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State	Finished
Completed on	Wednesday, 6 April 2022, 5:21 PM
Time taken	6 hours 18 mins
Marks	11.00/12.00
Grade	**13.75** out of 15.00 (**92**%)

Question **1**

Correct

Mark 1.00 out of 1.00

Test your understanding with the problem given below. You could either use the same method we used in Lecture for the factories and defective part example (i.e., construct a table assuming that there are 100 programs), or you could use the formulas learned for Law of total probability and Bayes theorem. When I discuss the answer with you, I will use this notation for events:

N The event that a program runs successfully.

R The event that the software used is R

{"SAS"} Event that the software used is SAS

{"SPSS"} The event that the software used is SPSS

If you need more material to review, study sections 3.5 and 3.6 in the textbook, where you will find additional examples.

Exercise

In a US campus, 20% of data analysis hires use R, 30% use SPSS and 50% use SAS. 20% of the R programs run successfully as soon as they are typed, 70% of the SPSS programs run sucessfully as soon as they are typed and 80% of the SAS programs run successfully as soon as they are typed.

Match the following.

What is the probability that a program runs successfully as soon as it is typed?	0.65 ✔
If a randomly selected progran runs successfully as soon as it is typed, what is the probability that it has been written is SAS?	0.6154 ✔

Your answer is correct.

N = run successfully

(a)

$$P(N) = P(R \cap N) + P(\{"\ SPSS\ "\} \cap N) + P(\{"\ SAS\ "\} \cap N)$$

$$= P(R)P(N|R) + P(\{"\ SPSS\ "\})P(N|\{"\ SPSS\ "\}) + P(\{"\ SAS\ "\})P(N|\{"\ SAS\ "\})$$

$$= 0.2 \times 0.2 + 0.3 \times 0.7 + 0.5 \times 0.8$$

$$= 0.65$$

(b) $P(\{"\ SAS\ "\}|N) = \dfrac{P(\{"\ SAS\ "\} \cap N)}{P(N)} = \dfrac{P(\{"\ SAS\ "\})P(N|\{"\ SAS\ "\})}{P(\{"\ SAS\ "\} \cap N) + P(R \cap N) + P(\{"\ SPSS\ "\} \cap N)}$

$$= \frac{0.5 \times 0.8}{0.65} = 0.6154$$

The correct answer is: What is the probability that a program runs successfully as soon as it is typed? → 0.65, If a randomly selected progran runs successfully as soon as it is typed, what is the probability that it has been written is SAS? → 0.6154

Question **2**

Correct

Mark 1.00 out of 1.00

In lecture 5, we introduced the *product rule* to calculate the joint probability of two events and the *law of total probability* to calculate the probability of one event, using information about the joint probability of two events. Test your understanding by doing the following exercise.

As methodology, you could construct a table, like we did for the factory and defective part example in lecture 5, or you could use a probability tree, like the one we use in Section 3.4.2. This is a very useful tool as well. Notice how in the tree, it is more obvious that we are calculating the joint probability using the product rule.

Exercise

Of the graduate students in a University, 70% are engineering (E) and 30% are other sciences (O). Suppose that 20% and 25% of the Engineering and the other population, respectively, smoke cigarettes (C). What is the probability that a randomly selected graduate student is

An engineer who smokes?	0.14 ✔
An other student who smoke?	0.075 ✔
A smoker?	0.215 ✔

Your answer is correct.

(a) $P(E \cap C) = P(E)P(C|E) = 0.7 \times 0.2 = 0.14$

(b) $P(O \cap C) = P(O)P(C|O) = 0.3 \times 0.25 = 0.075$

(c) $P(C) = P(E \cap C) + P(O \cap C) = 0.215$

The correct answer is: An engineer who smokes? → 0.14, An other student who smoke? → 0.075, A smoker? → 0.215

Question **3**

Correct

Mark 1.00 out of 1.00

Not all taxpayers want to finance new infrastructure projects. For that reason, public opinion is constantly measured by local governments in order to determine the chance of new projects implementation. Public opinion in a city regarding the opening of a car pool lane in its most congested highway is reflected in the Table

	Yes (event A)	No (event A^c.)
Center of the city (C)	0.150	0.250
Suburbs (B)	0.250	0.150
Rural areas (R)	0.050	0.150

The table reflects the opinion of adults eligible to vote and is saying, for example, that 15 percent of the town adults eligible to vote live in the center of the city and are in favor of the car pool lane.

0.05/0.2 is equal to

Select one:

- a. P(A | R) ✔
- b. P(R |A)
- c. P(AR)
- d. P(R)

Your answer is correct.

The correct answer is: P(A | R)

Question **4**

Incorrect

Mark 0.00 out of 1.00

Automobile recalls by car manufacturers are associated mostly with three defects: engine (E), brakes (B) and seats (Seats). A database of all recalls indicates that the probability of each of these defects is

P(E) = 0.05, P(B) =0.50 and P(Seats) =0.3

Let R be the event that a car is recalled. It is known that P(R|E) = 0.9, P(R|B)=0.8, P(R|Seats)=0.4 and P(R|other defect)= 0.4. What is the probability that a randomly chosen automobile is recalled?

Select one:

- a. 0.565 ✖
- b. 0.5
- c. 0.375
- d. 0.625

Your answer is incorrect.

It follows from the information given that $P(Other) = 1-(0.05+0.5+0.3)= 0.15

By law of total probability,

P(R) = P(R|E)P(E) + P(R|B)P(B) + P(R|seats)P(seats) + P(R|O)P(O)

= (0.9)(0.05) + (0.8)(0.5) + (0.4)(0.3) (0.4)(0.15)

= (0.9)(0.05) + (0.8)(0.5) + (0.4)(0.3) (0.4)(0.15)

=0.625

The correct answer is: 0.625

Question 5

Correct

Mark 1.00 out of 1.00

(Chowdhury, Flentje, and Bhattacharya 2010, 132). An earth dam may fail due to one of three causes, namely: (a) overtopping; (b) slope failure; (c) piping and subsurface erosion. The probabilities of failure due to these causes are respectively 0.7, 0.1, and 0.2. The probability that overtopping will occur within the life of the dam is 10^{-5}. The probability that slope failure will take place is 10^{-4} and the probability that piping and subsurface erosion will occur is 10^{-3}. What is the probability of failure of the dam, assuming that there are no other phenomena which can cause failure?

Let F denote the event "failure of the dam" ; O="overtopping"; L=slope failure; E=erosion.

Select one:

- a. 0.1
- b. P(F |L)
- c. 0.0001
- d. 0.000217 ✔

Your answer is correct.

Let F denote the event "failure of the dam" ; O="overtopping"; L=slope failure; E=erosion.

By law of total probability,

$P(F)= P(F|O)P(O) + P(F|L)P(L) + P(F|E)P(E)$

$= 0.7 (10^{-5})+ 0.1(10^{-4})+ 0.2(10^{-3})$

$= 0.000217$

The correct answer is: 0.000217

Question **6**

Correct

Mark 1.00 out of 1.00

The exercise given below is a good exercise to practice the use of Bayes theorem in medicine. I will translate for you the probabilities given, because I would like you to use the formula for Bayes theorem learned in Lecture 5.

Exercise

In 2002, a group of medical researchers reported that on average, 30 out of every 10,000 people have colorectal cancer. Of these 30 people with colorectal cancer, 18 will have a positive hemoccult test. Of the remaining 9970 people without colorectal cancer, 400 will have a positive test. If a randomly chosen person has a negative test result, what is the probability that the person is free of colorectal cancer?

Hint:

Let C denote the event "having colorectal cancer." Let + denote the event "having a positive hemoccult test result" and - denote the complement.

Then, based on the information given,

P(C) = 30/10000= 0.003 This is the prior probability (a total probability), which reveals that colorectal cancer is not very frequent;

P(+ | C)= 18/30=0.6 This is known from the factory producing the test, because they test it with colorectal cancer patients to see if the test is good;

$P(+\mid C^c) = 400/9970 = 0.0401$ The latter is also known from the factory.

Notice how these allow you to calculate complements of the conditional probabilities given. For example,

$$P(-\mid C) = 1 - P(+\mid C).$$

Notice that the complement when doing conditional probability has the same event after the bar |.

The answer you must find using Bayes theorem is:

$$P(C^c\mid -) = \frac{P(-\mid C^c)P(C^c)}{P(-)}$$

Use this formula to do the problem. Then, if you want, you can double check your work using a table (assuming 100 patients) or a probability tree. It is good to practice the formula.

Select one:

- a. 0.9987 ✔
- b. 0.043
- c. 0.0013
- d. 0.957

Your answer is correct.

Let C denote the event "having colorectal cancer." Let + denote the event "having a positive hemoccult test result" and - denote the complement.

Then, based on the information given,

P(C) = 30/10000= 0.003; P(+ | C)= 18/30=0.6; $P(+\mid C^c) = 400/9970 = 0.0401$

$P(C^c\mid -) = \frac{P(-\mid C^c)P(C^c)}{P(-)} = 0.957/0.9582 = 0.9987$

\(P(- \mid C^c) = 1- P(+ \mid C^c) = 1- 0.0401= 0.9599 \) ;

$P(C^c) = 1 - P(C) = 0.997$;

\(P(- \mid C^c) P(C^c)= (0.9599)(0.997) =0.9570 \) ;

$P(-) = P(-\mid C^c)P(C^c) + P(-\mid C)P(C) = 0.9570 + (1-0.6)(0.003) = 0.9582$

$= \frac{1-0.04012036}{(1-0.04012036)(1-0.003)+(1-0.6)(0.003)}$

The correct answer is: 0.9987

Question 7

Correct

Mark 1.00 out of 1.00

This exercise below is a good opportunity for you to practice using the table method used in Lecture 5 in the factories and defective part. The difference now is that instead of you having to decide the total to use (the 100), you are given the total, 937, because it is referring to a specific finite population. So instead of 100 used 937 as total. Then plug in the counts given in the exercise and complete the table. This will give you all the information that you need to answer almost any probability question regarding this context.

Exercise

(This problem is from the Society of Actuaries, 2007). A public health researcher examines the medical records of a group of 937 men who died in 1999 and discovers that 210 of the men died from causes related to heart disease. Moreover, 312 of the 937 men had at least one parent who suffered from heart disease, and, of these 312 men, 102 died from causes related to heart disease. Determine the probability that a man randomly selected from this group died of causes related to heart disease, given that neither of his parents suffered from heart disease.

Select one:

a. 0.667

b. 0.1728

c. 0.514

d. 0.1152

Your answer is correct.

Use the information provided to make a table, for example, if you wish. The information given is indicated in bold face.

	At least one parent died of HD (A)	None of the parents died of HD (A^c)	Total
Died of HD (H)	**102**	108	**210**
Did not die of HD (H^c)	210	517	727
Total	**312**	625	937

Then the probability asked is $P(H \mid A^c) = 108/625 = 0.1728$

The correct answer is: 0.1728

Question 8

Correct

Mark 1.00 out of 1.00

The product rule studied in lecture 5 was the general product rule. This is the rule you must apply to calculate a joint probability when there is no independence. The exercise given below is a good exercise to practice that rule for three events. Use this notation:

Let A denote the event "People live in urban area" , U the event "upper middle class"; M the event "middle class" ; E the event "purchased electronics."

The product rule for the three events in this problem, A, U, E would be

$$P(A \cap U \cap E) = P(A)P(U|A)P(E|A \cup U)$$

Exercise

About 52% of the population of China lived in urban areas in 2012. In 2012, the upper-middle class accounted for just 14% of urban households, while the middle-middle class accounted for almost 50%. About 56% of the urban upper-middle class bought electronics and household appliances, as compared to 36% of the middle-middle class. If this continued like this in the near future, what would be the probability that a randomly chosen household in China is an upper-middle class urban person that purchases appliances and electronics? This information was obtained from https://www.mckinsey.com/industries/retail/our-insights/mapping-chinas-middle-class.

Select one:

- a. 0.13
- b. 0.67
- c. 0.0407 ✔
- d. 0.07

Your answer is correct.

Let A denote the event "People live in urban area" , U the event "upper middle class"; M the event "middle class" ; E the event "purchased electronics"

Given is

P(A) =0.52; P(U|A) =0.14; P(M|A) = 0.5; $P(E|U \cap A) = 0.56$; $P(E|M \cap A) = 0.36.$

Want $P(A \cap U \cap E) = P(A)P(U|A)P(E|U \cap A) = 0.52(0.14)(0.56) = 0.0407$

The correct answer is: 0.0407

Question 9

Correct

Mark 1.00 out of 1.00

The Pell grant example in Lecture 5 talks about 2 types of probability. Later on in the lecture and the material studied this week, we have also talked about different probabilities. In the Pell grant example, the 0.32 is a [Prior probability] ✔ and the 0.34 is a [Conditional Probability] ✔ . If we had been interested in calculating the probability that a Pell grant receiver was from UCLA, we could not calculate it because we do not have the information about [P(B)] ✔ , which is needed to apply [Bayes] ✔ theorem.

[Prior probability] [Conditional Probability] [P(B)] [Bayes] [the addition rule]

Your answer is correct.

The correct answer is:
The Pell grant example in Lecture 5 talks about 2 types of probability. Later on in the lecture and the material studied this week, we have also talked about different probabilities. In the Pell grant example, the 0.32 is a [Prior probability] and the 0.34 is a [Conditional Probability]. If we had been interested in calculating the probability that a Pell grant receiver was from UCLA, we could not calculate it because we do not have the information about [P(B)], which is needed to apply [Bayes] theorem.

Question **10**

Correct

Mark 1.00 out of 1.00

In the factories and defective part example of Lecture 5, there are several probabilities. When calculating

$$P(B \cap D)$$

we are calculating the [joint] ✔ probability that a part randomly chosen among the [100] ✔ parts has the two characteristics of being from factory [B] ✔ and being [defective] ✔ . On the other hand, when we are calculating

$$P(D \mid B)$$

we are calculating the probability that a part randomly chosen among [30] ✔ parts is defective. That last probability is [a conditional probability] ✔ . The

$$P(B \cap D)$$

equals [0.09] ✔ but the

$$P(D \mid B)$$

equals [0.3] ✔ . If we were just calculating the probability that a randomly chosen part is defective, that probability is [0.19] ✔ , and that is a [total probability] ✔ .

To make a decision as to which factory a defective part comes we need to compare the conditional probabilities [P(A|D)] ✔ , [P(B|D)] ✔ , [P(C|D)] ✔ . The defective part, we decide, comes from the factory that has the highest probability among those. This does not mean we know exactly where the defective part came from, but we are less likely to be wrong if we make our prediction using this method.

joint	100	B	defective	30
a conditional probability	0.09	0.3	0.19	total probability
P(A|D)	P(B|D)	P(C|D)		

Your answer is correct.

The correct answer is:
In the factories and defective part example of Lecture 5, there are several probabilities. When calculating $$P(B \cap D)$$ we are calculating the [joint] probability that a part randomly chosen among the [100] parts has the two characteristics of being from factory [B] and being [defective]. On the other hand, when we are calculating $$P(D \mid B)$$ we are calculating the probability that a part randomly chosen among [30] parts is defective. That last probability is [a conditional probability]. The $$P(B \cap D)$$ equals [0.09] but the $$P(D \mid B)$$ equals [0.3]. If we were just calculating the probability that a randomly chosen part is defective, that probability is [0.19], and that is a [total probability].

To make a decision as to which factory a defective part comes we need to compare the conditional probabilities [P(A|D)], [P(B|D)], [P(C|D)]. The defective part, we decide, comes from the factory that has the highest probability among those. This does not mean we know exactly where the defective part came from, but we are less likely to be wrong if we make our prediction using this method.

Question **11**

Correct

Mark 1.00 out of 1.00

In lecture 5, we revisited again the hit and run accident in which a taxicab was involved. Use the following notation to match questions and answers.

B Event that the cab is actually blue

G Event that the cab is actually green

I Event that the witness identified the cab as blue

What is the probability that the cab is blue given the evidence? Use Bayes theorem	P(B | I)=[P(I |B)P(B)]/[P(I | B)P(B) + P(I | G)P(G)]=0.4137 ✔
Calculate the total probability needed in the denominator of Bayes theorem.	P(I | B)P(B) + P(I |G)P(G) ✔
Which total probability is needed in the denominator of Bayes formula for this problem?	The total probability that the witness identified the cab as blue ✔
The prior probability (unconditional probability) that the taxicab was blue is	0.15 ✔
What is the probability that the witness identified a blue cab correctly?	0.8 ✔

Your answer is correct.

The correct answer is: What is the probability that the cab is blue given the evidence? Use Bayes theorem → P(B | I)=[P(I |B)P(B)]/[P(I | B)P(B) + P(I | G)P(G)]=0.4137, Calculate the total probability needed in the denominator of Bayes theorem. → P(I | B)P(B) + P(I |G)P(G), Which total probability is needed in the denominator of Bayes formula for this problem? → The total probability that the witness identified the cab as blue, The prior probability (unconditional probability) that the taxicab was blue is → 0.15, What is the probability that the witness identified a blue cab correctly? → 0.8

Question **12**

Correct

Mark 1.00 out of 1.00

In lecture 6, we defined independence of events. More material on it can be found in Section 3.3. of the textbooks, which has more examples. The following exercise requires that you use the product rule when there is independence.

Exercise

(This example is from Parzen, page 90.) Consider an automobile accident on a city street in which car I stops suddenly and is hit from behind by car II. Suppose that three persons, whom we call A', B', C', witness the accident. Suppose the probability that each witness has correctly observed that car I stopped suddenly is estimated by having the witnesses observe a number of contrived incidents about which each is then questioned. Assume that it is found that A' has probability 0.9 of stating that car I stopped suddenly, B' has probability 0.8 of stating that car I stopped suddenly, and C' has probability 0.7 of stating that car stopped suddenly. Let A, B, and C denote, respectively, the events that persons A', B', C' will state that car I stopped suddenly. Assuming that A, B, and C are independent events, calculate the probability that A', B', and C' will state that car I stopped suddenly.

- a. 0.902
- b. 0.504 ✔ Calculate P(A)P(B)P(C)
- c. 0.398
- d. 0.5

Your answer is correct.

The correct answer is:
0.504