

Practice final (not for credit) - Work questions not included but there will be some in the exam

Started: Jun 1 at 7:28am

Quiz Instructions

- Reproduction of this quiz outside our Bruinlearn space is prohibited. The material is copyrighted.
- This is a practice final to help you review some of the course content. It is comprehensive, like the final exam will be. You have 5 attempts. There are things you have to take into account though:
 - Its length and content is different from the actual final exam. There is no room in a practice final to ask everything that could be asked. The practice final is to give you an idea of the format of the exam. Read the final exam folder for instructions for the final exam, and suggestions for review.
 - This practice final does not have work questions, **to help you see the feedback right after the quiz closes**, but your actual final exam will have questions where you must show work.
 - This practice final is in Canvas and the actual final exam will be also in Canvas.

Check the Final Exam module and read the instructions for the final exam there before Thursday, 8 PM. Ask any questions you may have about the instructions. All questions regarding the course must be asked before Thursday, June 2nd, 8 PM.

Question 1

1 pts

When we say that a set of random variables, X_1, X_2, \dots, X_n , are independent and identically distributed, what we mean is

- ☐ that all of them have the same distribution, but the observed value could be different for each of them
- ☐ that the observed value of the random variable will be the same for all of them.
- ☐ that the product of their densities is not equal to their joint density
- ☐ that they necessarily do not have the same probability density function.

Question 2

1 pts

X, Y are the duration (in years) of a hard drive for the CS department's office and the Stats Department's office, respectively. X, Y are independent and exponential with

parameter $\lambda = 0.1$ for CS and $\lambda = 0.083333$ for the Stats department, respectively. The store selling these hard drives exaggerates a little, and says to the CS department that the life of the hard drive sold to CS will be twice as long as its true expected value. The store tells the Statistics Department that the hard drive sold to it will last 3 times longer than its true expected value. From the point of view of the store, what is the expected total life of the two hard drives together?

- ☐ 56 years
- ☐ 45 years
- ☐ 0.45 years
- ☐ 19 years.

Question 3

1 pts

Let X and Y be two random variables. What is the following expression equal to?

$$E[(aX + bY) - (a\mu_x + b\mu_y)]^2$$

- ☐ $a^2\sigma_X^2 + b^2\sigma_Y^2 + 2abCov(X, Y)$
- ☐ $\text{Var}(aX+bY)$
- ☐ $\text{Cov}(X, Y)$
- ☐ $E\left((X - \mu_x) + (Y + a + \mu_y)^2 + (X - b)\right)$

Question 4

1 pts

We are interested in random variable X , the number of miles run per week by an individual training for the LA marathon. Consider n individuals, training for the LA marathon and let

$$X_1, X_2, \dots, X_n$$

denote the number of miles run per week by each of these individuals.

If these random variables are independent, and all of them have the same Expected value and Variance, i.e.,

$$E(X_i) = E(X) = \mu, \quad i = 1, \dots, n; \quad \text{Var}(X_i) = \text{Var}(X) = \sigma^2, \quad i = 1, \dots, n$$

which of the following statement is NOT true?

☐ $\text{Var}(10X) = 10\sigma^2$

☐ $\text{Var}\left(\frac{\sum_{i=1}^n X_i}{n}\right) = \frac{\sigma^2}{n}$

☐ $\text{Var}\left(\sum_{i=1}^{10} X_i\right) = 10\sigma^2$

☐ The expected value of $\frac{\sum_{i=1}^n X_i}{n}$ will be the same whether n is 100 or n is 5

Question 5

1 pts

Baseball fans and the media are fascinated with the occurrence of streaky behavior among players and teams. In the 202 baseball season, the Oakland Athletics won 20 games in a row that was the longest winning streak since the Chicago Cub's winning streak of 21 games in 1935. In the 2003 season, the Detroit Tigers lost 11 games in a row, and Albert Pujols hit successively in 30 consecutive games. The above information was provided by Jim Albert, in an article written in 2004. Which of the following statements is correct? Check all that applies.

☐ Winning streaks do not contradict the law of large numbers.

☐ Winning streaks contradict the law of large numbers

☐ The law of large numbers tells us that if the teams keep playing the probability that the observed frequency of games won approaches the true probability of winning a game is 1.

Question 6

1 pts

A certain process for production of an industrial chemical yields a product that contains two main types of impurities. For a certain volume of sample from this process, let X denote the proportion of total impurities in the sample, and let Y denote the proportion of type I impurity among all impurities found. Suppose that under investigation of many such samples, the joint distribution of X and Y can be adequately modeled by the following function.

$$f(x, y) = 2(1 - x), \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1$$

The $\text{Var}(X+Y)$ equals

- ☐ 0.1944
- ☐ 0.1388
- ☐ 0.999
- ☐ 0.833

Question 7

1 pts

According to the Law of Large Numbers,

- ☐ social media can not predict what a single individual will do, but has high probability of knowing what a large number of individuals will do.
- ☐ the behavior of a large number of individuals mimics the random behavior of a single individual
- ☐ the limit, as sample size goes to infinity, of $\frac{\text{Var}(\bar{X})}{\epsilon}$ goes to 1.

Question 8

1 pts

Suppose we have a set of independent and identically distributed random variables X_1, X_2, \dots, X_n , $E(X_i) = \mu$, $\text{Var}(X_i) = \sigma^2$, $i = 1, 2, \dots, n$.

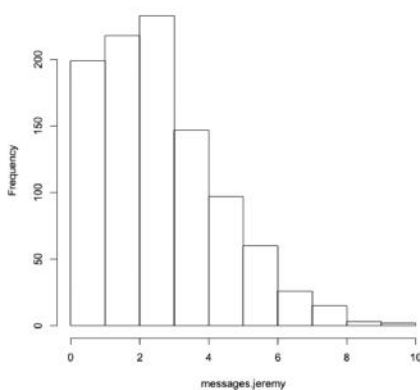
Let define $S_n = \sum_{i=1}^n x_i$.

The expected value and variance of $\frac{S_n}{n}$ are, respectively,

- ☐ $\mu, \frac{\sigma^2}{n}$
- ☐ $\mu, \frac{\sigma^2}{n^2}$
- ☐ $\frac{\mu}{n}, n^2\sigma^2$
- ☐ $n\mu, n\sigma$

Question 9**1 pts**

We take a random sample of 1000 days in the life of Rabindranath. For each day, we track the number of junk emails received that contained the word "join me in this venture." The frequency histogram indicating the number of messages with that characteristic per day, in the horizontal axis, and the number of days in which that number of messages were observed, in the vertical axis, is given below.



By how much, approximately, does the empirical probability of receiving less than two messages differ from what the true probability would be if the number of messages received was $\text{Poisson}(\lambda=3)$ (approximately). Assume in the histogram that if X is 2, the number of days with 2 messages will be counted in the bar that goes from 2 to 4, not in the bar that goes from 1 to 2. Choose closest answer.

- ☐ 0.2509 approximately
- ☐ 0.85 approximately
- ☐ 80 approximately
- ☐ 0.0002 approximately

Question 10**1 pts**

The daily number of cases of a devastating disease at a certain country averages 130, with a standard deviation of 5. What can we say about the fraction of days on which the number of cases falls between 120 and 140?

- ☐ At least in 3 out of 4 days we will observe between 120 and 140 cases
- ☐ At most in 3 out of 4 days we will observe between 120 and 140 cases
- ☐ Between 68 and 70% of the days we will observe between 120 and 140 cases
- ☐ We can not tell without knowing the distribution of the number of cases per day.

Question 11**1 pts**

Let \bar{X} denote the average of a set of independent and identically distributed random variables, X_1, X_2, \dots, X_n . Convergence in probability of \bar{X} to the $E(X_i)$ means that

- ☐ the limit, as n goes to infinity, of $P(\mu - \epsilon < \bar{X} < \mu + \epsilon)$ is 1
- ☐ the limit, as n goes to infinity, of $P(\mu - \epsilon < \bar{X} < \mu + \epsilon)$ is 0
- ☐ The limit of \bar{X} as n goes to infinity is 0
- ☐ The limit of \bar{X} as n goes to infinity is 1.

Question 12**1 pts**

Random variables X, Y are independent. Which of the following is true?

- ☐ $W = aX + b$ and $T = c + dY$ are independent, where a,b,c,d are constants.
- ☐ $\text{Covariance}(X,Y) = E(X)E(Y)$
- ☐ $\text{Var}(W+T) = \text{Var}(X) + \text{Var}(Y)$
- ☐ $E(XY)=E(X)$

Question 13**1 pts**

Consider two discrete random variables X, Y. Their joint probability mass function is

x \ y	20	25	30
20	0.05	0.05	0.1
25	0.05	0.1	0.35
30	0	0.2	0.1

For these two random variables

- ☐ The conditional probability mass function $P(Y|X)$ is identical to the marginal (also known as total) probability mass function $P(Y)$
- ☐ The conditional probability mass function $P(Y|X)$ does not equal the marginal (also known as total) probability mass function of Y.
- ☐ The conditional probability mass function $P(Y|X)$ equals the marginal probability mass function $P(X)$
- ☐ It is the case that the random variables are independent.

Question 14**1 pts**

The conditional probability mass function of random variable Y given X is

$$f(y | x) = \frac{x^2 + y^2}{2x^2 + 1} \text{ for } x=0,1,2,3, \text{ and } y=0,1$$

The marginal density functions of X and Y, respectively, are

$$f(x) = \frac{1}{32}(2x^2 + 1), \quad x = 0, 1, 2, 3$$

$$f(y) = \frac{1}{16}(2y^2 + 7), \quad y = 0, 1$$

Thus, $P(X < 2, Y = 1)$ equals.

☐ 3/32

☐ 1/32

☐ 1/16

☐ 2/16

Question 15

2 pts

Let X be the random variable representing how full a container of crude oil is and let Y represent how much of the container is demanded in a fixed period of time. X and Y both take values between 0 and 1 because they denote the proportion of the container. The joint density function of X and Y is

$$f(x, y) = 3x, \quad 0 \leq y \leq x \leq 1$$

Match the following:

$E(X)$

[Choose]



$E(Y)$

[Choose]



$E(XY)$

[Choose]



$\text{Var}(X)$

[Choose]



$\text{Var}(Y)$

[Choose]



Correlation(X, Y) -what we call rho

[Choose]



$E(2X+4Y)$

[Choose]

 $E(Y|X=1/2)$

[Choose]

 $P(X>0.8|Y=1/2)$

[Choose]

 $P(0.3 < X < 0.5 | Y = 1/2)$

[Choose]

**Question 16****1 pts**

The correlation between students' homework scores and final exam scores is 0.55. Homework scores have a expected value of 270 and standard deviation 30. The expected value of final exam scores 70 with standard deviation 9. Homework and final exam scores are random variables that follow a Bivariate Normal density. What is the probability that a student who gets 310 in homework gets a score in the final larger than 70?

☐ 0.8101☐ 0.5☐ 0.1899☐ 0.701**Question 17****1 pts**

According to a consumer organization observing Myseas department store, every consumer returns 1 gift on average. We survey 6 individuals accounts with the store randomly chosen to gather evidence about the consumer organization's claim. We record

how many gifts they returned last holiday season. If the consumer organization is correct, what would be the probability that each of the 6 individuals returned 1 gift?

☐ 0.002478

☐ 0.015625

☐ 0.5

☐ 0.7819

Question 18

1 pts

Let X be a discrete random variable. We were asked to calculate the following expression:

$$\sum_{x=0}^{\infty} \frac{e^{-7} 7^x}{x!} + \sum_{y=0}^{10} \frac{10!}{y!(10-y)!} 0.3^y (0.7)^{10-y}$$

The expression is equal to

☐ 2

☐ 3

☐ 10

☐ ∞

Question 19

1 pts

Suppose that random variable X follows a Poisson distribution with parameter $\lambda=4$. What is the expression

$$\sum_x [2P(X=x)] + \sum_x [3xP(X=x)] + \sum_x [4x^2P(X=x)]$$

equal to?

- ☐ 94
- ☐ 1124
- ☐ 16
- ☐ 334

Question 20**1 pts**

If X is a geometric random variable with parameter $p = 1/3$, what is the value of $P(X=15 | X>10)$?

- ☐ 0.0658
- ☐ 0.9341
- ☐ 0.1975
- ☐ 0.7194

Question 21**1 pts**

A second-stage smoke alert has been called in an area of Los Angeles County in which there are 50 industrial firms. An inspector will visit 10 randomly selected firms to check for violations of regulations. If 15 of the firms are actually violating at least one regulation, then the probability that 2 of the randomly selected firms violates at least one regulation is

- ☐ 0.2405758
- ☐ 0.2334744
- ☐ 0.3
- ☐ 0.005188321

Question 22**1 pts**

A salesman has scheduled two appointments to sell encyclopedias. The first appointment will lead to a sale with probability 0.3, and the second attempt will lead independently to a sale with probability 0.6. Any sale made is equally likely to be either for the deluxe model, which costs \$1000, or the standard model, which costs \$500. Let X denote sales. What is the probability that the total money made is 500?

☐ 0.27☐ 0.315☐ 0.09☐ 0.28**Question 23****1 pts**

The sensitivity of the SARS-CoV-2 diagnostic PCR tests is said to be 80%. If 1000 people are tested in a region and the probability that a random person has the disease is 1% in that region, what proportion of people with the disease will test negative?

☐ 0.2☐ 0.8☐ 0.9☐ 0.3**Question 24****1 pts**

A random variable Y has cumulative distribution function

$$F(y) = y^{\frac{1}{2}}, \quad 0 \leq y \leq 1$$

The density function of Y is

☐ $f(y) = \frac{1}{2}y^{-\frac{1}{2}}, 0 \leq y \leq 1$

☐ $f(y) = 2y^{-\frac{1}{2}}, 0 \leq y \leq 1$

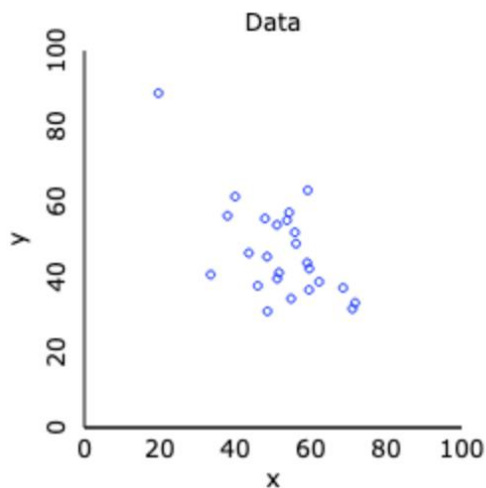
☐ $f(y) = \frac{1}{2}y^{\frac{1}{2}}, 0 \leq y \leq 1$

☐ $f(y) = 2y, 0 \leq y \leq 1$

Question 25**1 pts**

In the "guess the correlation app" that we used this quarter, we learned to estimate (albeit informally) the ρ of a bivariate distribution using a sample of data and its scatter plot.

Guess the ρ of the bivariate distribution that generated the sample observed in the following scatter plot (or the closest that you can get to it)



☐ -0.6

☐ 0.9

☐ -0.007

☐ 0.51

Question 26**1 pts**

The probability of getting at least one six in the roll of 10 fair 12-sided dice equals

- ☐ 0.4189039
- ☐ 0.3519956
- ☐ 0.9
- ☐ 0.000197

Question 27

1 pts

Assuming that the classical definition of probability to calculate the probability of an event applies,

- ☐ is like assuming that all outcomes of the sample space of an experiment are equally likely.
- ☐ is like assuming a discrete uniform probability model for each outcome in the sample space.
- ☐ is something that the gamblers that we talked about in this course used to calculate the probabilities that we saw were not well calculated.
- ☐ is the correct assumption for each of the outcomes in a sampling experiment consisting of sampling three Brazilian adults to ask them whether they know capoeira or not.
- ☐ is very appropriate if the sample space is continuous, e.g., the whole nonnegative real line.

Not saved

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