MATH 33A Final

CHARLES ZHANG

TOTAL POINTS

92 / 100

QUESTION 1

1 Problem 1 5 / 10

✓ **+ 5 pts Correct basis for orthogonal complement of L**

+ 5 pts Correctly uses Gram Schmidt on the basis obtained from the previous point, it doesn't matter whether it's the right basis

- 2 pts small mistake in computing orthonormal vectors

+ 0 pts Incorrect basis for orthogonal complement of L

✓ **+ 0 pts Incorrectly uses Gram-Schmidt (on basis, obtained from previous point, it doesn't matter whether it's the right basis)**

+ 0 pts Missing answer

+ 0 pts Does not use Gram-Schmidt on the basis (correct or not) obtained from the previous point, but on a collection of vectors that form a basis of a different subspace

The two vectors u_1 and u_2 are not orthogonal to each other!

QUESTION 2

2 Problem 2 10 / 10

✓ **+ 5 pts Correct basis for the image**

✓ **+ 5 pts Correct basis for the kernel (based on obtained RREF, it doesn't matter if it is the correct RREF)**

+ 3 pts Derives the basis for the kernel from the RREF making a small mistake

+ 0 pts Wrong basis for the kernel

- 2 pts Small mistake in computing RREF

+ 0 pts Wrong basis for the image

+ 0 pts Missing basis for the kernel

+ 0 pts Missing basis for the image

QUESTION 3

3 Problem 3 10 / 10

✓ **+ 6 pts Main Solution: Correctly set up system of equations. Includes setting up transition matrix solution.**

✓ **+ 4 pts Main Solution: Correctly solve system.**

- 1 pts Minor arithmetic errors.

- 2 pts Transition matrix solution incorrect. Did not invert.

+ 0 pts Incorrect.

+ 4 pts +4: Set up wrong system, but solved.

QUESTION 4

4 Problem 4 7 / 10

✓ **+ 5 pts Reasonable explanation of why det(A^-1)=1/det(A) using interpretation as expansion factor.**

+ 3 pts Reasonable explanation why A not invertible implies det(A)=0 via expansion factor interpretation.

✓ **+ 2 pts Reasonable explanation why det(A)=0 implies A not invertible using expansion factor interpretation.**

+ 2 pts Bonus Points: Explained what the expansion factor is.

+ 2 pts Bonus Points: Explained why det(AB)=det(A)det(A) using expansion factors.

+ 3 pts Reasonable explanation of the first property, but not using expansion factors.

+ 3 pts Reasonable explanation of second property but did not use expansion factors.

+ 4 pts Incorrect, but exhibited some understanding of determinant as expansion factor interpretation.

+ 0 pts incorrect

QUESTION 5

5 Problem 5 10 / 10

✓ **+ 3 pts Sets up system of equations correctly**

✓ + **3 pts** **Solves system of equations correctly**

✓ + **4 pts** **Geometric interpretation; it is a line in R3**

+ **2 pts** Partially credit for geometric interpretation

+ **0 pts** No credit

QUESTION 6

6 Problem 6 **12 / 12**

✓ + **2 pts** **Explains why the matrix is diagonalizable [spectral thm or other correct argument]**

+ **1 pts** Partial credit on 1st part

+ **0 pts** Incorrect answer to 1st part

✓ + **10 pts** **Full credit for second part**

+ **1 pts** Finds characteristic polynomial

+ **1 pts** Eigenvalues

+ **2 pts** Multiplicities

+ **3 pts** Eigenspaces

+ **3 pts** S

+ **0 pts** No credit

QUESTION 7

7 Problem 7 **13 / 13**

✓ + **13 pts** **Completely Correct**

+ **4 pts** Full Credit Criterion 1: Student describes the eigenspace for eigenvalue 1 and gives its geometric multiplicity.

+ **4 pts** Full Credit Criterion 2: Student describes the eigenspace for eigenvalue 0 and gives its geometric multiplicity.

+ **2 pts** Full Credit Criterion 3: Student shows that A is diagonalizable.

+ **3 pts** Full Credit Criterion 4: Student shows A^k = A, or just
A^k =SB S^{-1}.

+ **0 pts** Incorrect/ no progress made

QUESTION 8

8 Problem 8 **10 / 10**

✓ + **10 pts** **Completely Correct**

+ **5 pts** Full Credit Criterion 1: Student describes the linear transformation given by the matrix A. They need only indicate what happens to the standard basis, but they could also describe it geometrically.

+ **5 pts** Full Credit Criterion 2: The correct inverse matrix is provided. Ideally the student should just undo the linear transformation described above, but they can also use row reduction.

+ **0 pts** Completely Incorrect

+ **3 pts** Potential Partial Credit: If the student makes a small error in one of the parts (e.g. forgets a negative sign or writes an index incorrectly) they can get partial credit.

QUESTION 9

Problem 9 15 pts

9.1 **5 / 5**

✓ + **2 pts** **Correct Answer**

✓ + **3 pts** **Correct counter example.**

+ **0 pts** Incorrect

9.2 **5 / 5**

✓ + **2 pts** **Correct answer**

✓ + **3 pts** **Essentially correct explanation; states that row operations do not change the solution space or that RREF doesn't change it, but this has to be explicitly stated**

+ **2 pts** Circular explanation; we use the RREF of A to find the kernel because they have the same kernel, not the other way around

+ **1 pts** Example instead of explanation

+ **1 pts** Inaccurate explanation

+ **0 pts** False / No credit

9.3 **5 / 5**

✓ + **5 pts** **Correct, and justification given**

+ **2 pts** Correct, but incorrect or no justification given

+ **0 pts** Incorrect

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Final (Math 33A, Fall 2019)

Your Name: Charles Zhang

UCLA id: 305413659

Date: 12/9/19

The rules: You can answer using a pencil or ink pen. You are allowed to use only this paper, pencil or pen, and the scratch paper provided. You should not hand the scratch paper in. No calculators. No books, no notebooks, no notes, no mobile phones, no web access. You must write your name and UCLA id. You have exactly 180 minutes.

Problem 1	10 points
Problem 2	10 points
Problem 3	10 points
Problem 4	10 points
Problem 5	10 points
Problem 6	12 points
Problem 7	13 points
Problem 8	10 points
Problem 9	15 points
Total	100 points

Good luck!

DO NOT TURN THIS PAGE UNTIL INSTRUCTED TO DO SO

Problem 1 (10 points)

Let L be the line in $\mathbb{R}^3$ spanned by the vector $\begin{bmatrix}1\\1\\1\end{bmatrix}$. Find a basis for the orthogonal complement $L^\perp$ of L. Then use Gram-Schmidt to obtain an orthonormal basis for $L^\perp$.

Solution:

$\dim(L)+\dim(L^\perp)=3$

$\dim(L^\perp)=2$

$\vec{v}_1\cdot\vec{w}_1=0$

$\vec{v}_1\cdot\vec{w}_2=0$

$\begin{bmatrix}1\\1\\1\end{bmatrix}\cdot\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=0$

$x_1+x_2+x_3=0$

$x_1=1 \quad x_1=1$

$x_2=-1 \quad x_3=-1$

$\begin{bmatrix}1\\-1\\0\end{bmatrix},\begin{bmatrix}1\\0\\-1\end{bmatrix}$ is a basis for $L^\perp$

$\vec{w}_1=\begin{bmatrix}1\\-1\\0\end{bmatrix}$

$\vec{w}_2=\begin{bmatrix}1\\0\\-1\end{bmatrix}$

$\vec{u}_1=\frac{\vec{w}_1}{\|\vec{w}_1\|}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\\0\end{bmatrix}$

$\mathrm{proj}_{V_1}(\vec{w}_2)=(\vec{u}_1\cdot\vec{w}_2)\vec{u}_1$

$=0$

$\vec{w}_2^\perp=\vec{w}_2$

$\vec{u}_2=\frac{\vec{w}_2}{\|\vec{w}_2\|}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\-1\end{bmatrix}$

$\boxed{\text{orthonormal basis of } L^\perp=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\\0\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\-1\end{bmatrix}}$

Solution:

Problem 2 (10 points)

Let

$$A = \begin{bmatrix} 0 & -1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find a basis for the image of A and a basis for the kernel of A.

Solution:

$$\boxed{\text{basis for } \operatorname{im}(A) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}}$$

$$A = \begin{bmatrix} 0 & -1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times -1$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \text{II}$$

$$\begin{bmatrix} 1 & 0 & 2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \operatorname{rref}(A)$$

$x_1 + 2x_3 - x_4 = 0$

$x_2 + x_3 - x_4 = 0$

$x_3 = t$

$x_4 = s$

$x_5 = r$

$x_1 = -2t + s$

$x_2 = -t + s$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2t+s \\ -t+s \\ t \\ s \\ r \end{bmatrix}$$

Next page →

Solution:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2t+s \\ -t+s \\ t \\ s \\ r \end{bmatrix} = t\begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + r\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{\text{basis of } \ker(A) = \begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}$$

$\dim(\text{im}) + \dim(\ker) = 5$ ✓

Problem 3 (10 points)

Let $\mathcal{B}$ be a basis of $\mathbb{R}^2$. You know that $\left[\begin{bmatrix}1\\2\end{bmatrix}\right]_{\mathcal{B}} = \begin{bmatrix}1\\1\end{bmatrix}$ and $\left[\begin{bmatrix}-1\\1\end{bmatrix}\right]_{\mathcal{B}} = \begin{bmatrix}2\\0\end{bmatrix}$. What is the basis $\mathcal{B}$?

Hint: if you are unsure on how to proceed, first recall (and write down) how $\mathcal{B}$-coordinates are defined. You should set up a linear system in four variables and four equations.

Solution:

$\vec{b}_1 = \begin{bmatrix}a\\b\end{bmatrix}$ $\vec{b}_2 = \begin{bmatrix}c\\d\end{bmatrix}$

$\left[\begin{bmatrix}1\\2\end{bmatrix}\right]_{\mathcal{B}} = \begin{bmatrix}1\\1\end{bmatrix}$ $\left[\begin{bmatrix}-1\\1\end{bmatrix}\right]_{\mathcal{B}} = \begin{bmatrix}2\\0\end{bmatrix}$

$\vec{b}_1 + \vec{b}_2 = \begin{bmatrix}1\\2\end{bmatrix}$ $2\vec{b}_1 = \begin{bmatrix}-1\\1\end{bmatrix}$

$\begin{bmatrix}a+c\\b+d\end{bmatrix} = \begin{matrix}1\\2\end{matrix}$

$2a = -1$

$2b = 1$

$a + c = 1$

$b + d = 2$

$a = -1/2$

$b = 1/2$

$c = 3/2$

$d = 3/2$

$\boxed{\mathcal{B} = \left\{\begin{bmatrix}-1/2\\1/2\end{bmatrix}, \begin{bmatrix}3/2\\3/2\end{bmatrix}\right\}}$

Solution:

Check:

$\vec{x} \to [\vec{x}]_B$, use S^{-1}

$$\mathcal{B} = \left\{ \begin{bmatrix} -1/2 \\ 1/2 \end{bmatrix}, \begin{bmatrix} 3/2 \\ 3/2 \end{bmatrix} \right\}$$

$$\begin{tikzcd}
{[\vec{x}]} \arrow[r, "A"] & {[T(x)]} \arrow[d, "S"] \\
{[x]_B} \arrow[u, "S^{-1}"] \arrow[r, "B"'] & {[T(x)]_B}
\end{tikzcd}$$

$$S = \begin{bmatrix} -1/2 & 3/2 \\ 1/2 & 3/2 \end{bmatrix}$$

$$\begin{bmatrix} -1/2 & 3/2 \\ 1/2 & 3/2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} -1/2 & 3/2 \\ 1/2 & 3/2 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix} \checkmark$$

$$\left[\begin{array}{cc|cc} -1/2 & 3/2 & 1 & 0 \\ 1/2 & 3/2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & -3 & -2 & 0 \\ 1 & 3 & 0 & 2 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & -3 & -2 & 0 \\ 0 & 6 & 2 & 2 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & -3 & -2 & 0 \\ 0 & 1 & 1/3 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & \cancel{2} & -1 & 1 \\ 0 & 1 & 1/3 & 1/3 \end{array}\right]$$

$$S^{-1} = \begin{bmatrix} -1 & 1 \\ 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} -1 & 1 \\ 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \checkmark$$

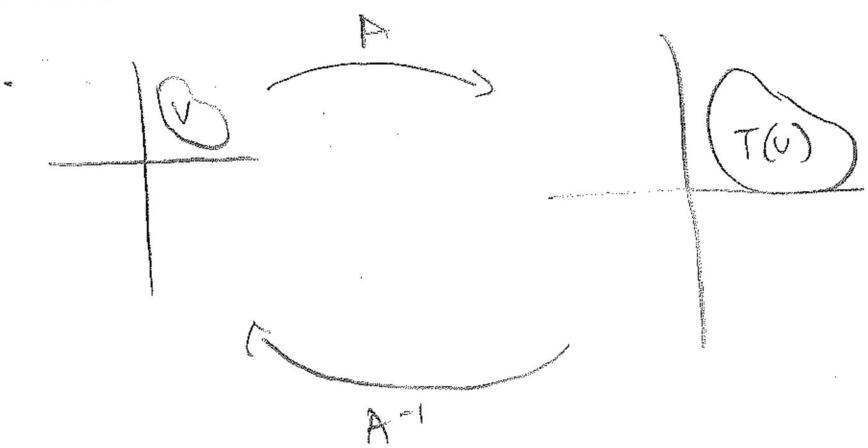
Problem 4 (10 points)

Use the interpretation of the determinant as an expansion factor to explain why the following properties hold:

- For any invertible 2×2 matrix A we have: $\det(A^{-1}) = 1/\det(A)$
- For any 2×2 matrix A we have: $\det(A) = 0 \Leftrightarrow A$ is not invertible

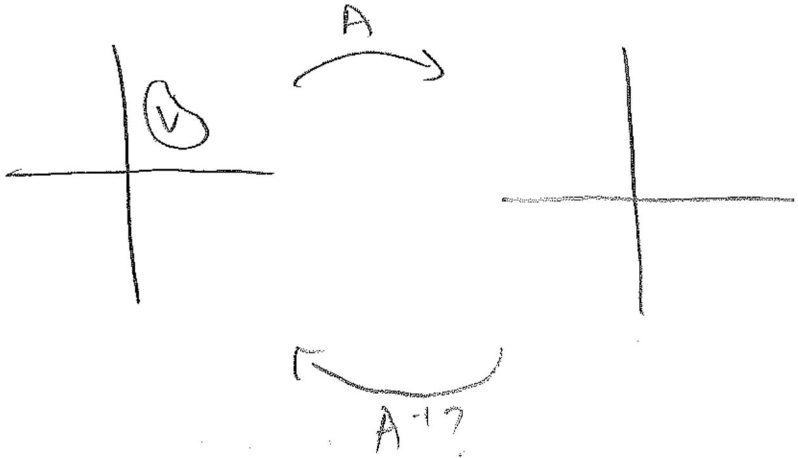
A drawing and/or a brief explanation are sufficient.

Solution:



If transforming V multiplies its area by some number n, then its inverse must revert it back to its original area by multiplying it by 1/n, therefore $\det(A^{-1}) = 1/\det(A)$ if looking at determinant as an expansion factor n, where n is not 0

Solution:



If $\det(A) = 0$, then the expansion factor is 0, meaning it would multiply the area of V by 0, creating a result that is impossible to invert back to its original.

Problem 5 (10 points) Consider the two planes in $\mathbb{R}^3$ defined by the equations

$$x_1 - x_2 + x_3 = 0$$

and

$$x_2 + x_3 = 0\,.$$

Find all points of intersection of these two planes. Then interpret the points of intersection geometrically (a brief description or a drawing are sufficient).

Solution:

$$\begin{array}{r} x_1 - x_2 + x_3 = 0 \\ x_2 + x_3 = 0 \\ \hline x_1 + 2x_3 = 0 \end{array}$$

$x_1 = -2x_3 \quad x_2 = -x_3$

$x_3 = t$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2t \\ -t \\ t \end{bmatrix},\ t \in \mathbb{R}}$$

the intersection is the line in $\mathbb{R}^3$ that spans $\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$

Solution:

Problem 6 (12 points in total)

$A\vec{v} = \lambda\vec{v}$

Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$.

- Without computing eigenvalues and eigenvectors: why is A diagonalisable?
- Find the eigenvalues of A, compute their algebraic and geometric multiplicities and give a matrix S such that $S^{-1}AS$ is diagonal.

Solution:

$\boxed{\text{A is diagonalisable because it is symmetric}}$

$$f_A(\lambda) = \det(A - \lambda I_n) = \begin{vmatrix} -\lambda & 0 & -1 \\ 0 & -\lambda & 0 \\ -1 & 0 & -\lambda \end{vmatrix} = -\lambda^3 + \lambda = 0$$

$\lambda(\lambda^2 - 1) = 0$

$\lambda(\lambda+1)(\lambda-1) = 0$

$\boxed{\lambda = 0, -1, 1}$

$\boxed{\begin{array}{l} \text{almu}(0) = 1 \\ \text{almu}(-1) = 1 \\ \text{almu}(1) = 1 \end{array}}$

$E_0 = \ker(A)$

$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ t \\ 0 \end{bmatrix}$

$\ker(A) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\boxed{\text{gemu}(0) = 1}$

Solution:

$$E_{-1} = \ker(A + I_n) = \ker\left(\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\right)$$

$$\text{rref}(A+I_n) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_3 = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ 0 \\ t \end{bmatrix}$$

$$\ker(A+I_n) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \boxed{\text{gemu}(-1) = 1}$$

$$E_1 = \ker(A - I_n) = \ker\left(\begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}\right)$$

$$\text{rref}(A-I_n) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_3 = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -t \\ 0 \\ t \end{bmatrix}$$

$$\ker(A-I_n) = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \boxed{\text{gemu}(1) = 1}$$

if $S^{-1}AS = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\boxed{S = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}}$$

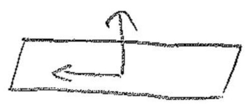
V has dim 2

Problem 7 (13 points) Let V be a plane in $\mathbb{R}^4$, and let A be the matrix that represents the orthogonal projection onto V.

- Is A diagonalisable? If yes, use geometric arguments to find its eigenvalues, and their geometric multiplicities.
- Without computing A: what is A^k, where k is any positive integer?

Solution:

A is diagonalisable $\qquad A\vec{v} = \lambda\vec{v}$



All vectors in V are projected onto themselves

$\therefore \boxed{\lambda_1 = 1}$

All vectors in $V^{\perp}$ are projected to $\vec{0}$

$\therefore \boxed{\lambda_2 = 0}$

$\boxed{\text{gemu}(1) = 2}$ $\qquad \sum \text{gemu} = n$

$\boxed{\text{gemu}(0) = 2}$

Solution:

• A^k is A, because A^2 is multiplying $A\vec{v}$ (the vector $\vec{v}$ after projection onto V) by A, essentially projecting $A\vec{v}$ onto V. Since $A\vec{v}$ is already in V, it will be projected to itself. Therefore, $A^k\vec{v}$ is the same vector as $A\vec{v}$, and A^k must equal A.

Problem 8 (10 points)

Let $A = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$.

- Describe what the linear transformation represented by this matrix does (it is enough if you describe what the standard basis vectors in $\mathbb{R}^5$ are sent to).
- Compute the inverse of A. **Hint:** there is a simple way to find the inverse using the geometric description from the previous point, without having to use Gauss-Jordan.

Solution:

$\vec{e}_1$ is sent to $\vec{e}_5$
$\vec{e}_2$ is sent to $\vec{e}_3$
$\vec{e}_3$ is sent to $\vec{e}_4$
$\vec{e}_4$ is sent to $\vec{e}_2$
$\vec{e}_5$ is sent to $-\vec{e}_1$

$$\boxed{A^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix}}$$

A^{-1} sends
$\vec{e}_5 \to \vec{e}_1$
$\vec{e}_3 \to \vec{e}_2$
$\vec{e}_4 \to \vec{e}_3$
$\vec{e}_2 \to e_4$
$-\vec{e}_1 \to e_5$

Solution:

Check

$$\begin{bmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = I_5$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]$$

Problem 9 (15 points total; 5 points each)

Let A be any $n \times n$ matrix. Which of the following are true? Give a brief explanation, or provide a counterexample. Note that for each question you receive 2 points for the correct answer and 3 points for the explanation or counterexample.

1. $\mathrm{im}(A) = \mathrm{im}(\mathrm{RREF}(A))$.

Solution:

False

$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad$ $\mathrm{rref}(A) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$\mathrm{im}(A) = \mathrm{span}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$ $\quad$ $\mathrm{im}(\mathrm{rref}(A)) = \mathrm{span}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$

$\neq$

2. $\ker(A) = \ker(\mathrm{RREF}(A))$.

Solution:

True

RREF preserves solutions to systems of A, therefore $A\vec{x} = 0$ (definition of kernel) has the same solutions, regardless of if A is in RREF or not.

3. $\det(A) = \det(\text{RREF}(A))$

Solution:

False

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad \text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\det(A) = 8 \qquad \det(\text{rref}(A)) = 1$$

$$\neq$$

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Use this sheet of paper if you need additional space.