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Disc. 1D

Math 33B HW #4

Chapter 4.1

14) $y'' + 4y = 0,\ y_1(t) = \cos(2t),\ y_2(t) = \sin(2t)$

$y_1'' + 4y_1 = 0$

$-4\cos(2t) + 4\cos(2t) = 0$ ✓

$y_2'' + 4y_2 = 0$

$-4\sin(2t) + 4\sin(2t) = 0$ ✓

$y(t) = C_1 y_1(t) + C_2 y_2(t)$

$= C_1\cos(2t) + C_2\sin(2t)$

$y''(t) + 4y(t) = 0$

$-4C_1\cos(2t) - 4C_2\sin(2t) + 4C_1\cos(2t) + 4C_2\sin(2t) = 0$

$0 = 0$ ✓

16) $y'' + 4y' + 4y = 0,\ y_1(t) = e^{-2t},\ y_2(t) = te^{-2t}$

$4e^{-2t} + (-8)e^{-2t} + 4e^{-2t} = 0$

$8e^{-2t} - 8e^{-2t} = 0$ ✓

$y_2' = e^{-2t} - 2te^{-2t}$

$y_2'' = -2e^{-2t} - 2e^{-2t} + 4te^{-2t}$

$= -4e^{-2t} + 4te^{-2t}$

$-4e^{-2t} + 4te^{-2t} + 4e^{-2t} - 8te^{-2t} + 4te^{-2t} = 0$

$0 = 0$ ✓

$y(t) = C_1e^{-2t} + C_2te^{-2t}$

$y'(t) = -2C_1e^{-2t} + C_2e^{-2t} - 2C_2te^{-2t}$

$y''(t) = 4C_1e^{-2t} - 2C_2e^{-2t} - 2C_2e^{-2t} + 4C_2te^{-2t}$

$4C_1e^{-2t} - 2C_2e^{-2t} - 2C_2e^{-2t} + 4C_2te^{-2t} - 8C_1e^{-2t}$

$+ 4C_2e^{-2t} - 8C_2te^{-2t} + 4C_1e^{-2t} + 4C_2te^{-2t} = 0$

$0 = 0$ ✓

18) $y'' + 9y = 0,\ y_1(t) = \cos(3t),\ y_2(t) = \sin(3t)$

$Cy_1(t) = y_2(t)$ — Def. 1.22

$C(\cos(3t)) = \sin(3t)$

$C = \frac{\sin(3t)}{\cos(3t)} = \tan(3t)$

$\tan(3t)$ is not constant

$W(t) = y_1(t)y_2'(t) - y_1'(t)y_2(t)$

$= 3\cos^2(t) + 3\sin^2(t) = 3$

$\boxed{3 \neq 0}$

20) $y'' + 6y' + 9y = 0,\ y_1(t) = e^{-3t},\ y_2(t) = te^{-3t}$

$\frac{y_1(t)}{y_2(t)} = \frac{1}{t}$

$1/t$ is not a constant → lin. independent

$W(t) = y_1(t)y_2'(t) - y_1'(t)y_2(t)$

$= e^{-3t}(e^{-3t} - 3te^{-3t}) + 3e^{-3t}(te^{-3t})$

$= e^{-6t} - 3te^{-6t} + 3te^{-6t}$

$= \boxed{e^{-6t} \neq 0}$

24) $y'' + 2y' + 5y = 0,\ y_1(t) = e^{-t}\cos(2t)$

$y_2(t) = e^{-t}\sin(2t)$

$\frac{y_1(t)}{y_2(t)} = \frac{\cos(2t)}{\sin(2t)} = \cot(2t)$

$\cot(2t)$ is not constant

y_1 and y_2 are a fundamental solution

$y(t) = C_1e^{-t}\cos(2t) + C_2e^{-t}\sin(2t)$

$y'(t) = C_1(-e^{-t}\sin(2t) - e^{-t}\cos(2t)) +$

$C_2(-e^{-t}\sin(2t) + e^{-t}\cos(2t))$

$y(0) = -1$

$-1 = C_1e^0 = C_1$

$C_1 = -1$

$0 = C_1 + 2C_2$

$C_2 = -1/2$

$\boxed{y(t) = -e^{-t}\cos(2t) - \frac{1}{2}e^{-t}\sin(2t)}$

Chapter 4.3

4) $y'' + y' - 12y = 0$

$y = e^{rt},\ y' = re^{rt},\ y'' = r^2e^{rt}$

$r^2e^{rt} + re^{rt} - 12e^{rt} = 0$

$e^{rt}(r^2 + r - 12) = 0$

$r^2 + r - 12 = 0$

$(r+4)(r-3) = 0$

$r = -4, 3$

$y_1(t) = e^{-4t},\ y_2(t) = e^{3t}$

$\boxed{y(t) = C_1e^{-4t} + C_2e^{3t}}$

8) $6y'' + 5y' - 6y = 0$

$6\lambda^2 e^{\lambda t} + 5\lambda e^{\lambda t} - 6e^{\lambda t} = 0$

$e^{\lambda t}(6\lambda^2 + 5\lambda - 6) = 0$

$6\lambda^2 + 5\lambda - 6 = 0$

$\lambda = \frac{-5 \pm \sqrt{25+144}}{12} = \frac{-5 \pm \sqrt{169}}{12}$

$\lambda = \frac{-18}{12}, \frac{8}{12}$

$(\lambda + \frac{3}{2})(\lambda - \frac{2}{3}) = 0$

$\lambda_1 \neq \lambda_2$

$y_1(t) = e^{-3/2 t}, \quad y_2(t) = e^{2/3 t}$

$\boxed{y(t) = C_1 e^{-3/2 t} + C_2 e^{2/3 t}}$

12) $y'' + 2y' + 17y = 0$

$\lambda^2 e^{\lambda t} + 2\lambda e^{\lambda t} + 17 e^{\lambda t} = 0$

$\lambda^2 + 2\lambda + 17 = 0$

$\lambda = \frac{-2 \pm \sqrt{4-68}}{2} = \frac{-2 \pm 8i}{2} = -1 \pm 4i$

$z(t) = e^{(-1+4i)t}, \quad \bar{z}(t) = e^{(-1-4i)t}$

$= e^{-t}(\cos 4t + i \sin 4t)$

$\boxed{y(t) = C_1 e^{-t} \cos 4t + C_2 e^{-t} \sin 4t}$

16) $y'' + 2y' + 2y = 0$

$\lambda^2 + 2\lambda + 2 = 0$

$\lambda = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm 2i}{2}$

$\lambda = -1 \pm i$

$z = e^{(-1+i)t}, \quad \bar{z} = e^{(-1-i)t}$

$\boxed{y(t) = C_1 e^{-t} \cos t + C_2 e^{-t} \sin t}$

20) $4y'' + 12y' + 9y = 0$

$4\lambda^2 + 12\lambda + 9 = 0$

$(2\lambda + 3)^2 = 0$

$\lambda = -3/2$

$y_1(t) = e^{-3/2 t}, \quad y_2(t) = t e^{-3/2 t}$

$\boxed{y(t) = C_1 e^{-3/2 t} + C_2 t e^{-3/2 t}}$

24) $y'' + 8y' + 16y = 0$

$\lambda^2 + 8\lambda + 16 = 0$

$(\lambda + 4)^2 = 0$

$\lambda = -4$

$y_1(t) = e^{-4t}, \quad y_2(t) = t e^{-4t}$

$\boxed{y(t) = C_1 e^{-4t} + C_2 t e^{-4t}}$