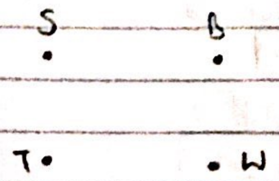
Math 61 HW#7

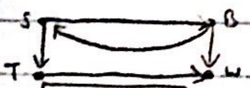
1.



i)

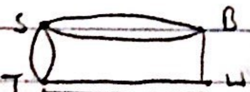
$\boxed{\text{Undirected, simple}}$

ii)



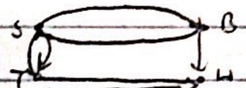
$\boxed{\text{Directed}}$

iii)



$\boxed{\text{Undirected}}$

iv)

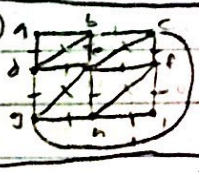


$\boxed{\text{Directed}}$

2. a)

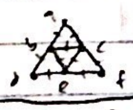
$\boxed{a,b,d,c,e,d,f,e,b,c,a}$

b)



$\boxed{a,d,g,e,b,d,e,f,h,e,c,f,i,h,g,c,b,a}$

c)



$\boxed{a,b,d,e,f,c,b,e,c,a}$

3. a) The verticies c and d have an odd number of incident edges

b) Vertex d has an odd number of incident edges

c) Vertex b and d have an odd number of incident edges

4. i) $V = \{v_1, v_2, v_3, v_4\}$

$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$

Parallel: e_1 and e_6

Loops: e_5

Iso. Verticies: v_4

$\boxed{\text{Not simple}}$

$\boxed{e_1 \text{ is incident on } v_1 \text{ and } v_2}$

ii) $V = \{v_1, v_2, v_3, v_4, v_5\}$

$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$

Parallel: None

Loops: None

$\boxed{\text{Simple}}$

$\boxed{e_1 \text{ is incident on } v_3 \text{ and } v_4}$

iii) $V = \{v_1, v_2, v_3\}$

$E = \emptyset$

Parallel/loops: None

$\boxed{\text{Simple}}$

$\boxed{e_1 \text{ DNE}}$

5.* K_n has N verticies

Each vertex has $(n-1)$ incident edges

Each edge is counted 2x

$\boxed{\frac{N(N-1)}{2}}$

6. i) $\boxed{\text{Not bipartite - loop}}$

ii) $\boxed{\text{Not bipartite - triangle}}$

iii) $\boxed{\text{Bipartite} \cdot \{v_1, v_2, v_5\} \text{ and } \{v_3, v_4\}}$

iv) $\boxed{\text{Bipartite}: \{v_1, v_6, v_7\} \text{ and } \{v_2, v_3, v_4, v_5, v_8, v_9, v_{10}\}}$

7.* M verticies with N edges

$\boxed{M \vee N}$

8. i) $v=b, u=e$

$\boxed{b, c, a, d, e \to 16}$

ii) $v=a, u=b$

$\boxed{a, e, d, c, b \to 17}$

iii) $v=c, u=d$

$\boxed{c, b, a, e, d \to 24}$

9. $\boxed{(a,b), (b,f), (b,d), (d,c), (c,g), (g,e)}$

↳ 55

10. $f(G)=|E|$ is both 1-to-1 and onto, as a simple graph has no loops or parallel edges, so a simple graph with n vertices has exactly n-1 edges. Therefore ~~n~~ 2 simple graphs in $\mathcal{G}$ have the same # of edges (1-to-1), and there exist some simple graph in $\mathcal{G}$ with $n \in \mathbb{N}$ edges.

11. i) (a, d, c, d, e)

$\boxed{\text{None}}$

ii) (e, d, c, b)

$\boxed{\text{simple path}}$

iii) (b, c, d, e, b, b)

$\boxed{\text{cycle}}$

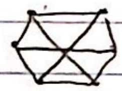
iv) $(b, c, d, a, b, e, d, c, b)$

$\boxed{\text{none}}$

v) (d, c, d)

$\boxed{\text{none}}$

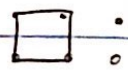
12. i).



ii)

iii) Total degree is odd - not possible

iv)



v) Handshaking Theorem requires at least 5 edges, not possible

vi) Impossible, requirement of 2 verticies w/ degree 5 violates the 1 vertex w/ degree 1.

13. Cycles: $\{(a,a), (b,c,g), (c,b,g), (g,b,c)$

↳ this is worthless

Paths: $\{(a,b,c,d,e), (a,b,g,f,e), (a,b,c,g,f,d,e), (a,b,c,d,f,e), (a,b,g,c,d,f,e), (a,b,g,f,d,e)\}$

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14. i) $\delta(v_1)=4,\ \delta(v_2)=4,\ \delta(v_3)=4,$
$\delta(v_4)=4,\ \delta(v_5)=4$

ii) $\delta(v_1)=2,\ \delta(v_2)=2,\ \delta(v_3)=3,$
$\delta(v_4)=6,\ \delta(v_5)=2,\ \delta(v_6)=2,$
$\delta(v_7)=4,\ \delta(v_8)=4,\ \delta(v_9)=4,$
$\delta(v_{10})=2$

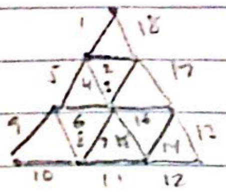
15. i) $V'=\{v_1\}, E'=\emptyset$ $\quad V'=\{v_1,v_3\}, E'=\{(v_1,v_3)\}$
$V'=\{v_2\}, E'=\emptyset$ $\quad V'=\{v_1,v_2,v_3\}, E'=\emptyset$
$V'=\{v_3\}, E'=\emptyset$ $\quad V'=\{v_1,v_2,v_3\}, E'=\{(v_1,v_2)\}$
$V'=\{v_1,v_2\}, E'=\emptyset$ $\quad V'=\{v_1,v_2,v_3\}, E'=\{(v_2,v_3)\}$
$V'=\{v_2,v_3\}, E'=\emptyset$ $\quad V'=\{v_1,v_2,v_3\}, E'=\{(v_1,v_2),(v_2,v_3)\}$
$V'=\{v_1,v_3\}, E'=\emptyset$
$V'=\{v_1,v_2\}, E'=\{(v_1,v_2)\}$

ii) $V'=\{v_1\}, E'=\emptyset$ $\quad V'=\{v_2\}, E'=\emptyset$
$V'=\{v_1,v_2\}, E'=\emptyset$, $\quad V'=\{v_1,v_2\}, E'=\{e_1\}$
$V'=\{v_1,v_2\}, E'=\{e_2\}$, $\quad V'=\{v_1,v_2\}, E'=\{e_1,e_2\}$

iii) Nope fuck that

16. i) $\boxed{(f,b,a,h,j,g,f,j,e,i,h,c,i,d,c,b,d,e,f)}$
ii) $\boxed{(a,b,c,b,d,e,g,h,e,f,h,i,j,k,i,f,g,d,c,a)}$

17.



18. i) $K_n \rightarrow$ $\boxed{\text{when } n \text{ is odd}}$
ii) $K_{m,n} \rightarrow$ $\boxed{\text{when } m \text{ and } n \text{ are even}}$

19. 11 verticies
degree = 9
Total games play = $\frac{99}{2}$

$\boxed{\text{You cannot have a non-whole \# of games}}$

20. $\boxed{(d,b,a,d,f,g,d,c,h,g,j,h,i,e,b,c,e)}$

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