Name:

StudentID:

Practice Final, Physics 1B

• Please write your name and UID on the front page, if you separate pages or use extra paper please write your name and UID on each one of them.

• Closed book, **three** 5x3in note cards in your own writing allowed.

• calculators allowed, no laptops or smartphones

• If a problem is ambiguous, notify the instructor. Clarifications will be written on the blackboard. Check the board occasionally.

• Time for exam: 3 hours

• There are 7 questions, check that your exam has all n sheets.

Good Luck !!

Please do not write in this box

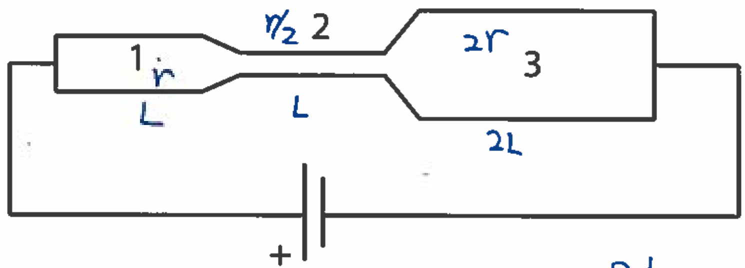
Problem	Points	of Total
1		30
2		30
3		30
4		20
5		30
6		30
7		20
Total		190

Useful quantities

$$
\begin{aligned}
\epsilon_0 &= 8.85 \times 10^{-12} C^2 m^{-2} N^{-1} \\
g &= 9.81 m/s^2 \\
m_{electron} &= 9.11 \times 10^{-31} kg \\
m_{proton} &= 1.67 \times 10^{-27} kg \\
q_e &= -1.602 \times 10^{-19} C
\end{aligned}
$$

Problem 1: [25pts] Concept questions

a) [5pts] A resistor is made up of three cylindrical segments of stainless steel (The figure shows a cross section of the resistor). Segment 1 has radius r and length L, segment 2 has radius $r/2$ and length L, segment 3 has radius $2r$ and length $2L$. A battery is connected as shown.



$R_1 \quad R_2 \quad R_3$

$R = \rho \frac{L}{S} \quad \frac{L}{r^2} \quad \frac{4L}{r^2} \quad \frac{L}{2r^2}$

Rank the resistance

(1) $R_1 = R_2 = R_3$

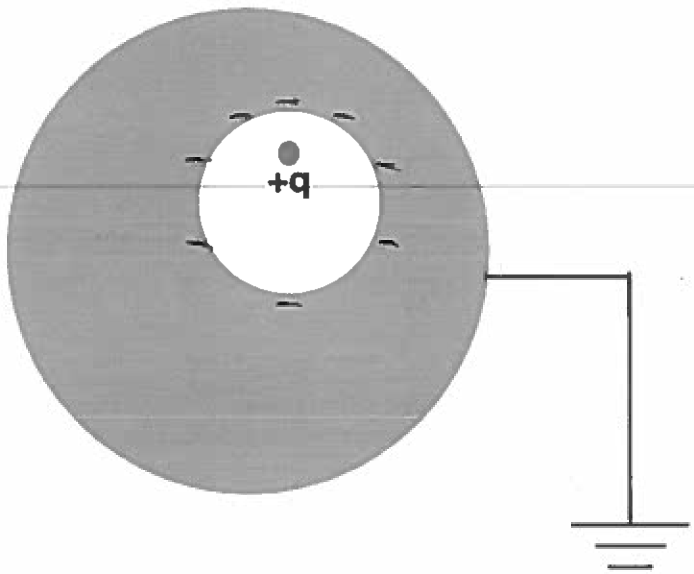
(2) $R_2 > R_1 > R_3$

(3) $R_2 > R_3 > R_1$

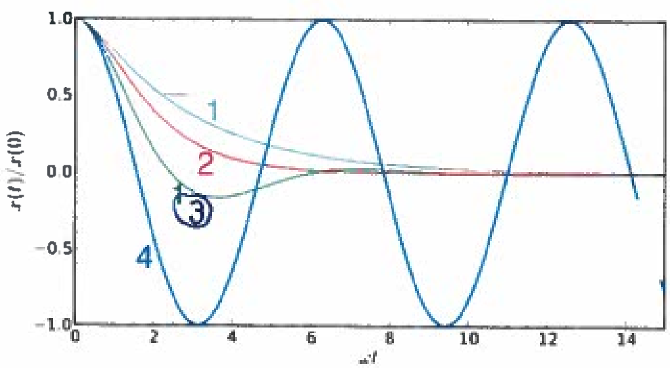
(4) $R_1 > R_2 > R_3$

(5) $R_3 > R_1 > R_2$

b) [5pts] A conducting sphere has an internal spherical cavity which is offset from the center of the sphere. Inside the cavity, but offset from the center of the cavty, I place a positive point charge $+q$. The conducting sphere is connected to ground via a thin wire as shown. Draw a qualitative picture of the location of charges in the conductor and on the surfaces of the conductor.



c) **[5pts]** Consider the the plot below, showing position versus time for four damped oscillators. All four have the same mass and spring constant, but have different damping coefficients. In which case is the damping coefficient the largest?



(1)

(2)

(3)

(4)

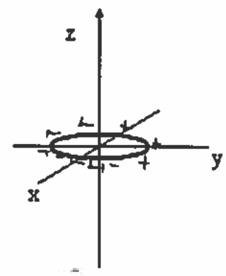
d) [5pts] For which of the following displacement functions for sinusoidal waves is the speed of wave propagation the largest?

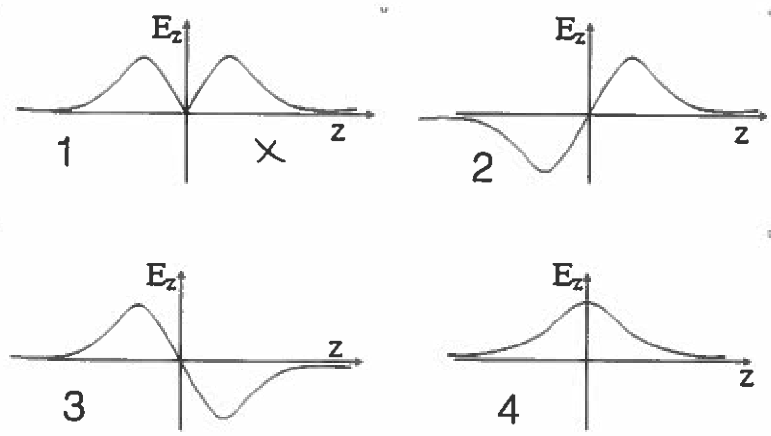
	ω/k
(1) $u(x,t) = 5\cos\left(\frac{1}{2}(6x - 4t + 3)\right)$	$4/6$
(2) $u(x,t) = 12\sin(3x + t)$	$1/3$
(3) $u(x,t) = 42\cos(53x - \frac{1}{10}t + 73)$	$\frac{1}{10}/53$
(4) $u(x,t) = 6\sin(3x - t - 1)$	$1/3$
(5) $u(x,t) = 5\cos\left(\frac{1}{20}x + 3t + 3\right)$	$3/\frac{1}{20} = 60$ largest ✓

$(kx + \omega t)$

$V_{speed} = \frac{\lambda}{T} = \frac{\omega}{k}$

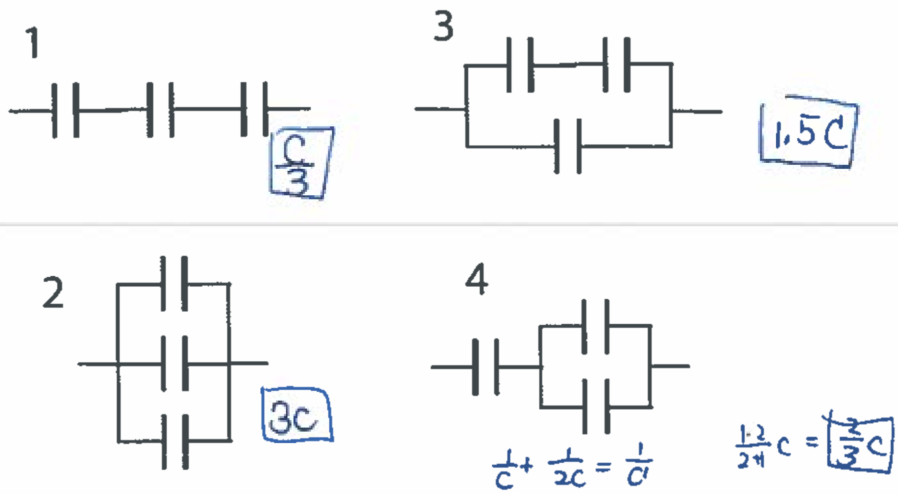
e) **[5pts]** A circular uniformly charged ring with positive charge Q is placed in the xy-plane as shown in the figure. Which graph most accurately represents the electric field component E_z on the z-axis ?





(1)
(2)
(3)
(4)

f) [5pts] I have three capacitors with equal capacitance. Which of the below arrangements of these three will result in the most stored energy if I give each arrangement the same amount of charge (i.e. I connect a battery for just long enough to transfer the same amount of charge Q from the right terminal to the left)



(1)

(2)

(3)

(4)

(5) They all store the same amount of energy

$U = \frac{Q^2}{2C}$

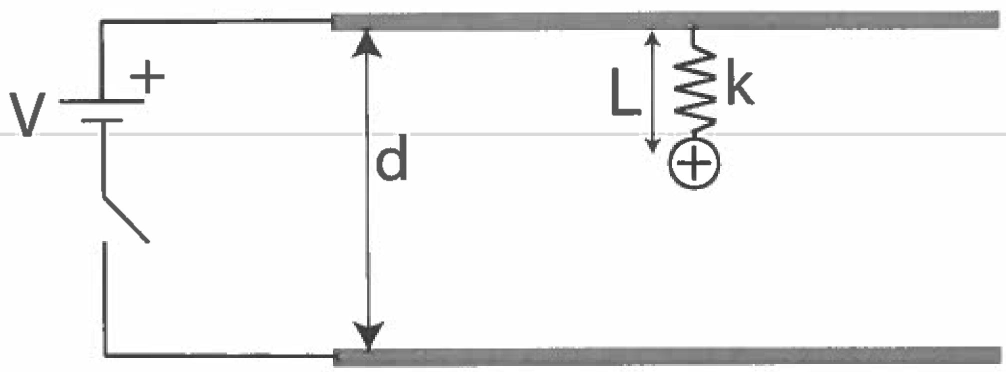
Q is same

smaller C has large U

-additional space for calculation-

Problem 2: [30pts]

An insulating spring of spring constant k is attached to one plate of a parallel plate capacitor. A charged sphere of mass M and charge $+q$ is attached to the spring. The capacitor plates are initially uncharged and the spring is at its unstretched length, L. (Note: (1) the sides of each plate are much much longer than the separation between the plates, d; (2) there is no gravity; (3) you can treat the charged sphere as a point particle)



The charged sphere is held in place so that the spring remains unstretched and the switch is closed and the capacitor is connected to a ideal battery with emf V. After a long time, the capacitor is charged and the switch is opened again. The charged sphere is then released and begins to oscillate.

a) [10pts] What is the frequency of the oscillation?

b) [10pts] What is the amplitude of the oscillation ?

c) [10pts] The plates are moved apart (with the switch open) so that their new separation is $2d$. What happens to the amplitude of the oscillation?

Solutions:
a)

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

b) After the plates are charged, there is an electric field in the region between the plates. This field will tend to pull the mass downwards (the top plate is positive). So, this problem is identical to one where you release a mass attached to an unstretched spring in a gravitational field. The key to the problem is to realize that the starting position of the mass is above the equilibrium position (where the electric force on the mass is balanced by the spring force) and therefore this problem is identical to pulling back a mass on a horizontal spring and letting go – the distance between the starting point and the equilibrium point is the amplitude. So, we just need to find the distancem to the equilibrium point: $ky_{eq} = qE$, and $E = V/d$. Therefore $y_{eq} = qV/dk$, and:

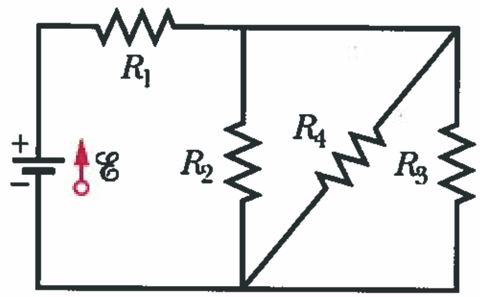
$$A = \frac{qV}{kd}$$

c) If we move the bottom plate away, the electric field in the region does not change (note that the switch is open and therefore the charge is constant, and as the distance increases, so does the voltage and therefore E is constant). Since the electric field does not change, the oscillator does not notice a difference and therefore the amplitude does not change.

-additional space for calculation-

Problem 3: [30pts]

The ideal battery with EMF ϵ is connected as shown in the figure to the three resistors, $R_1 = R_2 = R_3 = R_4 = R$.



a) [10pts] Find the resitance of a single equivalent resistor that replaces the four resistors.

b) [10pts] What is the current in resistor R_1?

c) [10pts] What is the power released in resistor R_4 ?

a) R_2, R_4, R_3 are in parallel. $\longleftrightarrow$ $R_{234} = \frac{R}{3}$

then it is in series with R_1

$R_{total} = R + \frac{R}{3} = \boxed{\frac{4}{3}R}$

b) $I_1 = \frac{\varepsilon}{R_{total}} = \boxed{\frac{3\varepsilon}{4R}}$

c) the current of R_2 R_3 R_4 should be equal

$I_4 = \frac{1}{3} I_1 = \frac{1}{4}\frac{\varepsilon}{R}$

$P = I_4^2 \cdot R_4 = \frac{\varepsilon^2}{16R^2} R = \boxed{\frac{\varepsilon^2}{16R}}$

-additional space for calculation-

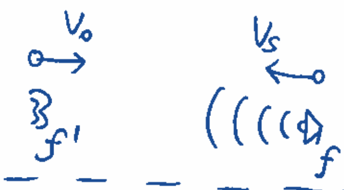
Problem 4: [20pts] A stationary siren gives of a pure sine-wave with frequency $f = 700Hz$ and is opposite a wall 20m away. Assume the speed of sound is $v_s = 340m/s$.

a) [10pts] A microphone is moved with velocity $v_1 = 5m/s$ away from the siren towards the wall. What is the frequency the microphone picks up for the wave coming directly from the microphone and the reflected wave from the wall ?

b) [10pts] The microphone now moves with an unknown velocity v_2. When you play the recording of the microphone you hear a beat frequency of $f_b = 1.5Hz$. What is v_2 ?

Doppler equation

$$f' = \frac{V + V_o}{V - V_s} f$$



a) from microphone

$$f_{micro} = \frac{340 - 5}{340} \cdot 700 = \boxed{689.7 \text{ Hz}}$$

from wall

$$f_{wall} = \frac{340 + 5}{340} \cdot 700 = \boxed{710.3 \text{ Hz}}$$

b) $f_{beat} = |f_2 - f_1|$

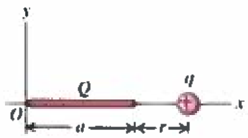
$$1.5 \text{ Hz} = \left| \frac{340 - V_2}{340} - \frac{340 + V_2}{340} \right| \cdot 700 \text{ Hz}$$

$$\frac{1.5}{700} = 2 \cdot \frac{V_2}{340}.$$

$$V_2 = \boxed{0.36 \text{ m/s}}$$

-additional space for calculation-

Problem 5: [30pts] Positive charge Q is distributed uniformly along the x-axis from $x = 0$ to $x = a$. A positive point charge q is located on the positive x-axis at $x = a + r$, a distance r to the right of the end of Q as shown in the figure.



a) [10pts] Calculate the x- and y-components of the electric field produced by the charge distribution Q on the points on the positive x-axis where $x > a$.

b) [10pts] Calculate the force (magnitude and direction) that the charge distribution Q exerts on q.

c) [10pts] Show that if $r \gg a$, the magnitude of the force in part (b) is approximately $\frac{Qq}{4\pi\epsilon_0 r^2}$. Explain why this result is obtained.

$$-\int_0^a \frac{d\,x-x'}{(x-x')^2}$$

$$\frac{1}{(x-x')}\Big|_0^a$$

$$\frac{1}{x-a} - \frac{1}{x}$$

$$\frac{x-(x-a)}{(x-a)x} = \frac{a}{(x-a)x}$$

a) $E_y = 0$

$$E_x = \frac{Q}{4\pi\epsilon_0 \cdot a}\int_0^a \frac{dx'}{(x-x')^2}$$

$$= \frac{Q}{4\pi\epsilon_0 \cdot a}\left(\frac{1}{x-a} - \frac{1}{x}\right)$$

b) $$F = E \cdot q = \frac{Qq}{4\pi\epsilon_0 a}\left(\frac{1}{x-a} - \frac{1}{x}\right) = \frac{Qq}{4\pi\epsilon_0 \cdot a}\left(\frac{1}{r} - \frac{1}{r+a}\right)$$

direction is $\hat{x}$

c) if $r \gg a$

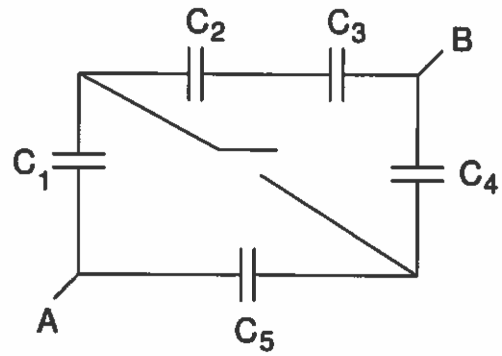
$$\frac{1}{a}\left(\frac{1}{r} - \frac{1}{r+a}\right) = \frac{1}{r(r+a)} \approx \frac{1}{r^2}$$

$$F = \frac{Qq}{4\pi\epsilon_0}\frac{1}{r^2}$$

Q can be approximated as a point charge.

-additional space for calculation-

Problem 6: [30pts] Five capacitors all have identical capacitance $C_i = C, i = 1, 2, 3, 4, 5$. Express all your answers in terms of the capacitance C.



a) [10pts] Calculate the equivalent capacitance between points A and B if the switch is open.

b) [10pts] Calculate the equivalent capacitance between A and B when the switch is closed.

c) [10pts] If the potential difference $V_{AB} = V$ is applied between points A and B when the switch is open, what is the charge Q_1 on capacitor C_1 after all capacitors are charged ?

Solutions:

a) C_1, C_2, C_3 are in series and parallel to C_4, C_5 which are in series. Hence equivalent capacitance is

$$C_{eq} = \frac{C}{3} + \frac{C}{2} = \frac{5}{6}C$$

b) The circuit is equivalent to $C_1 + C_5 = 2C$ and $(1/2)C + C = (3/2)C$ connected in a series. Hence,

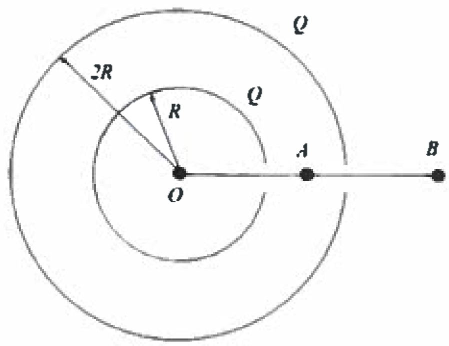
$$C_{eq} = (6/7)C$$

c) The capacitors C_1, C_2, C_3 in series have equivalent capacitance C_3 and the total charge stored in all three is therefore $Q_{eq} = CV_{AB}/3$. Capacitors in series all carry the same charge hence $Q_1 = CV_{AB}/3$.

-additional space for calculation-

Problem 7: [20pts] Two concentric thin non-conducting spherical shells have radii R and 2R, and each has a small hole along the line OB as shown. Each shell carries a positive charge Q, uniformly distributed with a constant charge density.

Assume $R = 0.5m$ and $Q = 0.04C$.



a) [10pts] What is the electric potential at point A, at distance $3R/2$ from the center, assuming the potential is zero at infinity? (Any effect of the holes should be neglected.)

b) [10pts] A proton with charge q_p and mass m_p is released from point O in the direction of point B with an initial speed $v_0 = 200m/s$. What is the speed of the proton at point B at distance $3R$ from the center? (Assume the motion is non-relativistic, and there are no forces other than the electrostatic forces.)

$\frac{3}{2} - \frac{2}{3}$

$\frac{9-4}{6} = \frac{5}{6}$

a) The potential of A comes from two parts,

$$U_A = U_{inner} + U_{outer}$$

$$= \frac{Q}{4\pi\varepsilon_0 \cdot \frac{3}{2}R} + \frac{Q}{4\pi\varepsilon_0 \cdot 2R} = \boxed{\frac{Q}{4\pi\varepsilon_0 R}\left(\frac{2}{3} + \frac{1}{2}\right)}$$

$$= \frac{0.04}{4\pi \cdot 8.85\times10^{-12} \cdot 0.5}(0.6+0.5)$$

$$= 8.4\times10^{8}\ V \quad ??$$

since C is a large unit.

b) energy conservation:

$$\frac{1}{2}mv_0^2 + \frac{Qq_p}{4\pi\varepsilon_0 R} + \frac{Qq_p}{4\pi\varepsilon_0 \cdot 2R} = \frac{1}{2}mV_1^2 + \frac{Qq_p}{4\pi\varepsilon_0 \cdot 3R} + \frac{Qq_p}{4\pi\varepsilon_0 \cdot 3R}$$

$$\frac{1}{2}m(v_0^2 - v_1^2) = \frac{Qq_p}{4\pi\varepsilon_0 R}\left(\frac{2}{3} - 1 - \frac{1}{2}\right)$$

$$\frac{1}{2}\cdot 1.67\times10^{-27}\cdot(200^2 - v_1^2) = \frac{0.04 \quad 1.6\times10^{-19}}{4\pi\times8.85\times10^{-12}\cdot 0.5}\left(\frac{2}{3} - 1.5\right)$$

Q is too large!

-additional space for calculation-