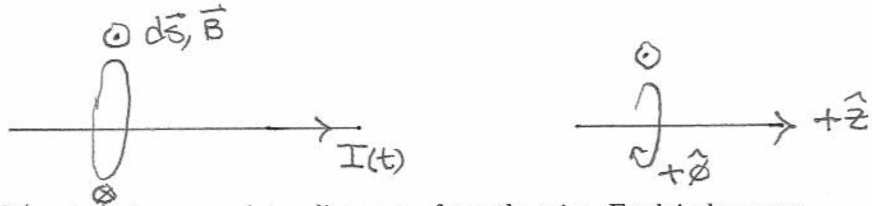
A long, straight wire carries a current $I(t)$...



- 2a) (5 points) *Derive* the magnetic field (vector) at some point a distance r from the wire. Explain how you obtained the direction of that vector.

Ampere's Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

$$B\, 2\pi r = \mu_0 I(t)$$

$$\boxed{\vec{B} = \frac{\mu_0 I(t)}{2\pi r}\hat{\phi}}$$

- No Sources or sinks - the lines of $\vec{B}$ must form Closed loops that enclose Current
- Symmetry - Those closed loops are Circles in planes that are perpendicular to the current and they are centered on the Current
- $\vec{B}$ points in the right-handed sense around the current ($+\hat{\phi}$ if $\hat{z}$ is aligned with the Current)

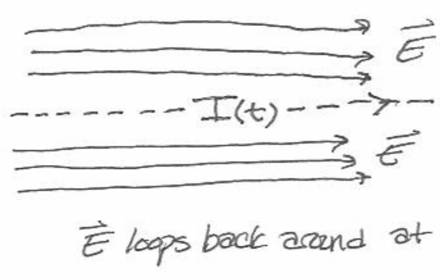
- 2b) (10 points) Under what conditions will that wire give rise to an electric field? Assume these conditions are met and use symmetry (as I did in class) to obtain a mathematical description of the electric field so generated. The more correct detail you provide, the more points you will receive. Do a quick, qualitative plot of the electric field for points near the wire.

Electric field is created when magnetic flux varies in time, In this Case, So long as $I(t)$ Varies in time, we should get an electric field.

$\vec{E}(r, \phi, z)\ (\hat{r}, \hat{\phi}, \hat{z})$

- No Sources or Sinks - the lines of $\vec{E}$ must form Closed loops that enclose magnetic flux
- Since one Cannot distinguish one position in ϕ from another, $\vec{E}$ can't depend on ϕ. Similar reasoning for z
- $\vec{E}$ Can't really have a $\hat{\phi}$ Component <u>and</u> enclose magnetic flux without introducing dependence on ϕ or z
- $\vec{E}$ Can't have an $\hat{r}$ component without introducing dependence on ϕ or z

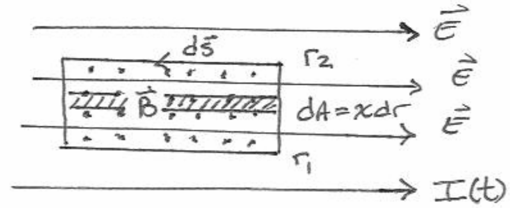
$\Rightarrow$ taken together $\boxed{\vec{E} = E(r)\hat{z}}$



$\vec{E}$ loops back around at ∞

($\vec{E}$ Could point in opposite direction)

- 2c) (10 points) Show that in the region right around the wire, the electric field varies logarithmically with distance from the wire. What direction will the electric field point near the wire if the current is increasing in time?



$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\Phi_B = \int \frac{\mu_0 I}{2\pi r} x\,dr$$

$$\Phi_B = \frac{\mu_0 I x}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$

$$\mathcal{E}_i = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E}_i = -\frac{\mu_0 x}{2\pi} \ln\left(\frac{r_2}{r_1}\right)\frac{dI}{dt}$$

$$\oint \vec{E} \cdot d\vec{s} = \mathcal{E}_i = -\frac{\mu_0 x}{2\pi} \ln\left(\frac{r_2}{r_1}\right)\frac{dI}{dt}$$

$$x\left(E(r_1) - E(r_2)\right) = -\frac{\mu_0 x}{2\pi} \ln\left(\frac{r_2}{r_1}\right)\frac{dI}{dt}$$

$$\boxed{E(r) = \frac{\mu_0}{2\pi} \ln(r) \frac{dI}{dt} + C}$$

If $\frac{dI}{dt} > 0$, $\mathcal{E}_i < 0 \Rightarrow \mathcal{E}_i$ is clockwise around the loop (consistent with Lenz's law). Assuming $|E(r_2)| < |E(r_1)|$, the electric field would be opposite the current direction.

which feels consistent with Lenz's law

(for the curious, $C = -\frac{\mu_0}{2\pi} \ln(r_0) \frac{dI}{dt}$ where $\vec{E}(r_0) \equiv 0$)

- 2d) (5 points) Assuming you've done everything correctly, there's still a problem with your solution. What is that problem? With the time that remains (remember, you only get 30 minutes to take the quiz!) discuss a possible shortcoming in your approach and/or a way to address the problem.

The problem is $|\vec{E}(r)|$ ultimately grows without bounds... There's nothing we can do to keep $|\vec{E}|$ from going to infinity (and that's not physical!)

It's not really clear what it means to change current in an infinite wire ¿ That is, it takes time for that information to travel along the wire and it takes time for that information to propagate outward from the wire. Is it appropriate to talk about the fields response to changes in current at points so far away that the information could not have reached them? The solution, then, at distant points, must contain some knowledge of relativity...