Math 33A Sheet 5

Chapter 3.2

Ex.22) $\begin{bmatrix}1&3\\2&6\end{bmatrix}$ $-2(I)$

$\begin{bmatrix}1&3\\0&0\end{bmatrix}$

$3\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}3\\6\end{bmatrix}$

$3\begin{bmatrix}1\\2\end{bmatrix}-\begin{bmatrix}3\\6\end{bmatrix}=0$

$\boxed{\ker(A)=\text{span}\left(\begin{bmatrix}3\\-1\end{bmatrix}\right)}$

Ex.24) $\begin{bmatrix}1&0&2&0\\0&1&3&0\\0&0&0&1\end{bmatrix}$

$\left[\begin{array}{ccc|c}1&0&0&2\\0&1&0&3\\0&0&1&0\end{array}\right]$

$2\begin{bmatrix}1\\0\\0\end{bmatrix}+3\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}2\\3\\0\end{bmatrix}$

$2\begin{bmatrix}1\\0\\0\end{bmatrix}+3\begin{bmatrix}0\\1\\0\end{bmatrix}-\begin{bmatrix}2\\3\\0\end{bmatrix}+0\begin{bmatrix}0\\0\\1\end{bmatrix}=0$

$\boxed{\ker(A)=\text{span}\left(\begin{bmatrix}2\\3\\-1\\0\end{bmatrix}\right)}$

Ex.29) $\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}$ $-4(I)$

$\begin{bmatrix}1&2&3\\0&-3&-6\end{bmatrix}$ $\times -\frac{1}{3}$

$\begin{bmatrix}1&2&3\\0&1&2\end{bmatrix}$ $-2(II)$

$\begin{bmatrix}1&0&-1\\0&1&2\end{bmatrix}$

$-\begin{bmatrix}1\\4\end{bmatrix}+2\begin{bmatrix}2\\5\end{bmatrix}=\begin{bmatrix}3\\6\end{bmatrix}$

$\boxed{\text{basis of A is } \begin{bmatrix}1\\4\end{bmatrix},\begin{bmatrix}2\\5\end{bmatrix}}$

Ex30)* $\begin{bmatrix}1&1&1\\1&2&5\\1&3&7\end{bmatrix}$ $\begin{matrix}\\-I\\-I\end{matrix}$

$\begin{bmatrix}1&1&1\\0&1&4\\0&2&6\end{bmatrix}$ $\begin{matrix}-II\\\\-2II\end{matrix}$

$\begin{bmatrix}1&0&-3\\0&1&4\\0&0&-2\end{bmatrix}$ $\times -\frac{1}{2}$

$\begin{bmatrix}1&0&-3\\0&1&4\\0&0&1\end{bmatrix}\rightarrow\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$\boxed{\text{basis of A is } \begin{bmatrix}1\\1\\1\end{bmatrix},\begin{bmatrix}1\\2\\3\end{bmatrix},\begin{bmatrix}1\\5\\7\end{bmatrix}}$

Ex39) They must be linearly independent, because $\vec{u}$ isn't in the original span, adding it would change the span, and removing any vector from $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m, \vec{u}$ would change the new span.

Ex 52)* $\begin{bmatrix}a\\0\\0\\0\end{bmatrix},\begin{bmatrix}b\\c\\0\\0\end{bmatrix},\begin{bmatrix}d\\e\\f\\0\end{bmatrix}$

$a\neq 0$, otherwise the 1st column vector would be $\vec{0}$, which is linearly dependent

$c\neq 0$ and $f\neq 0$ for the same reasons in the 2nd and 3rd columns

$\boxed{a,c,f\neq 0}$

Chapter 3.3

Ex22) $\begin{bmatrix}2&4&8\\4&5&1\\7&9&3\end{bmatrix}$ $\times\frac{1}{2}$

$\begin{bmatrix}1&2&4\\4&5&1\\7&9&3\end{bmatrix}$ $\begin{matrix}\\-4(I)\\-7(I)\end{matrix}$

$\begin{bmatrix}1&2&4\\0&-3&-15\\0&-5&-25\end{bmatrix}$ $\times -\frac{1}{3}$

$\begin{bmatrix}1&2&4\\0&1&5\\0&-5&-25\end{bmatrix}$ $\begin{matrix}-2II\\\\+5II\end{matrix}$

$\begin{bmatrix}1&0&-6\\0&1&5\\0&0&0\end{bmatrix}=\text{rref}(A)$

$\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}6t\\-5t\\t\end{bmatrix}=t\begin{bmatrix}6\\-5\\1\end{bmatrix}$

$\boxed{\text{basis of }\ker(A)=\begin{bmatrix}6\\-5\\1\end{bmatrix}}$

$\boxed{\text{basis of }\text{im}(A)=\begin{bmatrix}2\\4\\7\end{bmatrix},\begin{bmatrix}4\\5\\9\end{bmatrix}}$

Ex 23)* $\begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 3 & 4 & -6 & 8 \\ 0 & -1 & 3 & 1 \end{bmatrix} -3\mathrm{I}$

$\begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 0 & 4 & -12 & -4 \\ 0 & -1 & 3 & 1 \end{bmatrix} \begin{matrix} \\ \\ -4\mathrm{II} \\ +\mathrm{II} \end{matrix}$

$\begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \text{rref}(A)$

$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} -2t-4s \\ 3t+s \\ t \\ s \end{bmatrix}$

$= t \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -4 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

$\boxed{\text{basis of } \ker(A) = \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ 0 \\ 1 \end{bmatrix}}$

$\boxed{\text{basis of } \operatorname{im}(A) = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 4 \\ -1 \end{bmatrix}}$

Ex 28) $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 2 & 3 & 4 & k \end{bmatrix} -2\mathrm{I}-3\mathrm{II}-4\mathrm{III}$

$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & k-29 \end{bmatrix}$

$k-29 \neq 0$

$\boxed{k \neq 29}$

Ex 29)* $2x_1+3x_2+x_3=0$

$2x_1 = -3x_2 - x_3$

$x_1=-3, x_2=2, x_3=0$

$\begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix}$

$x_1=1, x_2=0, x_3=-2$

$\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix}$ are linearly independent

$\boxed{\text{basis of subspace} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix}}$

Ex 31) $x_1 - x_2 + 2x_3 + 4x_4 = 0$

$x_1 = x_2 - 2x_3 - 4x_4$

$x_1=1, x_2=1, x_3=0, x_4=0$

$x_1=-2, x_2=0, x_3=1, x_4=0$

$x_1=-4, x_2=0, x_3=0, x_4=1$

$A = \begin{bmatrix} 1 & -2 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\boxed{T(\vec{v}) = \begin{bmatrix} 1 & -2 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \vec{v}}$

Ex 36) $\dim(\ker A) + \dim(\operatorname{im} A) = m$

$\dim(\ker A) + \dim(\operatorname{im} A) = 3$

$2\dim(\ker A) = 3$

$\dim(\ker A) = \frac{3}{2}$

$\boxed{\text{No, the dimension of the kernel and image cannot be } 3/2}$

Ex 3.1) if $A = BC$, then $\ker(C) \subseteq \ker(A)$ and $\ker(B) \subseteq \ker(A)$

$\ker(C) = \{\vec{0}\}$ only if it is an $n \times m$ matrix where $m \leq n$

↳ C is a 4×5, 5 is not less than or equal to 4

↳ $\ker(A)$ cannot equal zero, $\ker(C)$ has nonzero vectors

↳ since $\ker(A) \neq 0$, $\dim(\ker A) \neq 0$

↳ $\dim(\ker A) + \operatorname{rank}(A) = m = 5$

↳ $\operatorname{rank}(A) < 5$

↳ A is not invertible