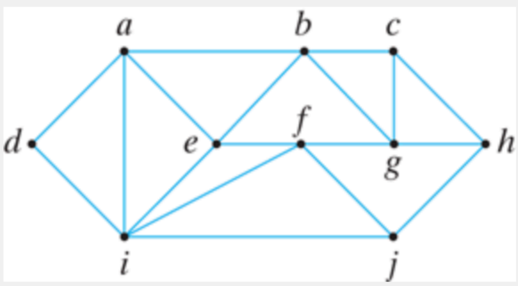
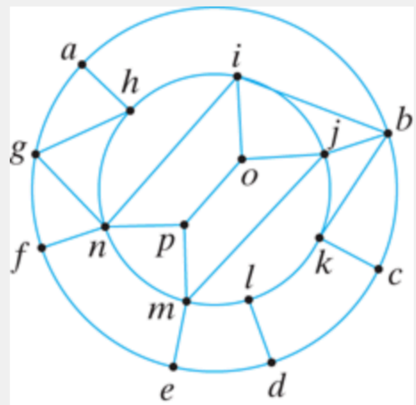
Hamiltonian Cycles

> Section 8.3 in course textbook.

1. In the following exercises, find a Hamiltonian cycle in the corresponding graph.



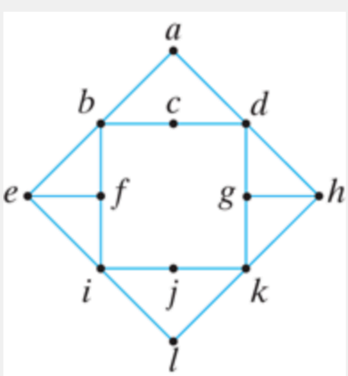
Graph 1



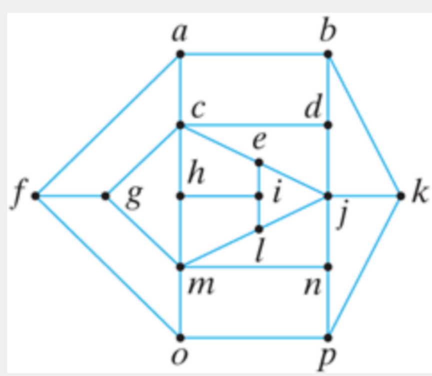
Graph 2

   i. Graph 1 (above)
   ii. Graph 2 (above)

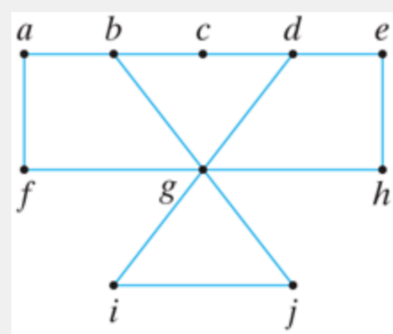
2. Explain why none of the graphs below has a Hamiltonian cycle.



Graph 3



Graph 4



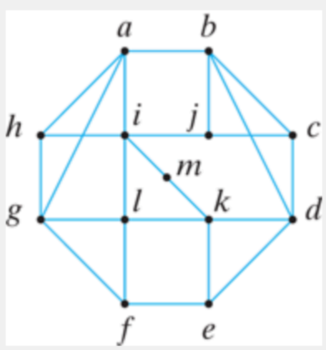
Graph 5

i. Graph 3 (above)

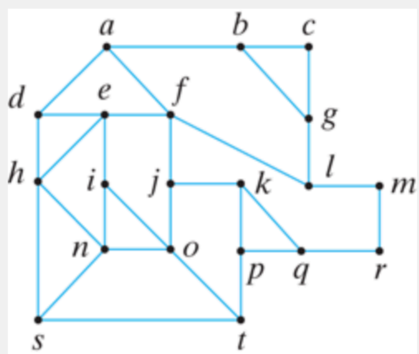
ii. Graph 4 (above)

iii. Graph 5 (above)

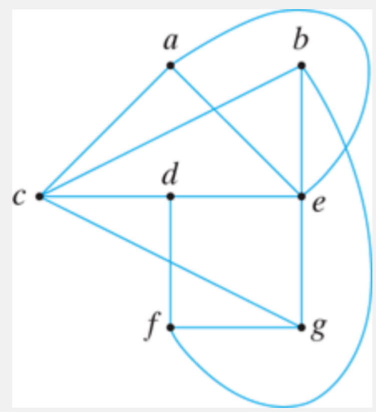
3. For each of the graphs below, determine whether or not a Hamiltonian cycle exists. If there is one, exhibit it; otherwise, give an argument that shows there is none.



Graph 6



Graph 7



Graph 8

i. Graph 6 (above)

ii. Graph 7 (above)

iii. Graph 8 (above)

4. Give an example of a graph that has

i. an Euler cycle but contains no Hamiltonian cycle.

ii. an Euler cycle that is also a Hamiltonian cycle.

5. A *Hamiltonian path* in a graph G is a simple path that contains every vertex in G exactly once.

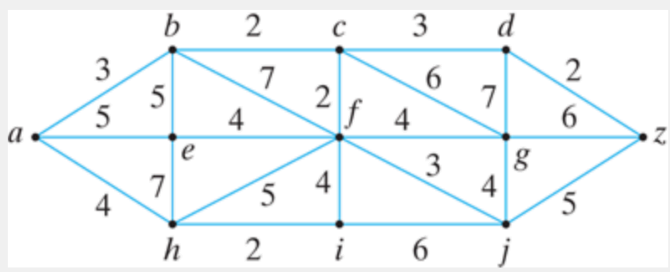
i. If a graph has a Hamiltonian cycle, must it have a Hamiltonian path? Explain.

ii. If a graph has a Hamiltonian path, must it have a Hamiltonian cycle? Explain.

A shortest path algorithm

> Section 8.4 in course textbook.

6. For the pairs of vertices below, use the following graph to find the length of a shortest path, as well as a shortest path between each pair of vertices.



Graph 9

   i. Vertices a and f.

   ii. Vertices b and j.

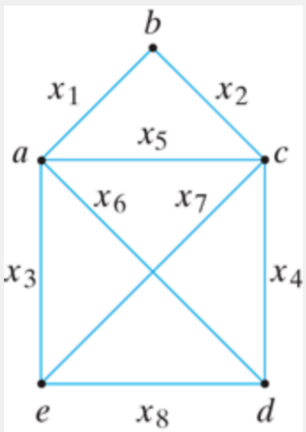
   iii. Vertices h and d.

7. Modify *Dijkstra's* algorithm so that it finds the lengths of the shortest paths from a given vertex to every other vertex in a connected, weighted graph G.
8. *Dijkstra's* algorithm is to be applied to connected weighted graphs. Describe the values of the labels $L(v)$ for all vertices of G, if the graph is not connected. Assume G has at least two components, and that both components have at least one vertex each.

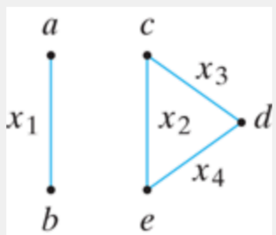
Representation of graphs

> Section 8.5 in course textbook.

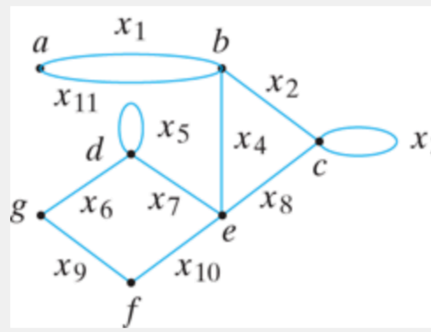
9. Write the adjacency matrix for each graph.



Graph 10



Graph 11



Graph 12

   i. Graph 10 (above)
   ii. Graph 11 (above)
   iii. Graph 12 (above)
   iv. The complete bipartite graph $K_{2,3}$.
   v. The complete graph on five vertices K_5.
10. Write the incidence matrix for each graph.
   i. The complete bipartite graph $K_{2,3}$.
   ii. The complete graph on five vertices K_5.
   iii. Graph 10 (above)
   iv. Graph 11 (above)
   v. Graph 12 (above)

11. Draw the graph represented by each adjacency matrix.

$$\mathbb{A} = \begin{pmatrix} 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}, \qquad \mathbb{B} = \begin{pmatrix} 4 & 1 & 1 & 1 & 0 & 2 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 3 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 2 & 0 & 3 & 1 & 1 & 0 \end{pmatrix}.$$

i. Order: a, b, c, d, e. Matrix $\mathbb{A}$ above.

ii. Order: a, b, c, d, e, f. Matrix $\mathbb{B}$ above.

12. Suppose that a graph G has an adjacency matrix of the form

$$\mathbb{A} = \begin{pmatrix} \mathbb{A}_{1,1} & 0 \\ 0 & \mathbb{A}_{2,2} \end{pmatrix}$$

where $\mathbb{A}_{1,1}$ and $\mathbb{A}_{2,2}$ are non-zero $n_1 \times n_1$ and $n_2 \times n_2$ square matrices, respectively. What must the graph of G look like?