Midterm 2 Review notes, Physics 1B, Winter 2020

These are review notes for some of the material covered in lectures 9-15. **Caveat emptor**: this is not all the material you are responsible for (see exam information) and there could be mistakes !

1 Basic properties of charges

• There are two types of charges: positive and negative.

• Like charges (++ or - -) repel and unlike charges (+-) attract.

• The SI unit of charge is the Coulomb (C)

• The basic quantum of electric charge is that of an electron ($-1.602 \times 10^{-19}C$) and that of a proton ($+1.602 \times 10^{-19}C$).

• Charge is conserved, charges can be separated and transported, but the net amount of charge (positive - |negative|) remains the same in an isolated system.

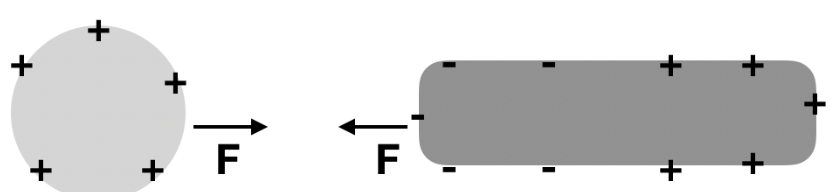
• A conductor has freely moving charges, if you put an excess charge on a conductor the repulsive force between them will make them spread out until an equilibrium is reached.

• Charge in an insulator or on its surface cannot move. Charge can be put/removed on an insulator by contact (e.g. rubbing a plastic rod with silk).

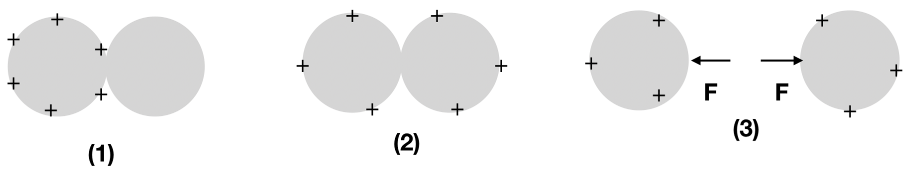
• "Ground" is an infinite reservoir of positive or negative charges which can be accessed at no energy cost.

2 Basic electrostatic phenomena

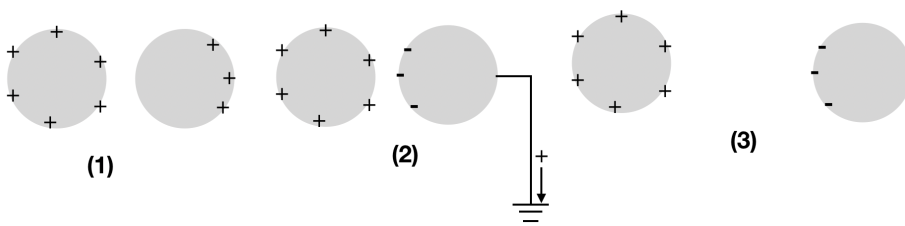
• Separation of charge in a conductor due to presence of an external charge, leads to an attractive force.



• Transfer of charge to a neutral conductor by contact, leads to a repulsive force



• Charging by induction, only works for conductors: 1) Bringing an external charge near a neutral conductor leads to a separation of charge. 2. Grounding removes one type of charge and leaves the conductor charged (3).



• The presence of an external charge nearby an insulator leads to polarization, where the "center of charge " for positive and negative charge in each individual atom is moved in opposite directions, leading to a net attractive force.

3 Coulomb's law

• Force that two point charges exert on each other.

• Magnitude F of force depends on charges q_1 and q_2 and distance r between the charges.

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} \tag{1}$$

where ϵ_0 is the dielectric constant

$$\epsilon_0 = 8.854 \times 10^{-12}\frac{C^2}{Nm^2} \tag{2}$$

Direction: pointing towards the other charge for unlike charges (attractive), pointing away from other charge for like charges (repulsive).

• Vector form: $F_{1\leftarrow 2}$ force of charged object 2 acting on charged object 1 (i.e. the force experienced by object 1). $\vec{r}_1$ location of charged object 1, $\vec{r}_2$ location of charged object 2

$$\vec{F}_{1\leftarrow 2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2}\hat{r}_{12}, \qquad \hat{r}_{12} = \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} \tag{3}$$

Here $\hat{r}_{12}$ is the unit vector pointing from object 2 to object 1. A simple special case is when $\vec{r}_2 = 0$, then $\hat{r}_{12} = \frac{\vec{r}_1}{|\vec{r}_1|}$ is the radial unit vector pointing from the origin to $\vec{r}_1$.

• Note that the force of 1 acting on 2 and 2 acting on one are force pairs obeying Newton's third law

$$\vec{F}_{1\leftarrow 2} = -\vec{F}_{2\leftarrow 1} \tag{4}$$

• For more than one point charge acting on a point charge q_1 the superposition principle holds: the total force acting on q_1 is the sum of all the forces

$$\vec{F}_{1,tot} = \sum_{i=2}^{n} \vec{F}_{1\leftarrow i} = \sum_{i=2}^{n} \frac{1}{4\pi\epsilon_0} \frac{q_1 q_i}{|\vec{r}_1 - \vec{r}_i|^2} \hat{r}_{1i} \tag{5}$$

• For a charge distribution with a 3dim charge density (charge per volume) $\rho(\vec{x})$ the sum over point charges turns into a integral over the charge density

$$\vec{F}_{1,tot} = \frac{1}{4\pi\epsilon_0} \int d^3x \frac{q_1 \rho(\vec{x})}{|\vec{r}_1 - \vec{x}|^2} \hat{r}_{1x}, \qquad \hat{r}_{1x} = \frac{\vec{r}_1 - \vec{x}}{|\vec{r}_1 - \vec{x}|} \tag{6}$$

Note that this formula needs to be modified when the volume charge density $\rho(\vec{x})$ is replaced with a linear charge density $\mu(\vec{x})$ (charge/length, sometimes also denoted $\lambda(\vec{x})$), or a surface charge density $\sigma(\vec{x})$ (charge/area). In these cases the volume integral over d^3x is replaced by a suitable line or surface integral.

• The direction of the total force or the vanishing of the total force can often be determined by a symmetry argument. If you can perform a transformation (for example a reflection about a surface or a point, a rotation or a translation) which leaves configuration of all the charges unchanged, then the resulting force must be unchanged by the transformation too.

4 Electric field

• Introduce a (very small) test charge q_t at a location $\vec{r}$, it feels a force produced by the electric field $\vec{E}(\vec{r})$ defined by

$$\vec{F}_{q_t,tot} = q_t \vec{E}(\vec{r}), \qquad \vec{E}(\vec{r}) = \frac{\vec{F}_{q_t,tot}}{q_t} \tag{7}$$

• $\vec{E}$ is a vector field, each point in space ($\vec{r}$) is assigned a vector $\vec{E}(\vec{r})$

• Electric field produced by a point charge q at $\vec{r} = 0$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \tag{8}$$

Where $\hat{r}$ is the unit vector pointing radially outward from 0. The electric field produced by a point charge points away from a positive point charge and points towards a negative point charge.

• If you know the electric field, then the force on a test charge is given by (7).

• For several point charges the electric field is the superposition (vector sum) of the electric field produced by the individual point charges

$$\vec{E}(\vec{r}) = \sum_{i=1}^{n} \frac{1}{4\pi\epsilon_0} \frac{q_i}{|\vec{r} - \vec{r}_i|^2} \hat{r}_{ri}, \qquad \hat{r}_{ri} = \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|} \tag{9}$$

Here $\hat{r}_{ri}$ is the unit vector pointing from the location $\vec{r}$ where you want to evaluate the electric field and the location $\vec{r}_i$ of the i-th point charge.

• Superposition for a three dimensional charge density $\rho(\vec{x})$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int d^3x \frac{\rho(\vec{x})}{|\vec{r} - \vec{x}|^2} \hat{r}_{rx}, \qquad \hat{r}_{rx} = \frac{\vec{r} - \vec{x}}{|\vec{r}_1 - \vec{x}|} \tag{10}$$

As above this is replaced by an appropriate surface or line integral if ρ is replaced by a surface charge density σ or a linear charge density μ (or λ).

• Electric field from an infinite thin sheet (extended in x,y direction) with uniform charge density (charge per area) σ, at $z = 0$

$$\vec{E}(\vec{r}) = \begin{cases} \frac{\sigma}{2\epsilon_0} \hat{e}_z & z > 0 \\ -\frac{\sigma}{2\epsilon_0} \hat{e}_z & z < 0 \end{cases} \tag{11}$$

• Electric field from inside two infinite thin sheets (extended in x,y direction) with uniform charge density $+\sigma$, at $z = 0$ and $-\sigma$, at $z = d, d > 0$

$$\vec{E}(\vec{r}) = \begin{cases} 0 & z > d \\ \frac{\sigma}{\epsilon_0} \hat{e}_z & 0 < z < d \\ 0 & z < 0 \end{cases} \tag{12}$$

• Infinite line charge with linear charge density (charge per length) μ, wire is extending in the z-direction and is at $r = 0$ in cylindrical coordinates (z, r, θ)

$$\vec{E}(z, r) = \frac{1}{2\pi\epsilon_0} \frac{\mu}{r} \hat{e}_r \tag{13}$$

where $\hat{e}_r$ is the unit vector pointing radially outward in cylindrical coordinates.

• A spherically symmetric charge distribution centered around the origin with total charge q, has an electrical field (<u>outside</u> the charge distribution)

$$\vec{E}(z, r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{e}_r \tag{14}$$

Where $\hat{e}_r$ is the unit vector pointing radially outward in spherical coordinates, and r is the distance from the origin

• A efficient way to visualize electric fields is using electric field lines, they follows these rules

1. Electric field $\vec{E}$ at a point is tangent to the field line at that point, pointing in the direction the field line points.

2. The higher the density of field lines the larger the magnitude of the electric field.
3. Electric field lines originate at positive charges and terminate at negative charges.
4. In conductors carrying charge electric field lines are orthogonal to the surface. (There is no electric field, and hence no field lines, inside conductors.)
5. field lines do not cross
6. field lines do not form closed loops
7. charges test particle do not necessarily follow field lines, the direction of the field line gives the direction of the force (and hence the acceleration).

• A dipole is positive charge $+q$ separated by a distance d from a negative charge $-q$. In a constant electric field $\vec{E}$. There is no net force on the dipole but a torque on the dipole

$$\tau = q|\vec{E}|d\sin\theta \tag{15}$$

where θ is the angle between the dipole axis and the electric field vector. Defining the dipole moment as

$$\vec{p} = q\vec{d} \tag{16}$$

where $\vec{d}$ is the distance vector pointing from the negative charge to the positive charge. The torque is then given by

$$\vec{\tau} = \vec{p} \times \vec{E} \tag{17}$$

5 Electric flux and Gauss's law

• For a rectangular surface with area A, one defines the area vector $\vec{A} = A\hat{n}$ where $\hat{n}$ is the unit normal vector (a vector with unit length and orthogonal to the surface).

• For a constant electric field $\vec{E}$ and a rectangular surface A, the flux of the electric field through the surface A is given by

$$\Phi = \vec{A} \cdot \vec{E} = |\vec{A}||\vec{E}|\cos\theta \tag{18}$$

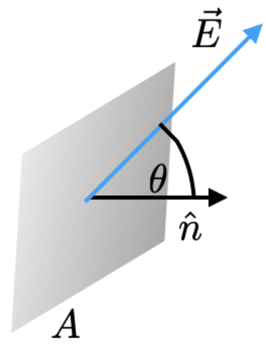
where θ is the angle between the normal vector $\hat{n}$ and the electric field $\vec{E}$.

• For an infinitesimal area dA the infinitesimal flux $d\Phi$ through the surface is defined the same way

$$d\Phi = d\vec{A} \cdot \vec{E} = |dA||\vec{E}|\cos\theta \tag{19}$$

• For a curved surface or non-constant electric field one sums up infinitesmal flux by a surface integral

$$\Phi = \oint_S d\vec{A} \cdot \vec{E} \tag{20}$$



• For Gauss's law we are particularly interested in closed surfaces, which are surfaces which don't have any holes or boundaries. For such a surface there is an obvious notion of inside and outside. The normal vector $\hat{n}$ to the surface is the one which points outward.

• Using flux lines we can get an intuitive understanding of electric flux: Count a fluxlines which pierces the closed surface and goes out of the surface as $+1$ and count a flux line which pierces the closed surface and goes into the surface as -1.

• Fluxes are additive, say a closed surface S is made by combining two surfaces S_1 and S_2 (for example make a sphere by putting together the northern and southern hemisphere). Then the we have

$$\Phi_s = \Phi_1 + \Phi_2 \tag{21}$$

• Gauss's law relates the electric flux through a surface S to the enclosed charge q_{encl} inside S

$$\Phi_S = \frac{q_{encl}}{\epsilon_0} \tag{22}$$

• It does not matter how the charge is distributed inside S, only the total enclosed charge matters

• Gauss's law is very often a good way to determine the electric field following the steps, especially is symmetric configurations

1. Use the symmetry to determine the electric field pattern (which way is $\vec{E}$ pointing)
2. Choose a closed surface (called Gaussian surface), a good choice is usually informed by symmetry, especially if $d\vec{A}$ and $\vec{E}$ are either parallel or orthogonal.
3. Work out the flux integral $\Phi = \oint_S d\vec{A} \cdot \vec{E}$
4. work out the enclosed charge q_{encl}
5. determine E using Gauss's law

• Good choices for Gaussian surfaces:

1. spherical symmetric charge distribution: concentric spherical surfaces
2. infinite line charge, or charge distribution with cylindrical symmetry: concentric cylinder
3. infinite sheet with constant charge density: rectangular prism parallel to surface.

• In electrostatics we define electrostatic equilibrium as a situation where no charge moves (in the absence of batteries or generators etc. a charge configuration will settle into this state)
• Consequences of Gauss's law for conductors (in electrostatic equilibrium)

1. Inside a conductor the electric field vanishes everywhere.
2. All the excess charge (charge in excess of neutral) resides on the surface of a conductor
3. The electric field on the surface of a conductor is orthogonal to the surface

6 Electrostatic potential

• The electrostatic potential V of a charge distribution is defined in analogy with the electric field: For a very small test charge q_t define the potential energy $U(\vec{x})$ as the work it takes to move the test charge from infinity to $\vec{x}$. (Recall $U > 0$ is you have to do positive work on the system, and $U < 0$ if the work is negative). The the electrostatic potential $V(\vec{x})$ is then defined by

$$V(\vec{x}) = \frac{U(\vec{x})}{q_t} \tag{23}$$

• The unit of electrostatic potential is $Volt = Joule/Coulomb$.

• Given V the potential energy of a test charge is given by $U(\vec{x}) = q_t V(\vec{x})$. In particular this potential energy can be used to solve motion problem of test charges using energy conservation

$$U_i + K_i + U_{i,other} = U_f + K_f + U_{f,other} \tag{24}$$

Here $U_{i,f}$ is the electrostatic potential energy at the initial and final location and $U_{i,f,other}$ are other potential energies (such as spring or gravitational potential energies).

• The electrostatic potential of a point charge q located at the origin is

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \tag{25}$$

where $r = |\vec{r}|$ is the distance to the origin

• The electrostatic potential for several point charges q_i located at $\vec{r}_i$ is given by the superposition principle

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{|\vec{r} - \vec{r}_i|} \tag{26}$$

• For a charge distribution with charge density $\rho(x)$ the potential is given

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int d^3x \frac{\rho(\vec{x})}{|\vec{r}-\vec{x}|} \tag{27}$$

As for electric fields, this needs to be changed to a surface or line integral over any surface or linear charge distributions that may be present (i.e. $\rho(\vec{x})d^3x \to \sigma(\vec{x})dA$ or $\mu(\vec{x})d\ell$.)

• If the charge distribution goes all the way to infinity (infinite line charge, infinite sheet) the integral (27) can be divergent, but the divergence will only be in an overall constant (which is immaterial in the definition of V).

• Potential in between infinitely large charged sheets at $z = 0$ with charge density $-\sigma$ and $z = d$ with charge density $+\sigma$.

$$V(z) = \begin{cases} \frac{\sigma}{\epsilon_0}d & z > d \\ \frac{\sigma}{\epsilon_0}z & 0 < z < d \\ 0 & z < 0 \end{cases} \tag{28}$$

• Potential of uniform spherical shell of radius R and total charge q

$$V(r) = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{q}{r} & r > R \\ \frac{1}{4\pi\epsilon_0}\frac{q}{R} & r \le R \end{cases} \tag{29}$$

• Potential difference of a infinite line charge along the z direction with charge density μ

$$V(r_b) - V(r_a) = \frac{\mu}{2\pi\epsilon_0}\ln\left(\frac{r_a}{r_b}\right) \tag{30}$$

• Note that the superposition principle means that if the charge distribution is build up from several simple ones the total potential is the sum of the potentials from the individual charge distributions calculated as if the other charge distributions were not there.

• In order to check signs it is useful to use a rule of thumb: roughly speaking, the electrostatic potential decreases in the direction of the field lines (i.e. in the direction the electric field is pointing). Note that "decrease" includes signs, i.e. -100 is smaller than +10 !

• A second way to obtain the electrostatic potential is by a line integral.

$$V(\vec{r}_b) - V(\vec{r}_a) = -\int_{C_{a\to b}} \vec{E}\cdot d\vec{x} \tag{31}$$

Here $C_{a\to b}$ is a path in space going from $\vec{r}_a$ to $\vec{r}_b$.

• The potential difference (31) does not depend on the choice of path. Often it's possible to choose a path which makes the calculation easy (For example the electric field can be calculated using Gauss's law or path goes through a conductor and henc $\vec{E}$ is zero everywhere along the path).

• The line integral of the electric field along a closed loop C vanishes (the electrostatic force is conservative).

$$\oint_C \vec{E} \cdot d\vec{x} = 0 \tag{32}$$

• In a conductor in electrostatic equilibrium all points have the same value of the potential.

• A way to visualize the potential is to use equipotential surfaces (or equipotential lines in two dimensional diagrams).

1. All points on an equipotential surfaces have the same potential
2. The denser equipotential surface lie together the larger the change of the potential over a distance.
3. equipotential surfaces are orthogonal to electric field lines
4. the surfaces of conductors in electrostatic equilibrium are equipotential surfaces
5. the electric field points towards equipotentials which decrease.

• One can obtain the electric field $\vec{E}$ from the potential V by taking the gradient

$$\vec{E} = -\vec{\nabla} V \tag{33}$$

or in components

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z} \tag{34}$$

Useful formulas when evaluating gradients. For $r = \sqrt{x^2 + y^2 + z^2}$

$$\frac{\partial r}{\partial x} = \frac{x}{r}, \quad \frac{\partial r}{\partial y} = \frac{y}{r}, \quad \frac{\partial r}{\partial z} = \frac{z}{r}, \tag{35}$$

• The potential energy of a charge configuration is defined as follows. Bring the first charge to its location from infinity: that does not produce any potential energy. Then bring the second charge in from infinity in the field of the first charge and calculate the potential energy, the bring in the third charge in from infinity in the field of the two charges and calculate the potential energy etc, at the end add all the potential energies. The final result for n charges q_i is

$$U_{tot} = \frac{1}{2} \sum_{i \neq j} \frac{q_i q_j}{|\vec{r_i} - \vec{r_j}|} \tag{36}$$