Started on	Monday, 11 April 2022, 12:10 PM
State	Finished
Completed on	Wednesday, 13 April 2022, 10:01 PM
Time taken	2 days 9 hours

Question **1**

Correct

Marked out of 1.00

Let Y be a random variable denoting the sum of the roll of two fair six-sided dice. Posted below is a table for the pmf. The table is incomplete. Before doing the table, remember that if the die is fair, then each outcome in the roll of the two dice is equally likely ✓ and therefore it is appropriate to use the classical definition of probability ✓ to calculate the probability of each outcome (Review Lecture 1 and chapter 1 if you do not remember that). Also, each roll of two fair six sided dice is a random sample of size 2 from an urn containing numbers 1 to 6 with replacement ✓ . And after studying independence, we can calculate the probability of rolling, for example, (2,3) using the product rule for independent events ✓ . Now that we have reviewed that, here is the table. You will complete what is missing. A couple of examples are included.

y	P(Y=y)	Outcomes in the event mapping to that y
2	1/36	{(1,1)}
3	2/36	{(1,2), (2,1)}
4	3/36 ✓	
5	4/36 ✓	
6	5/36 ✓	
7	6/36	{(1,6), (6,1),(2,5),(5,2), (3,4), (4,3)
8	5	{(2,6), (6,2), (3,5), (5,3), (4,4)} ✓
9	4/36	{(3,6),(6,3), (4,5),(5,4)} ✓
10	3/36	{(4,6), (6,4), (5,5)}
11	2/36	{(6,5), (5,6) } ✓
12	1/36	{(6,6) }

Notice that if you revisit the app studied together with Lecture 1 https://www.randomservices.org/random/apps/DiceExperiment.html

you will be able to construct other probability mass functions for the sum of 2 dice when the dice are not fair. Practice on your own those other distributions, because in those cases you will not be able to use the classical definition, but you can use the product rule for independent events to calculate the probability of each outcome.

Question **2**

Correct

Marked out of 1.00

CycleTheWorld sells bicycles. Based on the history of the store, it is known that in May it is equally likely that the store will sell 0,1,2,3,or 4 bicycles a day. The story has never sold more than 4 bicycles in a day. It is February and CycleTheWorld needs to start planning for the months ahead. CycleTheWorld has only one salesperson in the story, whose income depends on the number of bicycles the sales person sells per day. Specifically, there is no commission on the first bicycle sold in a day, a $20 commission on the second bicycle sold in a day, a $30 commission on the third, and a $40 commission on the fourth (thus if there are three bicycles sold in a given day, the commission for that day is $50). The income of this person is certainly a random variable because it depends on the random amount sold. We will assume no deductions or income taxes).

The probability that the number of bicycles sold in a given May day will be less than 3	3/5	✓
The probability that the commission in a randomly chosen May day is $50 or more	2/5	✓
The expected daily bonus on a randomly chosen May day	$32	✓
The variability around the expected value (in $) that should be anticipated (i.e., the standard deviation)	$34.2928	✓

Question **3**

Correct

Marked out of 1.00

Daily tooth brushing by residents in a remote country was found to follow the probability mass function given below, where the random variable Y represents the number of times brushing teeth per day

y	0	1	2	3
P(Y=y)	0.325	0.474	0.15	0.051

The value 0.474 means that 47.4% of the residents brush their teeth once per day.

What is the probability that a randomly chosen resident of this country brushes teeth at least twice a day?

● a. 0.201 ✓

○ b. 0.15

○ c. 0.051

○ d. 0.0225

Question **4**

Correct

Marked out of 1.00

Daily tooth brushing by residents in a remote country was found to follow the probability mass function given below, where the random variable Y represents the number of times brushing teeth per day

y	0	1	2	3
P(Y=y)	0.325	0.474	0.15	0.051

The value 0.474 means that 47.4% of the residents brush their teeth once per day.

The statement "F(2) equals 1-0.051 " is

Select one:

● True ✓

○ False

Question **5**

Correct

Marked out of 1.00

Let Y be a random variable representing the sum of the roll of two fair six-sided die. The pmf of Y was studied in class on July 2nd. Let F(y) be the cumulative distribution function of Y. To calculate correctly the following probability,

$$P(4 \leq Y \leq 7),$$

using the cumulative distribution function, we would calculate

Select one:

○ a. F(7)-F(4)

● b. F(7) - F(3) ✓

○ c. F(8)-F(4)

○ d. F(8)-F(3)

Question **6**

Correct

Marked out of 1.00

The probability mass function of a random variable X is

x	**0**	**1**	**2**
P(X=x)	a	a	0.4

a is a number

What is the expected value of this random variable?

Select one:

● a. 1.1 ✓

○ b. 1.2

○ c. 1.3

○ d. 1.4

Question **7**

Correct

Marked out of 1.00

Expectations are used to make decisions. Based on what is expected, we act. The exercise below is a dilemma often encountered by authorities. You will have to use the definition of expected value of a discrete random variable under two scenarios. Then compare expectations and decide based on the comparison.

Exercise

You are the forecaster responsible for hurricane warnings on the southeast corner of the United States in September. Anyone that has lived in North Carolina for a while knows what is it is like at that time of the year, for example. The cost of issuing a warning like a hurricane involves people taking shelter, business stopping, the areas's economy paused - a moderate cost of C dollars. You also know the preventable loss should a hurricane come and the area be unprepared: property damage, lives lost -an extremely high loss of L dollars. Your weather forecast indicates hurricane with a probability of p.

So you weigh the options:

(1) If a warning is issued, the expected cost is Cp+C(1-p) ✓

(2) If warning is not issued, the expected cost is Lp ✓

Therefore, it is clear that you will issue a warning if Lp > C ✓ .

Based on that, we can conclude that a warning is not likely to be given if p< C/L ✓ .

p>(C/L)+(1-p) Lp+C(1-p)

Question **8**

Correct

Marked out of 1.00

CHAPTER 7 SECTION 7.2.1 Ex 2 TEXTBOOK

Let X be the time that it takes to drive between point A and point B during the afternoon rush hour period on highway 4005. The density function of X is

$$f(x) = \frac{1}{2}x, \qquad 0 \leq x \leq 2$$

Calculate the value of the 70th percentile 1.67332 ✓

Calculate the interquartile range 0.732 ✓

Calculate $P(0.5 \leq x \leq 1.5)$ 0.5 ✓

Find the median 1.414 ✓

Question **9**

Correct

Marked out of 1.00

Survival time in years (X) after lung transplant has the following pdf:

$$f(x) = 5e^{-5x}, \qquad x \geq 0$$

We are interested in the median survival time.

Select one:

○ a. 1.289341

○ b. 0.89189

● c. 0.1386294 ✓

○ d. 5

Question **10**

Correct

Marked out of 1.00

The cumulative distribution function of a random variable X defined in the interval 2 to 4 is

F(x) = (1/26)[$2x^2$+x -10]

To find the density function of X I would have to:

Select one:

○ a. Find the moment generating function and take the first derivative and evaluate at 0

● b. Take the derivative of F(x) with respect to x ✓

○ c. Compute the integral of F(x)

○ d. Nothing, the F(x) is the density function of X.

Question **11**

Correct

Marked out of 1.00

Let

$$f(x) = 3x^2, \qquad 0 \le x \le 1,$$

and f(x) = 0 for any other value of X in the real line.

The cumulative distribution function of X (cdf) is

Select one:

○ a. $$F(x) = 6x, \qquad 0 \le x \le 1$$

○ b. $$F(x) = 2x + 1, \qquad 0 \le x \le 1$$

● c. $$F(x) = x^3, \qquad 0 \le x \le 1$$ ✓

○ d. $$F(x) = 3, \qquad 0 \le x \le 1$$

Question **12**

Correct

Marked out of 1.00

The proportion of time X during a 40-hour week that a drug has effect on blood pressure is a r.v. with pdf of

f(X) = 2X, $0 < X < 1$

What is the expected value of

$$X^3$$

?

Select one:

● a. 0.4 ✓

○ b. 0.8

○ c. 0

○ d. 0.61

Question **13**

Correct

Marked out of 1.00

A gambling book recommends the following "winning strategy" for the game of roulette. It recommends that a gambler bet $1 on red.

If red appears (which has probability 18/38), then the gambler should take her 1 dollar profit and quit. If the gambler loses this bet (which has probability 20/38 of occurring), she should make additional 1 dollar bets on red on each of the next two spins of the roulette wheel and then quit. Let X denote the gambler's winnings when she quits. What is the expected value of

$$X^2$$

?

Select one:

- ● a. 2.16635 ✓
- ○ b. -0.1080331
- ○ c. 0.1325
- ○ d. 0. 9811
- ○ e. 10.112091

Question **14**

Correct

Marked out of 1.00

Suppose that you and a friend are matching balanced coins (i.e., each coin has probability 1/2 of landing head). Each of you tosses a coin. If the upper faces match, you win 1.00 dollar; if they do not match, you lose 1.00 dollar (your friend wins 1.00 dollar). The probability of a match is 0.5. Let X= your winnings. The variance of X is

Select one:

- ○ a. 0
- ○ b. 0.5
- ● c. 1 ✓
- ○ d. 0.25
- ○ e. 8

Question **15**

Correct

Marked out of 1.00

Let X be the time that it takes to drive between point A and point B during the afternoon rush hour period on highway 4005. The density function of X is

$$f(x) = \frac{1}{2}x, \qquad 0 \le x \le 2$$

Calculate the value of the 70th percentile	1.67332	✓
Calculate the interquartile range	0.732	✓
Calculate $P(0.5 \le x \le 1.5)$	0.5	✓
Find the median	1.414	✓

Question **16**

Correct

Marked out of 1.00

The response time at an online computer terminal follows, approximately, a gamma distribution, with expected value 4 seconds and variance of 8 seconds. Which of the following is the probability density function for the response times (all functions below have domain of X from 0 to \(\infty\)?

(A) $$ f(x) = \frac{xe^{-1/2x}}{4} $$
(B) $$f(x) = \frac{e^{-1/2x}}{2} $$
(C) $$f(x) = \frac{xe^{-1/2x}}{2} $$

Select one:

- ● a. A ✓
- ○ b. B
- ○ c. C