Midterm 1

Last Name: _____

First Name: _____

Student ID: _____

Signature: _____

Section:

Tuesday:	Thursday:	
1A	1B	TA: Khang Huynh
1C	1D	TA: Eli Sadovnik
1E	1F	TA: Jason Snyder

Instructions: Do not open this exam until instructed to do so. You will have 50 minutes to complete the exam. Please print your name and student ID number above, and circle the number of your discussion section. **You may not use calculators,** books, notes, or any other material to help you. Please make sure **your phone is silenced** and stowed where you cannot see it. You may use any available space on the exam for scratch work. If you need more scratch paper, please ask one of the proctors. You must **show your work** to receive credit. Please circle or box your final answers.

Please do not write below this line.

Question	Points	Score
1	15	
2	15	
3	10	
4	10	
Total:	50	

1. (a) (5 points) Find the solution y_h to the differential equation:

$$y' = \frac{1}{x}y$$

.

(b) (10 points) Solve the initial value problem:

$$y' = \frac{1}{x}y + \sqrt{x}, y(1) = 0$$

.

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2. (a) (5 points) Find the general solution $y_h = C_1 y_1 + C_2 y_2$ to the differential equation:

$$y'' + y = 0$$

(b) (10 points) Use undetermined coefficient or variation of parameters, find the general solution to the differential equations

$$y'' + y = t + e^t$$

3. (10 points) Solve the homogeneous equation:

$$(y^2 + 2xy)dx - x^2dy = 0$$

(Hint: Using $y = vx$ change the differential equation to a seperable equation)

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4. Consider the autonomous equation:

$$y'' = y(y-2)e^y$$

(a) (2 points) Find the equilibrium solutions of the above differential equations.

(b) (3 points) Determine the stability of the equilibrium solutions.

(c) (5 points) Prove that if $y(t)$ is a solution and $y(0) = 1$, then $0 < y(t) < 2$ for all $t \in (-\infty, \infty)$.

Scratch Paper

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Scratch Paper

Some useful formulas, etc:

Integrating factor $u(x)$ of a 1st Order Linear DE $x' = ax + f$:

$$u(x) = e^{-\int a(t)dt}$$

Single variable integrating factor μ for $Pdx + Qdy = 0$

- If $h = \frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$,
$$\mu(x) = e^{\int h(x)dx}$$
- If $g = \frac{1}{P}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$,
$$\mu(x) = e^{-\int g(x)dx}$$

Variation of Parameters, (2nd Order Differential Equations)

$$v_1(x) = -\int \frac{1}{W} y_2(x) f(x) \mathrm{d}x$$

$$v_2(x) = \int \frac{1}{W} y_1(x) f(x) \mathrm{d}x$$