In Exercises 21 through 26, find a redundant column vector of the given matrix A, and write it as a linear combination of preceding columns. Use this representation to write a nontrivial relation among the columns, and thus find a nonzero vector in the kernel of A. (This procedure is illustrated in Example 8.)

21. 
$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$
 22.  $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$  23.  $\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$ 
24.  $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$  25.  $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ 

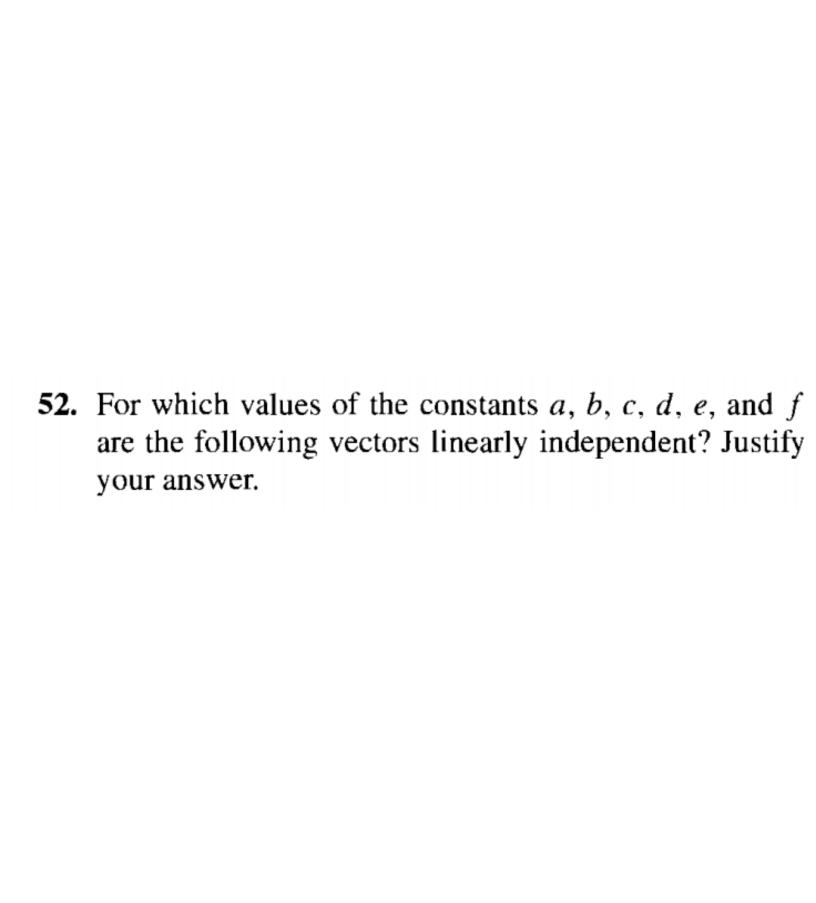
**26.** 
$$\begin{bmatrix} 1 & 3 & 6 \\ 1 & 2 & 5 \\ 1 & 1 & 4 \end{bmatrix}$$

Find a basis of the image of the matrices in Exercises 27 through 33.

**27.** 
$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$$
 **28.** 
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$
 **29.** 
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

30. 
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 3 & 7 \end{bmatrix}$$
 31. 
$$\begin{bmatrix} 2 & 6 \\ 3 & 7 \\ 5 & 8 \end{bmatrix}$$

**39.** Consider some linearly independent vectors  $\vec{v}_1, \vec{v}_2, \ldots$ ,  $\vec{v}_m$  in  $\mathbb{R}^n$  and a vector  $\vec{v}$  in  $\mathbb{R}^n$  that is not contained in the span of  $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m$ . Are the vectors  $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m, \vec{v}$  necessarily linearly independent? Justify your answer.



$$\begin{bmatrix} a \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} b \\ c \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} d \\ e \\ f \\ 0 \end{bmatrix}$$

In Exercises 21 through 25, find the reduced row-echelon form of the given matrix A. Then find a basis of the image of A and a basis of the kernel of A.

**21.** 
$$\begin{bmatrix} 1 & 3 & 9 \\ 4 & 5 & 8 \\ 7 & 6 & 3 \end{bmatrix}$$

$$22. \begin{bmatrix} 2 & 4 & 8 \\ 4 & 5 & 1 \\ 7 & 9 & 3 \end{bmatrix}$$

**28.** For which value(s) of the constant k do the vectors below form a basis of  $\mathbb{R}^4$ ?

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \\ k \end{bmatrix}$$

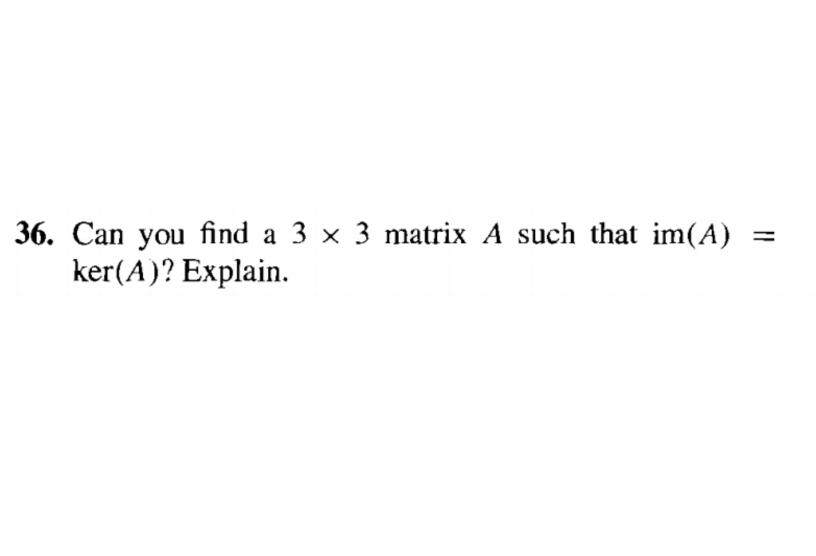
**29.** Find a basis of the subspace of  $\mathbb{R}^3$  defined by the equation

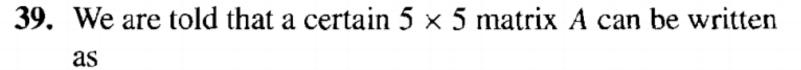
$$2x_1 + 3x_2 + x_3 = 0.$$

31. Let V be the subspace of  $\mathbb{R}^4$  defined by the equation

$$x_1 - x_2 + 2x_3 + 4x_4 = 0.$$

Find a linear transformation T from  $\mathbb{R}^3$  to  $\mathbb{R}^4$  such that  $\ker(T) = \{\vec{0}\}$  and  $\operatorname{im}(T) = V$ . Describe T by its matrix A.





$$A = BC$$

where B is a  $5 \times 4$  matrix and C is  $4 \times 5$ . Explain how you know that A is not invertible.