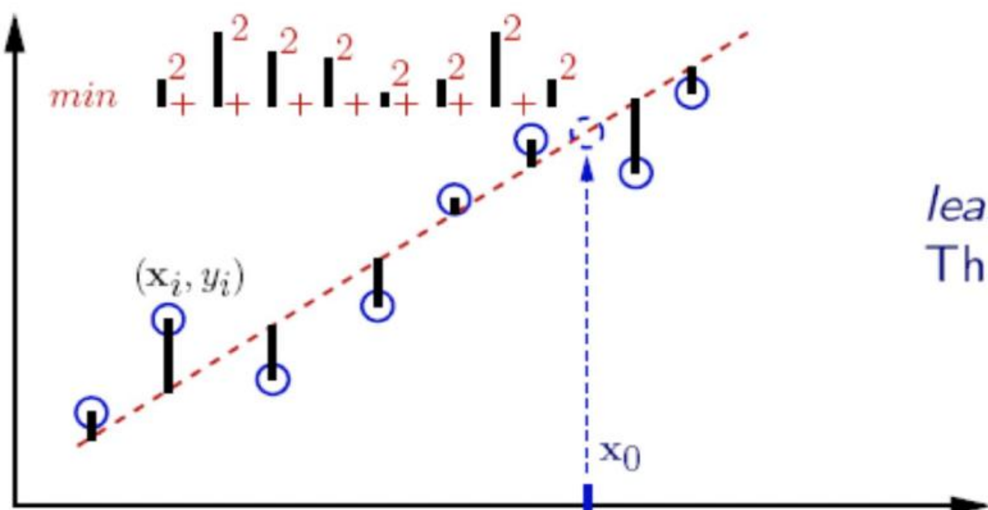


Least Mean Squares (LMS) Regression

Given a dataset $S = \{(x_i, y_i)\}_{i=1..m}, x_i \in \mathbb{R}^n, y \in \mathbb{R}$

$$\arg\min_{w,b} \frac{1}{2}\sum_{i}^{m}(y_i - (w^T x_i + b))^2$$

Learning: minimizing mean squared error



least squares (LSQ)
The fitted line is used as a predictor

Exercise

- Derive the stochastic gradient descent algorithm for solving LMS regression

$$\arg\min_{w,b} \frac{1}{2} \sum_{i}^{m} (y_i - (w^T x_i + b))^2$$

What you will learn today

- Optimization
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- Neural network / Deep learning
  - Non-linear classifier
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Accuracy and Error Rate

True label	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-
Predicted label	-	-	-	-	+	-	+	-	-	-	-	+	-	+	-	-

Error rate = 3/16

Accuracy = 13/16

Accuracy and Error Rate

True label	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-
Predicted label	-	-	-	-	+	-	+	-	-	-	-	+	-	+	-	-

Error rate = 3/16 = 19%

Accuracy = 13/16 = 81%

When data is unbalanced, check the performance of majority baseline

True label	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-
Predicted label	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Confusion Matrix

True label	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-
Predicted label	-	-	-	-	+	-	+	-	-	-	-	+	-	+	-	-

	True Label Positive	True Label Negative
Predicted Label Positive	2 True Positive (TP)	2 False Positive (FP)
Predicted Label Negative	1 False Negative (FN)	16 True Negative (TN)

Accuracy = (TP+TN)/(TP+TN+FN+FP)

Precision, Recall

True label	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-
Predicted label	-	-	-	-	+	-	+	-	-	-	-	+	-	+	-	-

	True Label Positive	True Label Negative
Predicted Label Positive	2 True Positive (TP)	2 False Positive (FP)
Predicted Label Negative	1 False Negative (FN)	16 True Negative (TN)

Accuracy = (TP+TN)/(TP+TN+FN+FP)

Precision = (TP)/(TP+FP)

Recall = (TP)/(TP+FN)

F1 Score

- Harmonic mean of precision and recall:

$$\frac{1}{F_1} = \left(\frac{1}{P} + \frac{1}{R}\right)/2$$

$$F_1 = \frac{2TP}{2TP + FP + FN}$$

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- Neural network / Deep learning
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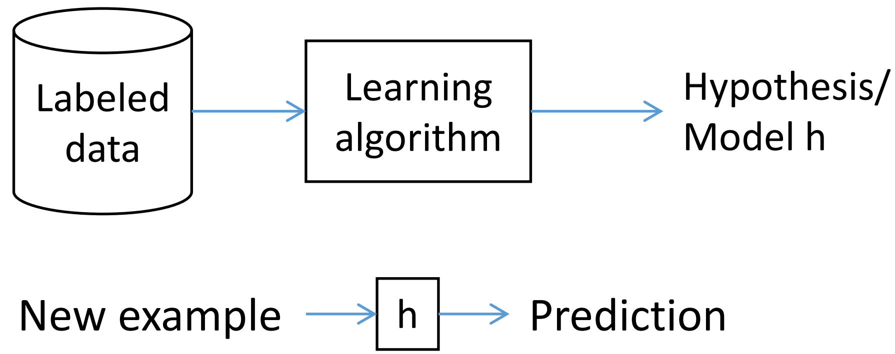
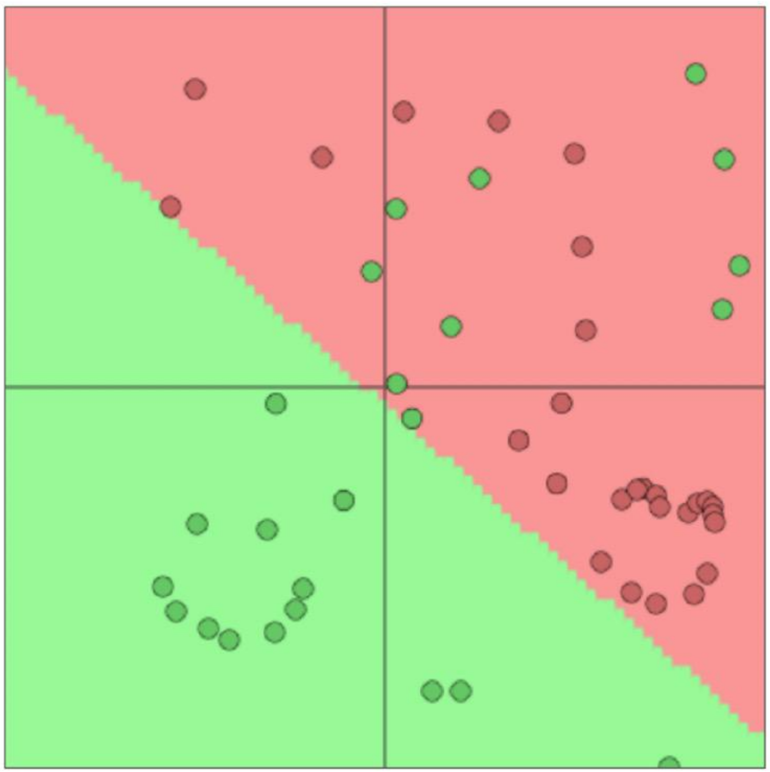
Checkpoint: The bigger picture

- Supervised learning: instances, concepts, and hypotheses
- Specific learners
  - Decision trees
  - K-NN
  - Perceptron
  - Logistic regression
- General ML ideas
  - Feature vectors
  - Overfitting
  - Probabilistic model



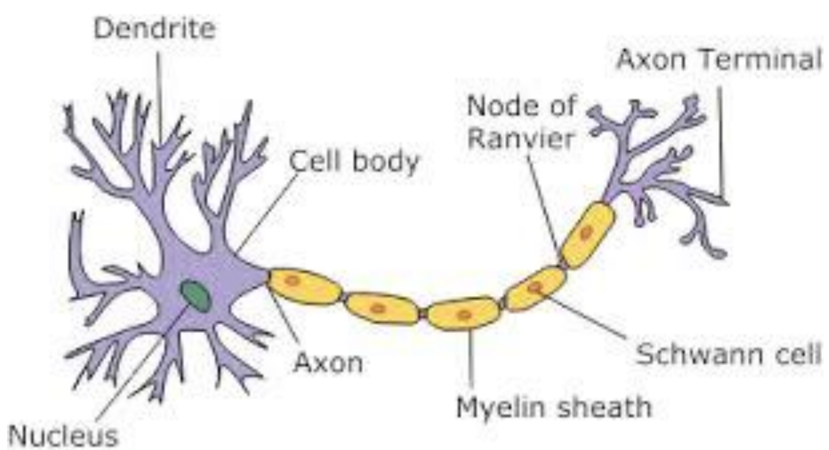
Non-Linear Decision Boundary

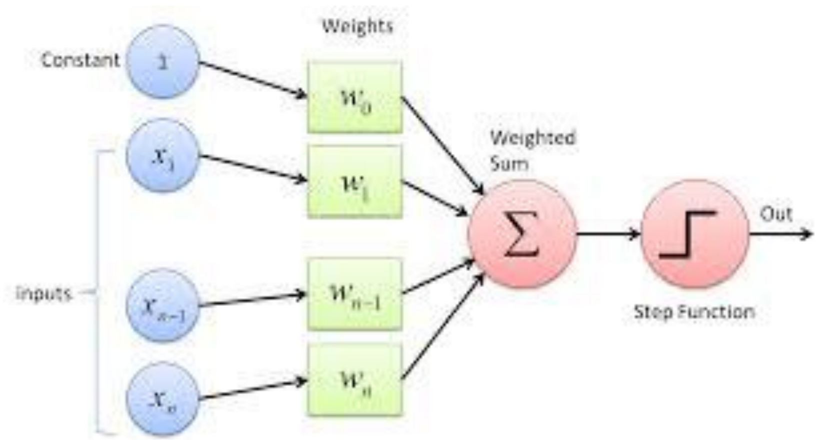


Neural Networks

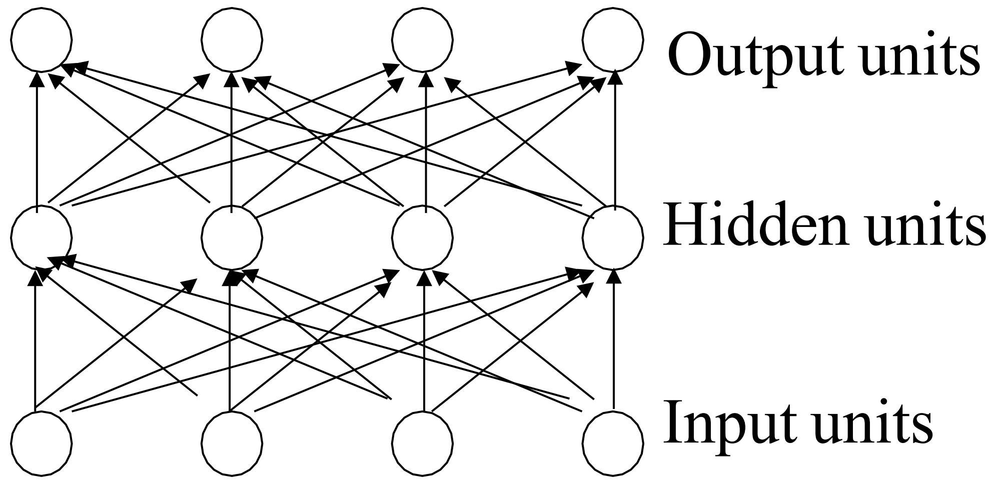
- Design to mimic the brain.

- Artificial neural networks are not nearly as complex or intricate as the actual brain structure





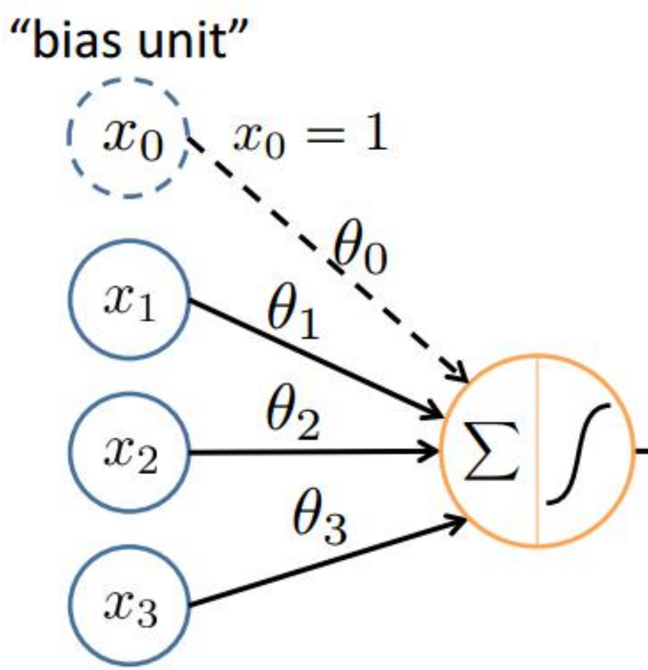
Feed-forward neural network



Layered feed-forward network

- Neural networks are made up of **nodes** or **units**, connected by **links**
- Each link has an associated **weight** and **activation level**
- Each node has an **input function** (typically summing over weighted inputs), an **activation function**, and an **output**

Neuron Model Example: Logistic Unit

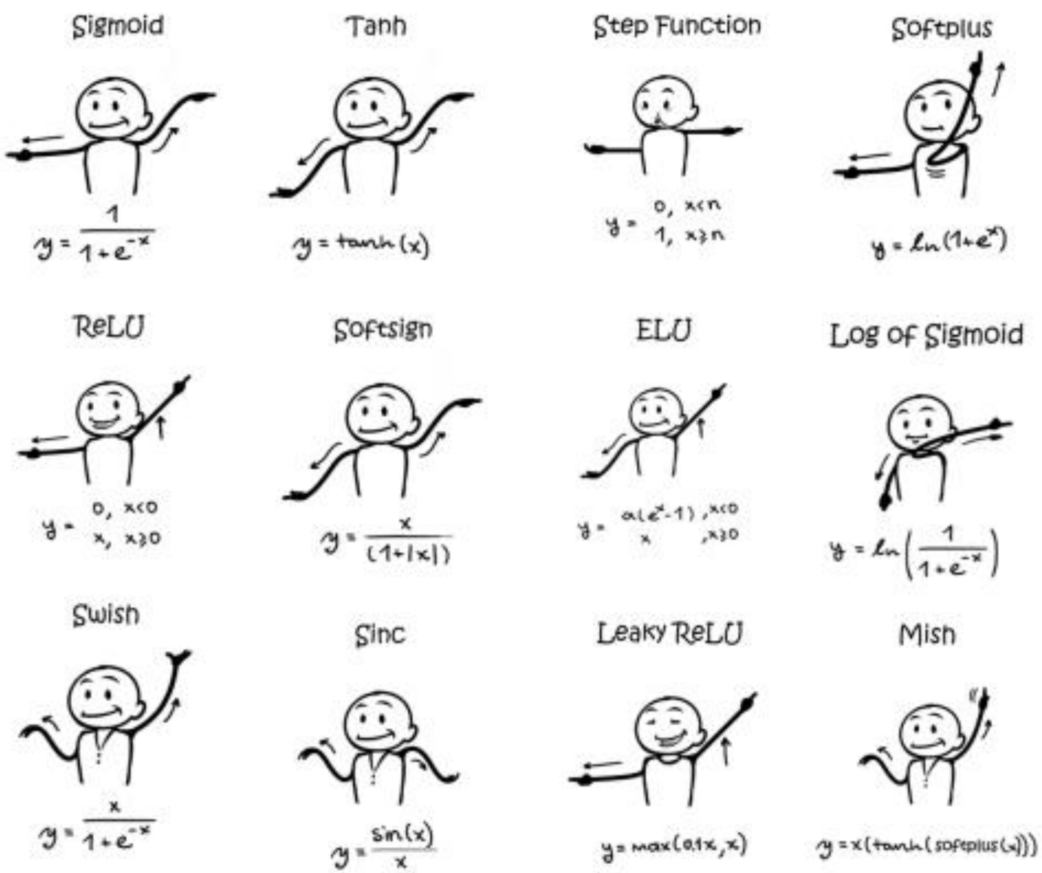


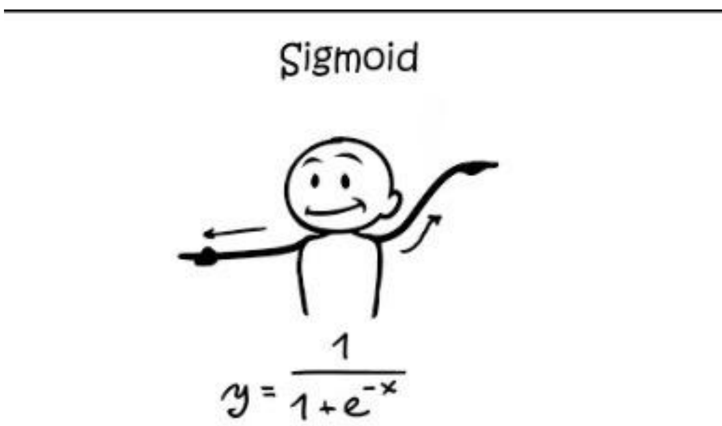
$$\mathbf{x} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad \boldsymbol{\theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

$$h_{\boldsymbol{\theta}}(\mathbf{x}) = g\left(\boldsymbol{\theta}^\intercal \mathbf{x}\right) = \frac{1}{1 + e^{-\boldsymbol{\theta}^\intercal \mathbf{x}}}$$

Sigmoid (logistic) activation function: $g(z) = \dfrac{1}{1 + e^{-z}}$

Activation function

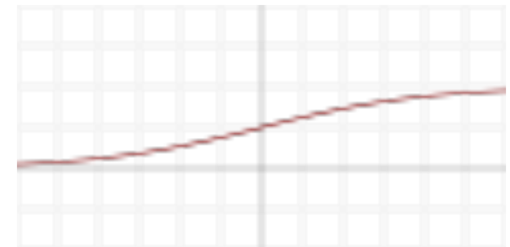




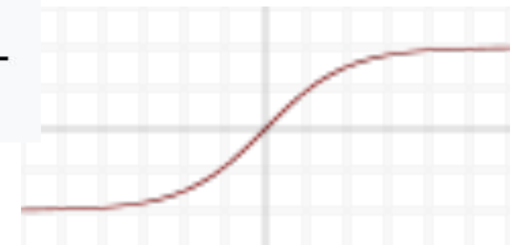
https://sefiks.com/2020/02/02/dance-moves-of-deep-learning-activation-functions/
by Sefik Ilkin Serengil

Activation functions

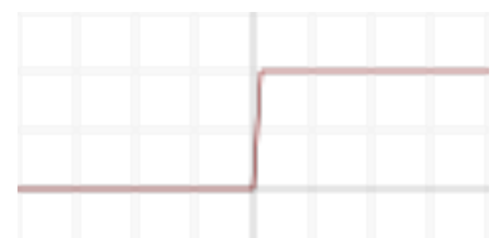
- sigmoid function $$f(x) = \frac{1}{1+e^{-x}}$$



- hyperbolic tangent $$\tanh(x) \doteq \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

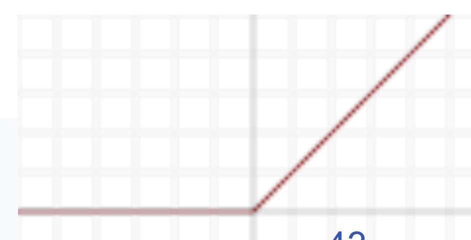


- step function $$\begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

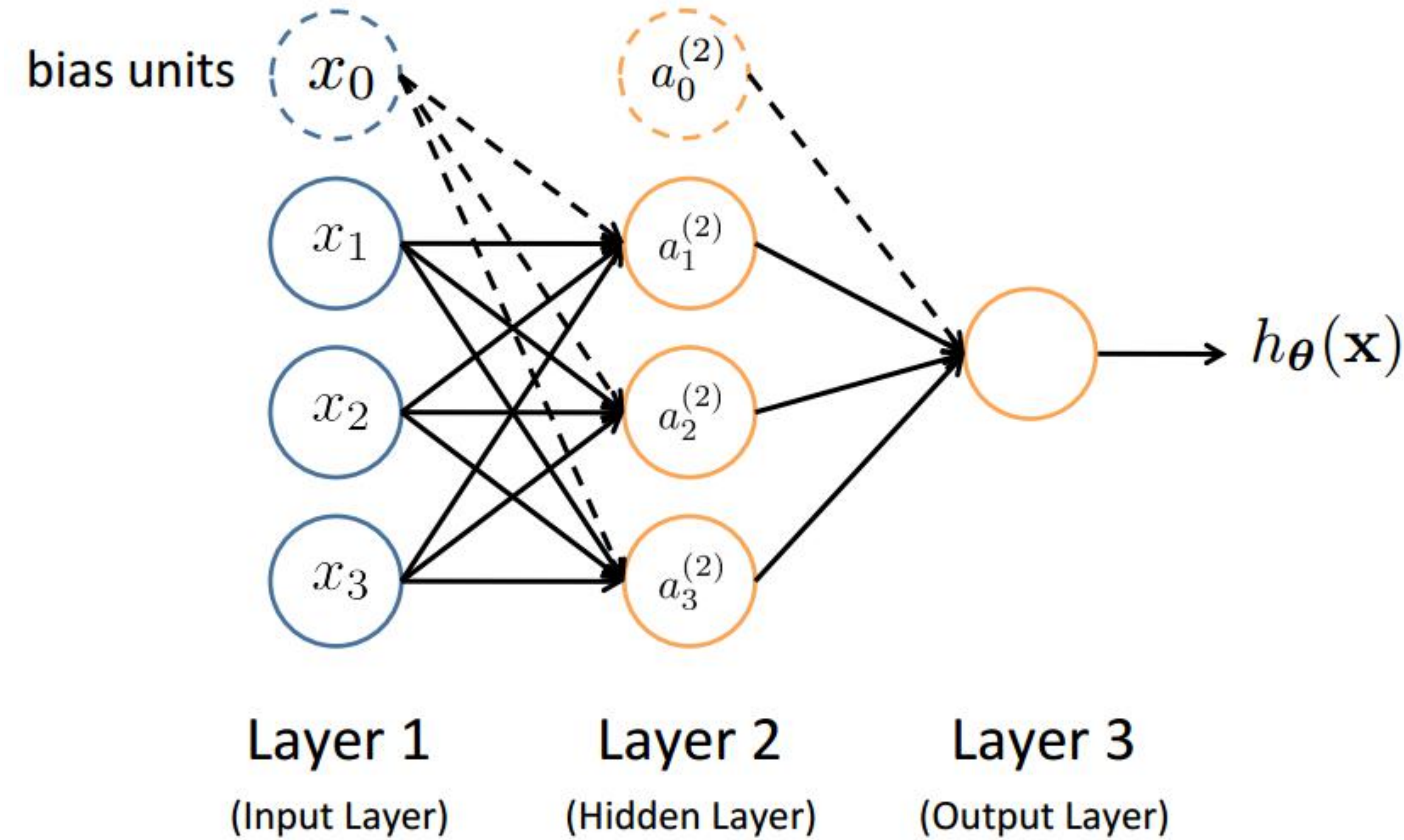


- Rectified linear unit (ReLU)

$$(x)^{+} \doteq \begin{cases} 0 & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$$



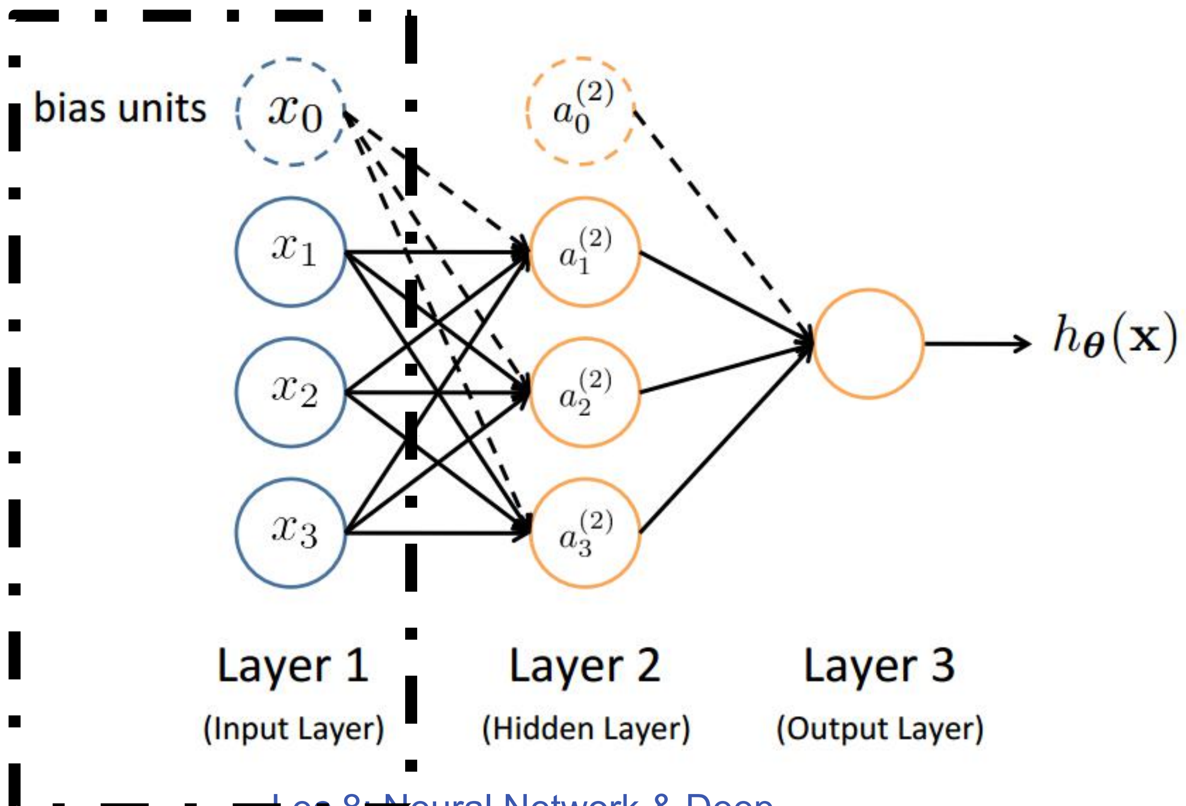
Neural Network



Slide by Andrew Ng

Feed-Forward Process

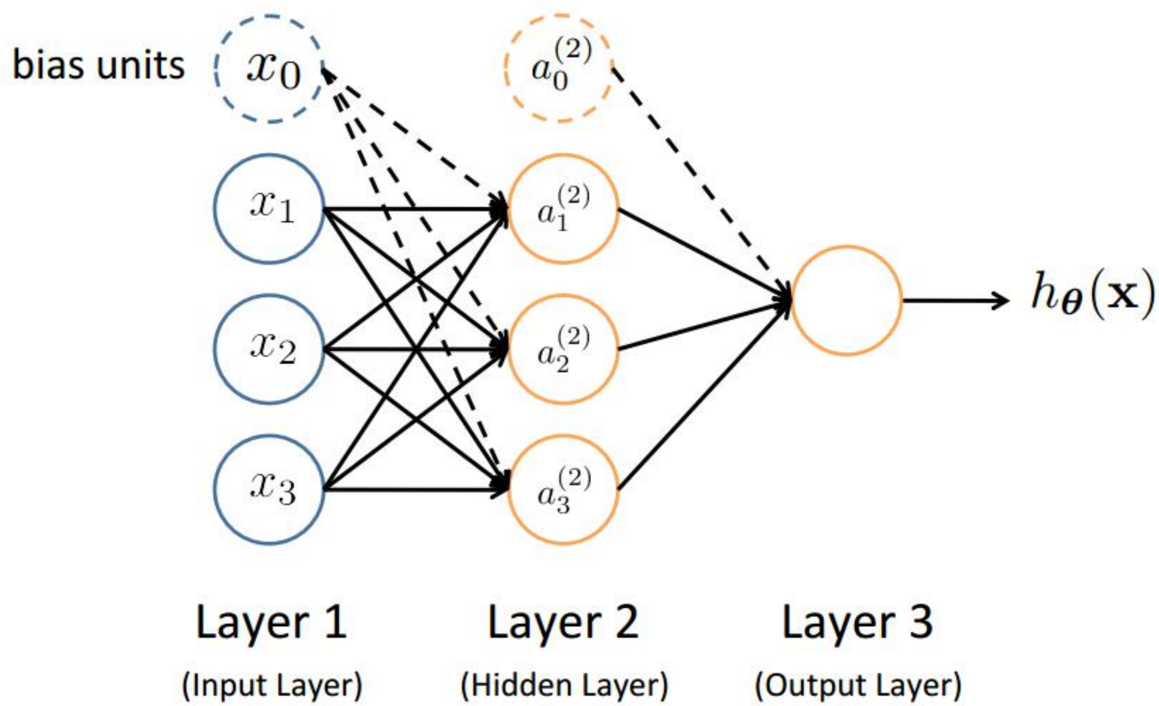
- Input layer units are set by some exterior function (think of these as **sensors**), which causes their output links to be **activated** at the specified level



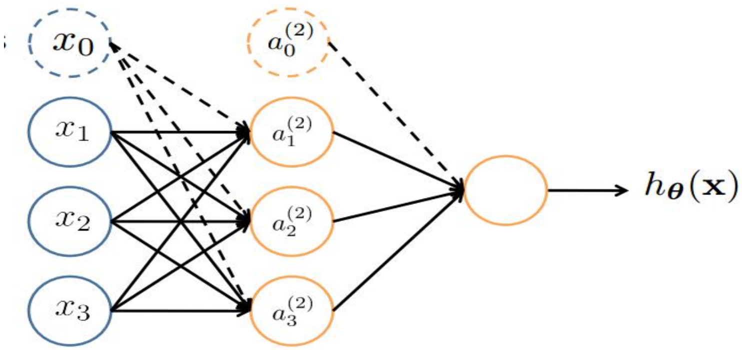
Based on slide by T. Finin, M. desJardins, L Getoor, R. Par

Feed-Forward Process

- Working forward through the network, the **input function** of each unit is applied to compute the input value
- The **activation function** transforms this input function into a final value



Neural Network



$a_i^{(j)}$ = "activation" of unit i in layer j

$\Theta^{(j)}$ = weight matrix controlling function mapping from layer j to layer j + 1

$$a_1^{(2)} = g(\Theta_{10}^{(1)} x_0 + \Theta_{11}^{(1)} x_1 + \Theta_{12}^{(1)} x_2 + \Theta_{13}^{(1)} x_3)$$
$$a_2^{(2)} = g(\Theta_{20}^{(1)} x_0 + \Theta_{21}^{(1)} x_1 + \Theta_{22}^{(1)} x_2 + \Theta_{23}^{(1)} x_3)$$
$$a_3^{(2)} = g(\Theta_{30}^{(1)} x_0 + \Theta_{31}^{(1)} x_1 + \Theta_{32}^{(1)} x_2 + \Theta_{33}^{(1)} x_3)$$
$$h_\Theta(x) = a_1^{(3)} = g(\Theta_{10}^{(2)} a_0^{(2)} + \Theta_{11}^{(2)} a_1^{(2)} + \Theta_{12}^{(2)} a_2^{(2)} + \Theta_{13}^{(2)} a_3^{(2)})$$

If network has s_j units in layer j *and* s_{j+1} units in layer $j+1$, then $\Theta^{(j)}$ has dimension $s_{j+1} \times (s_j+1)$.

$$\Theta^{(1)} \in \mathbb{R}^{3\times 4} \qquad \Theta^{(2)} \in \mathbb{R}^{1\times 4}$$

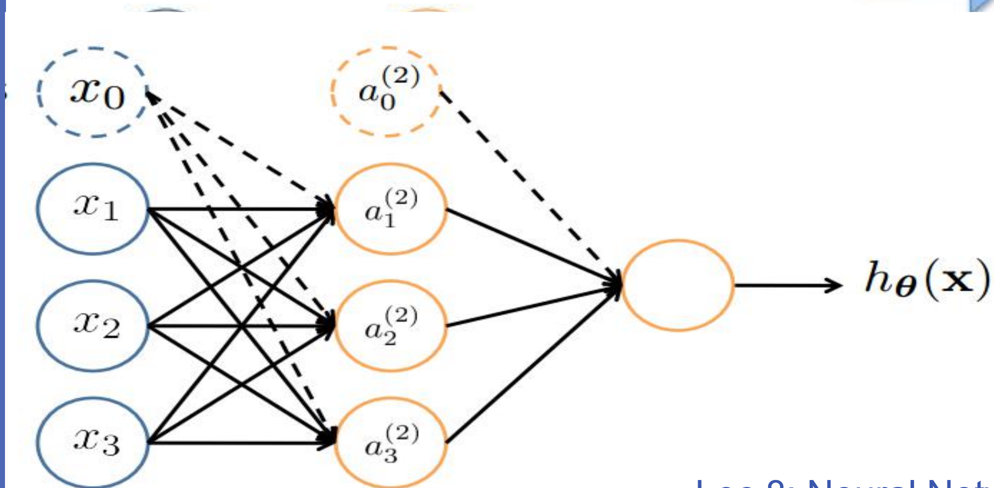
Vectorization

$$a_1^{(2)} = g\left(\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3\right) = g\left(z_1^{(2)}\right)$$

$$a_2^{(2)} = g\left(\Theta_{20}^{(1)}x_0 + \Theta_{21}^{(1)}x_1 + \Theta_{22}^{(1)}x_2 + \Theta_{23}^{(1)}x_3\right) = g\left(z_2^{(2)}\right)$$

$$a_3^{(2)} = g\left(\Theta_{30}^{(1)}x_0 + \Theta_{31}^{(1)}x_1 + \Theta_{32}^{(1)}x_2 + \Theta_{33}^{(1)}x_3\right) = g\left(z_3^{(2)}\right)$$

$$h_\Theta(\mathbf{x}) = g\left(\Theta_{10}^{(2)}a_0^{(2)} + \Theta_{11}^{(2)}a_1^{(2)} + \Theta_{12}^{(2)}a_2^{(2)} + \Theta_{13}^{(2)}a_3^{(2)}\right) = g\left(z_1^{(3)}\right)$$



Feed-Forward Steps:

$$\mathbf{z}^{(2)} = \Theta^{(1)}\mathbf{x}$$

$$\mathbf{a}^{(2)} = g(\mathbf{z}^{(2)})$$

$$\text{Add } a_0^{(2)} = 1$$

$$\mathbf{z}^{(3)} = \Theta^{(2)}\mathbf{a}^{(2)}$$

$$h_\Theta(\mathbf{x}) = \mathbf{a}^{(3)} = g(\mathbf{z}^{(3)})$$

Exercise

- Why do we need non-linear activation functions?
  - What happen if $g(z) = z$

Feed-Forward Steps:

$$\mathbf{z}^{(2)} = \Theta^{(1)}\mathbf{x}$$

$$\mathbf{a}^{(2)} = g(\mathbf{z}^{(2)})$$

$$\text{Add } a_0^{(2)} = 1$$

$$\mathbf{z}^{(3)} = \Theta^{(2)}\mathbf{a}^{(2)}$$

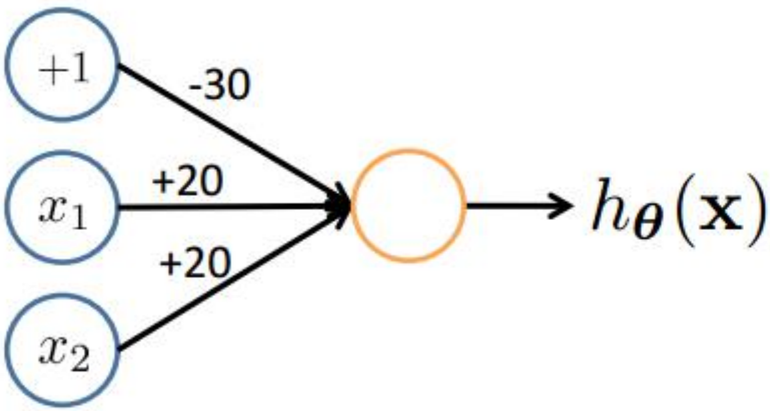
$$h_\Theta(\mathbf{x}) = \mathbf{a}^{(3)} = g(\mathbf{z}^{(3)})$$

Non-Linear Representations

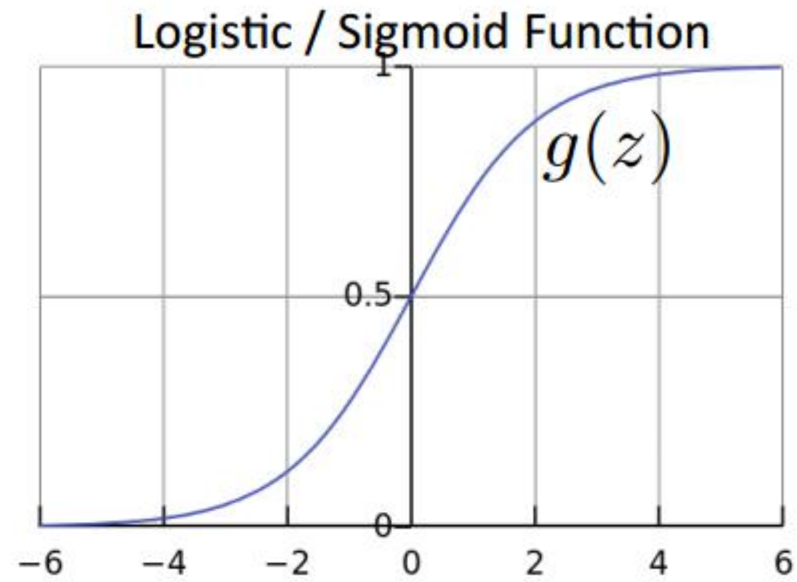
Simple example: AND

$x_1, x_2 \in \{0, 1\}$

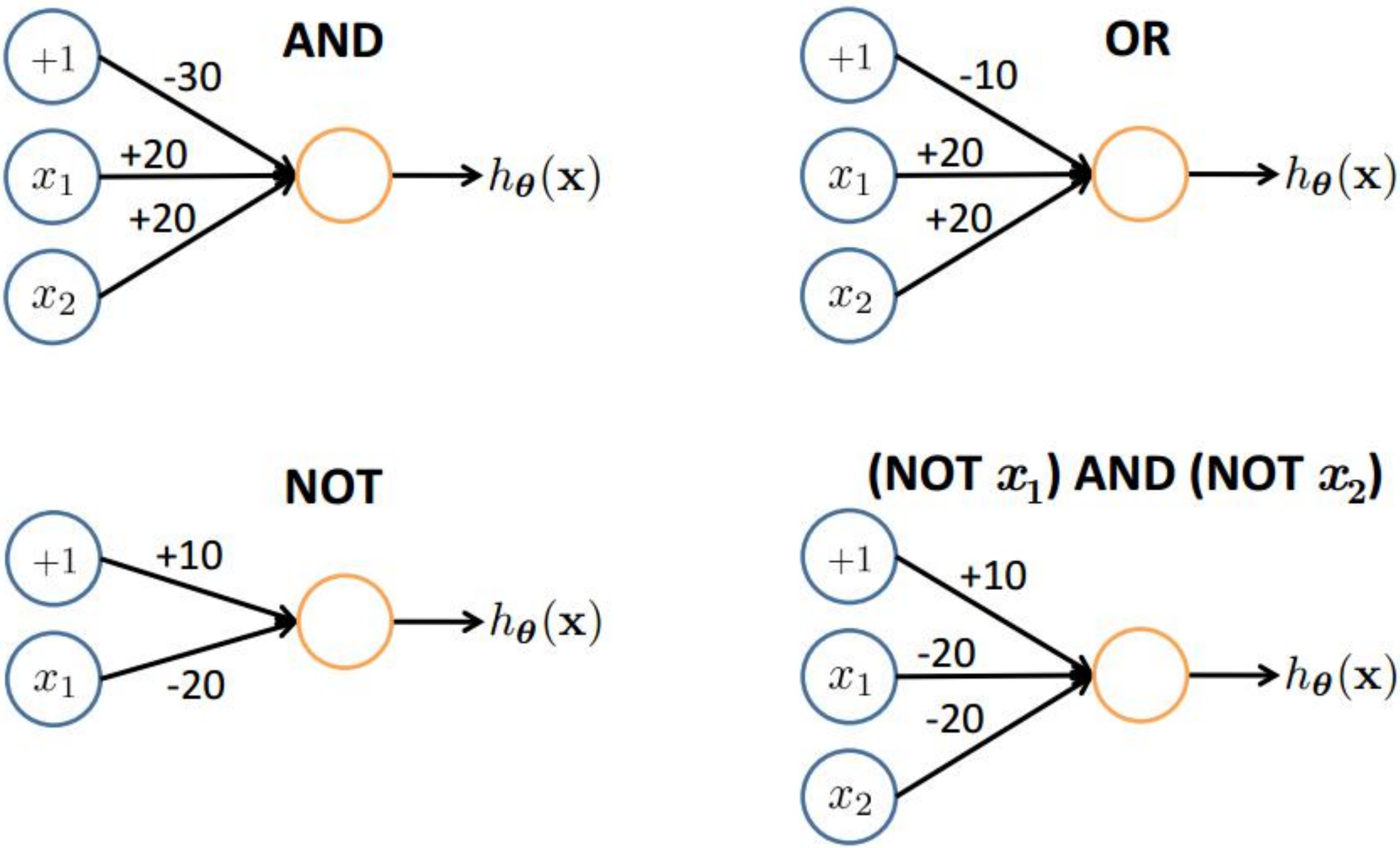
$y = x_1 \text{ AND } x_2$

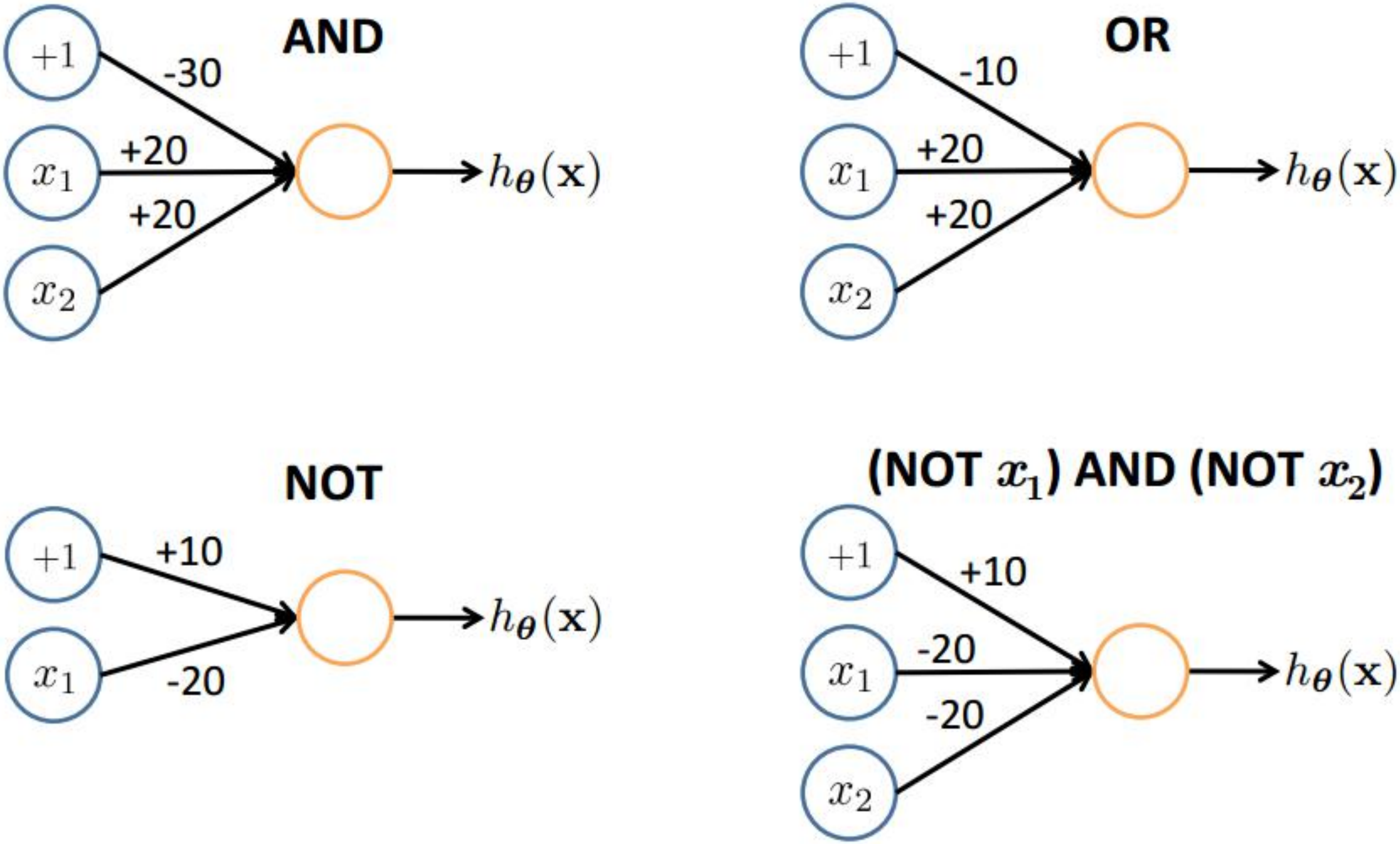


$$h_{\Theta}(\mathbf{x}) = g(-30 + 20x_1 + 20x_2)$$



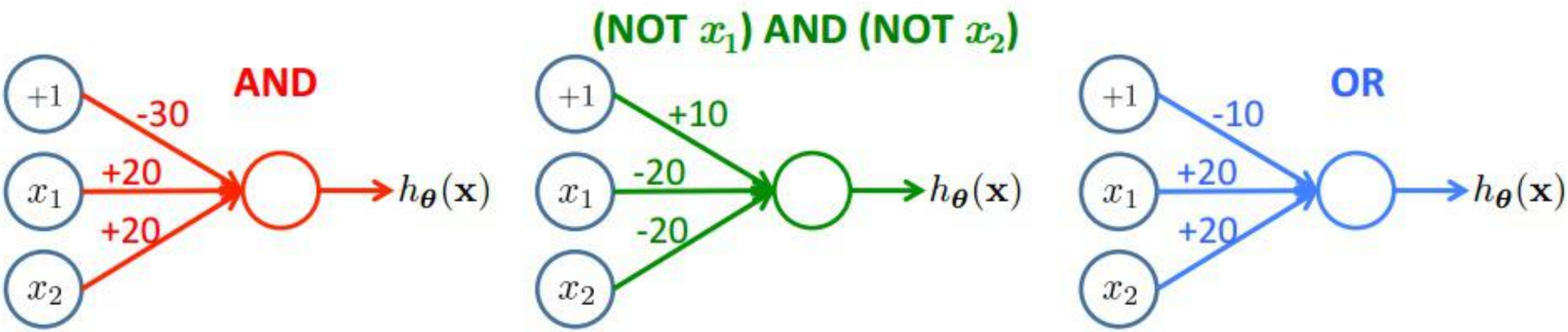
x_1	x_2	$h_{\Theta}(\mathbf{x})$
0	0	$g(-30) \approx 0$
0	1	$g(-10) \approx 0$
1	0	$g(-10) \approx 0$
1	1	$g(10) \approx 1$


AND
$+1$
x_1
x_2
-30
+20
+20
$h_\theta(\mathbf{x})$
OR
$+1$
x_1
x_2
-10
+20
+20
$h_\theta(\mathbf{x})$
NOT
$+1$
x_1
+10
-20
$h_\theta(\mathbf{x})$
(NOT x_1) AND (NOT x_2)
$+1$
x_1
x_2
+10
-20
-20
$h_\theta(\mathbf{x})$

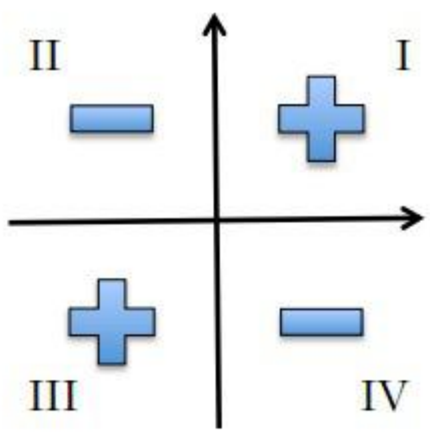


AND
+1
-30
x_1
+20
+20
x_2
$h_\theta(\mathbf{x})$
OR
+1
-10
x_1
+20
+20
x_2
$h_\theta(\mathbf{x})$
NOT
+1
+10
x_1
-20
$h_\theta(\mathbf{x})$
(NOT x_1) AND (NOT x_2)
+1
+10
x_1
-20
-20
x_2
$h_\theta(\mathbf{x})$

Combining Representations to Create Non-Linear Functions

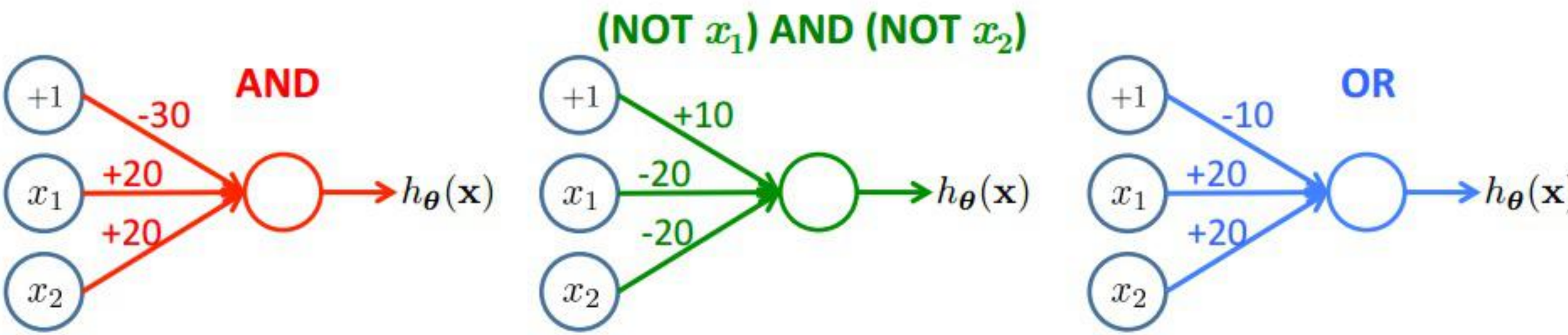


XNOR

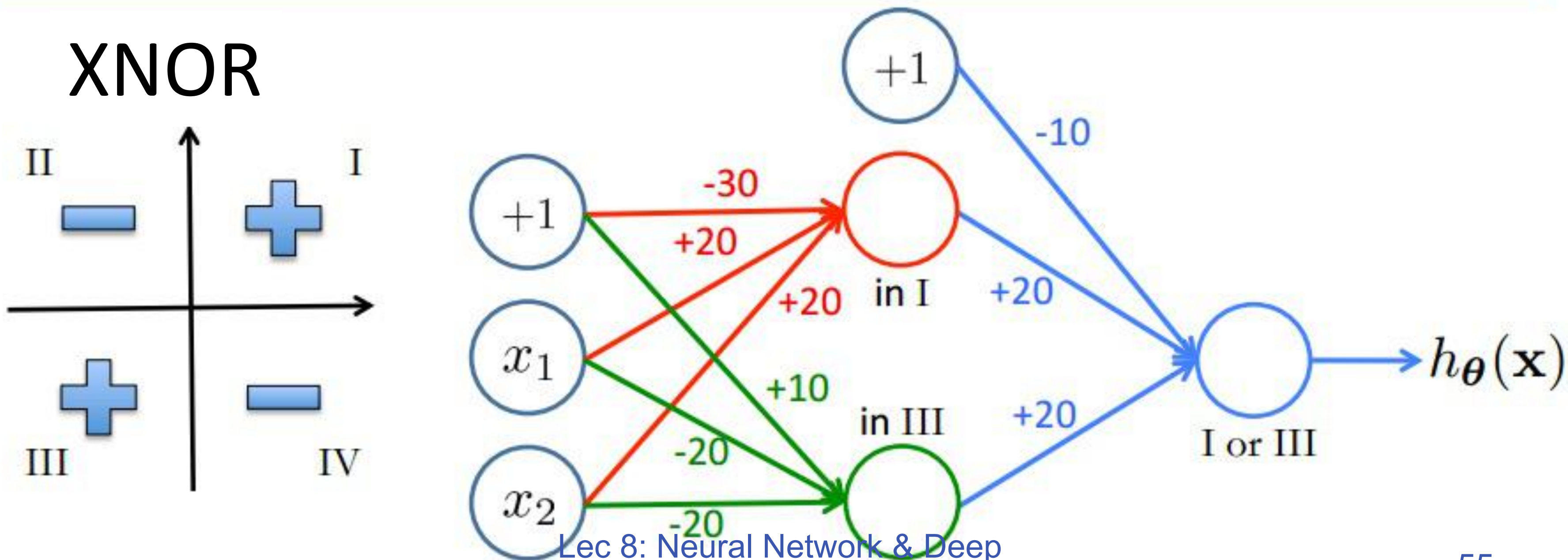


XNOR

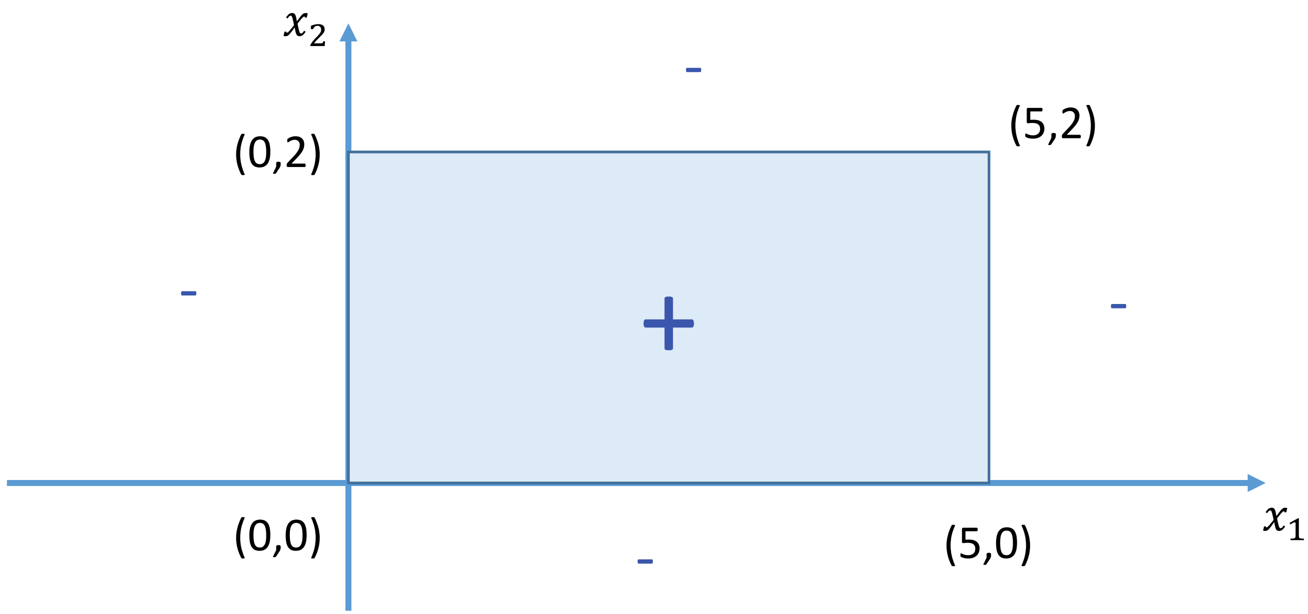
Combining Representations to Create Non-Linear Functions



XNOR



Exercise



Feed-Forward Steps:

$$\mathbf{z}^{(2)} = \Theta^{(1)}\mathbf{x}$$

$$\mathbf{a}^{(2)} = g(\mathbf{z}^{(2)})$$

$$\text{Add } a_0^{(2)} = 1$$

$$\mathbf{z}^{(3)} = \Theta^{(2)}\mathbf{a}^{(2)}$$

$$h_\Theta(\mathbf{x}) = \mathbf{a}^{(3)} = g(\mathbf{z}^{(3)})$$

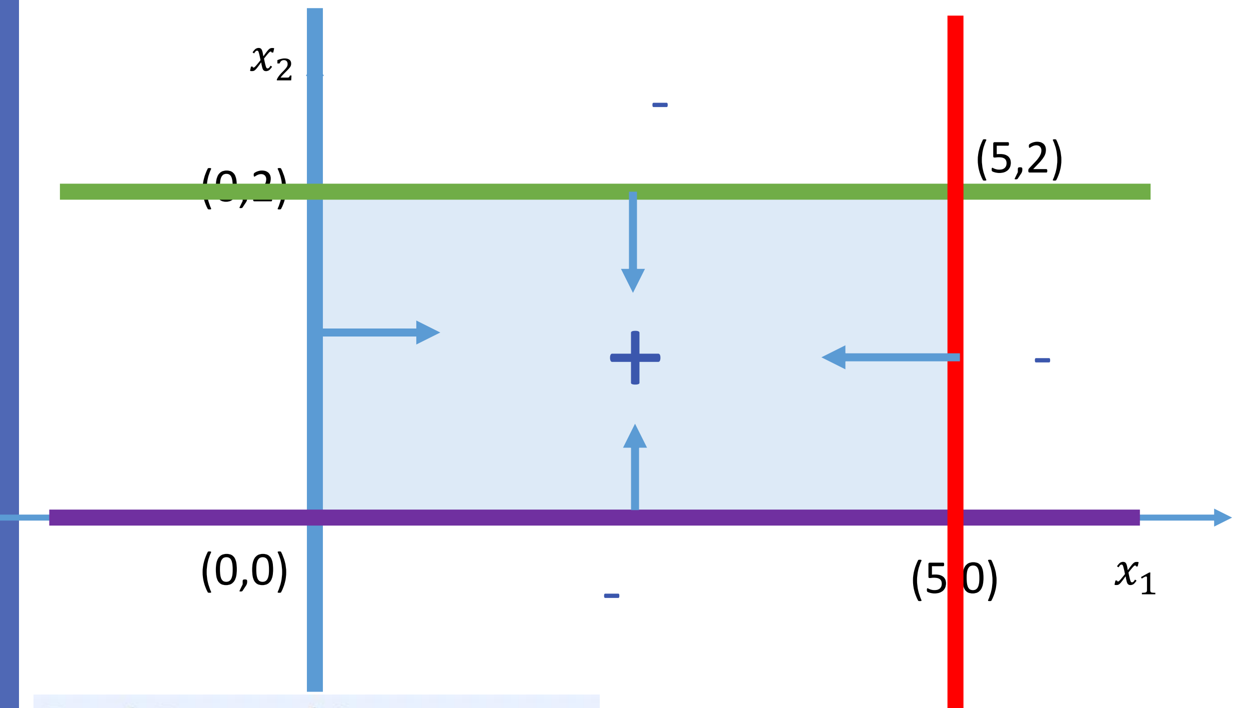
If all the samples inside the rectangle are positive; otherwise are negative

Show a feedforward NN can classify all the samples correctly

For simplicity, we assume g(z) is a step function.

What are $\Theta^{(1)}$ and $\Theta^{(2)}$

Exercise



$$\begin{cases} x_1 > 0 \\ 5 - x_1 > 0 \\ x_2 > 0 \\ 2 - x_2 > 0 \end{cases}$$

$$\Rightarrow \Theta^{(1)} = \begin{array}{c} \begin{bmatrix} 0 & 1 & 0 \\ 5 & -1 & 0 \\ 0 & 0 & 1 \\ 2 & 0 & -1 \end{bmatrix} \\ \Theta_0^{(1)} \ \Theta_1^{(1)} \ \Theta_2^{(1)} \end{array}$$

$$-3.5 + a_1 + a_2 + a_3 + a_4 > 0$$

$$\Theta^{(2)} = \begin{array}{c} [-3.5 \quad 1 \quad 1 \quad 1 \quad 1] \\ \Theta_0^{(2)} \ \Theta_1^{(2)} \ \Theta_2^{(2)} \Theta_3^{(2)} \ \Theta_4^{(2)} \end{array}$$

Feed-Forward Steps:

$$\mathbf{z}^{(2)} = \Theta^{(1)}\mathbf{x}$$

$$\mathbf{a}^{(2)} = g(\mathbf{z}^{(2)})$$

$$\text{Add } a_0^{(2)} = 1$$

$$\mathbf{z}^{(3)} = \Theta^{(2)}\mathbf{a}^{(2)}$$

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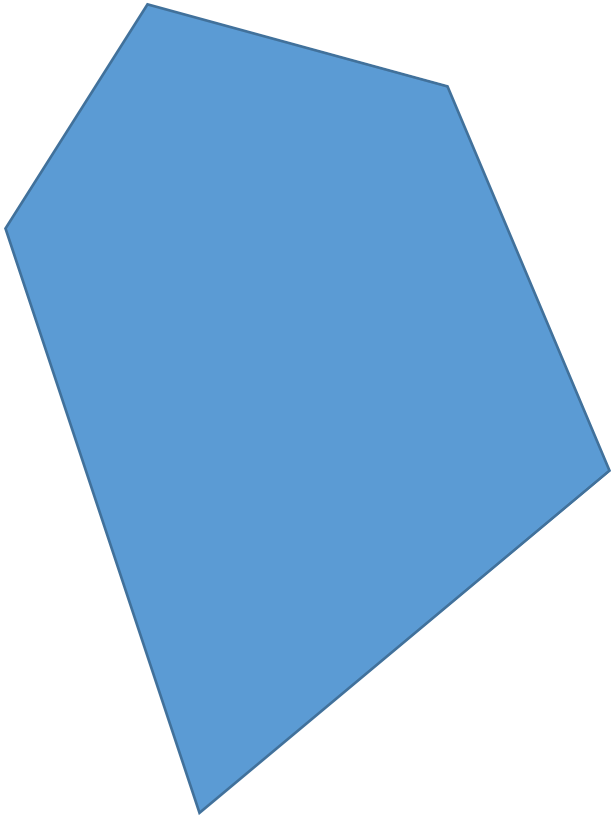
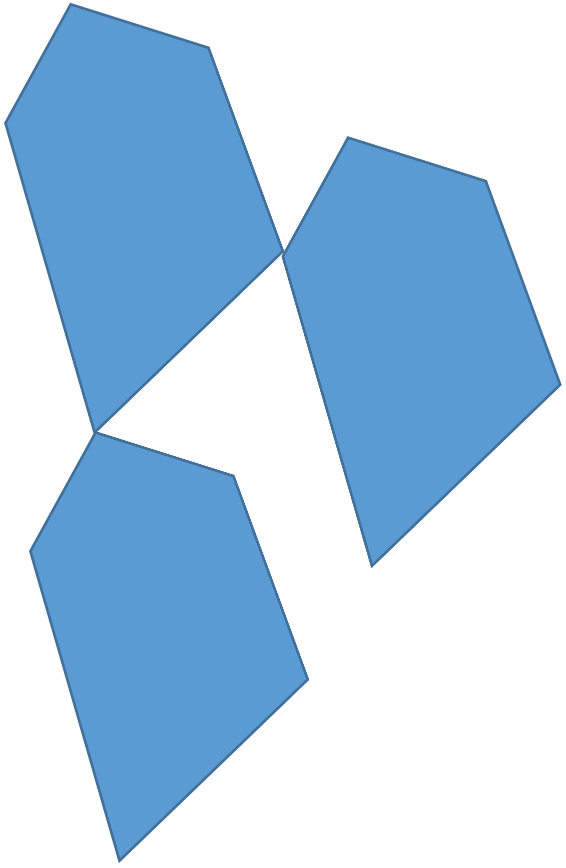
Arbitrary Decision Boundary



Arbitrary Decision Boundary



Neural Network Training Animation

- https://playground.tensorflow.org/