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Dis. 1D

Math 33B HW#3

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Chapter 2.7

4) $w' = w\sin w + s$, $w(0) = 1$

$w\sin w + s$ is continuous for all w and s

$\frac{\partial F}{\partial w} = w\cos w + \sin w$

$\frac{\partial F}{\partial w}$ is continuous for all w and s

$\boxed{\text{Unique solution}}$

6) $y' = \frac{1}{x}y + 2$, $y(0) = 1$

y' is continuous for $x \neq 0$

the initial condition is $x = 0$, therefore there is no solution containing this point that is continuous

$\boxed{\text{No unique solution guaranteed}}$

8) $ty' = 2y - t$, $y(0) = 2$

$y' = \frac{2}{t}y - 1$

$y' = a(t)y + f$

$a(t) = \frac{2}{t}$

$IF = e^{-\int a(t)\,dt} = e^{-\int 2/t\,dt}$

$= e^{-2\int 1/t\,dt} = e^{-2\ln|t|}$

$= \frac{1}{t^2}$

$y' - \frac{2}{t}y = -1$

$\frac{y'}{t^2} - \frac{2}{t^3}y = -\frac{1}{t^2}$

$\int \left(\frac{y}{t^2}\right)' = \int -\frac{1}{t^2}$

$\frac{y}{t^2} = \frac{1}{t}$

$y_p = t$

$y' = \frac{2}{t}y$

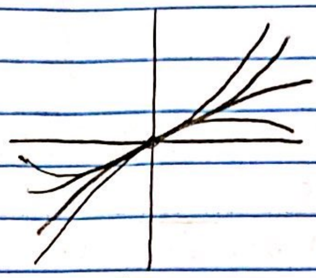
$\frac{dy}{dt} = \frac{2}{t}y$

$\frac{dy}{y} = \frac{2}{t}dt$

$\ln|y| = 2\ln|t|$

$y_h = t^2$

$\boxed{\text{Sol'n set: } Ct^2 + t}$



ii) all solutions have $y(0) = 0$

$ty' = 2y - t$

$y' = \frac{2y - t}{t}$

$t \neq 0$, therefore it is not continuous.

14) $\frac{dy}{dt} = \frac{1}{(t+2)(y-3)}$, $y(0) = 1$

$dy(y-3) = \frac{dt}{(t+2)}$

$\frac{y^2}{2} - 3y = \ln|t+2| + C$

$y^2 - 6y = 2\ln|t+2| + C$

$1 - 6 = 2\ln|2| + C$

$C = -5 - 2\ln|2|$

$y^2 - 6y = 2\ln|t+2| - 5 - 2\ln|2|$

$(y^2 - 6y + 9) = 2\ln|t+2| - 2\ln|2| + 4$

$(y-3)^2 = 2\ln|t+2| - 2\ln|2| + 4$

$y - 3 = \sqrt{2\ln|t+2| - 2\ln|2| + 4}$

$\boxed{y = 3 + \sqrt{2\ln|t+2| - 2\ln|2| + 4}}$

The solver has issues at $t \approx -1.7$
interval of existence $\approx (-1.7, \infty)$

ii) $2\ln|t+2| = 2\ln|2| - 4$

$\ln|(t+2)^2| = 2\ln|2| - 4$

$|t+2 = 0.27$

$t = -1.729$

$\boxed{(-1.729, \infty)}$

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26) $x_1(\frac{\pi}{2}) = 0$

$x_2(\frac{\pi}{2}) = 0$

$x' = f(t,x)$ has a unique solution

$\cos t \neq 1 - \sin t$

$\boxed{\therefore \text{ It is not possible}}$

30) $x' = \frac{x^3 - x}{1+t^2x^2}$, $x(0) = 1/2$

$x_1(0) = 0$, $x_2(0) = 1$

$x(0) < 1/2 = x(0) = x_2(0)$

$\frac{x^3 - x}{1+t^2x^2}$, $t^2x^2 \geq 0$

$\therefore f(t,x)$ is continuous

$\frac{\partial f}{\partial x} = \frac{(3x^2-1)(1+t^2x^2) - (x^3-x)(t^2)}{(1+t^2x^2)^2}$

$(1+t^2x^2)^2 \geq 1$

$\frac{\partial f}{\partial x}$ is continuous

uniqueness is satisfied
$x(t)$ doesn't cross with $x_1(0)$ or
$x_2(0)$, and is therefore between
0 and 1

32) $y' = y^2 - \cos^2 t - \sin t$, $y(0) = 2$

$\cos t \rightarrow y_1(0) = 1 < y(0) = 2$

$y^2 - \cos^2 t - \sin t$ is continuous for all y and t

$\frac{\partial f}{\partial y} = 2y \rightarrow$ continuous for all y and t

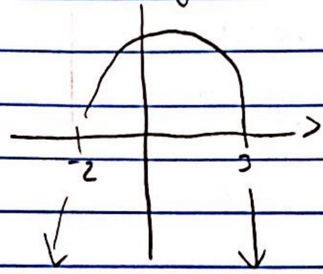
uniqueness is satisfied, since no
solutions can intersect and $y(0)$
is greater than $y_1(0)$, $y(t)$ must
be greater than all $y_1(t) = \cos t$

Chapter 2.9

18) $y' = 6 + y - y^2 = f(y)$

$y' = (-y+3)(y+2)$

zeros at $y = 3$ and $y = -2$

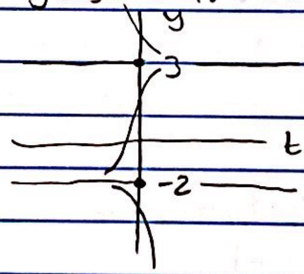


ii)

$y(t) = -2$ is unstable

$y(t) = 3$ is stable

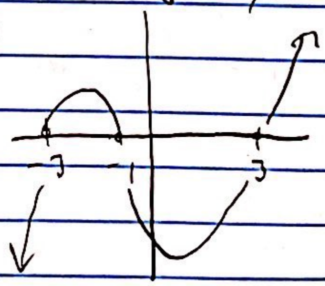
iii)



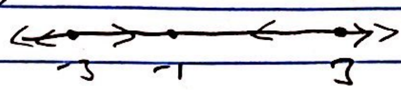
20) $y' = (y+1)(y^2 - 9)$

zeros at $y = -1$, $y = 3$, $y = -3$

i)



ii)



$y(t) = -3$ is unstable

$y(t) = -1$ is stable

$y(t) = 3$ is unstable

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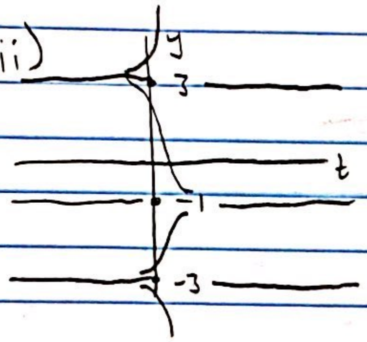
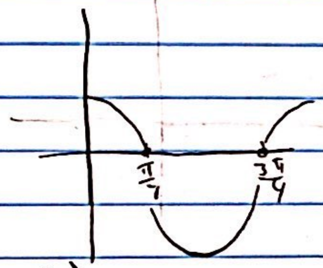
iii)



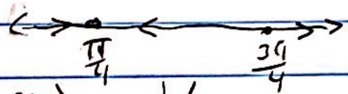
22) $y' = \cos 2y$

i) zeros at $2y = \frac{\pi}{2}$

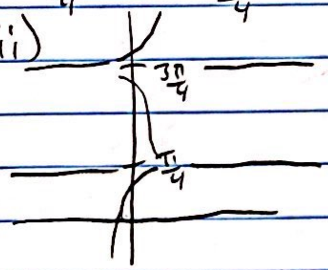
zeros at $y = n\frac{\pi}{4}$



ii)



iii)



24) $y' + 2y = 5, \quad y(0) = 0$

$y' = 5 - 2y$

zeros are $y = \frac{5}{2}$

$\frac{dy}{dt} = 5 - 2y$

$dy = (5-2y)dt$

$\frac{dy}{5-2y} = dt$

$\frac{dy}{-2y+5} = dt$

$u = -2y + 5$

$du = -2dy$

$-\frac{1}{2}\ln|2y+5| = t + C$

$\ln\left|\frac{1}{\sqrt{2y+5}}\right| = t + C$

$\frac{1}{\sqrt{2y+5}} = Ke^t$

$\frac{1}{Ke^t} = \sqrt{2y+5}$

$\frac{1}{Ae^{2t}} = 2y+5$

$\frac{1}{A} = 5$

$A = 1/5$

$\frac{1}{\frac{1}{5}e^{2t}} = 2y+5$

$5e^{-2t} = 2y+5$

$\boxed{\frac{5(e^{-2t}-1)}{2} = y}$

ii) $\lim_{t\to\infty} y(t) \to$ approaches $5/2$

iii) The qualitative analysis agrees and was easier.

26) $y' = (3+y)(1-y), \quad y(0) = 2$

i) $\frac{1}{(3+y)(1-y)}dy = dt$

$\frac{A}{3+y} + \frac{B}{1-y} = \frac{1}{(3+y)(1-y)}$

$A - Ay + 3B + By = 1$

$A = B$

$A + 3B = 1$

$4B = 1$

$B = 1/4, \ A = 1/4$

$\frac{1}{4}\left[\frac{1}{(3+y)} + \frac{1}{(1-y)}\right] = dt$

$4t + C = \ln|3+y| - \ln|1-y|$

$4t + C = \ln\left|\frac{3+y}{1-y}\right|$

$Ke^{4t} = \frac{3+y}{1-y}$

$K = -5$

$-5e^{4t} = \frac{3+y}{1-y}$

$-5e^{4t} + 5e^{4t}y = 3 + y$

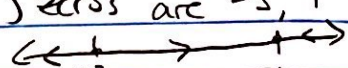
$3 + 5e^{4t} = 5e^{4t}y - y$

$3 + 5e^{4t} = y(5e^{4t} - 1)$

$\boxed{y = \frac{3+5e^{4t}}{5e^{4t}-1}}$

ii) $\frac{3e^{-4t}+5}{5-e^{-4t}} = \frac{0+5}{5-0} = \boxed{1}$

iii) zeros are $-3, 1$



$\boxed{\text{agrees, was easier}}$