Math 61 HW #4

1. i) $8 \times 4 \times 5$

   160

   ii) $2 \times 2 \times 2 \times 2 \times 2 \times 2 - 1$

   $2^6 - 1$

   63 characters

   iii) 6×6

   36 outcomes

2. $X = \{A, B, C, D, E, F\}$

   i) $\ell = 5$, Begin w/ F, end w/ A

   $1 \times 5 \times 4 \times 3 \times 1$

   60 strings

   ii) $\ell = 5$, Begin w/ F, end w/ A

   $1 \times 6 \times 6 \times 6 \times 1$

   216 strings

   iii) $\ell = 5$, Begin w/ F, don't end w/ EB

   $1 \times 5 \times 4 \times 2 \times 1$

   40 strings

3. i) Is pres: $1 \times 3 = 3$ selections

   Is not officer: $3 \times 2 = 6$ selections

   9 selections

   ii) Is pres.: $1 \times 3 = 3$ selections

   Is sec.: $3 \times 1 = 3$ selections

   6 selections

4. i) S7: 6 ways

   S11: 2 ways

   8 ways

   ii) R2: 5 ways

   D2: 5 ways

   10 ways

   iii) R2: 6 ways

   D2: 5 ways

   11 ways

5. i) No Citlali

   $5 \times 4 \times 3 =$

   60 selections

   ii) Daneli is, Frida is not

   $1 \times 4 \times 3 = 12$ selections

   $4 \times 1 \times 3 = 12$ selections

   $4 \times 3 \times 1 = 12$ selections

   36 selections

   iii) Beatriz is chair or treasurer

   $1 \times 5 \times 4 = 20$ selections

   $5 \times 4 \times 1 = 20$ selections

   40 selections

6. i) # of symmetric relations

   $\begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}$

   n diagonal, $n^2 - n$ non

   $2^n \cdot 2^{\frac{n^2-n}{2}}$

   $2^{\left(\frac{n^2+n}{2}\right)}$

   $2^6 =$ **64 relations**

   ii) # of anti symmetric relations

   $\begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}$

   n diagonal, $n^2 - n$ non

   $2^n \cdot 3^{\left(\frac{n^2-n}{2}\right)}$

   $2^3 \cdot 3^3 = 8 \cdot 27$

   216 relations

   iii) $2^{(n^2+n/2)}$

   2^{10}

   1024 relations

   iv) $2^n \cdot 3^{\left(\frac{n^2-n}{2}\right)}$

   $2^4 \cdot 3^{(6)} =$

   11664 relations

7. Beatriz chair: $1 \times 5 \times 4 = 20$

Abigail scc.: $5 \times 1 \times 4 = 20$

40 selections

Both counted 2x

$40 - 1 =$ **39 selections**

8. 1-10,000, divisible by 5 or 7

$10000/5 = 2000$

$10000/7 = 1428$

$5 \times 7 \rightarrow 35$

$10000/35 = 285$

$2000 + 1428 - 285 =$ **3142 numbers**

9. i) permutations of a, b, c, d

abcd, acdb, abdc, acbd, adbc, adcb,
bacd, badc, bdac, bdca, bcad, bcda,
cabd, cadb, cbad, cbda, cdab, cdba,
dabc, dacb, dbac, dbca, dcba, dcab

ii) 11!

$11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

39916800 permutations

iii) $\frac{11!}{6!} =$

$11 \cdot 10 \cdot 9 \cdot 8 \cdot 7$

55440 permutations

10. i) 5+5=10 distinct

$10! =$ **3628800 ways**

ii) 10 orderings / way in line that are equal

$3628800 / 10$

362880 ways

11. $X = \{w, x, y, z\}$

$C(4,3) = \frac{4!}{1!3!} =$ **4 combinations**

wxy, wxz, xyz, wyz

12. Uruk-hai: $C(10,4) = \frac{10!}{6!4!}$

$= 210$

Haradrim: $C(12,3) = \frac{12!}{9!3!}$

$= 220$

Nazgul: $C(4,2) = \frac{4!}{2!2!}$

$= 6$

$210 \cdot 220 \cdot 6$

277200 ways

13. i) 4A

$1 \times 1 \times 1 \times 48$

48 hands

ii) Quads

13 diff. quads

48×13

624 hands

iii) Spades flush

$C(13,5) = \frac{13!}{8!5!}$

1287 hands

14. i) SALESPERSONS, $\ell = 12$

$\frac{12!}{4!2!} =$ **7979200 strings**

ii) [illegible]

$\ell = 21 \rightarrow \frac{21!}{2!2!2!3!2!}$

2.66×10^{17} strings

iii) [illegible]

$\ell = 19 \rightarrow \frac{19!}{5!3!2!2!2!}$

1.056×10^{17} strings

15. $C(10,7) = \frac{10!}{9!} =$ **10 ways**