Midterm 2 (draft)

This exam consist of 4 base question. Each question comes in 4 different versions: a CCLE quiz component will randomly select one (out of possibly 16) different exam for every student enrolled in the course. The exam will be split into 2 different parts.

Both parts of the exam are set to be released on Wednesday, May 27 at 5:00 pm PDT. Students will have 24 hours to start any of the component parts of the midterm. Once a part has been started, it has to completed (CCLE submission) within 1 hour and 30 minutes. Even though in principle students could take 3 hours to work on their exams, I am not too concerned about Chegg.com and the like, because of their average response time. In addition I have the ability to request answers from the site; moreover I can check against CCLE logs to pinpoint possible question leaks.

> Note: At a later time (up to 2 days later), students are expected to upload a high quality version of their answers to Gradescope.

Base question 1

- **Part 1A:** Assume there are 10 identical *"There, and back again"* (by *Bilbo Baggins*) books available for *hobbits* to borrow at their local library.

  Count the number of ways

  a) to choose 7 of them and arrange them in a line on a bookshelf.
  b) to choose 7 of them to place them inside a backpack.
  c) to choose 7 of them and distribute them among *Frodo*, *Sam*, *Merry*, and *Pippin*, if a valid distribution allows for one or more hobbits to receive no books at all.

- **Part 1B:** There are 8 different kinds of evil creatures in *Middle Earth*: *Balrogs*, *Nazgul*, *Dragons*, *Spiders*, *Wargs*, *Trolls*, *Orcs*, and *Crebain*.

  Count the number of ways

  a) to pick 6 of them and arrange them in a line.
  b) to pick 6 of them and place them into lines named M (for *Melkor*) and S (for *Sauron*) with 3 creatures in each line.
  c) to pick 6 of them to make a mini fighting squad.

- **Part 2A:** Thirteen dwarves (*Dwalin*, *Balin*, *Kili*, *Fili*, *Dori*, *Nori*, *Ori*, *Oin*, *Gloin*, *Bifur*, *Bofur*, *Bombur*, and *Thorin*), are heading to *The Prancing Pony* to have some

drinks. The place sells 5 different styles of beer: *Brown Ale*, *Pale Ale*, *India Pale Ale*, *Porter*, and *Stout*.

The dwarves will order exactly 13 drinks but they want to make sure the sample includes at least one beer of each type. In how many ways can they order their beer?

- **Part 2B:** Nine friends (*Gandalf*, *Aragorn*, *Boromir*, *Legolas*, *Gimli*, *Mr. Baggins*, *Mr. Gamgee*, *Mr. Brandybuck*, and *Mr. Took*), are heading to *The Prancing Pony* to have some drinks. The place sells 5 different styles of beer: *Brown Ale*, *Pale Ale*, *India Pale Ale*, *Porter*, and *Stout*.

  These fellows will order exactly 9 drinks but they want to make sure the sample includes at least one beer of each type. In how many ways can they order their beer?

  Different versions:

  i. Part 1A, and part 2A.
  ii. Part 1A, and part 2B.
  iii. Part 1B, and part 2B.
  iv. Part 1B, and part 2A.

Base question 2

Let K_n denote the complete graph with n vertices, and let $K_{m,n}$ denote the complete bipartite graph with n vertices in V_1, and m vertices in V_2.

Find

a) An *Euler cycle* in K_5.
b) A *Hamiltonian cycle* in K_6.
c) An *Euler path* in $K_{2,7}$.

Different versions:

i. As above.
ii. Order of component parts: c) a) b)
iii. Order of component parts: b) c) a)
iv. Order of component parts: c) b) a)

Base question 3

Let $\theta \neq 1$. Find the solution of the recurrence relation

$$a_{n+1} = (\theta + 1)a_n - \theta a_{n-1}, \qquad n \geq 1,$$

with initial conditions $a_0 = 1$, $a_1 = 2 - \theta$.

Different versions:

i. As above
ii. Replace θ above with φ.
iii. Replace θ above with ε, $a_0 = -1$, and $a_1 = \varepsilon - 2$.
iv. Replace θ above with μ, $a_0 = -1$, and $a_1 = \mu - 2$.

Base question 4

Part 1A: *Prof. Pien Sapoco* gives a multiple choice quiz that has 3 questions, each with four possible answers: A, B, C, and D. He suspects his class with 130 enrolled students is cheating after he finds 3 answer sheets that are identical.

Explain to professor *Sapoco* why this is completely normal.

Part 1B: *Prof. Pien Sapoco* gives a multiple choice quiz that has 3 questions, each with five possible answers: `i`, `ii`, `iii`, `iv`, and `v`. He suspects his class with 130 enrolled students is cheating after he finds 2 answer sheets that are identical.

Explain to professor *Sapoco* why this is completely normal.

Part 2A: A row of houses are randomly assigned distinct numbers between 50 and 99 (inclusive). Show that if there are at least 41 houses in this row, 5 of them will be numbered consecutively.

Part 2B: A row of houses are randomly assigned distinct numbers between 1 and 60 (inclusive). Show that if there are at least 41 houses in this row, 3 of them will be numbered consecutively.

Different versions:

i. Part 1A, and part 2A.
ii. Part 1A, and part 2B.
iii. Part 1B, and part 2B.
iv. Part 1B, and part 2A.