

Math 33A Sheet 5

Chapter 3.2

Ex. 22) $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} - 2(I)$

$$\begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$$

$$3 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$3 \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ 6 \end{bmatrix} = 0$$

$$\ker(A) = \text{span} \left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} \right)$$

Ex. 24) $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$2 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

$$2 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$\ker(A) = \text{span} \left(\begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix} \right)$$

Ex. 29) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} - 4(I)$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \end{bmatrix} \times -\frac{1}{3}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

$$-\begin{bmatrix} 1 \\ 4 \end{bmatrix} + 2 \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$\text{basis of } A \text{ is } \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

Ex. 30)* $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 3 & 7 \end{bmatrix} - I$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 4 \\ 0 & 2 & 6 \end{bmatrix} - II$$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 4 \\ 0 & 0 & -2 \end{bmatrix} \times -\frac{1}{2}$$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{basis of } A \text{ is } \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \\ 7 \end{bmatrix}$$

Ex 39) They must be linearly independent, because \vec{v} isn't in the original span, adding it would change the span, and removing any vector from $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n, \vec{v}$ would change the new span.

Ex 52)* $\begin{bmatrix} a \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} b \\ c \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} d \\ e \\ f \\ 0 \end{bmatrix}$

$a \neq 0$, otherwise the 1st column vector would be 0, which is linearly dependent

$c \neq 0$ and $f \neq 0$ for the same reasons in the 2nd and 3rd columns

$$\boxed{a, c, f \neq 0}$$

Chapter 3.3

Ex 22) $\begin{bmatrix} 2 & 4 & 8 \\ 4 & 5 & 1 \\ 7 & 9 & 3 \end{bmatrix} \times \frac{1}{2}$

$$\begin{bmatrix} 1 & 2 & 4 \\ 4 & 5 & 1 \\ 7 & 9 & 3 \end{bmatrix} - 4(I)$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & -3 & -15 \\ 0 & -5 & -25 \end{bmatrix} \times -\frac{1}{3}$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ 0 & -5 & -25 \end{bmatrix} - 2II$$

$$\begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix} + 5II$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 6t \\ -5t \\ t \end{bmatrix} = t \begin{bmatrix} 6 \\ -5 \\ 1 \end{bmatrix}$$

$$\text{basis of } \ker(A) = \begin{bmatrix} 6 \\ -5 \\ 1 \end{bmatrix}$$

$$\text{basis of } \text{im}(A) = \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 9 \end{bmatrix}$$

$$\text{Ex 23)} \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 3 & 4 & -6 & 8 \\ 0 & -1 & 3 & 1 \end{bmatrix} \xrightarrow{-3I}$$

$$\begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 0 & 4 & -12 & -4 \\ 0 & -1 & 3 & 1 \end{bmatrix} \xrightarrow{-4II}$$

$$\begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{+II}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} -2t-4s \\ 3t+s \\ t \\ s \end{bmatrix}$$

$$= t \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -4 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{basis of } \ker(A) = \left\{ \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$\text{basis of } \text{im}(A) = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 4 \\ -1 \end{bmatrix} \right\}$$

$$\text{Ex 28)} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 2 & 3 & 4 & k \end{bmatrix} \xrightarrow{-2I-3II-4III}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & k-29 \end{bmatrix}$$

$$k-29 \neq 0$$

$$k \neq 29$$

$$\text{Ex 29)} 2x_1 + 3x_2 + x_3 = 0$$

$$2x_1 = -3x_2 - x_3$$

$$x_1 = -3, x_2 = 2, x_3 = 0$$

$$\begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix}$$

$$x_1 = 1, x_2 = 0, x_3 = -2$$

$$\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$\text{and } \begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix} \text{ are}$$

linearly independent

$$\text{basis of subspace} = \left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix} \right\}$$

$$\text{Ex 31)} x_1 - x_2 + 2x_3 + 4x_4 = 0$$

$$x_1 = x_2 - 2x_3 - 4x_4$$

$$x_1 = 1, x_2 = 1, x_3 = 0, x_4 = 0$$

$$x_1 = -2, x_2 = 0, x_3 = 1, x_4 = 0$$

$$x_1 = -4, x_2 = 0, x_3 = 0, x_4 = 1$$

$$A = \begin{bmatrix} 1 & -2 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(\vec{v}) = \begin{bmatrix} 1 & -2 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \vec{v}$$

$$\text{Ex 36)} \dim(\ker A) + \dim(\text{im} A) = n$$

$$\dim(\ker A) + \dim(\text{im} A) = 3$$

$$2 \dim(\ker A) = 3$$

$$\dim(\ker A) = \frac{3}{2}$$

No, the dimension of the kernel and image cannot be $\frac{3}{2}$

(r) Ex 39) if $A = BC$, then $\ker(C) \subseteq \ker(A)$ and $\ker(B) \subseteq \ker(A)$
 $\ker(C) = \{\vec{0}\}$ only if it is an $n \times m$ matrix where $m \leq n$
 $\hookrightarrow C$ is a 4×5 , 5 is not less than or equal to 4
 $\hookrightarrow \ker(A)$ cannot equal zero, $\ker(C)$ has nonzero vectors
 \hookrightarrow since $\ker(A) \neq 0$, $\dim(\ker A) \neq 0$
 $\hookrightarrow \dim(\ker A) + \text{rank}(A) = m = 5$
 $\hookrightarrow \text{rank}(A) < 5$
 $\hookrightarrow A$ is not invertible