In Exercises 21 through 26, find a redundant column vector of the given matrix A, and write it as a linear combination of preceding columns. Use this representation to write a nontrivial relation among the columns, and thus find a nonzero vector in the kernel of A. (This procedure is illustrated in Example 8.)

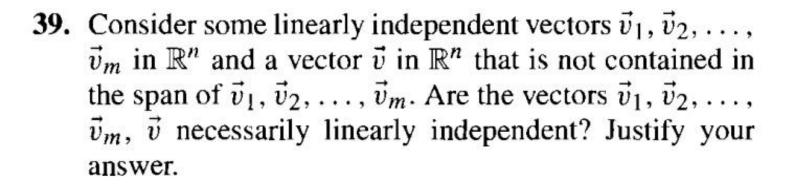
21.
$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$
 22. $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ 23. $\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$
24. $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ 25. $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

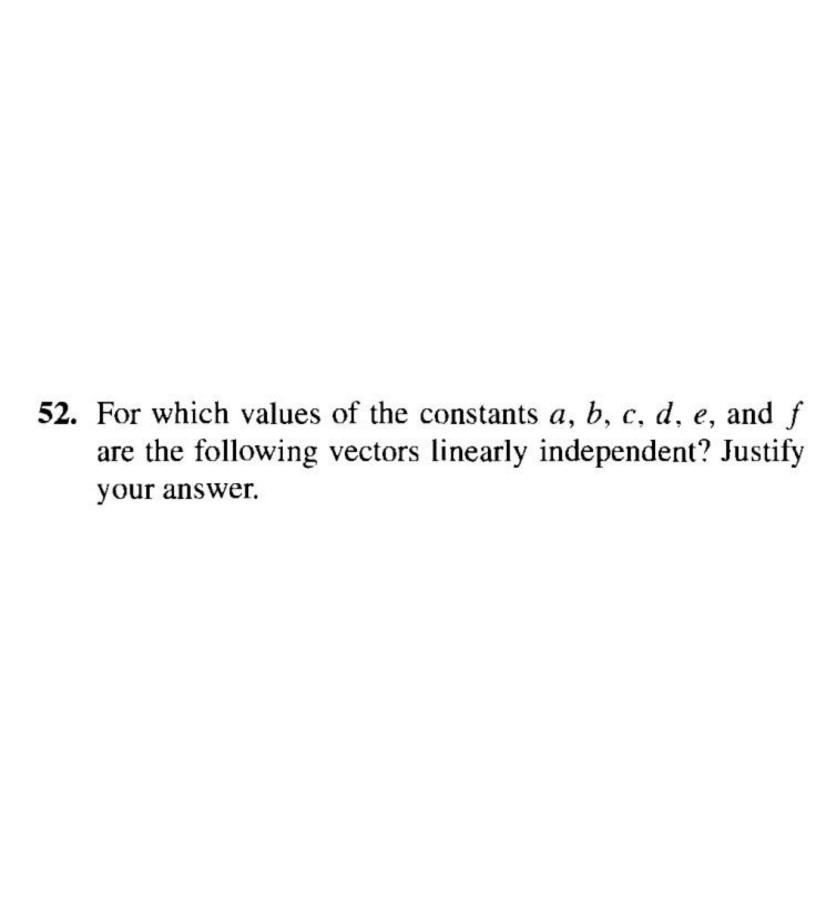
$$\mathbf{26.} \begin{bmatrix} 1 & 3 & 6 \\ 1 & 2 & 5 \\ 1 & 1 & 4 \end{bmatrix}$$

Find a basis of the image of the matrices in Exercises 27 through 33.

27.
$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$$
 28.
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$
 29.
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

30.
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 3 & 7 \end{bmatrix}$$
 31.
$$\begin{bmatrix} 1 & 3 \\ 2 & 6 \\ 3 & 7 \\ 5 & 8 \end{bmatrix}$$





$$\begin{bmatrix} a \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} b \\ c \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} d \\ e \\ f \\ 0 \end{bmatrix}$$

In Exercises 21 through 25, find the reduced row-echelon form of the given matrix A. Then find a basis of the image of A and a basis of the kernel of A.

$$\begin{array}{c|cccc}
\mathbf{21.} & \begin{bmatrix} 1 & 3 & 9 \\ 4 & 5 & 8 \\ 7 & 6 & 3 \end{bmatrix}
\end{array}$$

$$22. \begin{bmatrix} 2 & 4 & 8 \\ 4 & 5 & 1 \\ 7 & 9 & 3 \end{bmatrix}$$

28. For which value(s) of the constant k do the vectors below form a basis of \mathbb{R}^4 ?

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \\ k \end{bmatrix}$$

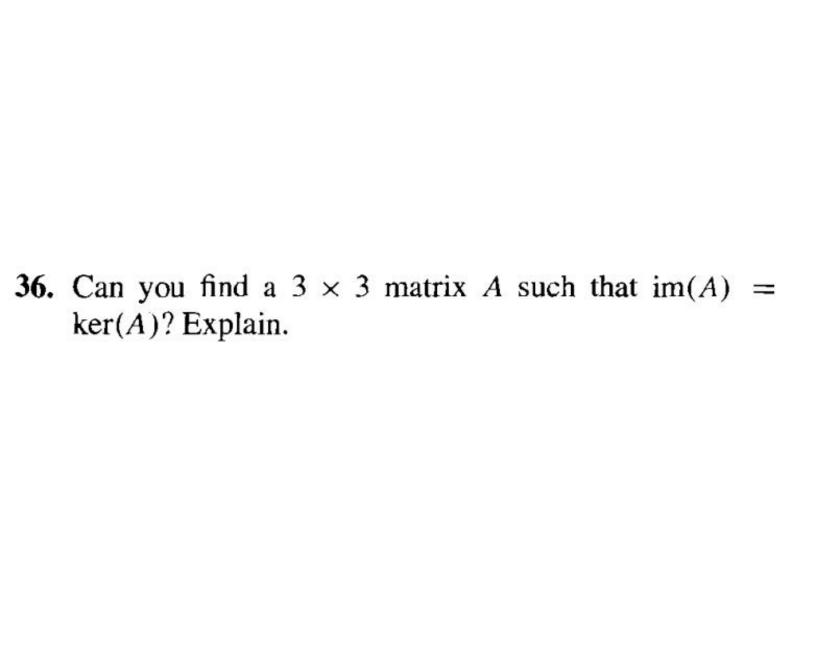
29. Find a basis of the subspace of \mathbb{R}^3 defined by the equation

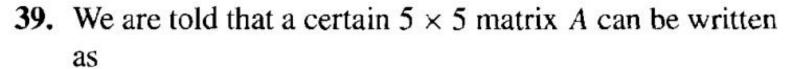
$$2x_1 + 3x_2 + x_3 = 0.$$

31. Let V be the subspace of \mathbb{R}^4 defined by the equation

$$x_1 - x_2 + 2x_3 + 4x_4 = 0.$$

Find a linear transformation T from \mathbb{R}^3 to \mathbb{R}^4 such that $\ker(T) = \{\vec{0}\}$ and $\operatorname{im}(T) = V$. Describe T by its matrix A.





$$A = BC$$

where B is a 5×4 matrix and C is 4×5 . Explain how you know that A is not invertible.