26 miles across the sea, on a hilltop overlooking Santa Catalina Island, a transmitter broadcasts radio signals at a frequency f from the top of a tower at an altitude d above sea-level. The sky is clear and the ocean is very calm. On the other side of the channel, somewhere on the coast near Palos Verdes (a horizontal distance D from the tower on Catalina), a technician is climbing up another tall tower.

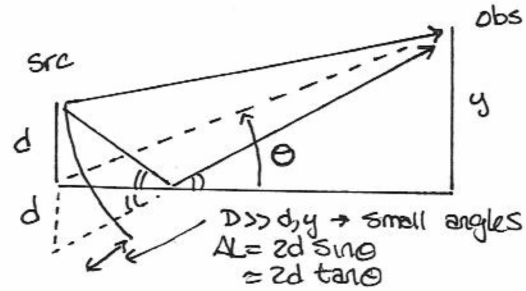
At some point, rather high on the tower, the technician decides to turn on a portable receiver she's brought along. Watching the signal strength climb as she ascends the tower, she makes note of the point at which the signal from Catalina is the strongest and marks the tower.

- 5a) (5 points) Given the distance from the technician to the transmitter site doesn't change significantly as she climbs the tower, how do you account for the change in signal strength that she observes? The more correct, relevant, clear and detailed your answer is, the more points you are likely to receive.

Two otherwise identical waves travel two different paths to arrive at the location of the observer → Interference!

$$\Delta\theta_{TOTAL} = \Delta\theta_{path} + \Delta\theta_{ic} + \Delta\theta_{ref}$$
$$= k\Delta L + 0 + \pi$$
$$= \frac{2\pi f}{c}\frac{2dy}{D} + \pi$$

$A_R = 2A\cos\left(\frac{\Delta\theta_{TOT}}{2}\right)$ → Since $\Delta\theta_{TOT}$ varies with height on the tower (y), the amplitude of the recieved signal varies with height on the tower



$D \gg d, y$ → small angles
$\Delta L = 2d\sin\theta$
$\approx 2d\tan\theta$
$\approx 2d\,y/D$

$n_{water} > n_{air}$ so reflected wave picks up an extra π

- 5b) (10 points) How much further up the tower will our technician have to climb to find a point where the signal strength drops to its lowest value?

Max: $\Delta\theta_{TOT} = 2N\pi$

$$\frac{2\pi f}{c}\frac{2dy}{D} + \pi = 2N\pi$$
$$y_1 = (N - \tfrac{1}{2})\frac{cD}{2fd}$$

Min: $\Delta\theta_{TOT} = (2m+1)\pi$

$$\frac{2\pi f}{c}\frac{2dy}{D} + \pi = (2m+1)\pi$$
$$y_2 = m\frac{cD}{2fd}$$

→ From the context, the minimum occurs just above the maximum, so $m = N$ ...

$$y_2 - y_1 = (m - N + \tfrac{1}{2})\frac{cD}{2fd} = \frac{cD}{4fd}$$

→ She must climb an extra distance
$$\boxed{\Delta y = \frac{cD}{4fd}}$$
to observe a minimum...

- 5c) (5 points) As the morning warms up, water begins to evaporate and a vapor forms over the surface of the ocean. As our technician descends the tower, will the point at which she observes maximum signal strength be above the mark she made on the way up or below it? Explain.

The amount of phase contained in a path of length L through a medium of index n is given by $\Theta_L = \frac{2\pi n}{\lambda} L$. $n_{vapor} > n_{air}$, so the phase contained in the lower path has increased. The physical length of the lower path must shorten to compensate, this means the location of maximum signal strength must fall below the mark on the tower.

- 5d) (10 points) If the distance between the original mark made by the technician and the new point at which she observes maximum signal strength is given by δ, how much phase difference did the water vapor introduce into the relevant path?

- we're considering the same maximum, so N doesn't change.
- we're lumping all the phase difference into $\Delta\Theta$ so we don't have to adjust $k = \frac{2\pi f}{c}$ for n_{vapor}

Going up: $$\frac{2\pi f}{c}\frac{2dy_1}{D} + \pi = 2N\pi$$

Coming down: $$\frac{2\pi f}{c}\frac{2dy_3}{D} + \pi + \Delta\Theta = 2N\pi$$

$$\frac{2\pi f}{c}\frac{2d}{D}(y_1 - y_3) - \Delta\Theta = 0$$

$$\delta = y_1 - y_3$$

$$\boxed{\Delta\Theta = \frac{4\pi f d}{cD}\delta}$$