Physics 1C Chapter 28 Problems

28.28) $B = \frac{\mu_0 I}{2\pi R}$

$R = d$

a) $\frac{F}{L} = \frac{\mu_0 I I'}{2\pi R}$

$\frac{F}{L} = \frac{\mu I(-I)}{2\pi d} + \frac{\mu_0 I^2}{2\pi(2d)}$

$\frac{F}{L} = \frac{-\mu_0 I^2}{2\pi d} + \frac{\mu_0 I^2}{4\pi d}$

$\boxed{\frac{F}{L} = \frac{\mu_0 I^2}{4\pi d}}$

b) upward

c) $\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$

$\frac{F}{L} = \frac{F_1}{L} - \frac{F_2}{L}$

$\frac{F_1}{L} = \frac{F_2}{L}$

$\boxed{\frac{F}{L} = 0}$

d) zero

e) force on bottom = force on top

$\boxed{\frac{F}{L} = \frac{\mu_0 I^2}{4\pi d}}$

f) down

28.32) $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{L} \times \hat{r}}{r^2}$

a) $d\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{I dL}{R^2}\right)$

only y component of $\vec{B}$ matters

$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{I}{R^2}\right)\int dL$

$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{I}{R^2}\right)(\pi R)$

$\boxed{B = \frac{\mu_0 I}{4R}}$

b) Into the page

28.49) a) $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$

$B = B_1 + B_2$

$B_1 = \frac{\mu_0}{4\pi}\left(\frac{qv}{r^2}\right)$

$B_2 = \frac{\mu_0}{4\pi}\left(\frac{q'v'}{r'^2}\right)$

$B = \frac{\mu_0}{4\pi}\left(\frac{qv}{r^2} + \frac{q'v'}{r'^2}\right)$

$\boxed{B = 1.05\times10^{-6}\,T}$

b) Into the page

c) $\vec{F_B} = q\vec{v}\times\vec{B}$

$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{I\vec{v}\times\hat{r}}{r^2}\right)$

$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{q'v'}{r^2}\right)(0.8)$

$F_B = qv\left(\frac{\mu_0}{4\pi}\right)\left(\frac{q'v'}{r^2}\right)(0.8)$

$\boxed{F_B = 7.96\times10^{-8}\,N}$

d) $\boxed{+y\text{-direction}}$

28.59) $I_1 = 6A$

a) $B = \frac{\mu_0 I}{2\pi r}$

$B_1 = \frac{\mu_0 I_1}{2\pi r_1}$

$B_1 - B_2 = 0$

$B_2 = \frac{\mu_0 I_2}{2\pi r_2}$

$\frac{\mu_0 I_1}{2\pi r_1} = \frac{\mu_0 I_2}{2\pi r_2}$

$\frac{I_1}{r_1} = \frac{I_2}{r_2}$

$I_2 = \frac{r_2 I_1}{r_1}$

$\boxed{I_2 = 2A}$

b) into the page

c) $B_1 + B_2 = B$

$\frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} = B$

$B = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right)$

$\boxed{B = 2.13\times10^{-6}\,T}$

d) upward

e) $B_x = B_{1x} - B_{2x}$

$B_y = B_{1y} + B_{2y}$

$\theta_1 = 53.13°$

$\theta_2 = 36.86°$

$B_{1x} = B_1\cos\theta_1 = \frac{\mu_0 I_1}{2\pi r_1}\cos\theta_1$

$B_x = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1}\cos\theta_1 - \frac{I_2}{r_2}\cos\theta_2\right)$

$B_x = 8\times10^{-7}\,T$

$B_{1y} = B_1\sin\theta = \frac{\mu_0 I_1}{2\pi r_1}\sin\theta_1$

$B_y = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1}\sin\theta_1 + \frac{I_2}{r_2}\sin\theta_2\right)$

$B_y = 1.9\times10^{-6}\,T$

$\boxed{B = 2.06\times10^{-6}\,T}$

28.63) $\vec{J} = \left(\frac{b}{r}\right) e^{(r-a)/\delta} \hat{k}$

$a = 5cm = 0.05m$

$b = 600 \frac{A}{m}$

$\delta = 2.5cm = 0.025m$

a) $dI = J\,dA$

$dA = 2\pi r\,dr$

$I = \int \left(\frac{b}{r}\right) e^{(r-a)/\delta} 2\pi r\,dr$

$I = 2\pi b \int_0^a e^{(r-a)/\delta}\,dr$

$u = \frac{r-a}{\delta}, \quad du = \frac{1}{\delta} dr$

$I = 2\pi b\delta \int_0^a e^u\,du$

$I = 2\pi b\delta \left[e^{\frac{r-a}{\delta}}\right]_0^a$

$\boxed{I = 2\pi b\delta \left[1 - e^{-a/\delta}\right]}$

b) $\boxed{I = 81.49A}$

c) $B = \frac{\mu_0 I_{encl}}{2\pi R}$

$\boxed{B = \frac{\mu_0 I_0}{2\pi r}}$

d) $I = \int \vec{J}\,d\vec{A}$

$I_0 = 2\pi b\delta \left[1 - e^{-a/\delta}\right]$

$I = 2\pi b\delta \left[e^{(r-a)/\delta} - e^{-a/\delta}\right]$

$2\pi b\delta = \frac{I_0}{1 - e^{-a/\delta}}$

$\boxed{I = I_0 \frac{e^{(r-a)/\delta} - e^{-a/\delta}}{1 - e^{-a/\delta}}}$

e) $\int B \cdot d\vec{s} = \mu_0 I_{enc}$

$B(2\pi r) = \mu_0 I_{enc}$

$\boxed{B = \frac{\mu_0 I_0}{2\pi r}\left(\frac{e^{(r-a)/\delta} - e^{-a/\delta}}{1 - e^{-a/\delta}}\right)}$

f) $\boxed{B = 1.8 \times 10^{-4} T}$

g) $\boxed{B = 3.3 \times 10^{-4} T}$

h) $r = 2a = 0.1m$

$\boxed{B = 1.64 \times 10^{-4} T}$

28.64) $r_1 = 30cm = 0.3m$

$r_2 = 20cm = 0.2m$

a) $dB = \frac{\mu_0}{4\pi}\left(\frac{I\,dL \times r}{r^2}\right)$

$I = 12A$

$B_{bt} = B_{inner} - B_{outer}$

$dB = \frac{\mu_0}{4\pi}\left(\frac{I\,dL}{r^2}\right)$

$\int dB = \frac{\mu_0}{4\pi}\left(\frac{I}{r^2}\right)\int dL$

$B_{inner} = \frac{\mu_0}{4\pi}\left(\frac{I}{r_2^2}\right) r_2 \theta = 1.26 \times 10^{-5} T$

$B_{outer} = \frac{\mu_0}{4\pi}\left(\frac{I}{r_1^2}\right) r_1 \theta = 8.37 \times 10^{-6} T$

$\boxed{B_{bt} = 4.22 \times 10^{-6}}$

b) $\boxed{\text{into the page}}$

28.65) radius $= R$, current $= I$

$J = \alpha r$

a) $I = \int_0^R J\,dA$

$I = \int_0^R \alpha r (2\pi r)\,dr$

$I = 2\pi\alpha \int_0^R r^2\,dr$

$I = \frac{2}{3}\pi\alpha R^3$

$\boxed{\alpha = \frac{3I}{2\pi R^3}}$

b) $\int \vec{B} \cdot d\vec{s} = \mu_0 I_{encl}$

$I_{encl} = \int_0^r J\,dA = \int_0^r \frac{3I}{2\pi R^3} r (2\pi r)\,dr$

$I_{encl} = \frac{3I}{R^3}\int_0^r r^2\,dr$

$I_{encl} = \frac{I}{R^3} r^3 = I\left(\frac{r^3}{R^3}\right)$

$B(2\pi r) = I\left(\frac{r^3}{R^3}\right)\mu_0$

$\boxed{B(r) = \frac{\mu_0 I r^2}{2\pi R^3}}$

c) $\int \vec{B} \cdot d\vec{s} = \mu_0 I_{encl}$

$I_{encl} = I$

$B(2\pi r) = \mu_0 I$

$\boxed{B = \frac{\mu_0 I}{2\pi r}}$

28.67) $\vec{J} = \frac{2I_0}{\pi a^2}\left[1 - \left(\frac{r}{a}\right)^2\right]\hat{k}$

a) $\int \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$

$I_{encl} = I_0$

$B(2\pi r) = \mu_0 I_0$

$\boxed{B = \frac{\mu_0 I_0}{2\pi r}}$

b) $I = \int \vec{J}\cdot d\vec{A} = \int J\,dA$

$dA = 2\pi r\,dr$

$I = \int_0^r \frac{2I_0}{\pi a^2}\left[1 - \left(\frac{r}{a}\right)^2\right] 2\pi r\,dr$

$I = \frac{4I_0}{a^2}\int_0^r \left(r - \frac{r^3}{a^2}\right) dr$

$\boxed{I = \frac{4I_0}{a^2}\left(\frac{1}{2}r^2 - \frac{1}{4a^2}r^4\right)}$

c) $\int \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$

$B(2\pi r) = \mu_0\left(\frac{4I_0}{a^2}\left(\frac{1}{2}r^2 - \frac{1}{4a^2}r^4\right)\right)$

$\boxed{B = \frac{2\mu_0 I_0}{\pi a^2}\left(\frac{1}{2}r - \frac{1}{4a^2}r^3\right)}$

28.75) radius = R, length = L, velocity = $\vec{v}$

ions/vol = n, charge = q

a) $\rho = \frac{Q}{V}$

$\boxed{\rho = qn}$

b) $\sigma = \frac{Q}{A}$

$Q = qnV$

$V = \pi r^2 h$, $SA = 2\pi r^2 + 2\pi r h$

$Q = \sigma A$

$\sigma A = qnV$

$\sigma = \frac{2nV}{A} \rightarrow \frac{V}{A}$ ratio

$\frac{V}{A} = n^{-1/3}$

$\sigma = qn^{-1/3}$

$\boxed{\sigma = qn^{2/3}}$

c) $\vec{J} = \rho\vec{v}$

$\int \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$

$B(2\pi R) = \mu_0 I_{encl}$

$I_{encl} = \int_0^R \vec{J}\,d\vec{A}$

$I_{encl} = \int_0^R \rho v\,(2\pi r)\,dr$

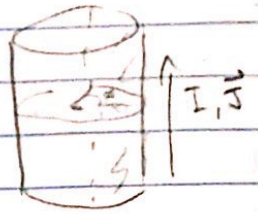
$I_{encl} = \pi\rho v R^2$

$B(2\pi R) = \mu_0 \pi \rho v R^2$

$B = \frac{1}{2}\mu_0 \rho v R$

$\boxed{B = \frac{1}{2}\mu_0 nqvR\,\hat{\phi}}$

d)



$dI = J\,dA$

$J = \rho\vec{v}$

$\rho = \sigma(n^{1/3})$

$dA = R\,d\phi\,(L)$

$\sigma = \frac{Q}{A}$

$dq = \sigma\,dA$

$\vec{J} = \sigma\vec{v}$

$dI = \sigma v\,dA$

$dI = \sigma v R\,d\phi$

$\boxed{dI = qn^{2/3} v R\,d\phi}$

e) $F_B = I\vec{L}\times\vec{B} = ILB$

$dF_B = dI\,LB$

$dF_B = (qn^{2/3} vR\,d\phi)\,L\left(\frac{1}{2}\mu_0 nqvR\right)$

$\boxed{dF_B = \frac{1}{2}\mu_0 n^{5/3} q^2 v^2 R^2 L\,d\phi}$

f) $F_B = \int_0^{2\pi} \frac{1}{2}\mu_0 n^{5/3} q^2 v^2 R^2 L\,d\phi$

$\boxed{F_B = \pi\mu_0 n^{5/3} q^2 v^2 R^2 L}$

g) $P = F_B/(2\pi R \cdot L)$

$\boxed{P = \frac{1}{2}\mu_0 n^{5/3} q^2 v^2 R}$

h) $R = 2cm$

$n = 8.0\times10^{16}\ \frac{ions}{cm^3}$

$q = 1.6\times10^{-19}\ C$

$v = 20\frac{m}{s} = 2000\frac{cm}{s}$

$\mu_0 = 4\pi\times10^{-7}\ \frac{m\cdot kg}{s^2 A^2} \rightarrow \frac{m\cdot kg}{C^2}$

$A = \frac{C}{s} \rightarrow A^2 = \frac{C^2}{s^2}$

$\mu_0 = 1.26\times10^{-4}\ \frac{cm\cdot kg}{C^2}$

$\left(\frac{cm\cdot kg}{C^2}\right)\left(\frac{ions}{cm^3}\right)\left(C^2\right)\left(\frac{cm^2}{s^2}\right)\left(\frac{1}{ions}\right)$

$\boxed{1.9\times10^{-7}\ \frac{kg}{cm\cdot s^2}}$

28.77) charge = Q, mass = M

$\vec{H} = \gamma \vec{L}$

$\gamma = g\left(\frac{Q}{2m}\right)$

a) $dA = R\,d\theta\,(R\sin\theta\,d\phi)$

$dA = R^2 \sin\theta\, d\theta\, d\phi$

$dI = \frac{dq}{dt}$

$\sigma = \frac{Q}{A} \rightarrow dq = \sigma\, dA$

$\sigma = \frac{Q}{4\pi R^2}$

$dq = \frac{Q}{4\pi R^2} R^2 \sin\theta\, d\theta\, d\phi$

$\boxed{\frac{dq}{dt} = dI = \frac{Q}{4\pi} \sin\theta\, d\theta\, \omega}$

b) $dH = A\,dI$, $A = \pi r^2$

$A = \pi(R^2 \sin^2\theta)$

$\boxed{dH = \frac{1}{4} G\omega R^2 \sin^3\theta\, d\theta}$

c) $H = \frac{1}{4} G\omega R^2 \int_0^{\pi/2} \sin^3\theta\, d\theta$

$\int \sin^3\theta\, d\theta = \int \sin\theta(\sin^2\theta) = \int \sin\theta(1-\cos^2\theta)\, d\theta$

$= \int \sin\theta - \int \sin\theta\cos^2\theta\, d\theta$

$u = \cos\theta$, $du = \sin\theta\, d\theta$

$= -\cos\theta - \int u^2 du = -\cos\theta - \frac{\cos^3\theta}{3}$

$H = \frac{1}{4} G\omega R^2 \left[-\cos\theta - \frac{\cos^3\theta}{3}\right]_0^{\pi}$

$\boxed{H = \frac{1}{3} G\omega R^2}$

d) $\vec{L} = I\omega$

$I = \frac{2}{3} MR^2$

$\boxed{\vec{L} = \frac{2}{3} MR^2 \omega}$

e) $\gamma = \frac{H}{L}$

$\gamma = \frac{\frac{1}{3} G\omega R^2}{\frac{2}{3} MR^2 \omega}$

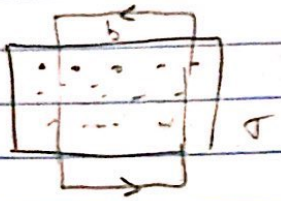
$\gamma = \frac{Q\omega}{2M\omega} = \boxed{\frac{Q}{2m}}$

f) $\gamma = g\left(\frac{Q}{2m}\right)$

$\boxed{g = 1}$

28.80) $\int \vec{B} \cdot d\vec{s} = \mu_0 I_{encl}$

a)



$\int \vec{B} \cdot d\vec{s} + \int \vec{B} \cdot d\vec{s} = \mu_0 I_{encl}$

$2Bx = \mu_0 I_{encl}$

$\sigma = \frac{Q}{A}$

$I_{encl} = I\left(\frac{x}{b}\right) = \frac{Q}{T}$

$2Bx = \mu_0 I\left(\frac{x}{b}\right)$

$B = \frac{\mu_0 I}{2b}$

$I = \frac{Q}{T} = \frac{\sigma(wL)}{\frac{L}{v}} = \sigma w v$

$w = b$

$B = \frac{\mu_0 I}{2b} = \frac{\mu_0 \sigma b v}{2b}$

$\boxed{B = \frac{\mu_0 \sigma v}{2}}$

b) $\boxed{\text{out of the page}}$