EXERCISES

In Exercises 1–4, use the technique demonstrated in Example 5.6 to find a particular solution for the given differential equation.

1. $y'' + 3y' + 2y = 4e^{-3t}$
2. $y'' + 6y' + 8y = -3e^{-t}$
3. $y'' + 2y' + 5y = 12e^{-t}$
4. $y'' + 3y' - 18y = 18e^{2t}$

In Exercises 5–8, use the form $y_p = a\cos\omega t + b\sin\omega t$, as in Example 5.8, to help find a particular solution for the given differential equation.

5. $y'' + 4y = \cos 3t$
6. $y'' + 9y = \sin 2t$
7. $y'' + 7y' + 6y = 3\sin 2t$
8. $y'' + 7y' + 10y = -4\sin 3t$
9. Suppose that $z(t) = x(t) + iy(t)$ is a solution of

$$z'' + pz' + qz = Ae^{i\omega t}. \tag{5.31}$$

Substitute $z(t)$ into equation (5.31). Then compare the real and imaginary parts of each side of the resulting equation to prove two facts:

$$x'' + px' + qx = A\cos\omega t,$$
$$y'' + py' + qy = A\sin\omega t.$$

Write a short paragraph summarizing the significance of this result.

In Exercises 10–13, use the complex method, as in Example 5.12, to find a particular solution for the differential equation.

10. $y'' + 4y = \cos 3t$
11. $y'' + 9y = \sin 2t$
12. $y'' + 7y' + 6y = 3\sin 2t$
13. $y'' + 7y' + 10y = -4\sin 3t$

In Exercises 14–17, use the technique shown in Example 5.17 to find a particular solution for the given differential equation.

14. $y'' + 5y' + 4y = 2 + 3t$
15. $y'' + 6y' + 8y = 2t - 3$
16. $y'' + 5y' + 6y = 4 - t^2$
17. $y'' + 3y' + 4y = t^3$

In Exercises 18–23, use the technique of Section 4.3 to find a solution of the associated homogeneous equation; then use the technique of this section to find a particular solution. Use Theorem 5.2 to form the general solution. Then find the solution satisfying the given initial conditions.

18. $y'' + 3y' + 2y = 3e^{-4t}, \quad y(0) = 1, \quad y'(0) = 0$
19. $y'' - 4y' - 5y = 4e^{-2t}, \quad y(0) = 0, \quad y'(0) = -1$
20. $y'' + 2y' + 2y = 2\cos 2t, \quad y(0) = -2, \quad y'(0) = 0$
21. $y'' - 2y' + 5y = 3\cos t, \quad y(0) = 0, \quad y'(0) = -2$
22. $y'' + 4y' + 4y = 4 - t, \quad y(0) = -1, \quad y'(0) = 0$
23. $y'' - 2y' + y = t^3, \quad y(0) = 1, \quad y'(0) = 0$

In Exercises 24–29, the forcing term is also a solution of the associated homogeneous solution. Use the technique of Example 5.20 to find a particular solution.

24. $y'' - 3y' - 10y = 3e^{-2t}$
25. $y'' - y' - 2y = 2e^{-t}$
26. $y'' + 4y = 4\cos 2t$
27. $y'' + 9y = \sin 3t$
28. $y'' + 4y' + 4y = 2e^{-2t}$
29. $y'' + 6y' + 9y = 5e^{-3t}$
30. If $y_f(t)$ is a solution of

$$y'' + py' + qy = f(t)$$

and $y_g(t)$ is a solution of

$$y'' + py' + qy = g(t),$$

show that $z(t) = \alpha y_f(t) + \beta y_g(t)$ is a solution of

$$y'' + py' + qy = \alpha f(t) + \beta g(t),$$

where α and β are any real numbers.

Use the technique suggested by Examples 5.23 and 5.26, as well as Exercise 30, to help find particular solutions for the differential equations in Exercises 31–38.

31. $y'' + 2y' + 2y = 2 + \cos 2t$

32. $y'' - y = t - e^{-t}$

33. $y'' + 25y = 2 + 3t + \cos 5t$

34. $y'' + 2y' + y = 3 - e^{-t}$

35. $y'' + 4y' + 3y = \cos 2t + 3\sin 2t$

36. $y'' + 2y' + 2y = 3\cos t - \sin t$

37. $y'' + 4y' + 4y = e^{-2t} + \sin 2t$

38. $y'' + 16y = e^{-4t} + 3\sin 4t$

39. Use the form $y_p(t) = (at + b)e^{-4t}$ in an attempt to find a particular solution of the equation $y'' + 3y' + 2y = te^{-4t}$.

Use an approach similar to that in Exercise 39 to find particular solutions of the equations in Exercises 40–43.

40. $y'' - 3y' + 2y = te^{-3t}$ **41.** $y'' + 2y' + y = t^2e^{-2t}$

42. $y'' + 5y' + 4y = te^{-t}$ **43.** $y'' + 3y' + 2y = t^2e^{-2t}$

44. Use the form $y_p = e^{-2t}(a\cos t + b\sin t)$ in an attempt to find a particular solution of $y'' + 2y' + 2y = e^{-2t}\sin t$.

45. If $z(t) = x(t) + iy(t)$ is a solution of

$$z'' + pz' + qz = Ae^{(a+bi)t},$$

show that $x(t)$ and $y(t)$ are solutions of

$$x'' + px' + qx = Ae^{at}\cos bt$$

and

$$y'' + py' + qy = Ae^{at}\sin bt,$$

respectively.

46. Use the technique suggested by Exercise 45 to find a particular solution of the equation in Exercise 44.

47. Prove that the imaginary part of the solution of $z'' + z' + z = te^{it}$ is a solution of $y'' + y' + y = t\sin t$. Use this idea to find a particular solution of $y'' + y' + y = t\sin t$.

48. Prove Theorem 5.22.

EXERCISES

For Exercises 1–12, find a particular solution to the given second-order differential equation.

1. $y'' + 9y = \tan 3t$
2. $y'' + 4y = \sec 2t$
3. $y'' - y = t + 3$
4. $x'' - 2x' - 3x = 4e^{3t}$
5. $y'' - 2y' + y = e^t$
6. $x'' - 4x' + 4x = e^{2t}$
7. $x'' + x = \tan^2 t$
8. $x'' + x = \sec^2 t$
9. $x'' + x = \sin^2 t$
10. $y'' + 2y' + y = t^5 e^{-t}$
11. $y'' + y = \tan t + \sin t + 1$
12. $y'' + y = \sec t + \cos t - 1$
13. Verify that $y_1(t) = t$ and $y_2(t) = t^{-3}$ are solutions to the homogeneous equation
$$t^2 y''(t) + 3ty'(t) - 3y(t) = 0.$$
Use variation of parameters to find the general solution to
$$t^2 y''(t) + 3ty'(t) - 3y(t) = \frac{1}{t}.$$
14. Verify that $y_1(t) = t^{-1}$ and $y_2(t) = t^{-1} \ln t$ are solutions to the homogeneous equation
$$t^2 y''(t) + 3ty'(t) + y(t) = 0.$$
Use variation of parameters to find the general solution to
$$t^2 y''(t) + 3ty'(t) + y(t) = \frac{1}{t}.$$