20S-MATH61-2 Midterm 1

CHARLES ZHANG

TOTAL POINTS

44 / 50

QUESTION 1

1 Question 1 6 / 10

- **0 pts** Correct, or mostly correct.
- **4 pts** Major algebraic/conceptual error.

✓ **- 2 pts** **Misleading notation, circular reasoning, or wrong proof direction (necessary instead of sufficient).**

✓ **- 2 pts** **Conceptual error.**

- **1 pts** Minor conceptual error/typo, or unproven claim.
- **6 pts** Not clear what is happening, or incorrect proof.

1 No. It is "at least". But it is better to write an inequality.

2 Since this is what you want to prove, you cannot use it yet....

3 ... and here you are using it.

QUESTION 2

Question 2 10 pts

2.1 Part i. 5 / 5

✓ **- 0 pts** **Correct**

- **1 pts** Minor mistake.
- **2.5 pts** Incorrect.

2.2 Part ii. 5 / 5

✓ **- 0 pts** **Correct**

- **1 pts** Minor mistake.
- **2 pts** Conceptual mistake.
- **2.5 pts** Incorrect.
- **5 pts** No submission.

QUESTION 3

Question 3 10 pts

3.1 Part i. 6 / 6

✓ **- 0 pts** **Correct**

- **4 pts** Incorrect assumption/proof (see note).
- **2 pts** Incorrect.
- **1 pts** Minor mistakes/misuse of notation.
- **5 pts** Vague/ambiguous statement(s).
- **6 pts** No submission.

3.2 Part ii. 4 / 4

✓ **- 0 pts** **Correct**

- **2 pts** Incorrect assumption/explanation/proof.
- **4 pts** No submission.
- **2 pts** False/Unproven/unexplained claim.
- **0.5 pts** Unproven/unexplained believable claim.
- **1 pts** Correct idea, but incomplete explanantion/proof.
- **3.5 pts** Vague/ambiguous statement(s).

QUESTION 4

Question 4 10 pts

4.1 Part i. 5 / 5

✓ **- 0 pts** **Correct**

- **0 pts** Correct, but see note.
- **1 pts** Poor/Inaccurate argumentation (see note).
- **4 pts** Not a proof, or incorrect argumentation (see note).
- **2 pts** Missing argument(s).

4.2 Part ii. 5 / 5

✓ **- 0 pts** **Correct**

- **0 pts** Correct, but see note.
- **2 pts** Incorrect. See note.
- **0.5 pts** Little to no arguments provided.

- **0.5 pts** Correct idea, but poor argumentation.
- **4 pts** Not a proof, or incorrect argumentation (see note).
- **1 pts** Incorrect claim (see note).
- **2 pts** Missing argument(s).

QUESTION 5

5 Question 5 8 / 10

- **0 pts** Correct

✓ - **2 pts** **Incorrect/Incomplete answer (see note).**

- **3 pts** Partial, or incorrect answer (see note).
- **4 pts** Incorrect, but partial credit awarded.
- **0.5 pts** Incorrect answer, but [mostly] correct procedure.
- **2 pts** Incorrect partial answer (see note).
- **0 pts** Attempted a different problem.
- **10 pts** Missing submission.
- **1 pts** Minor mistake.

4 This should be (3+2). Can you explain why?

Q1B: $3^{n+1} \geq 3n+3$, $n > 0$

$n=0$: $3^{0+1} \geq 3(0)+3$

$3 \geq 3$ ✓

$n=1$: $3^{1+1} \geq 3(1)+3$

$9 \geq 3+3=6$ ✓

Assume: $3^{n+1} \geq 3n+3$

Prove: $3^{n+2} \geq 3(n+1)+3 = 3n+6$

2

$3^{n+2} \geq 3n+6$

↳ $3^{n+1} = 3n+3$ → at most 1

$3^{n+2} = 3(3^{n+1})$

$3(3^{n+1}) \geq 3n+6$ 3

$3(3n+3) \geq 3n+6$

$9n+9 \geq 3n+6$

$9n+3 \geq 3n$

$9n+3$ is greater than $3n$ for all positive n

∴ $3^{n+1} \geq 3n+3$ for all $n > 0$

1 Question 1 6 / 10

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✓ **- 2 pts** **Misleading notation, circular reasoning, or wrong proof direction (necessary instead of sufficient).**

✓ **- 2 pts** **Conceptual error.**

- **1 pts** Minor conceptual error/typo, or unproven claim.
- **6 pts** Not clear what is happening, or incorrect proof.

1. No. It is "at least". But it is better to write an inequality.
2. Since this is what you want to prove, you cannot use it yet....
3. ... and here you are using it.

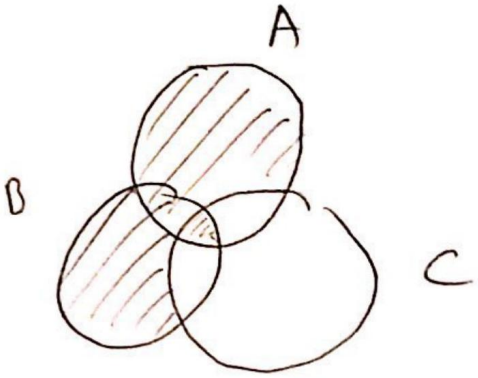
Q2D:

A = French

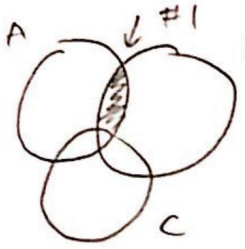
B = Business

C = Music

- $|A \cap B \cap C| = 10$
- $|A \cap B| = 36$
- $|A \cap C| = 20$
- $|B \cap C| = 18$
- $|A| = 65$
- $|B| = 76$
- $|C| = 63$



51 nothings?
35 only in music
86 students ✓



← $|A \cap B| - |A \cap B \cap C|$

1. Business + French, no music

Business + French = $|A \cap B|$

$|A \cap B| - |A \cap B \cap C|$

$36 - 10$

26 students

2. No Business or French

Students in Business or French = $|B| + |A| - |A \cap B|$

$= 76 + 65 - 36$

$= 105$

$191 - 105 =$ **86 students**

2.1 Part i. 5 / 5

✓ **- 0 pts Correct**

- 1 pts Minor mistake.

- 2.5 pts Incorrect.

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A = French

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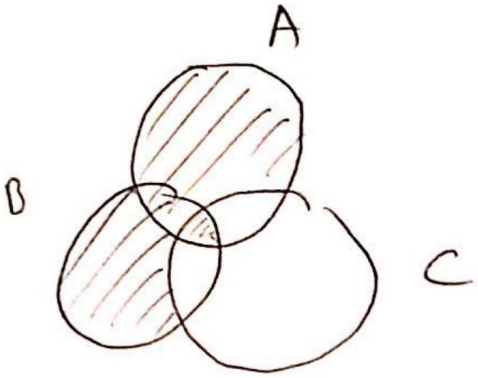

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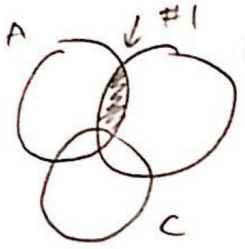

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$= 76 + 65 - 36$

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$191 - 105 =$ **86 students**

2.2 Part ii. 5 / 5

✓ **- 0 pts Correct**

- 1 pts Minor mistake.

- 2 pts Conceptual mistake.

- 2.5 pts Incorrect.

- 5 pts No submission.

Q30: If $x \in L$, then $axb \in L$ and $bqa \in L$

If $\alpha \in L$ and $\beta \in L$, then $\alpha\beta \in L$

Initial. $\phi \in L$

1. $aababb \in L$

↳ $\alpha = \phi$

$\therefore ab \in L$ and $ba \in L$ by rule #1

↳ $\alpha = ba$

$\therefore abab \in L$ by rule #1

↳ $\alpha = abab$

$\therefore aababb \in L$ by rule #1

2. abaab in L?

No.

$|abaab| = 5$, an odd number

- Applying rule #1 to ϕ, the resultant strings are of even length.
- Since rule #1 always adds an 'a' to the front of α and a 'b' to the front of α, it is impossible to alter the parity of strings in L with rule #1.
- Rule #2 is only capable of adding 2 strings together.
- Since the only strings in L thus far are even in length, any string coming from rule #2 will also be even in length.

$\therefore$ All strings in L are even in length, $abaab \notin L$

↳ it is impossible to create an odd length string using rule 1 or 2 unless you start with an odd length string to begin with.

↳ since the initial string is ϕ, this never happens, all strings in L are even length

3.1 Part i. 6 / 6

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Q4B: Equivalence relation = Symmetric, Reflexive, Transitive

R_1 = same first name

↳ Symmetric: If person 1 has the same first name as person 2, then person 2 has the same first name as person 1. Equality is symmetric. ✓

↳ Reflexive: All people have the same first name as themselves. Equality is reflexive ✓

↳ Transitive: If person 1 has the same first name as person 2 and person 2 has the same first name as person 3, then person 1 must have the same first name as person 3. Equality is transitive ✓

∴ R_1 is an equivalence relation

R_2 = speak same language

↳ Transitive: If person 1 speaks the same language as person 2, and person 2 speaks the same language as person 3, it is not necessarily true that person 1 speaks the same language as person 3. For instance, P1 is only an English speaker. P2 speaks Chinese and English. P3 is only a Chinese speaker. $(P_1, P_2) \in R_2$, and $(P_2, P_3) \in R_2$, but $(P_1, P_3) \notin R_2$

∴ R_2 is not an equivalence relation, as it is not transitive

4.1 Part i. 5 / 5

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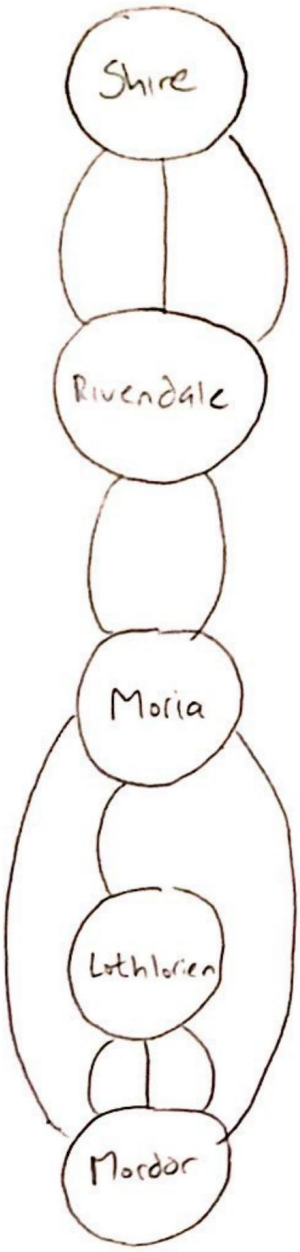
Q5c:



Ways from Shire → Mordor = 3 × 2 × 3 × 2 = 36 ways

Ways from Mordor → Shire = 36 − 1 = 35

(subtract 1 because of no repetition)

$$\begin{array}{r} 36 \\ \times\ 35 \\ \hline 180 \\ 1080 \\ \hline 1260 \end{array}$$

Total Round trip ways = 35 × 36 = **1260 ways**

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