EXERCISES

In Exercises 1–6, the matrix A has real eigenvalues. Find the general solution of the system $\mathbf{y}' = A\mathbf{y}$.

1. $A = \begin{pmatrix} 2 & -6 \\ 0 & -1 \end{pmatrix}$

2. $A = \begin{pmatrix} -1 & 6 \\ -3 & 8 \end{pmatrix}$

3. $A = \begin{pmatrix} -5 & 1 \\ -2 & -2 \end{pmatrix}$

4. $A = \begin{pmatrix} -3 & -6 \\ 0 & -1 \end{pmatrix}$

5. $A = \begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}$

6. $A = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$

In Exercises 7–12, find the solution of the initial-value problem for system $\mathbf{y}' = A\mathbf{y}$ with the given matrix A and the given initial value.

7. The matrix in Exercise 1 with $\mathbf{y}(0) = (0, 1)^T$
8. The matrix in Exercise 2 with $\mathbf{y}(0) = (1, -2)^T$
9. The matrix in Exercise 3 with $\mathbf{y}(0) = (0, -1)^T$
10. The matrix in Exercise 4 with $\mathbf{y}(0) = (1, 1)^T$
11. The matrix in Exercise 5 with $\mathbf{y}(0) = (3, 2)^T$
12. The matrix in Exercise 6 with $\mathbf{y}(0) = (1, 5)^T$

In Exercises 13 and 14, a complex vector valued function $\mathbf{z}(t)$ is given. Find the real and imaginary parts of $\mathbf{z}(t)$.

13. $\mathbf{z}(t) = e^{2it}\begin{pmatrix} 1 \\ 1+i \end{pmatrix}$

14. $\mathbf{z}(t) = e^{(1+i)t}\begin{pmatrix} -1+i \\ 2 \end{pmatrix}$

15. The system

$$\mathbf{y}' = \begin{pmatrix} 3 & 3 \\ -6 & -3 \end{pmatrix}\mathbf{y} \tag{2.47}$$

has complex solution

$$\mathbf{z}(t) = e^{3it}\begin{pmatrix} -1-i \\ 2 \end{pmatrix}.$$

Verify, by direct substitution, that the real and imaginary parts of this solution are solutions of system (2.47). Then use Proposition 5.2 in Section 8.5 to verify that they are linearly independent solutions.

In Exercises 16–21, the matrix A has complex eigenvalues. Find a fundamental set of real solutions of the system $\mathbf{y}' = A\mathbf{y}$.

16. $A = \begin{pmatrix} -4 & -8 \\ 4 & 4 \end{pmatrix}$

17. $A = \begin{pmatrix} -1 & -2 \\ 4 & 3 \end{pmatrix}$

18. $A = \begin{pmatrix} -1 & 1 \\ -5 & -5 \end{pmatrix}$

19. $A = \begin{pmatrix} 0 & 4 \\ -2 & -4 \end{pmatrix}$

20. $A = \begin{pmatrix} -1 & 3 \\ -3 & -1 \end{pmatrix}$

21. $A = \begin{pmatrix} 3 & -6 \\ 3 & 5 \end{pmatrix}$

In Exercises 22–27, find the solution of the initial value problem for system $\mathbf{y}' = A\mathbf{y}$ with the given matrix A and the given initial value.

22. The matrix in Exercise 16 with $\mathbf{y}(0) = (0, 2)^T$
23. The matrix in Exercise 17 with $\mathbf{y}(0) = (0, 1)^T$
24. The matrix in Exercise 18 with $\mathbf{y}(0) = (1, -5)^T$
25. The matrix in Exercise 19 with $\mathbf{y}(0) = (-1, 2)^T$
26. The matrix in Exercise 20 with $\mathbf{y}(0) = (3, 2)^T$