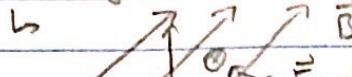


Physics 1C Lecture 1: Magnetism

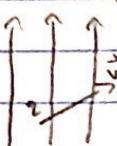
- Magnetic Fields \leftrightarrow electric charge

$\hookrightarrow \vec{F}_B = q\vec{v} \times \vec{B}$

\hookrightarrow only acts on charge if it's moving/moving in the right dir



$\hookrightarrow E_x$)



$$\vec{B} = B_z \hat{z}$$

$$q\vec{F} = m\vec{a}$$

$$q\vec{v} \times \vec{B} = m \frac{d\vec{v}}{dt}$$

$$qB_z \langle v_y, -v_x, 0 \rangle = m \langle \frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt} \rangle$$

$$\frac{dv_x}{dt} = \frac{qB_z}{m} v_y, \quad \frac{dv_y}{dt} = -\frac{qB_z}{m} v_x, \quad \frac{dv_z}{dt} = 0$$

$$\begin{aligned} \frac{\partial^2 v_x}{\partial t^2} &= \frac{qB_z}{m} \frac{\partial v_y}{\partial t} \\ \frac{\partial^2 v_y}{\partial t^2} &= -\frac{qB_z}{m} \frac{\partial v_x}{\partial t} \end{aligned}$$

$$\left. \begin{aligned} \frac{\partial^2 v_x}{\partial t^2} + \left(\frac{qB_z}{m}\right)^2 v_x &= 0 \\ \frac{\partial^2 v_y}{\partial t^2} + \left(\frac{qB_z}{m}\right)^2 v_y &= 0 \end{aligned} \right] \text{ SHO Equations} \rightarrow \frac{\partial^2 x}{\partial t^2} + \omega_0^2 x = 0$$

$$\frac{\partial v_z}{\partial t} = 0$$

$\hookrightarrow v_x$ and v_y oscillate, v_z is constant

\hookrightarrow Since $\vec{F}_B \perp \vec{v}$, $\omega_B = 0$, $\Delta K = 0$, $V = \text{constant}$

$$v_x = V_{xy} \cos(\omega t + \phi)$$

$$v_y = V_{xy} \sin(\omega t + \phi)$$

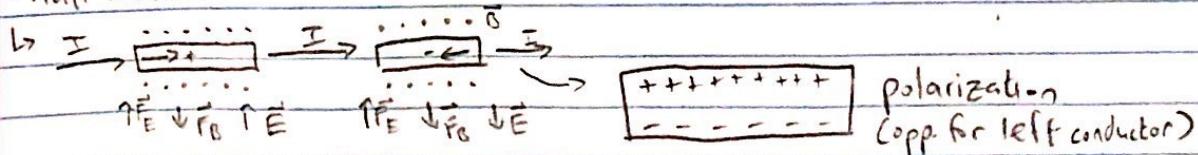
$$v_z = \text{constant}$$

$$V_{xy} = \sqrt{v_x^2 + v_y^2}$$

$$\omega = \frac{qB_z}{m} \leftarrow \text{cyclotron frequency}$$

\hookrightarrow Charged particles traversing a region of uniform \vec{B} will execute helical motion around a longitudinal axis aligned w/ that field

Hall Effect



\hookrightarrow Electric field created

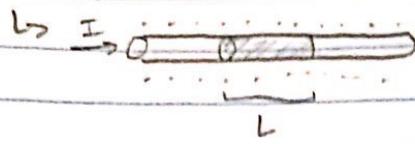
$$\hookrightarrow \text{In equilibrium: } |\vec{F}_E| = |\vec{F}_B| \rightarrow qE = qvB \sin(90^\circ) \rightarrow E = vB$$

$$\hookrightarrow \frac{dv}{dt} = vB$$

\hookrightarrow drift velocity, $\Delta v = \text{Voltage}$

Physics 1C Lecture 2: Magnetism (cont.)

- Magnetic force has a more fundamental relationship w/ current rather than charge carriers



$\hookrightarrow \vec{F}_B = n(q\vec{v} \times \vec{B})$, n is the # of charge carriers in L

\hookrightarrow think of amnt. of charge in segment as amnt. of charge that initially fills it up

$\hookrightarrow nq = I \frac{L}{\Delta x} \Delta t$

$$nqv = IL$$

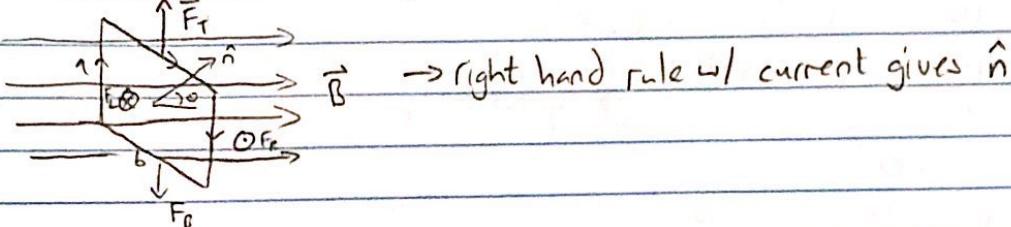
$$nq\vec{v} = I\vec{E}$$

\hookrightarrow dir. of \vec{E} is dir. of I

$\hookrightarrow \vec{F}_0 = q\vec{v} \times \vec{B} \rightarrow$ point charge

$\hookrightarrow \vec{F}_B = IL \times \vec{B} \rightarrow$ segment of charge

\hookrightarrow



$$|\vec{F}_T| = ILB \sin(90^\circ - \theta) = IbB \cos \theta$$

$$|\vec{F}_0| = ILB \sin(90^\circ + \theta) = IbB \cos \theta$$

$$|\vec{F}_T| = |\vec{F}_0| = IbB \cos \theta$$

$$|\vec{F}_L| = IaB \sin(90^\circ) \quad] \text{current } \perp \text{ to } \vec{B}$$

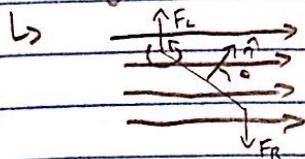
$$|\vec{F}_L| = IaB \sin(90^\circ)$$

$$|\vec{F}_L| = |\vec{F}_R| = IaB$$

$$\sum \vec{F} = \vec{F}_B + \vec{F}_T + \vec{F}_L + \vec{F}_R = 0 \rightarrow \sum F = 0 \text{ on the loop}$$

$\hookrightarrow \vec{a}$ at center of mass = 0

\hookrightarrow Left/right forces exert a torque that aligns \hat{n} with \vec{B}



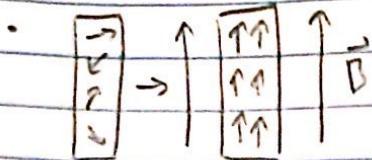
$$|\vec{T}| = r_L F \quad r_L = b \sin \theta, \quad F = F_R = IaB$$

$$|\vec{T}| = Ia'bB \sin \theta = IA B \sin \theta = IA ||B|| \sin \theta$$

$$\vec{p} = I\vec{A} \times \vec{B}, \quad \vec{T}_E = \vec{p} \times \vec{E}$$

$$\text{Magnetic dipole moment } \vec{\mu} = NI\vec{A} \rightarrow N = \# \text{ of turns in loop}$$

Physics 1C Lecture 3: Magnetic Dipole Moments

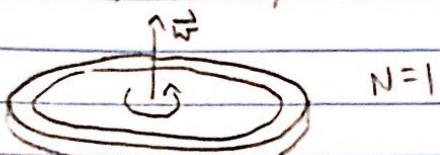


↳ internal dipoles become locked in orientation due to the magnetic fields they generate

↳ demagnetized by de-Gaussing, heat, etc.

• Uniform disk, charge Q, radius R, angular velocity $\vec{\omega}$

↳



$N=1$

↳ $\delta \vec{H} = N \delta I A \hat{n} \rightarrow A$ is area of current loop, not whole

$$\delta I = \frac{\partial q}{T} = \frac{\partial q r}{2\pi}$$

$$\delta q = \sigma \delta A = \frac{Q}{\pi R^2} \cdot 2\pi r dr = \frac{Q r dr}{R^2}$$

$$\delta I = \frac{Q r dr}{\pi R^2}$$

$$\delta \vec{H} = \frac{Q r dr}{\pi R^2} \pi r^2 = \frac{Q r^2 dr}{R^2} \hat{n} (r^3 dr)$$

$$\int \delta \vec{H} = \frac{Q r^2}{R^2} \hat{n} \int_0^R r^3 dr$$

$$\vec{H} = \frac{1}{4} Q r R^2 \hat{n} = \boxed{\vec{H} = \frac{1}{4} Q R^2 \vec{\omega}}$$

↳ Gyromagnetic Ratio: $\gamma \equiv \frac{H}{\vec{\omega}} = \frac{1/4 Q R^2 \vec{\omega}}{I} = \frac{1/4 Q R^2 \vec{\omega}}{I e m n^2 \vec{\omega}} = \frac{1}{2} \left(\frac{Q}{m} \right)$

↳ Ex)



sphere

$$\hookrightarrow \vec{H}_{\text{disk}} = \frac{1}{4} Q R^2 \vec{\omega}$$

↳ build the sphere with the disks: $\delta \vec{H} = \frac{1}{4} \delta q r^2 \vec{\omega}$

↳ Ex) non-uniform disk



$$\sigma(r) \propto r^n$$

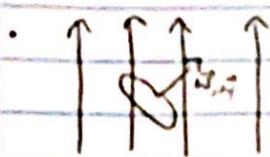
↳ 1st figure out proportionality constant

$$Q = \int \delta q = \int \sigma(r) dr = \int_0^R C r^n 2\pi r dr$$

$$\hookrightarrow \delta \vec{H} = \delta I A \hat{n}$$

$$\delta I = \frac{\partial q}{T} = \frac{\partial q}{2\pi} \vec{\omega}$$

$$\delta q = \sigma(r) 2\pi r dr = C r^n (2\pi r) dr$$

•  $\gamma \vec{B}_z = \langle L_y, L_x, 0 \rangle = \langle \frac{\partial L_y}{\partial t}, \frac{\partial L_x}{\partial t}, \frac{\partial L_z}{\partial t} \rangle$

$$\hookrightarrow \vec{L} \vec{T} = \frac{d\vec{L}}{dt}, \vec{L} = \gamma \vec{L}'$$

$$\vec{L}' \times \vec{B} = \frac{d\vec{L}'}{dt}$$

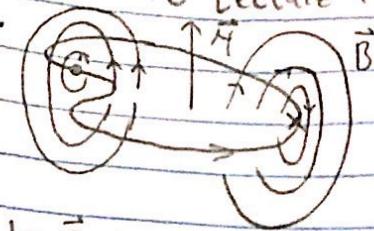
$$\gamma \vec{L}' \times \vec{B} = \frac{d\vec{L}'}{dt}$$

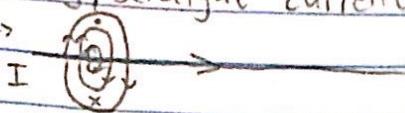
$$\frac{d\omega}{dt} = \gamma B_0 L_y, \quad \frac{dL_y}{dt} = \gamma B_0 L_x, \quad \frac{dL_x}{dt} = 0$$

$$L_x = L_{x0} \cos(\omega_0 t + \phi), \quad L_y = L_{y0} \sin(\omega_0 t + \phi), \quad L_z = \text{const}$$

$$L_{y0} = \sqrt{L^2 - L_z^2}, \quad \omega_0 = \gamma B_0 = \text{cyclotron frequency}$$

Physics 1C Lecture 4: Ampere's Law



- ↳ \vec{B} is right-handed with respect to the current
- ↳ \vec{B} lines form loops
- Long, straight current
- ↳ 
- ↳ $B(r, \phi, z)(\hat{r}, \hat{\phi}, \hat{z})$
- ↳ B cannot depend on ϕ or z , because there is no indication of where those would be
- ↳ B cannot depend on \hat{r} or \hat{z} because then it would depend on ϕ

↳ Sole reliance on r and $\phi \rightarrow \vec{B} = B(r) \hat{\phi} \rightarrow$ circular loops

$$\hookrightarrow B(r) = \frac{H_0}{2\pi} \left(\frac{I}{r} \right)$$

$$\hookrightarrow H_0 = 4\pi \times 10^{-7}$$

$$B(r) 2\pi r = H_0 I_{\text{enc}}$$

$$\oint \vec{B} \cdot d\vec{s} = H_0 I_{\text{enc}} \rightarrow \text{Ampere's Law}$$

$$\hookrightarrow \oint B(r) ds \cos 0^\circ = H_0 I$$

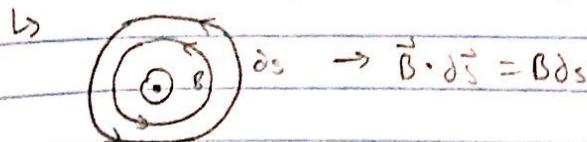
$$\oint B(r) ds = H_0 I \rightarrow B(r) 2\pi r = H_0 I \rightarrow B(r) = \frac{H_0 I}{2\pi r}$$

• Ampere's Law \longleftrightarrow Gauss' Law

↳ Gauss' Law uses symmetry to relate field to charge distribution

$$\hookrightarrow \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

↳ Best Ampere loops mimic the symmetry of current cross section



• Ex) Thick, long, straight wire with radius a , uniform current I



$$\hookrightarrow (r \leq a) \rightarrow \oint \vec{B} \cdot d\vec{s} = H_0 I_{\text{enc}} \rightarrow \oint B(r) ds = H_0 I_{\text{enc}}(r)$$

$$B(r) 2\pi r = H_0 I_{\text{enc}}(r) \rightarrow B(r) = \frac{H_0 I_{\text{enc}}(r)}{2\pi r}$$

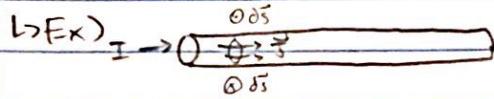
$$I \propto A \propto r^2 \rightarrow \frac{I_{\text{enc}}}{I} = \frac{r^2}{a^2} \rightarrow I_{\text{enc}} = I \frac{r^2}{a^2}$$

$$B(r) = \frac{H_0 I r}{2\pi a^2} \hat{\phi}$$

Physics 1C Lecture 5: Ampere's Law

- $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{encl}}$

↳ Shortcut for long wires: $\vec{B}(r) = \frac{\mu_0 I(r)}{2\pi r} \hat{\phi}$



$$J \propto r^2$$

$$I = \int \vec{J} \cdot d\vec{A} = \int J dA = \int cr^2 2\pi r dr$$

$$I_{\text{encl}} = 2\pi c \int_0^r r^3 dr = 2\pi c \left(\frac{r^4}{4}\right)$$

$$I_{\text{encl}}(a) = I$$

$$c = \frac{2I}{\pi a^2}$$

$$J = \frac{2I r^2}{\pi a^2}$$

$$I_{\text{encl}}(r) = \frac{2\pi c r^3}{\pi a^2} \left(\frac{r^4}{4}\right)$$

$$I_{\text{encl}}(r) = I \frac{r^3}{a^3}$$

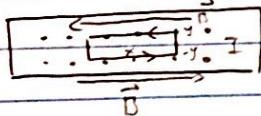
$$\vec{B}(r) = \frac{\mu_0 I r^3}{2\pi a^4} \hat{\phi}$$

- Current sheet: $H, U, I, w \gg H$

↳



↳ $B(x, y, z)(x, y, z) \rightarrow B(y)(x)$



↳ $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{encl}}$

$$\oint \vec{B} \cdot d\vec{s} + \oint \vec{B} \cdot d\vec{s} + \oint \vec{B} \cdot d\vec{s} + \oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{encl}}$$

$$B(y)x + B(-y)x = \mu_0 I_{\text{encl}} \rightarrow |\vec{B}(y)| = |\vec{B}(-y)|$$

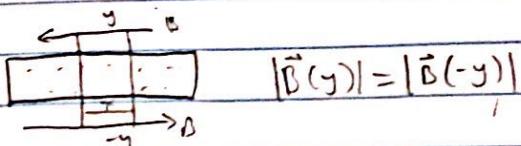
$$2B(y)x = \mu_0 I_{\text{encl}}$$

$$I_{\text{encl}} \propto A_{\text{encl}} \rightarrow \frac{I_{\text{encl}}}{I} = \frac{2xy}{HW} \rightarrow \text{uniform}$$

$$2B(y)x = \mu_0 \cdot I \left(\frac{2xy}{HW}\right)$$

$$\vec{B}(y) = -\mu_0 I \left(\frac{y}{HW}\right) \hat{x} \rightarrow \vec{B} \text{ inside the sheet}$$

↳ Outside:

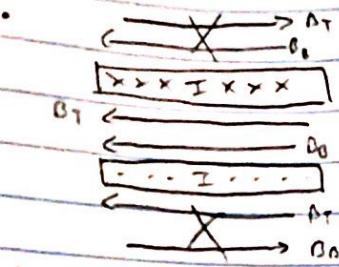


$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{encl}}$$

$$2B(y)x = \mu_0 I \frac{xH}{wH} = \mu_0 I \frac{x}{w}$$

$$\vec{B}(y) = \frac{\mu_0 I}{2w} \left(-\frac{y}{|x|} \hat{x}\right)$$

↳ independent of distance from sheet



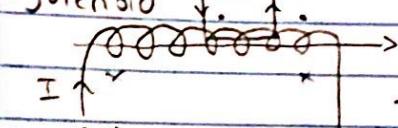
↳ All magnetic field is between the current sheets

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enc}}$$

$$B_x = \mu_0 \frac{I}{2d}$$

$$B = \mu_0 \frac{I}{2d}$$

• Solenoid



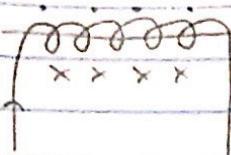
→ can approximate $\vec{B} = 0$ outside

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

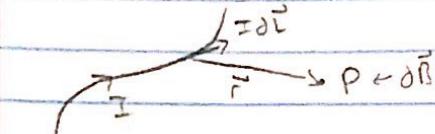
$$B_x = \mu_0 N_x I$$

$$B = \mu_0 n I \hat{z}$$

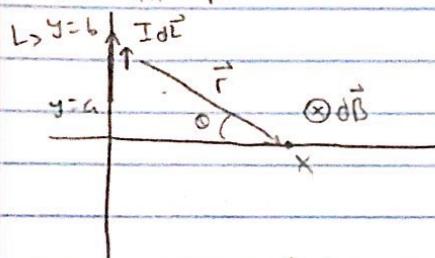
Physics 1C Lecture 6: Biot-Savart

-  $\vec{B} \rightarrow$ determined by RHR

Biot-Savart

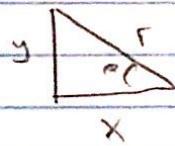


$$\hookrightarrow d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$$



$$\hookrightarrow d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \frac{I |d\vec{l}| \hat{r}_1}{r^2} (-\hat{z})$$

$$d\vec{B} = (-\hat{z}) \frac{\mu_0 I}{4\pi} \frac{d\sin(\theta) \hat{r}}{r^2} \rightarrow \int d\vec{B}_z = (-\hat{z}) \frac{\mu_0 I}{4\pi} \int_a^b \frac{dy \cos \theta}{r^2}$$



$$y = x \tan \theta \rightarrow dy = \frac{x d\theta}{\cos^2 \theta}$$

$$r = \sqrt{x^2 + y^2}$$

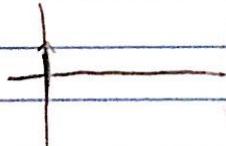
$$\vec{B}_z = (-\hat{z}) \frac{\mu_0 I}{4\pi} \int_a^b \frac{y d\theta}{\cos^2 \theta} \cdot \frac{\cos \theta}{x^2} \cos^2 \theta$$

$$\vec{B}_z = (-\hat{z}) \frac{\mu_0 I}{4\pi x} \int_a^b \cos \theta d\theta$$

$$\vec{B}_z = (-\hat{z}) \frac{\mu_0 I}{4\pi x} [\sin \theta_b - \sin \theta_a] = -(-\hat{z}) \frac{\mu_0 I}{4\pi x} \left[\frac{b}{\sqrt{b^2 + x^2}} - \frac{a}{\sqrt{a^2 + x^2}} \right]$$

\hookrightarrow special cases

$$\hookrightarrow a = -b$$



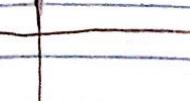
$$\vec{B}_z = (-\hat{z}) \frac{\mu_0 I}{2\pi x} \left(\frac{b}{\sqrt{b^2 + x^2}} \right)$$

$$\hookrightarrow a = -b, b = \infty$$



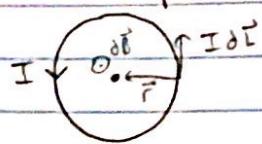
$$\vec{B}_z = (-\hat{z}) \frac{\mu_0 I}{2\pi x} \rightarrow \text{Ampere's Law}$$

$$\hookrightarrow a = 0, b = \infty$$



$$\vec{B}_z = (-\hat{z}) \frac{\mu_0 I}{4\pi x}$$

↳ Circular loop: I, R

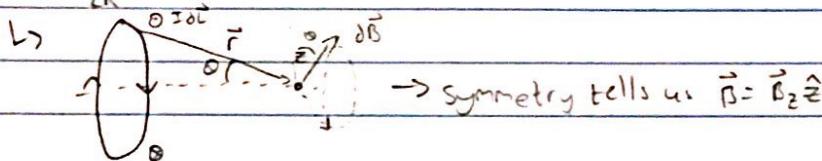


$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I dL \times \hat{r}}{r^2}$$

$$d\vec{B} = \hat{z} \frac{\mu_0}{4\pi} \frac{I dL \times \hat{r}}{R^2}$$

$$\int d\vec{B} = \frac{\mu_0 I}{4\pi R^2} \int ds \hat{z}$$

$$\vec{B} = \frac{\mu_0 I}{2R} \hat{z}$$



$$\vec{B} = B_z \hat{z} = \int d\vec{B}_z \hat{z}$$

$$d\vec{B}_z = dB \sin \theta = \frac{\mu_0 I}{4\pi} \frac{10L \times \hat{r}}{r^2} \sin \theta$$

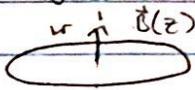
$$d\vec{B}_z = \frac{\mu_0 I}{4\pi} \frac{ds}{R^2 + z^2} \frac{R}{\sqrt{R^2 + z^2}}$$

$$\int d\vec{B}_z = \frac{\mu_0 I}{4\pi} \frac{R}{(R^2 + z^2)^{3/2}} \int ds$$

$$B_z = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}}$$

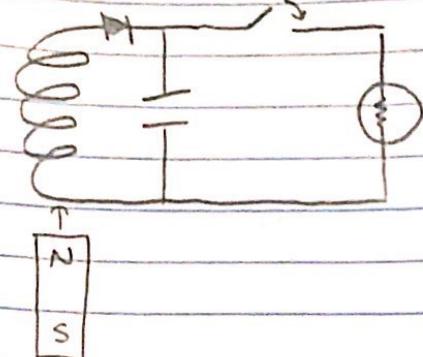
$$B = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}} \hat{z}$$

• Spinning uniform disk of Ω, R



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I dL \times \hat{r}}{r^2}$$

Physics 1C Lecture 7: Faraday's Law



• $\rightarrow \mathcal{E}_i = -\frac{\partial \Phi_B}{\partial t}$

↳ Ex) Copper loop: radius = a resistance = R

$\vec{B}(t) = B_0 e^{-t/T} \hat{z}$

$\mathcal{E}_i = I_i \cdot \vec{B}(t) \cdot \pi a^2 \cdot \vec{B} = \pi a^2 B_0 e^{-t/T}$

$\mathcal{E}_i = \frac{\pi a^2 B_0}{T} e^{-t/T} \rightarrow$ positive azimuthal direction

↳ not potential difference \rightarrow path dependent \rightarrow not conservative

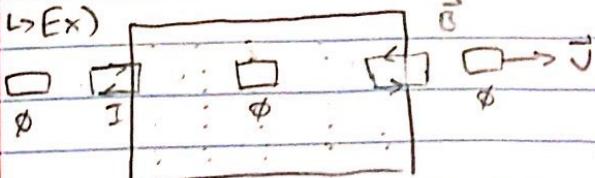
$I_i = \frac{\mathcal{E}_i}{R} = \frac{\pi a^2 B_0}{RT} e^{-t/T}$

↳ $B(t) \rightarrow$ increasing

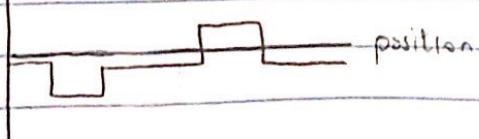
$\mathcal{E}_i, I_i \rightarrow$ against the current, must counter the effect that creates it

↳ Doesn't fully cancel original flux

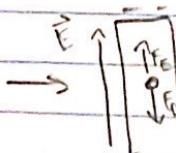
↳ Ex)



↳ I



↳ Ex) \vec{B} $\mathcal{E}_i?$



$$|\vec{F}_d| = qvB$$

$$|\vec{F}_d| = qE = q \frac{\Delta V}{L}$$

$$qvB = q \frac{\Delta V}{L}$$

$$\Delta V = E = BLv$$

$\Phi_B = BLv \rightarrow \mathcal{E}_i = -BLv \rightarrow$ motional EMF

$\Delta V = E = BLv$

Physics 1C Lecture 8: Inductance

• Generators:

$$\hookrightarrow \mathcal{E}_i = -\frac{\partial \Phi_B}{\partial t}$$

$$\mathcal{E}_i = -\frac{\partial}{\partial t} \int B \cdot d\vec{s}$$

$$\mathcal{E}_i = -\frac{\partial}{\partial t} \int B A \cos \theta$$

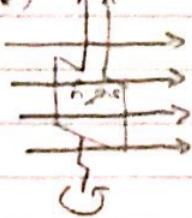
$$\mathcal{E}_i = - \int \frac{\partial B}{\partial t} A \cos \theta - B \frac{\partial}{\partial t} \cos \theta + \int A B \sin \theta \frac{\partial \theta}{\partial t}$$

\hookrightarrow 1st term = \mathcal{E}_i generated by changing $B \rightarrow$ Faraday's Law

\hookrightarrow 2nd term = \mathcal{E}_i generated by sweeping out area \rightarrow bar thru magnetic field

\hookrightarrow 3rd term = \mathcal{E}_i generated by changing orientation

$\hookrightarrow \mathcal{E}_i =$



$$\Phi_B = BA \cos \theta$$

$$\mathcal{E}_i = -BA \sin \theta \frac{\partial \theta}{\partial t}$$

$$\mathcal{E}_i = BA \omega \sin(\omega t)$$

$$P = \frac{\mathcal{E}^2}{R} = \frac{B^2 A^2 \omega^2}{2R} r^2$$

• 2 Kinds of EMF

\hookrightarrow Conservative (IB) Electrostatic (ΔV)

\hookrightarrow Non-Conservative Electrodynamics (\mathcal{E}_i)

$\hookrightarrow W_E = W_{\text{cons}} + W_{\text{non-cons}}$

$$\oint \vec{E} \cdot d\vec{s} = -q \Delta V + q \mathcal{E}_i$$

$\oint \vec{E} \cdot d\vec{s} = \Delta V + \mathcal{E}_i \rightarrow$ conservative piece lost in closed loop integral

$$\mathcal{E}_i = \oint \vec{E} \cdot d\vec{s}$$

$$\hookrightarrow \oint \vec{E} \cdot d\vec{s} = -\frac{\partial \Phi_B}{\partial t} \rightarrow$$
 non-conservative E-field

\hookrightarrow $\vec{E} = \mu_0 N I(t) \hat{z}$



$$\oint \vec{E} \cdot d\vec{s} = -\frac{\partial}{\partial t} \int B \cdot d\vec{A} \rightarrow \vec{E}$$
 has to form a closed loop (no charges)

$$E(2\pi r) = -\frac{\partial}{\partial t} [\mu_0 N I(t) r^2]$$

$$E = -\frac{\mu_0 N I}{2} \frac{\partial r}{\partial t} \hat{\theta} \quad (r > a)$$



$$\oint \vec{E} \cdot d\vec{s} = -\frac{\partial}{\partial t} \int B \cdot d\vec{A}$$

$$E(2\pi r) = -\frac{\partial}{\partial t} (\mu_0 N I \ln r^2) \rightarrow r \rightarrow$$
 magnetic field constrained by solenoid

$$\vec{E} = -\frac{1}{2} \mu_0 N \frac{\partial r}{\partial t} \hat{\theta} \rightarrow \vec{E}$$
 points in direction opposite of I increase

\hookrightarrow E resultant from $\frac{\partial I}{\partial t}$ \rightarrow magnetic field is changing

Physics IC Lecture 9: AC Circuits

- $\int_0^T f(t) dt = \bar{f} T \rightarrow \text{average}$

$$\langle f \rangle = \frac{1}{T} \int_0^T f(t) dt$$

$$\langle \sin \theta \rangle = 0 = \langle \cos \theta \rangle$$

$$\langle \sin \theta \cos \theta \rangle = 0$$

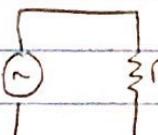
$$\langle \sin^2 \theta \rangle = \langle \cos^2 \theta \rangle = 1/2$$

- $I = I_{\max} \sin(\omega t) \rightarrow \text{AC circuit}$

$$\langle I \rangle = I_{\max} \langle \sin(\omega t) \rangle = 0$$

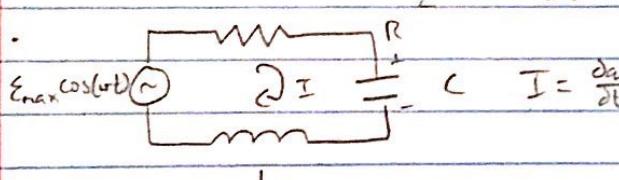
- $\langle I^2 \rangle = I_{\max}^2 \langle \sin^2(\omega t) \rangle = I_{\max}^2 / 2$

$$\hookrightarrow I_{\text{rms}} = \sqrt{\langle I^2 \rangle} = \frac{I_{\max}}{\sqrt{2}}$$

- 

$$\langle P \rangle = I_{\max}^2 R \langle \sin^2(\omega t) \rangle$$

$$\langle P \rangle = \frac{I_{\max}^2}{2} R = I_{\text{rms}}^2 R$$

- 

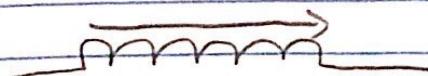
$$\hookrightarrow E_{\max} \cos(\omega t) - IR - \frac{q}{C} - L \frac{dI}{dt} = 0$$

$$\frac{d^2q}{dt^2} + \frac{R}{L} \frac{dq}{dt} + \frac{1}{LC} q = (E_{\max}/L) \cos(\omega t) \rightarrow q \propto x, R \propto b, L \propto m, C \propto 1/k, E \propto F$$

$$\hookrightarrow U = \frac{1}{2} kx^2 \rightarrow U = \frac{1}{2} \frac{q^2}{C}$$

$$\hookrightarrow K = \frac{1}{2} mv^2 \rightarrow K = \frac{1}{2} L I^2$$

$$\hookrightarrow P = F \cdot v = -b v^2 = -I^2 R$$

- 

$$\hookrightarrow P = IE_i = IL \frac{dI}{dt} = \frac{dU}{dt}$$

$$\int dU = \int_0^T L I dI$$

$$U = \frac{1}{2} L I^2 \rightarrow \text{energy stored in inductor}$$

$$\hookrightarrow U = \frac{1}{2} H_0 N^2 \pi a^2 \times I^2 = \frac{1}{2} \frac{Tia^2 X}{H_0} (N \omega I)^2 = \frac{1}{2} \frac{B^2}{H_0} (\pi a^2 X)$$

$$U_B \rightarrow \text{energy density} = \frac{1}{2} \mu_0 B^2 \rightarrow \text{can lead to inductance}$$

$$U = \int u_B dV$$

- $x(t) = \frac{I_{\max}/m}{\sqrt{(\frac{E}{m} - \omega^2)^2 + (\frac{b\omega}{m})^2}} \cos(\omega t + \phi)$

$$\hookrightarrow \tan \phi = -\frac{b\omega}{m(\frac{E}{m} - \omega^2)}$$

$$q(t) = \frac{E_{max}/L}{\sqrt{(\frac{1}{LC} - \omega^2)^2 + (\frac{\omega}{R})^2}} \cos(\omega t + \delta) \rightarrow \text{how much charge is on the capacitor}$$

$$\tan \delta = \frac{-R\omega}{L(\frac{1}{LC} - \omega^2)}$$

$$\hookrightarrow I(t) = \frac{E_{max}}{\sqrt{(\frac{1}{LC} - \omega^2)^2 + R^2}} \sin(\omega t + \delta) = \frac{E_{max}}{\sqrt{(\frac{1}{LC} - \omega^2)^2 + R^2}} \cos(\omega t + \delta - \frac{\pi}{2})$$

$$\tan \delta = \frac{-R}{1/\omega C - \omega L}$$

$$\hookrightarrow I(t) = \frac{E_{max}}{\sqrt{(\frac{1}{LC} - \omega^2)^2 + R^2}} \cos(\omega t - \phi)$$

$$\tan \phi = \frac{\omega L - 1/\omega C}{R}$$

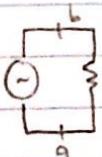
$$\hookrightarrow I_{max} = \frac{E_{max}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \leftrightarrow I = \frac{\Delta V}{R}$$

\hookrightarrow Impedance (Z) \rightarrow ohms

$$\hookrightarrow Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2} \rightarrow \text{opposition to flow of AC}$$

$$\hookrightarrow I_{max} = \frac{E_{max}}{Z}$$

\hookrightarrow Current lags Voltage by ϕ

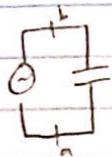


$$\Delta V_{AB} = \Delta V_{BA}$$

$$E_{max} \cos(\omega t) = IR$$

$$I = \frac{E_{max}}{R} \cos(\omega t)$$

$$I_{max} = \frac{E_{max}}{R} \rightarrow I \text{ in phase w/ voltage}$$



$$\Delta V_{AP} = \Delta V_{PA}$$

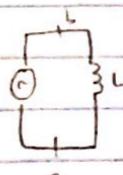
$$E_{max} \cos(\omega t) = \frac{q}{C}$$

$$q = E_{max} C \cos(\omega t)$$

$$I = -E_{max} \omega C \sin(\omega t) = E_{max} \omega C \cos(\omega t + 90^\circ)$$

$$I_{max} = \frac{E_{max}}{1/\omega C} \rightarrow I \text{ leads } E \text{ by } 90^\circ$$

$$\hookrightarrow \frac{1}{\omega C} \rightarrow R$$



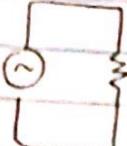
$$E_{max} \cos(\omega t) = L \frac{dI}{dt}$$

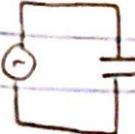
$$I = \frac{E_{max}}{\omega L} \sin(\omega t) = \frac{E_{max}}{\omega L} \cos(\omega t - 90^\circ) \rightarrow I \text{ lags } E \text{ by } 90^\circ$$

$$I_{max} = \frac{E_{max}}{\omega L}$$

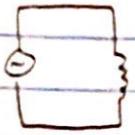
• ELI the ICE man

Physics 1C Lecture 10: Reactance

- 
 $P = IE = I_{\max} E_{\text{max}} \sin(\omega t) \rightarrow I/E \text{ in phase across resistor}$
 $\langle P \rangle = \frac{I_{\max} E_{\text{max}}}{2} = I_{\text{rms}} E_{\text{rms}}$

- 
 $P = IE = I_{\max} E_{\text{max}} \sin(\omega t) \cos(\omega t) \rightarrow 90^\circ \text{ out of phase}$
 $\langle P \rangle = I_{\max} E_{\text{max}} \langle \sin(\omega t) \cos(\omega t) \rangle = 0$

↳ store energy, get it back \rightarrow Avg. P consumption = 0

- 
 $P = IE = I_{\max} E_{\text{max}} \sin(\omega t) \cos(\omega t) \rightarrow 90^\circ \text{ out of phase}$
 $\langle P \rangle = 0$

↳ Energy pushed into B by motion of charge, released by same way

↳ Capacitors/inductors are conservative devices \rightarrow don't have resistance

• Reactance

↳ $X_C = \frac{1}{\omega C} \rightarrow$ Capacitor

↳ $X_L = \omega L \rightarrow$ Inductor

↳ $e^{i\theta} = \cos\theta + i\sin\theta \rightarrow$ unit phasor

$$\hookrightarrow \begin{array}{c|c|c} \text{Im} & \tilde{z} = x+iy & \bar{z} \\ \hline \text{Re} & \begin{matrix} \sin\theta \\ \cos\theta \end{matrix} & \begin{matrix} y \\ x \end{matrix} \end{array}$$

$$\hookrightarrow \tilde{z} = \sqrt{x^2+y^2} e^{i\theta}, \tan\gamma = \frac{y}{x}$$

$$\tilde{z}^* = x-iy$$

$$\tilde{z}^* = \sqrt{x^2+y^2} e^{-i\theta}$$

$$\tilde{z}\tilde{z}^* = |\tilde{z}|^2 = x^2+y^2$$

$$\cos\theta = \text{Re}(e^{i\theta}) \rightarrow \cos\theta \rightarrow e^{i\theta}$$

- 
 $I = I_{\max} \cos(\omega t)$

$$\hookrightarrow \Delta V_R = \Delta V_{\text{max}} \cos(\omega t)$$

$$I_{\text{max}} = I_{\text{max}} e^{i\omega t}$$

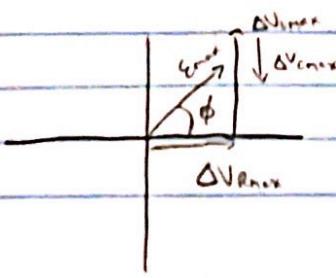
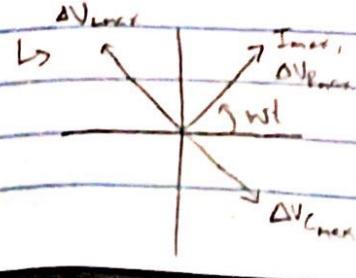
$$\Delta V_C = \Delta V_{\text{max}} \cos(\omega t - 90^\circ)$$

$$\Delta V_R = \Delta V_{\text{max}} e^{i\omega t}$$

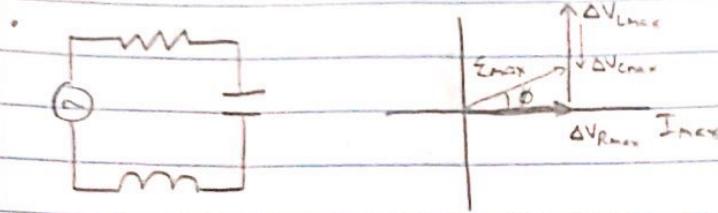
$$\Delta V_L = \Delta V_{\text{max}} \cos(\omega t + 90^\circ)$$

$$\Delta V_C = \Delta V_{\text{max}} e^{i(\omega t - 90^\circ)}$$

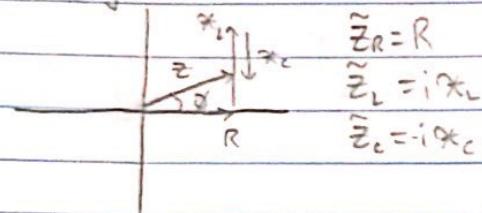
$$\Delta V_L = \Delta V_{\text{max}} e^{i(\omega t + 90^\circ)}$$



Physics IC Lecture 11: Resonance



↳ divide by I_{max}



$$Z = \sqrt{R^2 + (X_L - X_C)^2}, \tan \phi = \frac{X_L - X_C}{R}$$

↳ replace $X_L w / wL$ and $X_C w / wC$

↳ $P = IE \rightarrow$ Energy delivered by the source

$$P = I_{\text{max}} E_{\text{max}} \cos(\omega t) \cos(\omega t + \phi)$$

$$\langle P \rangle = I_{\text{max}} E_{\text{max}} [\langle \cos^2(\omega t) \rangle \cos \phi - \langle \cos(\omega t) \sin(\omega t) \rangle \sin \phi]$$

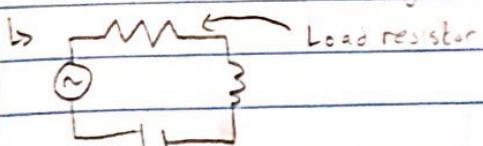
$$\langle P \rangle = I_{\text{rms}} E_{\text{rms}} \left(\frac{1}{2} \cos \phi \right) = I_{\text{rms}} E_{\text{rms}} \cos \phi \rightarrow \cos \phi = \text{power factor}$$

$$\hookrightarrow \langle P \rangle = I_{\text{rms}}^2 Z \cos \phi \rightarrow Z \cos \phi = R \rightarrow I_{\text{rms}}^2 R$$

↳ Power factor exists to take resistance (power consumer) out of Z

↳ Max power consumed when Z is purely resistance

• Resonance \rightarrow transmitting maximum current to load



$$I_{\text{max}} = \frac{E_{\text{max}}}{Z} = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$$

↳ maximized when $X_L = X_C \rightarrow Z$ is pure resistance

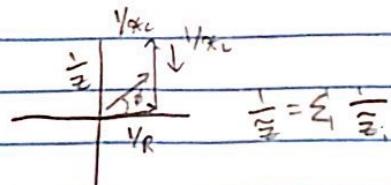
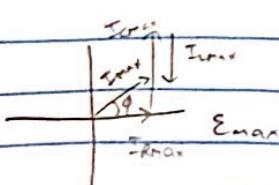
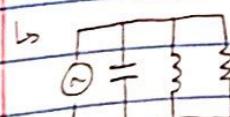
$$\hookrightarrow wL = \frac{1}{wC} \rightarrow w = 1/\sqrt{LC}$$

$$\hookrightarrow \frac{E_{\text{max}}}{Z} = I_{\text{max}} X_L = \frac{E_{\text{max}}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

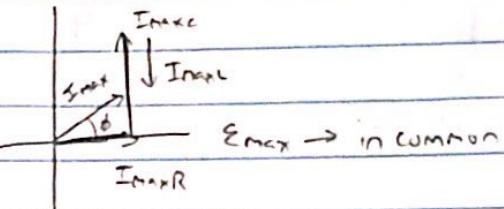
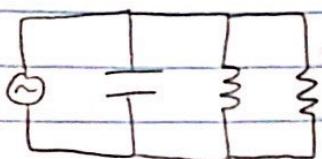
what if $X_L = X_C, X_L \gg R$

$$\hookrightarrow \Delta V_{L_{\text{max}}} \gg E_{\text{max}}$$

• Parallel Circuits



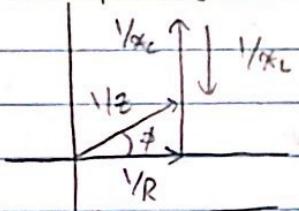
Physics 1C Lecture 12: Parallel AC Circuits



$$\hookrightarrow E_{max} = I_{max} Z$$

$$E_{max} = I_{max} R, E_{max} = I_{max} X_C, E_{max} = I_{max} X_L$$

\hookrightarrow rescale plot by $1/E_{max}$



$$\hookrightarrow \frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}$$

$$\tan \phi = \frac{(1/X_C - 1/X_L)}{1/R}$$

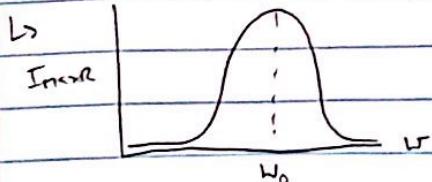
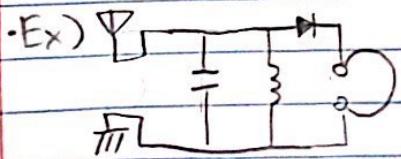
• Resonance: Max current to load

$$\hookrightarrow I_{max} R = \frac{E_{max}}{R}$$

$$I_{max} R = I_{max} \frac{Z}{R},$$

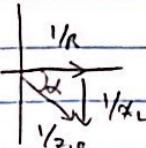
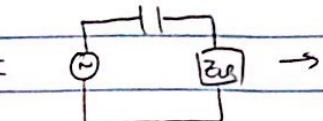
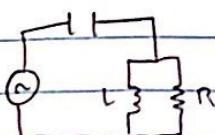
$$I_{max} R = I_{max} R \sqrt{R^2 + (1/X_C - 1/X_L)^2}$$

$$X_C = X_L \rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \rightarrow \text{series/parallel when load is 100% resistive}$$



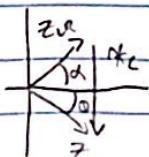
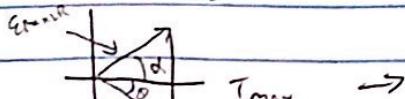
$$Q \propto \frac{1}{F_{WHM}}$$

• Ex)



$$\hookrightarrow \frac{1}{Z_{parallel}} = \sqrt{\frac{1}{R^2} + \frac{1}{X_C^2}}$$

$$\tan \phi = \frac{R}{X_C}$$



$$Z = \sqrt{(Z_{parallel} \cos \alpha)^2 + (Z_{parallel} \sin \alpha - X_C)^2}$$

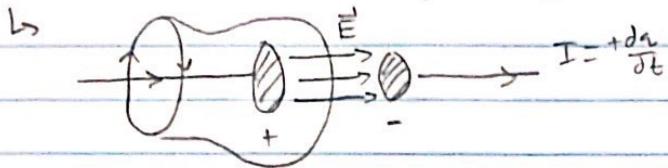
$$\tan \phi = \frac{Z_{parallel} \sin \alpha - X_C}{Z_{parallel} \cos \alpha}$$

Physics 1C Lecture 13: Maxwell's Equations

$$\oint \vec{B} \cdot d\vec{s} = H_0 I_{\text{enc}}$$

↳ path integral around closed loop on left

↳ current enclosed by loop on right



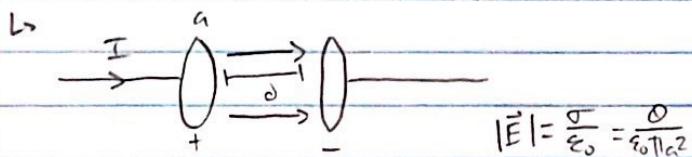
$$\oint \vec{B} \cdot d\vec{s} = H_0 I_{\text{pierce}} \rightarrow B = 0$$

$$\Phi_E = EA = \frac{\epsilon_0}{\epsilon_0} A$$

$$\vec{D}_E = q/\epsilon_0$$

$$I = \frac{\partial q}{\partial t} = \epsilon_0 \frac{\partial D_E}{\partial t}$$

$$\oint \vec{B} \cdot d\vec{s} = H_0 I_{\text{enc}} + H_0 \epsilon_0 \frac{\partial D_E}{\partial t}$$



$$\oint \vec{B} \cdot d\vec{s} = H_0 I + H_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$B(2\pi r) = 0 + H_0 \epsilon_0 \frac{1}{\partial t} \left[\frac{\sigma}{\epsilon_0 \pi a^2} \pi r^2 \right]$$

$$2\pi r B = H_0 \epsilon_0 \frac{1}{\partial t} \frac{r^2}{\pi a^2} I$$

$$\boxed{B = \frac{H_0 I}{2\pi r} \hat{\phi} \quad (r < a)}$$

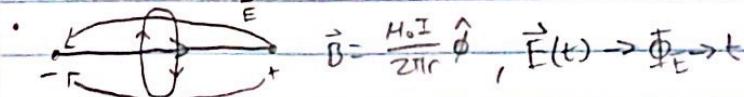
$$\text{for } r > a: \oint \vec{B} \cdot d\vec{s} = H_0 I + H_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$B(2\pi r) = H_0 \epsilon_0 \frac{1}{\partial t} \left[\frac{\sigma}{\epsilon_0 \pi a^2 \pi r^2} \right]$$

$$\boxed{\vec{B} = \frac{H_0 I}{2\pi r} \hat{\phi}}$$

$$\text{for } I_{\text{disp}} = \epsilon_0 \frac{\partial E}{\partial t} \rightarrow \text{displacement current}$$

$$\oint \vec{B} \cdot d\vec{s} = H_0 [I_{\text{polariz}} + I_{\text{disp}}]$$



$$\text{Gauss: } \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} \quad \text{Gauss: } \oint \vec{B} \cdot d\vec{A} = 0$$

$$\text{Faraday: } \oint \vec{E} \cdot d\vec{s} = -\frac{\partial \phi}{\partial t}$$

$$\text{Ampere: } \oint \vec{B} \cdot d\vec{s} = H_0 I + H_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$\cdot \vec{\nabla} = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z}$$

$$\cdot \text{Divergence Theorem: } \oint \vec{G} \cdot d\vec{A} \equiv \int (\vec{\nabla} \cdot \vec{G}) dV$$

$$\oint \vec{E} \cdot d\vec{A} = q_{in}/\epsilon_0 \rightarrow \int (\vec{\nabla} \cdot \vec{E}) dV = \int \frac{q}{\epsilon_0} dV \rightarrow \vec{\nabla} \cdot \vec{E} = q/\epsilon_0$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \rightarrow \vec{\nabla} \cdot \vec{B} = 0$$

$$\cdot \text{Stokes' Theorem: } \oint \vec{G} \cdot d\vec{s} \equiv \int (\vec{\nabla} \times \vec{G}) \cdot d\vec{A} \rightarrow \oint \vec{B} \cdot d\vec{s} = H_0 I + H_0 \epsilon_0 \frac{\partial E}{\partial t} \rightarrow \vec{\nabla} \times \vec{B} = H_0 \vec{J} + H_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{\partial \phi}{\partial t} \rightarrow \int (\vec{\nabla} \times \vec{E}) \cdot d\vec{A} = -\int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A} \rightarrow \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{E}}{\partial t}$$

Physics 1C Lecture 14: Free-Space

- $\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$ $\oint \vec{B} \cdot d\vec{A} = 0$ $\oint \vec{E} \cdot d\vec{s} = -\frac{\partial \phi_B}{\partial t}$ $\oint \vec{B} \cdot d\vec{E} = H_0 I_{\text{enc}} + H_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

- $\nabla \cdot \vec{E} = \frac{q}{\epsilon_0}$ $\vec{B} \cdot \vec{B} = 0$ $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\vec{B} \times \vec{B} = H_0 \vec{J} + H_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

- $\nabla = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \rightarrow \nabla \cdot \vec{G} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y} + \frac{\partial G_z}{\partial z} \rightarrow \text{Divergence of } \vec{G}$

- $\nabla \times \vec{G} = \hat{x} \left(\frac{\partial G_z}{\partial y} - \frac{\partial G_y}{\partial z} \right) + \hat{y} \left(\frac{\partial G_x}{\partial z} - \frac{\partial G_z}{\partial x} \right) + \hat{z} \left(\frac{\partial G_y}{\partial x} - \frac{\partial G_x}{\partial y} \right) \rightarrow \text{Curl of } \vec{G}$

- $\vec{F} = x\hat{x} + y\hat{y}$ 

- $\nabla \cdot \vec{F} = 1+1+0=2$ 

- $\nabla \times \vec{F} = \hat{x}(0-0) + \hat{y}(0-0) + \hat{z}(0-0) = 0$

- $\vec{G} = -y\hat{x} + x\hat{y}$ 

- $\nabla \cdot \vec{G} = 0+0+0=0$ 

- $\nabla \times \vec{G} = \hat{x}(0-0) + \hat{y}(0-0) + \hat{z}(1+1) = 2\hat{z}$

- Divergence \rightarrow tendency of a field to diverge from a point

- Curl \rightarrow tendency to curl around an axis

- In order to diverge, there must be something different about that point

- $\vec{E} \rightarrow$ source or sink of charge $\rightarrow \nabla \cdot \vec{E} = \frac{q}{\epsilon_0}$

- $\vec{B} \rightarrow$ no sources/sinks $\rightarrow \nabla \cdot \vec{B} = 0$.

- In order to curl, there must be an axis to curl around

- $\vec{E} \rightarrow$ loops around changing \vec{B} -fields $\rightarrow \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

- $\vec{B} \rightarrow$ current density/changing \vec{E} -fields $\rightarrow \nabla \times \vec{B} = H_0 \vec{J} + H_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

- statements about how \vec{E} and \vec{B} fill space given distribution of sources, sinks, and currents

- Free-Space: $q=0, \vec{J}=0$

- $\nabla \cdot \vec{E} = 0, \nabla \cdot \vec{B} = 0, \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \nabla \times \vec{B} = H_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

- Self-perpetuating fields ($\frac{\partial \vec{B}}{\partial t}$ causes $\frac{\partial \vec{E}}{\partial t}$ causes $\frac{\partial \vec{B}}{\partial t}$...)

- $\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$

- $\nabla \times (\nabla \times \vec{E}) = -H_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$

- $\nabla \times (\nabla \times \vec{B}) = H_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$

- $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

- $\nabla \times \vec{A} \times \vec{E} = \vec{A}(\nabla \cdot \vec{E}) - \vec{E}(\nabla \cdot \vec{A}) = -H_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$

- $\nabla \times \vec{A} \times \vec{B} = \vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A}) = -H_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$

- $\nabla^2 \vec{E} = H_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \rightarrow H_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{\partial^2 \vec{E}}{\partial x^2}$

- $\nabla^2 \vec{B} = H_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \rightarrow H_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = \frac{\partial^2 \vec{B}}{\partial x^2}$

- $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ } traveling waves

Physics 1C Lecture 15: EM Waves

$$\nabla \cdot \vec{E} = 0, \nabla \cdot \vec{B} = 0, \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}, \nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \rightarrow \vec{E}$ and \vec{B} propagate in free space as traveling waves

Assume waves are traveling along the x-axis (1-D)

$$\vec{E} = E_0 \sin(kx - vt + \phi) \quad \Rightarrow f = A \sin(kx - vt + \phi)$$

Vectors \rightarrow 

$$k = \frac{2\pi}{\lambda}, \omega = \frac{2\pi}{T} = k v_x, T = \frac{1}{f}, f \lambda = v_x, v_x = c$$

$$\vec{E} = (E_{ox}, E_{oy}, E_{oz}) \sin(kx - vt + \phi)$$

$$\nabla \frac{\partial \vec{E}}{\partial t} = -\omega (E_{ox}, E_{oy}, E_{oz}) \cos(kx - vt + \phi)$$

$$\nabla \frac{\partial \vec{E}}{\partial x} = k (E_{ox}, E_{oy}, E_{oz}) \cos(kx - vt + \phi), \frac{\partial \vec{E}}{\partial y} = 0, \frac{\partial \vec{E}}{\partial z} = 0$$

\vec{B} : $E \rightarrow B, \phi \rightarrow \theta$

$$\nabla \cdot \vec{E} = 0 \rightarrow \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

$$\frac{\partial E_x}{\partial x} = 0 \rightarrow k E_{ox} \cos(kx - vt + \phi) = 0 \rightarrow E_{ox} = 0$$

$$\nabla \cdot \vec{B} = 0 \rightarrow B_{ox} = 0$$

no component along propagation (x-axis)

\vec{E} and \vec{B} are transverse waves

$$\nabla \times \vec{G} = \hat{x} \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) \hat{y} \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) + \hat{z} \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right)$$

$$= -\hat{y} \frac{\partial E_z}{\partial y} + \hat{z} \frac{\partial E_y}{\partial x}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$k (0, -E_{oz}, E_{oy}) \cos(kx - vt + \phi) = \omega (0, B_{oy}, B_{oz}) \cos(kx - vt + \theta)$$

$\phi = \theta \rightarrow \vec{E}$ and \vec{B} in phase

$$-k E_{oz} = \omega B_{oy}, k E_{oy} = \omega B_{oz}$$

$$E_{oy} = c B_{oz}, E_{oz} = -c B_{oy}$$

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$$

$$k (0, -B_{oz}, B_{oy}) \cos(kx - vt + \phi) = -\frac{\omega}{c^2} (0, E_{oy}, E_{oz}) \cos(kx - vt + \phi)$$

$$-k B_{oz} = -\frac{\omega}{c^2} E_{oy}, k B_{oy} = -\frac{\omega}{c^2} E_{oz}$$

$$E_{oy} = c B_{oz}, E_{oz} = -c B_{oy}$$

\vec{E} and \vec{B} are transverse

\vec{E} and \vec{B} are in phase

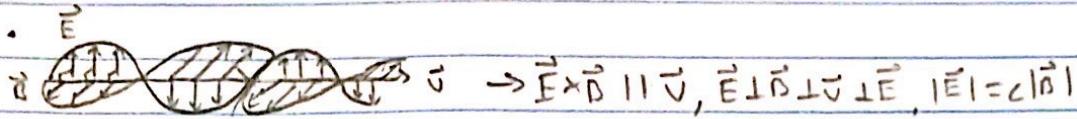
$$E_{oy} = c B_{oz}, E_{oz} = -c B_{oy}$$

$$\vec{E}_0 \cdot \vec{B}_0 = (0, E_{oy}, E_{oz}) \cdot (0, B_{oy}, B_{oz}) = c (0, B_{oz}, -B_{oy}) \cdot (0, B_{oy}, B_{oz}) = 0$$

$$\vec{E}_0 \perp \vec{B}_0, \vec{E} \perp \vec{B}$$

$$\vec{E}_0 \times \vec{B}_0 = \hat{x} (E_{oy} B_{oz} - E_{oz} B_{oy}) = c \hat{x} (B_{oz}^2 + B_{oy}^2) = \hat{x} B_0^2 c$$

$\hat{x} E_0^2 / c \rightarrow$ same direction as wave is traveling



↳ Polarization follows \vec{E} (vertically polarized above)

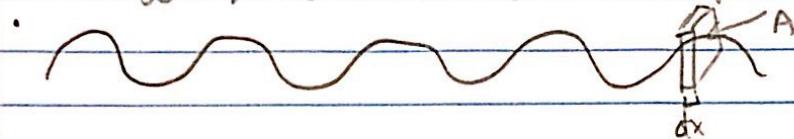
↳ If charge q in path of wave

$$F_E = q\vec{E}, F_{\text{max}} = qvB = qE \frac{v}{c}$$

↳ \vec{E} field does most of the work \rightarrow wires aligned w/ \vec{E}

$$\hookrightarrow U_E = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \epsilon_0 c^2 B^2 = \frac{1}{2 \mu_0} B^2 = U_B$$

↳ energy equally divided by electric/magnetic fields



↳ How much energy in the box?

$$dU = (U_B + U_E) dV$$

$$dU = 2U_E dV A$$

$$dU = \epsilon_0 E^2 A dx$$

$$dU = \epsilon_0 B C E dx$$

↳ What is the rate at which energy enters front plane

$$\frac{dU}{dt} = \epsilon_0 A B C E \frac{dx}{dt}$$

$$\frac{dU}{dt} = \epsilon_0 A B c^2 E$$

$$\frac{dU}{dt} = \frac{1}{\mu_0} |\vec{E} \times \vec{B}| |\vec{A}| \cos(\theta)$$

$$P = \frac{1}{\mu_0} (\vec{E} \times \vec{B}) \cdot \vec{A}$$

$$P = \int \frac{1}{\mu_0} (\vec{E} \times \vec{B}) \cdot d\vec{A} \rightarrow \text{flux of something}$$

$$\hookrightarrow \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \rightarrow \text{Poynting Vector}$$

↳ Points in direction energy is carried

$$\hookrightarrow |\vec{S}| = \text{intensity of wave} \left(\frac{W}{m^2} \right)$$

$$\hookrightarrow P = \oint S$$

Physics 1C Lecture 16: Poynting Vector

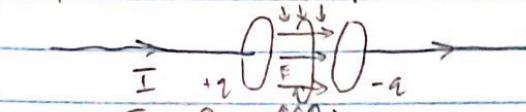
• Poynting Vector: $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$, $|S| = \frac{1}{\mu_0} EB$ (EM wave)

↳ Magnitude gives intensity (W/m^2)

↳ $\vec{S} \parallel$ to propagation of wave / direction energy flows

$$\hookrightarrow P = \vec{P} \cdot \vec{s} = \int \vec{S} \cdot d\vec{A}$$

• Capacitor: radius a , separation d , charge $q(t)$



$$\hookrightarrow E = \frac{\sigma}{\epsilon_0} = \frac{Q}{2\pi a^2}$$

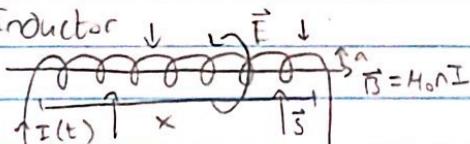
$$\hookrightarrow B(a) = \frac{\mu_0 I}{2\pi a}$$

$$\hookrightarrow S(a) = \frac{1}{\mu_0} EB = \frac{Q}{\epsilon_0 \pi a^2} \left(\frac{I}{2\pi a} \right) = \frac{QI}{2\epsilon_0 \pi^2 a^3}$$

$$\hookrightarrow P(a) = \int \vec{S} \cdot d\vec{A} = \int S(a) dA = \frac{QI}{2\epsilon_0 \pi^2 a^3} (2\pi a d) = \frac{QI}{c}$$

$$\hookrightarrow \frac{\partial}{\partial t} \left(\frac{1}{2} \frac{Q^2}{c} \right) = \frac{\partial U}{\partial t}$$

• Inductor



$$\hookrightarrow \oint \vec{E} \cdot d\vec{s} = - \frac{\partial \Phi_B}{\partial t}$$

$$E(2\pi a) = - \frac{d}{dt} [\mu_0 N I \pi a^2]$$

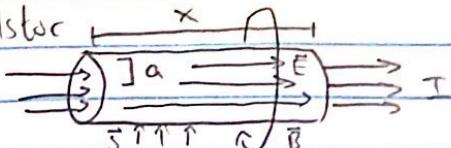
$$E = - \frac{\mu_0 N a}{2} \frac{\partial I}{\partial t}$$

$$\hookrightarrow S(a) = \frac{1}{\mu_0} EB = \frac{\mu_0 N a}{2} \frac{\partial I}{\partial t} = \frac{\mu_0 N^2 a}{2} I \frac{\partial I}{\partial t}$$

$$\hookrightarrow P = \int \vec{S} \cdot d\vec{A} = S(a) dA = \frac{\mu_0 N^2 a}{2} I \frac{\partial I}{\partial t} (2\pi a x)$$

$$P = \mu_0 N^2 \pi a^2 x I \frac{\partial I}{\partial t} = L I \frac{\partial I}{\partial t} = \frac{\partial}{\partial t} \left(\frac{1}{2} L I^2 \right) = \frac{\partial}{\partial t} U$$

• Resistor



$$\hookrightarrow \Delta V = IR$$

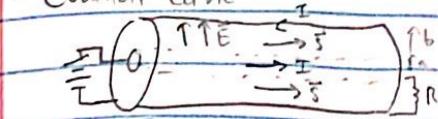
$$E_x = IR$$

$$E = \frac{IR}{x}$$

$$\hookrightarrow D = \frac{\mu_0 I}{2\pi a}$$

$$\hookrightarrow S(a) = \frac{1}{\mu_0} EB = \frac{R}{x} \frac{I^2}{2\pi a} = \frac{I^2 R}{2\pi a x} = \frac{P}{A}$$

• Coaxial Cable



$$\hookrightarrow B = \frac{\mu_0 I}{2\pi r}$$

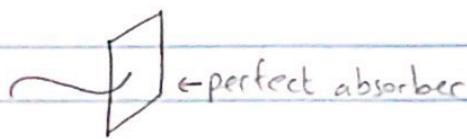
$$\hookrightarrow E = \frac{2}{\epsilon_0 \pi r}, \quad \epsilon = \Delta V = \frac{2}{\epsilon_0 \pi r} \ln \left(\frac{r_b}{r_a} \right) \rightarrow E = \frac{\epsilon}{r \ln(r/b/a)} \quad S(r) = \frac{I \epsilon}{2\pi r^2 \ln(r/b/a)}$$

$$\hookrightarrow S(r) = \frac{1}{\mu_0} EB = \frac{\epsilon}{r \ln(r/b/a)} \frac{I}{2\pi r}$$

$$P = \int \frac{I \epsilon}{2\pi r \ln(r/b/a)} \frac{2\pi r dr}{r^2} = \frac{I \epsilon}{\ln(r/b/a)} \ln(r/b/a) = I \epsilon = \frac{I \epsilon}{R}$$

Physics 1C Lecture 17: Light Pressure

- Light pressure



$$\hookrightarrow U = PC$$

$$\hookrightarrow P = \frac{mv}{\sqrt{1-v^2/c^2}} \rightarrow \text{obj.s at spd. of light} = \%$$

\hookrightarrow only massless particles can travel at speed of light

$$\hookrightarrow \Delta U = \Delta p C \rightarrow \text{divide by } \Delta t$$

$$\langle P \rangle \approx \langle F_{\text{net}} \rangle c \rightarrow \frac{\partial p}{\partial t} = F, \frac{\partial F}{\partial t} = \langle F \rangle$$

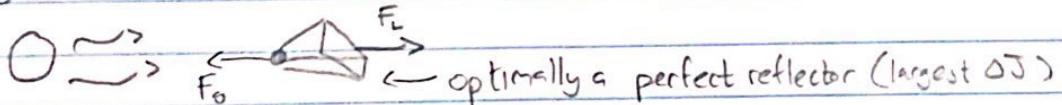
$$\langle S \rangle A = \langle F_{\text{net}} \rangle c$$

$$\frac{\langle F_{\text{net}} \rangle}{A} = \frac{\langle S \rangle}{c} = P \rightarrow \text{pressure}$$

\hookrightarrow Light is completely absorbed

$$\hookrightarrow \text{Light is perfectly reflected: } P = 2 \frac{\langle S \rangle}{c}$$

- Light Sail

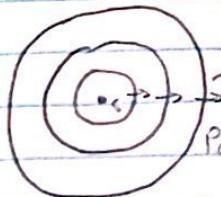


$$\hookrightarrow \sum F_x = \text{max}$$

$$F_L - F_B = \text{Max}$$

$$2 \frac{\langle S \rangle}{c} A - \frac{GMm}{R^2} = \text{Max} \rightarrow \langle S \rangle = \frac{P}{A} = \frac{P_0}{4\pi r^2}$$

\hookrightarrow



$$\frac{2P_0 A}{4\pi c r^2} - \frac{GMm}{r^2} = \text{Max}$$

$$\frac{1}{r^2} \left[\frac{2P_0 A}{4\pi c} - GMm \right] = m\ddot{A}$$

\hookrightarrow bracketed term must be positive to accelerate ($\frac{2P_0 A}{4\pi c} - GMm > 0$)

$$\hookrightarrow A > \frac{GMm \cdot 2\pi c}{P_0} \approx 6600 \text{ m}^2$$

- EM Spectrum

\hookrightarrow 0-3 MHz \approx 100m LF

430 THz-750 THz 700nm-400nm visible

3-30 MHz 100m-10m HF

750 THz- 3×10^4 THz 400nm-10nm UV

30-300 MHz 10m-1m VHF

3×10^4 THz- 3×10^6 THz 10nm-0.1nm X-ray

300 MHz-3 GHz 1m-10cm UHF

3×10^6 THz \rightarrow 0.1nm \rightarrow Gamma

3 GHz-700 GHz 10cm-1mm microwave

700 GHz-470 THz 1mm-700 nm IR

Physics 1C Lecture 18: Intro to Optics

• Empirical rules of optics:

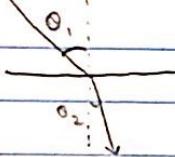
- ↳ In any given uniform medium, light travels in a straight line
- ↳ When light reflects off a medium, the angle of incidence is equal to the angle of reflection

↳  → measured with respect to the normal

- ↳ When light passes from one medium to another, it follows "Snell's Law"

$$\hookrightarrow n_1 \sin \theta_1 = n_2 \sin \theta_2, \text{ where } n \text{ is } \frac{c}{v}$$

$$\hookrightarrow n_{\text{vacuum}} = 1, n_{\text{air}} = 1, n_{\text{water}} = 4/3, n_{\text{glass}} = 3/2$$

↳ 

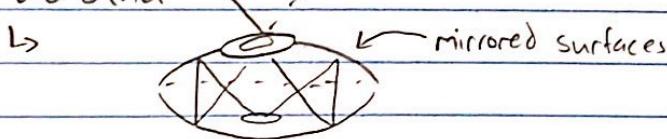
θ_1 = angle of incidence, θ_2 = angle of refraction

$$\sin \theta_1 \propto \frac{1}{n_1}$$

$$\hookrightarrow \text{small } \theta_1 \rightarrow \theta_1 \propto \frac{1}{n_1}$$

$$\hookrightarrow \theta_2 < \theta_1 \rightarrow n_2 > n_1 \rightarrow v_2 < v_1$$

- ↳ Light rays are reversible → a ray that works in one direction also works in the other



- ↳ Empirical: obtained by observation

- ↳ Fermat's principle of least time → light chooses the path that takes the least time

- ↳ Boundary conditions



$$\rightarrow \theta_1 = \theta_r$$



- ↳ Rays that are incident on a lens or mirror intersect at the location of the object for that lens or mirror

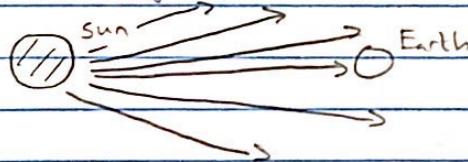
- ↳ If the object is on the same side as the incident light, it is a real object

- ↳ If the object is on the opposite side as the incident light, it is a virtual object

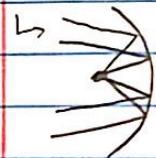
- ↳ Rays that emerge intersect at the image produced by that lens/mirror

- ↳ If the image is on the same side as the emerging light, it is a real image
- ↳ If on the opposite side \rightarrow virtual image
- ↳ Incident light \rightarrow object
- ↳ Emerging light \rightarrow image
- ↳ Same side \rightarrow real
- ↳ Opposite side \rightarrow virtual

• Paraxial Rays:



- ↳ The light from distant sources tends to arrive at an observation point as a set of parallel rays
- ↳ If the observational instrument is aligned with that distant source \rightarrow "optical axis" points at source \rightarrow rays run parallel to the optical axis and are called paraxial rays

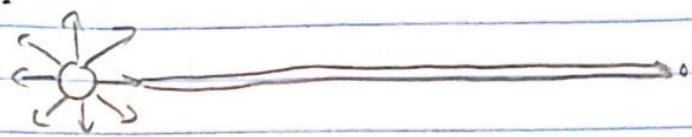


\rightarrow converging



\rightarrow diverging

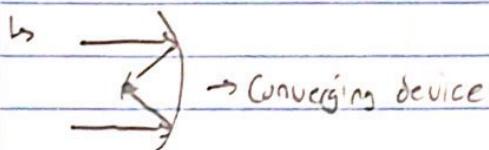
Physics 1C Lecture 19: Optic Conventions



↳ All rays that arrive are from a very limited range of angles

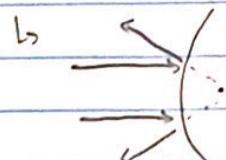
↳ Parallel rays

• Conventions:



↳ light from a distant source converges on a focal point

↳ Concave to incident light



↳ light from a distant source diverges from a focal point

↳ Convex to incident light

↳ Object distance (p) - dist. from object to optical device, measured along the optical axis

↳ $p = +$ for real objects, $p = -$ for virtual objects

↳ Image distance (q) - dist. from optical object to image along optical axis

↳ $q = +$ for real images, $q = -$ for virtual objects

↳ Focal length (f) - distance from focal point to lens or mirror, measured along OA

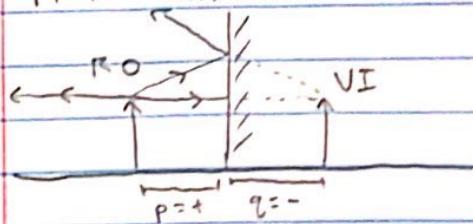
↳ $f = +$ for converging, $f = -$ for negative

↳ Magnification (m) - $M \equiv \frac{h_i}{h_o}$

↳ $M = +$ → i and o have same orientation (upright)

↳ $M = -$ → i and o have opposite orientation (inverted)

• Planar Mirror



↳ $h_i = h_o$

$$M = \frac{h_i}{h_o} = 1$$

$$M = -\frac{a}{p}$$

• Equation:

↳ $p = q$: reversibility of rays

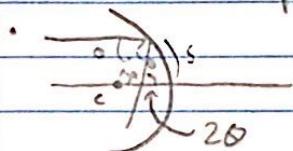
↳ $p \rightarrow \infty, q \rightarrow f \leftrightarrow p \rightarrow f, q \rightarrow \infty$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

• Technically, paraxial rays only focus to a point if the focusing surface is parabolic

↳ Limit consideration to small θ , we can treat spherical surfaces as parabolic surfaces

↳ Causes "spherical aberration"



↳ $S = R\theta$

$$R\theta = x \cdot 2\theta$$

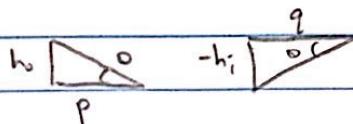
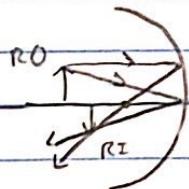
$$x = \frac{R}{2} = |f|$$

↳ Converging: $f = +\frac{R}{2}$, Diverging: $f = -\frac{R}{2}$

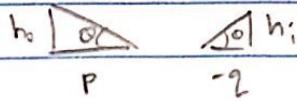
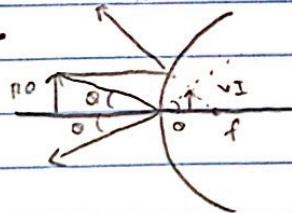
↳ Convention: $R = +$ for concave mirrors, $R = -$ for convex mirror

$$\hookrightarrow f = \frac{R}{2}$$

$$\hookrightarrow \frac{1}{P} + \frac{1}{Q} = \frac{1}{f} = \frac{2}{R}$$



$$\hookrightarrow M = \frac{h_i}{h_o} = -\frac{Q}{P}$$



$$\hookrightarrow M = \frac{h_i}{h_o} = -\frac{Q}{P}$$

↳ Spherical Mirrors:

$$\hookrightarrow \frac{1}{P} + \frac{1}{Q} = \frac{1}{f} = \frac{2}{R}$$

$$\hookrightarrow Q = \frac{PF}{P-f}$$

$$\hookrightarrow M = \frac{F}{F-P}$$

Physics 1C Lecture 20: Spherical Mirrors

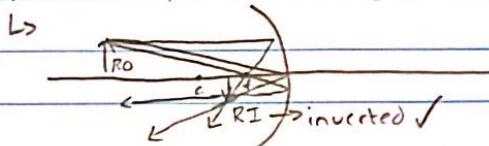
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R}, M = \frac{h_i}{h_o} = -\frac{q}{p}$$

↳ Ex) Concave mirrors, real objects $\rightarrow f = +, p = +$

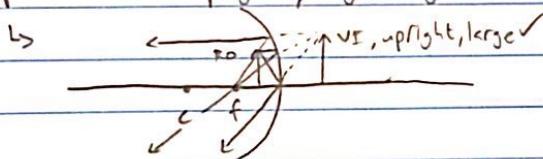
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$q = \frac{pf}{p-f}, M = \frac{-f}{p-f} \rightarrow 2 \text{ regions } (p > f \text{ and } p < f)$$

↳ $p > f \rightarrow RI, \text{ Inverted, any size}$



↳ $p < f \rightarrow VI, \text{ Upright, large image}$

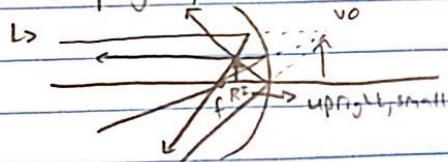


↳ Ex) Concave mirrors, virtual objects $\rightarrow f = +, p = -$

$$-\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$q = \frac{pf}{p+f}, M = \frac{f}{p+f} \rightarrow 1 \text{ region}$$

↳ RI, upright, small

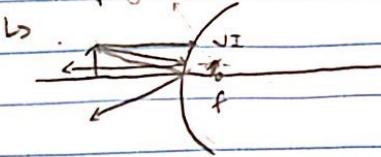


↳ Ex) Convex mirrors, real objects $\rightarrow f = -, p = +$

$$\frac{1}{p} + \frac{1}{q} = -\frac{1}{f}$$

$$q = -\frac{pf}{f+p}, M = \frac{f}{f+p} \rightarrow 1 \text{ region}$$

↳ VI, upright, small



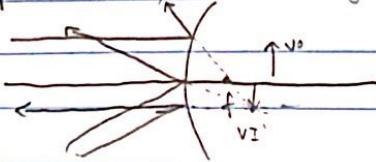
↳ Ex) Convex mirrors, virtual object, $\rightarrow f = -, p = -$

$$-\frac{1}{p} + \frac{1}{q} = -\frac{1}{f}$$

$$q = \frac{pf}{f-p}, M = -\frac{q}{p} = \frac{f}{f-p} \rightarrow 2 \text{ regions } (p < f \text{ and } p > f)$$

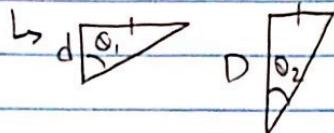
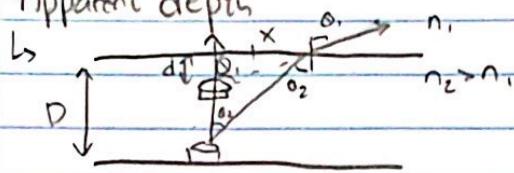
↳ $p < f \rightarrow RI, \text{ upright, large}$

↳ $p > f \rightarrow VI, \text{ inverted, any size}$



Physics 1C Lecture 21: Refraction

Apparent depth



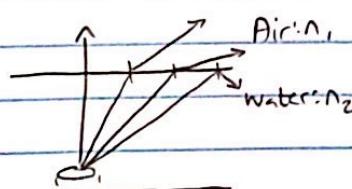
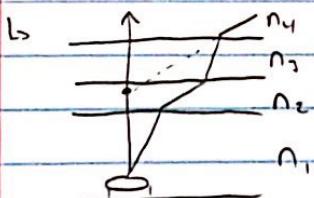
$$\hookrightarrow d \tan \theta_1 = D \tan \theta_2$$

$$d = D \frac{\tan \theta_2}{\tan \theta_1}$$

$$\text{Small Angles: } d = D \frac{\sin \theta_2}{\sin \theta_1}$$

$$d = D \frac{n_1}{n_2}$$

$$d = D \frac{1}{4/3} = \frac{3}{4}D$$

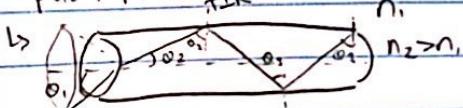


→ Total internal reflection

→ $\theta_{\text{critical}} \rightarrow$ leads to 90° refracted ray

$$\sin \theta_c = \frac{n_1}{n_2}$$

Optical Fibers



$$\theta_3 > \theta_c$$

$$\sin \theta_3 > \sin \theta_c$$

$$\sin \theta_2 > \frac{n_1}{n_2}$$

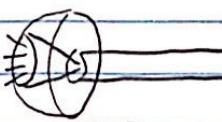
$$\cos \theta_2 > \frac{n_1}{n_2}$$

$$\sqrt{1 - \sin^2 \theta_2} > \frac{n_1}{n_2}$$

$$\sin^2 \theta_2 < 1 - \left(\frac{n_1}{n_2}\right)^2$$

$$\left(\frac{n_1}{n_2}\right)^2 \sin^2 \theta_1 < 1 - \left(\frac{n_1}{n_2}\right)^2$$

$$\sin \theta_1 < \sqrt{\left(\frac{n_1}{n_2}\right)^2 - 1}$$



$$\hookrightarrow E_{\text{eff}} = \frac{\int_L \cos \theta}{\int L (\pi/2)}$$

$$\hookrightarrow \int_{\rho \cos \theta}^{\infty} 2\pi R \sin \theta R d\theta = \partial A$$

$$\partial A = 2\pi R^2 \sin \theta d\theta$$

$$\partial L = \frac{\partial A}{R^2}$$

$$\Omega(\theta) = 2\pi \int_0^\theta \sin \theta d\theta$$

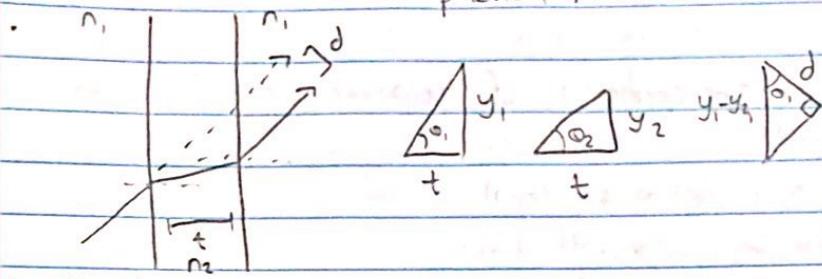
$$\Omega(0) = 2\pi (1 - \cos \theta)$$

$$\Omega(\pi/2) = 2\pi$$

$$E_{\text{eff}} = 1 - \cos \theta$$

$$E_{\text{eff}} = 1 - \sqrt{2 - \left(\frac{n_1}{n_2}\right)^2}$$

Physics 1C Lecture 21: Spherical Refractive Surfaces



$$\hookrightarrow d = (y_1 - y_2) \cos \theta_1$$

$$d = t \cos \theta_1, (\tan \theta_1, -\tan \theta_2)$$

$$d = t \sin \theta_1, [1 - \frac{\tan \theta_2}{\tan \theta_1}]$$

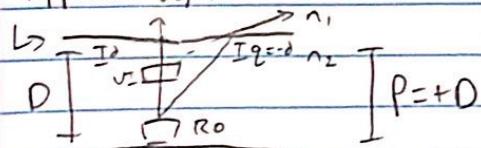
$$d = t \sin \theta_1, [1 - \frac{n_1 \cos \theta_1}{n_2 \cos \theta_2}]$$

$$\cos \theta_2 = \sqrt{1 - \sin^2 \theta_2} = \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2 \theta_1}$$

$$d = t \sin \theta_1, \left[1 - \frac{\cos \theta_1}{\sqrt{\left(\frac{n_1}{n_2}\right)^2 - \sin^2 \theta_1}} \right]$$

$$\hookrightarrow d \propto t$$

• Apparent depth



$$\hookrightarrow d = \frac{n_1}{n_2} D$$

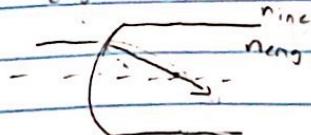
$$\frac{n_1}{d} = \frac{n_2}{P}$$

$$\frac{n_2}{D} - \frac{n_1}{d} = 0$$

$$\frac{n_{inc}}{P} + \frac{n_{eng}}{D} = \frac{n_{eng}}{d} \rightarrow \text{spherical refractive surfaces}$$

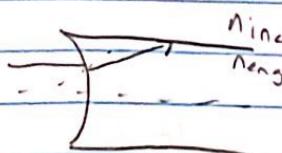
• SRS

$$\hookrightarrow n_{eng} > n_{inc}$$



Converging: $f = +f$

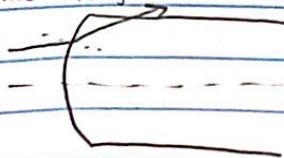
$$R = +R$$



Diverging: $f = -f$

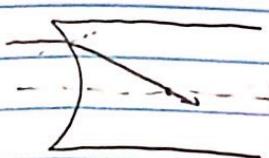
$$R = -R$$

$$\hookrightarrow n_{inc} > n_{eng}$$



Diverging: $f = -f$

$$\frac{n_{eng}}{f} = \frac{n_{inc} - n_{eng}}{R}$$



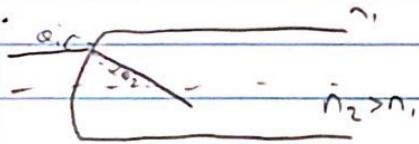
Converging: $f = +f$

↳ focusing power: RHS of optics equation

↳ mirror / thin lens $\rightarrow \frac{1}{f}$

↳ $\frac{n_{\text{eng}} - n_{\text{inc}}}{R} \rightarrow \text{SRS}$

↳ property of device and not in direction light comes from



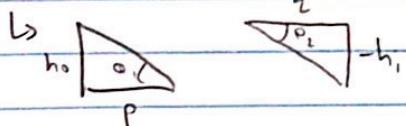
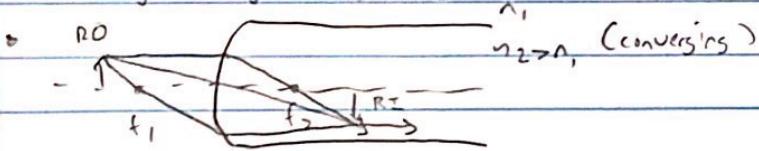
↳ $\frac{1}{f_2} = \frac{n_2 - n_1}{n_2} \left(\frac{1}{R} \right) \rightarrow \text{from left}$

↳ $\frac{1}{f_1} = \frac{n_1 - n_2}{n_1} \left(\frac{1}{-R} \right) \rightarrow \text{from right}$

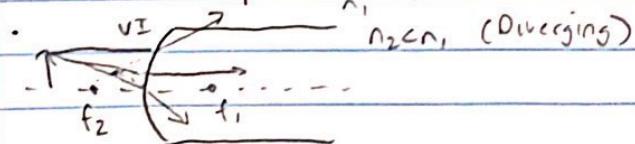
↳ $\frac{1}{f} = \frac{n_2 - n_1}{n_1} \left(\frac{1}{R} \right)$

↳ converging for left = converging from right

↳ $f_{\text{eng}} \propto n_{\text{eng}} \rightarrow \frac{f_1}{f_2} = \frac{n_1}{n_2}$



$$\hookrightarrow M = \frac{h_2}{h_1} = \frac{-q \tan \theta_2}{p \tan \theta_1} = \frac{-q \sin \theta_2}{p \sin \theta_1} = \frac{-q n_1}{p n_2} = \frac{-q / n_{\text{eng}}}{p / n_{\text{inc}}}$$

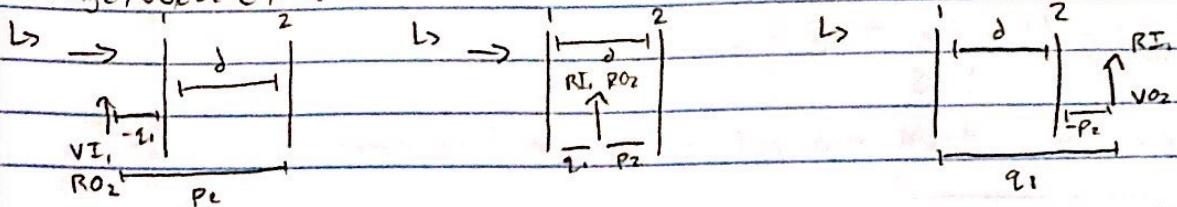


• Apparent Depth

$$\hookrightarrow M = \frac{-q / n_{\text{air}}}{p / n_{\text{water}}} = 1, q = -D \frac{n_{\text{air}}}{n_{\text{water}}}$$

Physics 1C Lecture 22: Thin Lenses

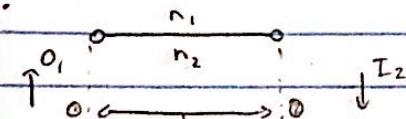
- Image/Object chains



$$\hookrightarrow d = p_2 + q_1$$

$$\hookrightarrow d = p_2 + q_1$$

$$\hookrightarrow d = p_2 + q_1$$



$$\begin{aligned} \hookrightarrow \frac{n_1}{p_1} + \frac{n_2}{q_1} &= \frac{n_2 - n_1}{R_1} \\ + \frac{n_2}{p_2} + \frac{n_1}{q_1} &= \frac{n_1 - n_2}{R_2} \end{aligned}$$

$$\frac{n_1}{p_1} + \frac{n_1}{q_1} + n_2 \left(\frac{1}{q_1} + \frac{1}{p_2} \right) = (n_2 - n_1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

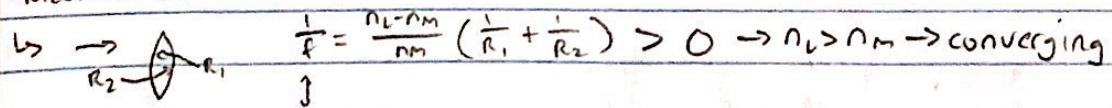
$$\hookrightarrow p_2 = d - q_1$$

$\hookrightarrow d \gg o \rightarrow$ thin lens

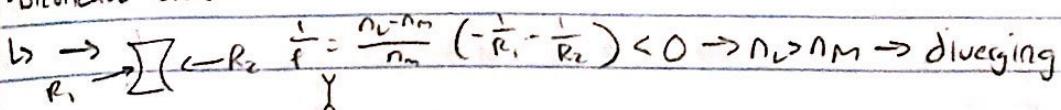
$$\hookrightarrow \frac{1}{p_1} + \frac{1}{q_2} = \frac{n_2 - n_1}{n_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\hookrightarrow \frac{1}{p} + \frac{1}{q} = \frac{n_{\text{out}} - n_{\text{in}}}{n_{\text{in}}} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f}$$

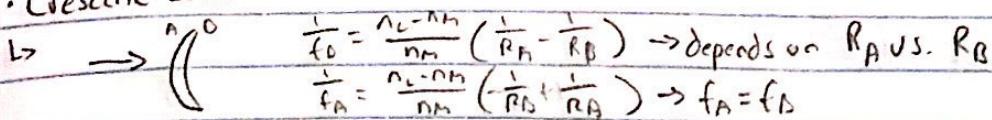
- Biconvex Lens



- Biconcave Lens



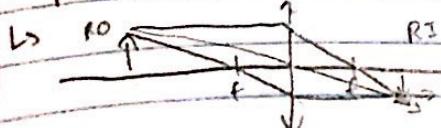
- Crescent Lens



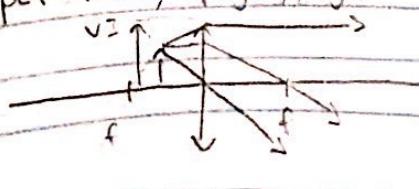
- Converging Lens, RO: $f = f$, $p = p$

$$\hookrightarrow \frac{1}{p} + \frac{1}{q} = \frac{1}{f}, M = -\frac{q}{p} \rightarrow q = \frac{|p| |M|}{|f|}, M = -\frac{q}{p-f}$$

$\hookrightarrow p > f \rightarrow$ RI, inverted, any-size



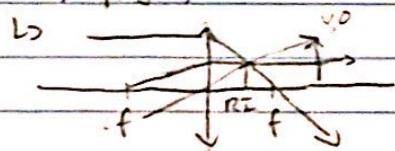
$\hookrightarrow p < f \rightarrow$ VI, upright, large



• Converging Lens, VO : $f = f$, $p = -p$

$$\hookrightarrow \frac{1}{p} + \frac{1}{q} = \frac{1}{f}, M = +\frac{q}{p} \rightarrow q = \frac{fp}{p+f}, M = \frac{f}{p+f}$$

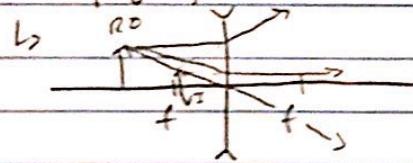
↳ RI, upright, small



• Diverging Lens, RO : $f = -f$, $p = p$

$$\hookrightarrow \frac{1}{p} + \frac{1}{q} = \frac{1}{f}, M = -\frac{q}{p} \rightarrow q = \frac{-pf}{p+f}, M = \frac{f}{p+f}$$

↳ VI, upright, small

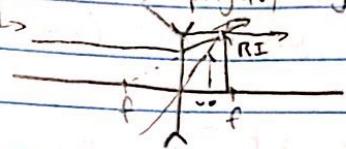


Physics 1C Lecture 24: Chained Devices

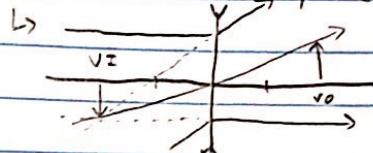
- Diverging lens, $V_0: f = -f, P = -P$

$\hookrightarrow \frac{1}{-P} + \frac{1}{q} = \frac{1}{f}, n = \frac{2}{P} \rightarrow q = \frac{Pf}{f-P}$, $M = \frac{f}{f-P}$

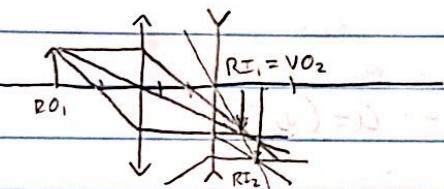
$\hookrightarrow p < f$, upright, large



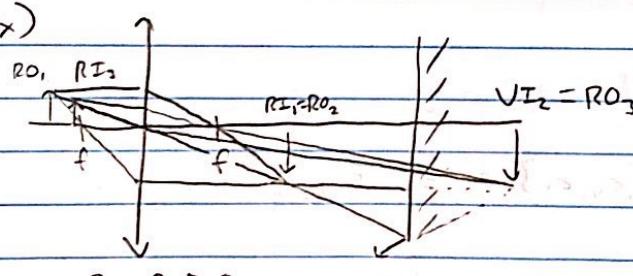
- $p > f$: V_I , inverted, any-size



Ex)



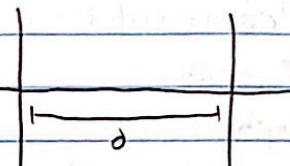
Ex)



$$p_1 > 0 \quad p_2 > 0 \quad p_3 > 0$$

$$z_1 > 0 \quad z_2 < 0 \quad z_3 > 0$$

Ex)



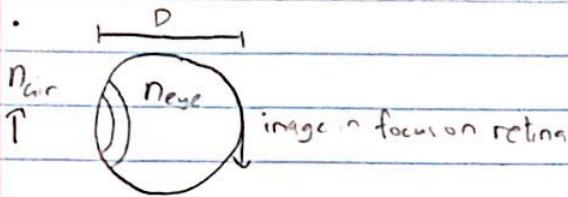
$$\frac{1}{P_1} + \frac{1}{z_1} = \frac{1}{f_1} \rightarrow \lim_{d \rightarrow 0} \rightarrow \text{adjacent lenses}$$

$$\frac{1}{P_2} + \frac{1}{z_2} = \frac{1}{f_2}$$

$$\hookrightarrow \frac{1}{P} + \frac{1}{z} = \frac{1}{f_1} + \frac{1}{f_2} \rightarrow \text{optical power is additive for adjacent thin lenses}$$

$$\hookrightarrow \frac{1}{P_{\text{eff}}} = \frac{1}{f_1} + \frac{1}{f_2} + \dots + \frac{1}{f_n}$$

Physics 1C Lecture 25: Interference



↳ Cornea \rightarrow fixed power, Truelens \rightarrow variable power, fine-tuning

$$\hookrightarrow \frac{n_{air}}{P} + \frac{n_{eye}}{D} = P_c + P_{TL}(P)$$

↳ P and $P_{TL}(P)$ only non-constants

↳ limits:

$$\hookrightarrow \frac{n_{air}}{P_{np}} + \frac{n_{eye}}{D} = P_c + P_{TL}(P_{np})$$

$$\hookrightarrow \frac{n_{air}}{\infty} + \frac{n_{eye}}{D} = P_c + P_{TL}(\infty)$$

$$\Delta\Phi = P_{TL}(P_{np}) - P_{TL}(\infty) = \frac{n_{air}}{P_{np}} \rightarrow \text{Accommodation}$$

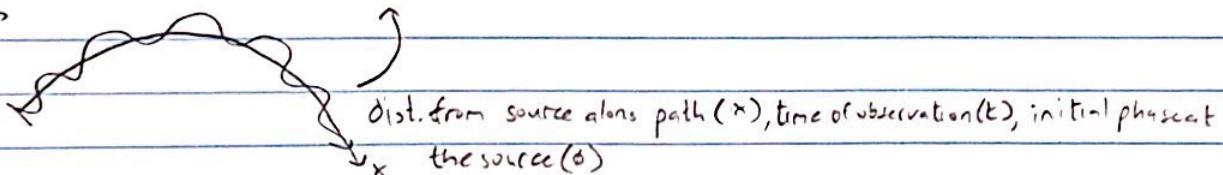
↳ Healthy: $\Delta\Phi = 12.5 \text{ diopters}$, $P_{np} = 8 \text{ cm}$

↳ Old: $\Delta\Phi = 2 \text{ diopters}$, $P_{np} = 50 \text{ cm}$

• Interference \rightarrow describes the effects of superposition

$$\hookrightarrow y(x, t) = A \sin(kx - \omega t + \phi)$$

↳



$$\hookrightarrow S_1 \xrightarrow{l_1} P \quad y_{1,p} = A \sin(kl_1 - \omega t + \phi_1)$$

$$\hookrightarrow S_2 \xrightarrow{l_2} P \quad y_{2,p} = A \sin(kl_2 - \omega t + \phi_2)$$

$$\hookrightarrow y_{rp} = 2A \cos\left(k \frac{l_2 - l_1}{2} + \frac{\phi_2 - \phi_1}{2}\right) \sin(kl_{avg} - \omega t + \phi_{avg})$$

$$A_{rp} = A_{rp} \left(\frac{\Delta\Phi_{bt}}{2} \right)$$

$$\hookrightarrow \phi_2 = kl_2 - \omega t + \phi_2$$

$$\phi_1 = kl_1 - \omega t + \phi_1$$

$$\Delta\Phi_{bt} = k(l_2 - l_1) + (\phi_2 - \phi_1)$$

$$\Delta\Phi_{tot} = \Delta\Phi_{path} + \Delta\Phi_{ic}$$

$$\hookrightarrow \Delta\Phi_{path} = \Delta(kL)$$

$$! \quad \Delta\Phi_{ic} = \phi_2 - \phi_1$$

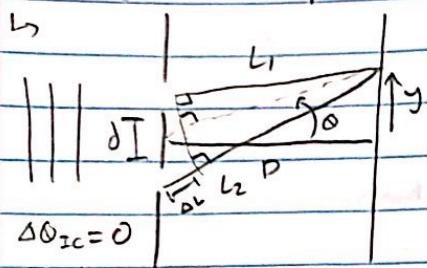
$$\hookrightarrow A_{rp} = 2A \cos\left(\frac{\Delta\Phi_{bt}}{2}\right)$$

$$\hookrightarrow \text{max. } \cos\left(\frac{\Delta\Phi_{bt}}{2}\right) = \pm 1 \rightarrow \Delta\Phi_{bt}/2 = n\pi \rightarrow \Delta\Phi_{bt} = 2n\pi \rightarrow \text{constructive}$$

$$\hookrightarrow \text{min. } \cos\left(\frac{\Delta\Phi_{bt}}{2}\right) = 0 \rightarrow \frac{\Delta\Phi_{bt}}{2} = (2n+1)\frac{\pi}{2} \rightarrow \Delta\Phi_{bt} = (2n+1)\pi \rightarrow \text{destructive}$$

Young's 2-Slit experiment

Young's 2-Slit experiments

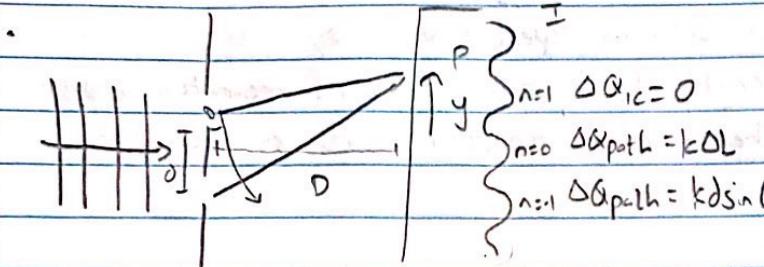


$$\Delta\phi_{Ic} = 0$$

$$\Delta\phi_{ph} = \Delta kL \rightarrow \Delta L = ds \sin \theta \rightarrow \Delta\phi_{ph} = kds \sin \theta = \frac{2\pi}{\lambda} ds \sin \theta$$

$$\hookrightarrow \text{Maxima: } \Delta\phi_{ph} = 2n\pi \rightarrow kds \sin \theta = 2n\pi \rightarrow \frac{2\pi}{\lambda} ds \sin \theta = 2n\pi \rightarrow ds \sin \theta = n\lambda$$

Physics 1C Lecture 26: Diffraction



↳ Max: $\Delta\theta_{bt} = 2n\pi \rightarrow$ inphase

$$kdsin\theta = 2n\pi$$

$$\frac{2\pi}{\lambda} dsin\theta = n2\pi$$

$$dsin\theta = n\lambda$$

$$\tan\theta = n\frac{\lambda}{D}$$

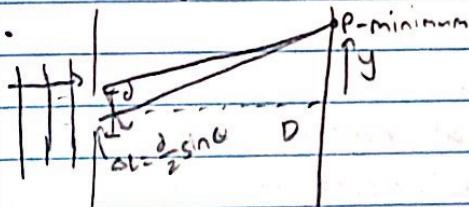
$$\frac{y}{D} = N\frac{\lambda}{D}$$

$$y_{max} = n\frac{\lambda D}{2} \rightarrow$$
 dist. between maxes

↳ slits closer = farther maxes

↳ smaller λ = closer maxes

↳ white light at central max, longer wavelengths farther from center



↳ min: $\Delta\theta_{bt} = (2n+1)\pi$

$$k\frac{d}{2}sin\theta = (2n+1)\pi \rightarrow$$
 denom. depends on slit separation

$$\frac{2\pi}{\lambda} \frac{d}{2}sin\theta = (2n+1)\pi$$

$$dsin\theta = (2n+1)\lambda$$

$$dsin\theta = m\lambda \rightarrow m$$
 is any integer except 0

↳ max: occurs between minima (approx. halfway)

↳ central max is 2x as large as others

↳ narrower slit = wider central max, spread out mins/maxes

- → resolvable (close)

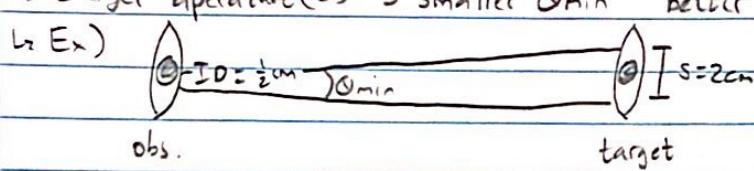


- → not resolvable (far)

- Rayleigh's Criterion - 2 objects are resolvable when the central max of one overlaps the 1st order min. of the other

Physics IC lecture 27: Reflection

- Rayleigh's Criterion - 2 objects are barely resolvable if the 1st order min from one overlaps with the central max on the other
 - ↳ for light traveling through a rectangular slit, the minimum angle of resolution $\theta_{\min} \approx \frac{\lambda}{d} \rightarrow d \text{ is width}$
 - ↳ for light passing through a circular aperture, $\theta_{\min} = 1.22 \frac{\lambda}{D}$
 - ↳ 1.22 comes from Bessel function
 - ↳ resolution worse when looking through circular aperture
 - ↳ Larger aperture (D) \rightarrow smaller $\theta_{\min} \rightarrow$ better resolution



$$\theta_{\min} = \frac{\lambda}{D} (1.22) = \frac{\lambda}{R}$$

$$R = \frac{s\lambda}{1.22\lambda} = 150 \text{ m}$$

- When light reflects off a medium of larger index, it flips ($\pm\pi$ in phase)

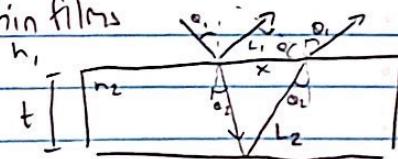
↳ $\Delta\phi_{\text{tot}} = \phi_{\text{int}} + \phi_{\text{path}} + \phi_{\text{ref}}$

↳ $\Delta\phi_{\text{ref}} = \begin{cases} 0-0 \rightarrow 0 \text{ flip} \\ 0-\pi \rightarrow 1 \text{ flip} \\ \pi-\pi \rightarrow 2 \text{ flips} \end{cases}$

↳ $k_f \frac{2\pi}{\lambda n}, f\lambda = \frac{c}{n} \rightarrow \lambda \text{ varies w/ medium, } k \text{ does too}$

↳ $f\lambda = \frac{c}{f} \rightarrow \frac{2\pi}{\lambda} = \frac{1}{n} \rightarrow \lambda_n = \frac{\lambda}{n}$

Thin films



↳ $L_1 = x \sin \theta_1$

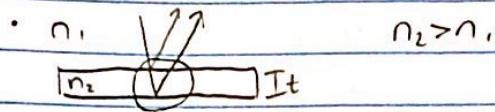
$x = 2t \tan \theta_2$

$L_2 = 2 \frac{t}{\cos \theta_2}$

$$\begin{aligned} \Delta\phi_{\text{path}} &= \Delta(kL) = k_2 L_2 - k_1 L_1 = \frac{2\pi}{\lambda} (n_2 L_2 - n_1 L_1) = \frac{2\pi}{\lambda} [n_2 \frac{2t}{\cos \theta_2} - n_1 2t \sin \theta_1 \tan \theta_2] \\ &= \frac{2\pi}{\lambda} \left(\frac{2tn_2}{\cos \theta_2} \right) [1 - \frac{n_1}{n_2} \sin \theta_1 \tan \theta_2] = \frac{2\pi}{\lambda} \left(\frac{2tn_2}{\cos \theta_2} \right) [1 - \sin^2 \theta_2] \\ &= \boxed{\frac{2\pi}{\lambda} (2tn_2) \cos \theta_2} \end{aligned}$$

↳ usually θ_1, θ_2 very small (≈ 0) $\rightarrow \Delta\phi_{\text{path}} \approx \frac{2\pi}{\lambda} (2tn_2)$

Physics 1C Lecture 28: Thin Films



$\Delta\phi$ path difference found here

$$\hookrightarrow \Delta\phi_{\text{path}} = k_2 2t = \frac{2\pi}{\lambda} n_2 (2t)$$

\hookrightarrow top ray inverts \rightarrow phase shift $+\pi$

\hookrightarrow bottom ray picks up no phase shift

\hookrightarrow Antireflective film \rightarrow leads to minima @ observer

$$\hookrightarrow \Delta\phi_{\text{tot}} = (2n+1)\pi$$

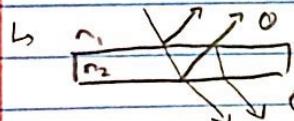
$$\Delta\phi_{\text{path}} + \Delta\phi_{\text{ref}} + \Delta\phi_{\text{int}} = (2n+1)\pi$$

\hookrightarrow When rays split, there is a guaranteed coherence of phase ($\Delta\phi_{\text{int}} = 0$)

$$\hookrightarrow \frac{2\pi}{\lambda} n_2 2t + 0 + \pi = (2n+1)\pi$$

$$\frac{2\pi}{\lambda} n_2 2t = 2n\pi$$

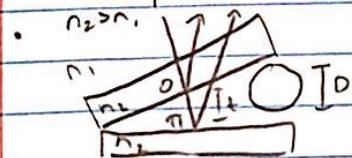
$t = n \left(\frac{\lambda}{2n_2} \right) \rightarrow$ half number of wavelengths in thin film \rightarrow destructive interference



$$\hookrightarrow \Delta\phi_{\text{tot}_1} = \frac{2\pi n_2}{\lambda} 2t + \pi$$

$$\hookrightarrow \Delta\phi_{\text{tot}_2} = \frac{2\pi n_2}{\lambda} 2t$$

\hookrightarrow top sees mins \rightarrow bottom sees maxes



$$\hookrightarrow \Delta\phi_{\text{tot}} = \frac{2\pi}{\lambda} 2t + \pi = (2n+1)\pi$$

$t = n \frac{\lambda}{2} \rightarrow$ dark fringes

$$m = n_{\text{max}} + 1 \rightarrow n_{\text{max}} = M - 1$$

$$t_{\text{max}} = n_{\text{max}} \frac{\lambda}{2}$$

$$t_{\text{max}} < D < t(n_{\text{max}} + 1)$$

$$(M-1) \frac{\lambda}{2} < D < M \frac{\lambda}{2}$$

\hookrightarrow We know D within $\pm \frac{\lambda}{4}$

$$\hookrightarrow \text{Ex: } \lambda = 550 \text{ nm} \rightarrow D = 160 \text{ nm}$$

Physics 1C Lecture 29: Lumineferous Ether

- 1D: $\frac{\partial^2 F}{\partial x^2} = \frac{1}{v_x^2} \frac{\partial^2 F}{\partial t^2}$ ← v_x = wave speed in a medium
- 1C: $\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$
- $\nabla^2 \vec{B} = \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2}$

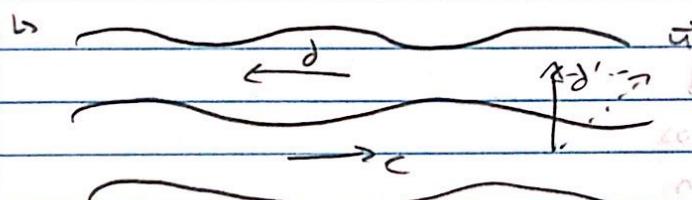
↳ c travels in the "lumineferous ether"

↳ must be everywhere → light is everywhere

↳ must be completely permeable → doesn't have anything interacting with it

↳ must be rigid → EM waves don't randomly refract

- Michelson and Morley



$$\Delta t_{\text{long}} = \frac{d}{c-u} + \frac{d}{c+u}$$

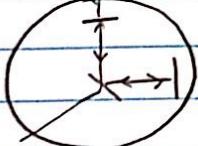
$$\Delta t_{\text{long}} = \frac{2cd}{c^2 - u^2}$$

$$\Delta t_{\text{trans}} = \frac{2d'}{c^2 - u^2}$$

↳ $\Delta t_{\text{long}} > \Delta t_{\text{trans}}$

$$\frac{\Delta t_{\text{long}}}{\Delta t_{\text{trans}}} = \frac{c}{\sqrt{c^2 - u^2}} = \frac{1}{\sqrt{1 - u^2/c^2}}$$

↳



→ look for interference

↳ $\Delta t_{\text{long}} > \Delta t_{\text{trans}} \rightarrow$ waves arrive out of phase

• Lorentz

↳ atoms = empty space

↳ LE pushes on the longitudinal arm, compressing it just enough to make the LE invisible

• Einstein

↳ Laws of physics are the same in all inertial frames of reference

↳ Speed of light in a vacuum is the same in all inertial frames

↳ Special relativity

• Transformations

$$\vec{r} = M(\phi) \vec{r}'$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$x = Ax' + By'$$

$$y = Cx' + Dy'$$



↳ Constraints:

$$1) \vec{r}'^2 = r^2$$

$$2) M(0) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$3) M(90^\circ) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\hookrightarrow \vec{r} \cdot \vec{r} = \vec{r}' \cdot \vec{r}'$$

$$(A^2 + C^2)x'^2 + (B^2 + D^2)y'^2 + (AD + CD)x'y' = x^2 + y^2$$

$$A^2 + C^2 = 1, B^2 + D^2 = 1, AD + CD = 0$$

$$\frac{A^2}{C^2} + 1 = \frac{1}{C^2}, 1 + \frac{D^2}{B^2} = \frac{1}{B^2}, AB = -CD \rightarrow \left(\frac{A}{C}\right)^2 = \left(\frac{D}{B}\right)^2$$

$$C = \pm B, A = \pm D$$

$$\hookrightarrow M = \begin{bmatrix} A & D \\ \pm B & \pm A \end{bmatrix} \rightarrow M = \begin{bmatrix} A & B \\ -B & A \end{bmatrix}$$

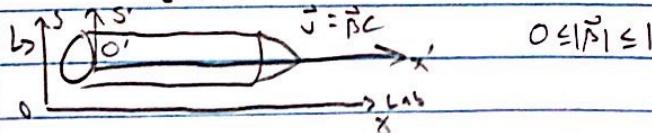
$$\hookrightarrow A(0) = 1, A(90^\circ) = 0 \rightarrow \cos$$

$$B(0) = 0, B(90^\circ) = 1 \rightarrow \sin$$

$$M = \begin{bmatrix} \cos \alpha & \sin \beta \\ -\sin \alpha & \cos \beta \end{bmatrix}$$

• Event 1:

$$\hookrightarrow \underline{r} = \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$



$$\hookrightarrow \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} l_{00} & l_{01} & l_{02} & l_{03} \\ \vdots & \vdots & \vdots & \vdots \\ l_{10} & l_{11} & l_{12} & l_{13} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix}$$

$$\hookrightarrow y = y', z = z' \rightarrow 0 = 0' @ t = t' = 0$$

$$\begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} l_{00} & l_{01} & 0 & 0 \\ l_{10} & l_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix}$$

Physics 1C Lecture 30: Transformations

$$\begin{bmatrix} ct \\ x \end{bmatrix} = \begin{bmatrix} l_{00} & l_{01} \\ l_{10} & l_{11} \end{bmatrix} \begin{bmatrix} ct' \\ x' \end{bmatrix}$$

↳ Classical (Galilean)

$$\hookrightarrow t = t'$$

$$\hookrightarrow at t = t' = 0 \rightarrow x = x'$$

↳ O' moves in S w/ \vec{v}

$$\hookrightarrow ct = l_{00}ct' + l_{01}x'$$

$$x = l_{10}ct' + l_{11}x'$$

$$\hookrightarrow t = t' \rightarrow l_{00} = 1, l_{01} = 0$$

$$\hookrightarrow x = x' \rightarrow l_{11} = 1$$

$$\hookrightarrow O' \text{ in rocket} \rightarrow \underline{\Sigma}' = \begin{bmatrix} ct' \\ x' \end{bmatrix}$$

$$O' \text{ in lab} \rightarrow \underline{\Sigma} = \begin{bmatrix} ct \\ x \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} ct \\ x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ l_{10} & 1 \end{bmatrix} \begin{bmatrix} ct' \\ x' \end{bmatrix}$$

$$ct = ct'$$

$$vt = l_{10}ct'$$

$$\hookrightarrow l_{10} = \frac{v}{c} = \beta$$

$$\hookrightarrow \begin{bmatrix} ct \\ vt \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \beta & 1 \end{bmatrix} \begin{bmatrix} ct' \\ x' \end{bmatrix}$$

↳ G → Galilean

↳ mouse moves thru rocket with velocity \vec{u}'

$$\hookrightarrow \text{rocket} = \underline{\Delta r}' = \begin{bmatrix} cot' \\ u'at' \end{bmatrix}$$

$$\hookrightarrow \text{lab: } \underline{\Delta r} = \begin{bmatrix} cot \\ uat \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} cot \\ uat \end{bmatrix} = G \begin{bmatrix} cot' \\ uat' \end{bmatrix}$$

$$\hookrightarrow cot = cot'$$

$$uat = \beta cot' + u'at' \rightarrow u = u + u'$$

$$\hookrightarrow V_{M,L} = V_{R,L} + V_{M,R} \rightarrow (\vec{V}_{A,C} = \vec{V}_{A,B} + \vec{V}_{B,C})$$

↳ flashlight in the rocket

$$\hookrightarrow \vec{V}_{0,L} = \vec{V}_{0,R} + \vec{V}_{R,L}$$

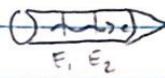
$c \neq ct + v \rightarrow$ Einstein's postulates not consistent w/ Galilean relativity

↳ redefine constraints

↳ light moves at c in all frames

↳ laws of physics are the same in all frames

$$\hookrightarrow \vec{v}$$



$$\hookrightarrow \underline{\Delta r} = \begin{bmatrix} \text{cat}' \\ \text{cat}' \end{bmatrix}$$

$$\underline{\Delta r} = \begin{bmatrix} \text{cat}' \\ \text{cat}' \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} \text{cat} \\ \text{cat} \end{bmatrix} = \begin{bmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{bmatrix} \begin{bmatrix} \text{cat}' \\ \text{cat}' \end{bmatrix}$$

$$\hookrightarrow \text{cat} = \text{cat}' (L_{00} + L_{01})$$

$$\text{cat} = \text{cat}' (L_{10} + L_{11})$$

$$L_{00} + L_{01} = L_{10} + L_{11}$$

\hookrightarrow now reverse the photon's direction

$$\hookrightarrow \underline{\Delta r'} = \begin{bmatrix} \text{cat}' \\ -\text{cat}' \end{bmatrix}$$

$$\hookrightarrow \underline{\Delta r} = \begin{bmatrix} \text{cat} \\ -\text{cat} \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} \text{cat} \\ -\text{cat} \end{bmatrix} = \begin{bmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{bmatrix} \begin{bmatrix} \text{cat}' \\ -\text{cat}' \end{bmatrix}$$

$$\hookrightarrow \text{cat} = \text{cat}' (L_{00} - L_{01})$$

$$-\text{cat} = \text{cat}' (L_{10} - L_{11})$$

$$L_{00} - L_{01} = -L_{10} + L_{11}$$

$$\hookrightarrow L_{00} = L_{11}, L_{01} = L_{10}$$

$$\hookrightarrow \begin{bmatrix} \text{cat} \\ \Delta x \end{bmatrix} = \begin{bmatrix} L_{00} & L_{01} \\ L_{01} & L_{00} \end{bmatrix} \begin{bmatrix} \text{cat}' \\ \Delta x' \end{bmatrix}$$

\hookrightarrow in S

$$\hookrightarrow \begin{bmatrix} \text{cat} \\ \Delta x \end{bmatrix} = \begin{bmatrix} L_{00} & L_{01} \\ L_{01} & L_{00} \end{bmatrix} \begin{bmatrix} \text{cat}' \\ 0 \end{bmatrix}$$

$$\hookrightarrow \text{cat} = L_{00} \text{cat}'$$

$$\hookrightarrow \Delta x = L_{01} \text{cat}'$$

$$\hookrightarrow \frac{\Delta x}{c} = \frac{L_{01}}{L_{00}} \rightarrow L_{01} = \beta L_{00}$$

$$\hookrightarrow \begin{bmatrix} \text{cat} \\ \Delta x \end{bmatrix} = L_{00} \begin{bmatrix} 1 & \beta \\ \beta & 1 \end{bmatrix} \begin{bmatrix} \text{cat}' \\ \Delta x' \end{bmatrix}$$

\hookrightarrow Laws of Physics

$$\hookrightarrow L = L(\rho) \underline{\Gamma} \rightarrow \underline{\Gamma}' = L(\rho) \underline{\Gamma}$$

$$\hookrightarrow \underline{\Gamma}' = L(-\rho) L(\rho) \underline{\Gamma} \rightarrow L(-\rho) L(\rho) = \underline{\Gamma}$$

$$\hookrightarrow L_{00}(\rho) = \frac{1}{\sqrt{1-\rho^2}} \equiv \delta$$

$$\hookrightarrow L_4 = \begin{bmatrix} \delta & \delta \rho & 0 & 0 \\ \delta \rho & \delta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Physics 1C Lecture 31: Space-Time

• $\beta = \frac{v}{c}, 0 \leq \beta \leq 1$

• $\gamma = \sqrt{\frac{1}{1-\beta^2}}, 1 \leq \gamma$

• $L_2 = \gamma \left(\begin{smallmatrix} 1 & \beta \\ \beta & 1 \end{smallmatrix} \right)$

$L_4 = \left[\begin{array}{cccc} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$

↳ β in top mixes space \rightarrow time, β in bottom mixes time \rightarrow space

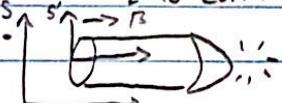
↳ $ct = \gamma(ct' + \beta x')$

$$x = \gamma(\beta ct' + x')$$

↳ if $\beta \ll 1 \rightarrow \gamma \approx 1$

↳ $ct = ct'$ \rightarrow results of G_2
 $x = \beta ct' + x'$

↳ G_2 is consistent in the limit of small speeds \rightarrow "real world"



↳ flashes every $\Delta t'$ seconds

↳ in rocket: $\Delta t' = \left[\begin{smallmatrix} c\Delta t' \\ 0 \end{smallmatrix} \right]$

in lab: $\Delta t = \left[\begin{smallmatrix} c\Delta t \\ \Delta x \end{smallmatrix} \right]$

↳ $\left[\begin{smallmatrix} c\Delta t \\ \Delta x \end{smallmatrix} \right] = \gamma \left[\begin{smallmatrix} 1 & \beta \\ \beta & 1 \end{smallmatrix} \right] \left[\begin{smallmatrix} c\Delta t' \\ 0 \end{smallmatrix} \right]$

↳ $c\Delta t = \gamma c\Delta t'$

$\Delta x = \gamma(\beta c\Delta t')$

$\Delta t = \gamma \Delta t'$ \leftarrow for every second in the rocket, γ seconds tick in the lab

↳ Ex) if $\gamma = 60 \rightarrow$ 1 yr in rocket = 60 yrs in lab

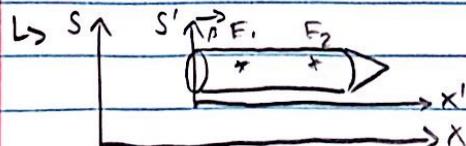
↳ "moving clocks run slow" \rightarrow time dilation

↳ works from both frames of reference \rightarrow meaning of time changed

↳ broken by acceleration \rightarrow position in rocket frame must be constant

↳ If the temporal interval between 2 events in the frame in which they're co-located ("rest frame") is $\Delta t'$, the temporal interval in which that rest frame moves w/ a speed β is given by $\gamma \Delta t'$.

• Simultaneous events



$$\hookrightarrow \Delta r' = \begin{bmatrix} 0 \\ \Delta x' \end{bmatrix}$$

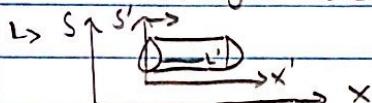
$$\Delta r = \begin{bmatrix} c\Delta t \\ \Delta x \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} c\Delta t \\ \Delta x \end{bmatrix} = \gamma \begin{bmatrix} 1 & \beta \\ \beta & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \Delta x' \end{bmatrix}$$

$c\Delta t = \gamma \beta \Delta x'$ ← not simultaneous in the lab (space → time)

$$\Delta x = \gamma \Delta x'$$

length measurement requires simultaneous measurement of front/end



$$\hookrightarrow \text{in lab: } \Delta r = \begin{bmatrix} 0 \\ L \end{bmatrix}$$

$$\text{in rocket: } \Delta r' = \begin{bmatrix} c\Delta t' \\ L' \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ L \end{bmatrix} = \gamma \begin{bmatrix} 1 & \beta \\ \beta & 1 \end{bmatrix} \begin{bmatrix} c\Delta t' \\ L' \end{bmatrix}$$

$$0 = \gamma(c\Delta t' + \beta L')$$

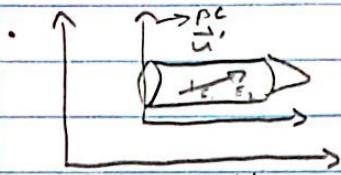
$$L = \gamma(\beta c\Delta t' + L')$$

$$L = \gamma(-\beta^2 + 1)L'$$

$L = \frac{L'}{\gamma}$ ← moving sticks are shortened along the direction of motion

- Time dilation → colocated events in rocket, $\Delta t = \gamma \Delta t'$
- Length contraction → simultaneous measurements in lab, $L = \frac{L'}{\gamma}$

Physics 1C Lecture 32: Relative Velocity



$$\Delta r' = \begin{bmatrix} c\Delta t \\ u'_x \Delta t \\ u'_y \Delta t \\ u'_z \Delta t \end{bmatrix}, \Delta r = \begin{bmatrix} c\Delta t \\ u_x \Delta t \\ u_y \Delta t \\ u_z \Delta t \end{bmatrix}$$

$$\begin{bmatrix} c \\ u_x \\ u_y \\ u_z \end{bmatrix} \Delta t = \begin{bmatrix} \gamma \gamma \beta^0 \\ \gamma \beta \gamma^0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} c \\ u'_x \\ u'_y \\ u'_z \end{bmatrix} \Delta t'$$

$$c\Delta t = \gamma(c + \beta u_x) \Delta t'$$

$$u_x \Delta t = \gamma(u'_x + \beta c) \Delta t'$$

$$u_y \Delta t = u'_y \Delta t'$$

$$u_z \Delta t = u'_z \Delta t'$$

$$\frac{u_x}{c} = \frac{\beta + u'_x/c}{1 + \beta u'_x/c}$$

$$\frac{u_z}{c} = \frac{u'_z}{\gamma(1 + \beta u'_x/c)} \rightarrow y \text{ and } z$$

Ex)

$$\textcircled{E} \quad \begin{array}{ccc} \textcircled{A} & \xrightarrow{\gamma_A=0.6} & \textcircled{B} \xrightarrow{\gamma_B=0.8} \end{array}$$

$$\frac{u_{11}}{c} = \frac{\beta + u_{11}/c}{1 + \beta u_{11}/c} = \frac{0.6 + 0.8}{1 + (0.6)(0.8)} = \frac{1.4}{1.48} = \boxed{0.95}$$

$$\underline{u} \cdot \underline{u} = \underline{u}' \cdot \underline{u}'$$

$$\underline{u} = \begin{bmatrix} u_t \\ u_x \\ u_y \\ u_z \end{bmatrix} \rightarrow \underline{u} \cdot \underline{u} = u_t^2 - (u_x^2 + u_y^2 + u_z^2)$$

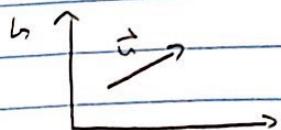
suggests space-time is saddle shaped

$$\underline{\Delta r} \cdot \underline{\Delta r} = \underline{\Delta r}' \cdot \underline{\Delta r}'$$

$$\underline{\Delta r} \cdot \underline{\Delta r} = (c\Delta t)^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2) = (c\Delta t')^2 - (\Delta x'^2 + \Delta y'^2 + \Delta z'^2)$$

$$\rightarrow c\Delta T^2 \rightarrow \text{proper time}$$

elapsed time between the events in the frame in which they are collocated



$$c^2 \Delta t^2 (1 - \frac{u^2}{c^2})$$

$$\Delta T = \Delta t \sqrt{1 - \frac{u^2}{c^2}} = \frac{\Delta t}{\gamma_u}$$

$$\underline{u} = \frac{\Delta r}{\Delta T} = \frac{\Delta r}{\Delta t} \frac{\Delta t}{\Delta T} = \begin{bmatrix} c \\ u_x \\ u_y \\ u_z \end{bmatrix} \gamma_u$$

Physics 1C Lecture 33: Relative Dynamics

$$\cdot \underline{u} = \gamma_u \left[\begin{smallmatrix} \underline{v} \\ \underline{v}_2 \end{smallmatrix} \right] = \gamma_u \left[\begin{smallmatrix} \underline{v} \\ \underline{u} \end{smallmatrix} \right]$$

$$\hookrightarrow \gamma_u = \frac{1}{\sqrt{1 - u^2/c^2}}$$

$$\hookrightarrow \underline{u} \cdot \underline{u} = \underline{u}' \cdot \underline{u}' = \left[\begin{smallmatrix} \underline{v} \\ \underline{u} \end{smallmatrix} \right] \cdot \left[\begin{smallmatrix} \underline{v} \\ \underline{u} \end{smallmatrix} \right] = c^2 \rightarrow \text{rest}$$

\hookrightarrow photon: $\left[\begin{smallmatrix} \underline{v} \\ \underline{c} \end{smallmatrix} \right] \cdot \left[\begin{smallmatrix} \underline{v} \\ \underline{c} \end{smallmatrix} \right] = 0 \rightarrow$ particle (nonmass)

$$\hookrightarrow \underline{P} = m\underline{v} = \gamma_u \left[\begin{smallmatrix} m\underline{v} \\ m\underline{u} \end{smallmatrix} \right] = \left[\begin{smallmatrix} E/c \\ \underline{p} \end{smallmatrix} \right]$$

$$\hookrightarrow \vec{p} = \frac{m\vec{u}}{\sqrt{1 - u^2/c^2}}$$

$$\hookrightarrow \underline{P} \cdot \underline{P} = \left(\frac{E}{c} \right)^2 - \vec{p} \cdot \vec{p} = \left(\frac{E}{c} \right)^2 - |\vec{p}|^2 = m^2 c^2$$

$$\cdot \quad \textcircled{1} \xrightarrow{m_1} \textcircled{2} \xrightarrow{m_2} \textcircled{3} \xrightarrow{m_3}$$

$$\hookrightarrow \underline{P}_1 + \underline{P}_2 = \underline{P}_3$$

$$\underline{P}_3 \cdot \underline{P}_3 = (\underline{P}_1 + \underline{P}_2) \cdot (\underline{P}_1 + \underline{P}_2)$$

$$\underline{P}_3 \cdot \underline{P}_3 = \underline{P}_1 \cdot \underline{P}_1 + \underline{P}_2 \cdot \underline{P}_2 + 2\underline{P}_1 \cdot \underline{P}_2$$

$$m_3^2 c^2 = m_1^2 c^2 + m_2^2 c^2 + 2 \left(\frac{E_1}{c} \frac{E_2}{c} - \vec{p}_1 \cdot \vec{p}_2 \right) \rightarrow E_2 = m_2 c^2, \vec{p}_2 = 0$$

$$m_3^2 c^2 = m_1^2 c^2 + m_2^2 c^2 + 2 E_1 m_2 \rightarrow E_1 = m_1 c^2 + K_1$$

$$m_3^2 c^2 = m_1^2 c^2 + m_2^2 c^2 + 2 m_2 (m_1 c^2 + K_1)$$

$$m_3^2 = (m_1^2 + m_2^2 + 2 m_1 K_1) / c^2$$

$$m_3 = \sqrt{(m_1 + m_2)^2 + \frac{2 m_1 K_1}{c^2}} \rightarrow \text{energy convert to mass}$$

$$\cdot \quad \textcircled{1} \xrightarrow{v_1} \textcircled{2} \xleftarrow{v_2} \textcircled{3}$$

$$\hookrightarrow \underline{P}_1 + \underline{P}_2 = \underline{P}_3$$

$$\underline{P}_1^2 + \underline{P}_2^2 + 2\underline{P}_1 \cdot \underline{P}_2 = \underline{P}_3^2 \leftarrow m_1 = m_2 = 0$$

$$0 + 0 + 2 \left(\frac{E_1}{c} \frac{E_2}{c} - |\vec{p}_1| |\vec{p}_2| \cos 180^\circ \right) = m_3^2 c^2$$

$$2 \frac{E_1}{c} \frac{E_2}{c} (1+1) = m_3^2 c^2$$

$$m_3^2 = \frac{4 E_1 E_2}{c^4}, \quad m_3 = \frac{2 \sqrt{E_1 E_2}}{c^2}$$

$$\cdot \quad \textcircled{1} \xrightarrow{m_0} \textcircled{2} \xrightarrow{m_1} \textcircled{3} \quad E_1 = ?$$

$$\hookrightarrow \underline{P}_3 = \underline{P}_1 + \underline{P}_2$$

$$\underline{P}_2 = \underline{P}_0 - \underline{P}_1$$

$$\underline{P}_2^2 = \underline{P}_0^2 + \underline{P}_1^2 - 2 \underline{P}_0 \cdot \underline{P}_1$$

$$m_2^2 c^2 = m_0^2 c^2 + m_1^2 c^2 - 2 \left(\frac{E_0 E_1}{c^2} - \vec{p}_0 \cdot \vec{p}_1 \right) \rightarrow \vec{p}_0 = 0, \quad E_0 = m_0 c^2$$

$$m_2^2 c^2 = m_0^2 c^2 + m_1^2 c^2 - 2 m_0 E_1$$

$$E_1 = \frac{(m_0^2 + m_1^2 - m_2^2) c^2}{2 m_0}$$

• $O^{n_1} \rightarrow O^{n_2} \vdash \overset{O^{n_1}}{O_0} \xrightarrow{m_3} \rightarrow \min E$, to produce m_3 ?

$$\hookrightarrow (\underline{P_{1f}} + \underline{P_{2f}})^2 = (\underline{P_{1f}} + \underline{P_{2f}} + \underline{P_{3f}})^2 = (\underline{P_{1f}'} + \underline{P_{2f}'} + \underline{P_{3f}'})_{cm}$$

$$\hookrightarrow \sum \vec{p}_i = M_{tot} \vec{v}_{cm} \rightarrow \vec{v}_{cm, cm} = 0$$

\hookrightarrow cm: $\sum \vec{p}_i = 0 \rightarrow$ smallest energy means no particles are moving in the end in cm frame

$$\hookrightarrow \underline{P_{1i}}^2 + \underline{P_{2i}}^2 + 2\underline{P_{1i}} \cdot \underline{P_{2i}} = \underline{P_{1f}'}^2 + \underline{P_{2f}'}^2 + \underline{P_{3f}'}^2 + 2(\underline{P_{1f}} \cdot \underline{P_{1f}'} + \underline{P_{2f}} \cdot \underline{P_{2f}'} + \underline{P_{3f}} \cdot \underline{P_{3f}'})$$

$$\hookrightarrow \underline{P_{1i}}^2 = \underline{P_{1f}'}^2, \underline{P_{2i}}^2 = \underline{P_{2f}'}^2$$

$$2\left(\frac{\epsilon_1}{c} \frac{m_2 c^2}{c} - \vec{p}_i \cdot \vec{0}\right) = m_3^2 c^2 + 2(m_1 m_2 c^2 + m_1 m_3 c^2 + m_2 m_3 c^2)$$

$$E_i = \frac{1}{2m_2} (m_3^2 + 2m_1 m_2 + 2m_1 m_3 + 2m_2 m_3) c^2$$

Physics 1C Lecture 34: Compton Scattering and Doppler Effect

• Compton scattering

$$\hookrightarrow \xrightarrow{\gamma_i} e^- : \xrightarrow{e^-} \gamma_f \rightarrow \gamma_i \neq \gamma_f$$

$$\hookrightarrow p_{\gamma_i} + p_{e^-} = p_{\gamma_f} + p_{e^-}$$

$$(p_{ef})^2 = (p_{\gamma_i} + p_{e^-} - p_{\gamma_f})^2$$

$$p_{ef}^2 = p_{\gamma_i}^2 + p_{e^-}^2 + p_{\gamma_f}^2 + 2(p_{\gamma_i} p_{e^-} - p_{\gamma_i} p_{\gamma_f} - p_{e^-} p_{\gamma_f})$$

$$m^2 c^2 = 0 + m^2 c^2 + 0 + 2 \left(\frac{E_{\gamma_i} m c^2}{c} - \frac{E_{\gamma_f} E_{\gamma_f}}{c^2} - \frac{E_{\gamma_f}^2}{c^2} - \vec{p}_{\gamma_i} \cdot \vec{p}_{e^-} + \vec{p}_{\gamma_i} \cdot \vec{p}_{\gamma_f} + \vec{p}_{e^-} \cdot \vec{p}_{\gamma_f} \right)$$

$$0 = m E_{\gamma_i} - \frac{E_{\gamma_i} E_{\gamma_f}}{c^2} - m E_{\gamma_f} + |\vec{p}_{\gamma_i}| |\vec{p}_{\gamma_f}| \cos \theta \rightarrow p_{\gamma} = \frac{E_{\gamma}}{c}$$

$$(E_{\gamma_i} - E_{\gamma_f}) m c^2 = E_{\gamma_i} E_{\gamma_f} (1 - \cos \theta)$$

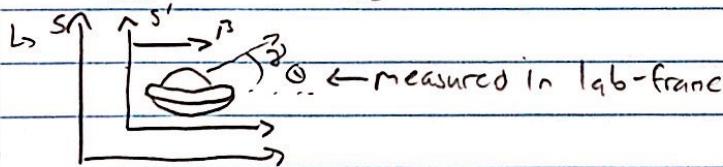
$$\left(\frac{1}{E_{\gamma_f}} - \frac{1}{E_{\gamma_i}} \right) m c^2 = 1 - \cos \theta \rightarrow \text{relates energy}$$

$$\hookrightarrow E = \frac{hc}{\lambda}$$

$$\boxed{\lambda_f - \lambda_i = \frac{h}{mc} (1 - \cos \theta)}$$

• Doppler Effect

$$\hookrightarrow \frac{f_{obs}}{f_{src}} = \frac{v_{src} - v_{obs}}{v_{src} + v_{obs}} \rightarrow \text{derived for a medium at rest}$$



$\hookrightarrow (\underline{u}) \sim 4\text{velocity of UFO}, (\underline{p}) \sim 4\text{momentum of photon}$

$$\hookrightarrow \text{Lab: } \underline{u} = \gamma \begin{bmatrix} \frac{c}{\gamma} \\ 0 \end{bmatrix}$$

$$\underline{p} = \frac{E}{c} \begin{bmatrix} \cos \theta \\ \sin \theta \\ 0 \end{bmatrix}$$

$$\hookrightarrow \text{UFO: } \underline{u}' = \gamma \begin{bmatrix} \frac{c}{\gamma} \\ 0 \end{bmatrix}$$

$$\underline{p}' = \frac{E'}{c} \begin{bmatrix} \cos \theta' \\ \sin \theta' \\ 0 \end{bmatrix}$$

$$\hookrightarrow \underline{u} \cdot \underline{p} = \underline{u}' \cdot \underline{p}'$$

$$\gamma E (1 - \beta \cos \theta) = E' \rightarrow E = h\nu$$

$$\gamma h\nu (1 - \beta \cos \theta) = h\nu'$$

$$\frac{\nu}{\nu'} = \frac{\gamma (1 - \beta \cos \theta)}{1 - \beta \cos \theta'} \rightarrow \frac{f_{obs}}{f_{emit}}$$

$$\boxed{\frac{\nu}{\nu'} = \frac{\sqrt{1 - \beta^2/c^2}}{1 - \beta^2/c^2 \cos \theta}}$$