

Math 33A Sheet 3

Chapter 2.1

Ex.2) $y_1 = 2x_2, y_2 = 3x_3, y_3 = x_1$

$\vec{v} = \langle 2x_2, 3x_3, x_1 \rangle$

$T(\vec{v}) = A\vec{v}$

$T(\langle x_1, x_2, x_3 \rangle) = A(\langle x_1, x_2, x_3 \rangle)$

$$\begin{bmatrix} 2x_2 \\ 3x_3 \\ x_1 \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \\ 1 & 0 & 0 \end{bmatrix}$$

Linear

Ex.4)* $y_1 = 9x_1 + 3x_2 - 3x_3$

$y_2 = 2x_1 - 9x_2 + x_3$

$y_3 = 4x_1 - 9x_2 - 2x_3$

$y_4 = 5x_1 + x_2 + 5x_3$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

4x1 4x3 3x1

$$T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \\ 4 \\ 5 \end{bmatrix} \quad T \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -9 \\ -9 \\ 1 \end{bmatrix}$$

$$T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -2 \\ 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 9 & 3 & -3 \\ 2 & -9 & 1 \\ 4 & -9 & -2 \\ 5 & 1 & 5 \end{bmatrix}$$

Ex.5) $T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$

$$T \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$$

$$T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -13 \\ 17 \end{bmatrix}$$

$$A = \begin{bmatrix} T(e_1) & T(e_2) & T(e_3) \\ 1 & 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 7 & 6 & -13 \\ 11 & 9 & 17 \end{bmatrix}$$

Chapter 2.2

Ex.2)* Theorem 2.2.3:

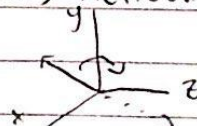
$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

60°:

$$\begin{bmatrix} \cos(\frac{\pi}{3}) & -\sin(\frac{\pi}{3}) \\ \sin(\frac{\pi}{3}) & \cos(\frac{\pi}{3}) \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$$

Ex.20) Reflection about x-z plane



$$T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ -y \\ z \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Chapter 1.3

Ex. 2) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$
 $3 \times 2 \quad 2 \times 2 \checkmark$
 $\begin{bmatrix} 1(3) + (-1)(1) & 1(2) + (-1)(0) \\ 0(3) + 2(1) & 0(2) + 2(0) \\ 2(3) + 1(1) & 2(2) + 1(0) \end{bmatrix}$
 $\begin{bmatrix} 2 & 2 \\ 2 & 0 \\ 7 & 4 \end{bmatrix}$

Ex. 3) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
 $2 \times 3 \quad 2 \times 2 \times$

Cannot be performed, # of columns of 1st matrix \neq # of rows of 2nd

Ex. 7) $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$
 $3 \times 3 \quad 3 \times 3 \checkmark$

$$\begin{bmatrix} 1(1) + 0(3) + (-1)(2) & 1(2) + 0(2) + (-1)(1) & 1(3) + 0(1) + (-1)(3) \\ 0(1) + 1(3) + 1(2) & 0(2) + 1(2) + 1(1) & 0(3) + 1(1) + 1(3) \\ 1(1) + (-1)(3) + (-2)(2) & 1(2) + (-1)(2) + (-2)(1) & 1(3) + (-1)(1) + (-2)(3) \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 5 & 3 & 4 \\ -6 & -2 & -4 \end{bmatrix}$$

Ex. 17) $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
 $B = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$

$$AB = \begin{bmatrix} x_1 & x_2 \\ 2x_3 & 2x_4 \end{bmatrix}$$

$$BA = \begin{bmatrix} x_1 & 2x_2 \\ x_3 & 2x_4 \end{bmatrix}$$

$$x_1 = x_1 \rightarrow x_1 = \mathbb{R}$$

$$x_2 = 2x_2 \rightarrow x_2 = 0$$

$$2x_3 = x_3 \rightarrow x_3 = 0$$

$$2x_4 = 2x_4 \rightarrow x_4 = \mathbb{R}$$

$$\begin{bmatrix} x_1 & 0 \\ 0 & x_4 \end{bmatrix}, x_1, x_4 \in \mathbb{R}$$

Ex. 55) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} X = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
 $2 \times 2 \quad 2 \times 2 \quad 2 \times 2$

$$X = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$$

$$\therefore x_1 + 2x_3 = 0 \quad \text{I} = \text{III}$$

$$x_2 + 2x_4 = 0 \quad \text{II} = \text{IV}$$

$$2x_1 + 4x_3 = 0$$

$$2x_2 + 4x_4 = 0$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \end{array} \right]$$

let $x_3 = t, x_4 = s$

$$X = \begin{bmatrix} -2t & -2s \\ t & s \end{bmatrix}, t, s \in \mathbb{R}$$

Ex 29) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & 4 & k^2 \end{bmatrix}$

-(I)

-(I)

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & k-1 \\ 0 & 3 & k^2-1 \end{bmatrix}$ -(II)

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & k-1 \\ 0 & 3 & k^2-1 \end{bmatrix}$ -3(II)

$\begin{bmatrix} 1 & 0 & -k+2 \\ 0 & 1 & k-1 \\ 0 & 0 & k^2-3k+2 \end{bmatrix} \times \frac{1}{k^2-3k+2}$

$k^2-3k+2=0$

$(k-2)(k-1)$

$k \neq 2, 1$

$\begin{bmatrix} 1 & 0 & -k+2 \\ 0 & 1 & k-1 \\ 0 & 0 & 1 \end{bmatrix}$

k can be any real number other than 1 and 2

Q2) Composition. $S \circ T(\vec{v})$

$S \circ T(\vec{v} + \vec{w}) = S \circ T(\vec{v}) + S \circ T(\vec{w})?$

$S \circ T(\vec{v} + \vec{w}) = S(T(\vec{v} + \vec{w}))$

$T(\vec{v} + \vec{w}) = T(\vec{v}) + T(\vec{w})$

$= S(T(\vec{v}) + T(\vec{w}))$

$= S(T(\vec{v})) + S(T(\vec{w}))$

$= S \circ T(\vec{v}) + S \circ T(\vec{w}) \checkmark$

$S \circ T(k\vec{v}) = k S \circ T(\vec{v})?$

$S(T(k\vec{v})) = S(kT(\vec{v}))$

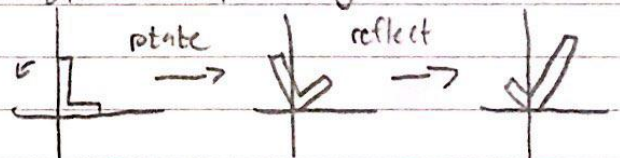
$= k S(T(\vec{v}))$

$= k S \circ T(\vec{v}) \checkmark$

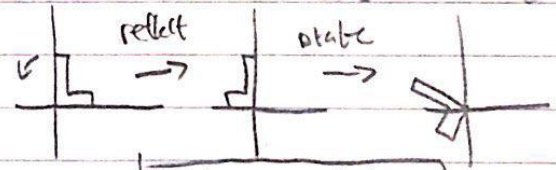
Q3) If the matrices do not

commute, then the transformations cannot be done in the reverse order.

An example of this is rotating and then reflecting:



vs. reflecting, then rotating:



not commutative

Q1) $T(\vec{v}) = A\vec{v}$

Conditions: $T(\vec{v} + \vec{w}) = T(\vec{v}) + T(\vec{w})$

$T(k\vec{v}) = kT(\vec{v})$

$T(\vec{0}_n) = \vec{0}_n$

$\vec{v} = \vec{0}_m \quad T(\vec{v}) = \vec{0}_n$

$T(\vec{v} + \vec{w}) = \begin{bmatrix} 0 + u_1 \\ \vdots \\ 0 + u_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$

$= T(\vec{v}) + T(\vec{w}) \checkmark$

$\checkmark T(k\vec{0}_m) = kT(\vec{0}_m) = \vec{0}$