Math 33A Sheet 1

Chapter. 1.1

Ex 11) $\left|\begin{matrix} x-2y=2 \\ 3x+5y=17 \end{matrix}\right|$

E.1 $(x-2y=2)-3$ E.5 $y=\frac{1}{2}x-1$

E.2 $3x+5y=17$ E.6 $y=-\frac{3}{5}x-\frac{17}{5}$

E.3 $-3x+6y=-6$

$11y=11$

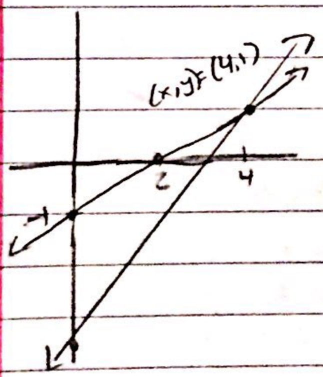
$y=1$

E.4 $x-2(1)=2$

$x-2=2$

$x=4$

$\boxed{(4,1)}$



Ex 12) $\left|\begin{matrix} x-2y=3 \\ 2x-4y=6 \end{matrix}\right|$

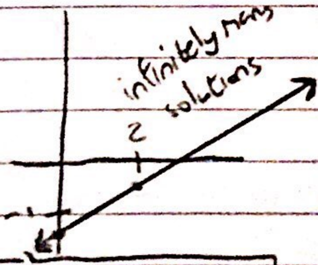
E.1 $(x-2y=3)-2$

E.2 $2x-4y=6$

E.3 $-2x+4y=-6$

$0=0$ $\boxed{\text{Infinitely many solutions}}$

E.4 $y=\frac{1}{2}x-\frac{3}{2}$



Ex 13) $\left|\begin{matrix} x-2y=3 \\ 2x-4y=8 \end{matrix}\right|$

E.1 $(x-2y=3)-2$

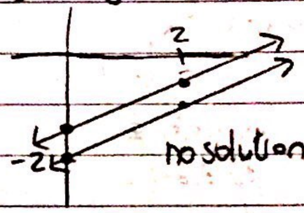
E.2 $2x-4y=8$

E.3 $-2x+4y=-6$

$0=2$ $\boxed{\text{no solution}}$

E.4 $y=\frac{1}{2}x-\frac{3}{2}$

E.5 $y=\frac{1}{2}x-2$



Ex 14) $\left|\begin{matrix} x+4y+z=0 \\ 4x+13y+7z=0 \\ 7x+22y+13z=1 \end{matrix}\right|$

E.1 $x+4y+z=0$

E.2 $4x+13y+7z=0$

E.3 $-4(x+4y+z=0)$

$-3y+3z=0$

E.4 $y-z=0$

E.5 $7x+22y+13z=1$

$-7(x+4y+z=0)$

$-6y+6z=1$

E.6 $y=z$

E.7 $-6y+6y=1$

$0=1$

no solution,
the 3 planes have no common intersection.

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Ex 15) $\left| \begin{array}{l} x+y-z=0 \\ 4x-y+5z=0 \\ 6x+y+4z=0 \end{array} \right|$

E.1 $x+y-z=0$

E.2 $4x-y+5z=0$

$-4(x+y-z=0)$

E.3 $-5y+9z=0$

E.4 $y-\frac{9}{5}z=0$

E.5 $6x+y+4z=0$

$-6(x+y-z=0)$

$-5y+10z=0$

E.6 $y-2z=0$

$-(y-\frac{9}{5}z=0)$

E.7 $\frac{4}{5}z=0$

$z=0$

E.8 $y-2(0)=0$

$y=0$

E.9 $x+0-0=0$

$x=0$

$\boxed{(x,y,z)=(0,0,0)}$

The 3 planes intersect at (0,0,0) only

★ Ex 16) $\left| \begin{array}{l} x+4y+z=0 \\ 4x+13y+7z=0 \\ 7x+22y+13z=0 \end{array} \right|$

E.1 $x+4y+z=0$

E.2 $4x+13y+7z=0$

$-4(x+4y+z=0)$

$-3y+3z=0$

E.3 $y-z=0$

E.4 $7x+22y+13z=0$

$-7(x+4y+z=0)$

$-6y+6z=0$

$y-z=0$

E.5 $y=z$

$x+4y+y=0$

E.6 $x+5y=0$

E.7 $x=-5y=-5z$

$z=t$

$y=t$

$x=-5t$

$\boxed{(x,y,z)=(-5t,t,t)=t(-5,1,1)}$

The 3 planes intersect at the line given by $(x,y,z)=t(-5,1,1)$ which runs through the origin

Ex 17)* $\left|\begin{array}{l} x+2y=a \\ 3x+5y=b \end{array}\right|$

E.1 $x+2y=a$

E.2 $3x+5y=b$

$-3(x+2y=a)$ $-3E.1$

$-y=b-3a$

E.3 $y=3a-b$

$x+2(3a-b)=a$

$x+6a-2b=a$

$x=-5a+2b$

$\boxed{(x,y)=(-5a+2b,\ 3a-b)}$

Ex 19) $\left|\begin{array}{l} x+y-z=-2 \\ 3x-5y+13z=18 \\ x-2y+5z=k \end{array}\right|$

a) E.1 $x+y-z=-2$

E.2 $3x-5y+13z=18$

$-3(x+y-z=-2)$ $-3E.2$

$-8y+16z=24$

E.3 $y-2z=-3$

E.4 $x-2y+5z=k$

$-(x+y-z=-2)$ $-E.1$

$-3y+6z=k+2$

E.5 $y-2z=-\frac{1}{3}k-\frac{2}{3}$

↳ cannot have one solution

↳ infinitely many when $-\frac{1}{3}k-\frac{2}{3}=-3$

$3(-\frac{1}{3}k-\frac{2}{3}=-3)$

$-k-2=-9$

$-k=-7$

$k=7$

$\boxed{\text{infinitely many solutions at } k=7}$

b) at $k=7$, there are infinitely many solutions, as shown in part a)

for all other values of k, no solution exists

c) if $k=7$:

E.1 $y-2z=-3$

$y-2z=-\frac{1}{3}(7)-\frac{2}{3}$

E.2 $y-2z=-3$

$y=2z-3$

$x+(2z-3)-z=-2$

$x+z-3=-2$

E.3 $x=-z+1$

Let $z=t$

$x=-t+1$, $y=2t-3$

$\boxed{(x,y,z)=(-t+1,\ 2t-3,\ t)}$

Chapter 1.2

Ex 2) $\left|\begin{array}{l} 3x+4y-z=8 \\ 6x+8y-2z=3 \end{array}\right|$

$\left[\begin{array}{ccc|c} 3 & 4 & -1 & 8 \\ 6 & 8 & -2 & 3 \end{array}\right]$ $-2(I)$

$\left[\begin{array}{ccc|c} 3 & 4 & -1 & 8 \\ 0 & 0 & 0 & -13 \end{array}\right]$

$\boxed{\text{no solution}}$

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Ex 20)

$$A = \begin{bmatrix} 0 & a & 2 & 1 & b \\ 0 & 0 & 0 & c & d \\ 0 & 0 & e & 0 & 0 \end{bmatrix}$$

↳ Row I has nonzeros → a must be a leading 1

↳ Row II has no leading 1s to the left of the column containing e, therefore e = 0

↳ c has to be either a 0 or 1, but if it were 1, it would be a leading 1, and leading 1s must have the rest of the column as 0, ∴ c = 0

$$\boxed{\begin{gathered} a=1,\ c=0,\ e=0,\ d=0,\ b=\mathbb{R} \\ \text{or} \\ a=1,\ c=0,\ e=0,\ d=1,\ b=0 \end{gathered}}$$

Ex26)* Yes, an operation can be done to turn B into A.

The types of operations are:

i) Dividing by a nonzero scalar

ii) subtracting a multiple of a row from another row

iii) swapping rows

If division by a nonzero scalar occurs, division by that nonzero scalar's reciprocal will revert B->A.

If subtracting a multiple of another row occurs, subtracting the negative of that same multiple will revert B->A.

If row swapping occurs, the rows can be swapped back.

In other words, the inverse operation swaps B->A.

Ex 20)* When you multiply both sides of an equation by the same value, the equation remains equivalent to its original value. It also remains equivalent when you subtract equivalent values from both sides of the equation, therefore, subtracting a multiple of an equation results in a system with the same solutions.

QI) In a system of 2 linear equations of 3 variables, the solution set either consists of no solution or infinitely many solutions. The intersection of 2 planes (3 variable equations) always forms a line, and therefore would be unable to provide a unique solution. They can also be parallel, having no intersection, and therefore no solution.

In a system of 3 linear equations of 3 variables, the solution set may consist of a unique solution, no solution, or infinitely many solutions. Once again, the 3 planes may intersect at a line, causing infinitely many solutions, or be parallel causing no solution. In addition, the planes may intersect at 2 parallel lines, also causing there to be no solution. However, the planes may also intersect at 2 lines that cross, which would mean their intersection is a point, and therefore a unique solution.