Math 61 HW #6

1. i) 3, 7, 11, 15 . . .

$\boxed{a_0 = 3}$

$\boxed{a_n = a_{n-1} + 4}$

ii) 3, 6, 9, 15, 24, 39 . . .

$\boxed{a_0 = 3, a_1 = 6}$

$\boxed{a_n = a_{n-1} + a_{n-2}}$

iii) 1, 1, 2, 4, 16, 128, 4096 . . .

$\boxed{a_0 = 1, a_1 = 1}$

$\boxed{a_n = 2(a_{n-1} \cdot a_{n-2})}$

2. \$2000, 14% interest annually

i) $A_1 = \$2000(1.14)$

$A_2 = (2000 \times 1.14)(1.14)$

$\boxed{A_n = 1.14(A_{n-1})}$

ii) $\boxed{A_0 = \$2000}$

iii) $A_1 = 1.14(A_0) = \boxed{\$2280}$

$A_2 = 1.14(A_1) = \boxed{\$2599.20}$

$A_3 = 1.14(A_2) = \boxed{\$2963.08}$

iv) $A_1 = 1.14^1 (A_0)$

$A_2 = 1.14(1.14 A_0) = 1.14^2 A_0$

$A_n = 1.14^n A_0$

$\boxed{A_n = 1.14^n (\$2000)}$

3. begin with 1 -> S_{n-1} strings

begin with 01 -> S_{n-2} strings

begin with 00 -> S_{n-3} strings

$\boxed{S_n = S_{n-1} + S_{n-2} + S_{n-3}}$

$\boxed{S_1 = 2, S_2 = 4, S_3 = 7}$

4. i) begin with 1 -> S_{n-1} strings

begin with 01 -> S_{n-2} strings

$\boxed{S_n = S_{n-1} + S_{n-2} \quad S_1 = 2, S_2 = 3}$

ii) $S_n = f_{n+2}$

$S_{n-1} + S_{n-2} = f_n + f_{n+1}$

$\boxed{2+3+5+8+13 \ldots = 2+3+5+8+13 \ldots ✓}$

5. Tower of Hanoi, $n = 3, 4$

$h_n = 2h_{n-1} + 1$

$h_1 = 1$

$2^n - 1 = h_n$

$\boxed{n=3: h_3 = 7}$

$\boxed{n=4: h_4 = 15}$

6. \$1 -> OJ, \$2 -> Milk or beer

Case 1: O, then there are R_{n-1} ways to spend rest

Case 2: M or B, then there are R_{n-2} ways to spend rest

$\boxed{\therefore R_n = R_{n-1} + 2R_{n-2}}$

7. $g_n = g_{n-1} + g_{n-2} + 1, \ n \geq 3$

$g_1 = 1, g_2 = 3$

$g_n = 2f_{n+1} - 1 \quad n \geq 1 = f_n$

Assume $g_{n-2} + g_{n-3} + 1 = 2f_n - 1$

Prove $g_{n-1} + g_{n-2} + 1 = 2f_{n+1} - 1$

$g_{n-1} = g_{n-2} + g_{n-3} + 1$

$g_{n-1} = 2f_n - 1$

$2f_n - 1 + g_{n-2} + 1 = 2f_{n+1} - 1$

$2f_n + g_{n-2} = 2f_{n+1} - 1$

$g_{n-2} = 2f_{n-1} - 1$

$g_{n-1} = 2f_n - 1$

$\boxed{g_{n-2} + g_{n-3} + 1 = 2f_n - 1 ✓}$

8. The assumption used for the inductive step is false; $t_k \neq 1$ for all $k < n$.

9. $L_n = L_{n-1} + L_{n-2}$, $L_1 = 1$, $L_2 = 3$

i) $L_3 = 3 + 1 = \boxed{4}$

$L_4 = 4 + 3 = \boxed{7}$

$L_5 = 7 + 4 = \boxed{11}$

ii) $L_{n+2} = f_{n+1} + f_{n+3}$

$L_3 = f_2 + f_4$

$4 = 1 + 3$ ✓

Assume $L_{n+1} = f_n + f_{n+2}$

Prove $L_{n+2} = f_{n+1} + f_{n+3}$

$L_{n+2} = L_{n+1} + L_n$

$L_{n+1} + L_n = f_{n+1} + f_{n+3}$

$L_n + f_n + f_{n+2} = f_{n+1} + f_{n+3}$

$L_n = f_{n-1} + f_{n+1}$

$\boxed{L_{n+2} = f_{n+1} + f_{n+3}}$ ✓

10. i) $a_n = -3a_{n-1}$

Linear homogeneous w/ constant coeff.

i) $a_n = 2na_{n-2} - a_{n-1}$

Linear homogeneous

iii) $a_n = 2na_{n-1}$

Linear homogeneous

iv) $a_n = a_{n-1} + n$

Linear with constant coefficients

v) $a_n = a_{n-1} + 1 + 2^{n-1}$

Linear with constant coefficients

11. i) $a_n = 2na_{n-1}$, $a_0 = 1$, $a_1 = 2$, $a_2 = 8$

$a_{n-1} = 2(n-1)a_{n-2} \Rightarrow a_n = 4n(n-1)a_{n-2}$

$a_{n-2} = 2(n-2)a_{n-3} \rightarrow a_n = 8n(n-1)(n-2)a_{n-3}$

$a_n = 2^n n!\, a_0$

$\boxed{a_n = 2^n n!}$

ii) $a_n = 6a_{n-1} - 8a_{n-2}$, $a_0 = 1$, $a_1 = 0$

$c_1 = 6$, $c_2 = -8$

$x^2 - 6x + 8 = 0$

$x = -4, -2$

$a_n = br_1^n + dr_2^n$

$a_n = b(-4)^n + d(-2)^n$

$a_0 = 1 = b + d$, $a_1 = 0 = -4b - 2d$

$b = 1 - d \rightarrow 0 = -4 + 4d - 2d$

$2d - 4 = 0$

$d = 2$, $b = -1$

$\boxed{a_n = -(-4)^n + 2(-2)^n}$

iii) $L_n = L_{n-1} + L_{n-2}$, $L_1 = 1$, $L_2 = 3$

$c_1 = c_2 = 1$

$x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{5}}{2}$

$x = \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$

$a_n = b\left(\frac{1+\sqrt{5}}{2}\right)^n + d\left(\frac{1-\sqrt{5}}{2}\right)^n$

$L_1 = 1 = b\left(\frac{1+\sqrt{5}}{2}\right) + d\left(\frac{1-\sqrt{5}}{2}\right)$

$L_2 = 3 = b\left(\frac{1+\sqrt{5}}{2}\right)^2 + d\left(\frac{1-\sqrt{5}}{2}\right)^2$

blah

12. $\sum_{k=1}^{n-1} nq^{k-1} = \frac{(n-1)q^n - nq^{n-1} + 1}{(q-1)^2}$

? problem makes no sense

13. $\sqrt{a_n} = \sqrt{a_{n-1}} + 2\sqrt{a_{n-2}}$

$a_0 = a_1 = 1$, $b_n = \sqrt{a_n}$

$b_n = b_{n-1} + 2b_{n-2}$

$c_1 = 1$, $c_2 = 2$

$x^2 - x - 2 = 0$

$x = 1, -2$

$b_n = b + d(2)^n$

$b_0 = 1 = b + d$

$b_1 = 1 = b - 2d$

!. Fuck this

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