Midterm 1 (draft)

This exam consist of 5 base question. Each question comes in 4 different versions: a CCLE quiz component will randomly select one (out of possibly 20) different exam for every student enrolled in the course.

The exam is set to be released on Monday, April 27 at 5:00 pm PDT. Students will have 24 hours to start the midterm. Once started, they will have to complete their initial CCLE submission within 1 hour and 15 minutes.

> Note: At a later time (up to 2 days later), students are expected to upload a high quality version of their answers to Gradescope.

Base question 1

Use induction to show that the inequality below holds for all $n > 0$.

> *Different versions:*
>
> a) $2^{n+1} \geq 2n + 2$
> b) $3^{n+1} \geq 3n + 3$
> c) $4^{n+1} \geq 4n + 4$
> d) $5^{n+1} \geq 5n + 5$

Base question 2

In a group of 191 students

- 10 are taking French, business, and music;
- 36 are taking French and business;
- 20 are taking French and music;
- 18 are taking business and music;
- 65 are taking French;
- 76 are taking business; and
- 63 are taking music.

Answer the following questions:

> *Different versions:*

a) How many students are
    i. taking music and business but not French?
    ii. not taking music or business (or both)?
b) How many students are
    i. taking music and business but not French?
    ii. not taking business or French (or both)?
c) How many students are
    i. taking business and French but not music?
    ii. not taking music or business (or both)?
d) How many students are
    i. taking business and French but not music?
    ii. not taking business or French (or both)?

Base question 3

Let L be the set of all strings, including the null string, that can be constructed by repeated application of the following rules:

- If $\alpha \in L$, then $a\alpha b \in L$, and $b\alpha a \in L$.
- If $\alpha \in L$ and $\beta \in L$, then $\alpha\beta \in L$.

Answer the following questions:

Different versions:

a) i. Show that *abbaab* is in L.
    ii. Is *abaab* in L? If yes, prove it. It not, explain why not.

b) i. Show that *aababb* is in L.
    ii. Is *abaab* in L? If yes, prove it. It not, explain why not.

c) i. Show that *aababb* is in L.
    ii. Is *aabab* in L? If yes, prove it. It not, explain why not.

d) i. Is *aabab* in L? If yes, prove it. It not, explain why not.
    ii. Show that *abbaab* is in L.

Base question 4

Let X be the set of all people. For the relations below, determine whether or not they are equivalence relations on X. If so, prove it. If not, explain why not.

Different versions:

a)

  i. $R_1 = \{(x, y) \mid x$ and y have the same first name$\}$.
  ii. $R_2 = \{(x, y) \mid x$ is younger than $y\}$.

b)

  i. $R_1 = \{(x, y) \mid x$ and y have the same first name$\}$.
  ii. $R_2 = \{(x, y) \mid x$ and y can speak the same language$\}$.

c)

  i. $R_1 = \{(x, y) \mid x$ and y own the same number of cars$\}$.
  ii. $R_2 = \{(x, y) \mid x$ and y can speak the same language$\}$.

d)

  i. $R_1 = \{(x, y) \mid x$ and y own the same number of cars$\}$.
  ii. $R_2 = \{(x, y) \mid x$ is younger than $y\}$.

Base question 5

Suppose there are

- n_1 roads from A to B;
- n_2 roads from B to C;
- n_3 roads from C to D; and
- n_4 roads from D to E.

In addition, assume there is also

- n_5 roads from C to E that do not pass through D.

In how many ways can a ~~hobbit~~ person make a round trip to E and back, if on the return trip they do not want to travel the reverse of the original route?

Different versions:

a) i. Cities: $A = $ *The Shire*, $B = $ *Rivendale*, $C = $ *Mirkwood*, $D = $ *Esgaroth*, and $E = $ *Erebor*.

ii. Roads: $n_1 = 2$, $n_2 = 3$, $n_3 = 2$, $n_4 = 2$, and $n_5 = 1$.

b) i. Cities: $A = $ *The Shire*, $B = $ *Rivendale*, $C = $ *Mirkwood*, $D = $ *Esgaroth*, and $E = $ *Erebor*.

ii. Roads: $n_1 = 3$, $n_2 = 2$, $n_3 = 3$, $n_4 = 1$, and $n_5 = 2$.

c) i. Cities: $A = $ *The Shire*, $B = $ *Rivendale*, $C = $ *Moria*, $D = $ *Lothlórien*, and $E = $ *Mordor*.

ii. Roads: $n_1 = 3$, $n_2 = 2$, $n_3 = 1$, $n_4 = 3$, and $n_5 = 2$.

d) i. Cities: $A = $ *The Shire*, $B = $ *Rivendale*, $C = $ *Moria*, $D = $ *Lothlórien*, and $E = $ *Mordor*.

ii. Roads: $n_1 = 2$, $n_2 = 3$, $n_3 = 2$, $n_4 = 2$, and $n_5 = 1$.