Chapter 1.2

Ex 2) $\left| \begin{array}{l} 3x + 4y - z = 8 \\ 6x + 8y - 2z = 3 \end{array} \right|$

$$\left[\begin{array}{ccc|c} 3 & 4 & -1 & 8 \\ 6 & 8 & -2 & 3 \end{array}\right] - 2(I)$$

$$\left[\begin{array}{ccc|c} 3 & 4 & -1 & 8 \\ 0 & 0 & 0 & -13 \end{array}\right]$$

$\boxed{\text{no solution}}$

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Ex3) $x+2y+3z=4$

$$[\,1\ 2\ 3 \mid 4\,]$$

$y=s$, $z=t$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & s \\ 0 & 0 & 1 & t \end{array}\right] -2(\text{II})$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 4-2s \\ 0 & 1 & 0 & s \\ 0 & 0 & 1 & t \end{array}\right] -3(\text{III})$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4-2s-3t \\ 0 & 1 & 0 & s \\ 0 & 0 & 1 & t \end{array}\right]$$

$$\boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4-2s-3t \\ s \\ t \end{bmatrix}}$$

Ex5)*

$$\left|\begin{array}{l} x_3+x_4=0 \\ x_2+x_3=0 \\ x_1+x_2=0 \\ x_1+x_4=0 \end{array}\right|$$

$$\left[\begin{array}{cccc|c} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ -(\text{I})-(\text{III}) \end{array}$$

$$\left[\begin{array}{cccc|c} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ +(\text{II}) \end{array}$$

$$\left[\begin{array}{cccc|c} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ -(\text{III}) \\ \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} -(\text{II}) \\ \\ \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_4=t$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & t \end{array}\right] \begin{array}{l} -(\text{V}) \\ +(\text{V}) \\ -(\text{V}) \\ \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -t \\ 0 & 1 & 0 & 0 & t \\ 0 & 0 & 1 & 0 & -t \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & t \end{array}\right]$$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -t \\ t \\ -t \\ t \end{bmatrix}}$$

Ex8) $\left| \begin{array}{l} x_2 + 2x_4 + 3x_5 = 0 \\ \quad 4x_4 + 8x_5 = 0 \end{array} \right|$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 0 & 4 & 8 & 0 \end{array}\right] -\tfrac{1}{2}(\mathrm{II})$$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 4 & 8 & 0 \end{array}\right] \times \tfrac{1}{4}$$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \end{array}\right]$$

$$\left| \begin{array}{l} x_2 - x_5 = 0 \\ x_4 + 2x_5 = 0 \end{array} \right|$$

$$\left| \begin{array}{l} x_2 = x_5 \\ x_4 = -2x_5 \end{array} \right|$$

$x_1 = r, \ x_3 = s, \ x_5 = t$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} r \\ t \\ s \\ -2t \\ t \end{bmatrix}}$$

Ex 25) Gauss-Jordan Elimination must be done until, all leading digits are 1, all values that share a column with a leading 1 are 0, all leading 1s are further to the right than leading 1s above, all digits before the leading 1s are 0s, and all rows with only 0s are at the bottom.

Ex 29)

$$\begin{array}{r} \\ -4(\mathrm{I}) \\ -7(\mathrm{I}) \end{array}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix}\begin{array}{l} \\ \times -\frac{1}{3} \\ -2(\mathrm{II}) \end{array}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{array}{l} -2(\mathrm{II}) \\ \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Each matrix only has one reduced row echelon form, so this transformation is not possible

Math 33A Sheet 2

Chapter 1.3

Ex. 1)* a) $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

$\boxed{\text{no solution, row III is of the form } [0 \ldots 0 \mid k],\ k \neq 0}$

b) $\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 6 \end{array}\right]$

$\boxed{\text{one solution, } x_1 = 5, x_2 = 6}$

c) $\left[\begin{array}{ccc|c} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

$\boxed{\text{infinitely many solutions, } x_1 \text{ is free}}$

Ex. 2) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{l} -2(\text{II}) \\ \\ \\ \end{array}$

$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{l} +(\text{III}) \\ -2(\text{III}) \\ \\ \end{array}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\boxed{\text{rank} = 3}$

Ex. 3)* $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{array}{l} \\ -(\text{I}) \\ -(\text{I}) \end{array}$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\boxed{\text{rank} = 1}$

Ex. 4) $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \begin{array}{l} \\ -2(\text{I}) \\ -3(\text{I}) \end{array}$

$\begin{bmatrix} 1 & 4 & 7 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix} \begin{array}{l} \\ \times -\frac{1}{3} \\ \\ \end{array}$

$\begin{bmatrix} 1 & 4 & 7 \\ 0 & 1 & 2 \\ 0 & -6 & -12 \end{bmatrix} \begin{array}{l} -4(\text{II}) \\ \\ +6(\text{II}) \end{array}$

$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$\boxed{\text{rank} = 2}$

Q.) i) rank(A) < n
Nothing can be said about the system's solution set. rank(A) < n implies there are less pivots than rows, which could be infinitely many solutions if there's a free variable, unique solution if there isn't, or no solution if a row containing [0 ... 0 | k] k ≠ 0 exists.

ii) rank(A) = n
This expression tells us that there are an equal number of pivots as rows. This tells us the system is consistent, or that the solution set either contains infinitely many solutions or a unique solution because a row with [0 ... 0 | k] k ≠ 0 cannot exist.

iii) rank(A) < m
This expression tells us that the system's solution set must either contain infinitely many solutions or no solution, since there are less pivots than variables. This means free variables must exist, and, therefore, a unique solution is impossible

iv) rank(A) = m
This expression tells us that there is a pivot for every variable, which means the solution set either contains a unique solution or no solution. There are no free variables, so ~~a~~ infinitely many solutions is impossible