20S-PHYSICS 4AL-12 Unit 2 report

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TOTAL POINTS

46.5 / 50

QUESTION 1

1 Total **46.5** / **50**

✓ **- 50 pts** Unit 2

+ 46.5 Point adjustment

Abstract/Introduction/Theory (15): 14
Methods/Experimental Setup/Technical Details (15): 14
Results (40): 37
Discussion + Conclusion (30): 28
Total (out of 100; divide by 2): 93 --> 46.5

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The Effects of Various Angles of Elevation on Acceleration

Claire Chung, Brendan Rossmango, Ryan Rossmango, Neil Vaishampayan, Charles Zhang
Physics 4AL, Lab 12, Group 2
May 10, 2020

Abstract

This experiment compared varying slope angles to examine the impact of gravity on an object's acceleration using either an ultrasonic sensor and an Arduino system or motion tracking software. The hypothesis was that increasing the sine of the angle of elevation would increase a rolling object's linear acceleration. It is expected for this relationship to be linear, since the force responsible for linear acceleration can be written as $mgsin(\theta)$. There are some discrepancies expected between the actual value of Earth's gravity and the results collected since the object will experience friction and drag that theoretical systems do not take into account.

The results support the hypothesis, showing a linear relationship between the sine of the angle of elevation and the linear acceleration of the object. However, all predicted values of acceleration were twice as large as what was derived from data. This was due to half of the object's energy being converted to rotational motion and therefore angular acceleration, an error that offset all of the predicted values. Linear acceleration cannot be predicted by the above quantity, because Newton's 2nd Law is rooted in principles of energy conservation, so rotational motion must be accounted for as well. Thus, acceleration can be more accurately predicted through equations of energy conservation.

Introduction

In accordance with Newton's Law of Universal Gravitation, gravity is a scientific principle that describes the tendency of bodies with mass to attract other bodies with mass (Cohen & Whitman, 1999). Although the way gravity acts on these bodies requires a very complicated analysis, these trials focus on the much simpler case of gravity's impact on objects close to the Earth's surface. One method to measure this impact is the use of kinematics, a mathematical analysis of the motion of objects. The kinematic equations allow position, velocity, acceleration, and time to be related to one another (Sivaraj, 2018). Assuming conditions of constant acceleration in a system only affected by gravity, this implies it is possible to derive the impact of gravity using position and time.

These trials involved the use of rolling objects on slopes of various angles to alter the impact of gravity on the object's acceleration. An ultrasonic sensor was placed at the end of the object's path in order to measure position and time. Through the use of kinematic equations, this positional data was then translated into velocity and acceleration for each of the test cases, and compared to the expected results derived from the canonical value of Earth's gravity, 9.81 m/s^2 (Sivaraj, 2018).

It is hypothesized that as the sine of the angle of elevation of the tilted surface increases, so will the linear acceleration. One can see this concept upon breaking down the force body diagram. There is a normal force (F_n) and a component of weight perpendicular to the surface ($mgcos(\theta)$) which balance out. Additionally, there is a sole force parallel to the surface (ignoring friction) which is responsible for the linear acceleration, $mgsin(\theta)$.

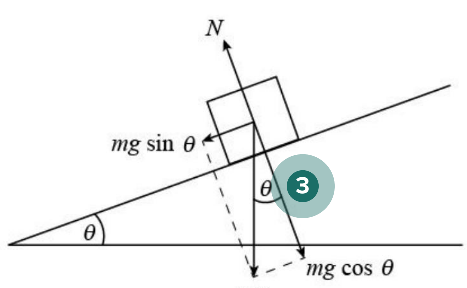

Figure 1. Force Body Diagram of object on frictionless surface.

Looking closer at this quantity, it can be observed that m and g are constants, and $sin(\theta)$ is the only quantity changing. In fact, in a range of arguments from 0 to 90 degrees, $sin(\theta)$ increases. Therefore, it is more precisely hypothesized that there will be a direct relationship between the angle of elevation and the linear acceleration.

Method

The first experiment set a baseline for the accuracy of using kinematics to track motion. An ultrasonic sensor was set up at one end of a plank of smooth wood, and a cylindrical container on the other. The ultrasonic sensor was connected to an Arduino, which would record the amount of time it took for the sound wave to be reflected back at the sensor, measuring distance. The container was then given an initial velocity and rolled in a straight line towards the sensor. The position data was then plotted and used to extrapolate the velocity.

The second experiment altered the plank that the container would roll on by changing the angle relative to the floor. The first such trial used a 10º angle, with the ultrasonic sensor parallel to the direction of motion. The container was then released from rest at the top of the plank, and allowed to roll down to the bottom. The trial was then repeated with angles of 15º and 20º. This positional data was then plotted and used to extrapolate velocity and acceleration of the container.

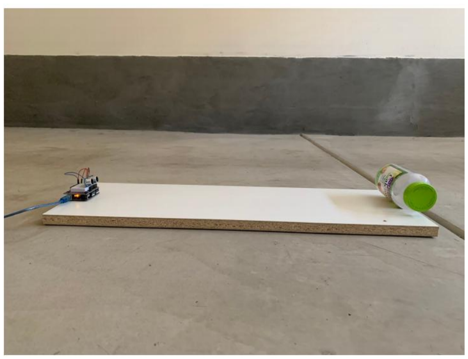

Fig 0b: Flat Ground Setup

Fig 0c: Tilted Slope Setup

Results

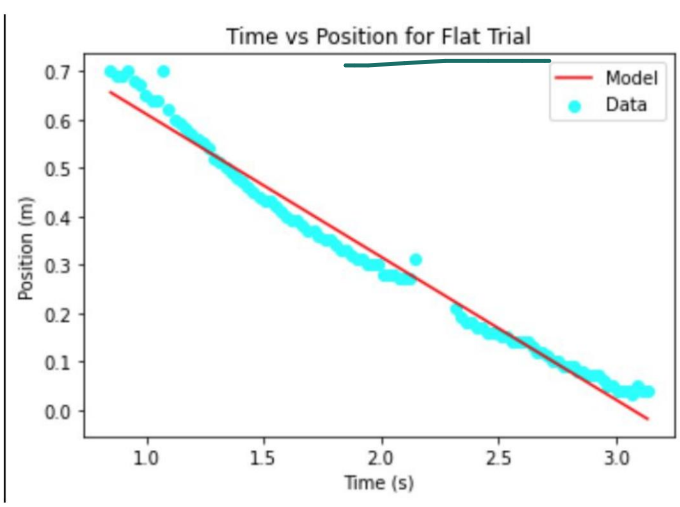

Fig 1: Time Series of Cylinder Position

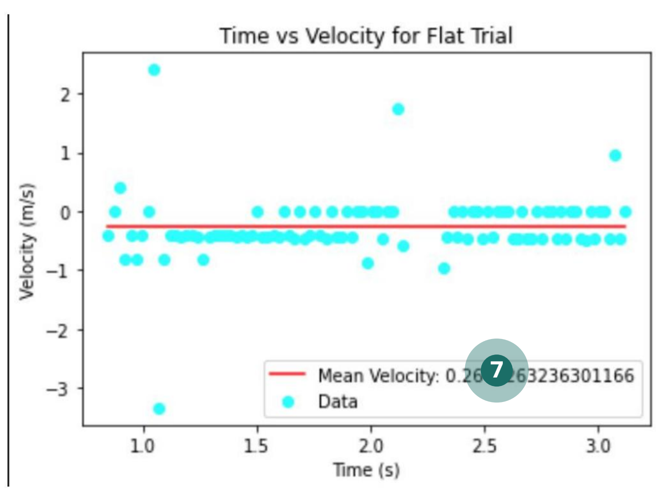

Fig 2: Time Series of Cylinder Velocity

Velocity from Slope: 0.295 m/s

Velocity from Mean: 0.265 m/s

Here, the reliability of the ultrasonic sensor instrument was tested to be adequate.

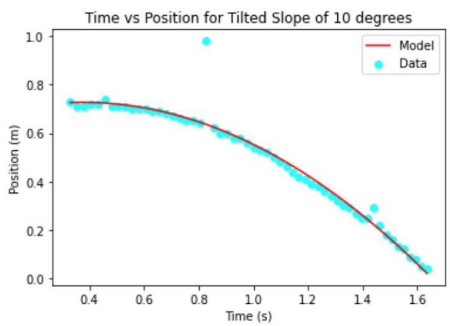

Fig 3: Time Series of Cylinder Position

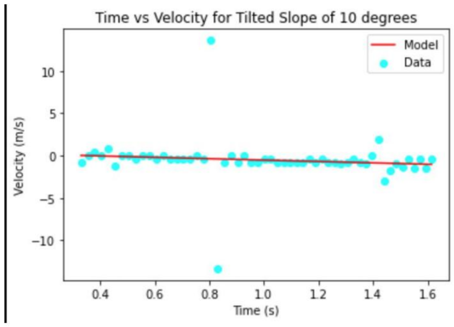

Fig 4: Time Series of Cylinder Velocity

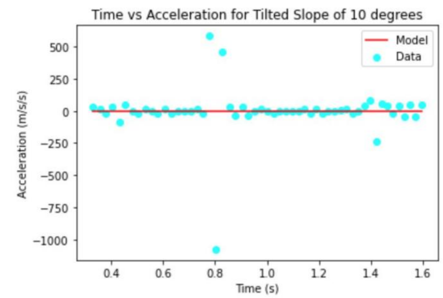

Fig 5: Time Series of Cylinder Acceleration

Acceleration from Curve: 0.887 m/s^2

Acceleration from Slope: 0.818 m/s^2

Acceleration from Mean: 0.301 m/s^2

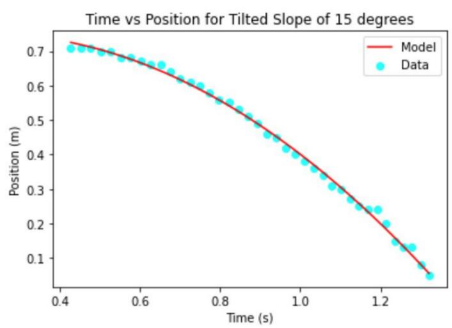

Fig 6: Time Series of Cylinder Position

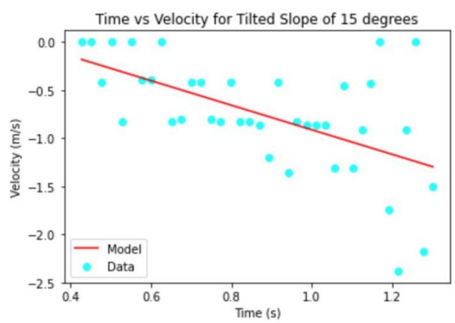

Fig 7: Time Series of Cylinder Velocity

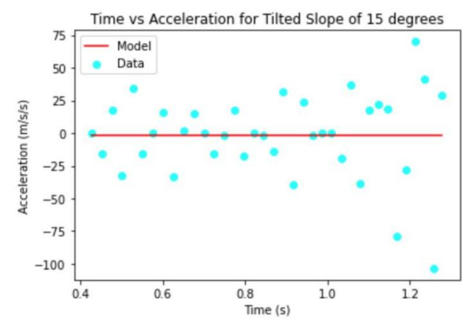

Fig 8: Time Series of Cylinder Acceleration

Acceleration from Curve: 1.138 m/s^2

Acceleration from Slope: 1.276 m/s^2

Acceleration from Mean: 1.73 m/s^2

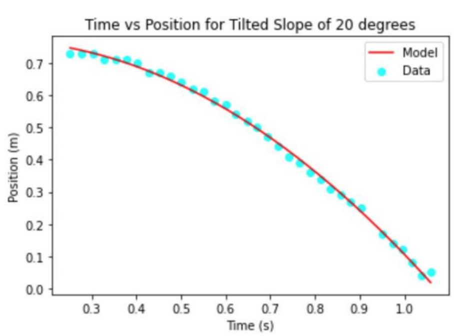

Fig 9: Time Series of Cylinder Position

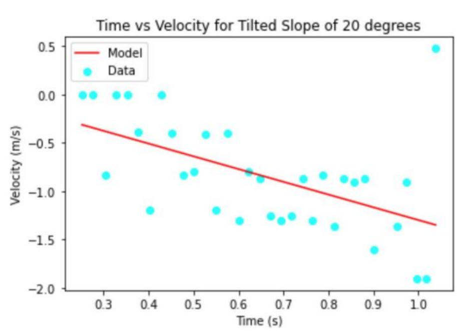

Fig 10: Time Series of Cylinder Velocity

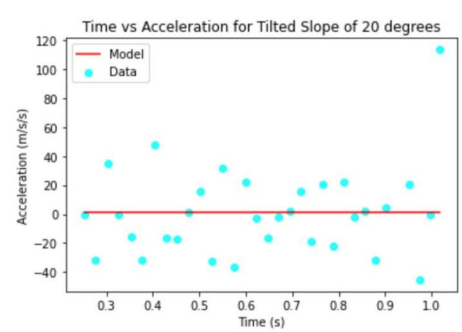

Fig 11: Time Series of Cylinder Acceleration

Acceleration from Curve: 1.564 m/s^2

Acceleration from Slope: 1.318 m/s^2

Acceleration from Mean: 0.985 m/s^2

It looks as if an increased angle leads to an increased linear acceleration.

Angle (degrees)	Predicted Acceleration (m/s^2)*	Derived acceleration (m/s^2)	Percentage difference
10	1.703	0.887	47.9 %

15	2.539	1.138	55.2 %
20	3.355	1.564	53.4 %

Table 1: Derived Acceleration vs Predicted Acceleration

*Note: the predicted acceleration is found by using Newton's second law: $a = g\sin(\theta)$

There seems to be a 50% discrepancy across the linear accelerations from the predicted values.

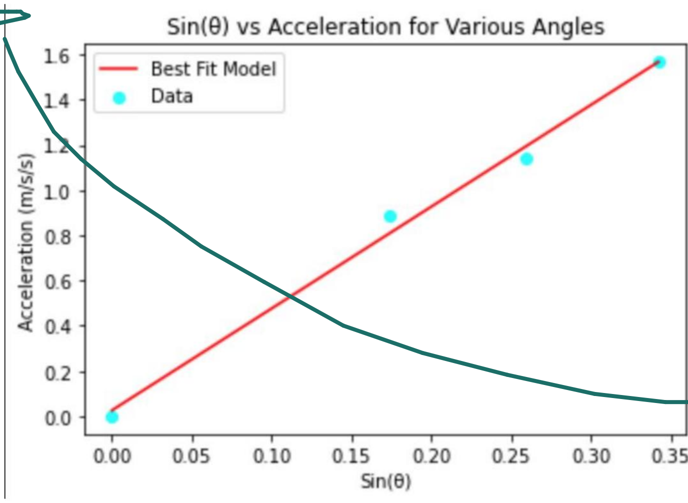

Fig 12: Relation of $sin(\theta)$ and Acceleration (m/s^2) for the Measured angles (correlation: 0.996)

However, there does seem to be a simple, linear relation between the sine of the angle and the linear acceleration.

Further Discussion

To explain this discrepancy in accelerations, let us further examine the presence of rotational motion in our cylindrical object. To do so, the topic of energy conservation will be outlined and analyzed below. With energy concepts, we can find not only the linear acceleration of the object, but also the angular acceleration of the object since as it rolls down the ramp, it rotates. The object starts with initial potential energy, and ends up with final linear and rotational kinetic energy. The work-energy theorem, $W = \Delta E$, where $\Delta E = \Delta K + \Delta U$, can be used to also find properties of kinematics such as final velocity, distance, and in tandem with basic kinematic equations, the acceleration. Initially, the object starts with gravitational potential energy only, as it starts on the ramp above the ground at an initial height with no velocity. Then, when it is released from rest, it has a final velocity (both linear and rotational) at the bottom of the ramp with no gravitational potential energy. Thus, we use the equation

$$W = K_f - U_i \text{ or } Fd = K_f - U_i$$

However, since we are ignoring friction, then the work done by nonconservative forces is 0, so we can set the initial and final energy together.

$$K_f = U_i$$

From the work-energy theorem, the initial gravitational potential energy and the final kinetic linear and rotational energy are equal.

$$0.5mv^2 + 0.5I\omega^2 = mgh$$

The moment of inertia of the object, which is like a hollow cylinder, is $I = mr^2$ (note, the object is not an ideal hollow cylinder, as it is a bottle, so $I = mr^2$ is only an estimation of the moment of inertia). The angular velocity, which is equal to v/r, is ω, the mass of the object is m in kilograms, and the height from which the object is initially positioned is h in meters. Finally, we use $g = 9.81$ m/s^2. To find the final velocity:

$$I\omega^2 = mr^2v^2/r^2 = mv^2$$
$$mv^2 = mgh$$
$$v_f^2 = gh$$

To obtain the acceleration, we use this equation of motion:

$$2ad = v_f^2 - v_i^2$$

With the object initially at rest, $v_i = 0$, so a is:

$$a = v_f^2/2d$$

Finally, angular acceleration is found by

$$\alpha = a/r$$

Angle (degrees)	Initial height (m)	Final velocity (m/s)	Acceleration (m/s^2)	Angular acceleration (rad/s^2)
10	0.130	1.13	0.849	20.9
15	0.194	1.39	1.28	31.5
20	0.257	1.59	1.68	41.3

Table 2: Theoretical Acceleration (m/s^2) using energy concepts

These findings are much more in line with the derived accelerations as shown in Table 1, and these findings take into account the final kinetic rotational energy, which the predicted accelerations do not. Initial kinetic energy is not only converted to kinetic translational energy, but also rotational energy, so it is important to also find this value. As a result of the initial gravitational energy, the object rolls and rotates with both linear acceleration and angular acceleration, not just linear acceleration a.

Conclusion

The purpose of the first experiment was to vet the accuracy of an ultrasonic sensor in regards to determining the kinematic properties of an object. The ultrasonic sensor seems to have held up well, looking at the results - in particular, the position plots - from both the first and second experiments. In another version of this experiment, the software Tracker could be used to analyze the accelerations of the rolling object instead of the sensor. Results from past labs (2B, sensor, and 2C, Tracker) show difficulties in measuring the acceleration due to gravity, with both derived g values having discrepancies. In short, the 2B lab had us take the ultrasonic sensor plugged into the Arduino and perform several trials of dropping an object. The sensor measured the time it took for a sound from the object hitting the ground to return to the sensor, and a plot with known heights vs. time was graphed. For 2C, projectile motion of some object was recorded and analyzed by Tracker. Vertical positions and time were recorded on a plot, and in both labs, acceleration due to gravity was taken from the polyfit curve. The best g value from the sensor experiment was 9.58 m/s^2 ± 0.31 m/s^2 (discrepancy of 0.23), while the Tracker experiment's best g value was 10.45 m/s^2 ± 0.26 m/s^2 (discrepancy of 0.64).

Across both experiments, it was observed that the g values from the sensor experiments were more accurate, contrary to our expectations. At first glance the ultrasonic sensor seems unreliable, because it is dependent on the holder of the sensor to position it parallel to the object. It is prone to jittering and detecting sounds that do not belong to the object-sensor system. However, Tracker was much more unreliable than the ultrasonic sensor, for a range of reasons. The sensor's advantage is that it is much more consistent in its data collection - it collects a sound that is emitted from the object hitting the ground, whose motion is directly monitored by the sensor that is directly above it. It also only gives us time, which we input into equations to give us g. With Tracker, we rely on it to give us all data values - time, position, velocity, etc, so Tracker thus has a much larger probability of collecting worse data. Oftentimes, Tracker struggles with following an object's motion because it becomes blurry or misplaces the point mass with something of similar color. Its advantage ~~is found in~~ its inherent ability to trace all of the kinematic properties of an object with much more efficiency, but this one advantage outweighs neither the sensor's simplicity and reliance on equations nor its own inconsistent properties.

The data supports the original hypothesis; the angle of elevation does increase along with the linear acceleration of the object. Looking at the three trials of rolling the object down the tilted surface, it is seen from the data that there is a linear relationship between the sine of the angle and the linear acceleration. At an angle of 10 degrees, the derived linear acceleration was found to be 0.887 m/s^2, at 15 degrees, the acceleration was found to be 1.138 m/s^2, and at 20 degrees, the acceleration was 1.564 m/s^2. These values were determined by analyzing the position vs. time plots, particularly the curve of the best-fit model, which represents $x = -0.5gt^2$. There is still a general increase in acceleration with increased angles when finding acceleration through velocity vs. time plots, although less clear. Acceleration data collapses when collected from acceleration vs. time plots, which can be attributed to the pronounced effect of numerical differentiation on imperfect data values.

Nonetheless, it can be seen that acceleration of a rolling object cannot be predicted with the simple g*sinθ. This miscalculation was an oversight we failed to see. The predicted accelerations were all twice as large as the derived accelerations; therefore, a distinction between linear acceleration and angular acceleration needed to be made. Because the object rolls, half of the mechanical energy goes to linear motion, and the other half goes to rotational motion. We should have taken this concept of energy into consideration; in fact, the linear acceleration should have been predicted with some form of energy conservation, such as the discourse outlined in the *Further Discussion* section. It is true that the object starts at rest with some energy due to its weight and height, but this energy is not all translated to energy of linear motion under energy conservation principles. The object rotates as it traverses down the surface, so just as some of the object's linear velocity is lowered due to the presence of rotational motion, linear acceleration is less than our predicted accelerations because of the presence of angular acceleration. As all of physics is interconnected - as in Newton's Second Law and energy conservation both combine concepts of force and motion - it can easily be seen now that kinematics needs to be addressed by looking at the full picture, and not by ignoring intrinsic properties of rotating objects.

References

Newton, Isaac. Principia: Mathematical Principles of Natural Philosophy, translated by I.B. Cohen & Whitman, University of California Press, 1999.

Sivaraj, Priyadharsini. A Study on Motion of a Free Falling Body in Kinematic Equation. International Journal for Research in Applied Science and Engineering Technology, 2018, 6. 3118-3124. 10.22214/ijraset.2018.1431.

Texas A&M Department of Physics. *Acceleration of Gravity*, Advanced Instructional Systems, Department of Physics at Texas A&M University, 2012, www.webassign.net/question_assets/tamucolphysmechl1/lab_2/manual.html.

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