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Disc. 1D

Math 33B HW#6

Chapter 4.5

4) $y''+3y'-18y=18e^{2t}$

$\lambda^2+3\lambda-18=0$

$(\lambda+6)(\lambda-3)=0$

$\lambda=-6,3$

$y(t)=Ae^{2t}$

$y'(t)=2Ae^{2t}$

$y''(t)=4Ae^{2t}$

$4Ae^{2t}+6Ae^{2t}-18Ae^{2t}=18e^{2t}$

$-8Ae^{2t}=18e^{2t}$

$A=-9/4$

$\boxed{y_p=-\frac{9}{4}e^{2t}}$

8) $y''+7y'+10y=-4\sin 3t$

$y(t)=a\cos 3t+b\sin 3t$

$y'(t)=-3a\sin 3t+3b\cos 3t$

$y''(t)=-9a\cos 3t-9b\sin 3t$

$-9(a\cos 3t+b\sin 3t)-21(a\sin 3t-b\cos 3t)$

$+10(a\cos 3t+b\sin 3t)=-4\sin 3t$

$-9a+21b+10a=0$

$a+21b=0$

$-9b-21a+10b=-4$

$-21a+b=4$

$a=\frac{42}{221},\ b=\frac{-2}{221}$

$\boxed{y_p=\frac{42}{221}\cos 3t-\frac{2}{221}\sin 3t}$

14) $y''+5y'+4y=2+3t$

$y(t)=at+b$

$y'(t)=a$

$y''(t)=0$

$5a+4at+4b=2+3t$

$5a+4b=2$

$4at=3t$

$a=\frac{3}{4},\ b=-\frac{7}{16}$

$\boxed{y_p=\frac{3}{4}a-\frac{7}{16}}$

20) $y''+2y'+2y=2\cos 2t,\ y(0)=-2,\ y'(0)=0$

$\lambda^2+2\lambda+2=0$

$\lambda=\frac{-2\pm\sqrt{4-8}}{2}=-1\pm i$

$y_h=c_1e^{-t}\cos t+c_2e^{-t}\sin t$

$z(t)=Ae^{i2t}$

$z'(t)=2i(Ae^{i2t})$

$z''(t)=-4(Ae^{i2t})$

$-4Ae^{i2t}+4i(Ae^{i2t})+2Ae^{i2t}=2e^{i2t}$

$-4A+4iA+2A=2$

$-A+2iA=1$

$A(-1+2i)=1$

$A=\frac{1}{-1+2i}\cdot\frac{-1-2i}{-1-2i}$

$A=\frac{-1-2i}{5}$

$A=-\frac{1}{5}-\frac{2}{5}i$

$z(t)=(-\frac{1}{5}-\frac{2}{5}i)e^{i2t}$

$y_p=-\frac{1}{5}\cos 2t+\frac{2}{5}\sin 2t$

$y=(c_1e^{-t}\cos t+c_2e^{-t}\sin t)-\frac{1}{5}\cos 2t+\frac{2}{5}\sin 2t$

$y'=e^{-t}(-c_1\sin t+c_2\cos t)-e^{-t}(c_1\cos t+c_2\sin t)$

$+\frac{2}{5}\sin 2t+\frac{4}{5}\cos 2t$

$0=c_2-c_1+\frac{4}{5}$

$-2=c_1-1/5$

$c_1=-9/5\quad c_2=-13/5$

$\boxed{y=(-\frac{9}{5}e^{-t}\cos t-\frac{13}{5}e^{-t}\sin t)-\frac{1}{5}\cos 2t+\frac{2}{5}\sin 2t}$

36) $y''+2y'+2y=3\cos t-\sin t$

$z(t)=Ae^{it}$

$z'(t)=iAe^{it}$

$z''(t)=-Ae^{it}$

$-Ae^{it}+2iAe^{it}+2Ae^{it}=e^{it}$

$A+2iA=1$

$A(1+2i)=1$

$A=\frac{1}{1+2i}\cdot\frac{1-2i}{1-2i}$

$A=\frac{1-2i}{5}$

$A=\frac{1}{5}-\frac{2}{5}i$

$z(t)=(\frac{1}{5}-\frac{2}{5}i)e^{it}$

$z(t)=(\frac{1}{5}-\frac{2}{5}i)(\cos t+i\sin t)$

$y_1''+2y_1'+2y_1=3\cos t$

$y_2''+2y_2'+2y_2=-\sin t$

$y_1(t)=\frac{1}{5}\cos t+\frac{2}{5}\sin t$

$y_2(t)=-\frac{2}{5}\cos t+\frac{1}{5}\sin t$

$y=3(\frac{1}{5}\cos t+\frac{2}{5}\sin t)-(-\frac{2}{5}\cos t+\frac{1}{5}\sin t)$

$\boxed{y=\cos t+\sin t}$

42) $y''+5y'+4y=te^{-t}$

$y_p(t)=(at-b)e^{-t}$

$(\lambda^2+5\lambda+4)=0$

$\lambda=-1,-4$

$y_h=C_1e^{-4t}+C_2e^{-t}$

$y_p(t)=t(at-b)e^{-t}=(at^2-bt)e^{-t}$

$y_p'(t)=(2at-b)e^{-t}+-(at^2-bt)e^{-t}$

$y_p''(t)=2ae^{-t}-(2at-b)e^{-t}+(b-2at)e^{-t}+(at^2-bt)e^{-t}$

$=2ae^{-t}-2ate^{-t}+be^{-t}+be^{-t}-2ate^{-t}+at^2e^{-t}-bte^{-t}$

$=e^{-t}(2a-2at+b+b-2at+at^2-bt)$

$=e^{-t}(at^2-4at-bt+2a+2b)$

$y_p'(t)=2ate^{-t}-be^{-t}-at^2e^{-t}+bte^{-t}$

$=e^{-t}(2at-b-at^2+bt)$

$(at^2-4at-bt+2a+2b)+5(2at-b-at^2+bt)+4(at^2-bt)=t$

$6at+2a+3b=t$

$6a=1,\ a=\frac{3}{2}b$

$a=\frac{1}{6},\ b=\frac{1}{9}$

$y=t(\frac{1}{6}t-\frac{1}{9})e^{-t}$

$\boxed{y=(\frac{1}{6}t^2-\frac{1}{9}t)e^{-t}}$

Chapter 4.6

4) $x''-2x'-3x=4e^{3t}$

$(\lambda-3)(\lambda+1)=0$

$\lambda=3,\lambda=-1$

$x_1(t)=e^{-t},\ x_2(t)=e^{3t}$

$x(t)=v_1x_1+v_2x_2$

$W(t)=3e^{2t}+e^{2t}=4e^{2t}$

$v_1=\int\frac{-e^{3t}(4e^{3t})}{4e^{2t}}=\int -e^{t}(e^{3t})dt$

$=\int -e^{4t}dt$

$=-\frac{1}{4}e^{4t}$

$v_2=\int\frac{e^{-t}(4e^{3t})}{4e^{2t}}=\int 1\,dt$

$v_2=t$

$\boxed{x(t)=-\frac{1}{4}e^{3t}+te^{3t}}$

5) $x''+x=\sec^2 t$

$(\lambda^2+1)=0$

$\lambda=\pm i$

$x_1=\cos t,\ x_2=\sin t$

$W(t)=\cos^2 t+\sin^2 t=1$

$v_2=\int\cos t(\sec^2 t)=\int\sec t$

$v_2=\ln|\sec t+\tan t|$

$v_1=\int-\sin t(\sec^2 t)=\int\frac{\sin t}{\cos^2 t}dt$

$u=\cos t,\ du=-\sin t$

$=-\int\frac{1}{u^2}du=\frac{1}{u}=-\sec t$

$-\sec t\cos t+\ln|\sec t+\tan t|\sin t$

$\boxed{-1+\ln|\sec t+\tan t|(\sin t)}$

12) $y''+y=\sec t+\cos t-1$

$(\lambda^2+1)=0$

$\lambda=\pm i$

$y_1=\cos t,\ y_2=\sin t$

$W(t)=1$

$v_1=\int -(\sin t)(\sec t+\cos t-1)\,dt$

$=-[\int \tan t+\int \sin t\cos t-\int \sin t]$

$=\ln|\cos t|+\int u\,du+\cos t$

$=\ln|\cos t|-\frac{1}{2}\cos^2 t+\cos t$

$v_2=\int \cos t(\sec t+\cos t-1)\,dt$

$=\int 1\,dt+\int \cos^2 t\,dt-\int \cos t\,dt$

$=\frac{3}{2}t+\frac{1}{2}\sin t\cos t-\sin t$

$y(t)=(\cos t)(\ln|\cos t|-\frac{1}{2}\cos^2 t+\cos t)+$

$(\sin t)(\frac{3}{2}t+\frac{1}{2}\sin t\cos t-\sin t)$

$\boxed{y(t)=\cos t\ln|\cos t|+\frac{3}{2}\sin t-1}$

14) $y_1(t)=t^{-1},\ y_2(t)=t^{-1}\ln t$

$t^2y''(t)+3ty'(t)+y(t)=0$

$y_1'(t)=-t^{-2}\quad y_2'(t)=t^{-2}-t^{-2}\ln t$

$y_1''(t)=2t^{-3}\quad y_2''(t)=-2t^{-3}+2t^{-3}\ln t-t^{-3}$

$t^2(2t^{-3})+3t(-t^{-2})+t^{-1}=0$

$2t^{-1}-3t^{-1}+t^{-1}=0$ ✓

$t^2(-2t^{-3}+2t^{-3}\ln t-t^{-3})+3t(t^{-2}-t^{-2}\ln t)+t^{-1}\ln t=0$

$(-2t^{-1}+2t^{-1}\ln t-t^{-1})+(3t^{-1}-3t^{-1}\ln t)+t^{-1}\ln t=0$ ✓

$y''(t)+3t^{-1}y'(t)+yt^{-2}=t^{-3}$

$y_1=t^{-1},\ y_2=t^{-1}\ln t$

$W(t)=t^{-1}(t^{-2}-t^{-2}\ln t)+t^{-2}(t^{-1}\ln t)$

$=t^{-3}-t^{-3}\ln t+t^{-3}\ln t$

$=t^{-3}$

$v_1=\int\frac{-t^{-1}\ln t\,(t^{-3})}{t^{-3}}=\int t^{-1}\ln t$

$u=\ln t,\ du=t^{-1}\rightarrow -\frac{1}{2}u^2$

$=-\frac{1}{2}(\ln t)^2$

$v_2=\int t^{-1}=\ln t$

$y_p=-\frac{1}{2}(\ln t)^2t^{-1}+(\ln t)(t^{-1}\ln t)$

$=\frac{1}{2}(\ln t)^2t^{-1}$

$\boxed{y=c_1t^{-1}+c_2t^{-1}\ln t+\frac{1}{2}t^{-1}(\ln t)^2}$