

20S-PHYSICS1C-1 1CS20 Final Exam

CHARLES ZHANG

TOTAL POINTS

82.5 / 150

QUESTION 1

1 30 pts

1.1 1a 0 / 10

- + 3 pts Correct Integral setup 1
- + 2 pts Current $r < a$
- + 3 pts Correct integral setup 2
- + 2 pts Current $r > a$
- ✓ + 0 pts Incorrect

1.2 1b 4 / 5

- ✓ + 1 pts Correct direction
- ✓ + 2 pts Correct magnetic field inside
- ✓ + 2 pts Correct magnetic field outside
- ✓ - 1 pts Mistake carried over
- + 0 pts Incorrect

1.3 1c 1 / 5

- ✓ + 2 pts J
- + 1 pts Net current
- + 1 pts Magnetic field
- + 1 pts Discussion
- ✓ - 1 pts Mistake carried over
- + 0 pts Incorrect

1.4 1d 5 / 10

- ✓ + 2.5 pts Correct Density
- + 5 pts Correct integral setup for energy/length
- ✓ + 2.5 pts Final answer
- + 0 pts Incorrect

QUESTION 2

30 pts

2.1 2a 4.5 / 10

- + 10 pts Correct

+ 5 pts kirchoffs setup

✓ + 3 pts kirchoff partly right lo used initially

+ 3 pts integral setup with the right limits

✓ + 1.5 pts integral setup partly right/limits wrong or missing

+ 2 pts right solution

+ 1 pts some attempt

+ 0 pts no attempt

2.2 2b 3 / 5

+ 5 pts Correct

✓ + 3 pts partly right/why should I_2 be zero at $t=0$

+ 1 pts some attempt

+ 0 pts no attempt

2.3 2c 5 / 10

+ 10 pts Correct

✓ + 4 pts voltage

+ 3 pts current

✓ + 3 pts direction

✓ - 2 pts error carried over/partial credit

+ 1 pts some attempt for the current and voltage

+ 0 pts wrong

2.4 2d (problem) 1 / 3

+ 3 pts magnitude correct

+ 2 pts identifying $I_2=0, dI_2/dt = e/I_2$ /typo/no explanation

✓ + 1 pts $I_2=0$ /write form of the equation

+ 0.25 pts some attempt

+ 0 pts no attempt

2.5 2d (polarity) 2 / 2

✓ + 2 pts Correct

+ 0 pts no attempt/wrong

QUESTION 3

3 30 pts

3.1 3a 8 / 10

- ✓ + 2 pts Correct X_C
- ✓ + 2 pts Correct X_L
- ✓ + 4 pts Z_{LC} expression
 - + 2 pts Modulus
 - + 0 pts Incorrect

3.2 3b 5 / 5

- ✓ + 3 pts Correct Condition on reactance
- ✓ + 2 pts Correct condition on ω
 - 1 pts Mistake carried over
 - + 0 pts Incorrect

3.3 3c 2 / 10

- + 5 pts Correct Impedance
- + 5 pts I_{max}
 - 1 pts Mistake carried over
 - + 0 pts Incorrect
- ✓ + 2 pts Partial steps in the correct direction

3.4 3d 5 / 5

- ✓ + 3 pts LC voltage lags LC current
- ✓ + 2 pts ω condition
 - + 0 pts Incorrect
 - 1 pts Mistake carried over

QUESTION 4

4 30 pts

4.1 4a 5.5 / 10

- + 10 pts Correct
- + 5 pts path
- ✓ + 2.5 pts path is partly right
 - + 3 pts reflected difference
- ✓ + 1 pts reflection mentioned just once (π instead of 2π)/zero mentioned but not why or it was wrong
- ✓ + 2 pts i_c
 - + 1 pts some attempt
 - + 0 pts no attempt

4.2 4b 5 / 5

- ✓ + 5 pts Correct
 - + 1.5 pts some attempt/mention of only in phase not the condition
 - + 0 pts no attempt

4.3 4c 3 / 10

- + 6 pts right wavelength relation
- + 4 pts right solution N not replaced/calculation error
 - + 2 pts $n-1$ and n
- ✓ + 4 pts correct substitution from part b
- ✓ - 1 pts error carried over
 - + 2 pts some attempt in the right direction
 - + 0 pts incorrect

4.4 4d 5 / 5

- + 0 pts Correct
- ✓ + 2.5 pts large δ
- ✓ + 2.5 pts proportional q
 - + 1 pts some attempt/no explanation
 - + 0 pts no attempt

QUESTION 5

30 pts

5.1 5a 1 / 5

- + 2.5 pts Length contraction
- + 2.5 pts Time dilation
- ✓ + 1 pts Partial steps
 - + 0 pts Incorrect

5.2 5b 10 / 10

- ✓ + 3 pts Realize NOT length contraction and time dilation
- ✓ + 1 pts Correct rocket frame vector
- ✓ + 1 pts Correct earth frame vector
- ✓ + 3 pts Correct transformation matrix
- ✓ + 2 pts Final answers
 - + 0 pts Incorrect
 - + 1 pts Partial steps in the correct direction

5.3 5c 5.5 / 10

- ✓ + 1 pts Correct parallel velocity wrt Earth
- ✓ + 1 pts Correct perpendicular velocity wrt Earth
 - + 1 pts Parallel formula to change frame
- ✓ + 1 pts Perpendicular formula to change frame
 - + 2.5 pts Correct parallel velocity wrt A
- ✓ + 2.5 pts Correct perpendicular velocity wrt A
 - + 1 pts Final answer
 - + 0 pts Incorrect

5.4 5d 2 / 5

- ✓ + 1 pts part c beta = 0 wrt earth
 - + 1 pts part c beta = 0 wrt A
- ✓ + 1 pts part c beta = 1 wrt earth
- ✓ + 1 pts part c beta = 1 wrt A
- ✓ - 1 pts Mistake carried over
 - + 0 pts Incorrect
 - + 1 pts Beta = 0 and beta = 1 sig

1CS20 Final Exam

Full Name (Printed) Charles Zhang

Full Name (Signature) 

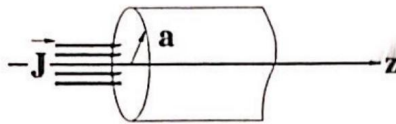
Student ID Number 305 413 659

- The exam is open-book and open notes. You will probably do better to limit yourself to a single page of notes you prepared well in advance.
- **All work must be your own.** You are not allowed to collaborate with anyone else, you are not allowed to discuss the exam with anyone until all the exams have been submitted (after the close of the submissions window for the exam).
- You have **120 minutes** to complete the exam and more than sufficient time to scan the exam and upload it to GradeScope. The exam *must* be uploaded to GradeScope within the time allotted (that is, by the end of the 3-hour finals slot). We will only except submissions through GradeScope and will not accept any exam submitted after the submission window closes (CAE students must contact Corbin for instructions).
- **Given the limits of GradeScope, you must fit your work for each part into the space provided.** You may work on scratch paper, but you will not be able to upload the work you do on scratch paper, so it is essential that you copy your complete solution onto the exam form for final submission. We can only consider the work you submit on your exam form.
- **For full credit the grader must be able to follow your solution from first principles to your final answer. *There is a valid penalty for confusing the grader.***
- It is **YOUR** responsibility to make sure the exam is scanned correctly and uploaded before the end of the submission window. The graders may refuse to grade pages that are significantly blurred, solutions to problems that are not written in the correct place, pages submitted in landscape mode and/or work that is otherwise illegible - if any of this occurs, you may not receive *any* credit for the affected parts.
- Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**

The following must be signed before you submit your exam:

By my signature below, I hereby certify that all of the work on this exam was my own, that I did not collaborate with anyone else, nor did I discuss the exam with anyone while I was taking it.

Signature 



1) An electrical current described by the current density

$$\vec{J} = J_0 \left(1 - 2 \frac{r^2}{a^2}\right) \hat{z}$$

flows in a cylindrical region of radius a aligned with and centered on the z -axis as shown.

- 1a) (10 points) How much current is enclosed by a circular loop of radius r centered on the symmetry-axis of the cylinder, oriented perpendicular to the current-density? Consider both $r \leq a$ and $r > a$.

$$I_{\text{encl}}(r) = \int \vec{J} \cdot d\vec{A}$$

$$r \leq a$$

$$I_{\text{encl}}(r) = \int_0^r J_0 \left(1 - 2 \frac{r^2}{a^2}\right) (\pi r^2) dr$$

$$I_{\text{encl}}(r) = \pi J_0 \int_0^r \left(1 - 2 \frac{r^2}{a^2}\right) r^2 dr$$

$$I_{\text{encl}}(r) = \pi J_0 \left[r - \frac{2}{5} \left(\frac{r^5}{a^2} \right) \right] \quad \text{for } r \leq a$$

$$r > a$$

$$I_{\text{encl}}(r) = \int_0^a J_0 \left(1 - 2 \frac{r^2}{a^2}\right) (\pi r^2) dr$$

$$I_{\text{encl}}(r) = \pi J_0 \int_0^a \left(1 - 2 \frac{r^2}{a^2}\right) r^2 dr$$

$$I_{\text{encl}}(r) = \pi J_0 \left[a - \frac{2}{5} a^3 \right] \quad \text{for } r > a$$

- 1b) (5 points) Find the (vector) magnetic field at a radial distance r from the symmetry-axis of the cylinder for points inside and outside the cylinder.

$$\int \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{encl}}$$

$$r \leq a$$

$$B(2\pi r) = \mu_0 \left(\pi J_0 \left[r - \frac{2}{5} \left(\frac{r^5}{a^2} \right) \right] \right)$$

$$B = \frac{\mu_0 J_0}{2r} \left[r - \frac{2}{5} \left(\frac{r^5}{a^2} \right) \right]$$

$$\vec{B} = \frac{\mu_0 J_0}{2} \left[1 - \frac{2}{5} \left(\frac{r^4}{a^2} \right) \right] \hat{\phi}$$

$$r > a$$

$$B(2\pi r) = \mu_0 \left[\pi J_0 \left[a - \frac{2}{5} a^3 \right] \right]$$

$$\vec{B} = \frac{\mu_0 J_0}{2r} \left[a - \frac{2}{5} a^3 \right] \hat{\phi}$$

- 1c) (5 points) At what value of r does \vec{J} change direction? At what value r does the net current through a loop of radius r change direction? At what value of r does \vec{B} change direction? Discuss.

$$1 - 2\frac{r^2}{a^2} = 0$$

$$1 = 2\frac{r^2}{a^2}$$

$$\frac{1}{2} = \frac{r^2}{a^2}$$

$$\frac{r}{a} = \sqrt{\frac{1}{2}}$$

$$r = a\sqrt{\frac{1}{2}}$$

$$\boxed{\vec{J} \text{ changes direction at } r = a\sqrt{\frac{1}{2}}}$$

$$r - \frac{2}{5}\left(\frac{r^5}{a^2}\right) = 0$$

$$r = \frac{2}{5}\left(\frac{r^5}{a^2}\right)$$

$$\frac{5}{2} = \frac{r^4}{a^2}$$

$$\frac{5}{2} = \frac{r^4}{a^2}$$

$$r^2 = a\left(\frac{5}{2}\right)^{1/2}$$

$$r = \sqrt{a}\left(\frac{5}{2}\right)^{1/4}$$

$$\boxed{\text{net current changes dir at } r = \sqrt{a}\left(\frac{5}{2}\right)^{1/4}}$$

\vec{D} also changes direction at $r = \sqrt{a}\left(\frac{5}{2}\right)^{1/4}$, since I_{enc} would be the only thing affecting \vec{D} 's direction

- 1d) (10 points) Find the energy per unit-length associated with the current.

$$U = \int u_B dV$$

$$u_B = \frac{B^2}{2\mu_0} \rightarrow \text{use } B \text{ at surface of wire}$$

$$u_B = \frac{\mu_0 J_0^2}{4} \left[1 - \frac{2}{5}\left(\frac{r^2}{a^2}\right)\right]^2$$

$$u_B = \frac{\mu_0 J_0^2}{8} \left[1 - \frac{2}{5}\left(\frac{r^2}{a^2}\right)\right]^2$$

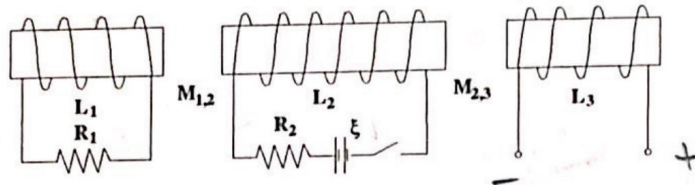
$$U = \int \frac{\mu_0 J_0^2}{8} \left[1 - \frac{2}{5}\left(\frac{r^2}{a^2}\right)\right]^2 dV$$

$$U = \int \frac{\mu_0 J_0^2}{8} \left[1 - \frac{2}{5}\left(\frac{r^2}{a^2}\right)\right]^2 (\pi r^2) dr (L)$$

$$\frac{U}{L} = \frac{\mu_0 J_0^2}{8} \left[1 - \frac{2}{5}\left(\frac{r^2}{a^2}\right)\right]^2 \int_0^a \pi r^2 dr$$

$$\boxed{\frac{U}{L} = \frac{\mu_0 J_0^2 \pi}{24} \left[1 - \frac{2}{5}\left(\frac{r^2}{a^2}\right)\right]^2 a^3}$$

$$\mathcal{E} = IR$$



2) All the values shown in the diagram above are known. The switch has been open a long time. It will be closed at $t = 0$.

- 2a) (10 points) Use one of Kirchhoff's laws to find the current through and the electromotive-force across L_2 as a function of time elapsed after the switch was closed. Don't make any assumptions about the initial current in the circuit just yet, call it I_0 for now.

$$\mathcal{E} - (i + I_0)R_2 - L_2 \frac{di}{dt} = 0$$

$$\frac{di}{dt} = \frac{\mathcal{E} - (i + I_0)R_2}{L_2}$$

$$\frac{di}{\mathcal{E} - (i + I_0)R_2} = \frac{1}{L_2} dt$$

$$\int \frac{di}{\mathcal{E} - (i + I_0)R_2} = -\frac{R_2}{L_2} t$$

$$\ln\left(\frac{i + I_0 - \mathcal{E}/R_2}{- \mathcal{E}/R_2}\right) = -\frac{R_2}{L_2} t$$

$$i = \frac{\mathcal{E}}{R_2} (1 - e^{-(R_2/L_2)t}) - I_0$$

$$\mathcal{E} = \frac{(i + I_0)R_2}{1 - e^{-(R_2/L_2)t}}$$

- 2b) (5 points) Take a close look at the Kirchhoff equation you wrote for the circuit and explain why I_0 has to be zero. It may be a good idea, at this point, to re-write your answers to part a with this new information taken into account.

The resistance cannot initially have any voltage drop across it since initially in an R-L circuit, all voltage drop comes across the inductor as the current increases.

$$i(t) = \frac{\mathcal{E}}{R_2} (1 - e^{-(R_2/L_2)t})$$

$$\mathcal{E}(t) = \frac{iR_2}{1 - e^{-(R_2/L_2)t}}$$

- 2c) (10 points) Find the potential difference across and the current through R_1 as a function of time elapsed after the switch was closed. In what direction does the current flow through R_1 ? Explain.

$$\left| \frac{\mathcal{E}_1^{(1)}}{\frac{\partial I_2}{\partial t}} \right| = \left| \frac{\mathcal{E}_1^{(2)}}{\frac{\partial I_1}{\partial t}} \right| \quad \mathcal{E}^{(1)} = -M \frac{\partial I_2}{\partial t}$$

~~$$\frac{dI_2}{dt} = \frac{\mathcal{E}_2 - IR_2}{L_2} = \frac{\mathcal{E}_2 - \frac{\mathcal{E}_2}{R} (1 - e^{-(R/L_2)t})}{L_2}$$~~

$$\frac{dI_2}{dt} = \frac{\mathcal{E}_2 - \mathcal{E}_2(1 - e^{-(R/L_2)t})}{L_2}$$

$$\mathcal{E}_1 = -M_{1,2} \left(\frac{\mathcal{E}_2 - \mathcal{E}_2(1 - e^{-(R/L_2)t})}{L_2} \right)$$

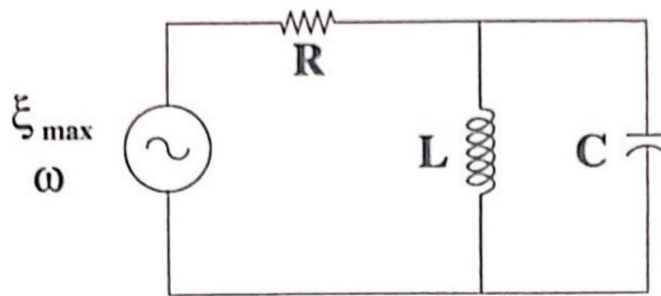
The current flows through R_1 to the left in order to counter the changes made by the changing current in the 2nd circuit

- 2d) (5 points) After a long time, the switch is once again opened. At that instant, estimate the magnitude of the electromotive force across L_3 and mark the polarity of that electromotive force on the open terminals in the diagram.

"Like many things in life, the answer to this question is yes."

— Brent Corbin

$$\mathcal{E}_3 = -M_{2,3} \frac{dI_2}{dt} = -M_{2,3} \left(\frac{\mathcal{E}_2 - \mathcal{E}_2(1 - e^{-(R/L_2)t})}{L_2} \right)$$



3) The quantities shown in the diagram above are all known. When you answer the following questions, make sure your final answers are written in terms of ω (in other words, don't leave any X 's lying around).

- 3a) (10 points) Find the impedance presented to the circuit by the L - C portion of the circuit.

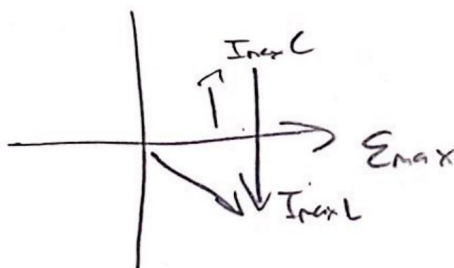
$$\frac{1}{Z} = \sqrt{\left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

$$\boxed{\frac{1}{Z} = \sqrt{(\omega C - \frac{1}{\omega L})^2}}$$

$$\boxed{\frac{1}{Z} = \omega C - \frac{1}{\omega L}}$$

$$\boxed{Z = \frac{1}{\omega C - \frac{1}{\omega L}}}$$

- 3b) (5 points) Under what conditions will the voltage across the L - C portion lead the current through the L - C portion of the circuit?



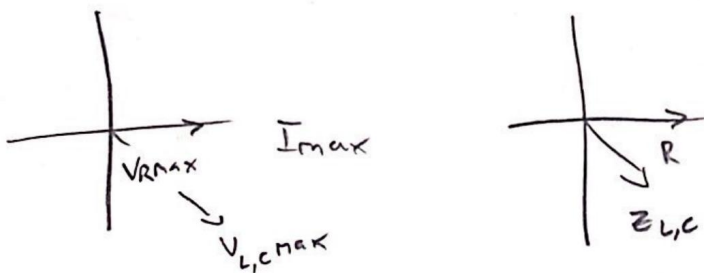
$$\boxed{\text{When } \frac{1}{\omega L} > \omega C, \mathcal{E} \text{ leads } I}$$

- 3c) (10 points) How large is the amplitude of the current drawn out of the source?

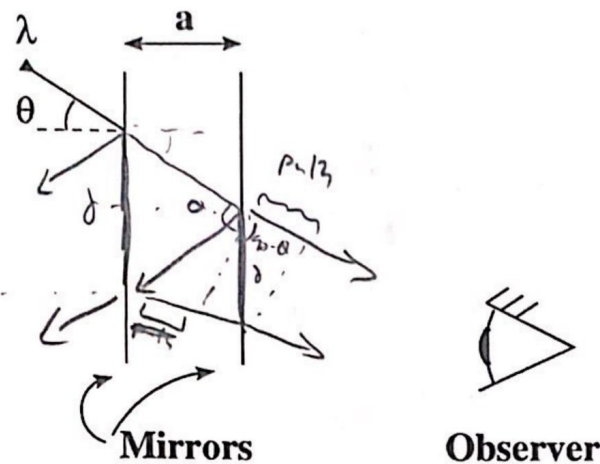
$$I_{\max} = \frac{\varepsilon_{\max}}{Z} = \frac{\varepsilon_{\max}}{R + \frac{1}{\omega C - \frac{1}{\omega L}}}$$

$$I_{\max} = \frac{\varepsilon_{\max}}{R + \frac{1}{\omega C - \frac{1}{\omega L}}}$$

- 3d) (5 points) Under what conditions will the current drawn from the source lead the voltage delivered by the source? Explain.



when $\omega C > \frac{1}{\omega L}$, I leads ε since the parallel section determines the phase between ε and I , so if we make I lead ε in that section, I leads ε in the circuit. I leads ε there if $\omega C > \frac{1}{\omega L}$



4) A Fabry-Perot interferometer consists of two parallel, half-silvered mirrors that are separated by a small distance a . Each of the mirrors transmits 50% of the incident light intensity and reflects the other 50% of the incident intensity. For the following, assume the angle of incidence is given by θ .

- 4a) (10 points) Derive the contributions to total phase difference at the location of the observer made by path difference, initial conditions and reflection. Be careful - the angles are not necessarily small.

$$\Delta\phi_{\text{tot}} = \Delta\phi_{\text{path}} + \Delta\phi_{\text{ic}} + \Delta\phi_{\text{ref}}$$

$$\Delta\phi_{\text{ic}} = 0 \quad \Delta\phi_{\text{ref}} = \pi$$

$$\tan \theta = \frac{d/2}{a}$$

$$\delta = 2a \tan \theta$$

$$\Delta\phi_{\text{path}} = k \delta \cos(90^\circ - \theta)$$

$$\Delta\phi_{\text{path}} = k \delta \sin \theta$$

$$\Delta\phi_{\text{tot}} = \frac{4\pi a}{\lambda} \tan \theta \sin \theta + \pi$$

- 4b) (5 points) Under what conditions will the light observed have maximum intensity?

$$\Delta\phi_{\text{tot}} = 2\pi n$$

$$2\pi n = \frac{4\pi a}{\lambda} \tan \theta \sin \theta + \pi$$

$$2n = \frac{4a}{\lambda} \tan \theta \sin \theta + 1$$

$$(2n-1)\lambda = 4a \tan \theta \sin \theta \rightarrow (2n-1)\frac{\lambda}{4} = a \tan \theta \sin \theta$$

When there are an odd number of quarter-wavelengths inside the mirror

- 4c) (10 points) If light of wavelength λ_0 , incident at an angle θ , produces a signal of maximum intensity at the location of the observer, how far from λ_0 will the next longer wavelength that produces a signal of maximum intensity at an angle of θ appear (that is, what is $\Delta\lambda$, if $\Delta\lambda \equiv \lambda_{\text{next}} - \lambda_0$)?

$$(2n-1)\frac{\pi}{4} = a \tan \theta \sin \theta$$

another maximum occurs at the next quarter-wavelength with an odd number

$$(2-1)\frac{\pi}{4} - (4-1)\frac{\pi}{4}$$

$$\frac{\pi}{4} \rightarrow \frac{3\pi}{4} \rightarrow \text{one maximum to another}$$

$$\lambda = \frac{4a \tan \theta \sin \theta}{2n-1}$$

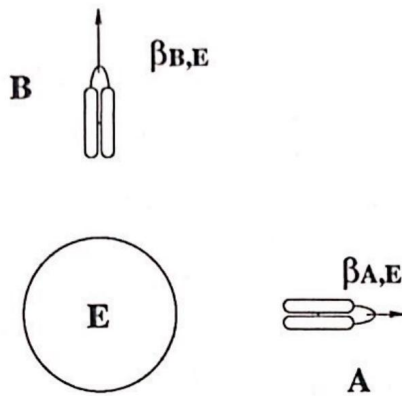
$$\Delta\lambda = \frac{4a \tan \theta \sin \theta}{1} + 4a \tan \theta \sin \theta$$

$$\boxed{\Delta\lambda = 8a \tan \theta \sin \theta}$$

- 4d) (5 points) Suppose you wanted to design a filter to extract light of wavelength λ_0 from a mixture with other signals flowing down an optical fiber. Would you want $\Delta\lambda$ to be large or small? Explain. If one were to define a quality factor for a Fabry-Perot filter, would it be proportional or inversely proportional to $\Delta\lambda$?

$\Delta\lambda$ should be large so that there are less maxima, limiting the chance that they mix with other signals' maxima.

$\Delta\lambda$ should be proportional to the quality factor since, higher $\Delta\lambda$ = higher quality.



5) Rocket A is moving with a velocity $(\beta_{A,E} C) \hat{x}$ relative to the Earth and Rocket B is moving with a velocity $(\beta_{B,E} C) \hat{y}$ relative to the Earth. Make sure your final answers to the following questions are in terms of the given information (no γ 's).

- 5a) (5 points) A stick of length L_A lies along the x axis in Rocket A. How long is the stick in the Earth's frame? If the stick ages a year in Rocket A, how much has it aged in the Earth's frame?

$$L = \frac{L'}{\gamma}$$

simultaneous \rightarrow measuring, co-located \rightarrow stick at rest

$$\Delta t = \frac{1}{\sqrt{1-\beta_{A,E}^2}} \Delta t'$$

$$L_A = L' \sqrt{1-\beta_{A,E}^2}$$

$$\Delta t' = \Delta t \sqrt{1-\beta_{A,E}^2}$$

$$L' = \frac{L_A}{\sqrt{1-\beta_{A,E}^2}}$$

$$\Delta t' = \sqrt{1-\beta_{A,E}^2} \text{ years}$$

- 5b) (10 points) It takes a bug a time Δt_A to walk along the stick (back-to-front) in (and relative to) Rocket A. How long did it take the bug in the Earth frame? How far did the bug travel in the Earth frame?

In rocket frame: $\Delta x = \Delta v \Delta t_A$

$$\Delta \underline{r} = \begin{bmatrix} c \Delta t \\ \Delta x \end{bmatrix} = \gamma \begin{bmatrix} 1 & \beta \\ \beta & 1 \end{bmatrix} \begin{bmatrix} c \Delta t_A \\ L_A \end{bmatrix}$$

$$c \Delta t = \gamma [c \Delta t_A + \beta_{A,E} L_A]$$

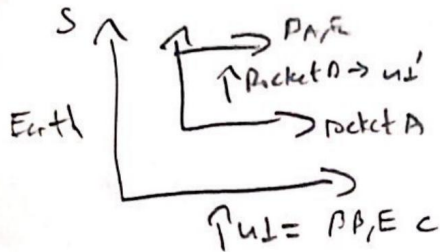
$$\Delta x = \gamma [\beta_{A,E} c \Delta t_A + L_A] \rightarrow \Delta x = \dots$$

$$\Delta t = \frac{1}{\sqrt{1-\beta_{A,E}^2}} [c \Delta t_A + \beta_{A,E} L_A]$$

$$\Delta x = \frac{1}{\sqrt{1-\beta_{A,E}^2}} [\beta_{A,E} c \Delta t_A + L_A]$$

- 5c) (10 points) How fast is Rocket B moving in Rocket A's frame?

$$\frac{u_{\perp}}{c} = \frac{u'_{\perp}/c}{\gamma(1 + \beta u'_{\parallel}/c)}$$



$$\beta_{B,E} = \frac{u'_{\perp}/c \sqrt{1 - \beta_{A,E}^2}}{(1 + \beta_{A,E}(0))}$$

$$u'_{\perp} = \frac{\beta_{B,E}}{\sqrt{1 - \beta_{A,E}^2}} c$$

- 5d) (5 points) What is the significance of $\beta = 0$ and $\beta = 1$? Evaluate your answer to part c in the limits $\beta_{B,E} = 0$ and $\beta_{B,E} = 1$ and discuss...

$\beta = 0$ represents an object at rest and $\beta = 1$ represents an object moving at the speed of light.

My equation behaves correctly. At $\beta_{B,E} = 0$, it's at rest, which is logical since $\beta \ll 1$, so it behaves according to a Galilean relationship. At $\beta_{B,E} = 1$, it behaves in line with γc .