Math 61 HW#5

* 1. i) A, 2, 3, 4, 5 → same suit

$\binom{4}{1}\cdot\binom{1}{1}\binom{1}{1}\binom{1}{1}\binom{1}{1}\binom{1}{1}$

$\boxed{4 \text{ hands}}$

ii) Straight flushes

$\binom{4}{1}\cdot\binom{9}{1}\binom{1}{1}\binom{1}{1}\binom{1}{1}\binom{1}{1}$

$\boxed{36 \text{ hands}}$

iii) $\binom{36}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}$

$\boxed{9216 \text{ hands}}$

iv) $\binom{13}{1}\binom{12}{1}\binom{4}{2}\binom{4}{3}$

$\boxed{3744 \text{ hands}}$

2. i) $\binom{4}{1}$

$\boxed{4 \text{ hands}}$

ii) $\binom{4}{2}\left[\binom{26}{13}-2\right]$

$\boxed{62403588 \text{ hands}}$

iii) $\binom{48}{9}$

$\boxed{1677106640 \text{ hands}}$

iv) $\binom{4}{1}\binom{13}{1}\binom{13}{4}\binom{13}{4}\binom{13}{4}$

$\boxed{19{,}007{,}345{,}500 \text{ hands}}$

3. i) 2^{10}

$\boxed{1024 \text{ outcomes}}$

ii) ${}_{10}C_3$

$\boxed{120 \text{ outcomes}}$

iii) $\binom{10}{1}+\binom{10}{2}+\binom{10}{3}$

$\boxed{175 \text{ outcomes}}$

* 4. i) ${}_{50}C_4$

$\boxed{230300 \text{ ways}}$

ii) ${}_{46}C_4$

$\boxed{163185 \text{ ways}}$

iii) $({}_4C_1)({}_{46}C_3)+({}_4C_2)({}_{46}C_2)+({}_4C_3)({}_{46}C_1)+1$

$60720+6210+185$

$\boxed{67115 \text{ ways}}$

5. i) $S_{n,k}=0$ if $k>n$

↳ Assume 1 person at each table

↳ after 1 seated → $k-1$, $n-1$

↳ n reaches 0 first because $k>n$

↳ there are tables without any people

$\therefore \boxed{S_{n,k}=0}$

ii) $S_{n,n}=1$ for all $n\geq 1$

↳ Assume one person at each table

n tables → n, people

all tables have a person → all people have a table

$\boxed{S_{n,n}=1}$

iii) $S_{n,1}=(n-1)!$ for all $n\geq 1$

↳ There are $n!$ ways to line n people in a straight line

↳ Each combination is repeated n times when the line is translated into a table

↳ $S_{n,1}=\frac{n!}{n}=\boxed{(n-1)!}$

iv) $S_{n,n-1}=\binom{n}{2}$ for all $n\geq 2$

↳ Each table has 1 person, except 1 table has 2

↳ There are ${}_nC_2$ ways to get that 1 table

$\boxed{S_{n,n-1}=\binom{n}{2}}$

6. SALESPERSONS

↳ Treat SSSS as 1 element

↳ 9 spots

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${}_9C_1$ ways for SSSS to be placed

$\frac{8!}{2!}$ ways to form rest of string

$\frac{9!}{2!} = \boxed{181440 \text{ ways}}$

7. i) ${}_{20}C_5$

$\boxed{15504 \text{ ways}}$

ii) ${}_7C_5$

$\boxed{21 \text{ ways}}$

iii) ${}_6C_2 \cdot {}_6C_3 \cdot {}_8C_2$

$\boxed{8400 \text{ ways}}$

8. i) There are 4 suits

↳ 5 cards chosen

↳ Assume one of each suit

↳ The 5th card must match one of the other 4

$\boxed{\therefore \text{ 2 cards will be of the same suit}}$

ii) There are 5 grades ($k=5$)

There are 6 students ($n=6$)

$\therefore$ Since $k<n$, by the pigeonhole principle, at least 2 students must have the same grade

iii) There are 31 days in January ($k=31$)

There are 32 people ($n=32$)

$\therefore$ Since $k<n$, by the pigeonhole principle, at least 2 people get checks on the same day

9.

1	10
2	9
3	8
4	7
5	6

There are 6 choices ($n=6$)

There are 5 choices in a column ($k=5$)

$\therefore$ Since $k<n$, by the pigeonhole principle, there will be at least one choice in each column

$\therefore$ 2 of the 6 choices will add up to 11

10. Paid every other week → 26 weeks

There are 12 months → $k=12$

2 pay periods → $n=13$

$\therefore$ Since $k<n$, by the pigeonhole principle, at least one month will have more than 2 pay periods

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