20S-PHYSICS 4AL-12 Unit 3 report

CHARLES XIAN ZHANG, CLAIRE CHUNG, Neil Vaishampayan, Brendan M Rossmango, RYAN ROSSMANGO

TOTAL POINTS

44 / 50

QUESTION 1

1 Name, Title, Abstract **6 / 6**

✓ **- 0 pts** **Correct**

QUESTION 2

2 Introduction **7 / 8**

✓ **+ 0 pts** **Correct**

+ 7 **Point adjustment**

QUESTION 3

3 Methods **11 / 12**

✓ **+ 0 pts** **Correct**

+ 11 **Point adjustment**

QUESTION 4

4 Results **14 / 18**

✓ **+ 0 pts** **Correct**

+ 14 **Point adjustment**

1 The small angle approximation will still yield mostly valid results for angles up to at least 45 degrees.

2 These aren't scaled correctly

QUESTION 5

5 Conclusion **6 / 6**

✓ **+ 0 pts** **Correct**

+ 6 **Point adjustment**

Energy and Behavior of Damped Oscillators

Claire Chung, Brendan Rossmango, Ryan Rossmango, Neil Vaishampayan, Charles Zhang
Physics 4AL, Lab 12, Group 2
May 25, 2020

Abstract

This experiment compared the damping of oscillations of various simple oscillations using Python and motion tracking software. The theoretical concept of damped oscillations is that such systems directly undergo energy loss over time. This relationship should be an exponential decrease, with a visual decrease in amplitude over time. Simple pendulums and cart motions were both analyzed to accurately portray this energy decay.

Moving carts on surfaces with varying elevations were also experimented with, supporting the hypothesis and suggesting that larger elevations lead to more initial energy, which then lead to a larger pool of energy that will eventually decay. Large-angle oscillations were also studied to determine deviations from simple harmonic oscillations, which largely depend on small-angle approximation. A pendulum with a large relative angle was used rather than an idealized pendulum with a small angle. The hypothesis for this case is that the larger angle will cause a greater rate of amplitude loss over time, no longer acting like an idealized simple undamped oscillator with relatively simple harmonic motion.

Introduction

Damped oscillations occur as energy is dissipated form an oscillating system by a damping force (Jain, 1997). This damping force results in a visual decrease in amplitude of the oscillation over time, and the position of such an oscillating system is theoretically described by

$$x = A_0 e^{-\frac{\gamma \cdot t}{2}} \cos(\omega t + \varphi)$$

where x is a function of time, A_0 describes the initial amplitude, and φ describes the initial phase shift (Triana & Fajardo, 2013). In these trials, this damping effect will be observed with regards to air resistance and friction, and the position of the oscillator will be used to calculate the total mechanical energy with respect to time.

The first set of trials involved the collection of data from a pendulum using the Tracker software to determine the position of the oscillator. The data was then fitted to the function described above, and the best-fit parameters were compared. From this fitted data, the kinetic energy per unit mass was then extrapolated and plotted.

The second set of trials involved the use of the Tracker software to analyze the motion of a cart over an air track across varying angles of elevation. This data was then used to determine and plot the kinetic energy, potential energy, and total energy in relation to time. Using this data, the amount of energy lost per oscillation was then determined. We hypothesised that an increase in angles of elevation would cause a faster energy decrease.

An additional set of trials was used to determine the effects of large-angle oscillations on the simple harmonic model. Since simple harmonic behavior is largely dependent on the small-angle approximation, these trials will track the position of the pendulum after it is released from an angle that does not fall within the bounds of the small-angle approximation. This data was then compared to the results of the first trial in order to distinguish between the special case of simple harmonic oscillation and regular oscillations.

Method

For the base pendulum experiment, we made the pendulum using a tennis ball and twine and tied it on a rod. We started the ball off with an initial displacement angle of less than 15°, began a recording, and then let the ball go to achieve simple harmonic motion. For the additional experiment, we used the same setup to test the effect of dropping the ball from a larger angle to analyze the difference between simple harmonic motion and non-simple harmonic motion.

1 Name, Title, Abstract **6 / 6**

✓ **- 0 pts** Correct

Energy and Behavior of Damped Oscillators

Claire Chung, Brendan Rossmango, Ryan Rossmango, Neil Vaishampayan, Charles Zhang
Physics 4AL, Lab 12, Group 2
May 25, 2020

Abstract

This experiment compared the damping of oscillations of various simple oscillations using Python and motion tracking software. The theoretical concept of damped oscillations is that such systems directly undergo energy loss over time. This relationship should be an exponential decrease, with a visual decrease in amplitude over time. Simple pendulums and cart motions were both analyzed to accurately portray this energy decay.

Moving carts on surfaces with varying elevations were also experimented with, supporting the hypothesis and suggesting that larger elevations lead to more initial energy, which then lead to a larger pool of energy that will eventually decay. Large-angle oscillations were also studied to determine deviations from simple harmonic oscillations, which largely depend on small-angle approximation. A pendulum with a large relative angle was used rather than an idealized pendulum with a small angle. The hypothesis for this case is that the larger angle will cause a greater rate of amplitude loss over time, no longer acting like an idealized simple undamped oscillator with relatively simple harmonic motion.

Introduction

Damped oscillations occur as energy is dissipated form an oscillating system by a damping force (Jain, 1997). This damping force results in a visual decrease in amplitude of the oscillation over time, and the position of such an oscillating system is theoretically described by

$$x = A_0 e^{-\frac{\gamma \cdot t}{2}} \cos(\omega t + \varphi)$$

where x is a function of time, A_0 describes the initial amplitude, and φ describes the initial phase shift (Triana & Fajardo, 2013). In these trials, this damping effect will be observed with regards to air resistance and friction, and the position of the oscillator will be used to calculate the total mechanical energy with respect to time.

The first set of trials involved the collection of data from a pendulum using the Tracker software to determine the position of the oscillator. The data was then fitted to the function described above, and the best-fit parameters were compared. From this fitted data, the kinetic energy per unit mass was then extrapolated and plotted.

The second set of trials involved the use of the Tracker software to analyze the motion of a cart over an air track across varying angles of elevation. This data was then used to determine and plot the kinetic energy, potential energy, and total energy in relation to time. Using this data, the amount of energy lost per oscillation was then determined. We hypothesised that an increase in angles of elevation would cause a faster energy decrease.

An additional set of trials was used to determine the effects of large-angle oscillations on the simple harmonic model. Since simple harmonic behavior is largely dependent on the small-angle approximation, these trials will track the position of the pendulum after it is released from an angle that does not fall within the bounds of the small-angle approximation. This data was then compared to the results of the first trial in order to distinguish between the special case of simple harmonic oscillation and regular oscillations.

Method

For the base pendulum experiment, we made the pendulum using a tennis ball and twine and tied it on a rod. We started the ball off with an initial displacement angle of less than 15°, began a recording, and then let the ball go to achieve simple harmonic motion. For the additional experiment, we used the same setup to test the effect of dropping the ball from a larger angle to analyze the difference between simple harmonic motion and non-simple harmonic motion.

2 Introduction 7 / 8

✓ + 0 pts Correct

+ 7 Point adjustment

Energy and Behavior of Damped Oscillators

Claire Chung, Brendan Rossmango, Ryan Rossmango, Neil Vaishampayan, Charles Zhang
Physics 4AL, Lab 12, Group 2
May 25, 2020

Abstract

This experiment compared the damping of oscillations of various simple oscillations using Python and motion tracking software. The theoretical concept of damped oscillations is that such systems directly undergo energy loss over time. This relationship should be an exponential decrease, with a visual decrease in amplitude over time. Simple pendulums and cart motions were both analyzed to accurately portray this energy decay.

Moving carts on surfaces with varying elevations were also experimented with, supporting the hypothesis and suggesting that larger elevations lead to more initial energy, which then lead to a larger pool of energy that will eventually decay. Large-angle oscillations were also studied to determine deviations from simple harmonic oscillations, which largely depend on small-angle approximation. A pendulum with a large relative angle was used rather than an idealized pendulum with a small angle. The hypothesis for this case is that the larger angle will cause a greater rate of amplitude loss over time, no longer acting like an idealized simple undamped oscillator with relatively simple harmonic motion.

Introduction

Damped oscillations occur as energy is dissipated form an oscillating system by a damping force (Jain, 1997). This damping force results in a visual decrease in amplitude of the oscillation over time, and the position of such an oscillating system is theoretically described by

$$x = A_0 e^{-\frac{\gamma \cdot t}{2}} \cos(\omega t + \varphi)$$

where x is a function of time, A_0 describes the initial amplitude, and φ describes the initial phase shift (Triana & Fajardo, 2013). In these trials, this damping effect will be observed with regards to air resistance and friction, and the position of the oscillator will be used to calculate the total mechanical energy with respect to time.

The first set of trials involved the collection of data from a pendulum using the Tracker software to determine the position of the oscillator. The data was then fitted to the function described above, and the best-fit parameters were compared. From this fitted data, the kinetic energy per unit mass was then extrapolated and plotted.

The second set of trials involved the use of the Tracker software to analyze the motion of a cart over an air track across varying angles of elevation. This data was then used to determine and plot the kinetic energy, potential energy, and total energy in relation to time. Using this data, the amount of energy lost per oscillation was then determined. We hypothesised that an increase in angles of elevation would cause a faster energy decrease.

An additional set of trials was used to determine the effects of large-angle oscillations on the simple harmonic model. Since simple harmonic behavior is largely dependent on the small-angle approximation, these trials will track the position of the pendulum after it is released from an angle that does not fall within the bounds of the small-angle approximation. This data was then compared to the results of the first trial in order to distinguish between the special case of simple harmonic oscillation and regular oscillations.

Method

For the base pendulum experiment, we made the pendulum using a tennis ball and twine and tied it on a rod. We started the ball off with an initial displacement angle of less than 15°, began a recording, and then let the ball go to achieve simple harmonic motion. For the additional experiment, we used the same setup to test the effect of dropping the ball from a larger angle to analyze the difference between simple harmonic motion and non-simple harmonic motion.

We used Tracker to collect the position data of the cart in the given videos. We then designed a Python script to automatically extract the collected data from the Tracker files, plotted the position and energy time series of the cart, and calculated the percent energy loss by bounce. Tracker was also employed to collect the angle data of the mass-string systems, and then through Python, best-fit plots and energy graphs were made.

Figure 1: The pendulum

Results

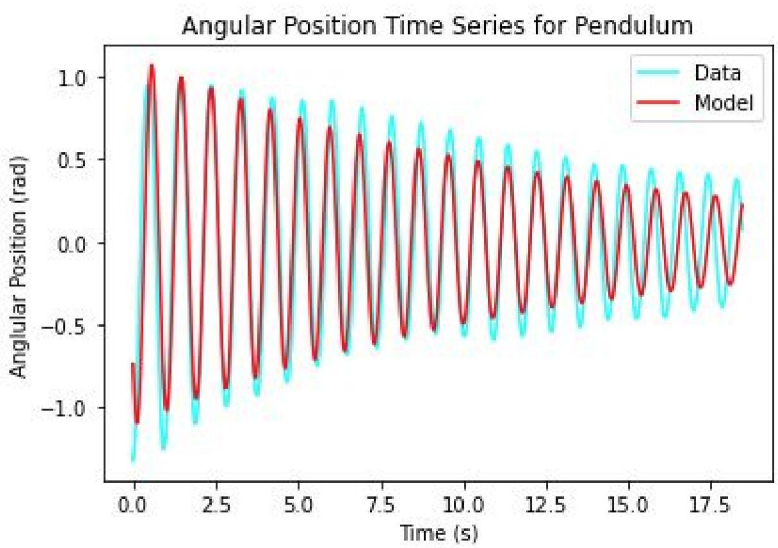

Figure 2: Angular Position (θ) vs. time (s) for Pendulum

Fit Parameter	**Value (Constants)**
Amplitude	1.11 rad
Omega	6.99 rad/s
Offset	0.00543 rad
Phi	-0.842 rad
Tau	12.51 sec

Table 1: Best Fit Curve Parameters

3 Methods 11 / 12

✓ + 0 pts Correct

+ 11 Point adjustment

We used Tracker to collect the position data of the cart in the given videos. We then designed a Python script to automatically extract the collected data from the Tracker files, plotted the position and energy time series of the cart, and calculated the percent energy loss by bounce. Tracker was also employed to collect the angle data of the mass-string systems, and then through Python, best-fit plots and energy graphs were made.

Figure 1: The pendulum

Results

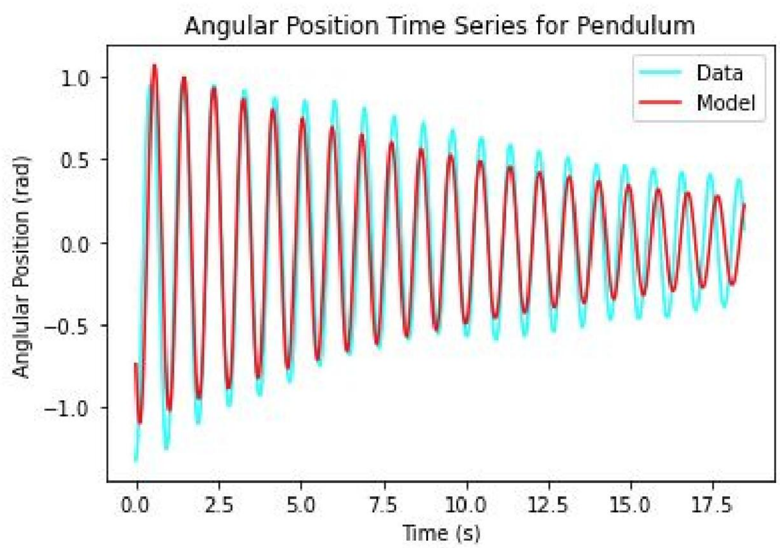

Figure 2: Angular Position (θ) vs. time (s) for Pendulum

Fit Parameter	**Value (Constants)**
Amplitude	1.11 rad
Omega	6.99 rad/s
Offset	0.00543 rad
Phi	-0.842 rad
Tau	12.51 sec

Table 1: Best Fit Curve Parameters

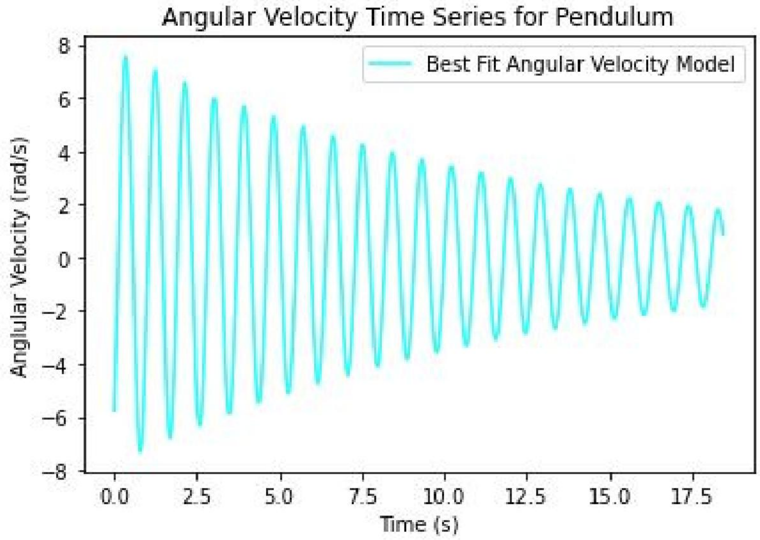

Figure 3: Angular Velocity (rad/s) vs. Time (s) for Pendulum

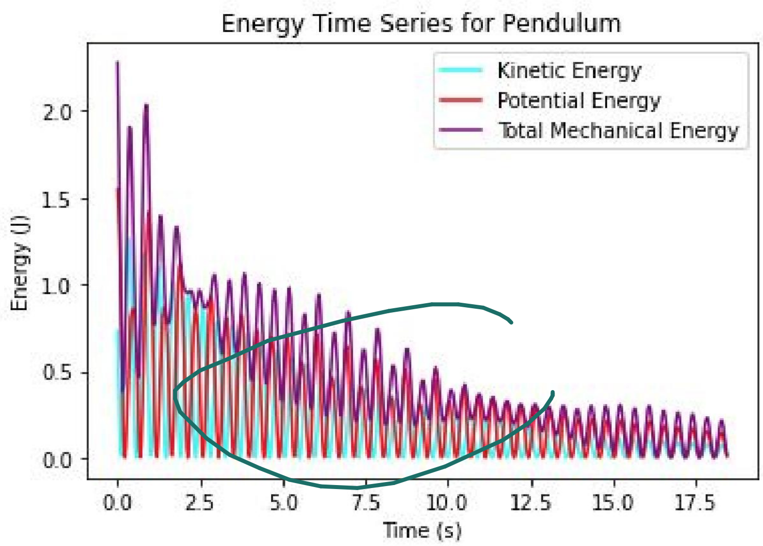

Figure 4: Mechanical Energy (J) vs. Time (s) for Pendulum

Further Discussion

The behavior of a damped oscillator was observed once again in the motion of a cart on an air track. The cart was determined to weigh 0.209 kg, and the oscillation was tracked using Tracker on 0.8º, 2.5º, and 3.8º slopes. The cart was released at a given amplitude and allowed to oscillate for approximately 30 seconds. The same exponential decay in energy was expected from these trials. Due to the variation of angle, it was also hypothesized that the steeper angles would result in a more rapid decline in energy. The goal of this experiment in particular was to gather data on the dissipation of energy relative to the number of bounces the cart experienced. The extrapolated data is displayed below.

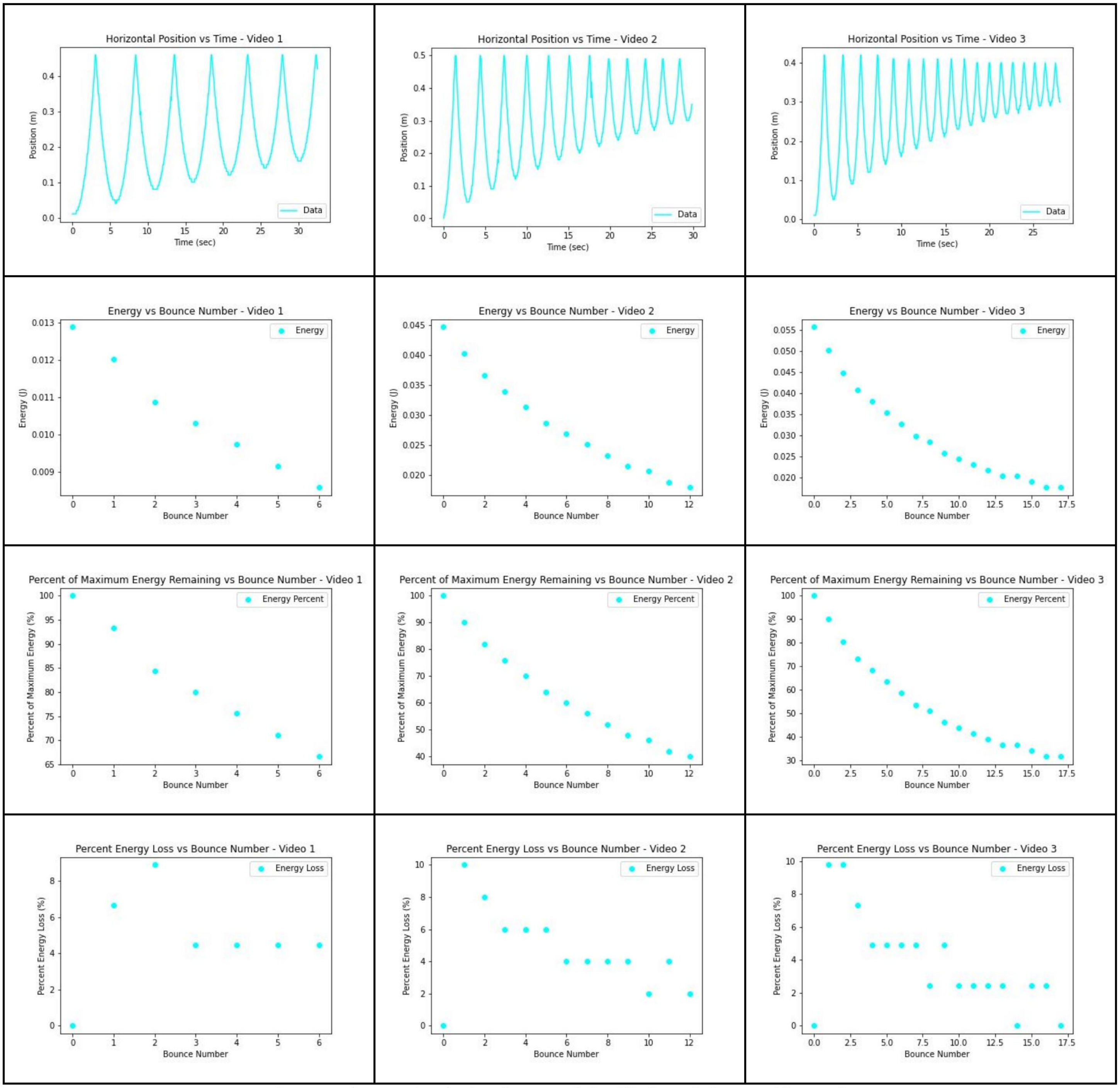

Figure 5a-l: Data Analysis of Damped Oscillation of Cart on Air Track

Our hypothesis that a steeper angle would cause a sharper decrease in energy was proven correct, as seen by the data above. All trials also exhibited an exponential decay of energy as predicted. Other trends between graphs were that the percent energy lost per bounce seemed to decrease as the total energy of the system decreased. This is consistent with our exponential decay model, and tells us that a larger momentum shift at the site of the bounce results in a greater energy loss. The exact data points recorded are listed below.

Bounce Number	Percent Energy Loss (%)
1	6.7
2	8.9
3	4.4
4	4.4
5	4.4
6	4.4

Table 2: Bounce Number vs. Percent Energy Loss for Video 1

Bounce Number	Percent Energy Loss (%)
1	10.0
2	8.0
3	6.0
4	6.0
5	6.0
6	4.0
7	4.0
8	4.0
9	4.0
10	2.0
11	4.0
12	2.0

Table 3: Bounce Number vs. Percent Energy Loss for Video 2

Bounce Number	Percent Energy Loss (%)
1	9.8
2	9.8
3	7.3
4	4.9
5	4.9
6	4.9
7	4.9
8	2.4
9	4.9
10	2.4
11	2.4
12	2.4
13	2.4
14	0.0
15	2.4
16	2.4
17	0.0

Table 4: Bounce Number vs. Percent Energy Loss for Video 3

Large Angles

Simple harmonic motion is able to be achieved due to the small angle approximation, where $\sin\theta \simeq \theta$ at small initial displacement angles. To get the simple harmonic motion equation, we start with the torque of the pendulum, $T = I\alpha$. The torque is the force that acts perpendicular to the lever arm, the string: $mg*\sin\theta$ multiplied the length of the string L. This is equal to the moment of inertia * acceleration: $mL^2 * d^2\theta/dt^2$. Simplifying this equation results in $d^2\theta/dt^2 + (g/L)\sin\theta = 0$, or $d^2\theta/dt^2 = -\omega^2\theta$, by the small angle approximation. However, when the angle is not small, the small angle approximation is not true, and simple harmonic motion is not achieved.

To analyze a pendulum not undergoing simple harmonic motion, the same pendulum was used from the base experiment; in the base case, the point mass was dropped from an initial displacement angle less than 15° to achieve simple harmonic motion, and in the other case, the angle was much greater than 15°, so it was not simple harmonic motion.

1

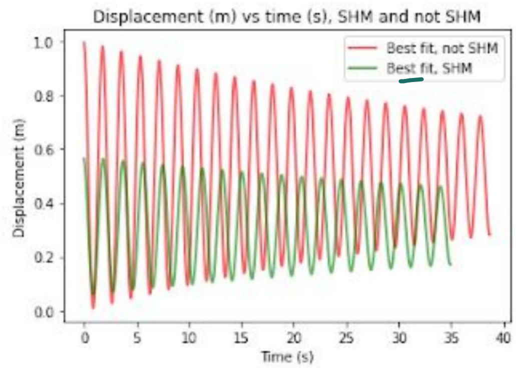

Figure 6a: Displacement (m) vs. time (s), comparing simple harmonic motion to non-SHM

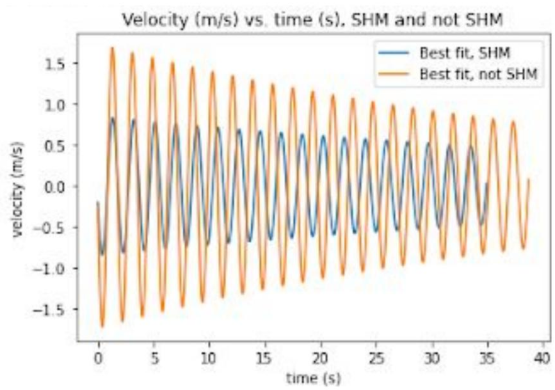

Figure 6b: Velocity (m/s) vs. time (s), comparing simple harmonic motion to non-SHM

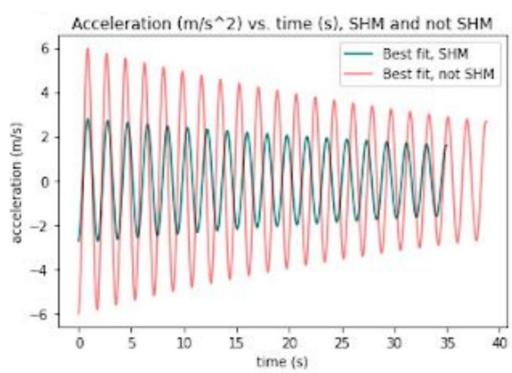

Figure 6c: Acceleration (m/s^2) vs. time (s), comparing simple harmonic motion to non-SHM

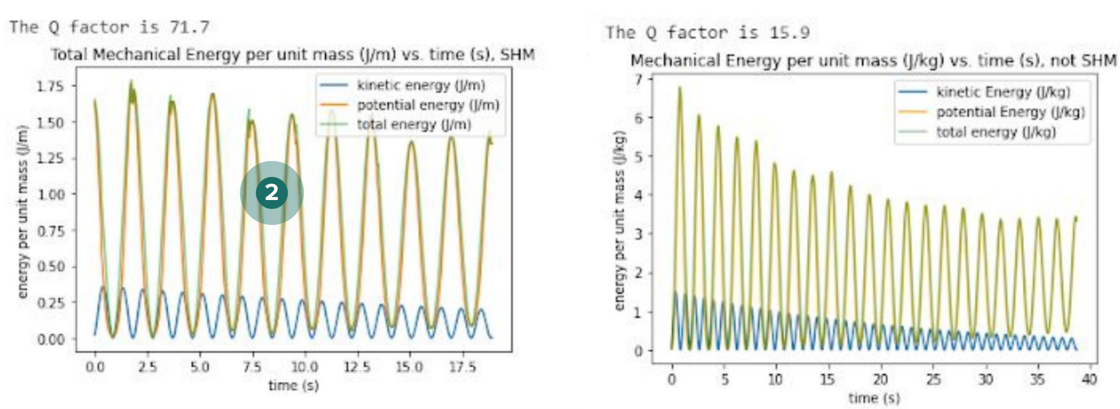

Figure 7a-b: Energy per unit mass (J) vs. time (s), comparing simple harmonic motion to non-SHM

It looks as if, at angles larger than 15°, the harmonic oscillator experiences much larger damping. The pendulum acts much less like a simple oscillator, as the motion dies out much faster. In contrast, at sufficiently small angles, the simple harmonic oscillator appears to "complete" its oscillations, or reach the maximum angle during each cycle.

Conclusion

In contrast to undamped simple harmonic oscillators, it can be seen through the above data of a simple pendulum that damped oscillation does entail energy loss, in line with what was theorized. The mass-string system elucidated this energy loss most transparently through the angle versus time plots, in which amplitude experiences exponential decay over time. Similar takeaways are displayed by energy versus time plots, as the total mechanical energy of the simple pendulum underwent a curve downwards. This research on a simple pendulum system adequately corroborates the damped oscillation equation, $x = A_0 e^{-\frac{\gamma \cdot t}{2}} \cos(\omega t + \varphi)$, suggesting that even our best attempts at creating undamped harmonic oscillations are futile, as nonconservative forces (most often friction) are always present to sap a system's energy.

Furthermore, experiments with moving carts on a track illustrated their damped oscillatory nature. This research once again supported the integral facets of damped oscillation, that over time less mechanical energy in the system remains. It was also seen in this portion of the experiment that higher angles of elevation lead to more energy loss, simply because there is a larger amount of energy to lose. With higher slopes, there is more initial gravitational potential energy to be transformed into kinetic energy, which then leads to more bounces and more proportional energy lost per bounce. Of course, the total energy decays due to the nature of damped oscillators; in this system, there most certainly was some degree of friction between the cart and surface, which played a role in diminishing the cart's energy. Also, within each bounce, some energy is lost due to sound and heat.

Lastly, changes were made to the same mass-string system, which was able to act as a simple harmonic oscillator with a maximum angle less than 15°, to explore the extent to which larger angles alter simple harmonic motion. The simple harmonic oscillator, when pushed to angles of less than 15° fulfills its maximum displacement during each cycle; its amplitude decays very little over time (ideally, not at all), ostensibly undergoing undamped motion. On the contrary, the non-simple harmonic oscillator, which reaches much larger angles, dies out much more quickly, with a much lower Q factor (15.9) than that of the simple harmonic oscillator (71.7)*. To explain such findings, one can look closer to the oscillatory equation which governs these mass-string systems: $d^2\theta/dt^2 = -[g/L]\sin\theta$. At small angles, or in simple harmonic motion, the $\sin\theta$ can be approximated to θ. Thus, the restoring force ($mg\theta$) acts directly opposite to the motion. Put differently, the angular acceleration is at every point directly opposite of angular displacement; $d^2\theta/dt^2$ is everywhere negative of θ multiplied by a constant. However, in non-simple harmonic motion, due to the small angle approximation being false, the restoring force ($mg\sin\theta$) is *not* directly opposite to the motion. Much less of the maximum angle is restored to its initial value before each new cycle begins, i.e. energy is lost. These results suggest that non-simple pendulums, when pushed outside of their boundaries (past angles of 15°), undergo damped motion.

*Note: A Q factor of closer to 0 would indicate an overwhelmingly damped system.

References

Jain, P.. (1997). Under-damped harmonic oscillator with large damping: Displacement, velocity, acceleration and energy. International Journal of Mathematical Education in Science and Technology. 28. 735-748. 10.1080/0020739970280510.

Triana, C.A., & Fajardo, F.. (2013). Experimental study of simple harmonic motion of a spring-mass system as a function of spring diameter. Revista Brasileira de Ensino de Física, 35(4), 1-8. https://doi.org/10.1590/S1806-11172013000400005

4 Results 14 / 18

✓ + 0 pts Correct

+ 14 Point adjustment

1 The small angle approximation will still yield mostly valid results for angles up to at least 45 degrees.

2 These aren't scaled correctly

It looks as if, at angles larger than 15°, the harmonic oscillator experiences much larger damping. The pendulum acts much less like a simple oscillator, as the motion dies out much faster. In contrast, at sufficiently small angles, the simple harmonic oscillator appears to "complete" its oscillations, or reach the maximum angle during each cycle.

Conclusion

In contrast to undamped simple harmonic oscillators, it can be seen through the above data of a simple pendulum that damped oscillation does entail energy loss, in line with what was theorized. The mass-string system elucidated this energy loss most transparently through the angle versus time plots, in which amplitude experiences exponential decay over time. Similar takeaways are displayed by energy versus time plots, as the total mechanical energy of the simple pendulum underwent a curve downwards. This research on a simple pendulum system adequately corroborates the damped oscillation equation, $x = A_0 e^{-\frac{\gamma \cdot t}{2}} \cos(\omega t + \varphi)$, suggesting that even our best attempts at creating undamped harmonic oscillations are futile, as nonconservative forces (most often friction) are always present to sap a system's energy.

Furthermore, experiments with moving carts on a track illustrated their damped oscillatory nature. This research once again supported the integral facets of damped oscillation, that over time less mechanical energy in the system remains. It was also seen in this portion of the experiment that higher angles of elevation lead to more energy loss, simply because there is a larger amount of energy to lose. With higher slopes, there is more initial gravitational potential energy to be transformed into kinetic energy, which then leads to more bounces and more proportional energy lost per bounce. Of course, the total energy decays due to the nature of damped oscillators; in this system, there most certainly was some degree of friction between the cart and surface, which played a role in diminishing the cart's energy. Also, within each bounce, some energy is lost due to sound and heat.

Lastly, changes were made to the same mass-string system, which was able to act as a simple harmonic oscillator with a maximum angle less than 15°, to explore the extent to which larger angles alter simple harmonic motion. The simple harmonic oscillator, when pushed to angles of less than 15° fulfills its maximum displacement during each cycle; its amplitude decays very little over time (ideally, not at all), ostensibly undergoing undamped motion. On the contrary, the non-simple harmonic oscillator, which reaches much larger angles, dies out much more quickly, with a much lower Q factor (15.9) than that of the simple harmonic oscillator (71.7)*. To explain such findings, one can look closer to the oscillatory equation which governs these mass-string systems: $d^2\theta/dt^2 = -[g/L]\sin\theta$. At small angles, or in simple harmonic motion, the $\sin\theta$ can be approximated to θ. Thus, the restoring force ($mg\theta$) acts directly opposite to the motion. Put differently, the angular acceleration is at every point directly opposite of angular displacement; $d^2\theta/dt^2$ is everywhere negative of θ multiplied by a constant. However, in non-simple harmonic motion, due to the small angle approximation being false, the restoring force ($mg\sin\theta$) is *not* directly opposite to the motion. Much less of the maximum angle is restored to its initial value before each new cycle begins, i.e. energy is lost. These results suggest that non-simple pendulums, when pushed outside of their boundaries (past angles of 15°), undergo damped motion.

*Note: A Q factor of closer to 0 would indicate an overwhelmingly damped system.

References

Jain, P.. (1997). Under-damped harmonic oscillator with large damping: Displacement, velocity, acceleration and energy. International Journal of Mathematical Education in Science and Technology. 28. 735-748. 10.1080/0020739970280510.

Triana, C.A., & Fajardo, F.. (2013). Experimental study of simple harmonic motion of a spring-mass system as a function of spring diameter. Revista Brasileira de Ensino de Física, 35(4), 1-8. https://doi.org/10.1590/S1806-11172013000400005

5 Conclusion 6 / 6

✓ + 0 pts Correct

+ 6 Point adjustment