Started on	Friday, 6 May 2022, 8:33 PM
State	Finished
Completed on	Saturday, 7 May 2022, 4:12 PM
Time taken	19 hours 38 mins

Question **1**

Complete

Marked out of 5.00

This question requires that you show work. Work is 80% of your credit and involves showing intermediate steps. The rest is for labeling and defining things so that we know what they are, making clear what probabilities you are calculating and labeling them as such (are they conditional, total, etc..?) and providing your final number.

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Let X and Y denote random variables denoting the proportion of contaminants of type A and the proportion of contaminants of type B present in a landfill, respectively. A student calculated the conditional density function

$$f(y|x) = \frac{2(x+y)}{3x^2}, \quad 0 \leq y \leq x,$$

and the marginal density function

$$f(x) = 3x^2, \quad 0 \leq x \leq 1$$

Are x and y independent? Make sure that you justify your answer with detailed work.

You may attach a file with work. But the file must contain only the answer to this question. If it contains answers for other questions, those questions will not be graded at all. You may not email your work to this question. It will not count that way.Alternatively, it would be best if you could use the editor and solve your math work using the equation editor in it, which you can see by clicking on the arrow in the menu and selecting the calculator icon.

See attached PDF

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Question **2**

Complete

Marked out of 3.00

This question requires that you show work. Work is 80% of your credit and involves showing intermediate steps. The rest is for labeling and defining things so that we know what they are, making clear what probabilities you are calculating and labeling them as such (are they conditional, total, etc..?) and providing your final number.

Y**ou may not attach a file to this question. Answer in the space provided. Only the work written in the space provided below will be read.** The editor has a table capability that will help you do nice tables. To expand the options in the editor, click on the arrow above in the menu to expand it, and choose the table option.

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Three randomly chosen students from UCLA are interviewed to determine their attitude towards changing from the current quarter system to a semester system. Let the random variable X denote the response **of the first student interviewed**, with X= 1 denote the answer "YES" (move to a semester system) and X=0 denote "NO" (do not move to a semester system). The probability that a randomly chosen UCLA student votes yes has been estimated by students in this class in another quarter to be approximately 0.45.

(a) List the sample space of the experiment consisting of observing the choice of three randomly chosen students to the question and calculate the probability of each single outcome in it, indicating the rules of probability and assumptions you are making.

(b) Write a table with the joint probability mass function of random variables X and Y, where Y denotes the number of students in the random sample of 3 students that are in favor of switching to the semester system.

(c) Determine and show whether X and Y are independent, using the definition of independence and also comparing the marginal distribution of Y with the conditional distributions of Y for given values of X.

Given: n = 3 (where n is the size of the sample), P(X = 1) = 0.45

a) S = { 000, 001, 010, 011, 100, 101, 110, 111 }

We assume X = 0 and X = 1 are mutually exclusive, X's domain is { 0, 1 }, and that the choice of each of the students in the sample is independent of one another

By the axioms of probability: $P(X=0) = 1 - P(X=1) = 1 - 0.45 = 0.55$

By the product rule for independent events, we can calculate the following probabilities:

$P(000) = P(0)P(0)P(0) = (0.55)^3 = 0.166$

$P(001) = P(0)P(0)P(1) = (0.55)^2(0.45) = 0.136$

$P(010) = P(0)P(1)P(0) = (0.55)^2(0.45) = 0.136$

$P(011) = P(0)P(1)P(1) = (0.55)(0.45)^2 = 0.111$

$P(100) = P(1)P(0)P(0) = (0.55)^2(0.45) = 0.136$

$P(101) = P(1)P(0)P(1) = (0.55)(0.45)^2 = 0.111$

$P(110) = P(1)P(1)P(0) = (0.55)(0.45)^2 = 0.111$

$P(111) = P(1)P(1)P(1) = (0.45)^3 = 0.091$

b)

Joint PMF of X and Y

X\Y	0	1	2	3
0	P(X=0,Y=0) = P(000) = 0.166	P(X=0,Y=1) = P(010) + P(001) = 0.136 + 0.136 = 0.272	P(X=0,Y=2) = P(011) = 0.111	P(X=0,Y=3) = 0
1	P(X=1,Y=0) = 0	P(X=1,Y=1) = P(100) = 0.136	P(X=1,Y=2) = P(101) + P(110) = 0.111 + 0.111 = 0.222	P(X=1,Y=3) = P(111) = 0.091

c)

Marginal Distribution of Y

y P(Y = y)

y P(Y = y)
0 P(Y=0) = P(X=0,Y=0) + P(X=1,Y=0) = 0.166 + 0 = 0.166
1 P(Y=1) = P(X=0,Y=1) + P(X=1,Y=1) = 0.272 + 0.136 = 0.408
2 P(Y=2) = P(X=0,Y=2) + P(X=1,Y=2) = 0.111 + 0.222 = 0.333
3 P(Y=3) = P(X=0,Y=3) + P(X=1,Y=3) = 0 + 0.091 = 0.091

By the definition of independence: $P(x, y) = P(x)P(y)$. We can therefore use this definition, checking that it holds for all possible values of X and Y. If X and Y were independent, it should be true that $P(X = 1, Y = 0) = P(X = 1)P(Y = 0)$. However, plugging in the given value for P(X=1) and the value of P(Y=0) from the marginal distribution table above, we see that $0 \neq (0.166)(0.45)$, showing that X and Y are not independent by the definition of independence.

Furthermore, we can prove this again using conditional distributions of Y for given values of X:

Conditional Distribution of Y given X=0

y P(Y=y|X=0)
0 P(Y=0|X=0) = P(X=0,Y=0)/P(X=0) = 0.166/0.55 = 0.302
1 P(Y=1|X=0) = P(X=0,Y=1)/P(X=0) = 0.272/0.55 = 0.495
2 P(Y=2|X=0) = P(X=0,Y=2)/P(X=0) = 0.111/0.55 = 0.202
3 P(Y=3|X=0) = P(X=0,Y=3)/P(X=0) = 0/0.55 = 0

Conditional Distribution of Y given X=1

y P(Y=y|X=1)
0 P(Y=0|X=1) = P(X=1,Y=0)/P(X=1) = 0/0.45 = 0
1 P(Y=1|X=1) = P(X=1,Y=1)/P(X=1) = 0.136/0.45 = 0.302
2 P(Y=2|X=1) = P(X=1,Y=2)/P(X=1) = 0.222/0.45 = 0.493
3 P(Y=3|X=1) = P(X=1,Y=3)/P(X=1) = 0.091/0.45 = 0.202
As seen from the tables above, the values for P(Y=y) vary depending on which table you are looking at. This tells us the probabilities for each value of Y depend on the given value for X, showing us that these two variables are indeed not independent.

Question **3**

Complete

Marked out of 1.00

We have two conditional distributions below:

y	P(Y=y|X=0)
0	$P(Y=0\|X=0) = \frac{P(X=0,Y=0)}{P(X=0)} = \frac{0.012}{0.3} = 0.04$
1	$P(Y=1\|X=0) = \frac{P(X=0,Y=1)}{P(X=0)} = \frac{0.096}{0.3} = 0.32$
2	$P(Y=2\|X=0) = \frac{P(X=0,Y=2)}{P(X=0)} = \frac{0.192}{0.3} = 0.64$
3	$P(Y=3\|X=0) = \frac{P(X=0,Y=3)}{P(X=0)} = \frac{0}{0.3} = 0$

y	P(Y=y|X=1)
0	$P(Y=0\|X=1) = \frac{P(X=1,Y=0)}{P(X=1)} = \frac{0}{0.7} = 0$
1	$P(Y=1\|X=1) = \frac{P(X=1,Y=1)}{P(X=1)} = \frac{0.028}{0.7} = 0.04$
2	$P(Y=2\|X=1) = \frac{P(X=1,Y=2)}{P(X=1)} = \frac{0.224}{0.7} = 0.32$
3	$P(Y=3\|X=1) = \frac{P(X=1,Y=3)}{P(X=1)} = \frac{0.448}{0.7} = 0.64$

The two random variables X and Y are

- [ ] a. complement of each other
- [x] b. not independent
- [ ] c. mutually exclusive
- [ ] d. independent

Question **4**

Complete

Marked out of 1.00

Office hours videos, supplementary lectures, module week 6.

The distribution of blood types in a certain country are such that 25% of the population are type O, 55% are type A, 20% are type B. Suppose that 30 people are selected at random from this country. What is the probability that 10 are type O, 10 are type A, and the other 10 are type B?

- a. 0.999
- b. 0.7814
- c. 0.33145
- d. 0.001373087

Question **5**

Complete

Marked out of 1.00

The random variable Y can take values 1 or 4, and the random variable X can take values $-1, -2, 1, 2$.
The joint probability mass function of X and Y, $P(X, Y)$, appears below.

		Y	
		1	**4**
	-2	0	1/4
X	**-1**	1/4	0
	1	1/4	0
	2	0	1/4

Select all that applies

Select one or more:

- a. X, Y are two independent random variables.
- b. $E(X) = 0$
- c. X, Y are not two independent random variables.
- d. $Var(X) = 1$

Question **6**

Complete

Marked out of 1.00

A small college is offering two meal plans for the undergraduate students in the dorms, plan A and plan B.
Let X and Y have the joint density function

$$f(x,y) = 2(x+y) \qquad 0 < y < x < 1,$$

where X is the proportion of students purchasing plan X and Y is the number of students purchasing plan B.
If 10% of the students purchase plan A, what is the probability that less than
5% buy plan B?

Select one:

- a. 0.013
- b. 0.010
- c. 0.5
- d. 0.4167

Question **7**

Complete

Marked out of 1.00

The conditional probability density function of random variable Y when X is 0 is given by

y	P(Y=y|X=0)
0	0.04
1	0.32
2	0.64
3	0

and the unconditional probability mass function of X is

x	P(X=x)
0	0.3
1	0.7

Then the joint probability P(Y=1, X=0) is equal to

- a. 0.096
- b. 0.89
- c. 0.124
- d. 0.028

Question **8**

Complete

Marked out of 3.00

This question requires that you show work. Work is 80% of your credit and involves showing intermediate steps. The rest is for labeling and defining things so that we know what they are, making clear what probabilities you are calculating and labeling them as such (are they conditional, total, etc..?) and providing your final number.

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Suppose we have a set of independent and identically distributed continuous random variables x, $x_1, x_2, \ldots\ldots, x_n$, each of them with the density given below. We would like to find the joint density function of these n random variables.

$$f(x) = \frac{\gamma}{\alpha}\left(\frac{x-\mu}{\alpha}\right)^{(\gamma-1)} \exp\left(-((x-\mu)/\alpha)^{\gamma}\right) \qquad x \geq \mu; \gamma, \alpha > 0$$

(a) Showing detailed intermediate work, provide a final formula where you have simplified the final expression as much as possible, as done in the pre-recorded lecture videos.

(b) Where in past modules, in pre-recorded lectures before module 6, did we use a joint density to solve a statistical estimation problem? Describe what problem that was.

(c) When we say that $x_1, x_2, \ldots\ldots, x_n$ are independent identically distributed, are we saying that all of those x_i have the same value? Use a random variable that applies to students at UCLA, a continuous random variable to explain.

You may attach a file with work. But the file must contain only the answer to this question. If it contains answers for other questions, those questions will not be graded at all. You may not email your work to this question. It will not count that way. Alternatively, it would be best if you could use the editor and solve your math work using the equation editor in it, which you can see by clicking on the arrow in the menu and selecting the calculator icon.

See attached PDF

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Question **9**

Complete

Marked out of 1.00

Let X be a random variable with density function

$$f(x, y) = 2, \quad 0 \leq y \leq x \leq 1$$

Which of the following is the marginal probability density function of X?

Select one:

- a. $$f(x) = 2x^2, \quad 0 \leq x \leq 1$$
- b. $$f(x) = 2x, \quad 0 \leq x \leq 1$$
- c. $$f(x) = 2x, \quad y \leq x \leq 1$$
- d. $$f(x) = 2xy, \quad y \leq x \leq 1$$

Question **10**

Complete

Marked out of 1.00

When we talk about the joint density function of two random variables, X, Y, (f(x,y)), for constants a and b,

$$P(X \leq a, Y \geq b)$$

is

Select one:

- a. always 1 to satisfy axioms
- b. an area
- c. the value of the first quartile
- d. a volume

Q1

$$f(y|x) = \frac{2(x+y)}{3x^2}, \quad 0 \le y \le x$$

$$f_x(x) = 3x^2, \quad 0 \le x \le 1$$

Independence : $f_x(x) f_y(y) = f(x,y)$

Find the joint density function $f(x,y)$:

$$f(y|x) = \frac{f(x,y)}{f(x)}$$

$$f(x,y) = f(x)\, f(y|x)$$

$$f(x,y) = 3x^2\left(\frac{2(x+y)}{3x^2}\right)$$

$$\underline{f(x,y) = 2(x+y), \quad 0 \le y \le x \le 1}$$

Find the marginal density function $f(y)$:

$$f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

$f(y) = \int_y^1 2(x+y)\,dx$ ⟵ apply domain of x

$$f(y) = 2\int_y^1 (x+y)\,dx$$

$$f(y) = 2\left[\tfrac{1}{2}x^2 + xy\right]_y^1$$

$$f(y) = 2\left[\left(\tfrac{1}{2} + y\right) - \left(\tfrac{1}{2}y^2 + y^2\right)\right]$$

$$f(y) = 2\left[\tfrac{1}{2} + y - \tfrac{3}{2}y^2\right]$$

$$\underline{f(y) = -3y^2 + 2y + 1}$$

Check using definition of indep.:

$$f(x)f(y) = f(x,y)?$$

$$3x^2(-3y^2 + 2y + 1) = 2(x+y)?$$

$$-9x^2y^2 + 6x^2y + 3x^2 = 2(x+y)?$$

$\boxed{\text{Not independent}} \rightarrow f(x)f(y) \neq f(x,y)$

Q8

$$f(x) = \frac{\gamma}{\alpha}\left(\frac{x-\mu}{\alpha}\right)^{(\gamma-1)} e^{-\left(\frac{x-\mu}{\alpha}\right)^{\gamma}}, \quad x \geq \mu;\ \gamma, \alpha > 0$$

a) $f(x_1, x_2, \ldots x_n) = f(x_1) f(x_2) \ldots f(x_n)$

↳ product rule for IID

$$f(x_1, \ldots, x_n) = \left(\frac{\gamma}{\alpha}\left(\frac{x_1-\mu}{\alpha}\right)^{\gamma-1} e^{-\left(\frac{x_1-\mu}{\alpha}\right)^{\gamma}}\right) \cdots \left(\frac{\gamma}{\alpha}\left(\frac{x_n-\mu}{\alpha}\right)^{\gamma-1} e^{-\left(\frac{x_n-\mu}{\alpha}\right)^{\gamma}}\right)$$

$$f(x_1, \ldots, x_n) = \left(\frac{\gamma}{\alpha} \cdots \frac{\gamma}{\alpha}\right)\left(\left[\frac{x_1-\mu}{\alpha}\right]^{\gamma-1} \cdots \left[\frac{x_n-\mu}{\alpha}\right]^{\gamma-1}\right)\left(e^{-\left(\frac{x_1-\mu}{\alpha}\right)^{\gamma}} \cdots e^{-\left(\frac{x_n-\mu}{\alpha}\right)^{\gamma}}\right)$$

$$\boxed{f(x_1 \ldots x_n) = \left(\frac{\gamma}{\alpha}\right)^n \left(\prod_{i=1}^{n} \left[\frac{x_i-\mu}{\alpha}\right]^{\gamma-1}\right)\left(e^{-\sum_{i=1}^{n}\left[\frac{x_i-\mu}{\alpha}\right]^{\gamma}}\right)}$$

b) In lecture 1, we use a joint distribution to solve the statistical estimation problem of the sum of dice rolls. We constructed a joint density table that examined the results of 2 events: dice #1's roll and dice #2's roll. From there, we summed up their results and investigated the probability of rolling various sums. Each sum is a sum of bivariate discrete random variables.

c) If $x_1, x_2, \ldots, x_n$ are IID, all x_i do not have to have the same value. For example, if we assume weight among UCLA male students is normally distributed and that each person's weight is independent of one another, then UCLA male student weight is IID, by definition. Each x_i is normally distributed and independent. However, it is clear that not all of the male student body is of the same weight. Some guys may be 150lbs while others may be 200lbs.