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|---|---|
| **Started on** | Monday, 9 May 2022, 11:29 PM |
| **State** | Finished |
| **Completed on** | Thursday, 12 May 2022, 9:58 PM |
| **Time taken** | 2 days 22 hours |

Question **1**

Complete

Marked out of 6.00

The app posted in Module 7 that gives correlations of data containing observations on two variables asks you to guess the correlation. Provide a screenshot of a negative but not very high correlation, another of a positive, but not very high correlation and another of a correlation very close to 0. Indicate which is which, and what is the correlation value.

Can you provide an example of your own about students at UCLA for each case. Justify why you chose those examples, based on what you see in the scatter plots that you created.

Put all your screenshots and examples in one file and upload.

See attached PDF

q1 (1).pdf

Question **2**

Complete

Marked out of 5.00

If the joint pmf of two random variables is

$$P(X=x, Y=y) = \frac{1}{32}(x^2+y^2), \quad x = 0,1,2,3, \;\; y = 0,1$$

Calculate the correlation between the two random variables using work.

You may write your work by hand and upload a pdf file with all the work.

The work must show how you obtain whatever distributions, expectations or variances needed to obtain the answer.

Note: if you choose to use the equation editor below, which you can obtain by clicking on the arrow and then clicking on the calculator and using the symbols there, make sure that your notation and writing is not ambiguous to avoid losing points.

See attached PDF

q2.pdf

Question **3**

Complete

Marked out of 1.00

Show detailed work. You may attach a pdf file. If you use the editor here, make sure your notation is clear and not ambiguous to avoid losing points.

The average price of a 3 star hotel room in small towns is $130.64 per night with standard deviation $20.36. The average cost of a typical family diner's dinner in small towns is $18.24 and the standard deviation is $8.7. It has been found that the correlation between hotel room and family dinner's dinner is 0.6. The two random variables follow a bivariate normal distribution.

(i) What would be the expected value of family diner's dinner in a small town where the 3 star hotel costs $120?

(ii) What is the probability that we would find a 3 start hotel room in a small town that costs less than $100 per night?

See attached PDF

q3.pdf

Question **4**

Correct

Marked out of 1.00

The ticket office offers two alternatives to go to Universal Studio, the thrifty and the luxurious alternatives.
We denote by X the proportion of students that buy the thrifty alternative and by Y the proportion of students that buy the luxurious one.

$$f(x, y) = 2(x + y) \qquad 0 < y < x < 1,$$

In a given weekend it is known that 10 percent of students purchased the luxurious alternative. Knowing that, what should we expect the proportion of students purchasing the thrifty alternative to be? Find out showing detailed work.

Select one:

- a. 0.167 approximately
- b. 0.654 approximately ✔
- c. 0.341 approximately
- d. 0.901 approximately

Question **5**

Correct

Marked out of 1.00

Let

$$f(x, y) = x + y$$

for

$$0 \leq x \leq 1$$

and

$$0 \leq y \leq 1$$

The Conditional Variance of

$$Y$$

when

$$X = \frac{1}{2}$$

is

Select one:

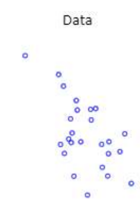
- a. 10/24
- b. 11/144 ✔
- c. 0.5 + x
- d. 7/12

**Q1**

**Positive Correlation (r = 0.566):**

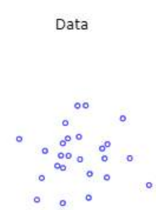

An example of this correlation could be the number of hours slept before an exam and the students' scores on that exam. While this is likely to have some positive correlation, as increased rest should lead to increased brain function, there are plenty of other factors (understanding of material, average test performance, etc.) that would throw this correlation off, leading to a slightly weaker positive correlation.

**Negative Correlation (r = -0.582):**




An example of this correlation could be the number of days partying in the week before an exam (x-axis) and the score on the exam (y-axis). Since the students that party more have less time to study, it makes sense that they would score lower on average, resulting in lower scores. However, as mentioned above, other factors make it so that this correlation isn't necessarily strong.

**Very Little Correlation (r = 0.040):**




An example of this correlation could be a student's GPA and the proportion of days in a month they wear the color green. Obviously, these factors should have very little to do with one another, and as a result, the data should show they have very little correlation.

Q2

$P(X=x, Y=y) = \frac{1}{32}(x^2+y^2),\ x=0,1,2,3,\ \ y=0,1$

Calculate expectations and variances:

$P(x) = \sum_y^1 P(x,y)$

$P(x) = \sum_y^1 \frac{1}{32}(x^2+y^2)$

$P(x) = \sum_y^1 \frac{1}{32}x^2 + \sum_y^1 \frac{1}{32}y^2$

$\underline{P(x) = \frac{2}{32}x^2 + \frac{1}{32}}$

$\mu_x = \sum_x^1 x P(x) = \sum_x x\left(\frac{2}{32}x^2 + \frac{1}{32}\right) = \sum_x^1 \frac{2}{32}x^3 + \frac{1}{32}x$

$\mu_x = 0 + \left(\frac{2}{32} + \frac{1}{32}\right) + \left(\frac{16}{32} + \frac{2}{32}\right) + \left(\frac{54}{32} + \frac{3}{32}\right) = \frac{78}{32} = \underline{2.44}$

$\sigma_x^2 = \sum_x^1 (x-\mu_x)^2 P(x) = \sum_x (x-2.44)^2\left(\frac{2}{32}x^2 + \frac{1}{32}\right)$

$\sigma_x^2 = \left(5.94 \cdot \frac{1}{32}\right) + \left(2.066 \cdot \frac{3}{32}\right) + \left(0.191 \cdot \frac{7}{32}\right) + \left(0.314 \cdot \frac{19}{32}\right)$

$\sigma_x^2 = 0.619$

$P(Y) = \sum_y^1 P(x,y) = \sum_y^1 \frac{1}{32}(x^2+y^2) = \underline{\frac{4}{32}y^2 + \frac{14}{32}}$

$\mu_y = \sum_y^1 \frac{4}{32}y^3 + \frac{14}{32}y = \frac{18}{32} = \underline{0.5625}$

$\sigma_y^2 = \sum_y^1 (y-\mu_y)^2 P(y) = \sum_y^1 (y-0.5625)^2\left(\frac{4}{32}y^2 + \frac{14}{32}\right)$

$\sigma_y^2 = \left(\frac{81}{256}\right)\left(\frac{14}{32}\right) + \left(\frac{49}{256}\right)\left(\frac{18}{32}\right) = \underline{0.2461}$

Calculate covariance:

$Cov(X,Y) = E(XY) - E(X)E(Y)$

$Cov(X,Y) = \left(\sum_x \sum_y^1 \left(xy\left(\frac{1}{32}(x^2+y^2)\right)\right)\right) - (2.44)(0.5625)$

$Cov(X,Y) = \left(0+0+0+0+0+\frac{1}{32}\cdot 2 + \frac{1}{32}\cdot 10 + \frac{1}{32}\cdot 30\right) - 1.3725$

$Cov(X,Y) = -0.06$

Calculate correlation:

$\rho_{X,Y} = \frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}}$

$\rho_{XY} = \frac{-0.06}{\sqrt{0.619}\sqrt{0.2461}} = \boxed{-0.1537}$

Q3

Let $X$ = cost of a 3 star hotel room

Let $Y$ = cost of a family diner's dinner

Given:

$\mu_x = 130.64$, $\sigma_x = 20.36$

$\mu_y = 18.24$, $\sigma_y = 8.7$

$\rho_{x,y} = 0.6$

i) $E(Y|X=120) = \mu_{Y|X=120}$

$\mu_{Y|x} = \mu_Y + \rho \frac{\sigma_Y}{\sigma_x}(x - \mu_x)$

$\mu_{Y|x} = 18.24 + 0.6\left(\frac{8.7}{20.36}\right)(x - 130.64)$

$\mu_{Y|x} = 18.24 + 0.2564\,(x - 130.64)$

$\mu_{Y|x=120} = 18.24 + 0.2564\,(120 - 130.64)$

$\boxed{\mu_{Y|x=120} = \$15.51}$

ii) $f(x<100)$

$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{1}{2\sigma_x^2}(x-\mu_x)^2}$

$f(x) = \frac{1}{51.03} e^{-\frac{1}{829.06}(x-130.64)^2}$

$f(x<100) = \int_{-\infty}^{100} f(x)\,dx = \boxed{0.066}$

Use App