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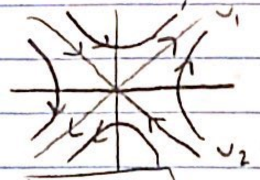
Math 33B HW #8

Chapter 9.3

12) $y(t) = C_1 e^{t}\begin{bmatrix}1\\1\end{bmatrix} + C_2 e^{-2t}\begin{bmatrix}1\\-1\end{bmatrix}$

$\lambda_1 = 1,\ \lambda_2 = -2$

$\vec{v}_1 = \begin{bmatrix}1\\1\end{bmatrix},\ \vec{v}_2 = \begin{bmatrix}1\\-1\end{bmatrix}$

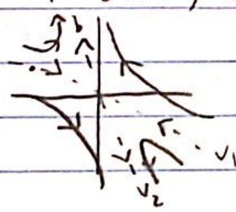


$\boxed{\text{saddle}}$

14) $y(t) = C_1 e^{-t}\begin{bmatrix}-5\\2\end{bmatrix} + C_2 e^{2t}\begin{bmatrix}-1\\4\end{bmatrix}$

$\lambda_1 = -1,\ \lambda_2 = 2$

$\vec{v}_1 = \begin{bmatrix}-5\\2\end{bmatrix},\ \vec{v}_2 = \begin{bmatrix}-1\\4\end{bmatrix}$



$\boxed{\text{saddle}}$

18) $y' = \begin{bmatrix}2 & 2\\-4 & -2\end{bmatrix}y$

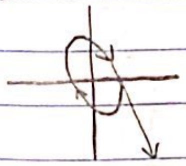
$T = 0,\ D = -4+8 = 4$

$p(\lambda) = \lambda^2 + 4$

$\lambda = 2i$

$a = 0 \rightarrow$ eq. is center

$\begin{bmatrix}2 & 2\\-4 & -2\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}2\\-4\end{bmatrix}$



20) $y' = \begin{bmatrix}-2 & 2\\-1 & 0\end{bmatrix}y$

$T = 2,\ D = 2$

$\lambda^2 + 2\lambda + 2 \rightarrow \lambda = -1 \pm i$

$\boxed{\text{spiral sink}}$

$\begin{bmatrix}-2 & 2\\-1 & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}-2\\-1\end{bmatrix}$

Chapter 9.4

2) $A = \begin{bmatrix}-16 & 9\\-18 & 11\end{bmatrix}$

$T = -5,\ D = (-16)(11) + (18)(9) = -14$

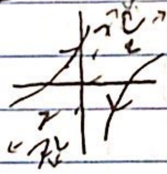
$D < 0 \rightarrow$ saddle

$p(\lambda) = \lambda^2 + 5\lambda - 14$

$\lambda_1 = -7,\ \lambda_2 = +2$

$\begin{bmatrix}-9 & 9\\-18 & 18\end{bmatrix} \rightarrow \vec{v}_1 = \begin{bmatrix}1\\1\end{bmatrix}$

$\begin{bmatrix}-18 & 9\\-18 & 9\end{bmatrix} \rightarrow \vec{v}_2 = \begin{bmatrix}1\\2\end{bmatrix}$



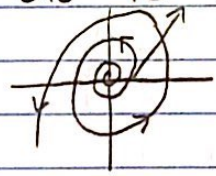
6) $A = \begin{bmatrix}6 & -5\\10 & -4\end{bmatrix}$

$T = 2,\ D = 26$

$T^2 - 4D = -100$

spiral source

$\begin{bmatrix}6 & -5\\10 & -4\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}6\\10\end{bmatrix}$



10) $A = \begin{bmatrix}-5 & 2\\-6 & 2\end{bmatrix}$

$T = -3,\ D = 2$

$T^2 - 4D = 1$

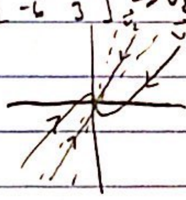
nodal sink

$p(\lambda) = \lambda^2 + 3\lambda + 2$

$\lambda_1 = -2,\ \lambda_2 = -1$

$\begin{bmatrix}-3 & 2\\-6 & 4\end{bmatrix} \rightarrow \vec{v}_1 = \begin{bmatrix}2\\3\end{bmatrix}$

$\begin{bmatrix}-4 & 2\\-6 & 3\end{bmatrix} \rightarrow \vec{v}_2 = \begin{bmatrix}1\\2\end{bmatrix}$



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Chapter 9.6

2) $A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

$e^A = I + A + \frac{1}{2!}A^2 + \ldots$

$A^2 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$e^A = I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

$\boxed{e^A = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}}$

4) $A = \begin{bmatrix} -2 & 1 & -3 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} -2 & 1 & -3 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 & -3 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} -2 & 1 & -3 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$e^A = I + A + \frac{1}{2}A^2$

$= \begin{bmatrix} -1 & 1 & -3 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1/2 & 1/2 \\ 0 & -1/2 & -1/2 \end{bmatrix}$

$\boxed{= \begin{bmatrix} -1 & 2 & -2 \\ -1 & 5/2 & -1/2 \\ 1 & -3/2 & 3/2 \end{bmatrix}}$

6) $e^{t\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}} = \begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$A^3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$A^4 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$e^{tA} = I + t\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} + \frac{t^2}{2}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + \frac{t^3}{6}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} +$

$\frac{t^4}{24}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ldots$

$= \begin{bmatrix} 1 - t^2/2 + t^4/24 & -t + t^3/6 \\ t - t^3/6 & 1 - t^2/2 + t^4/24 \end{bmatrix}$

$= \boxed{\begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix}}$ ✓

8) $A = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$

$A = a\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = X^2$

$e^{tA} = e^{t(aI + bX)}$

$e^{at}(I + btX)$

$e^{at}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & bt \\ 0 & 0 \end{bmatrix}\right)$

$\boxed{e^{at}\begin{bmatrix} 1 & bt \\ 0 & 1 \end{bmatrix}}$

10) $A = PDP^{-1}$, $e^{tA} = Pe^{tD}P^{-1}$

$e^{tA} = I + A + \frac{1}{2}A^2 + \frac{1}{6}A^3 \ldots$

$e^{tPDP^{-1}} = I + PDP^{-1} + \frac{1}{2}(PDP^{-1}PDP^{-1}) + \frac{1}{3!}(PDP^{-1}PDP^{-1}PDP^{-1}$

$e^{tA} = I + PtDP^{-1} + \frac{1}{2}(Pt^2D^2P^{-1}) + \frac{1}{3!}(Pt^3D^3P^{-1})$

$= P\left[I + tD + \frac{1}{2}t^2D^2 + \frac{1}{3!}t^3D^3\right]P^{-1}$

$= \boxed{Pe^{tD}P^{-1}}$ ✓

12) $A = \begin{bmatrix} -2 & 0 \\ -3 & -3 \end{bmatrix}$

$\lambda_1 = -2,\ \lambda_2 = -3$

$D = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 \\ -3 & -1 \end{bmatrix} \to \vec{v}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ -3 & 0 \end{bmatrix} \to \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$P = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

$\det(P) = 1$

$P^{-1} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

$e^{tA} = Pe^{tD}P^{-1}$

$= \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-3t} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} e^{-2t} & 0 \\ -3e^{-2t} & e^{-3t} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

$\boxed{= \begin{bmatrix} e^{-2t} & 0 \\ -3e^{-2t} + 3e^{-3t} & e^{-3t} \end{bmatrix}}$

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14) $A=\begin{bmatrix}-2 & 1\\ -1 & 0\end{bmatrix}$

$e^{At}=e^{\lambda t}[I+(A-\lambda I)t]$

$T=-2,\ D=1$

$p(\lambda)=\lambda^2+2\lambda+1$

$\lambda=-1$

$e^{At}=e^{-t}[I+(A+I)t]$

$=e^{-t}\left[\begin{bmatrix}1&0\\0&1\end{bmatrix}+\left(\begin{bmatrix}-2&1\\-1&0\end{bmatrix}+\begin{bmatrix}1&0\\0&1\end{bmatrix}\right)t\right]$

$=e^{-t}\left[\begin{bmatrix}1&0\\0&1\end{bmatrix}+\begin{bmatrix}-1&1\\-1&1\end{bmatrix}t\right]$

$\boxed{=e^{-t}\begin{bmatrix}1-t & t\\ -t & 1+t\end{bmatrix}}$

16) $A=\begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix}$

$T=-2,\ D=1$

$p(\lambda)=\lambda^2+2\lambda+1,\ \lambda=-1$

$e^{At}=e^{-t}\left[\begin{bmatrix}1&0\\0&1\end{bmatrix}+\left(\begin{bmatrix}0&1\\-1&-2\end{bmatrix}+\begin{bmatrix}1&0\\0&1\end{bmatrix}\right)t\right]$

$e^{At}=e^{-t}\left[\begin{bmatrix}1&0\\0&1\end{bmatrix}+\begin{bmatrix}1&1\\-1&-1\end{bmatrix}t\right]$

$=\boxed{e^{-t}\begin{bmatrix}1+t & t\\ -t & 1-t\end{bmatrix}}$

18) $A=\begin{bmatrix}-1&0&0\\-1&1&-1\\-2&4&-3\end{bmatrix}\begin{matrix}-1&0\\-1&1\\-2&4\end{matrix}$

$e^{tA}=e^{\lambda t}\left[I+t(A-\lambda I)+\frac{t^2}{2!}(A-\lambda I)^2\ldots\right]$

$T=-3,\ D=-1$

$(-1-\lambda)(1-\lambda)(-3-\lambda)-4(1-\lambda)$

$(-1+\lambda^2)(-3-\lambda)-4+4\lambda$

$(3-3\lambda^2+\lambda-\lambda^3)-4+4\lambda$

$-\lambda^3+5\lambda-3\lambda^2-1$

$\lambda^3+3\lambda^2-5\lambda-1=0$

$(\lambda+1)^3=0$

$\lambda=-1$

$A+I=\begin{bmatrix}0&0&0\\-1&2&-1\\-2&4&-2\end{bmatrix}$

$(A+I)^2=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$

$\boxed{k=2}$

$e^{tA}=e^{-t}\left[I+t\begin{bmatrix}0&0&0\\-1&2&-1\\-2&4&-2\end{bmatrix}\right]$

$=\boxed{e^{-t}\begin{bmatrix}1-t&0&0\\-1&1+2t&-1\\-2&4&1-2t\end{bmatrix}}$

20) $A=\begin{bmatrix}-2&-1&0\\0&0&1\\0&-4&-4\end{bmatrix}$

$(-2-\lambda)(-\lambda)(-4-\lambda)+(-2-\lambda)(+4)$

$(2\lambda+\lambda^2)(-4-\lambda)+(-8-4\lambda)$

$-8-4\lambda^2-2\lambda-\lambda^3-8-4\lambda$

$\lambda^3+4\lambda^2+6\lambda+16$

$(\lambda+2)^3=0$

$\lambda=-2$

$A+2I=\begin{bmatrix}0&-1&0\\0&2&1\\0&-4&-2\end{bmatrix}$

$(A+2I)^2=\begin{bmatrix}0&-2&-1\\0&0&0\\0&0&0\end{bmatrix}$

$\boxed{k=3}$

$e^{tA}=e^{-2t}\left[I+t(A+2I)+\frac{1}{2}t^2(A+2I)^2\right]$

$=e^{-2t}\left[\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}+t\begin{bmatrix}0&-1&0\\0&2&1\\0&-4&-2\end{bmatrix}+t^2\begin{bmatrix}0&-1&-1/2\\0&0&0\\0&0&0\end{bmatrix}\right]$

$=\boxed{e^{-2t}\begin{bmatrix}1&-t^2-t&-\frac{1}{2}t^2\\0&2t+1&t\\0&-4t&-2t+1\end{bmatrix}}$

22) $A=\begin{bmatrix}1&-1&2&0\\0&1&0&0\\0&0&1&0\\0&-1&2&1\end{bmatrix}$

$(\lambda-1)^4,\ \lambda=1$

$A-I=\begin{bmatrix}0&-1&2&0\\0&0&0&0\\0&0&0&0\\0&-1&2&0\end{bmatrix}$

$(A-I)^2=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$

$k=2$

$e^{tA}=e^t[I+t(A-I)]$

$=e^t\left[\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}+t\begin{bmatrix}0&-1&2&0\\0&0&0&0\\0&0&0&0\\0&-1&2&0\end{bmatrix}\right]$

$=\boxed{e^t\begin{bmatrix}1&-t&2t&0\\0&1&0&0\\0&0&1&0\\0&-t&2t&1\end{bmatrix}}$

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26) $A=\begin{bmatrix}-2&1&-1\\1&-3&0\\3&-5&0\end{bmatrix}\begin{matrix}-2&1\\1&-3\\3&-5\end{matrix}$

$(-2-\lambda)(-3-\lambda)(-\lambda)+(-3-\lambda)3$

$6+5\lambda+\lambda^2(-\lambda)-3\lambda-9$

$-\lambda^3-5\lambda^2-6\lambda-3\lambda-9$

$-\lambda^3-5\lambda^2-9\lambda-9$

$p(\lambda)=(\lambda+1)(\lambda+2)^2$

$\lambda_1=-1,\ \lambda_2=-2$

$\text{almu}(-1)=1,\ \text{almu}(-2)=2$

$\begin{bmatrix}-1&1&-1\\1&-2&0\\3&-5&-1\end{bmatrix}\xrightarrow{\text{rref}}\begin{bmatrix}1&0&2\\0&1&1\\0&0&0\end{bmatrix}$

$\text{gemu}(-1)=1$

$\begin{bmatrix}0&1&-1\\1&-1&0\\3&-5&2\end{bmatrix}\xrightarrow{\text{rref}}\begin{bmatrix}1&0&-1\\0&1&-1\\0&0&0\end{bmatrix}$

$\text{gemu}(-2)=1$

$\lambda_1\to k=1,\ \lambda_2\to k=2$

$\vec v_3=\begin{bmatrix}-1\\0\\1\end{bmatrix}$

$y_3(t)=e^{-2t}\begin{bmatrix}-1-t\\-t\\1-t\end{bmatrix}$

$\boxed{y(t)=e^{-t}\begin{bmatrix}-2\\-1\\1\end{bmatrix}+e^{-2t}\begin{bmatrix}1\\1\\1\end{bmatrix}+e^{-2t}\begin{bmatrix}-1-t\\-t\\1-t\end{bmatrix}}$

28) $A=\begin{bmatrix}-&1&0\\-4&4&0\\-2&0&1\end{bmatrix}$

$(\lambda-1)(\lambda^2-4\lambda+4)$

$(\lambda-1)(\lambda-2)^2$

$\lambda_1=1,\ \lambda_2=2$

$\text{almu}(1)=1,\ \text{almu}(2)=2$

$\begin{bmatrix}-1&1&0\\-4&3&0\\-2&0&0\end{bmatrix}\xrightarrow{\text{rref}}\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}$

$\text{gemu}(1)=1$

$\begin{bmatrix}-2&1&0\\-4&2&0\\-2&0&-1\end{bmatrix}\xrightarrow{\text{rref}}\begin{bmatrix}1&0&1/2\\0&1&1\\0&0&0\end{bmatrix}$

$\text{gemu}(2)=1$

$1\to k=1,\ 2\to k=2$

$\vec v_1=\begin{bmatrix}0\\0\\1\end{bmatrix}$

$y_3(t)=e^{2t}\begin{bmatrix}1-2t\\-4t\\-6+4t\end{bmatrix}$

$\boxed{y(t)=e^{t}\begin{bmatrix}0\\0\\1\end{bmatrix}+e^{2t}\begin{bmatrix}1\\2\\-2\end{bmatrix}+e^{2t}\begin{bmatrix}1-2t\\-4t\\-6+4t\end{bmatrix}}$

30) $A=\begin{bmatrix}11&-42&4&28\\-12&39&-4&-28\\0&0&-1&0\\-24&81&-8&-57\end{bmatrix}$

$p(\lambda)=(\lambda+3)^2(\lambda+1)^2$

$\text{almu}(-3)=2,\ \text{almu}(-1)=2$

$\begin{bmatrix}14&-42&4&28\\-12&42&-4&-28\\0&0&2&0\\-24&81&-8&-54\end{bmatrix}\xrightarrow{\text{rref}}\begin{bmatrix}1&0&0&0\\0&1&0&-2/3\\0&0&1&0\\0&0&0&0\end{bmatrix}$

$\text{gemu}(-3)=1$

$\begin{bmatrix}12&-42&4&28\\-12&40&-4&-28\\0&0&0&0\\-24&81&-8&-56\end{bmatrix}\xrightarrow{\text{rref}}\begin{bmatrix}1&0&1/3&7/3\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$

$\text{gemu}(-1)=2$

$k=2,\ k=1$

$\boxed{y(t)=e^{-3t}\begin{bmatrix}0\\-2\\0\\-3\end{bmatrix}+e^{-3t}\begin{bmatrix}-\\0\\0\\1\end{bmatrix}+e^{-t}\begin{bmatrix}-1\\0\\3\\0\end{bmatrix}+e^{-t}\begin{bmatrix}-7\\0\\0\\3\end{bmatrix}}$

36) $y'=\begin{bmatrix}8&3&2\\0&4&0\\-8&-6&0\end{bmatrix}y$

$\det(A)=-(\lambda-4)(\lambda^2-8\lambda+16)$

$p(\lambda)=(\lambda-4)^3$

$\begin{bmatrix}4&3&2\\0&0&0\\-8&-6&-4\end{bmatrix}\xrightarrow{\text{rref}}\begin{bmatrix}1&3/4&1/2\\0&0&0\\0&0&0\end{bmatrix}$

$\vec v_1=\begin{bmatrix}-3\\4\\0\end{bmatrix},\ \vec v_2=\begin{bmatrix}1\\0\\-2\end{bmatrix}$

$\vec v_3=\begin{bmatrix}1\\0\\0\end{bmatrix}$

$\boxed{y(t)=e^{4t}\left(\begin{bmatrix}-3\\4\\0\end{bmatrix}+\begin{bmatrix}1\\0\\-2\end{bmatrix}+\begin{bmatrix}1\\0\\0\end{bmatrix}\right)}$

38) $x' = -x + 5y + 3z$

$y' = y + z$

$z' = -2y - 2z$

$$\begin{bmatrix} -1 & 5 & 3 \\ 0 & 1 & 1 \\ 0 & -2 & -2 \end{bmatrix}$$

$p(\lambda) = (\lambda+1)(\lambda^2+\lambda)$

$= \lambda(\lambda+1)^2$

$\lambda_1 = 0, \lambda_2 = -1$

$$\begin{bmatrix} -1 & -5 & 3 \\ 0 & 1 & 1 \\ 0 & -2 & -2 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 5 & 3 \\ 0 & 2 & 1 \\ 0 & -2 & -1 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{(A+I)^2 = \begin{bmatrix} 0 & 4 & 2 \\ 0 & 2 & 1 \\ 0 & -2 & -1 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 0 & 1 & 1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}$$

$$\boxed{y(t) = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} + e^{-t}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + e^{-t}\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}}$$

40) $$A = \begin{bmatrix} -12 & -1 & 8 & 16 \\ -8 & 0 & -1 & 9 \\ 0 & 0 & 5 & 0 \\ -17 & -1 & 8 & 15 \end{bmatrix}$$

$p(\lambda) = (\lambda+1)^2(\lambda-5)^2$

$\lambda_1 = -1, \lambda_2 = 5$

$$\begin{bmatrix} -11 & -1 & 8 & 16 \\ -8 & 1 & -1 & 9 \\ 0 & 0 & 6 & 0 \\ -17 & -1 & 8 & 16 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -17 & -1 & 8 & 16 \\ -8 & -5 & -1 & 9 \\ 0 & 0 & 0 & 0 \\ -17 & -1 & 8 & 16 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -41/77 & -47/77 \\ 0 & 1 & 81/77 & -71/77 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{y(t) = e^{-t}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} + e^{-t}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} + e^{5t}\begin{bmatrix} 41 \\ -81 \\ 77 \\ 0 \end{bmatrix} + e^{5t}\begin{bmatrix} 47 \\ 71 \\ 0 \\ 77 \end{bmatrix}}$$

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