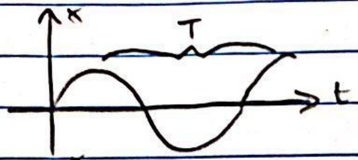
1/7 Lecture: Oscillations

- Period of a periodic process: smallest time interval between repetitions
  - ↳ denoted by $T$
- Ex) $x$ = some oscillating quantity

↳
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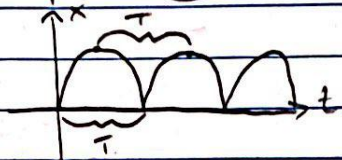


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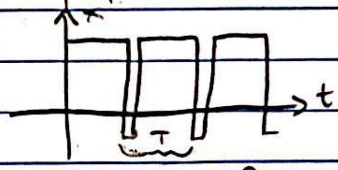



- The frequency $f$ is the inverse of the period → $T^{-1}$
  - ↳ $[T]$ = seconds $\therefore$ $[f] = sec^{-1} = Hz$
- Simple Harmonic Oscillator (SHO)
  - ↳ Model System: point mass ($m$) on an ideal spring with spring constant $k$
    - ↳ ideal springs are massless and obey Hooke's Law ($F=-kx$)
      - ↳ Using Newton's 2nd Law: $F=mA$, $A=\frac{\partial^2 x}{\partial t^2}$
        - ↳ $m\frac{\partial^2 x}{\partial t^2} = -kx$, $\omega = \sqrt{\frac{k}{m}}$
        - ↳ $\frac{\partial^2 x}{\partial t^2} = -\omega^2 x \rightarrow \frac{\partial^2 x}{\partial t^2} + \omega^2 x = 0$
          - ↳ $[\omega] = sec^{-1}$
    - ↳ Claim: $x(t) = A\cos(\omega t + \phi_0)$
      - ↳ $v(t) = x'(t) = -A\omega \sin(\omega t + \phi_0)$
      - ↳ $a(t) = x''(t) = -A\omega^2 \cos(\omega t + \phi_0) = \frac{\partial^2 x}{\partial t^2}$
        - ↳ $-A\omega^2\cos(\omega t + \phi_0) + \omega^2 x = 0$ ✓
      - ↳ $A$ = amplitude, $\phi_0$ = phase shift
      - ↳ Using $T$ of $\cos(\theta) = 2\pi$: $\omega T = 2\pi$, $\omega = \frac{2\pi}{T} = 2\pi f$
  - ↳ $T_{spring} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$
    - ↳ $f_{spring} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
  - ↳ Amplitude independent of frequency
  - ↳ $x_0 = x(t=0) = A\cos(\phi)$ } initial conditions
  - ↳ $v_0 = v(t=0) = -A\omega\sin(\phi)$ }
    - ↳ $\phi = 0 \rightarrow x_0 = A$, $v_0 = 0$ → at the amplitude (right)
    - ↳ $\phi = \frac{\pi}{2} \rightarrow x_0 = 0$, $v_0 = -A\omega$ → at the origin
    - ↳ $\phi = \pi \rightarrow x_0 = -A$, $v_0 = 0$ → at the amplitude (left)

↳ $\sin^2\alpha + \cos^2\alpha = 1$

↳ $\left(\frac{v(t)}{A\omega}\right)^2 + \left(\frac{x(t)}{A}\right)^2 = 1$

$A = \sqrt{[x(t)]^2 + [v(t)/\omega]^2} \rightarrow$ for any $t$

↳ at $t=0 \rightarrow A = \sqrt{x_0^2 + v_0/\omega^2} > 0$

↳ $-\frac{v_0}{x_0} = \omega \tan\phi_0$

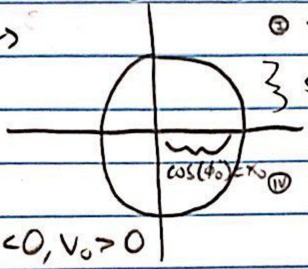
$-\frac{v_0}{\omega x_0} = \tan\phi_0$

$\phi_0 = \tan^{-1}\left(\frac{-v_0}{\omega x_0}\right)$

↳ often, but not always true, $\tan^{-1}$ can only output values $-\frac{\pi}{2} < x < \frac{\pi}{2}$

↳ $\phi_0 \in [-\pi, \pi) \rightarrow$ larger than range of $\tan^{-1}$

↳



Ⅰ $x_0 > 0,\ v_0 < 0$

} $\sin(\phi_0) = -v_0$

Ⅱ

$\cos(\phi_0) = x_0$

Ⅳ $x > 0,\ v_0 > 0$

Ⅲ $x_0 < 0,\ v_0 > 0$

↳ in QI, QIV, there are no problems, within the range of $\tan^{-1}$

↳ in QII, QIII $\phi_0 = \tan^{-1}\left(\frac{-v_0}{\omega x_0} - \pi\right)$ in QIII and $\phi_0 = \tan^{-1}\left(\frac{-v_0}{\omega x_0} + \pi\right)$ in QII

↳ Ex) $k = 200\ N/m,\ m = 0.5kg,\ x_0 = 0.015m,\ v_0 = 0.4\frac{m}{s}$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200N/m}{0.5kg}} = 20\ sec^{-1} = 20Hz$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{20Hz} = \frac{\pi}{10}\ sec$

$A = \sqrt{x_0^2 + (v_0^2/\omega^2)} = \sqrt{0.015m^2 + (0.4\frac{m}{s}^2/20Hz^2)} = 0.025m$

$\phi_0 = \tan^{-1}\left(\frac{-v_0}{\omega x_0}\right) = \tan^{-1}\left(\frac{-0.4\frac{m}{s}}{20Hz(0.015m)}\right) = \boxed{-0.927 rads}$

# 1/9 Lecture: Energy and Examples of SHM

• Summary:

↳ $\frac{\partial^2}{\partial t^2}x(t) + \omega^2 x(t) = 0$ (SHO eq.)

↳ $x(t) = A\cos(\omega t + \phi_0)$

↳ $v(t) = -A\omega\sin(\omega t + \phi_0)$

↳ $T = \frac{2\pi}{\omega}$, initial condition, $x_0 = x(t=0)$, $v_0 = v(t=0)$

↳ $A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}} = \sqrt{(x(t))^2 + \frac{(v(t))^2}{\omega^2}}$

↳ $$\phi_0 = \tan^{-1}\left(\frac{-v_0}{\omega x_0}\right) + \begin{cases} 0 & \text{if } x_0 > 0 \\ \pi & \text{if } x_0 < 0,\ v_0 < 0 \\ -\pi & \text{if } x_0 < 0,\ v_0 > 0 \end{cases}$$

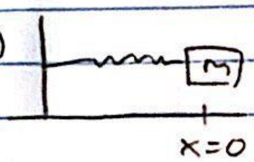
↳ $\omega = \sqrt{\frac{k}{m}}$

• Energy:

↳ $K = \frac{1}{2}m(v(t))^2$, $U = \frac{1}{2}k(x(t))^2$

↳ $E = \frac{1}{2}kA^2 = \frac{1}{2}m\underbrace{\omega^2 A^2}_{v_{max}}$

↳ Ex)




$k = 200\frac{N}{m}$

$m = 0.5kg$

$v_0 = 0.4\frac{m}{s}$

$x_0 = 0.015m$

$E = K_0 + U_0 = \frac{1}{2}(0.5kg)(0.4\frac{m}{s})^2 + \frac{1}{2}(200\frac{N}{m})(0.015m)^2$

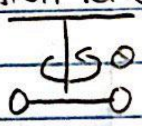
$E = 0.0625 J$

$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(0.0625J)}{200\frac{N}{m}}} = \boxed{0.025m}$

$v_{max} = \sqrt{\frac{2E}{m}} = \boxed{0.5\frac{m}{s}}$

• Torsion Pendulum

↳



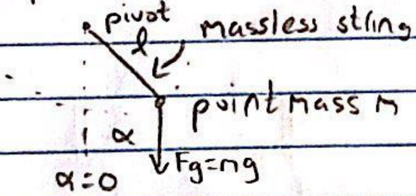

$\theta = 0$ is equilibrium

$\tau_{rest} = -\kappa\theta$

$I\frac{\partial^2\theta}{\partial t^2} = \tau = -\kappa\theta$

$\frac{\partial^2\theta}{\partial t^2} + \frac{\kappa}{I}\theta = 0$

• Simple Pendulum



↳ arclength: $l\alpha$

↳ $m\frac{\partial^2 x}{\partial t^2} = F^{\perp} = -mg\sin\alpha = ml\frac{\partial^2 \alpha}{\partial t}$

$\frac{\partial^2 \alpha}{\partial t^2} + \frac{g}{l}\sin\alpha = 0$

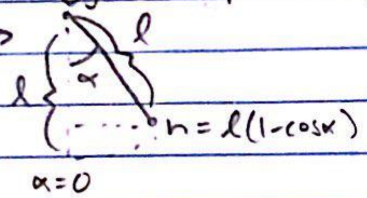
↳ small angle approximation: $\alpha \approx \sin\alpha$

↳ $\frac{\partial^2 \alpha}{\partial t^2} + \omega^2 \alpha = 0$

↳ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}} \to \omega^2 = g/l$

↳ $\alpha(t) = \alpha_{max}\cos(\omega t + \phi_0)$

↳ Energy of a pendulum

↳ 


$U = mgh = mgl(1-\cos\alpha)$

↳ small angle approximation $\to U \approx \frac{1}{2}mgl\alpha^2$

· Damping

↳ $F_f = -bv$, $b > 0$

↳ $mA = -kx - bv$

↳ 3 cases:

↳ Underdamping

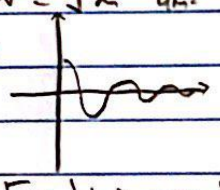
↳ Critical damping

↳ Overdamping

↳ Underdamped: $\frac{k}{m} > \frac{b^2}{4m^2}$ (small friction)

$x(t) = Ae^{-(\frac{b}{2m})t}\cos(\omega t + \phi_0)$

$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} < \sqrt{\frac{k}{m}}$

↳ 

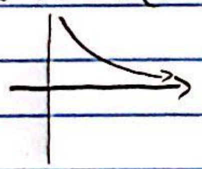
↳ Overdamped: $\frac{k}{m} < \frac{b^2}{4m^2}$

$x(t) = x_+ e^{(-\frac{b}{2m} + \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}})t} + x_- e^{(-\frac{b}{2m} - \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}})t}$

↳ $E = \frac{1}{2}kA \to \frac{1}{2}kA^2 e^{-\frac{b}{m}t}$

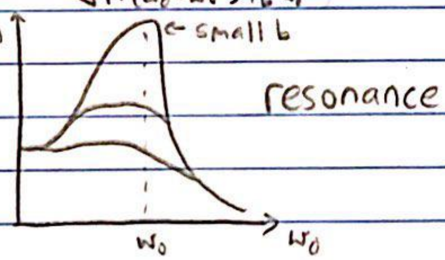
↳ Critical Damping: $\frac{k}{m} = \frac{b^2}{4m^2}$

$x(t) = x_0(1 + \frac{b}{2m}t)e^{-\frac{b}{2m}t}$

↳ 

# 1/14 Lecture: Driving Force and Waves

- All S.H.O: $\frac{\partial^2 x}{\partial t^2} + \omega^2 x(t) = 0$, $\omega \to$ angular frequency
  - ↳ $\omega^2 = \frac{\text{restoring force}}{\text{inertia}} = \frac{k}{m}$
  - ↳ Simple pendulum (length $\ell$), small angles ($\lesssim 20°$ or 0.35 rads)
    - ↳ $\omega^2 = \frac{g}{\ell}$, $T = 2\pi\sqrt{\frac{\ell}{g}}$
    - ↳ $K = \frac{1}{2}m\ell^2\left(\frac{d\alpha}{dt}\right)^2$, $U = \frac{1}{2}mg\ell\alpha^2$
- Damping
  - ↳ $F_f = -bv$, $b > 0$
  - ↳ Underdamped $\to \frac{k}{m} > \frac{b^2}{4m^2}$, $x(t) = Ae^{-b/2m}\cos(\omega' t + \phi_0)$
    - ↳ $\omega'$ is different, the damping slows the frequency
    - ↳ $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$
    - ↳ $x(t+T) = e^{(-b/2m)T}x(t)$
    - ↳ $E(t+T) = e^{(-b/2m)T}E(t)$
  - ↳ Critical damping $\to \frac{k}{m} = \frac{b^2}{4m^2}$
    - ↳ $v_0 = 0$, $x_0 = A \to x(t) = Ae^{-\frac{b}{2m}t}\left(1 + \frac{b}{2m}t\right)$
  - ↳ Overdamped $\to \frac{k}{m} < \frac{b^2}{4m^2}$
    - ↳ exponential decay that is slower than critical damping
    - ↳ $x(t) = A\exp\left[-\left(\frac{b}{2m} - \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}\right)t\right]$
- Driven (+ damped) SHO
  - ↳ SHO + $F_{ext} = F_0\cos(\omega_d t)$
  - ↳ Newton's 2nd Law: $m\frac{\partial^2 x}{\partial t^2} = F_{total}$
    - ↳ $m\frac{\partial^2 x}{\partial t^2} = -kx - b\frac{\partial x}{\partial t} + F_0\cos(\omega_d t)$, $k, b > 0$
  - ↳ Amplitude depends on $\omega_d$, maximized at resonance $\to$ phases are in sync
    - ↳ $x(t) = A\cos(\omega_d t + \phi_0)$
      - ↳ $A = \frac{F_0}{\sqrt{m^2(\omega_0^2 - \omega_d^2) + b^2\omega_d^2}}$
    - ↳ A

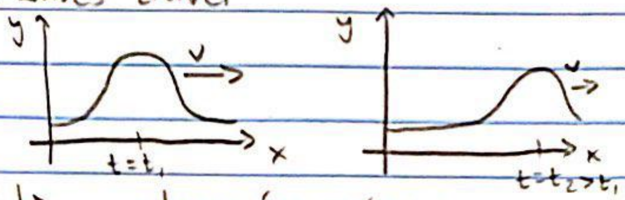



- ↳ peaks at $\omega_0$, shifts with change in b
- ↳ for larger values of b, the peak gets lower and wider

- Waves

↳ Transverse wave → wave where the particle's disturbance is perpendicular to the wave's travel

↳ Longitudinal wave → wave where the particle's disturbance is parallel to the wave's travel

↳ 


↳ $v = \frac{\text{change in maximums}}{t_2 - t_1}$ = wave speed

↳ Use of $y(x,t) = f(x - vt)$ to model wave's behavior

↳ $y(x,t) = g(x+vt)$ for wave moving left

↳ Wave equation

↳ $y(x,t) = f(x-vt)$

$$\frac{\partial}{\partial t} y(x,t) = -vf'(x-vt)$$

$$\frac{\partial^2 y}{\partial t^2} = v^2 f''(x-vt)$$

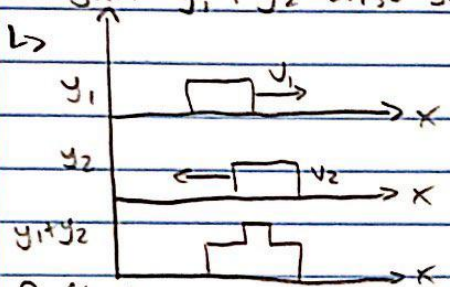
$$\frac{\partial^2 y}{\partial x^2} = f''(x-vt)$$

$$\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$$

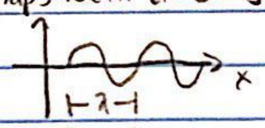
↳ Linear superposition: if $y_1(x,t)$ and $y_2(x,t)$ satisfy the wave equation, then $y_1 + y_2$ also does

# 1/16 Lecture: Wave Behavior

- Recap: Waves
  - ↳ After wave passes => equilibrium
  - ↳ speed $v$ doesn't depend on shape of wave
  - ↳ Shape of the wave is "rigid"
  - ↳ wave function: $y(x,t) = f_R(x-vt) + f_L(x+vt)$
    - displacement relative to equilibrium; location along the medium
  - ↳ $\frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial x^2}$
- Principle/Law of Linear Superposition
  - ↳ 2 waves $y_1(x,t)$ and $y_2(x,t)$ satisfy the wave equation -> their sum $y_1 + y_2$ also satisfies the wave equation
  - ↳

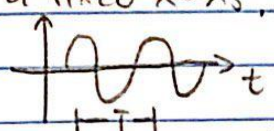



- Reflection
  - ↳ 2 simple examples:
    - ↳ free endpoint -> reflection
    - ↳ fixed endpoint -> reflection + inversion
      - ↳ changes direction and sign of wave
- High density -> low density string
  - ↳ free boundary transition
- Low density -> high density string
  - ↳ fixed boundary transition
- Sinusoidal Waves
  - ↳ Snapshot in time = sinusoidal function $y(t=t_0, x)$
    - ↳




$y(x+\lambda, t) = y(x,t)$

  - ↳ At a fixed $x=x_0$, $y(x=x_0,t)$ undergoes SHO
    - ↳




$y(x, t+T) = y(x,t)$

  - ↳ Going to the right: $y(x,t) = A\cos\left(\frac{2\pi}{\lambda}(x-vt) + \phi_0\right)$, $-\pi < \phi_0 < \pi$

↳Ex) $y(x,t) = 0.1m \, \sin\left(\frac{x}{0.2m} - \frac{t}{3s}\right)$

↳moving right or left? → right

↳ $A = 0.1m$, $\phi_0 = -\frac{\pi}{2}$

$\frac{\lambda}{0.2m} = 2\pi = \boxed{1.26m}$

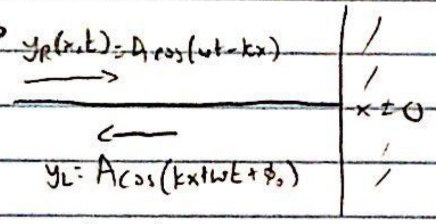
$\frac{T}{3s} = 2\pi = \approx 18s$

$f = \frac{1}{18s} \approx 0.05Hz$

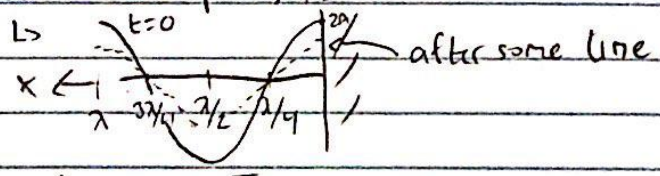
$v = \lambda f = 1.26m(0.05Hz) = \boxed{0.067 m/s}$

# Lecture 1/21: Wave Energy

- Speed of wave traveling on a string
  - ↳ $v^2 = \frac{\text{restoring force}}{\text{inertia}} = \frac{T}{\mu}$, $\mu = \frac{M}{L}$
  - ↳ Ex) $\overset{M}{\longrightarrow}$ $F_t = T$
- L/R moving sinusoidal wave
  - ↳ $y_{[L]}(x,t) = A\cos\left(\frac{2\pi}{\lambda}(x) + \frac{2\pi}{T}t + \phi_0\right)$
    - ↳ $x \to x + n\lambda$, $t \to t + nT$
    - ↳ $v = \frac{\lambda}{T} = \lambda f$
  - ↳ $y_R(x,t) = A\cos(kt - \omega t + \phi_0)$
    - ↳ $k = \frac{2\pi}{\lambda}$, $\omega = \frac{2\pi}{T} = 2\pi f$
- Power $= P = \frac{\text{Work}}{\text{time}} = \vec{F}\cdot\vec{v}$
  - ↳ $y_R(x,t) = A\cos\left(\frac{2\pi}{\lambda}x - \omega t\right)$
    - ↳ $P = \mu A^2\omega^2 v\sin^2\left(\frac{2\pi}{\lambda}x - \omega t\right)$
    - ↳ $\bar{P} \to$ time avg. of $P$, avg of full period
      - ↳ Let $x = 0$
      - ↳ $\frac{1}{T}\int_0^T P(t)\,dt = \frac{P_0}{T}\int_0^T dt\,\sin^2\left(\frac{2\pi t}{T}\right) = \frac{T}{2}\left(\frac{P_0}{T}\right) = \frac{P_0}{T} = \frac{1}{2}\mu A^2\omega^2 v$
- Standing Waves
  - ↳ reflection from boundaries + superposition
  - ↳

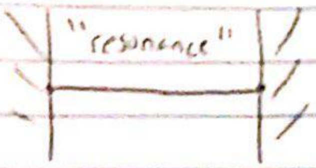



  - ↳ free boundary $\to \phi_0 = 0$, fixed boundary $\to \phi_0 = \pi$
  - ↳ $y_{tot} = y_L + y_R$
  - ↳ $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
    - ↳ $y_{tot} = 2A\cos\left(kx + \frac{\phi_0}{2}\right)\cos\left(\omega t + \frac{\phi_0}{2}\right)$
- ↳ Free exact point, $\phi_0 = 0$
  - ↳




  - ↳ at $t = \frac{T}{4}$, $y_{tot} = 0$
  - ↳ at $t = \frac{T}{2}$, the wave is inverted
  - ↳ points where $y_{total} = 0$ for all $t \to$ nodes
  - ↳ points where maxes occur $\to$ anti-nodes
- ↳ Fixed end, $\phi_0 = \pi$
  - ↳ $y_{total}(x,t) = 2A\sin(kx)\sin(\omega t)$

↳ "resonance"



fixed ends, nodes at boundaries

$x=0$ $x=L$

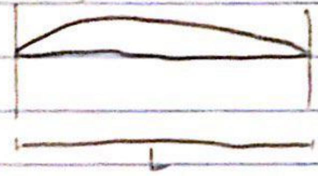
↳ $\sin\left(\frac{2\pi}{\lambda}L\right)=0$

$\frac{2\pi}{\lambda}L = n\pi$

$\lambda_n = \frac{2L}{n}, \; n \neq 0$

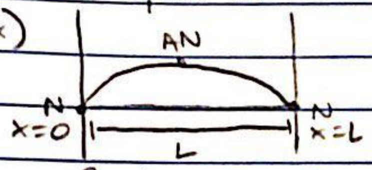
↳ wavelength is tuned so that the nodes are in the correct locations

↳ $n=1 \to \lambda = 2L$ $n=2 \to \lambda = L$ → harmonics



1/23 Lecture: Harmonics

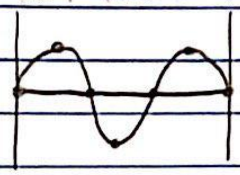
- Midterm Review: Mon. 5pm-6:50pm
  - ↳ this room
- Standing Waves
  - ↳ d between nodes and antinodes $= \frac{\lambda}{2}$
  - ↳ nodes and antinodes alternate
    - ↳ node → 0 amplitude, antinode → max amplitude
  - ↳ d between node and antinode $= \frac{\lambda}{4}$
  - ↳ Boundary conditions:
    - ↳ fixed end = node
    - ↳ open end = antinode ] at boundary
  - ↳ $v = \lambda f$ → valid for all sinusoidal waves
    - ↳ wave speed of a wave along a medium
  - ↳ Ex)




$\frac{\lambda}{2} = L \rightarrow \lambda_1 = 2L \rightarrow n=1 \rightarrow$ fundamental frequency / 1st harmonic

$L/2 = \lambda_2/2 \Rightarrow \lambda_2 = L \rightarrow$ Amplitude $\sim A\sin\left(\frac{2\pi x}{L}\right)$

↳



$\lambda_3 = \frac{2L}{3}$

- ↳ General formula: $\lambda_n = \frac{2L}{n}$, Amplitude $\sim A\sin\left(\frac{n\pi x}{L}\right)$

- Resonant frequencies
  - ↳ $f_n \lambda_n = v = \sqrt{\frac{F}{\mu}}$
    - $f_n = \left(\frac{v}{2L}\right)n$
- Full standing wave: $y(x,t) = A\sin\left(\frac{n\pi x}{L}\right)\cos(\omega_n t + \phi_0)$
  - ↳ $\omega_n = 2\pi f_n = \left(\frac{2\pi v}{2L}\right)n$
  - ↳ Ex) $A_2$ string, $\mu = 3.5\times10^{-3}$ kg/m, $f_1 = 110$ Hz, $T = ?$, $L = 63$ cm

$\lambda_1 = 2L$, $v = f\lambda$, $v = \sqrt{\frac{F}{\mu}}$

$\sqrt{\frac{F}{\mu}} = f2L$

$F = (f2L)^2\mu$

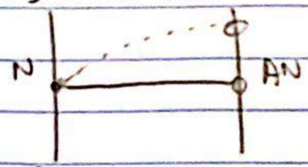
$F = (110\text{Hz} \cdot 2 \cdot 0.63\text{m})^2(3.5\times10^{-3}\text{kg/m})$

$\boxed{F = 67.2\text{N}}$

$f_2 = 220$ Hz

$f_3 = 330$ Hz

↳ Ex) 1 free end



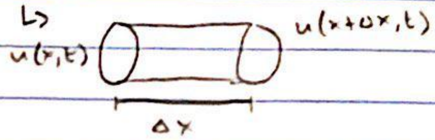

$\frac{\lambda_1}{4} = L$, $\lambda_1 = 4L$, Amplitude ~ $A\sin(\frac{\pi}{2L}x)$

↳ General formula: n must be odd, $\lambda_n = \frac{4L}{n} \to f_n = \frac{nv}{4L}$

• Longitudinal Waves

↳ displacement $u(x,t)$

↳




↳ Change in volume?

↳ $\Delta V = A(u(x+\Delta x,t) - u(x,t))$

↳ Fact: $P = -B\frac{\Delta V}{V}$

↳ $P = \bar{u}Bk\sin(kx-\omega t)$

↳ String: fixed end: displacement N, free end: displacement AN

Pipes: closed end: displacement N / pressure AN, open end: displacement AN / pressure N

# 1/28 Lecture: Sound Waves

- Standing Waves
  - ↳ Fixed/fixed → nodes @ ends
    - ↳ $\lambda_n = \frac{2L}{n}$, $f_n = n\left(\frac{v}{2L}\right)$
    - ↳ $n$ = # of AN
  - ↳ Fixed/free → 1 node/1 AN @ ends
    - ↳ $\lambda_n = \frac{4L}{n}$, $f_n = n\frac{v}{4L}$
    - ↳ $n = 1, 3, 5, \ldots$
  - ↳ Free/Free → 2AN @ ends
    - ↳ $\lambda_n = \frac{2L}{n}$, $f_n = n\left(\frac{v}{2L}\right)$
    - ↳ $n = 1, 2, 3, 4, \ldots$
  - ↳ Longer medium: $\lambda\uparrow$, $f\downarrow$
- Derivative of $u \rightarrow p$
- Wavespeed for Sound in Gases
  - ↳ $v = \sqrt{\frac{B}{\rho}}$
  - ↳ $v = \sqrt{\frac{\gamma R T}{M}}$, $R = 8.314$ J/mol·K
- Fixed length → fixed $\lambda$
- Wave intensity for sound:
  - ↳ $\bar{I} = \bar{P} = \frac{1}{2} v \mu \omega^2 A^2 = \frac{1}{2}\sqrt{\mu T}\,\omega^2 A^2$
  - ↳ $\bar{I} = \frac{1}{2}\sqrt{\rho B}\,\omega^2 \bar{u}^2$ ← displacement amplitude of sound wave
    - ↳ $p = -B\frac{\partial u}{\partial x} = -B\bar{u}k\cos(kx - \omega t)$

$$\frac{\omega}{k} = v = \frac{\lambda}{T}$$

$$p_{max} = -B\bar{u}k = \frac{B\bar{u}\omega}{v}$$

$$\bar{u} = \frac{v p_{max}}{B\omega} \rightarrow v^2 = \frac{B}{\rho}$$

$$\bar{I} = \frac{1}{2}\sqrt{\rho B}\,\frac{v^2 p_{max}^2}{B^2}$$

$$\bar{I} = \frac{p_{max}^2}{2\sqrt{\rho B}}$$

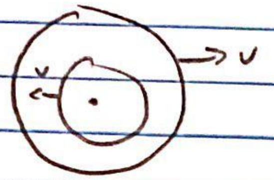
- Decibel scales
  - ↳ $\beta(I[dB]) = 10\,dB \log_{10}\left(\frac{I}{I_0}\right)$
    - ↳ $I_0$ = ref. Intensity = $10^{-12}\frac{W}{m^2}$ for hearing
    - ↳ $\beta_{whisper} \sim 25\,dB$, $\beta_{pain} \sim 120\,dB$
  - ↳ $\bar{I} \sim \sqrt{B\rho}\,\omega^2 \bar{u}^2$
    - ↳ $[B] = \frac{N}{m^2}$, $[\rho] = \frac{kg}{m^3}$, $[\bar{u}] = m$, $[\omega] = \frac{1}{s}$
    - ↳ $[I] = \frac{W}{m^2}$

· Plane waves – 1D waves in higher dimension

↳ $u(x,y,z,t) \sim A\cos(kx-\omega t)$

· Spherical Wave – generated by point sources

↳




↳ Power stays constant despite area of wave fronts getting larger

↳ Intensity × area ($4\pi r$) is constant

↳ $I(r) \times 4\pi r^2 = \text{const.}$

· Beats – interference of 2 waves with slightly diff. f

↳ $u_1(x,t) = A\sin(k_1x - \omega_1 t)$

$u_2(x,t) = A\sin(k_2x - \omega_2 t)$

$u_{tot}(x,t) = u_1 + u_2$

↳ $u_{tot} = 2A\cos\left[\frac{k_1-k_2}{2}x - \frac{\omega_1-\omega_2}{2}t\right]\sin\left[\frac{k_1+k_2}{2}x - \frac{\omega_1+\omega_2}{2}t\right]$

↳ $\omega_1 = 2\pi f_1$, $\omega_2 = 2\pi f_2$

↳ $|f_1 - f_2| << f_1$ or $f_2$

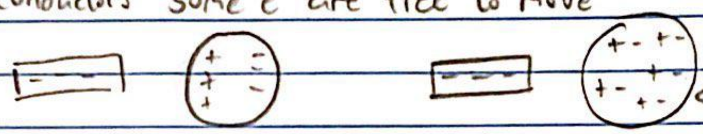
↳ $u_{tot} = 2A\sin(k_{av}x - \omega_{av}t)\cos\left(\frac{k_1-k_2}{2}x \pm \frac{2\pi f_{av}t}{2}\right)$

# 2/4 Lecture: Electricity

- $M_e = 9\times10^{-31}$ kg
- $M_n = 1.7\times10^{-27}$ kg
- $e^-$ → smallest − charge, $p^+$ − smallest + charge
- SI unit of charge − 1 Coulomb = charge of $6.242\times10^{18}$ protons
- Charge is conserved, but charge can move/be separated
- Observations:

↳ rubbing certain materials w/ a paper towel "charges" them

↳ there are 2 types of charge: + and −

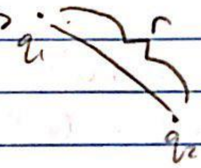
- Insulator − $e^-$ are more or less stuck to their atoms → not free to move
- Conductors − some $e^-$ are free to move
- 

← polarization

conductor    insulator

↳ Polarization → object isn't charged, but small attractive force present

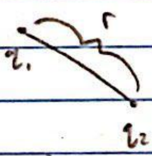
- Coulomb's Law: force between 2 point charges

↳ 

↳ $|\vec F| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2}$

↳ SI: $\varepsilon_0 = 8.854\times10^{-12}\ \frac{C^2}{Nm^2}$

- Force of charge $q_1$ acting on $q_2 = \vec F_{21}$

↳ 

like charges: $q_1q_2 > 0$ → repulsive → $\vec F_{21}$ away from $q_1$

opposite charges: $q_1q_2 < 0$ → attractive → $\vec F_{21}$ away from $q_2$

↳ $\vec F_{21} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2}\hat r_{21}$ ← points from 1 → 2

$r^2 = |\vec r_2{}^2 - r_1{}^2| = \vec r_{21}{}^2$

$\hat r_{21}$ → unit vector in direction of vector $\vec r_{21}$

↳ aligns correct direction

- Law of superposition of forces

↳ $q_1, \dots, q_n$, what is $\vec F_{1,tot}$?

↳ $\vec F_{1,total} = \sum_{i=2}^{n}\vec F_{1i} = \vec F_{12} + \vec F_{13} + \dots + \vec F_{1n}$

↳ $\vec F_{1,tot} = |F_{12}|\hat x - |F_{13}|\hat y$

↳ $\vec F_{2,tot} = \left(\frac{q q_2}{4\pi\varepsilon_0}\right)\left(\frac{-4dx}{(d^2-x^2)^2}\right)\hat x$

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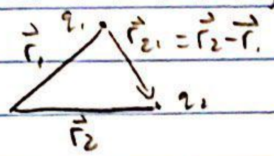
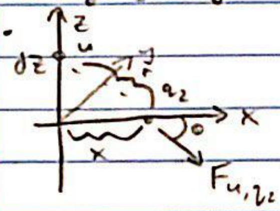
# 2/6 Lecture: Charges

- Charged stuff can attract neutral insulators ("static electricity")
- Force between conductors < force between insulators
- When a neutral object and charged object touch, both get charge $Q/2$
- 


$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^2} \hat{r}_{21}$$

- Superposition: $\vec{F}_{2\,total} = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{24} + \dots + \vec{F}_{2n}$
- Charge density $\rightarrow \frac{\text{charge}}{\text{length}} \rightarrow \frac{Q}{\ell}$
- 


$r = \sqrt{x^2+z^2}$

↳ $\cos\theta\,\hat{x} + \sin\theta\,\hat{z}$

$\cos\theta = \frac{x}{\sqrt{x^2+z^2}}$, $\sin\theta = \frac{z}{\sqrt{x^2+z^2}}$

↳ $\vec{F}_{1,tot} = \frac{\int dq \cdot q}{4\pi\varepsilon_0 r^2}(\cos\theta\hat{x} + \sin\theta\hat{z})$

$$= \int_{-\infty}^{\infty} \frac{(H_1\, dz)\, q_1}{4\pi\varepsilon_0 (x^2+z^2)} \left[\frac{x}{\sqrt{x^2+z^2}}\hat{x} - \frac{z}{\sqrt{x^2+z^2}}\hat{z}\right]$$

$$= \frac{H_1 q x}{4\pi\varepsilon_0}\hat{x} \int_{-\infty}^{\infty} \frac{dz}{(x^2+z^2)^{3/2}} = \left(\frac{H q \lambda \hat{x}}{4\pi\varepsilon_0}\right)\frac{x}{\lambda^3}\int_{-\infty}^{\infty}\frac{du}{(1+u^2)^{3/2}} = \frac{2H_q\hat{x}}{4\pi\varepsilon_0}$$

$z = x \cdot u$

$$\vec{F}_{1,tot} = \frac{2}{4\pi\varepsilon_0}\frac{H_q q_1}{x}\hat{x}$$

- Symmetry

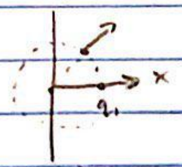
↳ Above force has no z component

↳ reflection in x-y plane: $z \rightarrow -z$

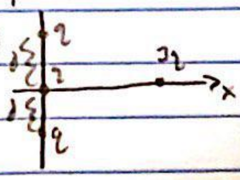
$F_z \rightarrow -F_z$

↳ $F_z = 0 \rightarrow$ appears in the integral

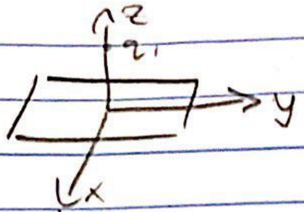
↳ $\vec{F} \sim \hat{x}$

↳ 


$\rightarrow \vec{F}_{1,tot} = \frac{1}{2\pi\varepsilon_0}\frac{H_1 q_1}{\rho}\hat{\rho} \rightarrow \rho = \text{radius}, \rho = \sqrt{x^2+y^2}$

↳ Ex) 


total force points along the x-axis (+)

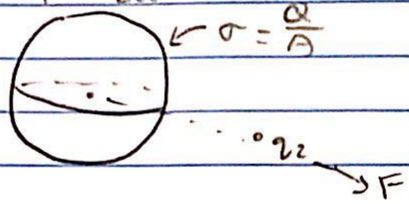
- 


$\sigma = \frac{\text{charge}}{\text{area}}$

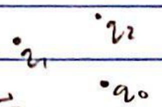
$\vec{F}_x = \vec{F}_y = 0$

↳ $\vec{F} = (\ldots)\,\hat{z}$

↳ $\vec{F} = \frac{q_1 \sigma}{2\varepsilon_0}\,\hat{z} \quad (z>0)$

- 


- Electric Fields

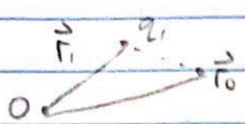



↳ $\frac{\vec{F}_{0,tot}}{q_0} = \sum_i \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_i}{r_{0i}^2}\,\hat{r}_{0i} = \vec{E}$ → electric field

# 2/11 Lecture: Electric Fields

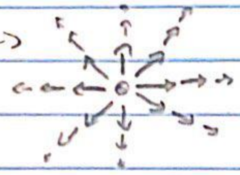
• Electric field: $\vec{E} = \frac{\vec{F}_{tot}}{q_0} = \frac{\text{coulomb force}}{\text{charge}}$

•

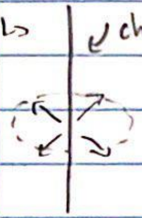



$= \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\vec{r}_i - \vec{r}_0|} \left( \frac{\vec{r}_0 - \vec{r}_i}{|\vec{r}_0 - \vec{r}_i|} \right)$ $q_i \rightarrow \vec{r}_i$

↳ $\vec{E}(\vec{r})$ : vectors → vectors → vector field
position vectors ↑ ↑ electric field vectors

↳ 

↳ away from +, towards −

↳ 

charge density $= H = \lambda = \frac{\text{charge}}{\text{length}}$

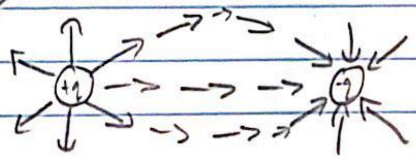
$\vec{F}_q = \frac{1}{2\pi\varepsilon_0}\left(\frac{H}{r}\right)$ where r is distance from wire → q

$\vec{E} = \left(\frac{1}{2\pi\varepsilon_0}\right)\left(\frac{H}{r}\right)\hat{r}$

↳ 

$\sigma = \frac{\text{charge}}{\text{area}} > 0$

$\vec{F} = \frac{q_0}{2\varepsilon_0}\sigma\hat{n}$ ← unit vector pointing away from σ surface

↳ 


• Field lines: instead of having separate vectors, we use contiguous lines with vectors

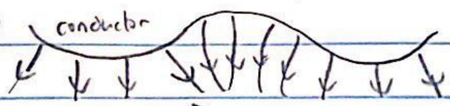
↳ 1) $\vec{E}$ is tangent/parallel to field lines

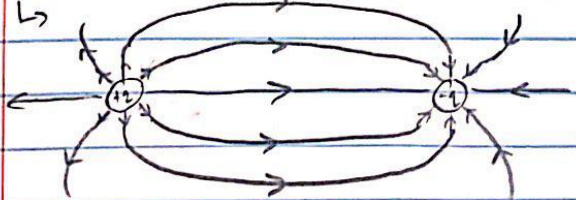
↳ 2) $|\vec{E}| \sim$ density of field line

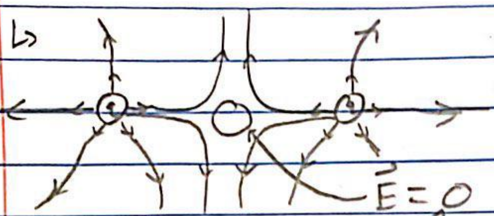
↳ 3) Field lines don't cross

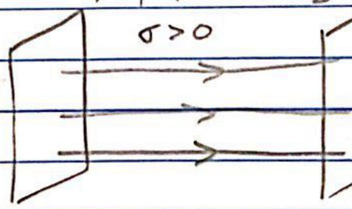
↳ 4) $\vec{E}$-fields must be ⊥ to conductors

↳ charges can rearrange until forces vanish

↳ 


↳ 

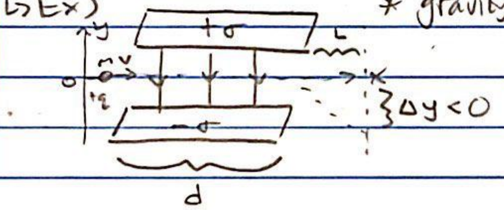
↳ 


↳ 


↳ outside $\vec{E}$-fields cancel → plates are infinitely big

↳ test particles do not necessarily follow the trajectory of the $\vec{E}$-fields

↳ Ex) * gravity ignored, calculate y-direction




$V_x = V$ → constant (no x-direction forces)

$\vec{E} = \frac{-\sigma}{\varepsilon_0}\hat{y}$ → ×2 because there are 2 plates

$\vec{F} = -\frac{q\sigma}{\varepsilon_0}\hat{y}$

$a_y = \frac{F_y}{m} = \frac{-q\sigma}{m\varepsilon_0}$

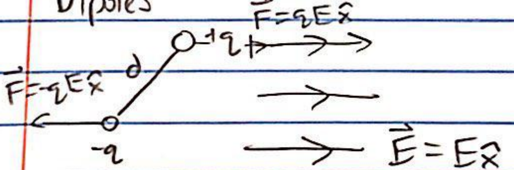
$t_{between} = \frac{d}{v}$, $\Delta y = \frac{1}{2}a_y(t_{between})^2 = \frac{-q\sigma}{2m\varepsilon_0}\left(\frac{d}{v}\right)^2$

$v_y = a_y t_{between}$, $t_{outside} = \frac{L}{v}$

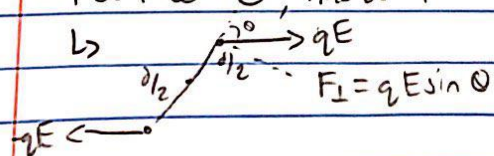
$\Delta y_{outside} = v_y t_{outside} = \frac{-q\sigma}{2m\varepsilon_0}\left(\frac{d}{v}\right)\left(\frac{L}{v}\right)$

$\Delta y = \Delta y_{in} + \Delta y_{out} = \left(-\frac{\sigma q d}{m\varepsilon_0 v}\right)\left(\frac{1}{2}d + L\right)$

· Dipoles




↳ Net force = 0, has a $\tau$

↳ 

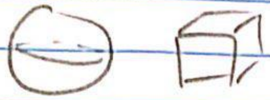

$\tau = 2\left(\frac{d}{2}\right)\cdot qE\sin\theta = dqE\sin\theta > 0$ (into the board)

↳ $\vec{p} = \vec{d}\cdot$ dipole moment, $|\vec{p}| = dq > 0$, $\vec{p}$ ∥ vector from $(-q) \to (+q)$

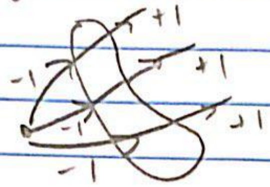
↳ $\ominus \xrightarrow[d]{\vec{p}} \oplus$

↳ $\vec{\tau} = \vec{p} \times \vec{E}$ , $|\vec{\tau}| = |\vec{p}||\vec{E}|\sin\theta$

• Electric Flux + Gauss' Law } reformation of Coulomb's Law using flux

↳ closed surface: 

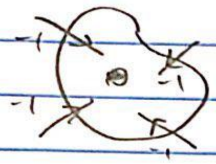
↳ Roughly: electric flux ~ # of $\vec{E}$-field lines going thru a closed surface

↳ 

flux = 0

going in → -1

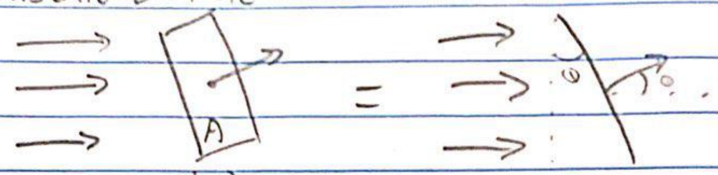
going out → +1

↳ 

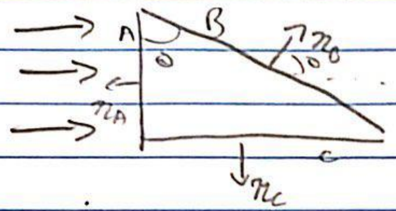
flux < 0

# 2/13 Lecture: Gauss' Law

· Constant E-field

↳ 

↳ def $\Phi = \vec{E}\cdot\vec{A} = |\vec{E}| A\cos\theta$

↳ 

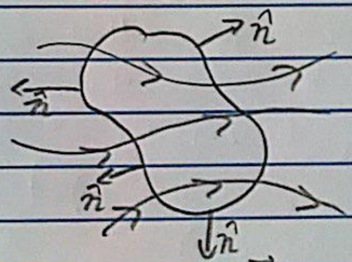
↳ $\phi_c = 0$

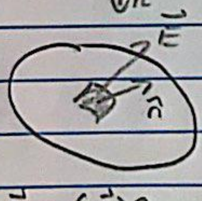
↳ $\phi_A = \vec{E}\cdot A\hat{n}_A = -\vec{E}A$

↳ $\phi_B = \vec{E}\cdot B\hat{n}_B = EB\cos\theta$

$B\cos\theta = A$

$\phi B = \vec{E}A$

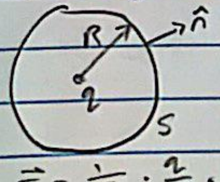
↳ 

↳ 

$\partial\vec{A} = (\partial\vec{A})\hat{n}$

$\partial\phi = \vec{E}\cdot\partial\vec{A}$

$\phi_s = \int_s \partial\vec{A}\cdot\vec{E} = \int_s \partial A(\hat{n}\cdot\vec{E})$

↳  $\hat{n} = \hat{e}_r = \hat{r}$

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\cdot\hat{e}_r$

$\phi_s = \int_s \partial A\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{e}_r\cdot\hat{e}_r\right) \rightarrow$ if $\phi/\theta$ dependence $\rightarrow \partial A = r^2\partial\theta\sin\theta\cos\phi$

$\phi_s = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\int\partial A \rightarrow \int\partial A = 4\pi R^2$

$\phi_s = \frac{q}{\varepsilon_0} \rightarrow$ thru sphere

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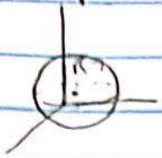
· Gauss' Law: given any closed surface $S$, any charge distribution $\Phi_s = \oint \vec{E} \cdot \partial\vec{A} = \frac{q_{encl}}{\varepsilon_0}$

↳ $= \vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0} \to \vec{\nabla} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \to \int \partial V \, \vec{\nabla}\vec{E} = \int_s \partial\vec{A} \cdot \vec{E}$

↳ Ex) Sphere of radius $R$, $\rho = \frac{Q}{\text{volume}} = \text{const} = \frac{Q}{4/3\pi r^3}$



$\vec{E}(\vec{r}) = E(r)\hat{r}$

$S$ = sphere of radius $r$

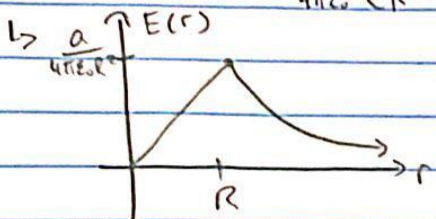
$\Phi_s = \int \partial A(\hat{r} \cdot \vec{E}) = \int \partial A \, E(r)$

$\Phi_s = E(r) \int \partial A = 4\pi r^2 E(r)$

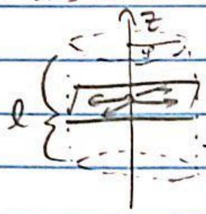
$q_{encl}(r) \begin{cases} r > R \to Q \\ r < R \to \rho\left(\frac{4\pi}{3} r^3\right) = Q\frac{r^3}{R^3} \end{cases}$

$\vec{E}(\vec{r}) = \begin{cases} r > R \to \frac{Q}{4\pi\varepsilon_0 r^2} \\ r < 0 \to \frac{Q}{4\pi\varepsilon_0}\left(\frac{r}{R^3}\right) \end{cases}$

↳




↳ Ex) Infinite line charge, $\lambda = \frac{\text{charge}}{\text{length}}$




$\vec{E} = E(\rho)\hat{e}_\rho$, $\rho = \sqrt{x^2+y^2}$

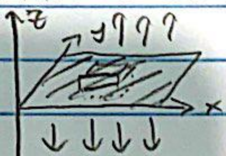
$\to \hat{n} = \hat{e}_\rho \to$ proportional to direction of $\vec{E}$

$\Phi = \int \vec{E} \cdot \partial\vec{A} = \int E(\rho)\cancel{\hat{e}_\rho \cdot \hat{e}_\rho} \cdot \partial A$

$= E(\rho)\int \partial A = E(\rho)(2\pi\rho)\ell = \frac{q_{encl}}{\varepsilon_0} = \frac{1}{\varepsilon_0}(\lambda\ell)$

$\boxed{E(\rho) = \frac{1}{2\pi\varepsilon_0}\left(\frac{\lambda}{\rho}\right)\hat{\rho}}$

↳ Ex) Infinite sheet of charge $\sigma = \frac{Q}{\text{area}}$




$E = \begin{cases} E\hat{e}_z & z > 0 \\ -E\hat{e}_z & z < 0 \end{cases}$

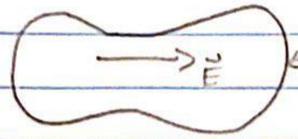
$\Phi = EA + EA = 2EA$

$q_{encl} = \sigma A \to \Phi = \frac{\sigma A}{\varepsilon_0}$

$E = \frac{\sigma}{2\varepsilon_0}$

## 2/18 Lecture: Electrostatic Potential

- Gauss' Law: $\Phi_s = \int \vec{E} \cdot dA = \frac{q_{encl}}{\varepsilon_0}$

- 

← conductor

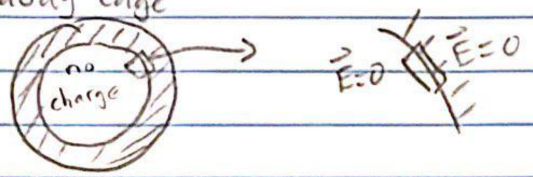
↳ charges will realign themselves to reach equilibrium → $\vec{E} = 0$

↳ there is no charge inside the conductor → no $\vec{E}$

↳ in electrostatic equilibrium → $\vec{E}_{\parallel}$ to the surface vanishes, $\vec{E}_{surface} \sim \hat{n}$

↳ $\vec{E}$ is perpendicular to the surface

- Faraday Cage

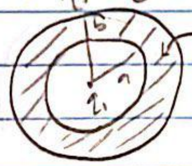
↳ 


↳ $\vec{E} = 0$ in the cavity and in the conductor

↳ no surface charge on the inner wall

↳ charge could be present on the outer wall

↳ $\Phi_n = 0 \rightarrow q_{encl} = 0 \rightarrow$ charge on the surface must equal 0

↳ 

$\vec{E} = 0$, $a < r < b$

$\vec{E} = \frac{q_1}{4\pi\varepsilon_0 r^2}\hat{r}$, $0 < r < a$

↳ $q_{enc} = 0 = q_1 +$ surface charge @ $r = a$

↳ $\sigma_a = \frac{\text{charge}}{\text{area}} = \text{const}$

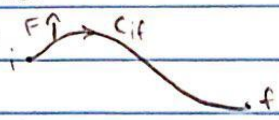
↳ charge of $r = a = \sigma_a (4\pi a^2)$

↳ $\sigma_a = \frac{-q_1}{4\pi a^2}$

↳ $-q_1 + (\sigma_{r=b}) 4\pi b^2 = 0$

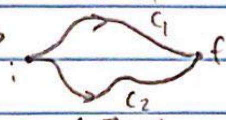
↳ $\sigma_{r=b} = \frac{q_1}{4\pi b^2}$

- Electrostatic Potential

↳ 

$W = \int_{c_{if}} \vec{F} \cdot d\vec{x}$

↳ $K_f - K_i = W$

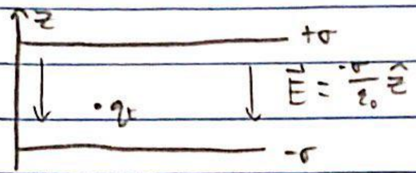
↳ 

$\int_{c_1} \vec{F} \cdot d\vec{x} = \int_{c_2} \vec{F} \cdot d\vec{x}$

↳ conservative force

↳ $\int_{c_{if}} \vec{F} \cdot d\vec{x} = U_i - U_f = -\Delta U$

↳ Ex)




$\vec{F} = q_t\vec{E} = \frac{-q_t\sigma}{\varepsilon_0}\hat{z}$

$W_z = \int_{z_1}^{z_2} \vec{F}\cdot d\vec{x} = \int_{z_1}^{z_2} F_z\, dz$

$W_z = \int_{z_1}^{z_2} \left(\frac{-q_t\sigma}{\varepsilon_0}\right)\hat{z}\, dz$

$W_{z_1\to z_2} = \frac{-q_t\sigma}{\varepsilon_0}(z_2 - z_1)$

$U(z) = \frac{q_t\sigma}{\varepsilon_0} z + U$ ⟵ const

↳ Ex)

$r_f$ • ↗ $q_t$

$r_i$ • $q$

$\vec{F} = \frac{qq_t}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r}$

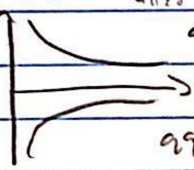
$W_{i\to f} = \int_{r_i}^{r_f} \left(\frac{qq_t}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\right) dx \;\to \hat{r}\,dx = dr$

$= \int_{r_i}^{r_f} \frac{dr}{r^2}\left(\frac{qq_t}{4\pi\varepsilon_0}\right)$

$= -\frac{qq_t}{4\pi\varepsilon_0}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$

$U(r) = \frac{qq_t}{4\pi\varepsilon_0}\frac{1}{r}$

↳



$qq_t > 0 \to$ repulsive

$qq_t < 0 \to$ attractive

↳ Ex)

$r_i = 0.1m$ $\quad q_t = -1\times10^{-6}C$

$v_i = 10\frac{m}{s}$

$q = 2\times10^{-4}C$ $\quad m = 50g$

$E_i = K_i + U_i = \frac{mv_i^2}{2} + \frac{qq_t}{4\pi\varepsilon_0 r_i}$

$E_f = U_f = \frac{qq_t}{4\pi\varepsilon_0 r_f}$

$\boxed{r_f = 0.225m}$

- Superposition: $U(\vec{r}) = \sum_{i=1}^{N} \frac{q_i q_t}{4\pi\varepsilon_0}\frac{1}{|\vec{r}-\vec{r}_i|}$
- $\vec{E} = \frac{F_{on\, q_t}}{q_t} \Longleftrightarrow V = \frac{U_{on\, q_t}}{q_t}$

↳ $|V|$ = Volts

2/20 Lecture: Potential Calculations

- test charge $q_t$ experiences a force $F(\vec{r})$

↳ $\vec{E}(\vec{r}) = \frac{F(\vec{r})}{q_t}$ → makes sense w/out $q_t$ in the picture

- $-\Delta U_{i\to f} = \int_{C_{if}} \vec{F} \cdot d\vec{x}$ → not dependent on path for conservative force

↳ $= U_i - U_f$ → potential energy

- Electric/electrostatic potential: $V(\vec{r})$

↳ $V(\vec{r}) = \frac{U(\vec{r})}{q_t}$

↳ $V_f - V_i = -\int_{C_{if}} E(\vec{r})\, d\vec{r}$

↳ 1V = 1 volt

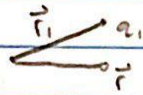
- $V_f = V_i = \int_{C_{i\to f}} \vec{E}(\vec{x}) \cdot d\vec{x}$

$i \xrightarrow{E} f \quad V_f < V_i$

$i \leftarrow f \quad V_f > V_i$

- point charge $q_1$ at $\vec{r}_1$

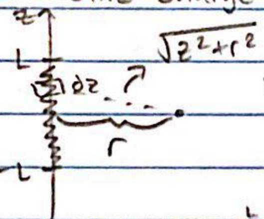
↳ $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{|\vec{r}-\vec{r}_1|}$



- $q_i$ at $\vec{r}_i$

↳ $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\vec{r}-\vec{r}_i|}$

- Ex) Line charge (linear charge density $\lambda$)



$V(\vec{r}) = ?$

↳ $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_{-L}^{L} \underbrace{dz\,(\lambda)}_{dq} \Big/ \sqrt{r^2+z^2}$

↳ $\frac{d}{dz}(f) = \frac{1}{\sqrt{r^2+z^2}}$

↳ $f = \ln(z + \sqrt{z^2+r^2})$

↳ $V(\vec{r}) = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{L+\sqrt{L^2+r^2}}{-L+\sqrt{L^2+r^2}}\right)$

- $\infty$ long wire → $L >> r$

↳ $\sqrt{1+\Delta} = 1 + \frac{1}{2}\Delta + n\Delta^2 + \ldots$

↳ $\frac{L(1+\sqrt{1+\frac{r^2}{L^2}})}{L(-1+\sqrt{1+\frac{r^2}{L^2}})} = \frac{2+\frac{r^2}{2L^2}}{-1+1+\frac{r^2}{2L^2}}$ ← much smaller than 2

$\simeq \frac{4L^2}{r^2}$

↳ $V(\vec{r}) \approx \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{4L^2}{r^2}\right)$

$\lim_{L\to\infty} V(\vec{r}) = \frac{\lambda}{4\pi\varepsilon_0}\left[-2\ln r + \lim_{L\to\infty} \ln(4L^2)\right]$ — doesn't matter

↳ $V_{12} = V(r_1) - V(r_2) = -\frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_1}{r_2}\right)$

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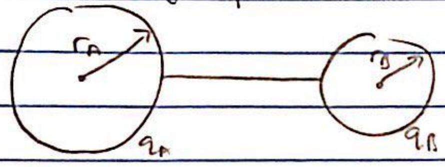
- $V(\vec{x}) = V_{\vec{x}} = V_1 - \int_{c_1 \to \vec{x}} \vec{E}(\vec{r}) \cdot d\vec{r}$
- Spherical conductor w/ $q$ and rad. $R$

↳ $\vec{E}(\vec{r}) = E(r)\hat{r}$

$$E(r) = \begin{cases} 0, & r < R \\ \frac{q}{4\pi\varepsilon_0 r^2}, & r > R \end{cases}$$

$$V(r) = \begin{cases} \frac{q}{4\pi\varepsilon_0 R}, & r < R \\ \frac{q}{4\pi\varepsilon_0 r}, & r > R \end{cases}$$

- Conducting spheres

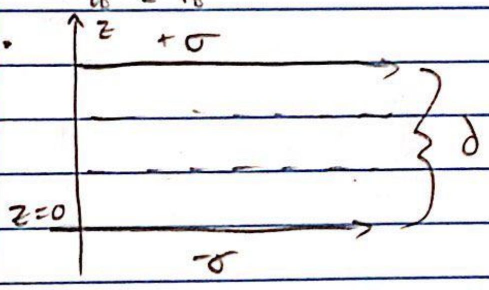



↳ $V_A = V_B$

$$\frac{q_A}{4\pi\varepsilon_0 r_A} = \frac{q_B}{4\pi\varepsilon_0 r_B}$$

$$\frac{q_A}{q_B} = \frac{r_A}{r_B}$$

-

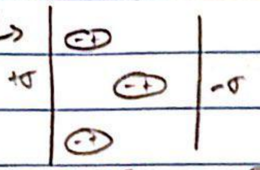



↳ equipotentials → surfaces of constant $V$

3/3 Lecture : Series and Parallel

· Dielectric Material + Capacitors

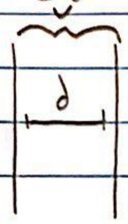
↳ $K > 1$, $C = KC_0$, $V = \frac{V_0}{K}$, $E = \frac{E_0}{K}$

↳ 


↳ $E_0 = \frac{\sigma}{\varepsilon_0}$, $E = \frac{\sigma - \sigma_i}{\varepsilon_0}$ (surface charge induced by dielectric) $= \frac{E_0}{K} = \frac{\sigma}{K\varepsilon_0}$

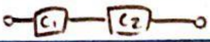
↳ $\sigma_i = \sigma - \frac{\sigma}{K} = \sigma(1 - \frac{1}{K})$

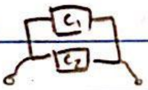
· $Q = CV$ → stops working when V or Q are huge → dielectric breakdown

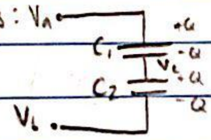
↳ 

$\frac{V}{d} \approx 3 \cdot 10^6 \frac{V}{m}$ for air

· Series/Parallel

↳ Series: 


↳ Parallel: 

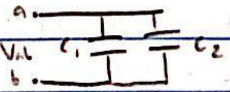

↳ Series: 


↳ $V_{ac} = V_a - V_c = \frac{Q}{C_1}$

↳ $V_{cb} = V_c - V_b = \frac{Q}{C_2}$

↳ $V_{ab} = V_{ac} + V_{cb} = \frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C_{eq}}$

↳ $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$

↳ Parallel : 


↳ $Q_1 = C_1 V_{ab}$, $Q_2 = C_2 V_{ab}$

↳ $C_{eq} = C_1 + C_2 + \ldots$

· Current = transport of charge

↳ Def. $I = \frac{\Delta Q}{\Delta t} \to \frac{dQ}{dt}$, $[I]$ = Ampere $= \frac{1C}{1s}$

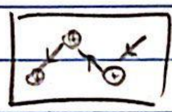
↳ $I = 40A$, $\Delta t = 1hr$

↳ Current density, $J = \frac{I}{A}$

↳ $I_{thru\ A} = \vec{J} \cdot \vec{A}$

↳ Conductors: $n = \frac{\#\ of\ e^-}{Vol}$, $v$ = speed of $e^-$

$J = q_e n v$

· Collisions : 

↳ $e^-$ don't go anywhere on avg.

↳ $T$ – collision time $\approx 3 \times 10^{-14}$

↳ $\vec{E} \neq 0$ → small, non-zero drift velocity

↳ $V_D = aT = \frac{q_e E}{m_e} T$

↳ $J = \left(\frac{q_e^2 T n}{m_e}\right) E = \sigma E$

↳ high $\sigma$ = good conductor

↳ $\frac{1}{\sigma} = \rho$ = resistivity

· (A ← I) , $I = JA = -AE$

↳ $E = V/\ell$

↳ $I = \frac{\sigma A}{\ell} V$, $V = RI$, $R = \frac{\ell}{\sigma A} = \rho\left(\frac{\ell}{A}\right)$

↳ $[R] = \frac{V}{A} = \Omega$

↳ $\rho_{cu} = 2 \times 10^{-8}\ \Omega m$, $\rho_{quartz} = 8 \times 10^{17}\ \Omega m$

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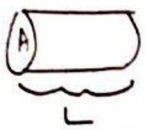
# 3/5 Lecture: Circuits

• Summary:




↳ $I = \frac{\partial q}{\partial t}$ = total current

↳ $J = \frac{I}{A} \to \vec{J} = J\hat{n}$ → dir. I flows in

↳ Ohm's Law:

↳ $\vec{J} = \sigma\vec{E}$

↳ resistivity = $\rho = \frac{1}{\sigma}$

↳ microscopic model for conductance

$\vec{J} = qn\vec{v}_D$

↳ $\vec{v}_D = \underbrace{\left(\frac{q\vec{E}}{m}\right)}_{a}\tau$ ← collision time

↳ $\sigma = \frac{q^2 n\tau}{m}$

↳ $E = \frac{V}{\ell}$

$J = \frac{1}{\rho}E = \frac{1}{\rho}\frac{V}{\ell}$

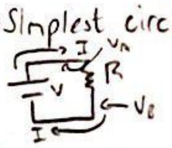
↳ $I = \left(\frac{A}{\rho\ell}\right)V = IR = V$

↳ $R = \frac{\rho\ell}{A}$, $[R] = \frac{V}{A} = \Omega$

• $\rho(T) = \rho_0[1+\alpha(T-T_0)]$

↳ colder temp → lower resistance (metals)

• Simplest circuit




↳ $V_{ab} = V_a - V_b = IR$

• Limits to how big I can be thru a battery

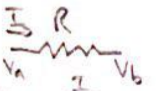
↳ $V_{ab} = \mathcal{E} - Ir$ ← Internal resistance of battery

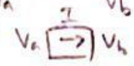
terminal voltage, ideal EMF } → $I = \frac{\mathcal{E}}{R+r}$

• AA battery, $\mathcal{E} = 1.5V$, $r \approx 0.1\Omega$

↳ $\mathcal{E} = V_{ab}$ → ideal battery

• Energy/Power

↳ Resistors dissipate energy  , $V_a > V_b$


↳ Batteries do work/input energy  , $V_a < V_b$


↳ charge q moves across $\Delta V$ $\Delta V > 0$, $\Delta K < 0$

↳ $\Delta U = q\Delta V$

↳ Resistor: $\Delta V < 0 \to \Delta U < 0$

• Rate at which a resistor dissipates energy: Power

↳ $dq = I\,dt$

↳ $P = \frac{dK}{dt} = -\frac{dU}{dt} = -\frac{dq}{dt}\Delta V = \frac{dq}{dt}(V_a - V_b)$

↳ $P = (V_a - V_b)I$

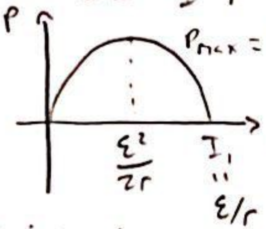
↳ $V_a > V_b \to P > 0$

↳ $V_a < V_b \to P < 0$

↳ $P = RI^2 = \frac{V^2}{R}$

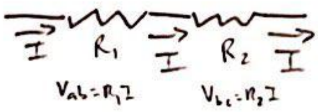
• Real battery: $V = \mathcal{E} - Ir$

↳ $P = \mathcal{E}I - I^2 r$




$P_{max} = \frac{\mathcal{E}^2}{2r} - \frac{\mathcal{E}^2}{4r} = \frac{\mathcal{E}^2}{4r}$

• Resistors in series

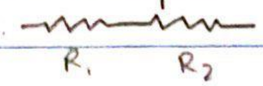
↳ 

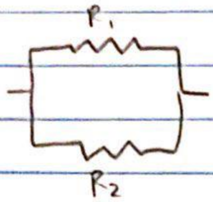

$V_{ac} = R_1 I + R_2 I$

$V_{ac} = IR_{eq}$

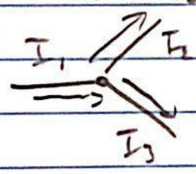
↳ $R_{eq} = R_1 + R_2$ → resistors in series add

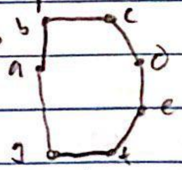
# 3/10 Lecture: Kirchhoff's Rules

- Resistors in series/parallel
  - ↳ Series: 

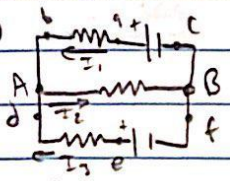
    →  $R_{eq} = R_1 + R_2$
  - ↳ Parallel: 

    →  $R_{eq} = \frac{1}{R_1} + \frac{1}{R_2}$

- Same V diff. and same resistors, parallel circuits dissipate more power
- In a steady state, capacitors are fully charged and no current flows through them
- General class of problems
  - ↳ circuits - steady state
  - ↳ resistors - $R_i$ } input
  - ↳ batteries - $\mathcal{E}$
  - ↳ output: all currents, all potential differences
- Junction Rule:
  - ↳ 

    $I_1 = I_2 + I_3 \rightarrow \Sigma I = 0$
    - ↳ Charge conservation
- Loop Rule:
  - ↳ 

    $V_{ab} + V_{bc} + V_{cd} + \ldots + V_{ga} = 0$

    $\oint \vec{E} \cdot d\vec{r} = 0 \rightarrow$ closed loop
- Ex) 


  $I_1 + I_2 - I_3 = 0$

  $I_3 - I_1 - I_2 = 0$