Started on	Monday, 23 May 2022, 12:49 PM
State	Finished
Completed on	Wednesday, 25 May 2022, 10:23 PM
Time taken	2 days 9 hours

Question **1**

Complete

Marked out of 1.00

If I sum a binomial random variable with n=10 and p =0.5 and a binomial random variable with m=17 and p =0.5, I get

Select one:

- a. A binomial random variable with number of Bernoulli trials =17 and p =1
- b. Something unfamiliar to us as yet
- c. A Binomial with number of Bernoulli trials =27 and p=0.5
- d. A normal distribution

Question **2**

Complete

Marked out of 1.00

Seoul is confered an average of 1370.5 mm of rainfall per year, with a standard deviation of approximately 200 mm. If we assume that annual rainfall in Seoul is Gamma distributed, what would be the moment generating function of annual rainfall in Seoul?

Select one or more:

- a. $$M(t) = \left(\frac{1370.5}{1370.5 - t}\right)^{200}$$
- b. $$M(t) = \left(\frac{0.0342625}{0.0342625 - t}\right)^{46.95676}$$
- c. $$M(t) = \left(\frac{200}{200 - t}\right)^{46.95676}$$
- d. $$M(t) = (1 - 29.18643)^{-46.95676}$$

Question **3**

Complete

Marked out of 1.00

As genetic theory shows, there is very close to an even chance that both children in a two-child family will be of the same sex. Here are two possibilities.

(i) 15 couples have two children each. In 10 or more of these families, it will turn out that both children are of the same sex.

(ii) 30 couples have two children each. In 20 or more of these families, it will turn out that both children are of the same sex.

According to the laws of probability, which of (i) and (ii) is more likely to be observed?

Select one:

- a. (i) is more likely to be observed than (ii).
- b. (ii) is more likely to be observed than (i).

Question **4**

Complete

Marked out of 1.00

According to the Law of Large Numbers,

Select one:

- a. The probability that a package sent within the state of California for 2-day delivery will actually arrive within one day can not be approximated by experimentation
- b. The probability that a package sent within the state of California for 2-day delivery will actually arrive within one day can be approximated by sending many packages, a large number, and observing how many of them arrive within one day
- c. The probability of at least one of several events happening is the complement of the union of those events.

Question **5**

Complete

Marked out of 1.00

Which of the following is most likely to happen?

Select one:

- a. at least one six when 6 six-sided fair dice are rolled
- b. at least two sixes when 12 six-sided fair dice are rolled
- c. at least three sixes when 18 six-sided fair dice are rolled

Question **6**

Complete

Marked out of 1.00

The law of large numbers (LLN) added only what to the belief that more observations obviously give more accurate estimates of the chances?

Select one or more:

- [x] a. The LLN (Law of Large Numbers) showed that the probability that the estimate is close to the truth increases with the number of trials.
- [x] b. The LLN tells us that we can be more certain that long observations give us accurate estimates the more the observations made.
- [x] c. The LLN legitimizes the frequentist definition of probability.
- [ ] d. The LLN is saying the same thing as the central limit theorem.

Question **7**

Complete

Marked out of 1.00

Suppose, in a certain region, the annual rainfall (in inches) is a normally distributed random variable with parameters $\mu = 40$ and $\sigma = 4$. Starting with this year, what is the probability that it will take over 5 years before a year occurs having a rainfall over 45 inches?

- a. 0.10565
- b. 0.03229
- c. 0.57219
- d. 0.00819

Question **8**

Complete

Marked out of 1.00

Assuming that each terminal in an interactive system has the same probability p of being in use during the peak period of the day (the load is evenly distributed over the terminals), we want to know how many observations n need to be made so that

$$P\left[\left|\frac{S_n}{n} - p\right| \geq 1\right] \leq 0.05$$

where S_n is the number of terminals in use. Call that answer (i)

If the first 100 observations indicate that p is approximately 0.2, how many more trials are needed? Call that answer (ii)

Which of the following is the answer?

(i) n=500, (ii) 220 False

(i) n=250; (ii) 560 True

Question **9**

Complete

Marked out of 3.00

The number of scooters observed per square mile, X, in minutes, is a random variable that can be modeled by an Poisson distribution with expected value 15 in minutes.

Match the probability models that must be used to find the probabilities

$f(x) = 15^x e^{-15}$	The probability that the total number of scooters in a block is larger than 140.
$N(\mu = 15, \sigma^2 = 5)$	probability that the average number of scooters per block is less than 5 when the number of blocks observed is 60
$$ Poisson (\lambda=900)$$	The probability that the total number of scooters in 60 blocks is larger than 140

Question **10**

Complete

Marked out of 5.00

\(X_1, X_2, X_3, X_4, X_5...., X_9 \) are a set of independent and identically distributed random variables, each with the moment generating function

$$M_x(t) = e^{\{4t+\frac{0.5t^2}{2}\}}.$$

Let the random variable Y be defined as $Y = \sum_{i=1}^{9} X_i$. Find the Expected value and variance of Y showing work. Attachments are not allowed.

By the properties of MGTs:

$Y = 9X \Rightarrow a = 0, b = 9$

$M_y(t) = M_x(9t)$

$M_y(t) = e^{4(9t)+\frac{0.5(9t)^2}{2}}$

$M_y(t) = e^{36t+20.25t^2}$

Find the first derivative of the MGT:

$M_y'(t) = \frac{d}{dt}[e^{36t+20.25t^2}]$

$M_y'(t) = (40.5t + 36)e^{20.25t^2+36t}$

Find the second derivative of the MGT:

$M_y''(t) = \frac{d}{dt}[(40.5t + 36)e^{20.25t^2+36t}]$

$M_y''(t) = (40.5t + 36)^2 e^{20.25t^2+36t} + 40.5e^{20.25t^2+36t}$

Find the first moment of Y:

$M'(0) = (40.5(0) + 36)e^{20.25(0)^2+36(0)}$

$M'(0) = 36$

Find the second moment of Y:

$M_y''(0) = (40.5(0) + 36)^2 e^{20.25(0)^2+36(0)} + 40.5e^{20.25(0)^2+36(0)}$

$M_y''(0) = 36^2 + 40.5$

$M_y''(0) = 1336.5$

Find the expected value of Y:

$\mu_y = M_y'(0)$

$\boxed{\mu_y = 36}$

Find the variance of Y:

$\sigma_y^2 = E(Y^2) - [E(Y)]^2$

$\sigma_y^2 = M_y''(0) - [M_y'(0)]^2$

$\sigma_y^2 = 1336.5 - 36^2$

$\boxed{\sigma_y^2 = 40.5}$

Question **11**

Complete

Marked out of 5.00

A random variable X is called a Bernoulli random variable if it can assume only two values, usually taken to be 1 and 0, the first with probability p and the second with probability q=1-p. Find the mean and variance of a Bernoulli random variable X, using its moment generating function. Show work. No attachments allowed.

Find the MGF of a Bernoulli Random Variable:

$M_x(t) = \sum_x e^{tx} P(x)$

$P(x) = \begin{cases} q, & x = 0 \\ p, & x = 1 \end{cases}$

$M_x(t) = e^{t(0)} q + e^{t(1)} p$

$M_x(t) = q + pe^t$

Find the first derivative of the MGF:

$M_x'(t) = \frac{d}{dt}[q + pe^t]$

$M_x'(t) = pe^t$

Find the second derivative of the MGF:

$M_x''(t) = \frac{d}{dt}[pe^t]$

$M_x''(t) = pe^t$

Find the first moment of X:

$M_x'(0) = pe^0 = p$

Find the second moment of X:

$M_x''(t) = pe^0 = p$

Find the mean of X:

$\mu_x = M_x'(0)$

$\boxed{\mu_x = p}$

Find the variance of X:

$\sigma_x^2 = E(X^2) - [E(X)]^2$

$\sigma_x^2 = M_x''(0) - [M_x'(0)]^2$

$\sigma_x^2 = pe^0 - [pe^0]^2$

$\sigma_x^2 = p - p^2$

$\sigma_x^2 = p(1 - p)$

$\boxed{\sigma_x^2 = pq}$

Question **12**

Not answered

Marked out of 5.00

The following was a multiple choice question in this quiz. Show work to justify your answer to (i) and (ii). You may attach 1 pdf file.

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Which of the following is the answer?