Started on	Friday, 29 April 2022, 6:00 PM
State	Finished
Completed on	Friday, 29 April 2022, 9:16 PM
Time taken	3 hours 15 mins
Grade	**51.37** out of 54.00 (**95%**)

Question **1**

Correct

Mark 2.00 out of 2.00

Consider the following distributions for a random variable X. In each case, match the distribution with the expectation of

$$Y = 10 - 2X^2$$

Geometric(p=0.6) 2.22222 ✔

Binomial(n=10,p=0.3) -12.2 ✔

Poisson distribution with

$\lambda = 5$ -50 ✔

$f(x) = \frac{1}{2}e^{-x/2}dx, \qquad 0 \leq x \leq \infty$ -6 ✔

Your answer is correct.

$$E(Y) = 10 - 2E(X^2) = 10 - 2[Var(X) + (E(X))^2]$$

Substitute where Var(X) the corresponding distribution Var(X) and where E(X) the corresponding distribution E(X)

The correct answer is: Geometric(p=0.6) → 2.22222, Binomial(n=10,p=0.3) → -12.2, Poisson distribution with

$$\lambda = 5$$

→ -50,

$$f(x) = \frac{1}{2}e^{-x/2}dx, \qquad 0 \leq x \leq \infty$$

→ -6

Question 2

Correct

Mark 1.00 out of 1.00

An n-tuple is a sequence of n numbers ordered in a particular way. For example, (2,2,3,4) is a 3-tuple. And (3,4,2,2) is a different 3-tuple. The n-tuple concept is different from the concept of set.

So thinking about n-tuples, the difference between the sample space of a binomial experiment and the sample space of a geometric experiment is, as we saw in lectures,

- a. There is no difference between the outcomes of the sample space in a binomial experiment and a geometric experiment, because they are all Bernoulli trials.
- b. All outcomes in the sample space of the binomial experiment are n-tuples of fixed length n, but in a geometric experiment all outcomes n-tuples differ in length n, some are length n-1, others length different from 1, etc. ✔
- c. The random variable in a Binomial experiment has values 0,1,2,......... and so does the random variable in a Geometric experiment, so since the random variable is the outcome in the sample space both the binomial and the geometric have identical n-tuples.
- d. Outcomes in the sample space of the binomial experiment differ in length, but in a geometric experiment all outcomes n-tuples have the same length n.

Your answer is correct.

The correct answer is:
All outcomes in the sample space of the binomial experiment are n-tuples of fixed length n, but in a geometric experiment all outcomes n-tuples differ in length n, some are length n-1, others length different from 1, etc.

Question 3

Correct

Mark 1.00 out of 1.00

Two different manufacturers supply a component with an exponentially distributed lifetime, that is, the length of service the component gives until if fails is an exponentially distributed random variable. Manufacturer A's device has expected lifetime 4 months and manufacturer B's has 10 months. A particular user has a batch of devices of which 40% came from manufacturer A and 60% from manufacturer B. If a randomly selected device from this batch is used, what is the probability that the lifetime of this device is more than 2 months?

- a. 0.4
- b. 0.6
- c. 0.18127
- d. 0.73385 ✔

Your answer is correct.

P(X=2) =P(X=2|lambda = 1/4) 0.4 + P(X=2 |lambda=1/10)0.6 =

0.73385

The correct answer is:
0.73385

Question 4

Correct

Mark 1.00 out of 1.00

A very complicated task, when given by a psychologist to randomly chosen children, takes an average 5 seconds to complete. But there is variability, and we know that there is a standard deviation of 1 second. The probability that a randomly chosen child takes between 4 and 7 seconds to complete the task is

Select one:

- a. 0.6148
- b. 0.00001
- c. 0.8185
- d. 0.8185 ✔

Your answer is correct.

P(4 < X < 7) =

$$P(-1 < Z < 2) = 0.9772 - 0.1587 = 0.8185$$

The correct answer is: 0.8185

Question **5**

Complete

Mark 3.00 out of 3.00

This is a question that requires showing work. That means, you must define your notation, must tell us what you are calculating, must show detailed work and provide final answers and explain.

No attachments are allowed for this question. You must answer in the space provided.

Not all taxpayers want to finance new infrastructure projects. For that reason, public opinion is constantly measured by local governments in order to determine the chance of new projects implementation. Public opinion in a city regarding the opening of a car pool lane in its most congested highway is reflected in the following table.

	Yes	No
Center of the city	0.150	0.250
Suburbs	0.250	0.150
Rural areas	0.050	0.150

The table reflects the opinion of adults eligible to vote and is saying, for example, that 15% of the town adults eligible to vote live in the center of the city and are in favor of the car pool lane.

With this information, answer the following questions:

(i) What is the probability that a randomly chosen eligible voter disapproves of the car pool lane?

(ii) What is the probability that a randomly chosen eligible voter does not live in the center of the city and disapproves of the car pool lane?

(iii) What is the probability that a voter from the suburbs disapproves of the car pool lane?

(i) Let N = the event that the selected voter disapproves of the carpool lane. We need to find $P(N)$, the total probability of selecting a voter that disapproves of the carpool lane. Let C = the event that the selected voter lives in the center of the city, S = the event that the selected voter lives in the suburbs, and R = the event that the selected voter lives in a rural area. To find $P(N)$, we can use the law of total probability to say that $P(N) = P(NC) + P(NS) + P(NR)$, or, in other words, we can sum up the "No" column of the table: $P(N) = 0.250 + 0.150 + 0.150 = \boxed{0.550}$.

(ii) Using the events defined in (i), we are now looking for the joint probability $P(C^C N)$. Knowing that if the voter doesn't live in the center of the city, they must live in either the suburbs or rural areas, we can calculate this joint probability as: $P(C^C N) = P(NS) + P(NR) = 0.150 + 0.150 = \boxed{0.300}$.

(iii) Using the events defined in (i), we are now looking for the conditional probability $P(N|S)$, as the prior knowledge that the voter is from the suburbs is given. Using the definition of conditional probability, we know that: $P(N|S) = \frac{P(NS)}{P(S)}$. We calculate $P(S)$ using the law of total probability as follows: $P(S) = P(SN) + P(SN^C) = 0.250 + 0.150 = 0.4$. Plugging this into our previous equation, we get: $P(N|S) = \frac{0.150}{0.4} = \boxed{0.375}$.

Let N= voted no; Y = voted yes; C=Lives in the Center; B=Lives in the suburbs; R=lives in rural areas.

(i) **(0.2pts, 0.6, 0.2)** P(N) = 0.250+0.150+0.150= 0.55

(ii) **(0.2pts, 0.6, 0.2)**

$$P(N \cap B) + P(N \cap R) = 0.150 + 0.150 = 0.3$$

(iii) **(0.2pts, 0.6, 0.2)**

$$P(N \mid B) = \frac{P(N \cap B)}{P(B)} = \frac{0.150}{0.250 + 0.150} = 0.375$$

Comment:

Question **6**

Correct

Mark 3.00 out of 3.00

Consider a random variable X with density function

$$f(x) = \frac{1}{25}(10 - 2x), \qquad 0 \le x \le 5$$

Match the expressions.

$\frac{1}{25}(10x - x^2), \qquad 0 \le x \le 5$ — F(x) ✓

0.669873 — First quartile ✓

1.66 — E(X) ✓

Your answer is correct.

The correct answer is:

$$\frac{1}{25}(10x - x^2), \qquad 0 \le x \le 5$$

→ F(x), 0.669873 → First quartile, 1.66 → E(X)

Question **7**

Complete

Mark 2.00 out of 3.00

This is a question that requires showing work. That means, you must define your notation, must tell us what you are calculating, must show detailed work and provide final answers and explain.

A complex system requires that a certain function be performed with reliability 0.9999. Devices that perform the function have reliability only 0.8, and so it is necessary to build redundancy. Several of the devices are to be used, and if at least one works then the function will be performed. Furthermore, we are willing to assume that the redundant devices function independently of each other. How many of the devices must be installed in order that the system will have the desired reliability?

No attachments are allowed for this question. The question must be answered in the space provided.

Since only one device needs to work for the function to be performed, we know that this is a parallel system. According to Horgan, the reliability of a parallel system is defined by: $\text{Rel} = 1 - \prod_{i=1}^{n}(1-p_i)$. We know that all devices that perform the function are the same and have $p = 0.8$, so Horgan's equation can be simplified to: $\text{Rel} = 1 - (0.2)^n$. From here, we can plug in the desired reliability to get the following:

$0.9999 = 1 - 0.2^n$

$0.2^n = 0.0001$

$\log_{0.2}(0.2^n) = \log_{0.2}(0.0001)$

$\boxed{n = 2.354 \text{ devices}}$

This question could be answered with the material from required reading by Horgan, posted right below Lecture 6, module 2.

Recognizes parallel system and defines reliability in such a system in order to justify computations that follow (1 pt).

Redundancy implies "parallel system"

Reliability of a parallel system with n components=P(system works) = P(at least one of the n components works) = 1-P(none of the n components works-all fail).

Identifies the correct probability to set up above formula and identifies probability rules used (0.5 pts)

Let

n=number of devices used ; p=prob(a device works)=0.8. By *complement rule,* (1-p)=P(a device does not work) = 1-0.8=0.2.

P({"all fail"}) =0.2^n --> *By product rule for independent events.*

P({"at least one works"}) = $1 - 0.2^n$

Sets up inequality correctly (0.5 pt)

We wish P({"at least one works"}) = $1 - 0.2^n \geq 0.9999$

So we need to have n large enough that $1 - 0.2^n \geq 0.9999$

Solve the inequality for n correctly showing steps of the work (1pt)

$1 - 0.2^n \geq 0.9999 \quad \Rightarrow 0.2^n \leq 0.0001 \quad \Rightarrow n log(0.2) \leq log(0.0001) \quad \rightarrow n \geq 5.72$

where log is natural log.

Since a whole number of devices is required, we should use

n=6 devices or more **(gives final number of components required, 0.5 pts)**

Comment:

We take natural logs.

What are the events for which you are calculating probabilities?

At least 6

Question **8**

Complete

Mark 1.00 out of 1.00

(This exercise is based on Horgan 2009, Section 6.3.) A microprocessor chip is a very important component of every computer. Once in a while tech news magazines report a defect in a chip and producers have to respond by giving some indication of the damage that we should expect due to the defect. In 1994, a flaw was discovered in the Intel Pentium chip. The chip would give an incorrect result when dividing two numbers. But Intel initially announced that such an error would occur in 1 in 9 billion divides. Consequently, it did not immediately offer to replace the chip. Horgan (2010) demonstrates what a bad decision that was. She shows, using the product rule for independent events, that the probability of an error can be as large as 0.28 in just 3 billion divides, which is not uncommon in many computer operations.

Calculate the probability of at least one error in 5 billion divides.

Show work. No attachments are allowed. Answer in the space provided.

Let E = the event that a divide causes an error and F = the event that at least one error occurs in 5 billion divides. By Intel's announcement, we know that $P(E) = \frac{1}{9 \times 10^9} = 0.111 \times 10^{-9}$. In order to calculate $P(F)$, we can first calculate $P(F^C)$ and then use complement rule to relate them by: $P(F) = 1 - P(F^C)$. To calculate $P(F^C)$, we can use the product rule for independent events, as each divide is independent from any other divide. Since the sample size we're looking at is 5 billion errors, we know that: $P(F^C) = [P(E^C)]^{5 \times 10^9}$. In order to calculate $P(E^C)$ we must again use complement rule to relate it to $P(E)$ as follows: $P(E^C) = 1 - P(E) = 1 - 0.111 \times 10^{-9}$. Plugging that value in, we get: $P(F^C) = (1 - 0.111 \times 10^{-9})^{5 \times 10^9} = 0.5738$. With this value, we can finally solve for $P(F)$: $P(F) = 1 - 0.5738 = \boxed{0.4262}$.

Solution

$$P(\texttt{"at least 1 error in 5 billion divides"} = 1 - \left(1 - \frac{1}{9\ billion}\right)^{5\ \ billion} = 0.4262466$$

Comment:

Question 9

Complete

Mark 1.00 out of 1.00

Suppose you were going to sample the opinion of 200 registered voters in the United States as a gauge of public opinion on some issue. Would it make a significant difference whether you chose sampling with replacement or sampling without replacement? Why or why not? Explain. **File attachments not allowed in this question.**

It shouldn't make a significant difference whether you chose sampling with replacement or sampling without replacement. Given the sample size of 200, we know that the total population of registered voters in the United States is far greater than the sample size. As a result, it is highly unlikely that we would select the same person twice in the first place, making sampling with replacement and sampling without replacement virtually indistinguishable.

The United States is a very large population and there are millions of registered voters. Sampling 200 will not significantly alter the probabilities the probability that a randomly chosen voter will vote favorably for an issue **(0.4pts)**. Independence can be assumed. **(0.3pts)**

Therefore, sampling with or without replacement will give the same results **(0.2 pts).** The hypergeometric or the binomial models are equally appropriate to use to gauge the probability that a given number of voters, X, favor or not an issue. **(0.1pts)**

Comment:

Question **10**

Complete

Mark 3.00 out of 3.00

Suppose that X is a random variable with Expected value μ_X and variance σ_X^2. Use rules of expectations to show what the following is equal to

$$E[(3X - \mu_{3X})]$$

Knowing μ_{3X} is a constant and the linearity of the expected operator, we can say that: $E(3X - \mu_{3X}) = -\mu_{3X} + E(3X)$. Since $\mu_{3X} = E(3X)$ by definition, we can substitute into the previous expression and find: $-\mu_{3X} + \mu_{3X} = \boxed{0}$.

$\mu_{3X} = E(3X) = 3E(X) = 3\mu_X$ by linearity of expectation operator. **Correctly shows this part (0.5pts)**

$E(\mu_{3X}) = E(E(3X)) = E(3X) = E(3\mu_X) = 3\mu_X$ because expectation is a constant. **(0.5pts)**

Thus,

$E[(3X - \mu_{3X})] = E(3X) - E(\mu_{3X})$ **Brings expectation operator inside (0.5pts)**

$= 3E(X) - 3\mu_X = 3\mu_x - 3\mu_X = 0$ **Applies rules of expectation for E(3X) (1 pts)**

Final answer is 0 **(0.5pts)**

Comment:

Question **11**

Correct

Mark 1.00 out of 1.00

In the factories and defective part example of one of lecture 5, there are several probabilities. When calculating

$$P(B \cap D)$$

we are calculating the [joint] ✔ probability that a part randomly chosen among the [100] ✔ parts has the two characteristics of being from factory [B] ✔ and being [defective] ✔ . On the other hand, when we are calculating

$$P(D \mid B)$$

we are calculating the probability that a part randomly chosen among [30] ✔ parts is defective. That last probability is [a conditional probability] ✔ . The

$$P(B \cap D)$$

equals [0.09] ✔ but the

$$P(D \mid B)$$

equals [0.3] ✔ . If we were just calculating the probability that a randomly chosen part is defective, that probability is [0.19] ✔ , and that is a [total probability] ✔ .

To make a decision as to which factory a defective part comes from we need to compare the conditional probabilities [P(A|D)] ✔ , [P(B|D)] ✔ , [P(C|D)] ✔ . The defective part, we decide, comes from the factory that has the highest probability among those. This does not mean we know exactly where the defective part came from, but we are less likely to be wrong if we make our prediction using this method.

The example just illustrated here and in the lectures resembles de example of [Mehul Jain] ✔

defective	100	P(A|D)	P(B|D)	joint
total probability	P(C|D)	Ryan Voda	Mehul Jain	0.09
a conditional probability	Noah Gardner	B	30	0.19
0.3				

Your answer is correct.

The correct answer is:
In the factories and defective part example of one of lecture 5, there are several probabilities. When calculating

$$P(B \cap D)$$

we are calculating the [joint] probability that a part randomly chosen among the [100] parts has the two characteristics of being from factory [B] and being [defective]. On the other hand, when we are calculating

$$P(D \mid B)$$

we are calculating the probability that a part randomly chosen among [30] parts is defective. That last probability is [a conditional probability]. The

$$P(B \cap D)$$

equals [0.09] but the

$$P(D \mid B)$$

equals [0.3]. If we were just calculating the probability that a randomly chosen part is defective, that probability is [0.19], and that is a [total probability].

To make a decision as to which factory a defective part comes from we need to compare the conditional probabilities [P(A|D)], [P(B|D)], [P(C|D)]. The defective part, we decide, comes from the factory that has the highest probability among those. This does not mean we know exactly where the defective part came from, but we are less likely to be wrong if we make our prediction using this method.

The example just illustrated here and in the lectures resembles de example of [Mehul Jain]

Question **12**

Correct

Mark 1.00 out of 1.00

Let

$$f(x) = 3x^2, \qquad 0 \le x \le 1,$$

and f(x) = 0 for any other value of X in the real line.

The cumulative distribution function of X (cdf) is

Select one:

a. $$F(x) = x^3, \quad 0 \le x \le 1$$

b. $$F(x) = 2x + 1, \qquad 0 \le x \le 1$$

c. $$F(x) = 3, \qquad 0 \le x \le 1$$

d. $$F(x) = 6x, \qquad 0 \le x \le 1$$

Your answer is correct.

The correct answer is:

$$F(x) = x^3, \quad 0 \le x \le 1$$

Question **13**

Correct

Mark 4.00 out of 4.00

According to https://data.census.gov/cedsci/profile?q&g=0100000US, 32.9% of the population 25 years and older in the United States have a Bachelor's Degree or Higher Education. We are going to sample from that population at random, without replacement, but being such a large population, we can assume independence. Answer the following questions:

The probability that in a sample of 300 hundred people there are at least 105 with a Bachelor's degree or higher education	0.24 approximately ✔
The probability that we need to sample less than 15 people to find the first 3 adult with a Bachelor's degree or higher education	0.95237 ✔
In calculating the probability that we need to sample less than 5 people to find the first person with a Bachelor's degree or higher education we are using a	cumulative probability calculation ✔
The probability that we need to sample less than 5 people to find the first person with a Bachelor's degree or higher education	0.86398 ✔

Your answer is correct.

The correct answer is:
The probability that in a sample of 300 hundred people there are at least 105 with a Bachelor's degree or higher education → 0.24 approximately,

The probability that we need to sample less than 15 people to find the first 3 adult with a Bachelor's degree or higher education → 0.95237,

In calculating the probability that we need to sample less than 5 people to find the first person with a Bachelor's degree or higher education we are using a → cumulative probability calculation,

The probability that we need to sample less than 5 people to find the first person with a Bachelor's degree or higher education → 0.86398

Question **14**

Partially correct

Mark 0.67 out of 1.00

According to the LOTUS results seen in lecture, the expected value of g(X) = 100+2X equals (select what it equals) for X in the following families

X lognormal, with parameters $\mu = 10$ and $\sigma = 2$	162754.8 ✖
X exponential with parameter λ equal 5	100.4 ✔
X Poisson with parameter λ equal 5	110 ✔

Your answer is partially correct.

You have correctly selected 2.
The correct answer is:
X lognormal, with parameters $\mu = 10$ and $\sigma = 2$

→ 325609.6,

X exponential with parameter λ equal 5

→ 100.4,

X Poisson with parameter λ equal 5

→ 110

Question **15**

Correct

Mark 1.00 out of 1.00

An analog signal received at a detector (measured in microvolts) is normally distributed with a mean of 100 and a variance of 256. What is the probability that the signal will be less than 120 microvolts given that it is larger than 110 microvolts.

Select one:

- a. 0.60538
- b. 2.58
- c. 0.7324
- d. 0.2211

Your answer is correct.

An analog signal received at a detector (measured in microvolts) is normally distributed with a mean of 100 and a variance of 256. What is the probability that the signal will be less than 120 microvolts given that it is larger than 110 microvolts.

X=signa;

$$P(X < 120 \mid X > 110) = \frac{P(110 < X < 120)}{P(X > 110)}$$

Use the normal tables to compute the probability on the numerator and the probability in the denominator.

Use normal tables that will be used in the exam (posted in CCLE lectures folder).

The correct answer is: 0.60538

Question **16**

Complete

Mark 3.00 out of 3.00

This is a question that requires showing work. That means, you must define your notation, must tell us what you are calculating, must show detailed work and provide final answers and explain.

For submission of your work, you may do the work on paper and upload a pdf file. Alternatively, if you know how to use the equation editor, you may use it and write your answers in the space provided below. If you do that, the work and equations must show clearly.

X is a continuous random variable with density

$$f(x) = \frac{1}{26}(4x+1), \qquad 2 \le x \le 4,$$

(a) Find the 90th percentile of X

(b) Calculate the expected value of X

(c) Calculate the probability that X is smaller than 3.2

See attached PDF

[CamScanner 04-29-2022 19.36.pdf]

a)

$$
\begin{aligned}
0.9 &= \int_2^c \frac{1}{26}(4x+1)dx \\
&= \frac{1}{26}\left[2x^2 + x|_2^c\right] \qquad (0.5\text{pt}) \\
23.4 &= 2c^2 + c - 10 \\
0 &= 2c^2 + c - 33.4
\end{aligned}
$$

Using the quadratic formula, we find that $c = 3.844$ (0.5pt)

b)

$$
\begin{aligned}
E(X) &= \int_2^4 \frac{1}{26}x(4x+1)dx \\
&= \frac{1}{26}\left[\frac{4}{3}x^3 + \frac{1}{2}x^2|_2^4\right] \qquad (0.5\text{pt})
\end{aligned}
$$

After simplifying, we obtain $E(X) = 3.103$ (0.5pt).

c)

$$
\begin{aligned}
P(X < 3.2) &= \int_2^{3.2} \frac{1}{26}(4x+1)dx \\
&= \frac{1}{26}\left[2x^2 + x|_2^{3.2}\right] \qquad (0.5\text{pt})
\end{aligned}
$$

After simplifying, we obtain $E(X) = 0.526$ (0.5pt)

Comment:

Question **17**

Correct

Mark 1.00 out of 1.00

Los Angeles County had 10.4 million people in 2019, and possibly much more in 2022. That is a very large population. And yet, some individuals are called for Jury Duty pretty often, almost every 2 or 3 years. More specifically, they are called to be in a panel from which jurors are chosen. So sometimes do not get selected to be in the jury. But they have to go and be in the panel for a day nonetheless. The article in our course web side titled: "The Binomial and Hypergeometric Probability Distributions in Jury Selection" by Jude T. Sommerfeld (a copy of which is available for view for your convenience and only for this exam) talks about panels and jury selection. The article considers that it is appropriate to use the hypergeometric distribution in which of the following scenarios? Provide details and formulas, and explain what they mean.

Attachments not allowed in this question. Use the space provided below.

- [ ] a. Both of the cases presented in the other two choices given in this question.
- [ ] b. Calculating the probability of having 8 black persons in a jury pool of 100 people drawn from a total population of 16000 men.
- [x] c. Calculating the probability of having 9 women in the choice of 100 potential jurors out of a jury panel of 350 people consisting of 102 women from a district's population which was 53% female. ✔

Your answer is correct.

The correct answer is:
Calculating the probability of having 9 women in the choice of 100 potential jurors out of a jury panel of 350 people consisting of 102 women from a district's population which was 53% female.

Question **18**

Correct

Mark 1.00 out of 1.00

The Pell grant example in Lecture 5 talks about 2 types of probability. Later on in the lecture and the material studied that week, we have also talked about different probabilities. In the Pell grant example, the 0.32 is a [Prior probability] ✔ and the 0.34 is a [Conditional Probability] ✔ . If we had been interested in calculating the probability that a Pell grant receiver was from UCLA, we could not calculate it because we do not have the information about [P(B)] ✔ , which is needed to apply [Bayes] ✔ theorem.

Bayes	P(B)	Prior probability	the addition rule	Conditional Probability

Your answer is correct.

The correct answer is:
The Pell grant example in Lecture 5 talks about 2 types of probability. Later on in the lecture and the material studied that week, we have also talked about different probabilities. In the Pell grant example, the 0.32 is a [Prior probability] and the 0.34 is a [Conditional Probability]. If we had been interested in calculating the probability that a Pell grant receiver was from UCLA, we could not calculate it because we do not have the information about [P(B)], which is needed to apply [Bayes] theorem.

Question **19**

Correct

Mark 1.00 out of 1.00

The Maxwell-Boltzman probability density function for the speed of air particles in a room does the following when the room temperature increases. Choose all that applies

- [ ] a. it shifts to the left of the speed horizontal axis
- [x] b. Is such that the speed has a higher expected value ✔
- [x] c. It shifts to the right of the speed horizontal axis ✔
- [ ] d. Is such that the speed has a lower expected value

Your answer is correct.

The correct answers are:
It shifts to the right of the speed horizontal axis ,

Is such that the speed has a higher expected value

Question **20**

Complete

Mark 3.00 out of 3.00

This is a question that requires showing work. That means, you must define your notation, must tell us what you are calculating, must show detailed work and provide final answers and explain.

For submission of your work, you may do the work on paper and upload a pdf file. Alternatively, if you know how to use the equation editor, you may use it and write your answers in the space provided below. If you do that, the work and equations must show clearly.

A box contains 6 bags of Crunchy Cheetos and 3 bags of Doritos. At the currently happening DataFest at UCLA, 3 bags are going to be randomly chosen without replacement and given to the student that guesses the answer to the midnight quiz question. We are interested in the number of Cheetos bags in the sample.

(a) Write a probability distribution table that contains in one column the possible values of the random variable of interest, in another column the probability of those values, and in another the work showing the mathematical calculations that are used to calculate the probabilities in each case. Work must be shown for full credit.

(b) After you obtain the table, show how to calculate the expected value of the random variable and its standard deviation using the values in your table. Use the appropriate Greek letter symbols to refer to the expected value and the standard deviation. Say in plain words and referring to the context (Cheetos and Doritos) of this problem to describe what the values you obtain are conveying.

(c) If you had drawn with replacement, what would the expected value have been. Use appropriate formulas for the context of this problem. You do not need to construct a table for this part.

For submission of your work, you may do the work on paper and upload a pdf file. Alternatively, if you know how to use the equation editor, you may use it and write your answers in the space provided below. If you do that, the work and equations must show clearly.

See attached PDF

CamScanner 04-29-2022 19.59.pdf

(a) Defines random variable so it is clear what the distribution is for (0.2 pts).

Must use formula for geometric distribution (0.5 pts)

The table has three columns and all are correct 0.3 pts. (last column must show calculation with the formula for hypergeometric)

We define X= number of Cheetos bags in a sample of size 3 from a box containing 6 bags of Crunchy Cheetos and 3 bags of Doritos.

X is hypergeometric, because the box we are drawing from is small, and the sample is large relative to the size of the box. The probabilities change when we draw a bag.

x P(X=x) Work

----

0 0.01190476

$$\frac{\binom{6}{0}\binom{3}{3}}{\binom{9}{3}}$$

1 0.21428571

$$\frac{\binom{6}{1}\binom{3}{2}}{\binom{9}{3}}$$

2 0.53571429

$$\frac{\binom{6}{2}\binom{3}{1}}{\binom{9}{3}}$$

3 0.23809524

$$\frac{\binom{6}{3}\binom{3}{0}}{\binom{9}{3}}$$

(b)

(appropriate greek letters, 0.2, formula and work 0.4 pts, final answer 0.2 pt. Explanation 0.2 points.

$$\mu_X = \sum_x xP(X=x) = 0(0.01190476) + 1(0.21428571) + 2(0.53571429) + 3(0.23809524) = 2$$

$$\sigma_X^2 = \sum_x (x-\mu_x)^2 P(X=x) = (0-2)^2(0.01190476) + (1-2)^2(0.21428571) + (2-2)^2(0.53571429) + (3-2)^2(0.23809524) = 0.5$$

$$\sigma_X = \sqrt(\sigma_X^2) = 0.7071068$$

We expect that there will be 2 bags of Crunchy Cheetos in the random sample, give or take approximately 1 bag of Crunchy Cheetos.

(c) Identifies the binomial formula for expected value (0.5 pts); Calculates it correctly (0.5 pts).

If we had drawn with replacement, witn n=sample size=3, and p=6/9=2/3, X ~ Bin(n=3, p=2/3)

$$\mu_X = np = 3(6/9) = 18/9 = 2$$

The expected value is the same as without replacement, but the standard deviation and the probabilities are different.

```
###### In case you want to program your hypergeometric distribution with R #####

hyper=function(x) {

  (choose(6,x))*(choose(3, 3-x))/(choose(9,3))

}

x=c(0,1,2,3)  # possible number of cheeto bags in sample of 3

prob=hyper(x); p

sum(hyper(x))  # should add up to 1.

dist=data.frame(x,prob); dist

sum(x*prob) # expected value

sum(prob*(x-sum(x*prob))^2 )

dbinom(2,3,6/9) # to check that probabilities will be different with a binomial.
```

Comment:

Question **21**

Correct

Mark 1.00 out of 1.00

The weight of anodized reciprocating pistons produced by Brown Company follows a normal distribution with mean 10 pounds and standard deviation 0.2 pounds.

The heaviest 2.5% of the pistons produced are rejected as overweight.

What is the probability that of 10 randomly chosen pistons at least 2 have weight larger than 10.392?

Select one:

- a. 0.0246115
- b. 0.981
- c. 0.3389
- d. 0.577

Your answer is correct.

X=weight in pounds

What weight, in pounds, determines the overweight classification? That is X=10.392

Y=number of pistons that are classified as overweight.

Find P(>=2) = 1-P(Y=0) - P(Y=1) =1-(choose(10,1))*(0.025)*(0.975)^9 -(choose(10,0))*((0.025)^0)*(0.975)^10

= 0.0246115

The correct answer is: 0.0246115

Question **22**

Complete

Mark 3.00 out of 3.00

This is a question that requires showing work. That means, you must define your notation, must tell us what you are calculating, must show detailed work and provide final answers and explain.

For submission of your work, you may do the work on paper and upload a pdf file. Alternatively, if you know how to use the equation editor, you may use it and write your answers in the space provided below. If you do that, the work and equations must show clearly.

The National Health and Nutrition Examination Survey (NHANES) examines about 5000 persons per year https://www.cdc.gov/nchs/nhanes/index.htm and constantly screens diabetic people to detect people with type I diabetes. Let X be a random variable measuring the number of diabetic people it takes to find the first Type I diabetic person. Assume that the probability of a diabetic person having type I diabetes is 0.05.

(i) List five or six outcomes of the sample space. Use as notation: 1 for type 1 and 0 for not type 1 diabetic. Also provide the probability of each of these outcomes and write a small table containing the first values of X and P(X). Indicate also what probability rule are you using to calculate the probabilities of these outcomes and what assumptions you are making to be allowed to apply the rule.

(ii) Granted that populations are finite. But would a finite number of outcomes in this sample space suffice to guarantee that all the axioms of probability hold? Explain why or why not.

(iii) Write in a table the cumulative probability mass function for the number of people it takes to find the first diabetic persons. Your table must have values of X in one column and the cumulative probability in the other column, with the column well labelled as cumulative probability. You may write the table for X=1 to X=5 and add another row to complete the table by writing the probability of X greater than 5, because you will not write a table that goes from X=1 to infinity, of course. Can you find a formula for that last probability, P(X>5) in terms of the parameter of the distribution? Hint: there is a handout posted with our lecture notes that will allow you to deduce the formula from one of the formulas given there and your knowledge of probability rules.

For submission of your work, you may do the work on paper and upload a pdf file. Alternatively, if you know how to use the equation editor, you may use it and write your answers in the space provided below. If you do that, the work and equations must show clearly.

See attached PDF

CamScanner 04-29-2022 20.20.pdf

(i) (1.5 pts)

P = probability that randomly chosen person has diabetes type I = 0.05

Some outcomes of the sample space S (5 outcomes is enough

S= { 1, 01, 001, 0001, 0001, ..., }

The values of the random variable X in the table are ordered according to the order of the list given above in S. X is defined in the problem. By its definition, we deduce that X ~ geom(p=0.05). That is why we calculate the probability using the product rule for independent events. Applying this rule, we obtain the formula for the geometric distribution.

------------------------ First term of geometric pmf

x P(X = x)

1 0.05 for outcome (1)

2 0.95(0.05)=0.0475 for outcome (01)

3 $0.95^2(0.05) = 0.045125$ for outcome (001)

4 $0.95)^3(0.05) = 0.04286875$ for outcome (0001)

5 $0.95^4(0.05) = 0.04072531$ for outcome (00001)

(ii) (0.5 pts)

$\sum_{x=1}^{\infty} q^{x-1}p = 1$ This must be satisfied by Axiom that says P(S) =1.

If we added only a finite number of terms like that,

$\sum_{k=1}^{x-1} q^{k-1}p = p[q^0 + q^1 + q^2 + \dots + q^x] = p[\frac{1-q^x}{1-q}] = (1-q)\frac{1-q^x}{1-q} = 1 - q^x < 1$

So adding only a finite number of terms would not give us a sum of probabilities equal to 1. The axioms would not be satisfied.

(iii) (1 pt)

------------------------ First term of geometric pmf

x	P(X = x)	$F(x) = P(X \leq x)$
1	0.05	0.05
2	0.95(0.05)=0.0475	0.05+0.0475 =0.0975
3	$0.95^2(0.05) = 0.045125$	0.05+0.0475+0.045125 =0.1426250
4	$0.95)^3(0.05) = 0.04286875$	0.1426250+0.04286875=0.1854938
5	$0.95^4(0.05) = 0.04072531$	0.1854938+0.04072531=0.2262191
>5	1-0.2262191 =0.7737809	

Comment:

Question **23**

Complete

Mark 2.70 out of 3.00

This is a question that requires showing work. That means, you must define your notation, must tell us what you are calculating, must show detailed work and provide final answers and explain.

No attachments are allowed in this question. You must enter your answer in the space provided.

We are now in a scenario where there is a disease floating around and a diagnostic test for the disease. Something like COVID and testing for COVID. In this scenario, 1.7% of the population are True Positive, 10% of the population test positive, and 80% of the population are True Negatives. What is the specificity of this test? Use a table to demonstrate your work.

	Has disease (D)	**Does not have disease (D^C)**	
Positive (+)	0.017	0.1 - 0.017 = 0.083	0.1
Negative (-)	0.9 - 0.8 = 0.1	0.8	1 - 0.1 = 0.9
	1 - 0.883 = 0.117	0.083 + 0.8 = 0.883	1

As defined in lecture, $\text{Specificity} = \frac{\text{TN}}{\text{TN+FP}}$. Knowing that true negatives are given as 0.8 and that false positives are people who tested positve, but do not have the disease ($P(+D^C) = 0.083$), we can calculate: $\text{Specificity} = \frac{0.8}{0.8+0.083} = \boxed{0.906}$.

(**See supplement right below lecture 5, module week 2, where similar problem was displayed with all conditional and joint probabilities written explicitly as such)**

Let D="A person has the disease"; += "the person tests positive"

P(True positive) = $P(+ \cap D) = 0.017;$ P(+) = 0.1; P(True negative)= $P(- \cap D^c) = 0.8$

Specificity is the probability that a person that does not have the disease tests negative, a conditional probability.

(Reference to what probabilities are being calculated. 0.5 pts)

Specificity =$P(- \mid D^c) = \frac{P(- \cap D^c)}{P(- \cap D^c)+P(+ \cap D^c)} = \frac{0.8}{0.8+(1-0.017)} = 0.9060023$

Tables (numbers bold is what we are given. Not bolded mean derived from known bolded. Probabilities and counts assuming 100 people (A table, complete **(with labels 0.5pts, correct joint probabilities 1 pts)**

Probability table **OR** **Counts table, assuming 100 in the population**

	D	D^c	
+	**0.017 P(TP)**	0.083 P(FP)	**0.1**
-	0.1 P(FN)	**0.8 P(TN)**	0.9
	0.117	0.883	1

	D	D^c	
+	**1.7 persons**	8.3 **persons**	**10**
-	10 persons	**80** persond	90
	10.7	88.3	100

Specificity=P(TN)/(P(TN)+P(FP)= 0.8/0.883=0.9060023 Specificity= 80/88.3=0.9060023 **(work and final number to calculate specificity, 1pt)**

Comment:

(Reference to what probabilities are being calculated not complete 0.3 pts)

Question **24**

Correct

Mark 1.00 out of 1.00

The following expression

$$\int_x 2\mu_x^2 x f(x) dx$$

, where f(x) is a density function and the integration is over all the domain of the random variable X,

equals

Select one:

- a. $2\mu_x^2$
- b. σ_x^2
- c. $2\mu_x^3$
- d. 1

Your answer is correct.

The correct answer is:

$$2\mu_x^3$$

Question **25**

Complete

Mark 3.00 out of 3.00

In this question you must show work and explain your work, like in all the work questions. Attachments are not allowed.

We observed several days at the door of the UCLA bookstore how many people entering the store it took to observe a famous person that we recognized entering the textbook store. This is the data we collected.

34, 21, 10, 108, 130, 66, 21, 56, 104, 147

Use this data to estimate the probability that it will take 100 people to observe a famous one that we recognize entering the store.

Describe the steps of your calculations and justify them based on methods used in lecture. No attachments are allowed in this question. Use the space provided below.

We are trying to find how many people entering the store it took to observe a famous person that we recognized entering the textbook store. In other words, we are looking for the number of trials it takes before the first success, and therefore, should use a geometric model. Let X = how many people entering the store it took to observe a famous person that we recognized entering the textbook store. We can start by finding the expected value from the discrete dataset. We can do this by finding the average of all days, just as was done in the baby births handout: $E(X) = \frac{34+21+10+108+130+66+21+56+104+147}{10} = \frac{697}{10} = 69.7$. From this expected value, we can now find p, the probability that the person walking in is a famous person: $p = \frac{1}{E(X)} = \frac{1}{69.7} = 0.0143$. Finally, we can plug the values into the geometric model to solve for $P(X = 100)$: $P(X = 100) = (1 - 0.0143)^{99}(0.0143) = \boxed{0.0342}$.

We calculate the average number of people per day by adding the numbers and dividing by the total number of days in which we conducted the experiment.

sample mean = $\frac{34+21+10+108+130+66+21+56+104+147}{10} = 69.7$ **(1pt)**

We define X= number of people it takes to observe the first famous person entering the store.

X ~ geometric(p) **(0.2pts)**

We do not know p, but we know that E(X) = 1/p **(0.6pts)**

So we make (1/p) = 69.7 and that gives p=0.0143472 **(0.2pts)**

With that p, we calculate, using the geometric probability model

$P(X = 100) = (1 - 0.0143472)^{99} 0.0143472 = 0.003431146$ **(1pt)**

Comment:

Question **26**

Correct

Mark 1.00 out of 1.00

A self driving car makes a forbidden U-turn and a regular car crashes against it. Three witnesses, whom we call Bison, Rabindranath, and Tom saw what happened. Suppose the reliability of the witnesses is estimated by having the witnesses observe a number of similar scenes. It is found that Bison has probability 0.9 of stating that the self-driving car did a forbidden U-turn, Rabindranath has probability 0.8 of stating that, and Tom has probability 0.7 of stating that. Let A, B, and C denote, respectively, the events that persons Bison, Rabindranath and Tom will state that the self driving car did the forbidden U-turn. Assuming that these events are independent events, calculate the probability that all three witnesses will testify that the self-driving car did the forbidden U-turn.

- a. 0.504 ✔ Calculate P(A)P(B)P(C)
- b. 0.902
- c. 0.398
- d. 0.5

Your answer is correct.

The correct answer is:
0.504

Question **27**

Correct

Mark 1.00 out of 1.00

According to the lognormal model fitted to radon data from Minnesota, the probability that a randomly chosen household has radon level larger than 4 is estimated as

- a. 0.574
- b. approximately 0
- c. 0.426 ✔
- d. 0.41006

Your answer is correct.

The correct answer is:
0.426

Question **28**

Correct

Mark 1.00 out of 1.00

The Old Faithful is a famous geyser in Yellowstone (https://www.yellowstonepark.com/things-to-do/geysers-hot-springs/about-old-faithful/)

The geyser varies in the time it makes visitors wait for its next eruption. Using past data we calculated that it takes an average of 35 minute waiting time from the moment the last eruption occurred to a new eruption, when the last eruption was short. You arrive at the site where you can watch Old Faithful, but you arrive right at the end of a short eruption. What is the probability that you will have to wait less than 30 minutes for its next eruption?

- [x] a. 0.57563
- [ ] b. $1 - e^{-35}$
- [ ] c. 1
- [ ] d. 0.42437

Your answer is correct.

$f(x) = \frac{1}{35}e^{-\frac{1}{35}x}, x \geq 0$

The correct answer is:
0.57563

Question **29**

Correct

Mark 1.00 out of 1.00

If X is uniformly distributed over (0,1), find the density function of

$$Y = e^x$$

.

Select one:

- (selected) a. $$1/y, \qquad y \in [1, e]$$
- b. exponential with lambda=1.
- c. $$N(\mu = 1/2, \sigma^2 = 1/12)$$

Your answer is correct.

The correct answer is:

$$1/y, \qquad y \in [1, e]$$

Question **30**

Partially correct

Mark 2.00 out of 3.00

A box contains 6 bags of Crunchy Cheetos and 3 bags of Doritos. At the currently happening DataFest at UCLA, 3 bags are going to be randomly chosen without replacement and given to the student that guesses the answer to the midnight quiz question. Match the following probabilities

Probability that two bags are Crunchy Cheetos bags.	0.5357143 ✔
The expected number of Doritos bags	1 ✖
The probability that one bag is Cruncy Cheetos and the other are Doritos.	0.2142857 ✔

Your answer is partially correct.

You have correctly selected 2.
Let Y =number of crunchy cheetos bags

Y hypergeometric, because we are drawing without replacement and the box is small, so everytime we draw a bag the probabilities will change.

$$P(Y=y|n=3)=\frac{\binom{6}{y}\binom{3}{3-y}}{\binom{9}{3}}, Y=0,1,2,3$$

$$P(Y=2|n=3)=\frac{\binom{6}{2}\binom{3}{1}}{\binom{9}{3}}=$$

Similarly,

\(P(Y=1|n=3) = \frac{{6 \choose 1}{3 \choose 2}}{{9 \choose 3}} =

The correct answer is:
Probability that two bags are Crunchy Cheetos bags. → 0.5357143,

The expected number of Doritos bags → 2,

The probability that one bag is Cruncy Cheetos and the other are Doritos. → 0.2142857