CS M51A, Winter 2021, Assignment 5
(Total Mark: 110 points, 11%)

Due: Wed Feb 10th, 10:00 AM Pacific Time

Student Name:

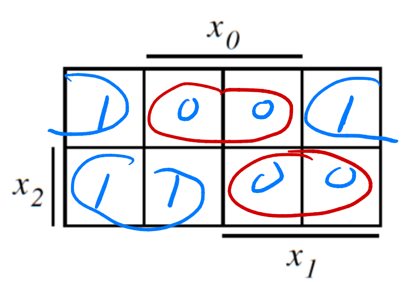
Student ID:

Note: You must complete the assignments entirely on your own, without discussing with others.

1. Given the following table:

x_2	x_1	x_0	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

(a) (4 points) Fill out the k-maps for this table.



(b) (4 points) Write the minimal sum of products for F.

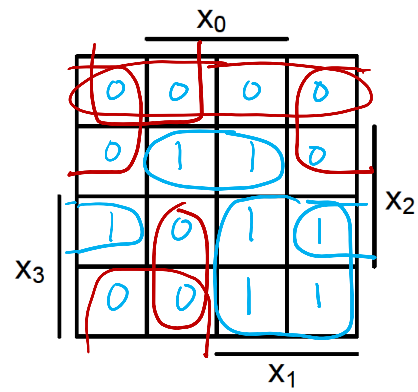
$F = x_2'x_0' + x_2x_1'$

(c) (4 points) Write the minimal product of sums for F.

$F = (x_0' + x_2)(x_1' + x_2')$

2. Given $f(x_3, x_2, x_1, x_0) = x_3x_2x_1x_0' + x_3x_2x_1x_0 + x_3'x_2x_1'x_0 + x_3'x_2x_1x_0 + x_3x_2x_1'x_0' + x_3x_2x_1x_0' + x_3x_2'x_1x_0 + x_3x_2'x_1x_0'$

(a) (8 points) Fill out the following K-maps.



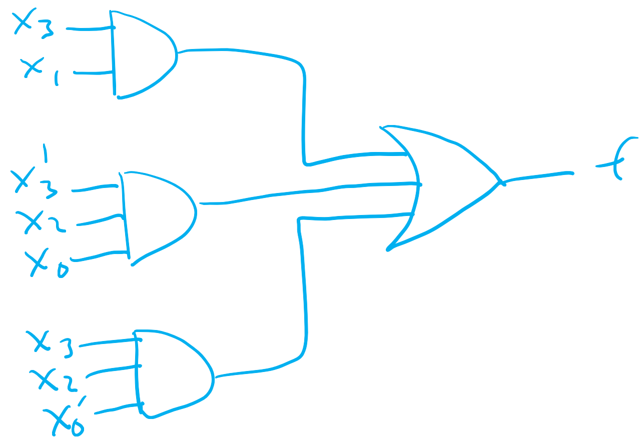
(b) (4 points) Write the minimal sum of products expression for f.

$f = x_3x_1 + x_3'x_2x_0 + x_3x_2x_0'$

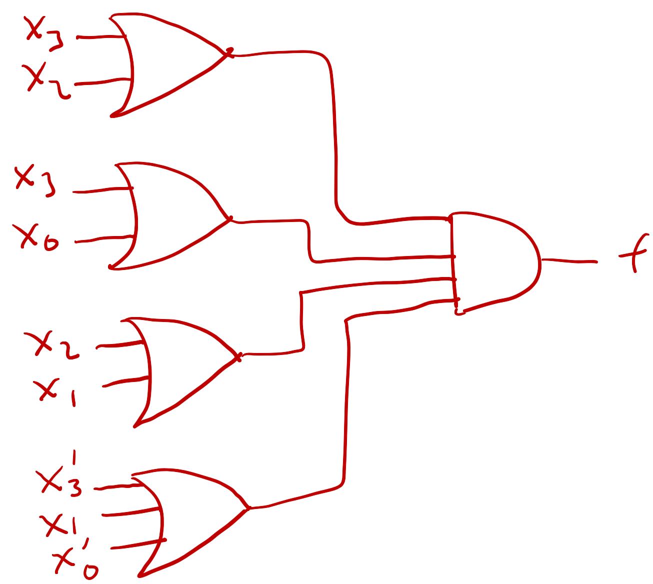
(c) (4 points) Write the minimal product of sums expression for f.

$f = (x_3 + x_2)(x_3 + x_0)(x_2 + x_1)(x_3' + x_1 + x_0')$

(d) (4 points) Draw the gate level design for (b)



(e) (4 points) Draw the gate level design for (c)



3. Consider a system that takes a decimal number (i.e. 0 to 9) as an input. The input is presented using a 4-bit unsigned binary code (x_3, x_2, x_1, x_0). For example, if the input is 7, $x_3 = 0, x_2 = 1, x_1 = 1, x_0 = 1$. The system has a 2-bit output (z_1, z_0). z_1 is one when the input is an odd number, otherwise it is zero. z_0 is one when the input is larger or equal to 5, otherwise it is zero.

(a) (8 points) Fill the truth table for this system.

$\cancel{x_1}$ x_3	x_2	x_1	x_0	z_1	z_0
0	0	0	0	0	0
0	0	0	1	1	0
0	0	1	0	0	0
0	0	1	1	1	0
0	1	0	0	0	0
0	1	0	1	1	1
0	1	1	0	0	1
0	1	1	1	1	1
1	0	0	0	0	1
1	0	0	1	1	1
1	0	1	0	x	x
1	0	1	1	x	x
1	1	0	0	x	x
1	1	0	1	x	x
1	1	1	0	x	x
1	1	1	1	x	x

(b) (4 points) Fill out the k-maps for this system (Left: z_1, Right: z_0).

z_1 (labels: x_0 over columns 2–3, x_1 under columns 3–4, x_2 beside rows 2–3, x_3 beside rows 3–4)

0	1	1	0
0	1	1	0
x	x	x	x
0	1	x	x

z_0 (labels: x_0 over columns 2–3, x_1 under columns 3–4, x_2 beside rows 2–3, x_3 beside rows 3–4)

0	0	0	0
0	1	1	1
x	x	x	x
1	1	x	x

(c) (4 points) Write the minimal sum of products expression for the outputs (for both z_1 and z_0).

$Z_1 = X_0$

$Z_0 = X_3 + X_0X_2 + X_1X_2$

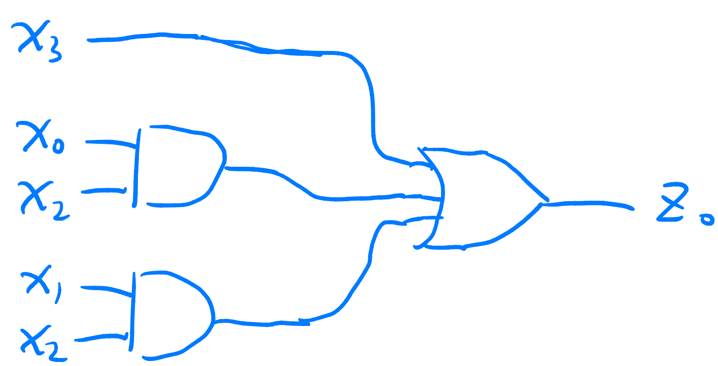
(d) (4 points) Write the minimal product of sums expression for the outputs (for both z_1 and z_0).

$z_1 = x_6$

$z_0 = (x_3 + x_2)(x_3 + x_1 + x_0)$

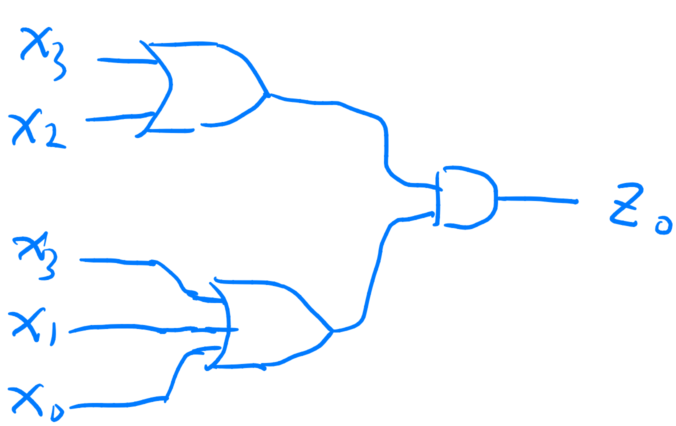
(e) (4 points) Draw a gate level design for (b) (for both z_1 and z_0)

x_6 — z_1



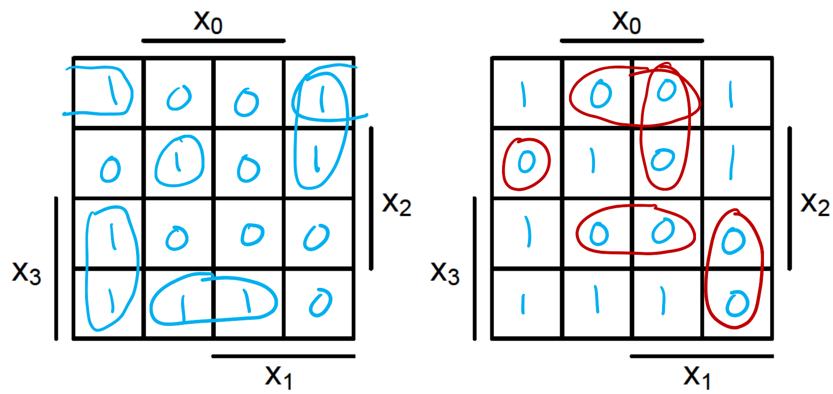
(f) (4 points) Draw a gate level design for (c) (for both z_1 and z_0)

x_6 — z_1



4. Using K-maps, find the minimal SOP and POS that are equivalent to the following expressions ($dc(\cdots)$ indicates the "don't care" terms):

(a) (5 Points) $F(x_3, x_2, x_1, x_0) = \Pi M(1, 3, 4, 7, 10, 13, 14, 15)$ Maxterm = put 0

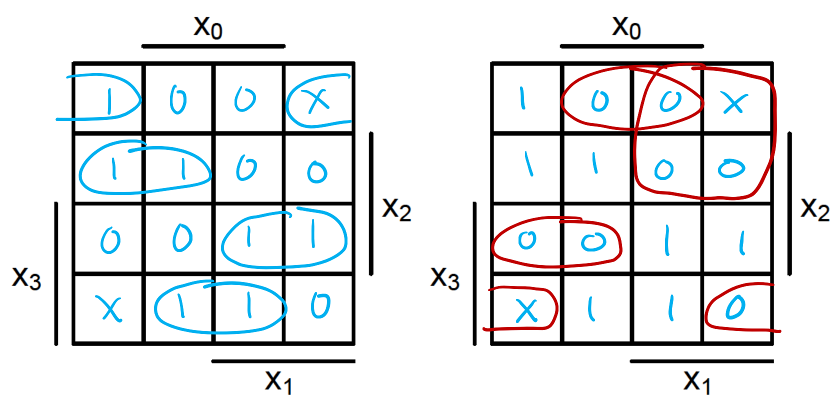


This can also be $x_2' x_1' x_0'$

$$F = \boxed{x_3' x_2' x_0'} + x_3' x_1 x_0' + x_3 x_1' x_0' + x_3 x_2' x_0 + x_3' x_2 x_1' x_0$$

$$F = (x_3 + x_2 + x_0')(x_3 + x_1' + x_0')(x_3' + x_2' + x_0')(x_3' + x_1' + x_0)(x_3 + x_2' + x_1 + x_0)$$

(b) (5 Points) $F(x_3, x_2, x_1, x_0) = \Sigma m(0, 4, 5, 9, 11, 14, 15)$, $dc(x_3, x_2, x_1, x_0) = \{m(2), m(8)\}$



This can also be $x_3' x_1' x_0'$

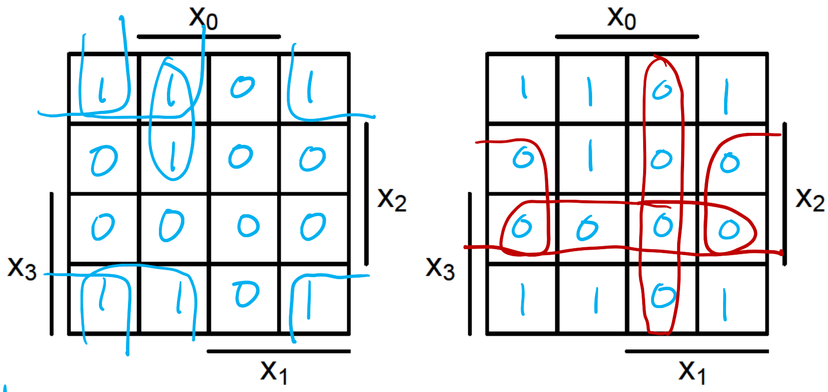
$$F = \boxed{x_3' x_2' x_0'} + x_3' x_2 x_1' + x_3 x_2 x_1 + x_3 x_2' x_0$$

$$F = \boxed{(x_3' + x_2 + x_0)}(x_3 + x_1')(x_3 + x_2 + x_0')(x_3' + x_2' + x_1)$$

This can also be

$(x_2 + x_1' + x_0)$

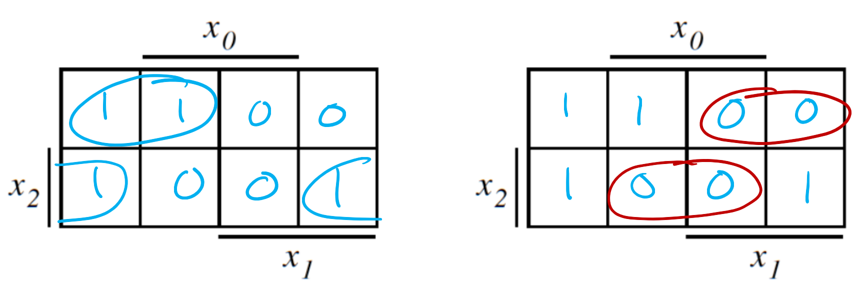
(c) (5 Points) $F(x_3, x_2, x_1, x_0) = \Sigma m(0, 1, 2, 5, 8, 9, 10)$



$F = x_2' x_0' + x_2' x_1' + x_3' x_1' x_0$

$F = (x_1' + x_0')(x_3' + x_2')(x_2' + x_0)$

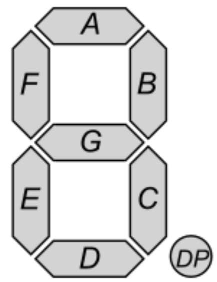
(d) (5 Points) $F(x_2, x_1, x_0) = \Sigma m(0, 1, 4, 6)$



$F = x_2' x_1' + x_2 x_0'$

$F = (x_2 + x_1')(x_2' + x_0')$

5. Given the following (uncompleted) high-level specification:
Input: $x \in \{0, 1, 2, \cdots, 9\}$, represented in unsigned binary code by 4 bits, $x = (x_3, x_2, x_1, x_0)$;
Output: $z \in \{0, 1\}$. z is one bit and indicates whether the "G" segment of the 7-segment display below is illuminated when the displayed number is x. **For example**: the "G" segment is illuminated when $x = 8$, while it will be off when $x = 0$.



(a) (8 Points) Write the sum of minterms and product of maxterms of z given $\{x_3, x_2, x_1, x_0\}$.

"G" is illuminated when $x = 2\ 3\ 4\ 5\ 6\ 8\ 9$

$Z = \Sigma m(2, 3, 4, 5, 6, 8, 9)$

$= \Pi M(0, 1, 7)$

$dc(x_3, x_2, x_1, x_0) = \Sigma m(10, 11, 12, 13, 14, 15)$

(b) (8 Points) Simplify the sum of minterms and product of maxterms in (a) using K-Map. (Hint: you may need to identify and utilize the "don't care" terms in this system)

x_0 (columns 2–3), x_1 (columns 3–4), x_2 (rows 2–3), x_3 (rows 3–4)

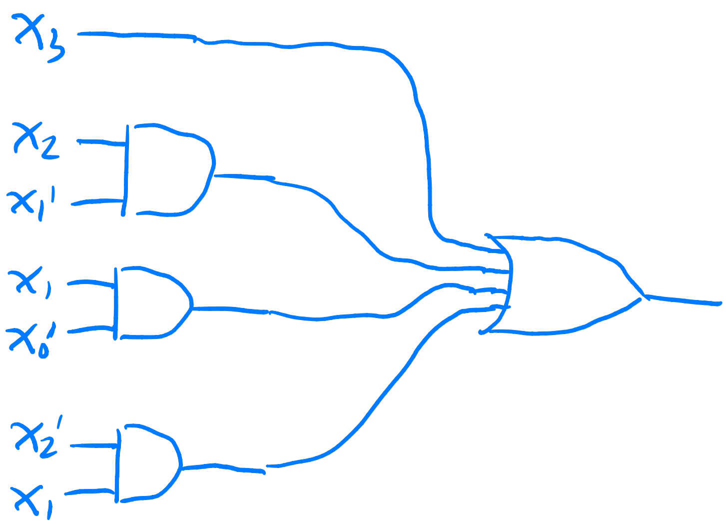
0	0	1	1
1	1	0	1
x	x	x	x
1	1	x	x

0	0	1	1
1	1	0	1
x	x	x	x
1	1	x	x

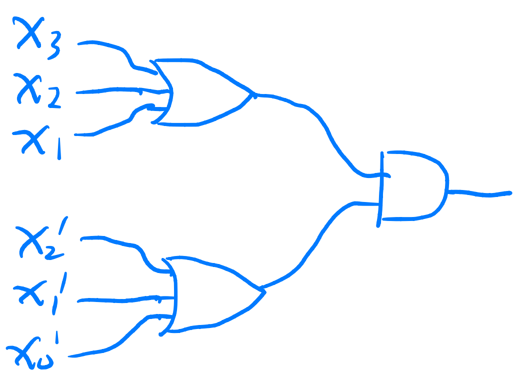
$Z = x_3 + x_2 x_1' + x_1 x_0' + x_2' x_1$ ---- SOP

$= (x_3 + x_2 + x_1)(x_2' + x_1' + x_0')$ --- POS

(c) (8 points) Draw a gate level design for the minimal SOP and POS in (b)



$Z = x_3 + x_2 x_1' + x_1 x_0' + x_2' x_1$



$Z = (x_3 + x_2 + x_1)(x_2' + x_1' + x_0')$

6. (2 Points) What is an advantage of a NAND-NAND network over a AND-OR network?

NAND-NAND networks are made from NAND gates, which use less transistors than AND gates and OR gates.