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Disc. 1D

Math 33B HW#1

Chapter 2.1

8) a) $t^2+y^2=C^2$

$\frac{d}{dt}(t^2+y^2)=0$

$2t+2y\frac{dy}{dt}=0$

$t+y\frac{dy}{dt}=0$

$\boxed{t+yy'=0}$ ✓

b) $t^2+y^2=c^2$

$y^2=c^2-t^2$

$y=\pm\sqrt{c^2-t^2}$

$y'=\mp\frac{2t}{2\sqrt{c^2-t^2}}$

$y'=\mp\frac{t}{\sqrt{c^2-t^2}}$

$$\boxed{\begin{array}{l} y=\sqrt{c^2-t^2},\ y'=-\frac{t}{\sqrt{c^2-t^2}} \\ y=-\sqrt{c^2-t^2},\ y'=\frac{t}{\sqrt{c^2-t^2}} \end{array}}$$

$t+\cancel{\sqrt{c^2-t^2}}\left(-\frac{t}{\cancel{\sqrt{c^2-t^2}}}\right)=0$

$t-t=0$ ✓

$t-\cancel{\sqrt{c^2-t^2}}\left(\frac{t}{\cancel{\sqrt{c^2-t^2}}}\right)=0$

$t-t=0$ ✓

c) $\sqrt{c^2-t^2}$ only when $t^2\le c^2$

since $y'=\mp\frac{t}{\sqrt{c^2-t^2}}$,

$\sqrt{c^2-t^2}\neq 0$

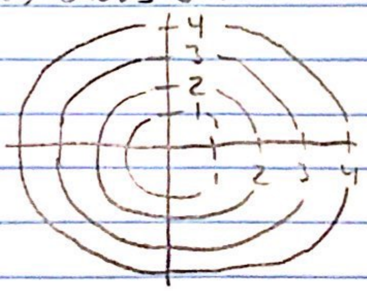
$\therefore c\neq t$

$t^2<c^2$

$t<c,\ t>-c$

$\boxed{-c<t<c}$

d) circles with $r=c$



10) $y(t)=\frac{3}{6t-11}$, $y'=-2y^2$, $y(2)=3$

$y'=-2y^2$

$\frac{dy}{dt}=-2y^2$

$y'(t)=\frac{d}{dt}\left(\frac{3}{6t-11}\right)$

$3\frac{d}{dt}(6t-11)^{-1}$

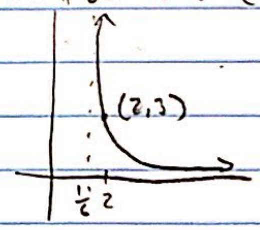
$y'=\frac{-18}{(6t-11)^2}$ ✓

$y'=-2\left(\frac{3}{6t-11}\right)^2=\frac{-18}{(6t-11)^2}$ ✓

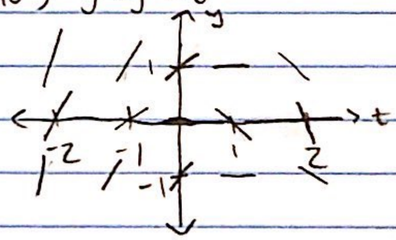
$6t-11\neq 0$

$t\neq\frac{11}{6}$

$\therefore$ interval is $\left(\frac{11}{6},\infty\right)$ -> $<\frac{11}{6}$ is negative



18) $y'=y^2-t$



Chapter 2.2

12) $y'=(2xy+2x)/(x^2-1)$

$\frac{dy}{dx}=2x(y+1)/(x^2-1)$

$\int\frac{dy}{y+1}=\int(2x/(x^2-1))dx$

$\ln|y+1|=\ln|x^2-1|+C$

$|y+1|=e^{\ln|x^2-1|+C}$

$|y+1|=C|x^2-1|$

$\boxed{y=C(x^2-1)-1}$

22) $y' = (y^2+1)/y,\ y(1)=2$

$\frac{dy}{dx} = \frac{(y^2+1)}{y}$

$\frac{y}{y^2+1}\,dy = dx$

$\frac{1}{2}\ln|y^2+1| = x + C$

$\ln|\sqrt{y^2+1}|$

$\ln\sqrt{y^2+1} = x + C$

$e^{\ln\sqrt{y^2+1}} = Ce^x$

$\sqrt{y^2+1} = Ce^x$

$y^2+1 = Ce^{2x}$

$y^2 = Ce^{2x}-1$

$y = \pm\sqrt{Ce^{2x}-1}$

$2 = \pm\sqrt{Ce^2-1}$

$Ce^2-1 = 4$

$C = \frac{5}{e^2}$

$\boxed{y = \sqrt{5e^{2x-2}-1}}$

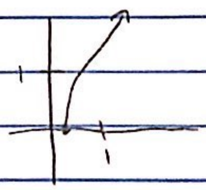
$5e^{2x-2}-1 > 0$

$e^{2x-2} > 1/5$

$2x-2 > -\ln 5$

$x > \frac{2-\ln 5}{2}$

$\boxed{\frac{2-\ln 5}{2} > x > \infty}$



36) $x = at+by+c$

$x' = a+b(y')$

$= a+f(at+by+c)$

$= a+f(x)$ ✓

$f(x) = x^2$

$x = y+t$

$x' = a+by'$

$x' = a+f(x)$

$x' = 1+x^2$

$\frac{dx}{1+x^2} = dt$

$\arctan x = t + C$

$x = \tan(t+C)$

$y = x-t$

$\boxed{y = \tan(t+C)-t}$

Chapter 2.4

6) $tx' = 4x+t^4$

$x' = \frac{4}{t}x + t^3 \quad t \neq 0$

$a(t) = \frac{4}{t}$

$IF = e^{\int -\frac{4}{t}dt}$

$= e^{-4\ln t}$

$= t^{-4}$

$x' - \frac{4}{t}x = t^3$

$\int t^{-4}x' - 4t^{-5}x = \int t^{-1}$

$\int (t^{-4}x)' = \ln|t| + C$

$t^{-4}x = \ln|t| + C$

$\boxed{x = t^4\ln|t| + Ct^4}$

8) $(1+x^3)y' = 3x^2y + x^2 + x^5$

$y' = (3x^2y + x^2 + x^5)/(1+x^3)$

$= x^2(3y+1+x^3)/(1+x^3)$

$y' = \frac{3x^2}{1+x^3}y + x^2$

$a(x) = \frac{3x^2}{1+x^3}$

$IF: e^{\int -\frac{3x^2}{1+x^3}dx}$

$= e^{-\ln|1+x^3|}$

$= \frac{1}{1+x^3}$

$y' - \frac{3x^2}{1+x^3}y = x^2$

$\frac{y'}{1+x^3} - \frac{3x^2}{(1+x^3)^2} = \frac{x^2}{1+x^3}$

$\int \left(\frac{y}{1+x^3}\right)' = \int \frac{x^2}{1+x^3}dx$

$\frac{y}{1+x^3} = \frac{1}{3}\ln|1+x^3| + C$

$\boxed{y = \frac{1}{3}(1+x^3)\ln|1+x^3| + C(1+x^3)}$

12) $x' - \left(\frac{n}{t}\right)x = e^t t^n$

$x' = \left(\frac{n}{t}\right)x + e^t t^n$

$a(t) = \left(\frac{n}{t}\right)$

$IF = e^{\int -n/t \, dt}$

$IF = e^{-n\ln t}$

$IF = t^{-n}$

$t^{-n}x' - \frac{n}{t^{n+1}}x = e^t t^n t^{-n}$

$(t^{-n}x)' = e^t$

$t^{-n}x = e^t + C$

$\boxed{x = t^n e^t + Ct^n}$

14) $y' = y + 2xe^{2x}, \; y(0) = 3$

$y' - y = 0$

$y' = y$

$\ln y = x$

$y_h = e^x$

$y = u(x)y_h$

$(u(x)y_h)' = u(x)y_h + 2xe^{2x}$

$u'(x)y_h + u(x)y_h' = u(x)y_h + 2xe^{2x}$

$u'(x)y_h + u(x)(y_h' - y_h) = 2xe^{2x}$

$u'(x)y_h = 2xe^{2x}$

$u'(x) = \frac{2xe^{2x}}{y_h} = \frac{2xe^{2x}}{e^x}$

$u'(x) = 2xe^x$

$u(x) = \int 2xe^x$

$u = 2x, \; dv = e^x$

$du = 2 \quad v = e^x$

$= 2xe^x - \int 2e^x$

$u = 2xe^x - 2e^x + C$

$u \cdot y_h = 2xe^{2x} - 2e^{2x} + Ce^x = y$

$3 = -2 + C$

$C = 5$

$\boxed{y = 2xe^{2x} - 2e^{2x} + 5e^x}$

16) $(1+t^2)y' + 4ty = (1+t^2)^{-2}, \; y(1) = 0$

$(1+t^2)y_h' + 4ty_h = 0$

$y_h' = \frac{-4ty_h}{(1+t^2)}$

$\ln y_h = -2\ln|1+t^2|$

$y_h = \frac{1}{(1+t^2)^2}$

$u(x)y_h = y$

$(1+t^2)y' = (1+t^2)^{-2} - 4ty$

$y' = (1+t^2)^{-3} - \frac{4ty}{(1+t^2)}$

$y' = \frac{-4t}{(1+t^2)}y + (1+t^2)^{-3}$

$(u(x)y_h)' = \frac{-4t}{(1+t^2)}u(x)y_h + (1+t^2)^{-3}$

$u'(x)y_h + u(x)y_h' + au(x)y_h' = (1+t^2)^{-3}$

$u'(x)y_h = (1+t^2)^{-3}$

$u'(x) = \frac{1}{1+t^2}$

$u(x) = \arctan(t) + C$

$y = u(x)y_h$

$y = (\arctan(t) + C)\frac{1}{(1+t^2)^2}$

$y = \frac{\arctan(t)}{(1+t^2)^2} + C(1+t^2)^{-2}$

$0 = \frac{\arctan(1)}{4} + C\left(\frac{1}{4}\right)$

$-\frac{\pi}{16} = C/4$

$C = -\frac{\pi}{4}$

$\boxed{y = \frac{\arctan(t)}{(1+t^2)^2} - \frac{\pi}{4}(1+t^2)^{-2}}$

20) $y' = \cos x - y\sec x, \; y(0) = 1$

$y' = -y\sec x + \cos x$

$a(x) = -\sec x$

$IF = e^{\int \sec x \, dx}$

$IF = e^{\ln(\sec x + \tan x)}$

$= \sec x + \tan x$

$y' + \sec x \, y = \cos x$

$y'(\sec x + \tan x) + y(\sec^2 x + \tan x) = 1 + \sin x$

$((\sec x + \tan x)y)' = 1 + \sin x$

$y\sec x + y\tan x = x - \cos x + C$

$1 + 0 = -1 + C$

$C = 2$

$\boxed{y = \frac{x - \cos x + 2}{(\sec x + \tan x)}}$

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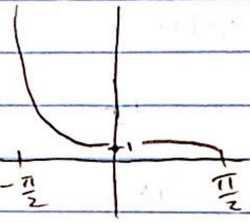
$\cos x \neq 0$

$x \neq \frac{\pi}{2}, -\frac{\pi}{2}$

$-\frac{\pi}{2} < x < \frac{\pi}{2}$



24) $x' = a(t)x + f(t)x^n$

$z = x^{1-n}$

$z' = (1-n)a(t)z + (1-n)f(t)$

$y' + y = y^2$

$z = y^{-1}$

$\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$

$= -y^{-2}(y^2 - y)$

$= -1 + \frac{1}{y} = 1 + z$

$IF = e^{-x}$

$z' = -1 + z$

$e^{-x}(z' - z) = -e^{-x}$

$(ze^{-x})' = -e^{-x}$

$ze^{-x} = e^{-x} + C$

$z = 1 + Ce^{x}$

$\boxed{y = \frac{1}{1 + Ce^{x}}}$

26) $P' = aP - bP^2$

$z = P^{-1}$

$\frac{dz}{dx} = \frac{dz}{dP}\frac{dP}{dx}$

$= -P^{-2}(aP - bP^2)$

$z' = -\frac{a}{P} + b = -az + b$

$IF = e^{\int a\,dx} = e^{ax}$

$e^{ax}(z' + az) = be^{ax}$

$(ze^{ax})' = be^{ax}$

$ze^{ax} = \frac{b}{a}e^{ax} + C$

$z = \frac{b}{a} + Ce^{-ax}$

$\boxed{P = \frac{1}{\frac{b}{a} + Ce^{-ax}}}$

40) $x' - \frac{2}{t^2}x = \frac{1}{t^2}, \ x(1) = 0$

$x' - \frac{2}{t^2}x = 0$

$x' = \frac{2}{t^2}x$

$\ln x = -\frac{2}{t}$

$x_h = e^{-2/t}$

$x = u(t)x_h$

$(u(t)x_h)' = \frac{2}{t^2}u(t)x_h + \frac{1}{t^2}$

$u'(t)x_h = \frac{1}{t^2}$

$u'(t) = \frac{e^{2/t}}{t^2} = e^{2/t}t^{-2}$

$u(t) = -\frac{1}{2}e^{2/t} + C$

$x = -\frac{1}{2}e^{2/t} \cdot e^{-2/t} + Ce^{-2/t}$

$x = -\frac{1}{2} + Ce^{-2/t}$

$0 = -1/2 + Ce^{-2}$

$C = \frac{e^2}{2}$

$\boxed{x = -\frac{1}{2} + \frac{e^2}{2}e^{-2/t}}$

42) a) $T' + kT = 0$

$\frac{T'}{T} = -k$

$\ln T = -kt$

$\boxed{T_h = Ce^{-kt}}$

b) $T' = 0$

$0 = -k(T_p - A)$

$\boxed{T_p = A}$

c) $T = T_h + T_p$

$\boxed{T = Ce^{-kt} + A}$

d) $T' + kT = -kA + H$

$T' + kT = 0$

$T_n = Ce^{-kt}$

$T' = 0$

$0 = -k(T_p - A) + H$

$T_p - A = \frac{H}{k}$

$T_p = \frac{H}{k} + A$

$\boxed{T = Ce^{-kt} + \frac{H}{k} + A}$