Practice Midterm Solutions

Amey Gaikwad

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1 Problem 1 a

The frequency and the amplitude is completely independent of the mass of the pendulum. For amplitude, we need to balance the energies and the mass cancels off.
Answer:
textbfE

2 Problem 1 b

Acceleration is maximum at maximum displacement.
Answer: **A**

3 Problem 1 c

The frequency depends on the source. It will remain unchanged. The wave speed definitely changes due to the change in the material for propagation. Hence, in order to keep the frequency constant, the wavelength also has to change.
Answer: **D**

4 Problem 2

a. Maximum amplitude $x_0 = 0.1$ m is related to the maximum velocity $v_0 = 0.01$ m/s through energy conservation which tells us that:

$$\begin{aligned} \frac{1}{2}mv_0^2 &= \frac{1}{2}kx_0^2 \\ \frac{m}{k} &= \frac{x_0^2}{v_0^2} \end{aligned} \tag{1}$$

Thus the time period is given by :

$$\begin{aligned} T &= 2\pi\sqrt{\frac{m}{k}} = 2\pi\frac{x_0}{v_0} \\ &= 0.628 \text{ s} \end{aligned} \tag{2}$$

b. Let $x_1 = 5$ cm and v_1 be the velocity at an extension of 5 cm. Use energy conservation again:

$$\frac{1}{2}kx_1^2 + \frac{1}{2}mv_1^2 = \frac{1}{2}kx_0^2 \tag{3}$$

Hence, on solving we get:

$$\begin{aligned} v_1^2 &= \frac{k}{m}(x_0^2 - x_1^2) \\ &= \frac{v_0^2}{x_0^2}(x_0^2 - x_1^2) \\ v_1 &= \frac{v_0}{x_0}\sqrt{x_0^2 - x_1^2} \\ &= 0.866 \text{ m/s} \end{aligned} \tag{4}$$

c. Damped oscillations :

$$\begin{aligned} \omega &= \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \\ T &= \frac{2\pi}{\omega} \end{aligned} \tag{5}$$

Mechanical energy is proportional to the amplitude (A) squared

$$\begin{aligned} x(t) &= Ae^{-\frac{b}{2m}t}\cos(\omega t + \phi) \\ E_{\text{mech}}(t) &= e^{-\frac{b}{m}t}E_{\text{mech}}(0) \end{aligned} \tag{6}$$

After one oscillation (time period T) the energy decays to $\frac{1}{e}$ of the original value. Hence,

$$\begin{aligned} e^{-\frac{b}{m}T} &= \frac{1}{e} \\ \implies \frac{b}{m}T &= 1 \\ \implies \frac{b^2}{m^2}4\pi^2 &= \frac{k}{m} - \frac{b^2}{4m^2} \\ \implies b^2 &= \frac{mk}{4\pi^2 + \frac{1}{4}} \\ \implies b &= \sqrt{\frac{mk}{4\pi^2 + \frac{1}{4}}} \\ &= 0.079 \text{ kg/s} \end{aligned} \tag{7}$$

5 Problem 3

1. For a tube (length L) open at both the ends, the frequency of the nth harmonic is given by:

$$f_n = \frac{nv}{2L} \tag{8}$$

Thus, we have the following relation:

$$\frac{f_{n_2}}{f_{n_1}} = \frac{n_2}{n_1} \tag{9}$$

Now, say we denote the three frequencies as $f_{n_1} = 315$ Hz, $f_{n_2} = 420$ Hz and $f_{n_1 3} = 525$ Hz. We have the ratios:

$$\begin{aligned}
&\frac{f_{n_3}}{f_{n_1}} = \frac{5}{3} \\
&\frac{f_{n_2}}{f_{n_1}} = \frac{4}{3} \\
&\Longrightarrow n_1 = 3, n_2 = 4, n_3 = 5
\end{aligned} \tag{10}$$

Thus,

$$\begin{aligned}
f_3 &= 315 \text{ Hz} \\
v &= f_3 \frac{2L}{3} \\
&= 399 \text{ m/s}
\end{aligned} \tag{11}$$

2. Open end = Pressure node ; n=3 $\longrightarrow$ 2 nodes inside ; n=5 $\longrightarrow$ 4 nodes inside.

Figure 1: 3rd harmonic: 2 nodes inside

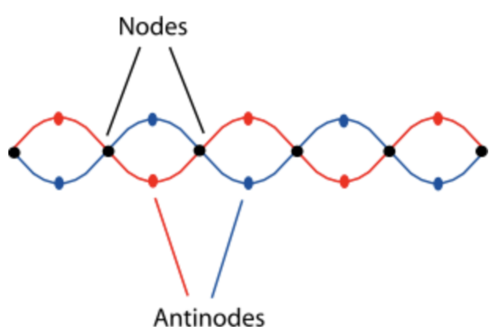

Figure 2: 5th harmonic: 4 nodes inside

3. With one end closed, the fundamental frequency is given by :

$$\begin{aligned} f &= \frac{v}{4L} \\ &= \frac{399}{4 \times 1.9} \\ &= 52.5 \text{ Hz} \end{aligned} \tag{12}$$

6 Problem 4

a. Standard form of the equation of a wave moving in the negative x direction is :

$$y(x,t) = A\sin(kx + \omega t + \phi) \tag{13}$$

Also,

$$\begin{aligned} k &= \frac{2\pi}{\lambda} \\ \omega &= \frac{\pi}{T} \end{aligned} \tag{14}$$

Thus reading off from the given equation, we get:

$$\begin{aligned} &\lambda = \frac{1}{40} \text{ m} = 0.025 \text{ m} \\ &f = 8000 \text{ Hz} \\ &v = \lambda \times f = 0.025 \times 8000 = 200 \text{ m/s} \end{aligned} \tag{15}$$

The wave moves in the negative x direction.

b. Since,

$$v = \sqrt{\frac{T}{\mu}} \tag{16}$$

we have,

$$\mu = \frac{T}{v^2} = \frac{200}{200 \times 200} = 0.005 \text{ kg}^{-1} \tag{17}$$

Thus, the weight of 1 m of string is 0.005 kg.

c. The amplitude of the wave is 0.1 cm. Average power transmitted by a string wave is given by:

$$\begin{aligned} P &= \frac{1}{2}\mu\omega^2 A^2 v \\ &= \frac{1}{2}(0.005)(2\pi 8000)^2(0.001)^2(200) \\ &= 1263 \text{ W} \end{aligned} \tag{18}$$