20S-MATH61-2 Final exam

CHARLES ZHANG

TOTAL POINTS

48 / 50

QUESTION 1

1 Question 1 **5 / 5**

✓ **- 0 pts Invalidated.**

QUESTION 2

2 Question 2 **5 / 5**

✓ **- 0 pts Invalidated.**

QUESTION 3

3 Question 3 (all three parts) **3.5 / 5**

- **- 0 pts** Correct
- **- 1 pts** One incorrect answer.
- ✓ **- 2 pts Two incorrect answers.**
- **- 3 pts** Three incorrect answers.
- **+ 0.5 Point adjustment**

201 - 140 = 61.

1) 8, 2) 93, 3) 61.

QUESTION 4

Question 4 5 pts

4.1 Part 1 **2 / 2**

- ✓ **- 0 pts Correct**
- **- 1.5 pts** Bad probability argument
- **- 0.5 pts** Minor error
- **- 1 pts** Counting error

4.2 Part 2 **2.5 / 3**

- **- 0 pts** Correct
- ✓ **- 1 pts Minor error**
- **- 2 pts** Major error
- **- 3 pts** Blank
- **+ 0.5 Point adjustment**

You should say that it is possible to split up the set of available numbers into sets of three consecutive in each.

QUESTION 5

5 Question 5 **5 / 5**

- ✓ **- 0 pts Correct**
- **- 1 pts** Incorrect computation.
- **- 0.5 pts** Solved a different problem.

QUESTION 6

6 Question 6 (all parts) **5 / 5**

- ✓ **- 0 pts Correct**
- **- 1 pts** One incorrect graph
- **- 2 pts** Two incorrect graphs

QUESTION 7

7 Question 7 (all parts) **5 / 5**

- ✓ **- 0 pts Correct**
- **- 3 pts** Only one correct answer.
- **- 2 pts** Two incorrect answers.
- **- 1 pts** One incorrect answer.
- **- 4 pts** No correct answers.

QUESTION 8

8 Question 8 **5 / 5**

- ✓ **- 0 pts Correct**
- **- 1 pts** Almost there.

QUESTION 9

9 Question 9 **5 / 5**

✓ **- 0 pts Invalidated.**

QUESTION 10

10 Question 10 **5 / 5**

- ✓ **- 0 pts Correct**
- **- 1 pts** Minor error

- **2.5 pts** Major error
- **5 pts** No answer

Q1B:

$(1+2x)^n \geq 1+2nx, \; x \geq -1/2, \; n \geq 1$

Base Case: $x = -1/2, \; n = 1$

$(1+2(-\frac{1}{2}))^1 \geq 1+2(1)(-\frac{1}{2})$

$0 \geq 0$ ✓

Inductive Step (taking x for granted):

Assume: $(1+2x)^n \geq 1+2nx$

Prove: $(1+2x)^{n+1} \geq 1+2(n+1)x$] =

Prove: $(1+2x)^{n+1} \geq 1+2nx+2x$

$(1+2x)^{n+1} = (1+2x)(1+2x)^n$

$(1+2x)^{n+1} \geq (1+2x)(1+2nx)$

$(1+2x)^{n+1} \geq 1+2nx+2x+4nx^2$ → For any given parameter within the constraints, $1+2nx+2x+4nx^2 > 1+2nx+2x$ since $4nx^2$ can't be negative if $n \geq 1$

$(1+2x)^{n+1} \geq 1+2nx+2x+4nx^2 \geq 1+2nx+2x$

$\therefore \; (1+2x)^{n+1} \geq 1+2nx+2x$ ✓

By induction, $(1+2x)^n \geq 1+2nx$ for $x \geq -1/2, \; n \geq 1$

1 Question 1 5 / 5

✓ **- 0 pts** Invalidated.

Q2:

13 dwarves, hobbit, wizard = 15 total people → $\boxed{\text{Irrelevant}}$

20 drinks total → 1 of each of the 3 ales, exactly 1 stout

$$\underset{\text{Stout}}{\underline{1}} \times \underset{\text{Brown Ale}}{\underline{1}} \times \underset{\text{Pale Ale}}{\underline{1}} \times \underset{\text{India Pale}}{\underline{1}} \times \underset{\text{Others may be anything other than Stout}}{\underline{4^{16}}}$$

$\boxed{\text{There are } 4^{16} \text{ ways to order their beer}}$

X

Scanned with CamScanner

2 Question 2 **5 / 5**

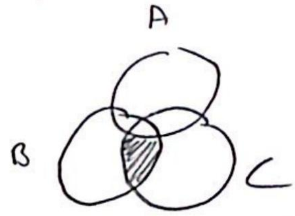
✓ **- 0 pts** **Invalidated.**

Q3: 201 students

A = French, B = Business, C = Music

$|A| = 65$
$|B| = 76$
$|C| = 63$
$|A \cap B| = 36$
$|A \cap C| = 20$
$|B \cap C| = 18$
$|A \cap B \cap C| = 10$

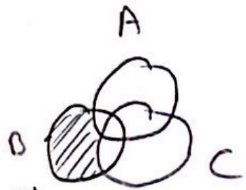
i) Music + Business, not French



$|B \cap C| - |A \cap B \cap C|$

$18 - 10 = \boxed{8 \text{ students}}$

ii) Not music or French



$|B| - |A \cap B| - |B \cap C| + |A \cap B \cap C|$ ← this group is subtracted 2x, adjust by adding 1x

$76 - 36 - 18 + 10$

$\boxed{32 \text{ students}}$

iii) No classes

↳ 1st, find # total in classes

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

$= 65 + 76 + 63 - 36 - 18 - 20 + 10$

$= 140$ students taking at least 1 class

↳ Now subtract from total

$201 - 140 = \boxed{71 \text{ students}}$

3 Question 3 (all three parts) **3.5 / 5**

- **- 0 pts** Correct
- **- 1 pts** One incorrect answer.
- ✓ **- 2 pts** **Two incorrect answers.**
- **- 3 pts** Three incorrect answers.
- **+ 0.5 Point adjustment**

201 - 140 = 61.

1) 8, 2) 93, 3) 61.

Q4c:

X

i) 3 Questions, 5 answers each

↳ 5^3 possible answer sheets = # of pigeonholes

Each student is a pigeon

Since there are 125 pigeonholes, and 130 pigeons, it's actually impossible for each student to have a unique set of answers; some pigeonholes will contain multiple pigeons, so some answer sheets will be submitted by multiple pigeons.

ii) Assume the best-case scenario: Every set of 3 houses has 2 consecutive #s, and then skips a #. In this case, there are $66/3 = 22$ groupings of houses/pigeonholes. Each grouping holds 2 houses/1 pigeon. If this were the case, only 44 houses would be accounted for, the 45th would have to fit in a pigeonhole, and there would be 3 consecutive house numbers.

4.1 Part 1 **2 / 2**

- ✓ **- 0 pts** Correct
- **- 1.5 pts** Bad probability argument
- **- 0.5 pts** Minor error
- **- 1 pts** Counting error

Q4c:

X

i) 3 Questions, 5 answers each
↳ 5^3 possible answer sheets = # of pigeonholes
Each student is a pigeon
Since there are 125 pigeonholes, and 130 pigeons, it's actually impossible for each student to have a unique set of answers; some pigeonholes will contain multiple pigeons, so some answer sheets will be submitted by multiple pigeons.

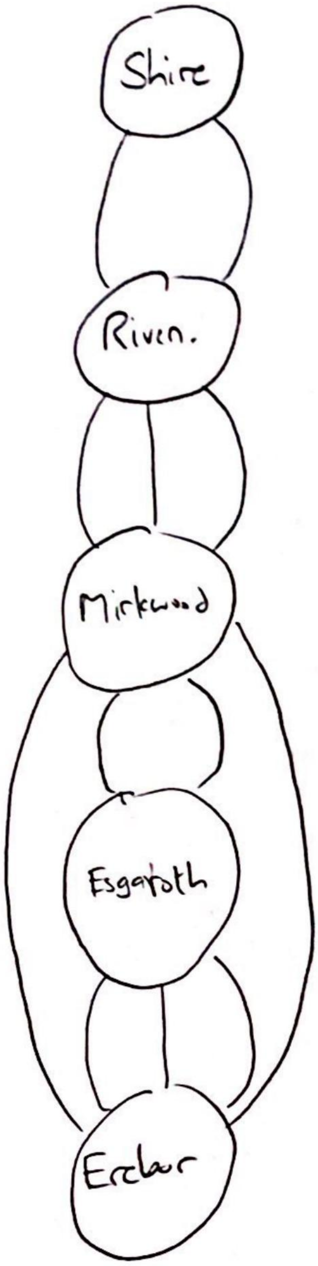
ii) Assume the best-case scenario: Every set of 3 houses has 2 consecutive #s, and then skips a #. In this case, there are 66/3 = 22 groupings of houses/pigeonholes. Each grouping holds 2 houses/1 pigeon. If this were the case, only 44 houses would be accounted for, the 45th would have to fit in a pigeonhole, and there would be 3 consecutive house numbers.

4.2 Part 2 **2.5 / 3**

- **- 0 pts** Correct
- ✓ **- 1 pts** **Minor error**
- **- 2 pts** Major error
- **- 3 pts** Blank
- **+ 0.5** **Point adjustment**

You should say that it is possible to split up the set of available numbers into sets of three consecutive in each.

Q5A:



2

3

$(2\times3+2)$ = 8 ways in this section

From Shire → Erebor = $2\times3\times(2\times3+2) = 48$ ways

Back = $48-1 = 47$ ways

Round Trip = $48\times47 = \boxed{2256 \text{ ways}}$

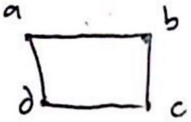
5 Question 5 **5 / 5**

✓ **- 0 pts** **Correct**

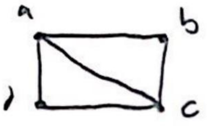
- 1 pts Incorrect computation.

- 0.5 pts Solved a different problem.

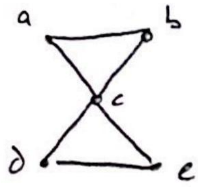
Q6:

i) 

{a, b, c, d, a} is a Hamiltonian and Euler cycle

ii) 

{a, b, c, d, a} is a Hamiltonian cycle, a and c have odd degrees, so an Euler cycle doesn't exist.

iii) 

{a, b, c, e, d, c, a} is an Euler cycle. Any cycle in this graph will intersect w/ c multiple times, so there is no Hamiltonian cycle.

iv) 

Hamiltonian cycles require all vertices to have >1 degree. Euler cycles require all vertices to have an even degree. The degree of both vertices here is 1, so neither type of cycle can exist.

6 Question 6 (all parts) **5 / 5**

- ✓ **- 0 pts** Correct
- **- 1 pts** One incorrect graph
- **- 2 pts** Two incorrect graphs

Q7c:

A is $n_2 \times n_2$

$$B = \begin{bmatrix} 0_{n_1 \times n_1} & 0_{n_1 \times n_2} \\ 0_{n_2 \times n_1} & A \end{bmatrix}$$

Connected: every vertex has a path to every other vertex

✗ i) True → The graph must be disconnected since some vertices will be adjacent to 0 edges, and therefore have no path to them

ii) False → $\begin{array}{c|c} & a \quad b \\ \begin{matrix} a \\ b \end{matrix} & \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \end{array}$ → a b

✗ iii) False → $\begin{array}{c|c} & a \quad b \\ \begin{matrix} a \\ b \end{matrix} & \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \end{array}$ → a b, $A = \begin{array}{c|c} & b \\ b & [2] \end{array}$ → b → diff. graphs

✗ iv) True

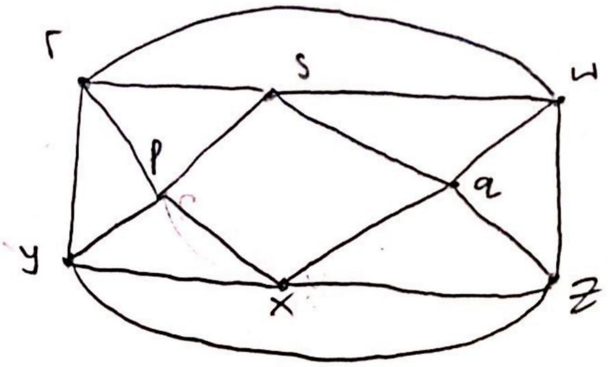
7 Question 7 (all parts) **5 / 5**

- ✓ **- 0 pts** **Correct**
- **- 3 pts** Only one correct answer.
- **- 2 pts** Two incorrect answers.
- **- 1 pts** One incorrect answer.
- **- 4 pts** No correct answers.

Q8B:

$V = \{p, q, r, s, w, x, y, z\}$

$E = \{(p,r), (p,s), (p,x), (p,y), (q,s), (q,w), (q,x), (q,z), (r,s), (r,w), (r,y), (s,w), (w,z), (x,y), (x,z), (y,z)\}$

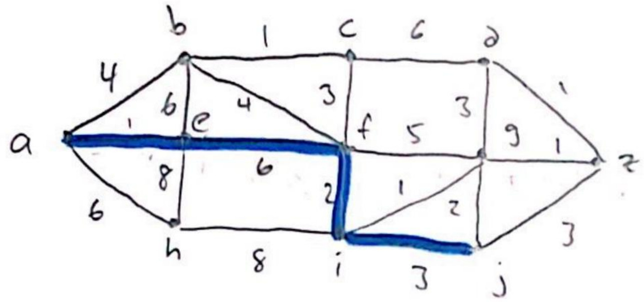


8 Question 8 **5 / 5**

✓ **- 0 pts** Correct

- 1 pts Almost there.

Q9D:



	a	b	c	d	e	f	g	h	i	j	z
a	0_a	4_a	∞_a	∞_a	1_a	∞_a	∞_a	6_a	∞_a	∞_a	∞_a
e	0_a	4_a	∞_a	∞_a	1_a	7_e	∞_a	6_a	∞_a	∞_a	∞_a
b	0_a	4_a	5_b	∞_a	1_a	7_e	∞_a	6_a	∞_a	∞_a	∞_a
c	0_a	4_a	5_b	11_a	1_a	7_e	∞_a	6_a	∞_a	∞_a	∞_a
h	0_a	4_a	5_b	11_a	1_a	7_e	∞_a	6_a	14_h	∞_a	∞_a
f	0_a	4_a	5_b	11_a	1_a	7_e	12_f	6_a	9_f	∞_a	∞_a
i	0_a	4_a	5_b	11_a	1_a	7_e	10_i	6_a	9_f	12_i	∞_a
g	0_a	4_a	5_b	11_a	1_a	7_e	10_i	6_a	9_f	12_i	11_g
d	0_a	4_a	5_b	11_a	1_a	7_e	10_i	6_a	9_f	12_i	11_g
z	0_a	4_a	5_b	11_a	1_a	7_e	10_i	6_a	9_f	12_i	11_g
j											

$\boxed{\text{The path } \{a, e, f, i, j\} \text{ has min. length } 12}$

9 Question 9 **5 / 5**

✓ **- 0 pts** **Invalidated.**

Q10D:

$e = 34$

$v = 13$

$f = e - v + 2$

each edge belongs to at most 2 faces

It's known that $2e \geq$ # of edge that bound faces

At best: a cycle is 3 edges (graph is simple, no parallel edges)

Therefore, each face is bounded by at least 3 edges

Therefore, $2e \geq 3f$

Assuming planarity:

$$2e \geq 3(e - v + 2)$$

$$2e \geq 3e - 3v + 6$$

$$68 \geq 102 - 39 + 6$$

$68 \geq 69$ is false

$\boxed{\text{By contradiction, a simple connected graph with 34 edges and 13 vertices cannot be planar}}$

Scanned with CamScanner

10 Question 10 **5 / 5**

- ✓ **- 0 pts** Correct
- **- 1 pts** Minor error
- **- 2.5 pts** Major error
- **- 5 pts** No answer