

Midterm 1

Last Name: _____

First Name: _____

Student ID: _____

Signature: _____

Section:

Tuesday:

Thursday:

1A

1B

TA: Khang Huynh

1C

1D

TA: Eli Sadovnik

1E

1F

TA: Jason Snyder

Instructions: Do not open this exam until instructed to do so. You will have 50 minutes to complete the exam. Please print your name and student ID number above, and circle the number of your discussion section. **You may not use calculators**, books, notes, or any other material to help you. Please make sure **your phone is silenced** and stowed where you cannot see it. You may use any available space on the exam for scratch work. If you need more scratch paper, please ask one of the proctors. You must **show your work** to receive credit. Please circle or box your final answers.

Please do not write below this line.

| Question | Points | Score |
|----------|--------|-------|
| 1 | 15 | |
| 2 | 15 | |
| 3 | 10 | |
| 4 | 10 | |
| Total: | 50 | |

1. (a) (5 points) Find the solution y_h to the differential equation:

$$y' = \frac{1}{x}y$$

$$y = C \cdot x.$$

- (b) (10 points) Solve the initial value problem:

$$y' = \frac{1}{x}y + \sqrt{x}, y(1) = 0$$

(b) Interval of Existence = ?

$$2x^{\frac{3}{2}} - 2x.$$

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2. (a) (5 points) Find the general solution $y_h = C_1y_1 + C_2y_2$ to the differential equation:

$$y'' + y = 0$$

- (b) (10 points) Use undetermined coefficient or variation of parameters, find the general solution to the differential equations

$$y'' + y = t + e^t$$

3. (10 points) Solve the homogeneous equation:

$$(y^2 + 2xy)dx - x^2 dy = 0$$

(Hint: Using $y = vx$ change the differential equation to a separable equation)

$$(v^2 x^2 + 2x^2 v)dx - x^2(x dv + v dx)$$

$$(v^2 + 2v) dx - x dv - v dx = 0.$$

$$(v^2 + v) dx = x dv.$$

$$\int \frac{dx}{x} = \int \frac{dv}{v(v+1)}$$

$$\ln|x| = \int \left(\frac{1}{v} - \frac{1}{v+1} \right) dv = \ln|v| - \ln|v+1| + C.$$

$$= \ln \left| \frac{v}{v+1} \right| + C.$$

$$\ln \left| \frac{x(v+1)}{v} \right| = C.$$

$$\frac{y+x}{\frac{y}{x}} = k.$$

$$xy + x^2 = ky, \quad k \in \mathbb{R}.$$

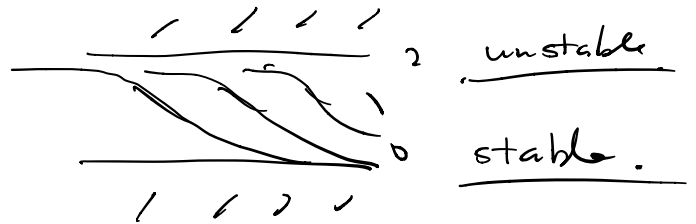
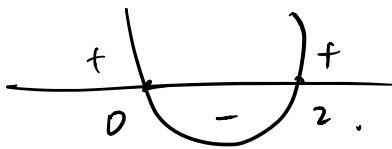
4. Consider the autonomous equation:

$$y' = y(y - 2)e^y$$

- (a) (2 points) Find the equilibrium solutions of the above differential equations.

$$y \equiv 2 \quad . \quad y \equiv 0.$$

- (b) (3 points) Determine the stability of the equilibrium solutions.



- (c) (5 points) Prove that if $y(t)$ is a solution and $y(0) = 1$, then $0 < y(t) < 2$ for all $t \in (-\infty, \infty)$.

$$(d) \quad \lim_{t \rightarrow \pm\infty} y(t) = ? \quad 0.$$

Scratch Paper

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Scratch Paper

Some useful formulas, etc:

Integrating factor $u(x)$ of a 1st Order Linear DE $x' = ax + f$:

$$u(x) = e^{-\int a(t)dt}$$

Single variable integrating factor μ for $Pdx + Qdy = 0$

- If $h = \frac{1}{Q} \left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} \right)$,

$$\mu(x) = e^{\int h(x)dx}$$

- If $g = \frac{1}{P} \left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} \right)$,

$$\mu(x) = e^{-\int g(x)dx}$$

Variation of Parameters, (2nd Order Differential Equations)

$$v_1(x) = - \int \frac{1}{W} y_2(x) f(x) dx$$

$$v_2(x) = \int \frac{1}{W} y_1(x) f(x) dx$$