

Math 33A Sheet 7

Chapter 5.1

Ex 17) $W = \text{span}\left(\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \\ 8 \end{bmatrix}\right)$

$W^\perp \rightarrow$ all vectors perpendicular to

W / basis of W

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = 0 \text{ and } \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 7 \\ 8 \end{bmatrix} = 0$$

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 0$$

$$5x_1 + 6x_2 + 7x_3 + 8x_4 = 0$$

$$-4x_2 - 8x_3 - 12x_4 = 0$$

$$x_2 + 2x_3 + 3x_4 = 0$$

$$x_1 - x_3 - 2x_4 = 0$$

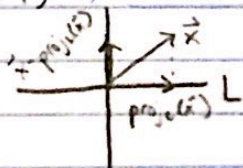
$$x_3 = t, x_4 = s$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} t+2s \\ -2t-3s \\ t \\ s \end{bmatrix}$$

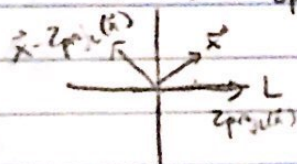
$$= t \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{basis of } W^\perp = \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

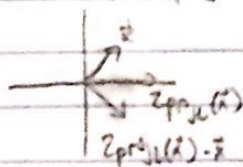
Ex 19) a) $T(\vec{x}) = \vec{x} - \text{proj}_L(\vec{x})$



b) $T(\vec{x}) = \vec{x} - 2\text{proj}_L(\vec{x})$



c) $T(\vec{x}) = 2\text{proj}_L(\vec{x}) - \vec{x}$



Chapter 5.2

Ex 1) $\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

$$\vec{u}_1 = \frac{\vec{v}}{\|\vec{v}\|}$$

$$\vec{u}_1 = \frac{\vec{v}}{3} = \frac{1}{3}\vec{v}$$

$$\vec{u}_1 = \begin{bmatrix} 2/3 \\ 1/3 \\ -2/3 \end{bmatrix}$$

Ex 10) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix}$

$$\vec{u}_1 = \frac{\vec{v}_1}{2} = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}$$

$$L = \text{span}(\vec{v}_1)$$

$$\text{proj}_L(\vec{v}_2) = (\vec{u}_1 \cdot \vec{v}_2) \vec{u}_1 = 10\vec{u}_1 = 5\vec{v}_1$$

$$\vec{v}_2^\perp = \vec{v}_2 - 5\vec{v}_1 = \begin{bmatrix} -4 \\ -4 \\ -4 \end{bmatrix}$$

$$\vec{u}_2 = \frac{\vec{v}_2^\perp}{\|\vec{v}_2^\perp\|} = \frac{1}{2}\vec{v}_2^\perp$$

$$\vec{u}_2 = \begin{bmatrix} -1/2 \\ -1/2 \\ -1/2 \end{bmatrix}$$

Ex 16) $\begin{bmatrix} 6 & 2 \\ 3 & -6 \\ 2 & 3 \end{bmatrix}$

$$\vec{u}_1 = \frac{\vec{v}_1}{7} = \frac{1}{7} \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$$\text{proj}_L(\vec{v}_2) = (\vec{u}_1 \cdot \vec{v}_2) \vec{u}_1 = \left(\frac{1}{7}(0)\right) \vec{u}_1 = \vec{0}$$

$$\vec{v}_2^\perp = \vec{v}_2 = \begin{bmatrix} 2 \\ -6 \\ 3 \end{bmatrix}$$

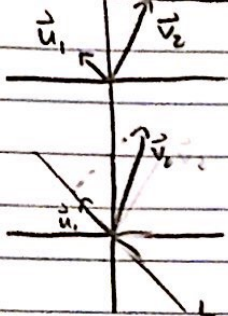
$$\vec{u}_2 = \frac{\vec{v}_2^\perp}{\|\vec{v}_2^\perp\|} = \frac{\vec{v}_2^\perp}{7} = \frac{1}{7}\vec{v}_2^\perp$$

$$Q = \frac{1}{7} \begin{bmatrix} 6 & 2 \\ 3 & -6 \\ 2 & 3 \end{bmatrix}$$

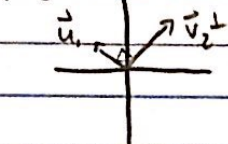
$$R = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

Ex 29) $\vec{v}_1 = \begin{bmatrix} -3 \\ 4 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$\vec{u}_1 = \frac{\vec{v}_1}{\|\vec{v}_1\|} = \frac{1}{5} \begin{bmatrix} -3 \\ 4 \end{bmatrix}$



$\text{proj}_{\vec{u}_1}(\vec{v}_2) = (\vec{u}_1 \cdot \vec{v}_2) \vec{u}_1 = 5 \vec{u}_1 = \vec{v}_1$



$\vec{v}_2 - \vec{v}_1 = \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \vec{v}_2^\perp$

$\vec{u}_2 = \frac{1}{5} \vec{v}_2^\perp = \frac{1}{5} \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

Ex 32) $x_1 + x_2 + x_3 = 0$

$x_1 = -x_2 - x_3$

$x_1 = -2, x_2 = 2, x_3 = 0$

$x_1 = -2, x_2 = 0, x_3 = 2$

$\text{span} \left(\begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} \right)$

$\vec{u}_1 = \frac{1}{2} \vec{v}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

$\text{proj}_{\vec{u}_1}(\vec{v}_2) = (\vec{u}_1 \cdot \vec{v}_2) \vec{u}_1 = (2) \vec{u}_1 = \vec{v}_1$

$\vec{v}_2^\perp = \vec{v}_2 - \vec{v}_1 = \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$

$\vec{u}_2 = \frac{1}{2} \vec{v}_2^\perp = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

Ex 33) *

$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & -2 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

$x_1 + x_4 = 0$

$x_2 + x_3 = 0$

$x_4 = s, x_3 = t$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -s \\ -t \\ t \\ s \end{bmatrix} = t \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$\vec{v}_1 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}$

$\vec{u}_1 = \frac{\vec{v}_1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$\text{proj}_{\vec{u}_1}(\vec{v}_2) = (\vec{u}_1 \cdot \vec{v}_2) \vec{u}_1 = 0$

$\vec{v}_2^\perp = \vec{v}_2$

$\vec{u}_2 = \frac{\vec{v}_2^\perp}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}$

Ex 36) $M = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 5 \\ 0 & -4 & 6 \\ 0 & 0 & 7 \end{bmatrix}$

↳ Q columns must be orthonormal ✓

↳ R diagonal must be positive ✓

↳ R must be upper diagonal ✓

Change signs of Q col. #2 and
R row #2

↳ $Q = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}, R = \begin{bmatrix} 2 & 3 & 5 \\ 0 & 4 & -6 \\ 0 & 0 & 7 \end{bmatrix}$

$M = \frac{1}{2} \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 5 \\ 0 & 4 & -6 \\ 0 & 0 & 7 \end{bmatrix}$

$= \frac{1}{\sqrt{12}} \left[\vec{i} \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} \right]$

$= \frac{1}{\sqrt{12}} \left[\vec{i}(-5) - \vec{j}(-4) + \vec{k}(-1) \right]$

$\vec{u}_3 = -\frac{1}{\sqrt{12}} \begin{bmatrix} 5 \\ -4 \\ 1 \end{bmatrix}$

Q1) i) $A \equiv S^{-1}AS$

let $S = I_n$

↳ invertible

$A = I_n A I_n$

↳ $A = A \checkmark$

↳ I_n is a neutral element

Ex 38) $A = \begin{bmatrix} 0 & -3 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

columns are orthogonal

$\vec{u}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$Q = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

ii) $A = S^{-1}BS \checkmark$

$B = S^{-1}AS \checkmark$

$B = S^{-1}(S^{-1}BS)S$

$B = S^{-1}S^{-1}BSS$

$B = B \checkmark$

Ex 39) $\text{span}(\vec{u}_1) = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right)$

$\text{span}(\vec{u}_1, \vec{u}_2) = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}\right)$

$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$\vec{u}_1 = \frac{\vec{v}_1}{\sqrt{14}} = \frac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\vec{u}_2 = \frac{\vec{v}_2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$\vec{u}_1 \times \vec{u}_2 = \vec{u}_3 \rightarrow$ cross product is orthonormal

$= \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1/\sqrt{14} & 2/\sqrt{14} & 3/\sqrt{14} \\ 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix} = \frac{1}{\sqrt{42}} \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$

iii) $A = S^{-1}BS$

$B = S^{-1}CS$

$\therefore A = S^{-1}(S^{-1}CS)S$

$A = S^{-1}S^{-1}CSS$

$A = C \checkmark$