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**Time taken** 41 mins 6 secs

### Question 1

Partially correct

Marked out of 4.00

Two aeronautics companies (I, II,) bid for contracts for space in a satellite navigation system. A company that bids for a contract gets funded for their contract by the European Union. Past information shows that firm I and firm II get each one contract with probability  $1/9$ , firm I and firm II can each get two contracts with probability  $1/9$  and firm I and firm II can each get 3 contracts with probability  $1/9$ . But any other distribution of the contracts between the two companies have also  $1/9$  probability of happening. None of the companies can get more than three contracts. There are then a total of 9 outcomes. The Sample space would be represented by

$S = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

where, for example, (2,3) denotes the outcome where firm I gets two contracts and firm II gets three contracts.

Thus, the first component of the pair indicates how many contracts company I gets and the second how many contracts company II gets. Consider random variable X denoting the number of contracts granted to firm I and random variable Y the number of contracts granted to firm II. Construct the joint probability mass function of X and Y on your own paper, and use it for finishing this exercise. Based on your table, the probability that I gets the same number of contracts as II is  $\frac{3}{9}$  ✓. The probability that I gets exactly 1 contract is  $\frac{3}{9}$  ✓. The expected number of contracts by company I is 2 ✓.

We can add that  ✗. Ignore this last one-everyone will lose some minor point for this, so it will not affect your grade differentially (Dr. Sanchez speaking here).

### Question 2

Partially correct

Marked out of 3.00

The multinomial probability mass function is the generalization of the binomial to non-binary choices. Section 6.10 in the textbook and the supplement to lecture 20 talks about it. The exercise below is about the multinomial.

#### Exercise

The demographic profile of Ecuador in 2018 is

- 27.08% of the population are 0-14 years old
- 18.35% are 15-24 years old
- 39.59 are 25-54 years old
- 7.53% are 55-64 years old
- 7.45% are 65 years and older.

We are interested in answering the following question: In a random sample of 20 people, what is the probability that 2 are 65 years or older, 5 are 55-64 years old, 6 are 25-54 years old, 4 are 15-24 years old and 3 are 0-14 years old?

The number of random samples that have a composition like the one described above is 97772875200 ✓. Each of those random samples have a probability of occurring of  $1.164905e-08$  ✗. The probability of the event containing all random samples that fit that description above is 0.319 ✗.

## Question 3

Correct

Marked out of 5.00

Table 6.2 in the textbook is the same example we discussed in lecture 20, but with different probabilities. The exercise that follows will be easier if you have reviewed up to Section 6.4 of Chapter 6 and watched lecture 20.

## Exercise

Find the following conditional probability mass functions obtained from Table 6.2

$x \setminus y$	0	1	2	3
0	$P(X=0, Y=0)=1/8$	$P(X=0, Y=1)=2/8$	$P(X=0, Y=2)=1/8$	$P(X=0, Y=3)=0$
1	$P(X=1, Y=0)=0$	$P(X=1, Y=1)=1/8$	$P(X=1, Y=2)=2/8$	$P(X=1, Y=3)=1/8$

What happens to the expected value of  $X$  as  $Y$  increases?

$P(X=1|Y=0)$

0



$P(X=1|Y=1)$

1/3



$P(X=1|Y=2)$

2/3



$P(X=1|Y=3)$

1



The expected value of  $X$  as  $Y$  increases

increases



## Question 4

Correct

Marked out of 2.00

This is a good exercise to practice calculating joint probabilities and covariance or calculating independence, to show understand of what independence and correlation means. In life we usually express ourselves with words. We use tables and formulas to model mathematically the problem in order to find an answer to our questions. Review Lecture 20, 21, 22 and Chapter 6 in the book if you are having trouble understanding this question.

## Exercise.

Two species, A and B, affected by the same environmental factors, are being studied to see if there is association between them. The species live in fruits. The random variable  $X$  measures the number of species A per fruit, and the random variable  $Y$  measures the number of species B per fruit. The joint probability mass function  $P(X,Y)$  is given by the following table.

$x \setminus y$	0	1	2
0	0.40	0.1	0.1
1	0.1	0.1	0.02
2	0.1	0.02	0.03
3	0.01	0.01	0.01

The probability that the number of species B is larger than the number of species A in a fruit is

found by adding the probability of the event  $\{(0,1), (0,2), (1,2)\}$  where the first number in a pair is the number of species A and the second number is the number of species B



We can say without doubt that

the number of species A is (albeit very slightly) related to the number of species B in the flower



## Question 5

Complete

Not graded

**The answer for the independence part of this question is incorrect. See feedback. Because of the two attempts nature of this quiz it is easier to just assign points 0. Look at key below to learn from this though.**

The joint probability mass function of two random variables X and Y is given by

$$P(X=x, Y=y) = k(2x+y), \quad x=1,2; y=1,2,3$$

where k is a constant.

- (i) What is the value of k?
- (ii) Find the marginal probability mass functions of X and Y
- (iii) Are X and Y independent?

for  $P(X,Y)$  to be a pmf k must be

$\mu_X$   
equals

$\mu_Y$   
equals

X and Y are

## Question 6

Correct

Marked out of 1.00

Suppose that 15% of the families in a certain community have no car, 20% have 1 car, 35% have 2, and 30% have 3. Suppose, further, that in each family, each car is equally likely (independently) to be a foreign or a domestic car. Let F be the number of foreign cars and D the number of domestic cars in a family.

The random variable denoting the number of cars in a family and the random variable denoting the number of foreign cars are

Select one:

☐ a.

mutually exclusive so one has to use the union rule for mutually exclusive events to calculate the joint probability that  $F=1$  and  $D=1$ .

☐ b.

independent, so one has to use the product rule for independent events to calculate the joint probability that  $F=1$  and  $D=1$

☒ c.

dependent, so one has to use the general product rule to calculate the joint probability that  $F=1$  and  $D=1$



☐ d.

partitioned, so one has to use Axiom 3 to calculate the joint probability that  $F=1$  and  $D=1$

## Question 7

Complete

Not graded

Suppose that 15% of the families in a certain community have no car, 20% have 1 car, 35% have 2, and 30% have 3. Suppose, further, that in each family, each car is equally likely (independently) to be a foreign or a domestic car. Let F be the number of foreign cars and D the number of domestic cars in a family.

The joint probability that the number of foreign cars in a family is 1 and the number of domestic cars is 2 is

**Note: the answer marked as correct for this question is not the correct answer. So even though you get it wrong in the first attempt, you could be right. Just mark what you think is right and we will grade it manually.**

Select one:

- ☐ a. 0.15
- ☐ b. 0.1
- ☒ c. 0.0375
- ☐ d. 0.6

## Question 8

Correct

Marked out of 1.00

When we talk about the joint density function of two random variables,  $X, Y$ ,  $(f(x,y))$ , for constants  $a$  and  $b$ ,

$$P(X \leq a, Y \geq b)$$

is

Select one:

- ☐ a. an area
- ☒ b. a volume
- ☐ c. always 1 to satisfy axioms
- ☐ d. the value of the first quartile



## Question 9

Correct

Marked out of 1.00

Chapter 8-textbook, mini quiz question 10.

A bank operates both a drive-up facility and a walk-up window. On a randomly selected day, let  $X$  = the proportion of time that the drive-up facility is in use (at least one customer is being served or waiting to be served) and  $Y$  = the proportion of time that the walk-up window is in use. Suppose the joint probability density function of  $X$  and  $Y$  is given by

$$f(x, y) = \frac{6}{5}(x + y^2), \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1$$

The probability that neither facility is busy more than one-quarter of the time is

Select one:

- ☐ a. 0.67
- ☒ b. 0.0109
- ☐ c. 0.0004
- ☐ d. 0.101968



## Question 10

Incorrect

Marked out of 1.00

Chapter 8, textbook, Mini quiz question number 6

Let  $X, Y$  be a random variable with density function

$$f(x, y) = 2, \quad 0 \leq y \leq x \leq 1$$

Which of the following is the marginal probability density function of  $X$ ?

Select one:

☒ a.

$$f(x) = 2x, \quad y \leq x \leq 1$$

✗

☐ b.

$$f(x) = 2x, \quad 0 \leq x \leq 1$$

☐ c.

$$f(x) = 2xy, \quad y \leq x \leq 1$$

☐ d.

$$f(x) = 2x^2, \quad 0 \leq x \leq 1$$

## Question 11

Partially correct

Marked out of 3.00

Suppose random variables  $X, Y$  are jointly distributed as  $f(x, y)$ .

Match the following:

$$\int_x \int_y x f(x, y) dy dx$$

Conditional expectation of  $X$  given  $Y$ 

✗

$$\int_x \int_y y f(x, y) dy dx$$

Marginal expectation of  $Y$ .

✓

$$\int_x (X - \mu_x)^2 \int_y f(x, y) dy$$

Marginal Variance of  $X$ 

✓

## Question 12

Incorrect

Marked out of 1.00

##

Suppose  $X$  is the number of hours it takes for a new pair of blue jeans mail ordered from a store to arrive to customers after ordering, and  $Y$  is the time it takes between ordering and the customer trying the new pair of jeans. The joint density of these two random variables is

$$f(x, y) = \frac{1}{125000}, \quad 0 \leq x \leq y \leq 500$$

what is the probability that the time it takes between ordering and the customer trying the new pair of jeans is less than 250 hours?

Select one or more:

- ☐ a. 0.35
- ☐ b. 0.75
- ☒ c. 0.5
- ☐ d. 0.25

