Started on	Friday, 6 May 2022, 8:33 PM
State	Finished
Completed on	Saturday, 7 May 2022, 4:12 PM
Time taken	19 hours 38 mins
Grade	**17.90** out of 18.00 (**99**%)

Question **1**

Complete

Mark 5.00 out of 5.00

This question requires that you show work. Work is 80% of your credit and involves showing intermediate steps. The rest is for labeling and defining things so that we know what they are, making clear what probabilities you are calculating and labeling them as such (are they conditional, total, etc..?) and providing your final number.

=======

Let X and Y denote random variables denoting the proportion of contaminants of type A and the proportion of contaminants of type B present in a landfill, respectively. A student calculated the conditional density function

$$f(y|x) = \frac{2(x+y)}{3x^2}, \quad 0 \le y \le x,$$

and the marginal density function

$$f(x) = 3x^2, \quad 0 \le x \le 1$$

Are x and y independent? Make sure that you justify your answer with detailed work.

You may attach a file with work. But the file must contain only the answer to this question. If it contains answers for other questions, those questions will not be graded at all. You may not email your work to this question. It will not count that way.Alternatively, it would be best if you could use the editor and solve your math work using the equation editor in it, which you can see by clicking on the arrow in the menu and selecting the calculator icon.

See attached PDF

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Students may attach a file with work. But the file must contain only the answer to this question. If it contains answers for other questions, deduct 1 pt from the total score for this problem, after it is graded.

If students use math editor, but notation ambiguous, deduct points proportional to ambiguity.

Must find the joint density f(x,y) first **(1.5 pts, giving 0.2 for saying f(x,y), 0.6 for multiplying the conditional by the marginal for x, 0.3 for the final formula 2(x+y) and 0.4 for the domain).**

$$f(x,y) = f(y \mid x)f(x) = \frac{2(x+y)}{3x^2}\left(3x^2\right) = 2(x+y), \quad 0 \le y \le x \le 1$$

Defines independence as the product of marginals and not as "f(y|x) = f(y)" which is a consequence of independent (1 pt for the definition below - deduct 0.5 points if they just say "f(y|x)f(y)"

The definition of independence is: two continuous random variables X,Y are independent if

$f(x,y) = f(x)f(y)$

So we need to calculate f(y). (Calculating f(y), 1.5 pts giving 0.2pt for f(y), 0.6 for setting up the correct work and solution,0.3 for final formula and 0.4 for domain)

$f(y) = \int_y^1 2(x+y)dx = x^2 \mid_y^1 + 2yx \mid_y^1 = 1 - 3y^2 + 2y, \ ; \quad 0 \le x \le 1$

We can see that **(demonstrates that f(x)f(y) not equal to f(x,y) 1 pt)**

$2(x+y) \neq \left(3x^2\right)\left(1 - 3y^2 + 2y\right)$,

therefore, X, Y are **NOT** two independent random variables.

Comment:

Question **2**

Complete

Mark 3.00 out of 3.00

This question requires that you show work. Work is 80% of your credit and involves showing intermediate steps. The rest is for labeling and defining things so that we know what they are, making clear what probabilities you are calculating and labeling them as such (are they conditional, total, etc..?) and providing your final number.

Y**ou may not attach a file to this question. Answer in the space provided. Only the work written in the space provided below will be read.** The editor has a table capability that will help you do nice tables. To expand the options in the editor, click on the arrow above in the menu to expand it, and choose the table option.

=======

Three randomly chosen students from UCLA are interviewed to determine their attitude towards changing from the current quarter system to a semester system. Let the random variable X denote the response **of the first student interviewed**, with X= 1 denote the answer "YES" (move to a semester system) and X=0 denote "NO" (do not move to a semester system). The probability that a randomly chosen UCLA student votes yes has been estimated by students in this class in another quarter to be approximately 0.45.

(a) List the sample space of the experiment consisting of observing the choice of three randomly chosen students to the question and calculate the probability of each single outcome in it, indicating the rules of probability and assumptions you are making.

(b) Write a table with the joint probability mass function of random variables X and Y, where Y denotes the number of students in the random sample of 3 students that are in favor of switching to the semester system.

(c) Determine and show whether X and Y are independent, using the definition of independence and also comparing the marginal distribution of Y with the conditional distributions of Y for given values of X.

Given: n = 3 (where n is the size of the sample), P(X = 1) = 0.45

a) S = { 000, 001, 010, 011, 100, 101, 110, 111 }

We assume X = 0 and X = 1 are mutually exclusive, X's domain is { 0, 1 }, and that the choice of each of the students in the sample is independent of one another

By the axioms of probability: $P(X=0) = 1 - P(X=1) = 1 - 0.45 = 0.55$

By the product rule for independent events, we can calculate the following probabilities:

$P(000) = P(0)P(0)P(0) = (0.55)^3 = 0.166$

$P(001) = P(0)P(0)P(1) = (0.55)^2(0.45) = 0.136$

$P(010) = P(0)P(1)P(0) = (0.55)^2(0.45) = 0.136$

$P(011) = P(0)P(1)P(1) = (0.55)(0.45)^2 = 0.111$

$P(100) = P(1)P(0)P(0) = (0.55)^2(0.45) = 0.136$

$P(101) = P(1)P(0)P(1) = (0.55)(0.45)^2 = 0.111$

$P(110) = P(1)P(1)P(0) = (0.55)(0.45)^2 = 0.111$

$P(111) = P(1)P(1)P(1) = (0.45)^3 = 0.091$

b)

Joint PMF of X and Y

X\Y	**0**	**1**	**2**	**3**
0	P(X=0,Y=0) = P(000) = 0.166	P(X=0,Y=1) = P(010) + P(001) = 0.136 + 0.136 = 0.272	P(X=0,Y=2) = P(011) = 0.111	P(X=0,Y=3) = 0
1	P(X=1,Y=0) = 0	P(X=1,Y=1) = P(100) = 0.136	P(X=1,Y=2) = P(101) + P(110) = 0.111 + 0.111 = 0.222	P(X=1,Y=3) = P(111) = 0.091

c)

Marginal Distribution of Y

y P(Y = y)

y	P(Y = y)
0	P(Y=0) = P(X=0,Y=0) + P(X=1,Y=0) = 0.166 + 0 = 0.166
1	P(Y=1) = P(X=0,Y=1) + P(X=1,Y=1) = 0.272 + 0.136 = 0.408
2	P(Y=2) = P(X=0,Y=2) + P(X=1,Y=2) = 0.111 + 0.222 = 0.333
3	P(Y=3) = P(X=0,Y=3) + P(X=1,Y=3) = 0 + 0.091 = 0.091

By the definition of independence: $P(x,y) = P(x)P(y)$. We can therefore use this definition, checking that it holds for all possible values of X and Y. If X and Y were independent, it should be true that $P(X=1, Y=0) = P(X=1)P(Y=0)$. However, plugging in the given value for P(X=1) and the value of P(Y=0) from the marginal distribution table above, we see that $0 \neq (0.166)(0.45)$, showing that X and Y are not independent by the definition of independence.

Furthermore, we can prove this again using conditional distributions of Y for given values of X:

Conditional Distribution of Y given X=0

y	P(Y=y|X=0)
0	P(Y=0|X=0) = P(X=0,Y=0)/P(X=0) = 0.166/0.55 = 0.302
1	P(Y=1|X=0) = P(X=0,Y=1)/P(X=0) = 0.272/0.55 = 0.495
2	P(Y=2|X=0) = P(X=0,Y=2)/P(X=0) = 0.111/0.55 = 0.202
3	P(Y=3|X=0) = P(X=0,Y=3)/P(X=0) = 0/0.55 = 0

Conditional Distribution of Y given X=1

y	P(Y=y|X=1)
0	P(Y=0|X=1) = P(X=1,Y=0)/P(X=1) = 0/0.45 = 0
1	P(Y=1|X=1) = P(X=1,Y=1)/P(X=1) = 0.136/0.45 = 0.302
2	P(Y=2|X=1) = P(X=1,Y=2)/P(X=1) = 0.222/0.45 = 0.493
3	P(Y=3|X=1) = P(X=1,Y=3)/P(X=1) = 0.091/0.45 = 0.202

As seen from the tables above, the values for P(Y=y) vary depending on which table you are looking at. This tells us the probabilities for each value of Y depend on the given value for X, showing us that these two variables are indeed not independent.

We do an example like this (the workstudy students example) in lecture of week 6 and Chapter 6 of the textbook.

No attachments or screenshots allowed and no answering in the pdf files of other questions or (-1 points from final score for this question)

(a) S= { 111, 110, 101, 100, 011, 010, 001, 000} where 1 denotes votes YES, and 0 votes NO, and, for example, 011 denotes that the first student interviewed voted NO and the last two voted YES.

(Sample space 0.5 pts; all or nothing. Must show individual outcomes, not a value of a random variable. For example, saying S={1 says yes, 2 say yes, 3 say yes, none says yes) does not get credit as this describes a random variable, a function from the top S to the reals.

$$P(111) = 0.45^3; \quad P(110) = P(011) = P(101) = 0.55(0.45^2); \quad P(100) = P(010) = P(001) = 0.45(0.55)^2; \quad P(000) = 0.55^3.$$

(probabilities 0.5pts-Note, if you write 111 = 0.45^3 that is not true, and we have to deduct points. Notation matters when writing math, and you must respect the notation.)

The probabilities for each outcome are calculated using the product rule for independent events. We are assuming that the students are independent. **(0.5pts)**

(b)

Work shown to identify the probabilities in the table (0.5pts)

S=	{ 111,	110,	101,	100,	011,	010,	001,	000}
X	1	1	1	1	0	0	0	0
Y	3	2	2	1	2	1	1	0

P(0) 0.0911 0.1113 0.1113 0.1361 0.1113 0.1361 0.1361 0.1663 rounded

We use the above information to put together the requested joint pmf table

Table (0.5 pts all or nothing. The P(Y=y) and P(X=)

x \y	0	1	2	3	
0	0.1663	0.2722=2(0.1361)	0.1113	0	
1	0	0.1361	0.2226=2(0.1113)	0.0911	0.4498
P(Y=y)	0.1663				

(c)

X, Y are independent if P(X=x)P(Y=y) = P(X=x, Y=y) **(defines independence 0.2 pts)**

So we check one case X=1, Y=0 where **(shows they are not independent using definition 0.2pts)**

P(X=1, Y=0) = 0

But P(X=1)P(Y=0) =(0.1361 +0.2226+0.0911)(0.1663)= (0.4498)(0.1663) =0.0748

So $P(X, y) \neq P(X)P(Y)$ and therefore, X and Y are not independent. **(Concludes 0.1pt)**

Comment:

Question **3**

Correct

Mark 1.00 out of 1.00

We have two conditional distributions below:

y	P(Y=y|X=0)
0	$P(Y=0\|X=0) = \frac{P(X=0,Y=0)}{P(X=0)} = \frac{0.012}{0.3} = 0.04$
1	$P(Y=1\|X=0) = \frac{P(X=0,Y=1)}{P(X=0)} = \frac{0.096}{0.3} = 0.32$
2	$P(Y=2\|X=0) = \frac{P(X=0,Y=2)}{P(X=0)} = \frac{0.192}{0.3} = 0.64$
3	$P(Y=3\|X=0) = \frac{P(X=0,Y=3)}{P(X=0)} = \frac{0}{0.3} = 0$

y	P(Y=y|X=1)
0	$P(Y=0\|X=1) = \frac{P(X=1,Y=0)}{P(X=1)} = \frac{0}{0.7} = 0$
1	$P(Y=1\|X=1) = \frac{P(X=1,Y=1)}{P(X=1)} = \frac{0.028}{0.7} = 0.04$
2	$P(Y=2\|X=1) = \frac{P(X=1,Y=2)}{P(X=1)} = \frac{0.224}{0.7} = 0.32$
3	$P(Y=3\|X=1) = \frac{P(X=1,Y=3)}{P(X=1)} = \frac{0.448}{0.7} = 0.64$

The two random variables X and Y are

- a. complement of each other
- b. not independent ✔
- c. mutually exclusive
- d. independent

Your answer is correct.

The correct answer is:
not independent

Question **4**

Correct

Mark 1.00 out of 1.00

Office hours videos, supplementary lectures, module week 6.

The distribution of blood types in a certain country are such that 25% of the population are type O, 55% are type A, 20% are type B. Suppose that 30 people are selected at random from this country. What is the probability that 10 are type O, 10 are type A, and the other 10 are type B?

a. 0.999

b. 0.7814

c. 0.33145

d. 0.001373087

Your answer is correct.

See section 6.10 of the textbook

See also the video on the multinomial.

The correct answer is:

0.001373087

Question **5**

Correct

Mark 1.00 out of 1.00

The random variable Y can take values 1 or 4, and the random variable X can take values $-1, -2, 1, 2$.
The joint probability mass function of X and Y, $P(X, Y)$, appears below.

		Y	
		1	**4**
	-2	0	1/4
X	**-1**	1/4	0
	1	1/4	0
	2	0	1/4

Select all that applies

Select one or more:

- [ ] a. X, Y are two independent random variables.
- [x] b. $E(X) = 0$ ✔
- [x] c. X, Y are not two independent random variables. ✔
- [ ] d. $Var(X) = 1$

The correct answers are: X, Y are not two independent random variables., $E(X) = 0$

Question **6**

Correct

Mark 1.00 out of 1.00

The ticket office offers two alternatives to go to Universal Studio, the thrifty and the luxurious alternatives.
We denote by X the proportion of students that buy the thrifty alternative and by Y the proportion of students that buy the luxurious one.

$$f(x,y) = 2(x+y) \qquad 0 < y < x < 1,$$

In a given weekend it is known that 10 percent of students purchased the luxurious alternative. Knowing that, what should we expect the proportion of students purchasing the thrifty alternative to be? Find out showing detailed work.

Select one:

- a. 0.341 approximately
- b. 0.901 approximately
- c. 0.654 approximately
- d. 0.167 approximately ✖

Key prepared by Stephen Smith

$$f_{X|Y=0.1}(x) = \frac{f(x, y = 0.1)}{f(y = 0.1)}$$

For the numerator, we have

$$f(x, y = 0.1) = 2(x + 0.1)$$

For the denominator, we must first find the marginal pdf for Y.

$$\begin{aligned} f(y) &= \int_y^1 2(x + y)dx \\ &= x^2 + 2xy \,|_y^1 \\ &= 1 + 2y - y^2 - 2y^2 \\ &= 1 + 2y - 3y^2 \end{aligned}$$

for $0 < y < 1$. Then, $f(y = 0.1) = 1.17$ and

$$f_{X|Y=0.1}(x) = \frac{2(x + 0.1)}{1.17}$$

5

where $0.1 < x < 1$. Now, we can find $\mathbb{E}(X \mid Y = 0.1)$, which is given by

$$\begin{aligned} \mathbb{E}(X \mid Y = 0.1) &= \int_{0.1}^1 \frac{2x(x + 0.1)}{1.17} dx \\ &= \frac{2}{1.17}\left[\frac{1}{3}x^3 + 0.05x^2 \,|_{0.1}^1\right] \\ &= \frac{2}{1.17}\left[\frac{1}{3} + 0.05 - \frac{0.001}{3} - 0.05(0.01)\right] \\ &= 0.65 \end{aligned}$$

The correct answer is: 0.654 approximately

Question 7

Correct

Mark 1.00 out of 1.00

The conditional probability density function of random variable Y when X is 0 is given by

y	P(Y=y|X=0)
0	0.04
1	0.32
2	0.64
3	0

and the unconditional probability mass function of X is

x	P(X=x)
0	0.3
1	0.7

Then the joint probability P(Y=1, X=0) is equal to

- a. 0.096
- b. 0.89
- c. 0.124
- d. 0.028

Your answer is correct.

The definition of joint probability is P(Y,X) = P(Y|X)P(X). So multiply those two distributions.

The correct answer is:
0.096

Question **8**

Complete

Mark 2.90 out of 3.00

This question requires that you show work. Work is 80% of your credit and involves showing intermediate steps. The rest is for labeling and defining things so that we know what they are, making clear what probabilities you are calculating and labeling them as such (are they conditional, total, etc..?) and providing your final number.

=======

Suppose we have a set of independent and identically distributed continuous random variables x, $x_1, x_2, \ldots\ldots, x_n$, each of them with the density given below. We would like to find the joint density function of these n random variables.

$$f(x) = \frac{\gamma}{\alpha}\left(\frac{x-\mu}{\alpha}\right)^{(\gamma-1)} \exp\left(-((x-\mu)/\alpha)^{\gamma}\right) \qquad x \geq \mu; \gamma, \alpha > 0$$

(a) Showing detailed intermediate work, provide a final formula where you have simplified the final expression as much as possible, as done in the pre-recorded lecture videos.

(b) Where in past modules, in pre-recorded lectures before module 6, did we use a joint density to solve a statistical estimation problem? Describe what problem that was.

(c) When we say that $x_1, x_2, \ldots\ldots, x_n$ are independent identically distributed, are we saying that all of those x_i have the same value? Use a random variable that applies to students at UCLA, a continuous random variable to explain.

You may attach a file with work. But the file must contain only the answer to this question. If it contains answers for other questions, those questions will not be graded at all. You may not email your work to this question. It will not count that way.Alternatively, it would be best if you could use the editor and solve your math work using the equation editor in it, which you can see by clicking on the arrow in the menu and selecting the calculator icon.

See attached PDF

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If using attachment, only the answer to this question allowed in the attachment. If you write in the editor the notation must be not ambiguous. If any of the above, subtract 1 pt from the total for the question.

(a) **(2 pts total)**

$f(x_1, x_2, \ldots\ldots, x_n) = f(x_1)f(x_2)\ldots\ldots f(x_n) =$ **Defines joint density function (0.5 pt)**

$\left(\frac{\gamma}{\alpha}\left(\frac{x_1-\mu}{\alpha}\right)^{\gamma-1} e^{-\left(\frac{x_1-\mu}{\alpha}\right)^{\gamma}}\right)\left(\frac{\gamma}{\alpha}\left(\frac{x_2-\mu}{\alpha}\right)^{\gamma-1} e^{-\left(\frac{x_2-\mu}{\alpha}\right)^{\gamma}}\right)\ldots\ldots\ldots\left(\frac{\gamma}{\alpha}\left(\frac{x_n-\mu}{\alpha}\right)^{\gamma-1} e^{-\left(\frac{x_n-\mu}{\alpha}\right)^{\gamma}}\right)$ **Shows work (1pt)**

$= \left(\frac{\gamma}{\alpha}\right)^n \prod_{i=1}^n \left(\frac{x_i-\mu}{\alpha}\right)^{\gamma-1} e^{-\sum_{i=1}^n \left(\frac{x_i-\mu}{\alpha}\right)^{\gamma}}$ **Simplifies (0.5pt)**

(b) **0.5 pts total.**

We used a joint pdf in the example of the radon data and the fitting of a log normal distribution to that data, in Lecture 11B in Module 4. (0.5 pts- **example 0.2pts, lecture 0.2pts, module 0.1pts)**

(c) **(0.5pts total)** (Answers will vary but the description of what iid means should be depicting the correct concepts.

A random variable that applies to students at UCLA has to be **a continuous random variable, (0.2pts) s**ome metric that can take values in the real line. For example, the distance from the dorms to the first classroom of the day on Mondays. Although the **distribution of distances could is identical for all students** (after all, we create the distribution by asking all students their distances), so all students contribute to the distribution, **each student**

has a different value of the random variable X. (last two bolded statements, 0.3 pts) Identically distributed only means that the student is one of the contributors to the making of that distribution. This is what we call in statistics a random draw from a probability distribution.

Comment:
need to mention which module -0.1

Question **9**

Correct

Mark 1.00 out of 1.00

Let X be a random variable with density function

$$f(x, y) = 2, \quad 0 \leq y \leq x \leq 1$$

Which of the following is the marginal probability density function of X?

Select one:

a. $$f(x) = 2x^2, \quad 0 \leq x \leq 1$$

b. $$f(x) = 2x, \quad 0 \leq x \leq 1$$

c. $$f(x) = 2x, \quad y \leq x \leq 1$$

d. $$f(x) = 2xy, \; \; \; \; y \leq x \leq 1 $$

Your answer is correct.

The correct answer is: $$f(x) = 2x, \; \; \; \; 0\leq x \leq 1 $$

Question **10**

Correct

Mark 1.00 out of 1.00

When we talk about the joint density function of two random variables, X, Y, (f(x,y)), for constants a and b,

$$P(X \leq a, Y \geq b)$$ is

Select one:

a. always 1 to satisfy axioms

b. an area

c. the value of the first quartile

d. a volume

Your answer is correct.

The correct answer is: a volume