# PHYS 1B Midterm 1 Solutions (Dumitrescu)

*Ian E. Powell*

**Problem 1**

**a)**

The analog of Hooke's law here is $\tau(\theta) = -k\theta$. Comparing this with the torque plot (which behaves like the force for our purposes) we identify point "C" as the only point which the torsion pendulum can execute simple harmonic motion. Furthermore, we note that simple harmonic motion can only occur around equilibrium points where $\tau$=0.

**b)**

We will treat the problem as if the incident wave is reflected perfectly off of a fixed end point because the second rope is very heavy–because it's inertia is large we can neglect any transmitted waves and disregard any small fluctuation of position at the point which connects the ropes. Thus the reflected wave is inverted and traveling to the left–the answer corresponding to this configuration is "B."

**c)**

Because it is a standing wave there is no net mechanical energy traveling along the string–thus "B" is true. Answers "A" and "C" are not true for all times. Answers "D" and "E" are not true for all times but this involves a more careful treatment of the problem and thus no deduction will be made for claiming that either of these choices is true.

**Problem 2**

**a)**

Given that $v = 340m/s$ and $\omega = 2000$ Hz we find the frequency $f = \omega/(2\pi) = 318.31$ Hz. The wavelength is $\lambda = v/f = 1.068$ m. This is a longitudinal wave.

**b)**

The displacement is

$$u(x,t) = (1.0\text{cm})\sin(\omega t + kx) \tag{1}$$

with $\omega$ listed above and $k = 2\pi/\lambda = 5.88(1/\text{m})$.

**c)**

The pressure as a function of position and time is related to the displacement via

$$P(x,t) = -B\frac{\partial}{\partial x}u(x,t) = -Bu_0k\cos(\omega t + kx). \tag{2}$$

To find the first time the pressure at $x = -1m$ is 0 implies that the argument of the cosine function must be $-3\pi/2$. This is because $-5\pi/2 < -5.88 < -3\pi/2$ and so the first time that the pressure vanishes must be when the argument of the cosine function is $-3\pi/2$. Thus we have that

$$2000(1/\text{s})t - (5.881/\text{m})(1\text{m}) = -3\pi/2 \to t = 0.00059\text{s}. \tag{3}$$

**Problem 3**

**a)**

The linear density is related to the density via $\mu = \rho A$ where $A = \pi r^2$ is the area of the cross section. Plugging in $\rho = 7850$ kg/m$^3$, and $r = 0.0003$ m we have that $\mu = 0.0022$ kg/m. Plugging in the tension given to us we yield the wave velocity

$$v = \sqrt{T/\mu} = 335.613\text{m/s} \tag{4}$$

**b)**

For a string with two fixed ends, or two free ends, the harmonic wavelengths are given by $\lambda_n = 2L/n$. Thus the frequencies are given via $f_n = v/\lambda_n = vn/(2L)$. Using the two given frequencies we have $f_2/f_1$ = 2, which indeed follows the case of the two ends of the string fixed, or two ends of the string free, i.e.

$$f_2/f_1 = 2/1 = 880/440. \tag{5}$$

The length of the string is related to the first harmonic wavelength $\lambda_1 = 2L$. We find the length of the string via $\lambda_1 = v/f_1 = 335.613/440 = 0.7628\text{m} \rightarrow L = 0.3814m$.

**c)**

Changing the boundary condition on one end to be free for the fixed-fixed case, or fixed for the free-free case implies that $\lambda_1 = 4L$, and $\lambda_2 = 4L/3 \rightarrow f_1^{\text{new}} = v/(4L)$, $f_2^{\text{new}} = 3v/(4L)$. Plugging in our calculated wave velocity and string length we find that $f_1^{\text{new}} = 220$ Hz, and $f_2^{\text{new}} = 660$ Hz.

**Problem 4**

**a)**

Here $\omega = \sqrt{k/m} = 3.9 \times 10^{12}$ 1/s. From conservation of energy (the "0" subscript indicates the initial condition) we have

$$R = (x_0^2 + v_0^2/\omega^2)^{1/2} = x_0 + 1.9 \times 10^{-16}\text{m} = 7.0000019 \times 10^{-10}\text{m} \tag{6}$$

Last, we calculate the phase $\phi_0$ via

$$\phi_0 = \tan^{-1}\left(\frac{-v_0}{x_0\omega}\right) = -0.00073\text{rad} \tag{7}$$

**b)**

The velocity as a function of time is

$$v(t) = -A\omega\text{sin}(\omega t + \phi_0), \tag{8}$$

and it reaches its highest speed when the argument of the sine function is $\pi/2$–note that the sign of the velocity does not matter because we are interested in the speed of the mass. Thus

$$\omega t + \phi_0 = \pi/2 \rightarrow t = (\pi/2 - \phi_0)/\omega = 4.03 \times 10^{-13}s. \tag{9}$$

**c)**

Stable equillibrium occurs when $U'(l) = 0$ and $U''(l) > 0$. The first derivative of the potential function is

$$U'(l) = u_0\left[-12\frac{(4 \times 10^{-10})^{12}}{l^{13}} + 12\frac{(4 \times 10^{-10})^6}{l^7}\right]. \tag{10}$$

Setting this equal to zero we solve for the equillibrium length $l_0$

$$U'(l_0) = 0 \rightarrow \frac{(4 \times 10^{-10})^{12}}{l_0^{13}} = \frac{(4 \times 10^{-10})^{12}}{l_0^7} \rightarrow l_0 = 4 \times 10^{-10}\text{m}. \tag{11}$$

To calculate the spring constant we perform a taylor expansion of $U(l)$ about its equillibrium up to quadratic order in $l$

$$U(l) \approx U(l_0) + \frac{dU(l = l_0)}{dl}(l - l_0) + \frac{1}{2}\frac{d^2U(l = l_0)}{dl^2}(l - l_0)^2. \tag{12}$$

We identify the spring constant via the potential for the spring $U(x) = \frac{1}{2}kx^2$, thus

$$k = \frac{d^2U(l = l_0)}{dl^2}. \tag{13}$$

Performing the second derivative and plugging in $l = l_0$ we yield

$$k = U_0\left(156\frac{(4 \times 10^{-10})^{12}}{l_0^{14}} - 84\frac{(4 \times 10^{-10})^6}{l_0^8}\right), \tag{14}$$

with $l_0$ given above. If we require $k = 1$ N/m we find $U_0$ via

$$1 = U_0\left(156\frac{(4 \times 10^{-10})^{12}}{l_0^{14}} - 84\frac{(4 \times 10^{-10})^6}{l_0^8}\right) \rightarrow U_0 = 2.22 \times 10^{-21}\text{J}. \tag{15}$$