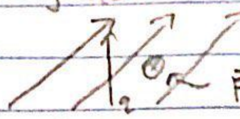
Physics 1C Lecture 1: Magnetism

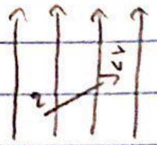
- Magnetic Fields $\leftrightarrow$ electric charge

↳ $\vec{F}_B = q\vec{v} \times \vec{B}$

↳ only acts on charge if it is moving/moving in the right dir

↳  $\vec{B}$

$\vec{F}_B \rightarrow$ into the page

↳ Ex) 

$\vec{B} = B_z\hat{z}$

$$\Sigma\vec{F} = m\vec{a}$$

$$q\vec{v}\times\vec{B} = m\frac{\partial\vec{v}}{\partial t}$$

$$qB_z\langle v_y, -v_x, 0\rangle = m\left\langle \frac{\partial v_x}{\partial t}, \frac{\partial v_y}{\partial t}, \frac{\partial v_z}{\partial t}\right\rangle$$

$$\frac{\partial v_x}{\partial t} = \frac{qB_z}{m}v_y, \quad \frac{\partial v_y}{\partial t} = -\frac{qB_z}{m}v_x, \quad \frac{\partial v_z}{\partial t} = 0$$

$$\frac{\partial^2 v_x}{\partial t^2} = \frac{qB_z}{m}\frac{\partial v_y}{\partial t}$$

$$\frac{\partial^2 v_y}{\partial t^2} = -\frac{qB_z}{m}\frac{\partial v_x}{\partial t}$$

$$\left.\begin{aligned}\frac{\partial^2 v_x}{\partial t^2} + \left(\frac{qB_z}{m}\right)^2 v_x = 0\\ \frac{\partial^2 v_y}{\partial t^2} + \left(\frac{qB_z}{m}\right)^2 v_y = 0\end{aligned}\right] \text{SHO Equations} \rightarrow \frac{\partial^2 x}{\partial t^2} + \omega_0^2 x = 0$$

$$\frac{\partial v_z}{\partial t} = 0$$

↳ v_x and v_y oscillate, v_z is constant

↳ Since $\vec{F}_B \perp \vec{v}$, $W_B = 0$, $\Delta K = 0$, $v =$ constant

↳ $v_x = v_{xy}\cos(\omega t + \phi)$

$v_y = v_{xy}\sin(\omega t + \phi)$

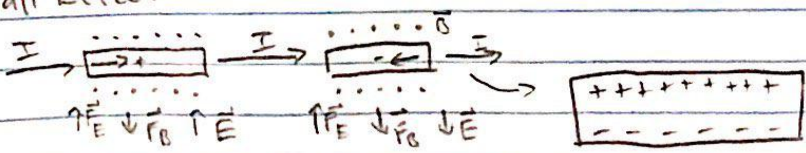
$v_z =$ constant

$v_{xy} = \sqrt{v^2 - v_z^2}$

$\omega = \frac{qB_z}{m}$ ← cyclotron frequency

↳ Charged particles traversing a region of uniform $\vec{B}$ will execute helical motion around a longitudinal axis aligned w/ that field

- Hall Effect

↳ 

polarization (opp. for left conductor)

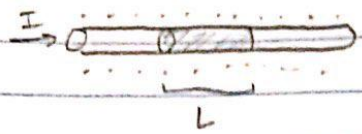
↳ Electric field created

↳ In equilibrium: $|\vec{F}_E| = |\vec{F}_B| \rightarrow qE = qvB\sin(90°) \rightarrow E = vB$

↳ $\frac{\Delta V}{d} = vB$

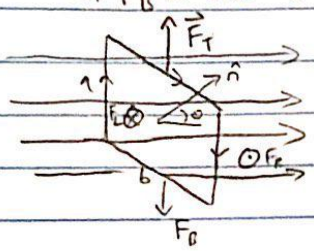
drift velocity, $\Delta V =$ Voltage

Physics 1C Lecture 2: Magnetism (cont.)

- Magnetic force has a more fundamental relationship w/ current rather than charge carriers
  - ↳ 
    - ↳ $\vec{F}_B = n(q\vec{v}\times\vec{B})$, n is the # of charge carriers in L
      - ↳ think of amnt. of charge in segment as amnt. of charge that initially fills it up
      - ↳ $nq = I\frac{L}{v}$] time

        $nqv = IL$

        $nq\vec{v} = I\vec{L}$

        ↳ dir. of $\vec{L}$ is dir. of I
    - ↳ $\vec{F}_B = q\vec{v}\times\vec{B}$ → point charge
    - ↳ $\vec{F}_B = I\vec{L}\times\vec{B}$ → segment of charge
  - ↳  → right hand rule w/ current gives $\hat{n}$

$|\vec{F}_T| = ILB\sin(90°-\theta) = IbB\cos\theta$

$|\vec{F}_B| = ILB\sin(90°+\theta) = IbB\cos\theta$

$|\vec{F}_T| = |\vec{F}_B| = IbB\cos\theta$

$|\vec{F}_R| = IaB\sin(90°)$] current ⊥ to $\vec{B}$

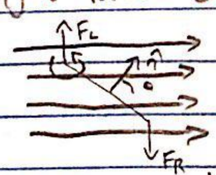
$|\vec{F}_L| = IaB\sin(90°)$

$|\vec{F}_L| = |\vec{F}_R| = IaB$

$\Sigma F = \vec{F}_B + \vec{F}_T + \vec{F}_L + \vec{F}_R = 0$ → $\Sigma F = 0$ on the loop

↳ $\vec{a}$ at center of mass $= 0$

↳ Left/right forces exert a torque that aligns $\hat{n}$ with $\vec{B}$

↳ 

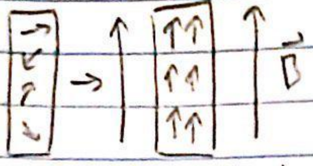
$|\vec{\tau}| = r_\perp F$, $r_\perp = b\sin\theta$, $F = F_R = IaB$

$|\vec{\tau}| = IabB\sin\theta = IAB\sin\theta = I|\vec{A}||\vec{B}|\sin\theta$

$\vec{\tau} = I\vec{A}\times\vec{B}$, $\vec{\tau}_E = \vec{p}\times\vec{E}$

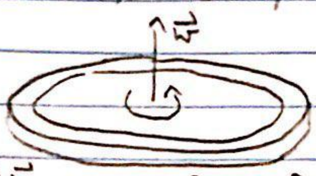
Magnetic dipole moment $\vec{\mu} = NI\vec{A}$ → N = # of turns in loop

Physics 1C Lecture 3: Magnetic Dipole Moments

- 

↳ internal dipole become locked in orientation due to the magnetic fields they generate
↳ demagnetized by de-Gausser, heat, etc

- Uniform disk, charge Q, radius R, angular velocity $\vec{\omega}$

↳ 

$N=1$

↳ $d\vec{\mu} = N\,dI\,A\hat{n}$ → A is area of current loop, not whole

$dI = \frac{dq}{T} = \frac{dq\,\omega}{2\pi}$

$dq = \sigma\,dA = \frac{Q}{\pi R^2}\cdot 2\pi r\,dr = \frac{Q\,2r\,dr}{R^2}$

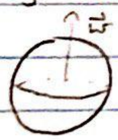
$dI = \frac{Q\omega r\,dr}{\pi R^2}$

$d\vec{\mu} = \frac{Q\omega r\,dr}{\pi R^2}\pi r^2 = \frac{Q\omega}{R^2}\hat{n}(r^3 dr)$

$\int d\vec{\mu} = \frac{Q\omega}{R^2}\hat{n}\int_0^R r^3 dr$

$\vec{\mu} = \frac{1}{4}Q\omega R^2\hat{n} = \boxed{\vec{\mu} = \frac{1}{4}QR^2\vec{\omega}}$

↳ Gyromag Ratio: $\gamma \equiv \frac{\vec{\mu}}{\vec{L}} = \frac{1/4\,QR^2\vec{\omega}}{1/2\,mR^2\vec{\omega}} = \frac{1}{2}\left(\frac{Q}{m}\right)$

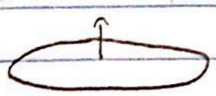
↳ Ex) 

sphere

↳ $\vec{\mu}_{disk} = \frac{1}{4}QR^2\vec{\omega}$

↳ build the sphere with the disks: $d\vec{\mu} = \frac{1}{4}dq\,r^2\vec{\omega}$

↳ Ex) non-uniform disk



$\sigma(r) \propto r^n$

↳ 1st figure out proportionality constant

$Q = \int dq = \int \sigma(r)\,dA = \int_0^R Cr^n\,2\pi r\,dr$

↳ $d\vec{\mu} = dI\,A\hat{n}$

$dI = \frac{dq}{T} = \frac{dq}{2\pi}\vec{\omega}$

$dq = \sigma(r)\,2\pi r\,dr = Cr^n(2\pi r)dr$

Scanned with CamScanner

• 

$\gamma B_z = \langle L_y, L_x, 0 \rangle = \langle \frac{\partial L_x}{\partial t}, \frac{\partial L_y}{\partial t}, \frac{\partial L_z}{\partial t} \rangle$

↳ $\sum \vec{T} = \frac{\partial \vec{L}}{\partial t}$, $\vec{\mu} = \gamma \vec{L}$

$\vec{\mu} \times \vec{B} = \frac{\partial \vec{L}}{\partial t}$

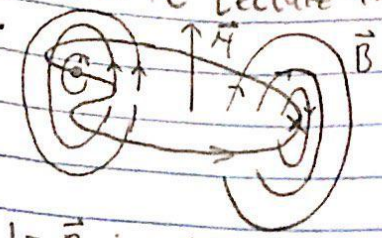
$\gamma \vec{L} \times \vec{B} = \frac{\partial \vec{L}}{\partial t}$

$\frac{\partial L_x}{\partial t} = \gamma B_z L_y$, $\frac{\partial L_y}{\partial t} = \gamma B_z L_x$, $\frac{\partial L_z}{\partial t} = 0$

$L_x = L_{xy} \cos(\omega_0 t + \phi)$, $L_y = L_{xy} \sin(\omega_0 t + \phi)$, $L_z = \text{const}$

$L_{xy} = \sqrt{L^2 - L_z^2}$, $\omega_0 = \gamma B_z$ = cyclotron frequency

Physics 1C Lecture 4: Ampere's Law

- 

  ↳ $\vec{B}$ is right-handed with respect to the current

  ↳ $\vec{B}$ lines form loops

- Long, straight current

  ↳ 

  ↳ $B(r, \phi, z)(\hat{r}, \hat{\phi}, \hat{z})$

  ↳ B cannot depend on ϕ or z, because there is no indication of where those would lie

  ↳ B cannot depend on $\hat{r}$ or $\hat{z}$ because then it would depend on ϕ

  ↳ Sole relliance on r and $\hat{\phi} \rightarrow \vec{B} = B(r)\hat{\phi} \rightarrow$ circular loops

↳ $B(r) = \frac{\mu_0}{2\pi}\left(\frac{I}{r}\right)$

↳ $\mu_0 = 4\pi \times 10^{-7}$

$B(r)2\pi r = \mu_0 I_{encl}$

$\oint \vec{B} \cdot \partial\vec{s} = \mu_0 I_{encl} \rightarrow$ Ampere's Law

↳ $\oint B(r)\partial s \cos\theta = \mu_0 I$

$B(r)\oint \partial s = \mu_0 I \rightarrow B(r)2\pi r = \mu_0 I \rightarrow B(r) = \frac{\mu_0 I}{2\pi r}$

- Ampere's Law $\longleftrightarrow$ Gauss' Law

↳ Gauss' Law uses symmetry to relate field to charge distribution

↳ $\oint \vec{E} \cdot \partial\vec{A} = \frac{q_{encl}}{\varepsilon_0}$

↳ Best Ampere loops mimic the symmetry of current cross section

↳ 

$\partial s \rightarrow \vec{B} \cdot \partial\vec{s} = B\partial s$

- Ex) Thick, long, straight wire with radius a, uniform current I

↳ 

↳ $(r \leq a) \rightarrow \oint \vec{B} \cdot \partial\vec{s} = \mu_0 I_{encl} \rightarrow \oint B(r)\partial s = \mu_0 I_{encl}(r)$

$B(r)2\pi r = \mu_0 I_{encl}(r) \rightarrow B(r) = \frac{\mu_0 I_{encl}(r)}{2\pi r}$

$I \propto A \propto r^2 \rightarrow \frac{I_{encl}}{I} = \frac{r^2}{a^2} \rightarrow I_{encl} = I\frac{r^2}{a^2}$

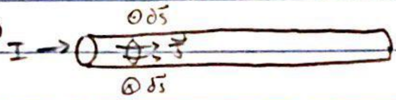
$B(r) = \frac{\mu_0 I r}{2\pi a^2}\hat{\phi}$

Physics 1C Lecture 5: Ampere's Law

- $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$

↳ Shortcut for long wires: $\vec{B}(r) = \frac{\mu_0 I(r)}{2\pi r}\hat{\phi}$

↳ Ex)



$J \propto r^2$

$I = \int \vec{J}\cdot d\vec{A} = \int J dA = \int Cr^2 2\pi r dr$

$I_{encl} = 2\pi C \int_0^r r^3 dr = 2\pi C\left(\frac{r^4}{4}\right)$

$I_{encl}(a) = I$

$C = \frac{2I}{\pi a^4}$

$J = \frac{2Ir^2}{\pi a^4}$

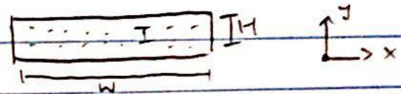
$I_{encl}(r) = \frac{2\pi 2I}{\pi a^4}\left(\frac{r^4}{4}\right)$

$I_{encl}(r) = I\frac{r^4}{a^4}$

$\vec{B}(r) = \frac{\mu_0 I r^3}{2\pi a^4}\hat{\phi}$

- Current sheet: H, W, I, $W \gg H$

↳



↳ $B(x,y,z)(\hat{x},\hat{y},\hat{z}) \to B(y)(\hat{x})$

↳ $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$

$\int \vec{B}\cdot d\vec{s} + \cancel{\int \vec{B}\cdot d\vec{s}} + \int \vec{C}\cdot d\vec{s} + \cancel{\int \vec{C}\cdot d\vec{s}} = \mu_0 I_{encl}$

$B(y)x + B(-y)x = \mu_0 I_{encl} \to |\vec{B}(y)| = |\vec{B}(-y)|$

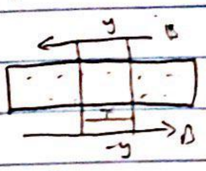
$2B(y)x = \mu_0 I_{encl}$

$I_{encl} \propto A_{encl} \to \frac{I_{encl}}{I} = \frac{2xy}{HW} \to$ uniform

$2B(y)x = \mu_0 \cdot I\left(\frac{2xy}{HW}\right)$

$\vec{B}(y) = -\mu_0 I\left(\frac{y}{HW}\right)\hat{x} \to \vec{B}$ inside the sheet

↳ Outside:



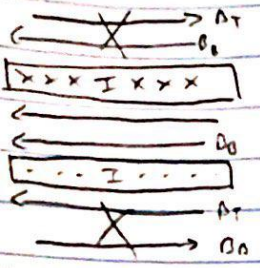
$|\vec{B}(y)| = |\vec{B}(-y)|$

$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$

$2B(y)x = \mu_0 I\frac{xH}{WH} = \mu_0 I\frac{x}{W}$

$$\boxed{\vec{B}(y) = \frac{\mu_0 I}{2W}\left(-\frac{y}{|y|}\hat{x}\right)}$$

↳ independent of distance from sheet

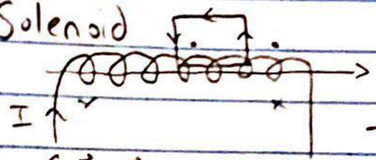
•


↳ All magnetic field is between the current sheets

↳ $\int \vec{B} \cdot d\vec{s} = \mu_0 I_{encl}$

$Bx = \mu_0 \frac{I}{w} x$

$B = \mu_0 \frac{I}{w}$

•Solenoid


→ can approximate $\vec{B} = 0$ outside

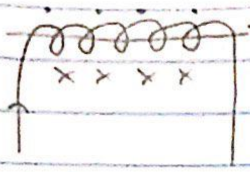
↳ $\oint \vec{B} \cdot d\vec{s} = \mu_0 I$

$Bx = \mu_0 N_x I$

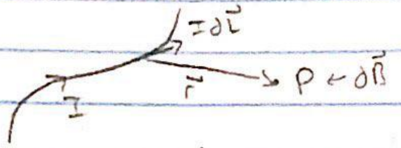
$B = \mu_0 n I \hat{z}$

Physics 1C Lecture 6: Biot-Savart

- $\rightarrow \vec{B} \rightarrow$ determined by RHR

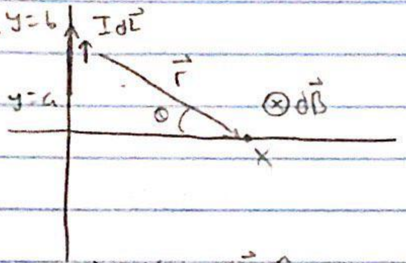


- Biot-Savart



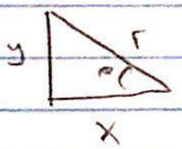
↳ $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\,d\vec{L} \times \hat{r}}{r^2}$

↳ $y = b$



↳ $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\,d\vec{L} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{I|d\vec{L} \times \hat{r}|}{r^2} (-\hat{z})$

$d\vec{B} = (-\hat{z}) \frac{\mu_0 I}{4\pi} \frac{dy \sin(90+\theta)}{r^2} \rightarrow \int d\vec{B}_z = (-\hat{z}) \frac{\mu_0 I}{4\pi} \int_a^b \frac{dy \cos\theta}{r^2}$



$y = x\tan\theta \rightarrow dy = \frac{x\,d\theta}{\cos^2\theta}$

$r = \frac{x}{\cos\theta}$

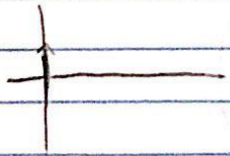
$\vec{B} = (-\hat{z}) \frac{\mu_0 I}{4\pi} \int_a^b \frac{x\,d\theta}{\cos^2\theta} \cdot \frac{\cos\theta}{x^2} \cos^2\theta$

$\vec{B} = (-\hat{z}) \frac{\mu_0 I}{4\pi x} \int_{\theta_a}^{\theta_b} \cos\theta\, d\theta$

$\vec{B} = (-\hat{z}) \frac{\mu_0 I}{4\pi x} [\sin\theta_b - \sin\theta_a] = \boxed{-(\hat{z}) \frac{\mu_0 I}{4\pi x} \left[\frac{b}{\sqrt{b^2+x^2}} - \frac{a}{\sqrt{a^2+x^2}}\right]}$

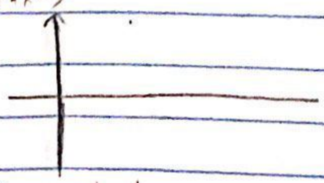
↳ special cases

↳ $a = -b$



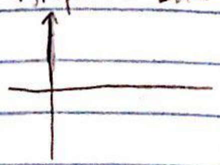
$\vec{B} = (-\hat{z}) \frac{\mu_0 I}{2\pi x} \left(\frac{b}{\sqrt{b^2+x^2}}\right)$

↳ $a = -b$, $b = \infty$



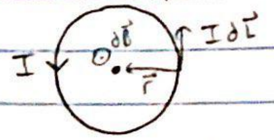
$\vec{B} = (-\hat{z}) \frac{\mu_0 I}{2\pi x} \rightarrow$ Ampere's Law

↳ $a = 0$, $b = \infty$



$\vec{B} = (-\hat{z}) \frac{\mu_0 I}{4\pi x}$

↳ Circular loop : I, R



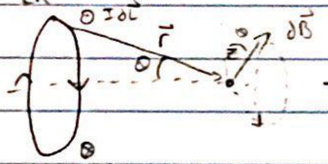
$$\partial\vec{B} = \frac{\mu_0}{4\pi}\frac{I\partial\vec{l}\times\hat{r}}{r^2}$$

$$\partial\vec{B} = \hat{z}\frac{\mu_0}{4\pi}\frac{I|\partial\vec{l}\times\hat{r}|}{R^2}$$

$$\int\partial\vec{B} = \frac{\mu_0 I}{4\pi R^2}\int\partial s\ \hat{z}$$

$$\vec{B} = \frac{\mu_0 I}{2R}\hat{z}$$

↳



→ symmetry tells us $\vec{B} = \vec{B}_z\hat{z}$

$$\vec{B} = B_z\hat{z} = \int\partial\vec{B}_z\hat{z}$$

$$\partial\vec{B}_z = \partial B\sin\theta = \frac{\mu_0 I}{4\pi}\frac{|\partial\vec{l}\times\hat{r}|}{r^2}\sin\theta$$

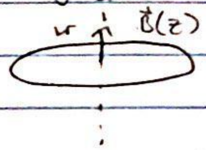
$$\partial B_z = \frac{\mu_0 I}{4\pi}\frac{\partial s}{R^2+z^2}\frac{R}{\sqrt{R^2+z^2}}$$

$$\int\partial B_z = \frac{\mu_0 I}{4\pi}\frac{R}{(R^2+z^2)^{3/2}}\int\partial s$$

$$B_z = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}$$

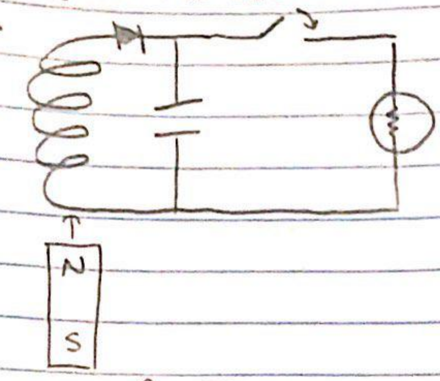
$$B = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}\hat{z}$$

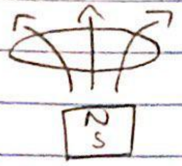
• Spinning uniform disk of Q, R



$$\partial\vec{B} = \frac{\mu_0}{4\pi}\frac{I\partial\vec{l}\times\hat{r}}{r^2}$$

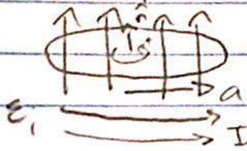
Physics 1C Lecture 7: Faraday's Law

- 

- 

$\rightarrow \mathcal{E}_i = -\frac{\partial \Phi_B}{\partial t}$

↳ Ex) Copper loop: radius $= a$ resistance $= R$



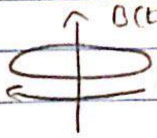
$\vec{B}(t) = B_0 e^{-t/T} \hat{z}$

$\Phi_B = BA = \pi a^2 (B_0 e^{-t/T})$

$\mathcal{E}_i = \frac{\pi a^2 B_0}{T} e^{-t/T} \rightarrow$ positive azimuthal direction

↳ not potential difference $\rightarrow$ path dependent $\rightarrow$ not conservative

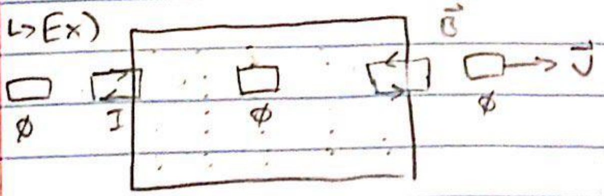
$I_i = \frac{\mathcal{E}_i}{R} = \frac{\pi a^2 B_0}{RT} e^{-t/T}$

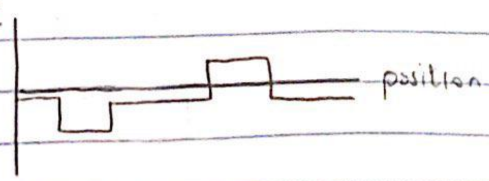
↳ 

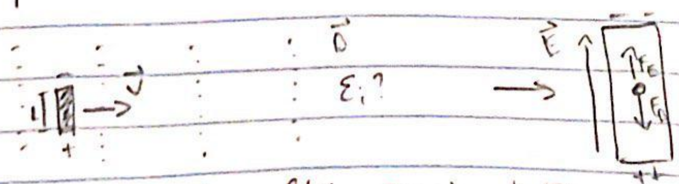
$\mathcal{E}_i, I_i \rightarrow$ against the current, must counter the effect that creates it

↳ Doesn't fully cancel original flux

↳ Ex)



↳ 

↳ Ex) 

$|\vec{F}_B| = qvB$

$|\vec{F}_E| = qE = q\frac{\Delta V}{L}$

$|\vec{F}_B| = |\vec{F}_E|$

$qvB = q\frac{\Delta V}{L}$

$\Delta V = \mathcal{E} = BLv_x$

$\Phi_B = BLX \rightarrow \mathcal{E}_i = -BLv_x \rightarrow$ motional EMF

Physics 1C Lecture 8: Inductance

- Generators

↳ $\mathcal{E}_i = -\frac{\partial \Phi_B}{\partial t}$

$\mathcal{E}_i = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{A}$

$\mathcal{E}_i = -\frac{\partial}{\partial t}\int B\,dA\cos\theta$

$\mathcal{E}_i = -\int \frac{\partial B}{\partial t}\,dA\cos\theta - B\frac{\partial A}{\partial t}\cos\theta + \int B\,dA\sin\theta\frac{\partial\theta}{\partial t}$

↳ 1st term = $\mathcal{E}_i$ generated by changing B → Faraday flashlight

↳ 2nd term = $\mathcal{E}_i$ generated by sweeping out area → bar thru magnetic field

↳ 3rd term = $\mathcal{E}_i$ generated by changing orientation

↳ Ex)

$\Phi_B = BA\cos\theta$

$\mathcal{E}_i = -BA\sin\theta\frac{\partial\theta}{\partial t}$

$\mathcal{E}_i = BA\omega\sin(\omega t)$

$P = \frac{\mathcal{E}^2}{R} = \frac{B^2A^2}{2R}\omega^2$

- 2 Kinds of EMF

↳ Conservative (1B) Electrostatic (ΔV)

↳ Non-Conservative Electrodynamic ($\mathcal{E}_i$)

↳ $W_E = W_{cons.} + W_{non\text{-}cons}$

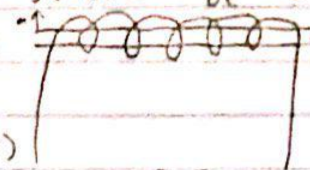
$\int q\vec{E}\cdot d\vec{s} = -q\Delta V + q\mathcal{E}_i$

$\oint \vec{E}\cdot d\vec{s} = \cancel{-\Delta V} + \mathcal{E}_i$ → conservative piece lost in closed loop integral

$\mathcal{E}_i = \oint \vec{E}\cdot d\vec{s}$

↳ $\oint \vec{E}\cdot d\vec{s} = -\frac{\partial\Phi_B}{\partial t}$ → non-conservative E-field

- Ex)



$\vec{B} = \mu_0 n I(t)\hat{z}$

$\oint \vec{E}\cdot d\vec{s} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{A}$ → $\vec{E}$ has to form a closed loop (no charges)

$E(2\pi r) = -\frac{\partial}{\partial t}\left[\mu_0 n I \pi r^2\right]$

$E = \frac{-\mu_0 n r}{2}\frac{\partial I}{\partial t}\hat{\phi}$ $(r<a)$

$(r>a)$

$\oint \vec{E}\cdot d\vec{s} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{A}$

$E(2\pi r) = -\frac{\partial}{\partial t}(\mu_0 n I \pi a^2)$ → a → magnetic field constrained by solenoid

$\vec{E} = -\frac{1}{2}\mu_0 n\frac{a^2}{r}\frac{\partial I}{\partial t}\hat{\phi}$ → E points in direction opposite of I increase

↳ E resultant from $\frac{\partial I}{\partial t}$ → magnetic field is changing

Physics 1C Lecture 9: AC Circuits

- $\int_0^T f(t)\,dt = \int_0^T \langle f \rangle\,dt \rightarrow$ average

  $\langle f \rangle = \frac{1}{T}\int_0^T f(t)\,dt$

  $\langle \sin\theta \rangle = 0 = \langle \cos\theta \rangle$

  $\langle \sin\theta\cos\theta \rangle = 0$

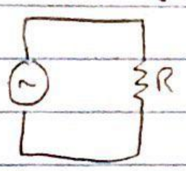
  $\langle \sin^2\theta \rangle = \langle \cos^2\theta \rangle = 1/2$

- $I = I_{max}\sin(\omega t) \rightarrow$ AC circuit

  $\langle I \rangle = I_{max}\langle \sin(\omega t)\rangle = 0$

- $\langle I^2 \rangle = I_{max}^2 \langle \sin^2(\omega t)\rangle = I_{max}^2/2$

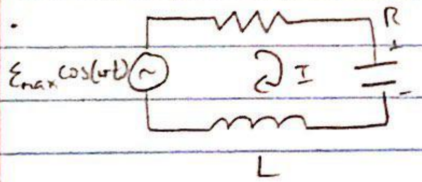
  ↳ $I_{rms} = \sqrt{\langle I^2 \rangle} = \frac{I_{max}}{\sqrt{2}}$

- 

  $P = I^2 R$

  $\langle P \rangle = I_{max}^2 R \langle \sin^2(\omega t)\rangle$

  $\langle P \rangle = \frac{I_{max}^2}{2} R = I_{rms}^2 R$

- 

  $I = \frac{dq}{dt}$

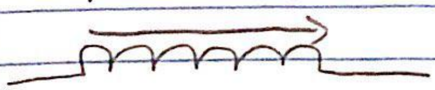
↳ $\mathcal{E}_{max}\cos(\omega t) - IR - q/C - L\frac{dI}{dt} = 0$

  $\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = (\mathcal{E}_{max}/L)\cos(\omega t) \rightarrow q \sim x,\ R \sim b,\ L \sim m,\ C \sim 1/k,\ \mathcal{E} \sim F$

  ↳ $U = \frac{1}{2}kx^2 \rightarrow U = \frac{1}{2}\frac{q^2}{C}$

  ↳ $K = \frac{1}{2}mv^2 \rightarrow K = \frac{1}{2}LI^2$

  ↳ $P = F\cdot v = -bv^2 = -I^2R$

- 

↳ $P = I\mathcal{E}_i = IL\frac{dI}{dt} = \frac{dU}{dt}$

  $\int dU = \int_0^I LI\,dI$

  $U = \frac{1}{2}LI^2 \rightarrow$ energy stored in inductor

↳ $U = \frac{1}{2}\mu_0 n^2 \pi a^2 x I^2 = \frac{1}{2}\frac{\pi a^2 x}{\mu_0}(\mu_0 n I)^2 = \frac{1}{2}\frac{B^2}{\mu_0}(\pi a^2 x)$

  $u_B \rightarrow$ energy density $= \frac{1}{2\mu_0}B^2 \rightarrow$ can lead to inductance

  $U = \int u_B\,dV$

- $x(t) = \frac{F_{max}/m}{\sqrt{(\frac{k}{m}-\omega^2)^2 + (\frac{b\omega}{m})^2}}\cos(\omega t + \xi)$

  ↳ $\tan\xi = -\frac{b\omega}{m(\frac{k}{m}-\omega^2)}$

$q(t) = \frac{\mathcal{E}_{max}/L}{\sqrt{(\frac{1}{LC}-\omega^2)^2 + (\frac{R\omega}{L})^2}} \cos(\omega t + \delta)$ → how much charge is on the capacitor

$\tan\delta = \frac{-R\omega}{L(1/LC - \omega^2)}$

↳ $I(t) = \frac{\mathcal{E}_{max}}{\sqrt{(\frac{1}{\omega C}-\omega L)^2 + R^2}} \sin(\omega t + \delta) = \frac{\mathcal{E}_{max}}{\sqrt{(\frac{1}{\omega C}-\omega L)^2 + R^2}} \cos(\omega t + \delta - \frac{\pi}{2})$

$\tan\delta = \frac{-R}{1/\omega C - \omega L}$

↳ $I(t) = \frac{\mathcal{E}_{max}}{\sqrt{(\frac{1}{\omega C}-\omega L)^2 + R^2}} \cos(\omega t - \phi)$

$\tan\phi = \frac{\omega L - 1/\omega C}{R}$

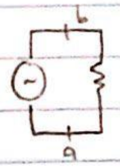
↳ $I_{max} = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \longleftrightarrow I = \frac{\Delta V}{R}$

↳ Impedance (Z) → ohms

↳ $Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$ → opposition to flow of AC

↳ $I_{max} = \frac{\mathcal{E}_{max}}{Z}$

↳ Current lags voltage by ϕ

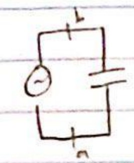
- 

$\Delta V_{AB} = \Delta V_{BA}$

$\mathcal{E}_{max}\cos(\omega t) = IR$

$I = \frac{\mathcal{E}_{max}}{R}\cos(\omega t)$

$I_{max} = \frac{\mathcal{E}_{max}}{R}$ → I in phase w/ voltage

- 

$\Delta V_{AB} = \Delta V_{BA}$

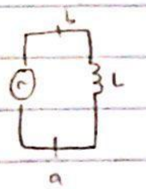
$\mathcal{E}_{max}\cos(\omega t) = \frac{q}{C}$

$q = \mathcal{E}_{max} C \cos(\omega t)$

$I = -\mathcal{E}_{max}\omega C \sin(\omega t) = \mathcal{E}_{max}\omega C \cos(\omega t + 90°)$

$I_{max} = \frac{\mathcal{E}_{max}}{1/\omega C}$ → I leads $\mathcal{E}$ by 90°

↳ $\frac{1}{\omega C} \rightarrow R$

- 

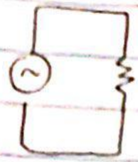
$\mathcal{E}_{max}\cos(\omega t) = L\frac{dI}{dt}$

$I = \frac{\mathcal{E}_{max}}{\omega L}\sin(\omega t) = \frac{\mathcal{E}_{max}}{\omega L}\cos(\omega t - 90°)$ → I lags $\mathcal{E}$ by 90°

$I_{max} = \frac{\mathcal{E}_{max}}{\omega L}$

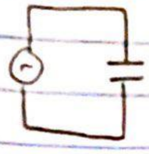
- ELI the ICE man

Physics 1C Lecture 10: Reactance

- 

$P = I\mathcal{E} = I_{max}\mathcal{E}_{max}\sin^2(\omega t) \rightarrow I/\mathcal{E}$ in phase across resistor

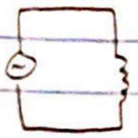
$\langle P \rangle = \frac{I_{max}\mathcal{E}_{max}}{2} = I_{rms}\mathcal{E}_{rms}$

- 

$P = I\mathcal{E} = I_{max}\mathcal{E}_{max}\sin(\omega t)\cos(\omega t) \rightarrow 90°$ out of phase

$\langle P \rangle = I_{max}\mathcal{E}_{max}\langle \sin(\omega t)\cos(\omega t)\rangle = 0$

↳ store energy, get it back → Avg. P consumption = 0

- 

$P = I\mathcal{E} = I_{max}\mathcal{E}_{max}\sin(\omega t)\cos(\omega t) \rightarrow 90°$ out of phase

$\langle P \rangle = 0$

↳ Energy pushed into B by motion of charge, released by same way

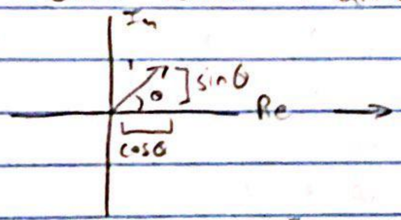
↳ Capacitors/inductors are conservative devices → don't have resistance

- Reactance

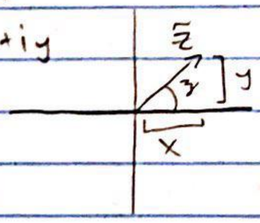
↳ $X_C = \frac{1}{\omega C} \rightarrow$ Capacitor

↳ $X_L = \omega L \rightarrow$ Inductor

↳ $e^{i\theta} = \cos\theta + i\sin\theta \rightarrow$ unit phasor

↳ 

$\tilde{z} = x + iy$



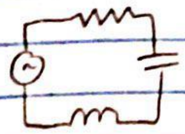
↳ $\tilde{z} = \sqrt{x^2+y^2}\, e^{i\gamma}$, $\tan\gamma = \frac{y}{x}$

$\tilde{z}^* = x - iy$

$\tilde{z}^* = \sqrt{x^2+y^2}\, e^{-i\gamma}$

$\tilde{z}\tilde{z}^* = |\tilde{z}|^2 = x^2 + y^2$

$\cos\theta = Re(e^{i\theta}) \rightarrow \cos\theta \rightarrow e^{i\theta}$

- 

$I = I_{max}\cos(\omega t)$

↳ $\Delta V_R = \Delta V_{Rmax}\cos(\omega t)$

$\Delta V_C = \Delta V_{Cmax}\cos(\omega t - 90°) \rightarrow$

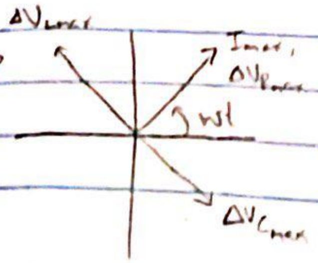
$\Delta V_L = \Delta V_{Lmax}\cos(\omega t + 90°)$

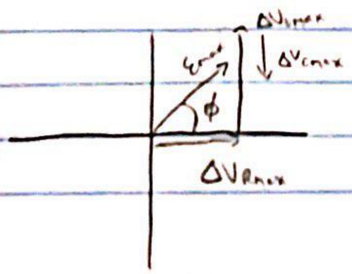
$I_{max} = I_{max}e^{i\omega t}$

$\Delta V_R = \Delta V_{Rmax}e^{i\omega t}$

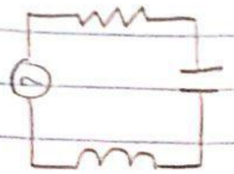
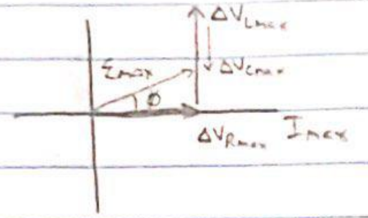
$\Delta V_C = \Delta V_{Cmax} = e^{i(\omega t - 90°)}$

$\Delta V_L = \Delta V_{Lmax} = e^{i(\omega t + 90°)}$

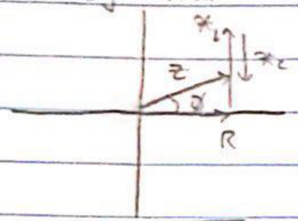
↳ 



Physics 1C Lecture 11: Resonance

- 


↳ divide by I_{max}



$\tilde{Z}_R = R$
$\tilde{Z}_L = iX_L$
$\tilde{Z}_C = -iX_C$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$, $\tan\phi = \frac{X_L - X_C}{R}$

↳ replace X_L w/ ωL and X_C w/ $\frac{1}{\omega C}$

↳ $P = I\mathcal{E}$ → Energy delivered by the source

↳ $P = I_{max}\mathcal{E}_{max}\cos(\omega t)\cos(\omega t + \phi)$

$P = I_{max}\mathcal{E}_{max}\cos(\omega t)[\cos(\omega t)\cos\phi - \sin(\omega t)\sin\phi]$

$\langle P\rangle = I_{max}\mathcal{E}_{max}[\langle\cos^2(\omega t)\rangle\cos\phi - \langle\cos(\omega t)\sin(\omega t)\rangle\sin\phi]$

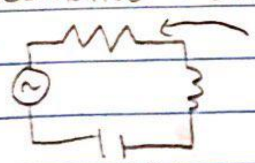
$\langle P\rangle = I_{max}\mathcal{E}_{max}(\frac{1}{2}\cos\phi) = I_{rms}\mathcal{E}_{rms}\cos\phi$ → $\cos\phi$ = power factor

↳ $\langle P\rangle = I_{rms}^2 Z\cos\phi$ → $Z\cos\phi = R$ → $I_{rms}^2 R$

↳ Power factor exists to take resistance (power consumer) out of Z

↳ Max power consumed when Z is purely resistance

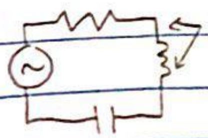
- Resonance → transmitting maximum current to load

↳ 

$I_{max} = \frac{\mathcal{E}_{max}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + (X_L - X_C)^2}}$

↳ maximized when $X_L = X_C$ → $\tilde{Z}$ is pure resistance

↳ $\omega L = \frac{1}{\omega C}$ → $\omega = 1/\sqrt{LC}$

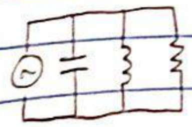
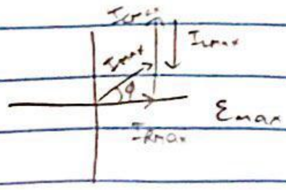
↳ 

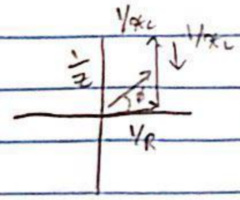
$\Delta V_{Lmax} = I_{max}X_L = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}$

what if $X_L = X_C$, $X_L >>> R$

↳ $\Delta V_{Lmax} >>> \mathcal{E}_{max}$

- Parallel Circuits

↳ 




$\frac{1}{\tilde{Z}} = \sum_i \frac{1}{\tilde{Z}_i}$

Physics 1C Lecture 12: Parallel AC Circuits

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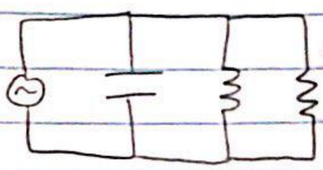
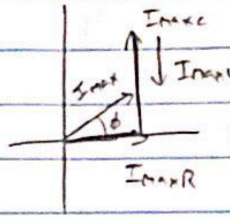


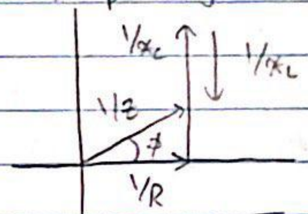


$\mathcal{E}_{max} \rightarrow$ in common

↳ $\mathcal{E}_{max} = I_{max} Z$

$\mathcal{E}_{max} = I_{maxR} R$, $\mathcal{E}_{max} = I_{maxC} X_C$, $\mathcal{E}_{max} = I_{maxL} X_L$

↳ rescale plot by $1/\mathcal{E}_{max}$



↳ $\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}$

$\tan\phi = \frac{(1/X_C - 1/X_L)}{1/R}$

• Resonance: Max current to load

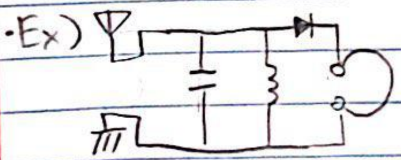
↳ $I_{maxR} = \frac{\mathcal{E}_{max}}{R}$

$I_{maxR} = I_{max} \frac{Z}{R}$

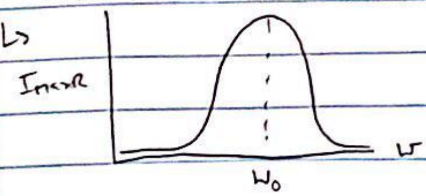
$I_{maxR} = I_{max} \frac{1}{R\sqrt{1/R^2 + (1/X_C - 1/X_L)^2}}$

$X_C = X_L \rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \rightarrow$ series/parallel when load is 100% resistive

• Ex)

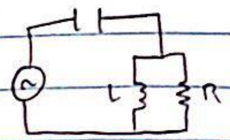


↳



$Q \propto \frac{1}{FWHM}$

• Ex)



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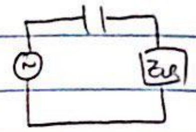

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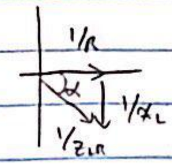
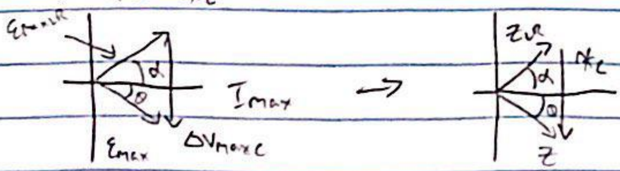


↳ $\frac{1}{Z_{LR}} = \sqrt{1/R^2 + 1/X_L^2}$

$\tan\alpha = \frac{R}{X_L}$



$Z = \sqrt{(Z_{LR}\cos\alpha)^2 + (Z_{LR}\sin\alpha - X_C)^2}$

$\tan\theta = \frac{Z_{LR}\sin\alpha - X_C}{Z_{LR}\cos\alpha}$

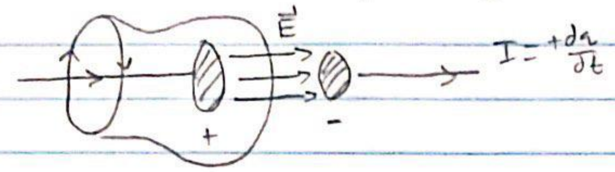
Physics 1C Lecture 13: Maxwell's Equations

- $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$

↳ path integral around closed loop on left

↳ current enclosed by loop on right

↳



↳ $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{pierce} \rightarrow B = 0$

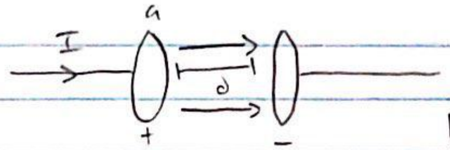
↳ $\Phi_E = EA = \frac{\sigma}{\varepsilon_0} A$

$\Phi_E = q/\varepsilon_0$

$I = \frac{dq}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt}$

↳ $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

↳



$|\vec{E}| = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 \pi a^2}$

↳

$\oint \vec{B}\cdot d\vec{s} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

$B(2\pi r) = 0 + \mu_0 \varepsilon_0 \frac{d}{dt}\left[\frac{Q}{\varepsilon_0 \pi a^2}\pi r^2\right]$

$2\pi r B = \mu_0 \varepsilon_0 \frac{1}{\varepsilon_0}\frac{r^2}{a^2} I$

$B2\pi = \frac{\mu_0 I r}{2\pi a^2}\hat{\phi}$ $(r<a)$

↳ $r>a$: $\oint \vec{B}\cdot d\vec{s} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

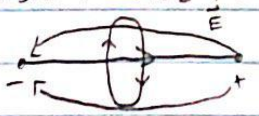
$B2\pi r = \mu_0 \varepsilon_0 \frac{d}{dt}\left[\frac{Q}{\varepsilon_0 \pi a^2}\pi a^2\right]$

$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}}$

↳ $I_{disp} = \varepsilon_0 \frac{d\Phi_E}{dt}$ → displacement current

↳ $\oint \vec{B}\cdot d\vec{s} = \mu_0 \left[I_{physical} + I_{disp}\right]$

-



$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$, $\vec{E}(t) \rightarrow \Phi_E \rightarrow t$

- Gauss: $\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$ Gauss: $\oint \vec{B}\cdot d\vec{A} = 0$

Faraday: $\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ Ampere: $\oint \vec{B}\cdot d\vec{s} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

- $\vec{\nabla} = \frac{\partial}{\partial \vec{r}} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$

- Divergence Theorem: $\oint \vec{G}\cdot d\vec{A} \equiv \int (\vec{\nabla}\cdot\vec{G})\, dV$

↳ $\oint \vec{E}\cdot d\vec{A} = q_{in}/\varepsilon_0 \rightarrow \int (\vec{\nabla}\cdot\vec{E})\, dV = \int \frac{\rho}{\varepsilon_0}\, dV \rightarrow \vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$

↳ $\oint \vec{B}\cdot d\vec{A} = 0 \rightarrow \vec{\nabla}\cdot\vec{B} = 0$

- Stoke's Theorem: $\oint \vec{G}\cdot d\vec{s} \equiv \int (\vec{\nabla}\times\vec{G})\cdot d\vec{A} \rightarrow \oint \vec{B}\cdot d\vec{s} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \rightarrow \vec{\nabla}\times\vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

↳ $\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt} \rightarrow \int (\vec{\nabla}\times\vec{E})\cdot d\vec{A} = \int -\frac{\partial \vec{B}}{\partial t}\cdot d\vec{A} \rightarrow \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

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Physics 1C Lecture 14: Free-Space

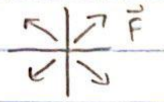
- $\oint \vec{E}\cdot d\vec{A} = q_{enc}/\varepsilon_0$ $\quad \oint \vec{B}\cdot d\vec{A} = 0$ $\quad \oint \vec{E}\cdot d\vec{s} = -\frac{\partial \Phi_B}{\partial t}$ $\quad \oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{\partial \Phi_E}{\partial t}$

↳ $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$ $\quad \vec{\nabla}\cdot\vec{B} = 0$ $\quad \vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$ $\quad \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial E}{\partial t}$

↳ $\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} \to \vec{\nabla}\cdot\vec{G} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y} + \frac{\partial G_z}{\partial z} \to$ Divergence of $\vec{G}$

↳ $\vec{\nabla}\times\vec{G} = \hat{x}\left(\frac{\partial G_z}{\partial y} - \frac{\partial G_y}{\partial z}\right) + \hat{y}\left(\frac{\partial G_x}{\partial z} - \frac{\partial G_z}{\partial x}\right) + \hat{z}\left(\frac{\partial G_y}{\partial x} - \frac{\partial G_x}{\partial y}\right) \to$ Curl of $\vec{G}$

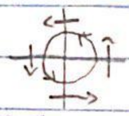
- $\vec{F} = x\hat{x} + y\hat{y}$



↳ $\vec{\nabla}\cdot\vec{F} = 1+1+0 = 2$

↳ $\vec{\nabla}\times\vec{F} = \hat{x}(0-0) + \hat{y}(0-0) + \hat{z}(0-0) = 0$

- $\vec{G} = -y\hat{x} + x\hat{y}$



- $\vec{\nabla}\cdot\vec{G} = 0+0+0 = 0$
- $\vec{\nabla}\times\vec{G} = \hat{x}(0-0) + \hat{y}(0-0) + \hat{z}(1+1) = 2\hat{z}$
- Divergence → tendency of a field to diverge from a point
- Curl → tendency to curl around an axis
- In order to diverge, there must be something different about that point

↳ $\vec{E}$ → source or sink of charge → $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$

↳ $\vec{B}$ → no sources/sinks → no divergence → $\vec{\nabla}\cdot\vec{B} = 0$

- In order to curl, there must be an axis to curl around

↳ $\vec{E}$ → loops around changing $\vec{B}$-fields → $\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$

↳ $\vec{B}$ → current density / changing $\vec{E}$-fields → $\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial E}{\partial t}$

↳ statements about how $\vec{E}$ and $\vec{B}$ fill space given distribution of sources, sinks, and currents

- Free-Space: $\rho = 0$, $\vec{J} = 0$

↳ $\vec{\nabla}\cdot\vec{E} = 0$, $\vec{\nabla}\cdot\vec{B} = 0$, $\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$, $\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial E}{\partial t}$

↳ Self-perpetuating fields ($\frac{\partial B}{\partial t}$ causes $\frac{\partial E}{\partial t}$ causes $\frac{\partial B}{\partial t}$ ...)

↳ $\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$

$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$

↳ $\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2}$

↳ $\vec{A}\times(\vec{B}\times C) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$

↳ $\vec{\nabla}\times\vec{\nabla}\times\vec{E} = \vec{\nabla}(\vec{\nabla}\cdot E) - \vec{E}(\vec{\nabla}\cdot\vec{\nabla}) = -\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$

↳ $\vec{\nabla}\times\vec{\nabla}\times\vec{B} = \vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \vec{E}(\vec{\nabla}\cdot\vec{\nabla}) = -\mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2}$

↳ $\nabla^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2} \to \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2} = \frac{\partial^2 E}{\partial x^2}$

↳ $\nabla^2\vec{B} = \mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2} \to \mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2} = \frac{\partial^2 B}{\partial x^2}$

traveling waves

↳ $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

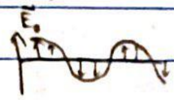
Physics 1C Lecture 15: EM Waves

- $\vec{\nabla}\cdot\vec{E}=0$, $\vec{\nabla}\cdot\vec{B}=0$, $\vec{\nabla}\times\vec{E}=-\frac{\partial \vec{B}}{\partial t}$, $\vec{\nabla}\times\vec{B}=\mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t}$

↳ $\vec{\nabla}^2\vec{E}=\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$, $\nabla^2\vec{B}=\mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$ → $\vec{E}$ and $\vec{B}$ propagate in free space as traveling waves

↳ Assume waves are traveling along the x-axis (1-D)

↳ $\vec{E}=\vec{E}_0\sin(kx-\omega t+\phi)$
$\vec{B}=\vec{B}_0\sin(kx-\omega t+\theta)$] → $f=A\sin(kx-\omega t+\phi)$

↳ vectors → 

↳ $k=\frac{2\pi}{\lambda}$, $\omega=\frac{2\pi}{T}=kv_x$, $T=\frac{1}{f}$, $f\lambda=v_x$, $v_x=c$

↳ $\vec{E}=\langle E_{0x},E_{0y},E_{0z}\rangle\sin(kx-\omega t+\phi)$

↳ $\frac{\partial\vec{E}}{\partial t}=-\omega\langle E_{0x},E_{0y},E_{0z}\rangle\cos(kx-\omega t+\phi)$

↳ $\frac{\partial\vec{E}}{\partial x}=k\langle E_{0x},E_{0y},E_{0z}\rangle\cos(kx-\omega t+\phi)$, $\frac{\partial\vec{E}}{\partial y}=0$, $\frac{\partial\vec{E}}{\partial z}=0$

↳ $\vec{B}$: $E\to B$, $\phi\to\theta$

↳ $\vec{\nabla}\cdot\vec{E}=0\to\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}=0$

↳ $\frac{\partial E_x}{\partial x}=0\to kE_{0x}\cos(kx-\omega t+\phi)=0\to E_{0x}=0$

↳ $\vec{\nabla}\cdot\vec{B}=0\to B_{0x}=0$

↳ no component along propagation (x-axis)

↳ $\vec{E}$ and $\vec{B}$ are transverse waves

↳ $\vec{\nabla}\times\vec{G}=\hat{x}\left(\frac{\partial G_z}{\partial y}-\frac{\partial G_y}{\partial z}\right)+\hat{y}\left(\frac{\partial G_x}{\partial z}-\frac{\partial G_z}{\partial x}\right)+\hat{z}\left(\frac{\partial G_y}{\partial x}-\frac{\partial G_x}{\partial y}\right)$

↳ $=-\hat{y}\frac{\partial G_z}{\partial x}+\hat{z}\frac{\partial G_y}{\partial x}$

↳ $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$

$k\langle 0,-E_{0z},E_{0y}\rangle\cos(kx-\omega t+\phi)=\omega\langle 0,B_{0y},B_{0z}\rangle\cos(kx-\omega t+\theta)$

↳ $\phi=\theta$ → $\vec{E}$ and $\vec{B}$ in phase

↳ $-kE_{0z}=\omega B_{0y}$, $kE_{0y}=\omega B_{0z}$

↳ $E_{0y}=cB_{0z}$, $E_{0z}=-cB_{0y}$

↳ $\vec{\nabla}\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}=\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$

$k\langle 0,-B_{0z},B_{0y}\rangle\cos(kx-\omega t+\phi)=\frac{-\omega}{c^2}\langle 0,E_{0y},E_{0z}\rangle\cos(kx-\omega t+\phi)$

$-kB_{0z}=\frac{-\omega}{c^2}E_{0y}$, $kB_{0y}=-\frac{\omega}{c^2}E_{0z}$

↳ $E_{0y}=cB_{0z}$, $E_{0z}=-cB_{0y}$

↳ $\vec{E}$ and $\vec{B}$ are transverse

↳ $\vec{E}$ and $\vec{B}$ are in phase

↳ $E_{0y}=cB_{0z}$, $E_{0z}=-cB_{0y}$

- $\vec{E}_0\cdot\vec{B}_0=\langle 0,E_{0y},E_{0z}\rangle\cdot\langle 0,B_{0y},B_{0z}\rangle=c\langle 0,B_{0z},-B_{0y}\rangle\cdot\langle 0,B_{0y},B_{0z}\rangle=0$

↳ $\vec{E}_0\perp\vec{B}_0$, $\vec{E}\perp\vec{B}$

- $\vec{E}_0\times\vec{B}_0=\hat{x}(E_{0y}B_{0z}-E_{0z}B_{0y})=c\hat{x}(B_{0z}^2+B_{0y}^2)=\hat{x}B_0^2c$

↳ $\hat{x}E_0^2/c$ → same direction as wave is traveling

-  $\rightarrow \vec{E}\times\vec{B} \parallel \vec{v}$, $\vec{E}\perp\vec{B}\perp\vec{v}\perp\vec{E}$, $|\vec{E}|=c|\vec{B}|$

↳ Polarization follows $\vec{E}$ (vertically polarized above)

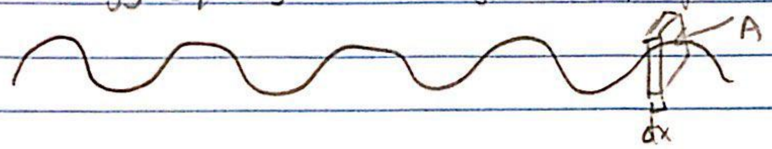
↳ If charge q in path of wave

↳ $F_E = qE$, $F_{B_{max}} = qvB = qE\frac{v}{c}$

↳ $\vec{E}$ field does most of the work → wires aligned w/ $\vec{E}$

↳ $u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 c^2 B^2 = \frac{1}{2\mu_0}B^2 = u_B$

↳ energy equally divided by electric/magnetic fields

- 

↳ How much energy in the box?

$du = (u_B + u_E)\,dV$

$du = 2u_E\,dx\,A$

$du = \varepsilon_0 E^2 A\,dx$

$du = \varepsilon_0 ABcE\,dx$

↳ What is the rate at which energy enters front plane

$\frac{du}{dt} = \varepsilon_0 ABcE\frac{dx}{dt}$

$\frac{du}{dt} = \varepsilon_0 ABc^2E$

$\frac{du}{dt} = \frac{1}{\mu_0}|\vec{E}\times\vec{B}||\vec{A}|\cos(0)$

$P = \frac{1}{\mu_0}(\vec{E}\times\vec{B})\cdot\vec{A}$

$P = \int\frac{1}{\mu_0}(\vec{E}\times\vec{B})\cdot d\vec{A}$ → flux of something

↳ $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ → Poynting Vector

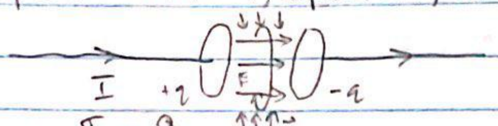
↳ Points in direction energy is carried

↳ $|\vec{S}|$ = intensity of wave ($\frac{W}{m^2}$)

↳ $P = \Phi_S$

Physics 1C Lecture 16: Poynting Vector

- Poynting Vector: $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$, $|\vec{S}| = \frac{1}{\mu_0}EB$ (EM wave)
  - ↳ Magnitude gives intensity (W/m^2)
  - ↳ $\vec{S}$ ∥ to propagation of wave / direction energy flows
  - ↳ $P = \Phi_{\vec{S}} = \int \vec{S}\cdot d\vec{A}$
- Capacitor: radius a, separation d, charge $q(t)$



↳ $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 \pi a^2}$

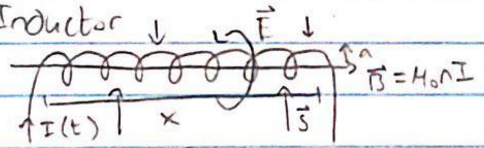
↳ $B(a) = \frac{\mu_0 I}{2\pi a}$

↳ $S(a) = \frac{1}{\mu_0}EB = \frac{Q}{\varepsilon_0\pi a^2}\left(\frac{I}{2\pi a}\right) = \frac{QI}{2\varepsilon_0\pi^2 a^3}$

↳ $P(a) = \int \vec{S}\cdot d\vec{A} = \int S(a)\,dA = \frac{QI}{2\varepsilon_0\pi^2a^3}(2\pi a d) = \frac{QI}{C}$

↳ $\frac{\partial}{\partial t}\left(\frac{1}{2}\frac{Q^2}{C}\right) = \frac{\partial U}{\partial t}$

- Inductor



↳ $\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$

$E(2\pi a) = -\frac{\partial}{\partial t}\left[\mu_0 n I \pi a^2\right]$

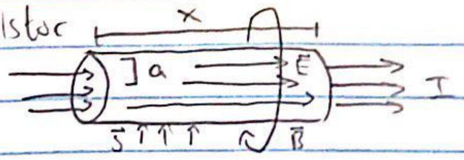
$E = -\frac{\mu_0 n a}{2}\frac{\partial I}{\partial t}$

↳ $S(a) = \frac{1}{\mu_0}EB = \frac{nI\mu_0 n a}{2}\frac{\partial I}{\partial t} = \frac{\mu_0 n^2 a}{2} I \frac{\partial I}{\partial t}$

↳ $P = \int \vec{S}\cdot d\vec{A} = S(a)\int dA = \frac{\mu_0 n^2 a}{2} I \frac{\partial I}{\partial t}(2\pi a x)$

$P = \mu_0 n^2 \pi a^2 x I \frac{\partial I}{\partial t} = LI\frac{\partial I}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}LI^2\right) = \frac{\partial}{\partial t}U$

- Resistor



↳ $\Delta V = IR$

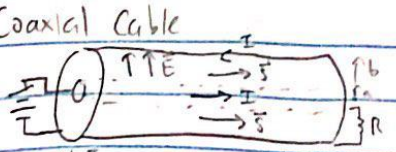
$Ex = IR$

$E = \frac{IR}{x}$

↳ $B = \frac{\mu_0 I}{2\pi a}$

↳ $S(a) = \frac{1}{\mu_0}EB = \frac{R}{x}\frac{I^2}{2\pi a} = \frac{I^2R}{2\pi a x} = \frac{P}{A}$

- Coaxial Cable



↳ $B = \frac{\mu_0 I}{2\pi r}$

↳ $E = \frac{\lambda}{2\pi\varepsilon_0 r}$, $\mathcal{E} = \Delta V = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right) \rightarrow E = \frac{\mathcal{E}}{r\ln(b/a)}$

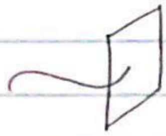
↳ $S(r) = \frac{1}{\mu_0}EB = \frac{\mathcal{E}}{r\ln(b/a)}\frac{I}{2\pi r}$

$S(r) = \frac{I\mathcal{E}}{2\pi r^2\ln(b/a)}$

$P = \int \frac{I\mathcal{E}}{2\pi\ln(b/a)}\frac{2\pi r\,dr}{r^2} = \frac{I\mathcal{E}}{\ln(b/a)}\ln(b/a) = I\mathcal{E} = \mathcal{E}^2/R$

Physics 1C Lecture 17: Light Pressure

- Light pressure


←perfect absorber

↳ $U = PC$

↳ $P = \frac{mv}{\sqrt{1-v^2/c^2}}$ → obj.s at spd. of light $= 0/0$

↳ only massless particles can travel at speed of light

↳ $\Delta U = \Delta p C$ → divide by Δt

$\langle P \rangle = \langle F_{net} \rangle c$ → $\frac{\Delta p}{\Delta t} = F$, $\frac{\Delta p}{\Delta t} = \langle F \rangle$

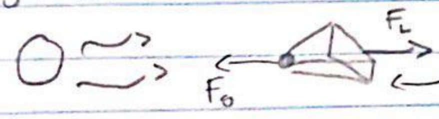
$\langle S \rangle A = \langle F_{net} \rangle c$

$\frac{\langle F_{net} \rangle}{A} = \frac{\langle S \rangle}{c} = \mathbb{P}$ → pressure

↳ Light is completely absorbed

↳ Light is perfectly reflected: $P = 2\frac{\langle S \rangle}{c}$

- Light Sail



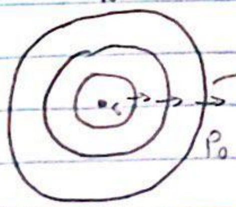
← optimally a perfect reflector (largest ΔJ)

↳ $\sum F_x = ma_x$

$F_L - F_g = ma_x$

$\frac{2\langle S \rangle}{c} A - \frac{GM_s m}{R^2} = mA_x$ → $\langle S \rangle = \frac{P}{A} = \frac{P_s}{4\pi r^2}$

↳



→ P_0 J/s must pass through each shell

$\frac{2P_s A}{4\pi c r^2} - \frac{GM_s m}{r^2} = mA_x$

$\frac{1}{r^2}\left[\frac{2P_s A}{4\pi c} - GM_s m\right] = mA_x$

↳ bracketed term must be positive to accelerate $\left(\frac{2P_s A}{4\pi c} - GM_s m > 0\right)$

↳ $A > \frac{GM_s m \, 2\pi c}{P_s} \approx 66000 m^2$

- EM Spectrum

↳ 0-3 MHz	∞ - 100m	LF	430 THz - 750 THz	700nm - 400nm	visible
3-30 MHz	100m - 10m	HF	750 THz - 3×10^4 THz	400nm - 10nm	UV
30-300 MHz	10m - 1m	VHF	3×10^4 THz - 3×10^6 THz	10nm - 0.1nm	X-ray
300MHz - 3GHz	1m - 10cm	UHF	3×10^6 THz →	0.1nm →	Gamma
3GHz - 300GHz	10cm - 1mm	microwave			
300GHz - 430THz	1mm - 700nm	IR			

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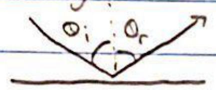
Physics 1C Lecture 18: Intro to Optics

- Empirical rules of optics:

↳ In any given uniform medium, light travels in a straight line

↳ When light reflects off a medium, the angle of incidence is equal to the angle of reflection

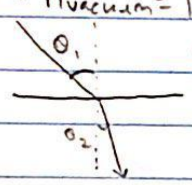
↳ 

→ measured with respect to the normal

↳ When light passes from one medium to another, it follows "Snell's Law"

↳ $n_1 \sin\theta_1 = n_2 \sin\theta_2$, where n is $\frac{c}{v_i}$

↳ $n_{vacuum} = 1$, $n_{air} = 1$, $n_{water} = 4/3$, $n_{glass} = 3/2$

↳ 

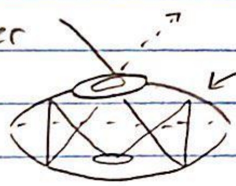
θ_1 = angle of incidence, θ_2 = angle of refraction

$\sin\theta_i \propto \frac{1}{n_i}$

↳ small θ_i → $\theta_i \propto \frac{1}{n_i}$

↳ $\theta_2 < \theta_1 \rightarrow n_2 > n_1 \rightarrow v_2 < v_1$

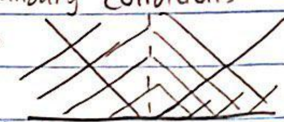
↳ Light rays are reversible → a ray that works in one direction also works in the other

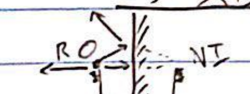
↳ 

↳ Empirical: obtained by observation

↳ Fermat's principle of least time → light chooses the path that takes the least time

↳ Boundary conditions

↳  → $\theta_i = \theta_r$

- 

↳ Rays that are incident on a lens or mirror intersect at the location of the object for that lens or mirror

↳ If the object is on the same side as the incident light, it is a real object

↳ If the object is on the opposite side as the incident light, it is a virtual object

↳ Rays that emerge intersect at the image produced by that lens/mirror

↳ If the image is on the same side as the emerging light, it is a real image

↳ If on the opposite side → virtual image

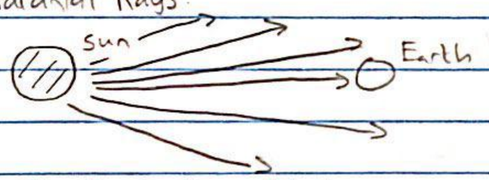
↳ Incident light → object

↳ Emerging light → image

↳ Same side → real

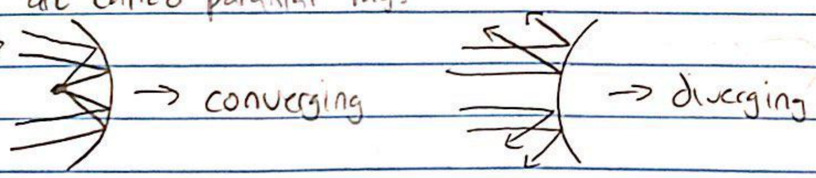
↳ Opposite side → virtual

• Paraxial Rays:

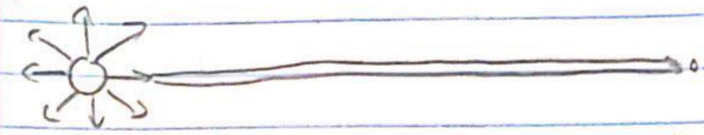


↳ The light from distant sources tends to arrive at an observation point as a set of parallel rays

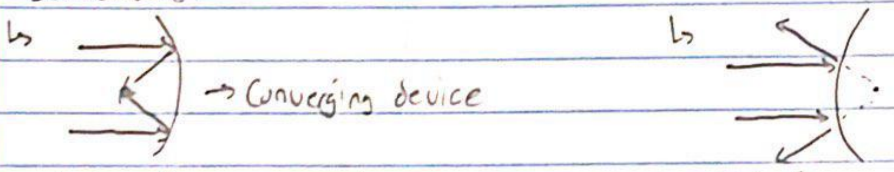
↳ If the observational instrument is aligned with that distant source → "optical axis" points at source → rays run parallel to the optical axis and are called paraxial rays

↳ 

Physics 1C Lecture 19: Optic Conventions

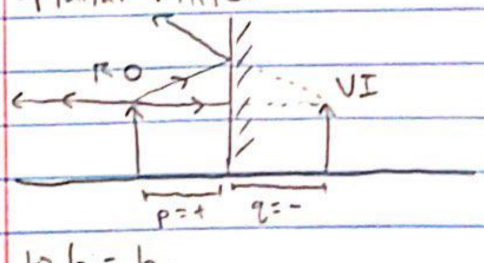


- all rays that arrive are from a very limited range of angles
  - Paraxial rays
- Conventions:



- light from a distant source converges on a focal point
  - concave to incident light
- light from a distant source diverges from a focal point
  - convex to incident light
- Object distance (p) - dist. from object to optical device, measured along the optical axis
  - $p=+$ for real objects, $p=-$ for virtual objects
- Image distance (q) - dist from optical object to image along optical axis
  - $q=+$ for real images, $q=-$ for virtual objects
- Focal length (f) - distance from focal point to lens or mirror, measured along OA
  - $f=+$ for converging, $f=-$ for negative
- Magnification (m) - $M \equiv \frac{h_i}{h_o}$
  - $M=+$ → i and o have same orientation (uplight)
  - $M=-$ → i and o have opposite orientation (inverted)

- Planar Mirror



- $h_i = h_o$

  $M = \frac{h_i}{h_o} = 1$

  $M = -\frac{q}{p}$

- Equation:
  - $p \leftrightarrow q$: reversibility of rays
  - $p \to \infty, q \to f \leftrightarrow p \to f, q \to \infty$
  - $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

·Technically, paraxial rays only focus to a point if the focusing surface is parabolic

↳ Limit consideration to small θ, we can treat spherical surfaces as parabolic surfaces

↳ causes "spherical aberration"

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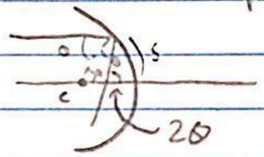

↳ $S = R\theta$

$R\theta = X 2\theta$

$x = \frac{R}{2} = |f|$

↳ Converging: $f = +\frac{R}{2}$, Diverging $f = -\frac{R}{2}$

↳ Convention: $R = +$ for concave mirrors, $R = -$ for convex mirror

↳ $f = \frac{R}{2}$

↳ $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R}$

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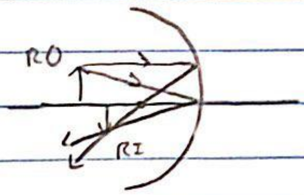

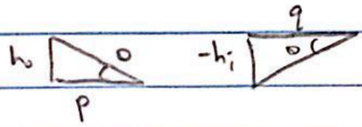

↳ $M = \frac{h_i}{h_o} = \frac{-q}{p}$

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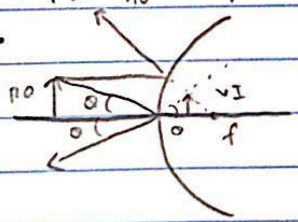
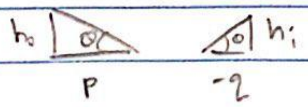




↳ $M = \frac{h_i}{h_o} = \frac{-q}{p}$

↳ Spherical mirrors:

↳ $1/p + 1/q = 1/f = 2/R$

↳ $q = \frac{pf}{p-f}$

↳ $M = \frac{f}{f-p}$

Physics 1C Lecture 20: Spherical Mirrors

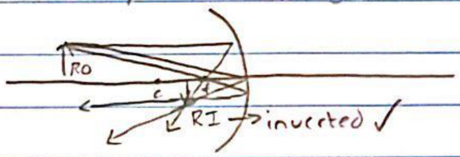
- $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{2}{R}$, $M=\frac{h_i}{h_o}=-\frac{q}{p}$

↳ Ex) Concave mirrors, real objects → $f=+$, $p=+$

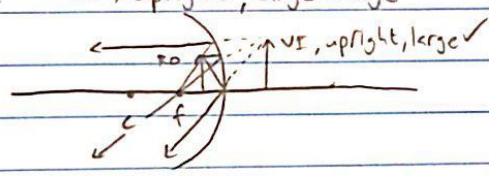
$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

$q=\frac{pf}{p-f}$, $M=\frac{-f}{p-f}$ → 2 regions ($p>f$ and $p<f$)

↳ $p>f$ → RI, Inverted, any size

↳ 

↳ $p<f$ → VI, Upright, large image

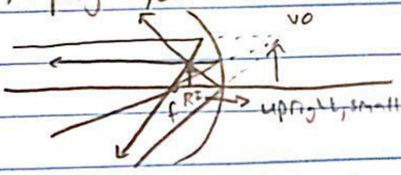
↳ 

↳ Ex) Concave mirrors, virtual objects → $f=+$, $p=-$

$-\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

$q=\frac{pf}{p+f}$, $M=\frac{f}{p+f}$ → 1 region

↳ RI, upright, small

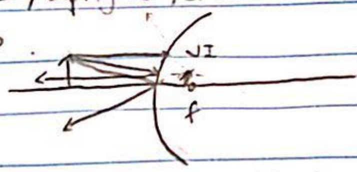
↳ 

↳ Ex) Convex mirrors, real objects → $f=-$, $p=+$

$\frac{1}{p}+\frac{1}{q}=-\frac{1}{f}$

$q=\frac{-pf}{f+p}$, $M=\frac{f}{f+p}$ → 1 region

↳ VI, upright, small

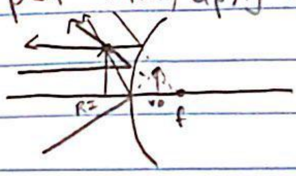
↳ 

↳ Ex) Convex mirrors, virtual object, → $f=-$, $p=-$

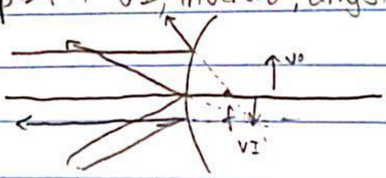
$-\frac{1}{p}+\frac{1}{q}=-\frac{1}{f}$

$q=\frac{pf}{f-p}$, $M=\frac{-q}{-p}=\frac{f}{f-p}$ → 2 regions ($p<f$ and $p>f$)

↳ $p<f$ → RI, upright, large

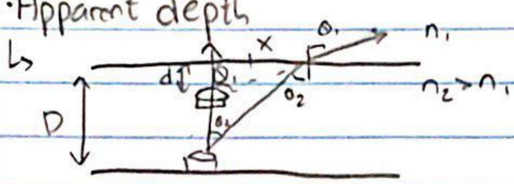


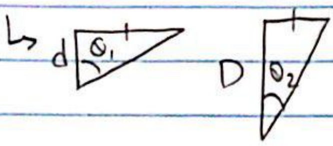
↳ $p>f$ → VI, inverted, anysize



Physics 1C Lecture 21: Refraction

· Apparent depth





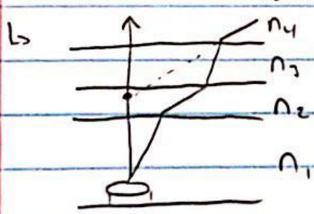
↳ $d\tan\theta_1 = D\tan\theta_2$

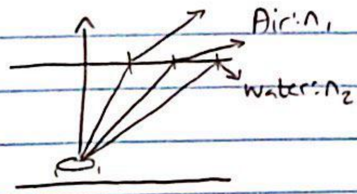
$d = D\frac{\tan\theta_2}{\tan\theta_1}$

Small Angles: $d = D\frac{\sin\theta_2}{\sin\theta_1}$

$d = D\frac{n_1}{n_2}$

$d = D\frac{1}{4/3} = \frac{3}{4}D$



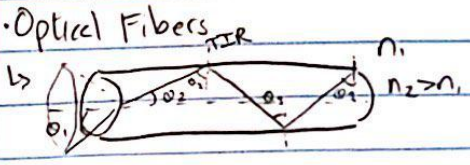


↳ Total internal reflection

↳ $\theta_{critical}$ → leads to 90° refracted array

↳ $\sin\theta_c = \frac{n_1}{n_2}$

· Optical Fibers



↳ $\theta_3 > \theta_c$

$\sin\theta_3 > \sin\theta_c$

$\sin\theta_3 > \frac{n_1}{n_2}$

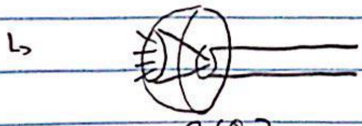
$\cos\theta_2 > \frac{n_1}{n_2}$

$\sqrt{1-\sin^2\theta_2} > \frac{n_1}{n_2}$

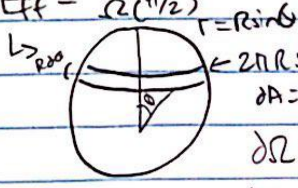
$\sin^2\theta_2 < 1-(\frac{n_1}{n_2})^2$

$(\frac{n_1}{n_2})^2\sin^2\theta_1 < 1-(\frac{n_1}{n_2})^2$

$\sin\theta_1 < \sqrt{(\frac{n_2}{n_1})^2-1}$



↳ $\text{Eff} = \frac{\Omega(\theta_1)}{\Omega(\pi/2)}$



← $2\pi R\sin\theta R d\theta = dA$

$dA = 2\pi R^2\sin\theta d\theta$

$d\Omega = \frac{dA}{R^2}$

$\Omega(\theta) = 2\pi\int_0^\theta \sin\theta d\theta$

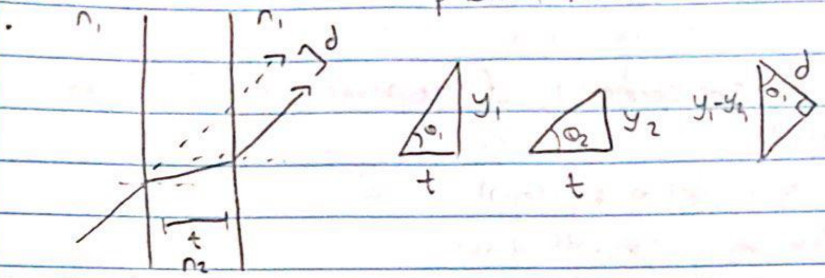
$\Omega(\theta_1) = 2\pi(1-\cos\theta_1)$

$\Omega(\pi/2) = 2\pi$

$\text{Eff} = 1-\cos\theta_1$

$\boxed{\text{Eff} = 1-\sqrt{2-(\frac{n_2}{n_1})^2}}$

Physics 1C Lecture 21: Spherical Refractive Surfaces



↳ $d = (y_1 - y_2)\cos\theta_1$

$d = t\cos\theta_1 (\tan\theta_1 - \tan\theta_2)$

$d = t\sin\theta_1 \left[1 - \frac{\tan\theta_2}{\tan\theta_1}\right]$

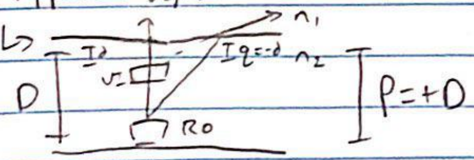
$d = t\sin\theta_1 \left[1 - \frac{n_1\cos\theta_1}{n_2\cos\theta_2}\right]$

$\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_1}$

$d = t\sin\theta_1\left[1 - \frac{\cos\theta_1}{\sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_1}}\right]$

↳ $d \propto t$

- Apparent depth

↳ 

↳ $d = \frac{n_1}{n_2} D$

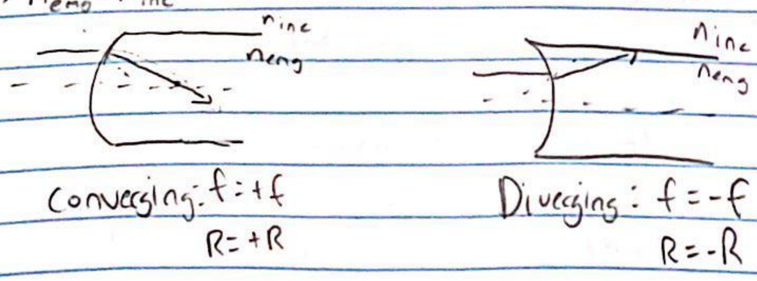
$\frac{n_1}{d} = \frac{n_2}{D}$

$\frac{n_2}{D} - \frac{n_1}{d} = 0$

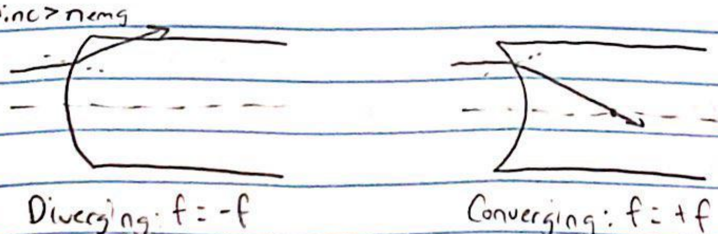
$\frac{n_{inc}}{p} + \frac{n_{emg}}{q} = \frac{n_{emg}}{f}$ → spherical refractive surfaces

- SRS

↳ $n_{emg} > n_{inc}$



↳ $n_{inc} > n_{emg}$



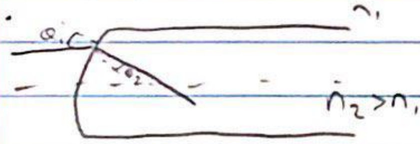
↳ $\frac{n_{emg}}{f} = \frac{n_{emg} - n_{inc}}{R}$

↳ focusing power: RHS of optics equation

↳ mirror / thin lens → $\frac{1}{f}$

↳ $\frac{n_{emg} - n_{inc}}{R}$ → SRS

↳ property of device and not in direction light comes from

• 

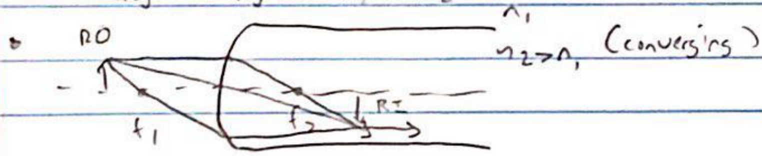
↳ $\frac{1}{f_2} = \frac{n_2 - n_1}{n_2}\left(\frac{1}{R}\right)$ → from left

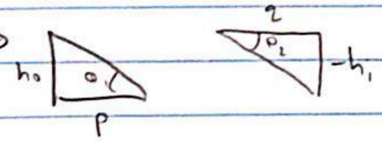
↳ $\frac{1}{f_1} = \frac{n_1 - n_2}{n_1}\left(\frac{1}{-R}\right)$ → from right

↳ $\frac{1}{f_1} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R}\right)$

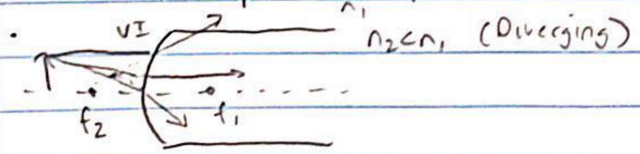
↳ converging from left = converging from right

↳ $f_{emg} \propto n_{emg}$ → $\frac{f_1}{f_2} = \frac{n_1}{n_2}$

• 

↳ 

↳ $M = \frac{h_i}{h_o} = \frac{-q\tan\theta_2}{p\tan\theta_1} = \frac{-q\sin\theta_2}{p\sin\theta_1} = \frac{-q n_1}{p n_2} = \frac{-q/n_{emg}}{p/n_{inc}}$

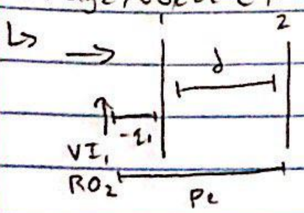
• 

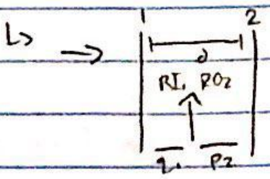
- Apparent Depth

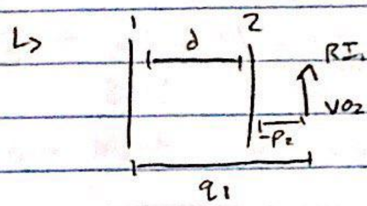
↳ $M = \frac{-q/n_{air}}{p/n_{water}} = 1$, $q = -D\,\frac{n_{air}}{n_{water}}$

Physics 1C Lecture 22: Thin Lenses

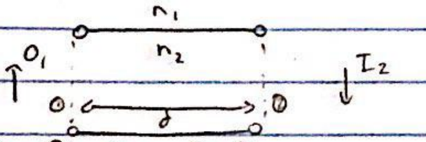
· Image/obect chains

↳ 

↳ 

↳ 

↳ $d = p_2 + q_1$ ↳ $d = p_2 + q_1$ ↳ $d = p_2 + q_1$

· 

↳ $\frac{n_1}{p_1} + \frac{n_2}{q_1} = \frac{n_2 - n_1}{R_1}$

$+\ \frac{n_2}{p_2} + \frac{n_1}{q_2} = \frac{n_1 - n_2}{R_2}$

$\frac{n_1}{p_1} + \frac{n_1}{q_2} + n_2\left(\frac{1}{q_1} + \frac{1}{p_2}\right) = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

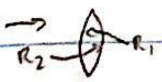
↳ $p_2 = d - q_1$

↳ $\lim_{d \to 0}$ → thin lens

↳ $\frac{1}{p_1} + \frac{1}{q_2} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

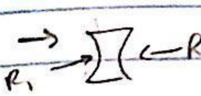
↳ $\frac{1}{p} + \frac{1}{q} = \frac{n_{lens} - n_{med}}{n_{med}}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$

• Biconvex Lens

↳ → 

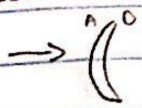
$\frac{1}{f} = \frac{n_L - n_M}{n_M}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) > 0 \rightarrow n_L > n_M \rightarrow$ converging

• Biconcave Lens

↳ → 

$\frac{1}{f} = \frac{n_L - n_M}{n_M}\left(-\frac{1}{R_1} - \frac{1}{R_2}\right) < 0 \rightarrow n_L > n_M \rightarrow$ diverging

• Crescent Lens

↳ → 

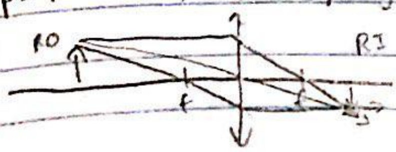
$\frac{1}{f_B} = \frac{n_L - n_M}{n_M}\left(\frac{1}{R_A} - \frac{1}{R_B}\right) \rightarrow$ depends on R_A vs. R_B

$\frac{1}{f_A} = \frac{n_L - n_M}{n_M}\left(\frac{1}{R_B} + \frac{1}{R_A}\right) \rightarrow f_A = f_B$

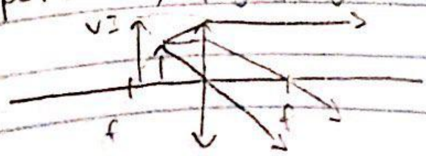
• Converging Lens, RO: $f = f$, $p = p$

↳ $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, $M = -\frac{q}{p} \rightarrow q = \frac{|p||f|}{p - f}$, $M = \frac{-f}{p - f}$

↳ $p > f \rightarrow$ RI, inverted, any-size

↳ 

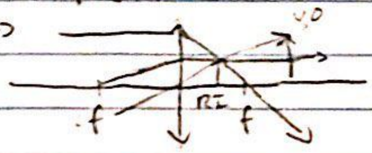
↳ $p < f \rightarrow$ VI, upright, large



· Converging Lens, VO : f = f , p = -p

↳ $\frac{1}{-p} + \frac{1}{q} = \frac{1}{f}$, $M = +\frac{q}{p}$ → $q = \frac{fp}{p+f}$, $M = \frac{f}{p+f}$

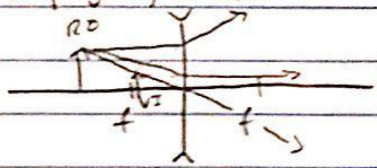
↳ RI, upright, small

↳ 

· Diverging Lens ; RO : f = -f , p = p

↳ $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, $M = \frac{-q}{p}$ → $q = \frac{-pf}{p+f}$, $M = \frac{f}{p+f}$

↳ VI, upright, small

↳ 

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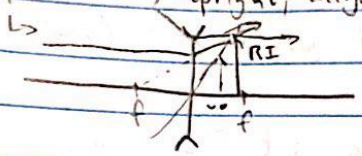
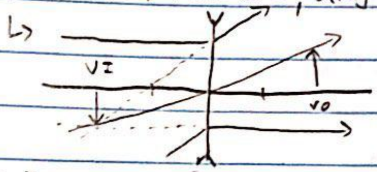
Physics 1C Lecture 24: Chained Devices

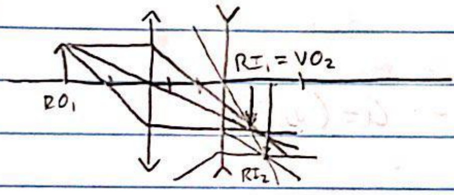
- Diverging lens, VO: $f = -f$, $p = -p$
  - ↳ $\frac{1}{-p} + \frac{1}{q} = \frac{1}{-f}$, $M = \frac{q}{p}$ → $q = \frac{pf}{f-p}$, $M = \frac{f}{f-p}$
  - ↳ $p < f$: RI, upright, large
    - ↳



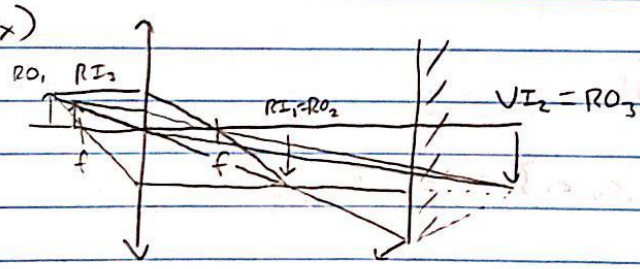
  - ↳ $p > f$: VI, inverted, any-size
    - ↳



- Ex)



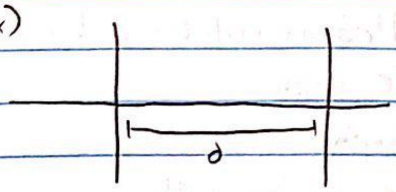
- Ex)



$p_1 > 0$ $p_2 > 0$ $p_3 > 0$

$q_1 > 0$ $q_2 < 0$ $q_3 > 0$

- Ex)



$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1}$ → $\lim_{d \to 0}$ → adjacent lenses

$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$

↳ $\frac{1}{p} + \frac{1}{q} = \frac{1}{f_1} + \frac{1}{f_2}$ → optical power is additive for adjacent thin lenses

↳ $\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} + \dots + \frac{1}{f_n}$

Physics 1C Lecture 25: Interference

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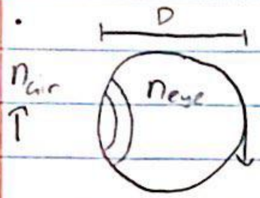


↳ Cornea → fixed power, True lens → variable power, fine-tuning

↳ $\frac{n_{air}}{p} + \frac{n_{eye}}{D} = P_c + P_{TL}(p)$

↳ p and $P_{TL}(p)$ only non-constants

↳ limits:

↳ $\frac{n_{air}}{P_{np}} + \frac{n_{eye}}{D} = P_c + P_{TL}(P_{np})$

↳ $\frac{n_{air}}{\infty} + \frac{n_{eye}}{D} = P_c + P_{TL}(\infty)$

$\Delta\Phi = P_{TL}(P_{np}) - P_{TL}(\infty) = \frac{n_{air}}{P_{np}}$ → Accomodation

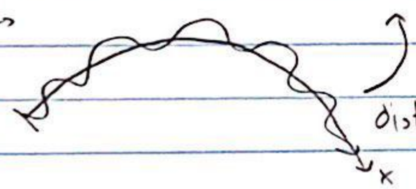
↳ Healthy: $\Delta\Phi = 12.5$ diopters, $P_{np} = 8cm$

↳ Old: $\Delta\Phi = 2$ diopters, $P_{np} = 50cm$

• Interference → describes the effects of superposition

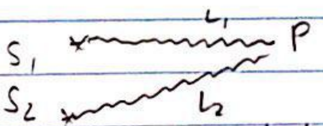
↳ $y(x,t) = A\sin(kx - \omega t + \phi)$

↳



Dist. from source along path (x), time of observation (t), initial phase at the source (ϕ)

↳



$y_{1p} = A\sin(kL_1 - \omega t + \phi_1)$

$y_{2p} = A\sin(kL_2 - \omega t + \phi_2)$

↳ $y_{rp} = \underbrace{2A\cos\left(k\frac{L_2 - L_1}{2} + \frac{\phi_2 - \phi_1}{2}\right)}_{A_{Rp} = A_{Rp}\left(\frac{\Delta\theta_{tot}}{2}\right)} \sin(kL_{avg} - \omega t + \phi_{avg})$

↳ $\theta_2 = kL_2 - \omega t + \phi_2$

$\theta_1 = kL_1 - \omega t + \phi_1$

$\Delta\theta_{tot} = k(L_2 - L_1) + (\phi_2 - \phi_1)$

$\Delta\theta_{tot} = \Delta\theta_{path} + \Delta\theta_{ic}$

↳ $\Delta\theta_{path} = \Delta(kL)$

$\Delta\theta_{ic} = \phi_2 - \phi_1$

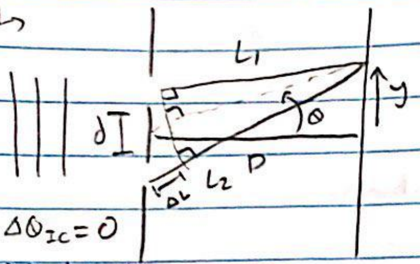
↳ $A_{Rp} = 2A\cos\left(\frac{\Delta\theta_{tot}}{2}\right)$

↳ max. $\cos\left(\frac{\Delta\theta_{tot}}{2}\right) = \pm 1 \rightarrow \Delta\theta_{tot}/2 = n\pi \rightarrow \Delta\theta_{tot} = 2n\pi$ → constructive

↳ min: $\cos\left(\frac{\Delta\theta_{tot}}{2}\right) = 0 \rightarrow \frac{\Delta\theta_{tot}}{2} = (2n+1)\frac{\pi}{2} \rightarrow \Delta\theta_{tot} = (2n+1)\pi$ → destructive

· Young's 2-Slit experiments

↳



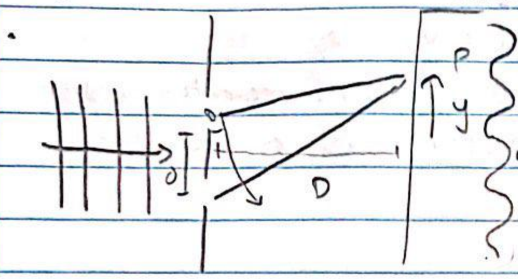
$\Delta\theta_{ic} = 0$

$\Delta\theta_{path} = \Delta kL \rightarrow \Delta L = d\sin\theta \rightarrow \Delta\theta_{path} = kd\sin\theta = \frac{2\pi}{\lambda} d\sin\theta$

↳ maximas: $\Delta\theta_{tot} = 2n\pi \rightarrow kd\sin\theta = 2n\pi \rightarrow \frac{2\pi}{\lambda} d\sin\theta = 2n\pi \rightarrow d\sin\theta = n\lambda$

Physics 1C Lecture 26: Diffraction

•



$n=1 \quad \Delta\phi_{ic}=0$

$n=0 \quad \Delta\phi_{path}=k\Delta L$

$n=1 \quad \Delta\phi_{path}=kd\sin\theta$

↳ Max: $\Delta\phi_{tot}=2n\pi$ → in phase

$$kd\sin\theta=2n\pi$$
$$\frac{2\pi}{\lambda}d\sin\theta=n2\pi$$
$$d\sin\theta=n\lambda$$
$$\tan\theta=n\frac{\lambda}{d}$$
$$\frac{y}{D}=N\frac{\lambda}{d}$$
$$y_{max}=n\frac{\lambda D}{d}$$ → dist. between maxes

↳ slits closer = farter maxes

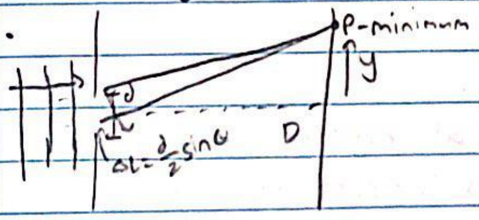
↳ smaller λ = closer maxes

↳ white light at central max, longer wavelengths farther from center

↳ Min: $\Delta\phi_{tot}=(2n+1)\pi$ → out of phase

$$kd\sin\theta=(2n+1)\pi$$
$$d\sin\theta=(n+\tfrac{1}{2})\lambda$$
$$d(\tfrac{y}{D})=(n+\tfrac{1}{2})\lambda$$
$$y_{min}=(n+\tfrac{1}{2})\frac{\lambda D}{d}$$

•



↳ min: $\Delta\phi_{tot}=(2n+1)\pi$

$$k\frac{d}{2}\sin\theta=(2n+1)\pi$$ → denom. depends on split of slit above
$$\frac{2\pi}{\lambda}\frac{d}{2}\sin\theta=(2n+1)\pi$$
$$d\sin\theta=(2n+1)\lambda$$
$$d\sin\theta=m\lambda$$ → m is any integer except 0

↳ max: occurs between minimas (approx. halfway)

↳ central max is 2x as large as others

↳ narrower slit = wider central max, spread out mins/maxes

•

→ resolvable (close)

→ not resolvable (far)

• Rayleigh's Criterion – 2 objects are resolvable when the central max of one overlaps the 1st order min. of the other

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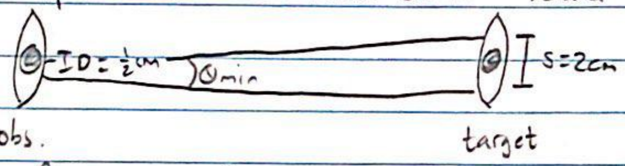
Physics 1C Lecture 27: Reflection

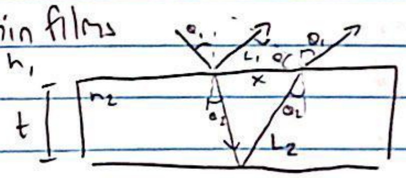
- Rayleigh's Criterion - 2 objects are barely resolvable if the 1st order min from one overlaps with the central max on the other
  - ↳ for light traveling through a rectangular slit, the minimum angle of resolution $\theta_{min} \approx \frac{\lambda}{d}$ → d is width
  - ↳ for light passing through a circular aperature, $\theta_{min} = 1.22\frac{\lambda}{D}$
    - ↳ 1.22 comes from Bessel function
    - ↳ resolution worse when looking through circular aperature
  - ↳ Larger aperature (d) → smaller θ_{min} → better resolution
  - ↳ Ex)



$$\theta_{min} = \frac{\lambda}{D}(1.22) = \frac{s}{R}$$

$$\boxed{R = \frac{sD}{1.22\lambda} = 150m}$$

- When light reflects off a medium of larger index, it flips ($+\pi$ to phase)
  - ↳ $\Delta\theta_{tot} = \theta_{ic} + \theta_{path} + \theta_{ref}$
    - ↳ $\Delta\theta_{ref} = \begin{cases} 0-0 \rightarrow \text{no flips} \\ \pi-0,\ 0-\pi \rightarrow 1 \text{ flip} \\ \pi-\pi \rightarrow 2 \text{ flips} \end{cases}$
  - ↳ $k = \frac{2\pi}{\lambda_n}$, $f\lambda = \frac{c}{n}$ → λ varies w/ medium, k does too
    - ↳ $f\lambda = \frac{c}{1}$ → $\frac{\lambda_n}{\lambda} = \frac{1}{n}$ → $\lambda_n = \frac{\lambda}{n}$
- Thin films



↳ $L_1 = x\sin\theta_1$

$x = 2t\tan\theta_2$

$L_2 = 2\frac{t}{\cos\theta_2}$

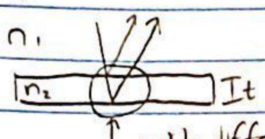
$\Delta\theta_{path} = \Delta(kL) = k_2L_2 - k_1L_1 = \frac{2\pi}{\lambda}(n_2L_2 - n_1L_1) = \frac{2\pi}{\lambda}\left[n_2\frac{2t}{\cos\theta_2} - n_1 2t\sin\theta_1\tan\theta_2\right]$

$= \frac{2\pi}{\lambda}\left(\frac{2tn_2}{\cos\theta_2}\right)\left[1 - \frac{n_1}{n_2}\sin\theta_1\sin\theta_2\right] = \frac{2\pi}{\lambda}\left(\frac{2tn_2}{\cos\theta_2}\right)\left[1 - \sin^2\theta_2\right]$

$= \boxed{\frac{2\pi}{\lambda}(2tn_2)\cos\theta_2}$

↳ usually θ_1, θ_2 very small (≈ 0) → $\Delta\theta_{path} \approx \frac{2\pi}{\lambda}(2tn_2)$

Physics 1C Lecture 28: Thin Films

- 

$n_2 > n_1$

path difference found here

↳ $\Delta\theta_{path} = k_2 2t = \frac{2\pi}{\lambda} n_2 (2t)$

↳ top ray inverts → phase shift $+\pi$

↳ bottom ray picks up no phase shift

↳ Antireflective film → leads to minimas @ observer

  ↳ $\Delta\theta_{tot} = (2n+1)\pi$

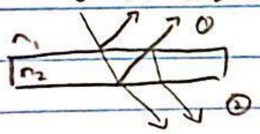
  $\Delta\theta_{path} + \Delta\theta_{ic} + \Delta\theta_{ref} = (2n+1)\pi$

  ↳ when rays split, there is a guaranteed coherence of phase ($\Delta\theta_{ic} = 0$)

  ↳ $\frac{2\pi}{\lambda} n_2 2t + 0 + \pi = (2n+1)\pi$

  $\frac{2\pi}{\lambda} n_2 2t = 2n\pi$

  $t = n\left(\frac{\lambda}{2n_2}\right)$ → half number of wavelengths in thin film → destructive interference

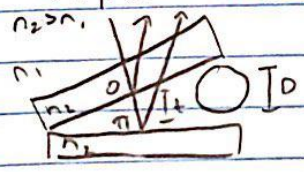
↳ 

→ neither waves flip, $n_2 > n_1$

  ↳ $\Delta\theta_{tot_1} = \frac{2\pi n_2}{\lambda} 2t + \pi$

  ↳ $\Delta\theta_{tot_2} = \frac{2\pi n_2}{\lambda} 2t$

  ↳ top sees mins → bottom sees maxes

- 

n_1

↳ $\Delta\theta_{tot} = \frac{2\pi}{\lambda} 2t + \pi = (2n+1)\pi$

  $t = n\frac{\lambda}{2}$ → dark fringes

  $m = n_{max} + 1 \rightarrow n_{max} = m - 1$

  $t_{max} = n_{max}\frac{\lambda}{2}$

  $t_{max} < D < t(n_{max}+1)$

  $(m-1)\frac{\lambda}{2} < D < m\frac{\lambda}{2}$

  ↳ We know D within $\pm\frac{\lambda}{4}$

    ↳ Ex) $\lambda = 550\text{nm} \rightarrow \sigma_D = 160\text{nm}$

Physics 1C Lecture 29: Lumineferous Ether

- 1B: $\frac{\partial^2 f}{\partial x^2} = \frac{1}{v_x^2}\frac{\partial^2 f}{\partial t^2}$ ← v_x = wave speed in a medium

  1C: $\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$

  $\nabla^2 \vec{B} = \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2}$

↳ c travels in the "luminiferous ether"

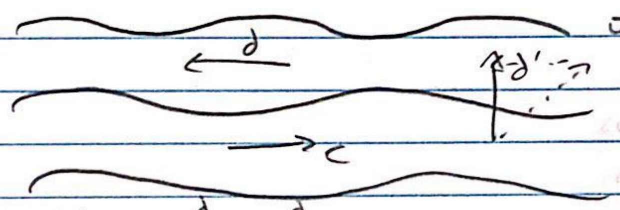
  ↳ must be everywhere -> light is everywhere

  ↳ must be completely permeable -> doesn't have anything interacting w/ it

  ↳ must be rigid -> EM waves don't randomly refract

- Michaelson and Morely

↳



↳ $\Delta t_{long} = \frac{d}{c-u} + \frac{d}{c+u}$

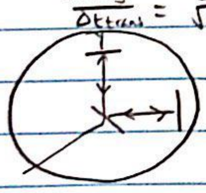
$\Delta t_{long} = \frac{2cd}{c^2-u^2}$

$\Delta t_{trans} = \frac{2d'}{\sqrt{c^2-u^2}}$

↳ $\Delta t_{long} > \Delta t_{trans}$

$\frac{\Delta t_{long}}{\Delta t_{trans}} = \frac{c}{\sqrt{c^2-u^2}} = \frac{1}{\sqrt{1-u^2/c^2}}$

↳



-> look for interference

↳ $\Delta t_{long} > \Delta t_{trans}$ -> waves arrive out of phase

- Lorentz

↳ atoms = empty space

↳ LE pushes on the longitudinal arm, compressing it just enough to make the LE invisible

- Einstein

↳ Laws of physics are the same in all inertial frames of reference

↳ Speed of light in a vacuum is the same in all inertial frames

  ↳ Special relativity

- Transformations

↳ $\vec{r} = M(\theta)\vec{r}'$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} x' \\ y' \end{bmatrix}$

$x = Ax' + By'$

$y = Cx' + Dy'$

↳ Constraints:

1) $\vec{r}'^2 = r^2$

2) $M(0) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3) $M(90°) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

↳ $\vec{r}\cdot\vec{r} = \vec{r}'\cdot\vec{r}'$

$(A^2+C^2)x'^2 + (B^2+D^2)y'^2 + (AD+CD)x'y' = x'^2+y'^2$

$A^2+C^2=1$, $B^2+D^2=1$, $AD+CD=0$

$\frac{A^2}{C^2}+1 = \frac{1}{C^2}$, $1+\frac{D^2}{B^2} = \frac{1}{B^2}$, $AB=-CD \rightarrow \left(\frac{A}{C}\right)^2 = \left(\frac{D}{B}\right)^2$

$C = \pm B$, $A = \pm D$

↳ $M = \begin{bmatrix} A & B \\ \pm B & \pm A \end{bmatrix} \rightarrow M = \begin{bmatrix} A & B \\ -B & A \end{bmatrix}$

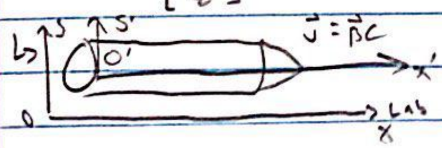
↳ $A(0)=1$, $A(90)=0 \rightarrow \cos$

$B(0)=0$, $B(90)=1 \rightarrow \sin$

$M = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

• Events

↳ $\underline{r} = \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$

↳ 

$0 \le |\vec{\beta}| \le 1$

↳ $\begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} L_{00} & L_{01} & L_{02} & L_{03} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix}$

↳ $y=y'$, $z=z' \rightarrow O=O'$ @ $t=t'=0$

$\begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} L_{00} & L_{01} & 0 & 0 \\ L_{10} & L_{11} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix}$

Physics 1C Lecture 30: Transformations

- $\begin{bmatrix} ct \\ x \end{bmatrix} = \begin{bmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{bmatrix} \begin{bmatrix} ct' \\ x' \end{bmatrix}$

↳ Classical (Galilean)

↳ $t = t'$

↳ at $t = t' = 0 \rightarrow x = x'$

↳ O' moves in S w/ $\vec{v}$

↳ $ct = L_{00}ct' + L_{01}x'$

$x = L_{10}ct' + L_{11}x'$

↳ $t = t' \rightarrow L_{00} = 1, L_{01} = 0$

↳ $x = x' \rightarrow L_{11} = 1$

↳ O' in rocket $\rightarrow \underline{r}' = \begin{bmatrix} ct' \\ 0 \end{bmatrix}$

O' in lab $\rightarrow \underline{r} = \begin{bmatrix} ct \\ vt \end{bmatrix}$

↳ $\begin{bmatrix} ct \\ vt \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ L_{10} & 1 \end{bmatrix} \begin{bmatrix} ct' \\ 0 \end{bmatrix}$

$ct = ct'$

$vt = L_{10}ct'$

↳ $L_{10} = \frac{v}{c} = \beta$

↳ $\begin{bmatrix} ct \\ vt \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \beta & 1 \end{bmatrix} \begin{bmatrix} ct' \\ 0 \end{bmatrix}$

↳ $G \rightarrow$ Galilean

↳ mouse moves thru rocket with velocity $\vec{u}'$

↳ rocket: $\Delta r' = \begin{bmatrix} c\Delta t' \\ u'\Delta t' \end{bmatrix}$

↳ lab: $\Delta \underline{r} = \begin{bmatrix} c\Delta t \\ u\Delta t \end{bmatrix}$

↳ $\begin{bmatrix} c\Delta t \\ u\Delta t \end{bmatrix} = G \begin{bmatrix} c\Delta t' \\ u\Delta t' \end{bmatrix}$

↳ $c\Delta t = c\Delta t'$

$u\Delta t = \beta c\Delta t' + u'\Delta t' \rightarrow u = v + u'$

↳ $v_{M,L} = v_{R,L} + v_{M,R} \rightarrow (\vec{v}_{A,C} = \vec{v}_{A,B} + \vec{v}_{B,C})$

↳ flashlight in the rocket

↳ $\vec{v}_{0,L} = \vec{v}_{0,R} + \vec{v}_{R,L}$

$c \neq c + v \rightarrow$ Einstein's postulates not consistent w/ Galilean relativity

↳ redefine constraits

↳ light moves at c in all frames

↳ laws of physics are the same in all frames

↳ $\vec{v}$

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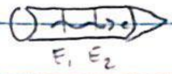


↳ $\underline{\Delta r}' = \begin{bmatrix} c\Delta t' \\ c\Delta t' \end{bmatrix}$

$\underline{\Delta r} = \begin{bmatrix} c\Delta t \\ c\Delta t \end{bmatrix}$

↳ $\begin{bmatrix} c\Delta t \\ c\Delta t \end{bmatrix} = \begin{bmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{bmatrix} \begin{bmatrix} c\Delta t' \\ c\Delta t' \end{bmatrix}$

↳ $c\Delta t = c\Delta t' (L_{00} + L_{01})$

$c\Delta t = c\Delta t' (L_{10} + L_{11})$

$L_{00} + L_{01} = L_{10} + L_{11}$

↳ now reverse the photon's direction

↳ $\underline{\Delta r}' = \begin{bmatrix} c\Delta t' \\ -c\Delta t' \end{bmatrix}$

$\underline{\Delta r} = \begin{bmatrix} c\Delta t \\ -c\Delta t \end{bmatrix}$

↳ $\begin{bmatrix} c\Delta t \\ -c\Delta t \end{bmatrix} = \begin{bmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{bmatrix} \begin{bmatrix} c\Delta t' \\ -c\Delta t' \end{bmatrix}$

↳ $c\Delta t = c\Delta t' (L_{00} - L_{01})$

$-c\Delta t = c\Delta t' (L_{10} - L_{11})$

$L_{00} - L_{01} = -L_{10} + L_{11}$

↳ $L_{00} = L_{11}$, $L_{01} = L_{10}$

↳ $\begin{bmatrix} c\Delta t \\ \Delta x \end{bmatrix} = \begin{bmatrix} L_{00} & L_{01} \\ L_{01} & L_{00} \end{bmatrix} \begin{bmatrix} c\Delta t' \\ \Delta x' \end{bmatrix}$

↳ $\vec{v}$ in S

↳ $\begin{bmatrix} c\Delta t \\ v\Delta t \end{bmatrix} = \begin{bmatrix} L_{00} & L_{01} \\ L_{01} & L_{00} \end{bmatrix} \begin{bmatrix} c\Delta t' \\ 0 \end{bmatrix}$

↳ $c\Delta t = L_{00} c\Delta t'$

↳ $v\Delta t = L_{01} c\Delta t'$

↳ $\frac{v}{c} = \frac{L_{01}}{L_{00}} \rightarrow L_{01} = \beta L_{00}$

↳ $\begin{bmatrix} c\Delta t \\ \Delta x \end{bmatrix} = L_{00} \begin{bmatrix} 1 & \beta \\ \beta & 1 \end{bmatrix} \begin{bmatrix} c\Delta t' \\ \Delta x' \end{bmatrix}$

↳ Laws of Physics

↳ $\underline{r} = L(\beta)\underline{r}' \rightarrow \underline{r}' = L(-\beta)\underline{r}$

↳ $\underline{r}' = L(-\beta)L(\beta)\underline{r}' \rightarrow L(-\beta)L(\beta) = I$

↳ $L_{00}(\beta) = \frac{1}{\sqrt{1-\beta^2}} \equiv \gamma$

↳ $L_4 = \begin{bmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Physics 1C Lecture 31: Space-Time

- $\beta = \frac{v}{c}$, $0 \leq \beta \leq 1$
- $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ $\quad 1 \leq \gamma$
- $L_2 = \gamma \begin{pmatrix} 1 & \beta \\ \beta & 1 \end{pmatrix}$

↳ $L_4 = \begin{bmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

↳ β in top mixes space → time, β in bottom mixes time → space

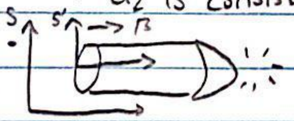
↳ $ct = \gamma(ct' + \beta x')$

$x = \gamma(\beta ct' + x')$

↳ if $\beta \ll 1 \rightarrow \gamma \approx 1$

↳ $\left.\begin{matrix} ct = ct' \\ x = \beta ct' + x' \end{matrix}\right] \rightarrow$ results of G_2

↳ G_2 is consistent in the limit of small speeds → "real world"



↳ flashes every $\Delta t'$ seconds

↳ in rocket: $\underline{\Delta r'} = \begin{bmatrix} c\Delta t' \\ 0 \end{bmatrix}$

in lab: $\underline{\Delta r} = \begin{bmatrix} c\Delta t \\ \Delta x \end{bmatrix}$

↳ $\begin{bmatrix} c\Delta t \\ \Delta x \end{bmatrix} = \gamma \begin{bmatrix} 1 & \beta \\ \beta & 1 \end{bmatrix} \begin{bmatrix} c\Delta t' \\ 0 \end{bmatrix}$

↳ $c\Delta t = \gamma c \Delta t'$

$\Delta x = \gamma(\beta c \Delta t')$

$\Delta t = \gamma \Delta t'$ ← For every second in the rocket, γ seconds tick in the lab

↳ Ex) if $\gamma = 60$ → 1yr in rocket = 60 yrs in lab

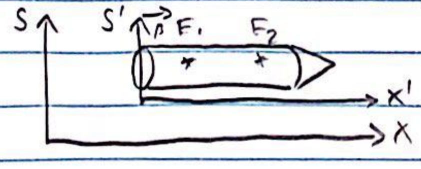
↳ "moving clocks run slow" → time dilation

↳ works from both frames of reference → meaning of time changed

↳ broken by acceleration → position in rocket frame must be constant

↳ If the temporal interval between 2 events in the frame in which they're co-located ("rest frame") is $\Delta t'$, the temporal interval in which that rest frame moves w/ a speed β is given by $\gamma \Delta t'$.

- Simultaneous events

↳ 

↳ $\underline{\Delta r}' = \begin{bmatrix} c \cdot 0 \\ \Delta x' \end{bmatrix}$

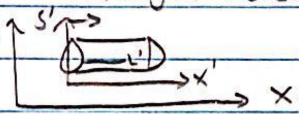
$\underline{\Delta r} = \begin{bmatrix} c\Delta t \\ \Delta x \end{bmatrix}$

↳ $\begin{bmatrix} c\Delta t \\ \Delta x \end{bmatrix} = \gamma \begin{bmatrix} 1 & \beta \\ \beta & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \Delta x' \end{bmatrix}$

$c\Delta t = \gamma \beta \Delta x'$ ← not simultaneous in the lab (space → time)

$\Delta x = \gamma \Delta x'$

↳ length measurement requires simultaneous measurement of font/end

↳ 

↳ in lab: $\underline{\Delta r} = \begin{bmatrix} 0 \\ L \end{bmatrix}$

in rocket: $\underline{\Delta r}' = \begin{bmatrix} c\Delta t' \\ L' \end{bmatrix}$

$\begin{bmatrix} 0 \\ L \end{bmatrix} = \gamma \begin{bmatrix} 1 & \beta \\ \beta & 1 \end{bmatrix} \begin{bmatrix} c\Delta t' \\ L' \end{bmatrix}$

↳ $0 = \gamma (c\Delta t' + \beta L')$

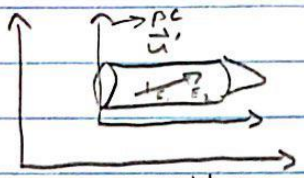
$L = \gamma (\beta c\Delta t' + L')$

$L = \gamma (-\beta^2 + 1) L'$

$L = \frac{L'}{\gamma}$ ← moving sticks are shortened along the direction of motion

- Time dilation → colocated events in rocket, $\Delta t = \gamma \Delta t'$
- Length contraction → simultaneous measurements in lab, $L = \frac{L'}{\gamma}$

Physics 1C Lecture 32: Relative Velocity

- 

↳ $$\underline{\Delta r}' = \begin{bmatrix} c\Delta t' \\ u_x'\Delta t' \\ u_y'\Delta t' \\ u_z'\Delta t' \end{bmatrix}, \quad \underline{\Delta r} = \begin{bmatrix} c\Delta t \\ u_x\Delta t \\ u_y\Delta t \\ u_z\Delta t \end{bmatrix}$$

↳ $$\begin{bmatrix} c \\ u_x \\ u_y \\ u_z \end{bmatrix}\Delta t = \begin{bmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c \\ u_x' \\ u_y' \\ u_z' \end{bmatrix}\Delta t'$$

$c\Delta t = \gamma(c + \beta u_x')\Delta t'$

$u_x\Delta t = \gamma(\beta c + u_x')\Delta t'$

$u_y\Delta t = u_y'\Delta t'$ — divide by

$u_z\Delta t = u_z'\Delta t'$

↳ $$\frac{u_x}{c} = \frac{\beta + u_x'/c}{1 + \beta u_x'/c}$$

$$\frac{u_\perp}{c} = \frac{u_\perp'}{\gamma(1 + \beta u_x'/c)} \rightarrow y \text{ and } z$$

↳ Ex)

$$\frac{u_{\parallel}}{c} = \frac{\beta + u_{\parallel}'/c}{1 + \beta u_{\parallel}'/c} = \frac{0.6 + 0.8}{1 + (0.6)(0.8)} = \frac{1.4}{1.48} = \boxed{0.95}$$

- $\underline{u} \cdot \underline{v} = \underline{u}' \cdot \underline{v}'$

↳ $$\underline{u} = \begin{bmatrix} u_t \\ u_x \\ u_y \\ u_z \end{bmatrix} \rightarrow \underline{u} \cdot \underline{u} = u_t^2 - (u_x^2 + u_y^2 + u_z^2)$$

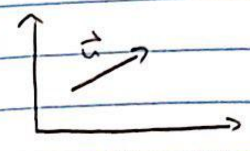
↳ suggests space-time is saddle shaped

↳ $\underline{\Delta r} \cdot \underline{\Delta r} = \underline{\Delta r}' \cdot \underline{\Delta r}'$

$\underline{\Delta r} \cdot \underline{\Delta r} = (c\Delta t)^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2) = (c\Delta t')^2 - (\Delta x'^2 + \Delta y'^2 + \Delta z'^2)$

↳ $= c\Delta T^2 \rightarrow$ proper time

↳ elasped time between the events in the frame in which they are colocated

↳ 

$$c^2\Delta t^2\left(1 - \frac{u^2}{c^2}\right)$$

$$\Delta T = \Delta t\sqrt{1 - \frac{u^2}{c^2}} = \frac{\Delta t}{\gamma_u}$$

↳ $$\underline{U} = \frac{\underline{\Delta r}}{\Delta T} = \frac{\underline{\Delta r}}{\Delta t}\frac{\Delta t}{\Delta T} = \begin{bmatrix} c \\ u_x \\ u_y \\ u_z \end{bmatrix}\gamma_u$$

Physics 1C Lecture 33: Relative Dynamics

- $\underline{u} = \gamma_u \begin{bmatrix} c \\ u_x \\ u_y \\ u_z \end{bmatrix} = \gamma_u \begin{bmatrix} c \\ \vec{u} \end{bmatrix}$

↳ $\gamma_u = \frac{1}{\sqrt{1-u^2/c^2}}$

↳ $\underline{u} \cdot \underline{u} = \underline{u}' \cdot \underline{u}' = \begin{bmatrix} c \\ 0 \end{bmatrix} \cdot \begin{bmatrix} c \\ 0 \end{bmatrix} = c^2 \rightarrow$ rest

↳ photon: $\begin{bmatrix} c \\ c \end{bmatrix} \cdot \begin{bmatrix} c \\ c \end{bmatrix} = 0 \rightarrow$ particle (no mass)

↳ $\underline{P} = m\underline{u} = \gamma_u \begin{bmatrix} mc \\ m\vec{u} \end{bmatrix} = \begin{bmatrix} E/c \\ \vec{p} \end{bmatrix}$

↳ $\vec{p} = \frac{m\vec{u}}{\sqrt{1-u^2/c^2}}$

↳ $\underline{P} \cdot \underline{P} = \left(\frac{E}{c}\right)^2 - \vec{p} \cdot \vec{p} = \left(\frac{E}{c}\right)^2 - |\vec{p}|^2 = m^2c^2$

- $m_1 \rightarrow \quad m_2 \quad \vdots \quad m_3 \rightarrow$

↳ $\underline{P_1} + \underline{P_2} = \underline{P_3}$

$\underline{P_3} \cdot \underline{P_3} = (\underline{P_1} + \underline{P_2}) \cdot (\underline{P_1} + \underline{P_2})$

$\underline{P_3} \cdot \underline{P_3} = \underline{P_1} \cdot \underline{P_1} + \underline{P_2} \cdot \underline{P_2} + 2\underline{P_1} \cdot \underline{P_2}$

$m_3^2c^2 = m_1^2c^2 + m_2^2c^2 + 2\left(\frac{E_1}{c}\frac{E_2}{c} - \vec{P_1} \cdot \vec{P_2}\right) \rightarrow E_2 = m_2c^2, \vec{P_2} = 0$

$m_3^2c^2 = m_1^2c^2 + m_2^2c^2 + 2E_1m_2 \quad \rightarrow E_1 = m_1c^2 + K_1$

$m_3^2c^2 = m_1^2c^2 + m_2^2c^2 + 2m_2(m_1c^2 + K_1)$

$m_3^2 = (m_1^2 + m_2^2 + 2m_1m_2) + 2m_2K_1/c^2$

$m_3 = \sqrt{(m_1+m_2)^2 + \frac{2m_2K_1}{c^2}} \rightarrow$ energy convert to mass

- $\gamma_1 \rightsquigarrow \quad \xleftarrow{\gamma_2}$

↳ $\underline{P_1} + \underline{P_2} = \underline{P_3}$

$\underline{P_1}^2 + \underline{P_2}^2 + 2\underline{P_1} \cdot \underline{P_2} = \underline{P_3}^2 \leftarrow m_1 = m_2 = 0$

$0 + 0 + 2\left(\frac{E_1}{c}\frac{E_2}{c} - |\vec{p_1}||\vec{p_2}|\cos 180\right) = m_3^2c^2$

$2\frac{E_1}{c}\frac{E_2}{c}(1+1) = m_3^2c^2$

$m_3^2 = \frac{4E_1E_2}{c^4}, \quad m_3 = \frac{2\sqrt{E_1E_2}}{c^2}$

- $m_0 \quad \vdots \quad m_1 \quad E_1 = ?$

m_2

↳ $\underline{P_0} = \underline{P_1} + \underline{P_2}$

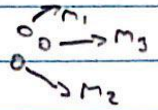
$\underline{P_2} = \underline{P_0} - \underline{P_1}$

$\underline{P_2}^2 = \underline{P_0}^2 + \underline{P_1}^2 - 2\underline{P_0} \cdot \underline{P_1}$

$m_2^2c^2 = m_0^2c^2 + m_1^2c^2 - 2\left(\frac{E_0E_1}{c^2} - \vec{P_0} \cdot \vec{P_1}\right) \rightarrow \vec{P_0} = 0, E_0 = m_0c^2$

$m_2^2c^2 = m_0^2c^2 + m_1^2c^2 - 2m_0E_1$

$E_1 = \frac{(m_0^2 + m_1^2 - m_2^2)c^2}{2m_0}$

· $0^{m_1} \to 0^{m_2}$: 

$\to$ min E_1 to produce m_3?

↳ $(\underline{p_{1i}} + \underline{p_{2i}})^2 = (\underline{p_{1f}} + \underline{p_{2f}} + \underline{p_{3f}})^2 = (\underline{p_{1f}'} + \underline{p_{2f}'} + \underline{p_{3f}'})_{cm}$

↳ $\sum_i \vec{p}_i = m_{tot}\vec{v}_{cm} \to \vec{v}_{cm,cm} = 0$

↳ cm: $\sum_i \vec{p}_f = 0$ → smallest energy means no particles are moving in the end in cm frame

↳ $\underline{p_{1i}}^2 + \underline{p_{2i}}^2 + 2\underline{p_{1i}} \cdot \underline{p_{2i}} = \underline{p_{1f}'}^2 + \underline{p_{2f}'}^2 + \underline{p_{3f}'}^2 + 2(\underline{p_{1f}'} \cdot \underline{p_{2f}'} + \underline{p_{1f}'} \cdot \underline{p_{3f}'} + \underline{p_{2f}'} \cdot \underline{p_{3f}'})$

↳ $\underline{p_{1i}}^2 = \underline{p_{1f}'}^2$, $\underline{p_{2i}}^2 = \underline{p_{2f}'}^2$

$2\left(\frac{E_1}{c}\frac{m_2c^2}{c} - \vec{p}_1 \cdot \vec{0}\right) = m_3^2c^2 + 2\left(m_1m_2c^2 + m_1m_3c^2 + m_2m_3c^2\right)$

$E_1 = \frac{1}{2m_2}\left(m_3^2 + 2m_1m_2 + 2m_1m_3 + 2m_2m_3\right)c^2$

Physics 1C Lecture 34: Compton Scattering and Doppler Effect

- Compton scattering
  - $\xrightarrow{\gamma,\lambda_i}$ $\underset{0}{e^-}$: γ, λ_f, θ, e^- $\rightarrow$ $\lambda_i \neq \lambda_f$
    - $p_{\gamma i} + p_{ei} = p_{\gamma f} + p_{ef}$

$$(p_{ef})^2 = (p_{\gamma i} + p_{ei} - p_{\gamma f})^2$$

$$p_{ef}^2 = p_{\gamma i}^2 + p_{ei}^2 + p_{\gamma f}^2 + 2(p_{\gamma i}p_{ei} - p_{\gamma i}p_{\gamma f} - p_{ei}p_{\gamma f})$$

$$m^2c^2 = 0 + m^2c^2 + 0 + 2\left(\frac{E_{\gamma i}}{c}\frac{mc^2}{c} - \frac{E_{\gamma i}E_{\gamma f}}{c^2} - \frac{mc^2}{c}\frac{E_{\gamma f}}{c} - \vec{p}_{\gamma i}\cdot\vec{p}_{ei} + \vec{p}_{\gamma i}\cdot\vec{p}_{\gamma f} + \vec{p}_{ei}\cdot\vec{p}_{\gamma f}\right)$$

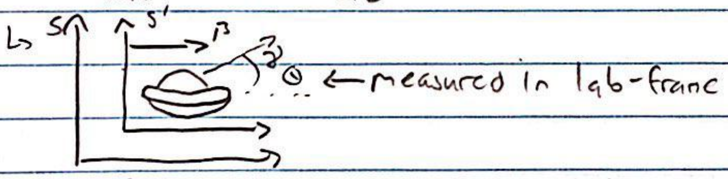
$$0 = mE_{\gamma i} - \frac{E_{\gamma i}E_{\gamma f}}{c^2} - mE_{\gamma f} + |\vec{p}_{\gamma i}||\vec{p}_{\gamma f}|\cos\theta \rightarrow p_\gamma = \frac{E_\gamma}{c}$$

$$(E_{\gamma i} - E_{\gamma f})mc^2 = E_{\gamma i}E_{\gamma f}(1-\cos\theta)$$

$$\left(\frac{1}{E_{\gamma f}} - \frac{1}{E_{\gamma i}}\right)mc^2 = 1 - \cos\theta \rightarrow \text{relates energy}$$

- $E = \frac{hc}{\lambda}$
- $\boxed{\lambda_f - \lambda_i = \frac{h}{m_e c}(1-\cos\theta)}$

- Doppler Effect
  - $\frac{f_{obs}}{f_{src}} = \frac{V_{snd} - V_{obs}}{V_{snd} - V_{src}}$ $\rightarrow$ ~~derived for a medium at rest~~



- $(\underline{u}) \sim$ 4 velocity of UFO, $(\underline{P}) \sim$ 4 momentum of photon
- Lab: $\underline{u} = \gamma\begin{bmatrix} c \\ \beta c \\ 0 \\ 0 \end{bmatrix}$

$$\underline{P} = \frac{E}{c}\begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix}$$

- UFO: $\underline{u}' = \gamma\begin{bmatrix} c \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$$\underline{P}' = \frac{E'}{c}\begin{bmatrix} \cos\theta' \\ \sin\theta' \\ 0 \end{bmatrix}$$

- $\underline{u}\cdot\underline{P} = \underline{u}'\cdot\underline{P}'$

$$\gamma E(1 - \beta\cos\theta) = E' \rightarrow E = h\nu$$

$$\gamma h\nu(1-\beta\cos\theta) = h\nu'$$

$$\frac{\nu}{\nu'} = \frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta} \rightarrow \frac{f_{obs}}{f_{emit}}$$

$$\boxed{\frac{\nu}{\nu'} = \frac{\sqrt{1-v^2/c^2}}{1-v^2/c^2\cos\theta}}$$

Scanned with CamScanner