Math 61 HW #2

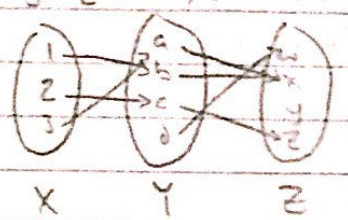
1. $g = \{(1,b),(2,c),(3,a)\}$

$X = \{1,2,3\}$, $Y = \{a,b,c,d\}$

$f = \{(a,x),(b,x),(c,z),(d,w)\}$

$Z = \{w,x,y,z\}$

$\boxed{f \circ g = \{(1,x),(2,z),(3,x)\}}$



2. $f(n) = 2n+1$ $g(n) = 3n-1$

$f \circ f = 2(2n+1)+1$

$\boxed{f \circ f = 4n+3}$

$g \circ g = 3(3n-1)-1$

$\boxed{g \circ g = 9n-4}$

$f \circ g = 2(3n-1)+1$

$\boxed{f \circ g = 6n-1}$

$g \circ f = 3(2n+1)-1$

$\boxed{g \circ f = 6n+2}$

3. $f = \{(a,b),(b,a),(c,b)\}$

i) $f \circ f = \{(a,a),(b,b),(c,a)\}$

$f \circ f \circ f = \{(a,b),(b,a),(c,b)\}$

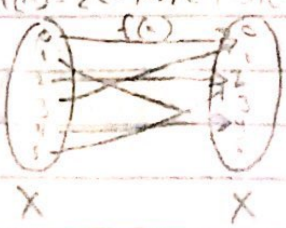
ii) $f^1 = \{(a,b),(b,a),(c,b)\}$

$f^{(2)} = \{(a,b),(b,a),(c,b)\}$

$f^{[illegible]} = \{(a,a),(b,b),(c,a)\}$

4 $X = \{0,1,2,3,4,5\}$, $f(x) = 4x \bmod 6$

$f(x) = \{(0,0),(1,4),(2,2),(3,0),(4,4),(5,2)\}$



$\boxed{\text{Neither}}$

5. $\lceil x+y \rceil = \lceil x \rceil + \lceil y \rceil$

$x = 1.5$, $y = 1.5$

$\lceil 1.5+1.5 \rceil = \lceil 1.5 \rceil + \lceil 1.5 \rceil$

$\lceil 3 \rceil = 2+2$

$3 \neq 4$

$\boxed{\text{Not true for all real \#s}}$

6. $s_n = 2^n - n^2$

i) $D = \{1,2,3,4\}$

$S_n = \{1,0,-1,0\}$

$\boxed{\text{None}}$

ii) $D = \{n \mid n \in \mathbb{Z}, n \geq 2020\}$

2^n is extremely high

n^2 is reasonable

$\boxed{\text{increasing, non-decreasing}}$

7. $t_n = 2n-1$, $n \geq 1$

i) $t_3 = 6-1 = \boxed{5}$

$t_7 = 14-1 = \boxed{13}$

$t_{100} = 200-1 = \boxed{199}$

$t_{2020} = 4040-1 = \boxed{4039}$

ii) $\prod_{i=1} t_i = 3 \cdot 4 \cdot 5 \cdot 6 = \boxed{360}$

$\sum_{i=1} t_i = 3+4+5+6+7 = \boxed{25}$

8. $w_n = \frac{1}{n} - \frac{1}{n+1}$, $n \geq 1$

i) $\sum_{i=1}^{3} w_i = (1-\frac{1}{2}) + (\frac{1}{2}-\frac{1}{3}) + (\frac{1}{3}-\frac{1}{4})$

$\boxed{\sum_{i=1}^{3} w_i = 3/4}$

$\sum_{i=1}^{10} w_i = (1-\frac{1}{2}) + \ldots + (\frac{1}{10}-\frac{1}{11})$

$\boxed{\sum_{i=1}^{10} w_i = \frac{10}{11}}$

ii) $\sum_{i=1}^{2020} w_i = (1-\frac{1}{2}) + \ldots + (\frac{1}{2020}-\frac{1}{2021})$

$\boxed{\sum_{i=1}^{2020} w_i = \frac{2020}{2021}}$

9. $r_n = 3 \cdot 2^n - 4 \cdot 5^n$

i) $\boxed{r_0 = -1}$

$r_1 = 3 \cdot 2 - 4 \cdot 5 = \boxed{-14}$

$r_2 = 3 \cdot 4 - 4 \cdot 25 = \boxed{-88}$

$r_3 = 3 \cdot 8 - 4 \cdot 125 = \boxed{-476}$

ii) $r_{n-1} = 3 \cdot 2^{n-1} - 4 \cdot 5^{n-1}$

$\boxed{r_{n-1} = \frac{3}{2} 2^n - \frac{4}{5} 5^n}$

$r_{n-2} = 3 \cdot 2^{n-2} - 4 \cdot 5^{n-2}$

$\boxed{r_{n-2} = \frac{3}{4} 2^n - \frac{4}{25} 5^n}$

iii) $r_n = 7r_{n-1} - 10r_{n-2}$

$3 \cdot 2^n - 4 \cdot 5^n = 7\left(\frac{3}{2} 2^n - \frac{4}{5} \cdot 5^n\right)$

$-10\left(\frac{3}{4} 2^n - \frac{4}{25} 5^n\right)$

$3 \cdot 2^n - 4 \cdot 5^n = \frac{21}{2} 2^n - \frac{28}{5} 5^n$

$-\frac{15}{2} 2^n + \frac{8}{5} 5^n$

$\boxed{3 \cdot 2^n - 4 \cdot 5^n = 3 \cdot 2^n - 4 \cdot 5^n \checkmark}$

10. $\sum_{i=1}^{n} i^2 r^{n-i}$, $i = k+1$

$k = i - 1$

$\boxed{\sum_{k=0}^{n-1} (k+1)^2 r^{n-k-1}}$

11. $\alpha = baab$, $\beta = caaba$, $\gamma = bbab$

i) $\boxed{\alpha\beta = baabcaaba}$

ii) $\boxed{\beta\alpha = caababaab}$

iii) $\boxed{\alpha\alpha = baabbaab}$

iv) $\boxed{\beta\beta = caabacaaba}$

v) $\boxed{|\alpha\beta| = 9}$

vi) $\boxed{|\beta\alpha| = 9}$

12. $X = \{0, 1\}$

i) $\boxed{01, 10, 11, 00}$

ii) $\boxed{01, 10, 11, 00, 1, 0, \lambda}$

iii) $\boxed{001, 010, 011, 100, 101, 110, 111, 000}$

iv) $\boxed{\begin{array}{l} 001, 010, 011, 100, 101, 110, 111, 000 \\ 00, 01, 10, 11, 0, 1, \lambda \end{array}}$

v) $\boxed{010, 011, 101, 110, 111}$

13. babc

$\boxed{\begin{array}{l} babc, bab, abc, ba, ab, bc, b, a, \\ b, c, \lambda \end{array}}$

14. $X = \{a, b\}$

$f(\alpha) = \alpha\alpha^R$

$f(\alpha) = f(\beta)$

$\alpha\alpha^R = \beta\beta^R$

by definition $\alpha = \beta$

$\boxed{f(\alpha) \text{ is one-to-one}}$

$|\alpha\alpha^R|$ is always even

$|a|$ and $|b|$ are odd

$\boxed{f(\alpha) \text{ is not onto}}$

15. If $\alpha \in L$, then $a\alpha b \in L$ and $ba \in L$

If $\alpha \in L$ and $\beta \in L$ then $\alpha\beta \in L$

i) $\alpha = \lambda \rightarrow ab \in L$

$\alpha = ab \rightarrow aabb \in L$

$\alpha = aabb \rightarrow \boxed{aaabbb \in L}$

ii) $\alpha = \lambda \rightarrow ab \in L$

$\alpha = \lambda \rightarrow ba \in L$

$ba \in L$ and $ab \in L \rightarrow baab \in L$

$baab \in L$ and $ab \in L \rightarrow \boxed{baabab \in L}$

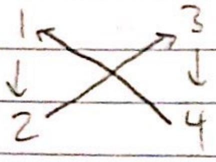
iii) $|a\alpha b|$ is always even (you always add 2 characters)

$|\alpha\beta|$ is always even $\rightarrow$ only even-length strings are in L (even + even = even)

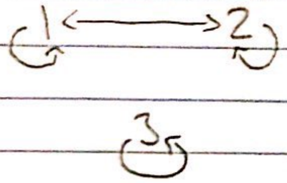
$|aab|$ is odd $\rightarrow \boxed{aab \notin L}$

Scanned with CamScanner

16. i) $R = \{(1,2),(2,3),(3,4),(4,1)\}$ on $\{1,2,3,4\}$



ii) $R = \{(1,2),(2,1),(3,3),(1,1),(2,2)\}$ on $\{1,2,3\}$



17. $\{1,2,3,4,5\}$, $(x,y) \in R$ if 3 divides $x-y$

i) $\boxed{R = \{(1,4),(4,1),(2,5),(5,2),(1,1),(2,2),(3,3),(4,4),(5,5)\}}$

ii) $\boxed{R^{-1} = \{(1,4),(4,1),(2,5),(5,2),(1,1),(2,2),(3,3),(4,4),(5,5)\}}$

iii) $(x,y) \in R$ if $x+y \leq 6$

$\boxed{R = \{(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(4,1),(4,2),(5,1)\}}$

$\boxed{R^{-1} = \{(1,1),(2,1),(3,1),(4,1),(5,1),(1,2),(2,2),(3,2),(4,2),(1,3),(2,3),(3,3),(1,4),(2,4),(1,5)\}}$

18. i) $(x,y) \in R$ if 3 divides $x-y$

$(1,1) \notin R \rightarrow$ not reflexive

$|x-y| = |y-x| \rightarrow$ $\boxed{\text{symmetric}}$ not anti

$\boxed{\text{transitive}}$

ii) $(x,y) \in R$ if $x \geq y$

$\boxed{\text{reflexive, anti-symmetric, transitive} \;\; \hookrightarrow \text{partial order}}$

19. $P(x) \rightarrow (A,B) \in R$ if $A \subseteq B$

reflexive ✓

symmetric X

antisymmetric ✓

transitive ✓

partial-order ✓

$\boxed{\text{reflexive, antisymmetric, transitive} \;\; \hookrightarrow \text{partial order}}$

20. $X = \{1,2,3,4\}$

$R_1 = \{(1,1),(1,2),(3,4),(4,2)\}$

$R_2 = \{(1,1),(2,1),(3,1),(4,4),(2,2)\}$

$\boxed{R_1 \circ R_2 = \{(1,1),(1,2),(2,1),(2,2),(3,1),(3,2),(4,2)\}}$

$\boxed{R_2 \circ R_1 = \{(1,1),(1,2),(3,4),(4,1),(4,2)\}}$