Question 1

Not yet answered

Marked out of 1.00

Suppose that $X_1, X_2, \ldots., X_n$ is a set of independent and identically distributed random variables and that

$$f(X_i) \sim Poisson(\lambda), \qquad i = 1, \ldots n$$

Consider

$$\bar{X} = \frac{\sum_{i-1}^{n} X_i}{n}$$

We define

$$Q = \sigma^2_{\bar{X}} + [E(\bar{X}) - \lambda]^2$$

The Q equals

Select one:

- a. λ
- b. λ/n
- c. 2n/λ
- d. nλ

Question **2**

Not yet answered

Marked out of 1.00

Consider the number of months since a patient had the last medical examination. This is a random variable that varies across patients. At a given point in time, this distribution can be assumed to be uniform between 4 and 20 months. Consider 150 patients randomly chosen. What is the probability that the average number of months since the last examination in this random sample is 14 or larger?

- [ ] a. 0.3325 approximately
- [ ] b. approximately 1
- [ ] c. 0.5
- [x] d. approximately 0

Question **3**

Not yet answered

Marked out of 1.00

Chapter 9, mini quiz question 2.

The distribution of income (in tens of thousands) of females in a small population can be modeled by a gamma distribution with mean 4 and standard deviation 8 dollars. A simulation is done, where each trial consists of drawing a random sample of 500 women from this population and computing the average salary of the women in this population. 1000 trials are done. Which of the following statements is true?

To answer this question, you must use the parameterization that we use in this class for the Gamma (there are different parameterizations out there in the internet, do not use those other ones). See the document posted at the bottom of the lectures folder on the Gamma. You also may want to review Dr. Sanchez's video on how to use the Rossman/Chance app on finding the distribution of the sample mean. See what we did there, which was a simulation similar to the one in this question.

Select one:

- a. The distribution of the sample obtained in one trial should be close to normal
- b. The distribution of the 1000 averages is close to normal.
- c. If we plotted the distribution of the sample obtained in each of the 1000 trials, we would have 1000 distributions that look like the normal.
- d. The mean of the 500 random variables in each sample is close to 2000.
- e. The distribution of the 1000 averages is exactly gamma.

Question **4**

Not yet answered

Marked out of 5.00

In one of the multiple choice questions we ask:

Suppose that $X_1, X_2, \ldots., X_n$ is a set of independent and identically distributed random variables and that

$$f(X_i) \sim Poisson(\lambda), \qquad i = 1, \ldots n$$

Consider

$$\bar{X} = \frac{\sum_{i-1}^{n} X_i}{n}$$

We define

$$Q = \sigma^2_{\bar{X}} + [E(\bar{X}) - \lambda]^2$$

The Q equals ?

Show work indicating how you obtained the answer to the multiple choice question.

Work must be detailed and show everything you had to compute to justify the final answer.
You may attach a pdf file but the file must contain only the answer to this question or it will not be graded.

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Question **5**

Not yet answered

Marked out of 5.00

Based on example 9.5.8

Approximately 16.2% of Americans purchase private individual health plans in the United States. We take a random sample of 200 Americans with the intention of finding the probability that there will be more than 14 percent with private individual health plan. To solve this question we need to first specify intermediate work. Lets start by defining the random variable Y,

Y=number of Americans in a random sample of 200 with private individual health plan

$P(\frac{Y}{n} > 0.14)$	Choose...	0.8
Var(Y)	Choose...	27.15
$E(\frac{Y}{n})$	Choose...	0.162
$Var(\frac{Y}{n})$	Choose...	0.00067878
E(Y)	Choose...	32.4

Question **6**

Not yet answered

Marked out of 1.00

Consider an 8-hour work day. Alice and Bob each check their email just once per day. Let X and Y denote the time, respectively, until Alice and Bob each check their emails during the day. Assume that Alice always checks her email first, and that X and Y are uniform on the region where 0 < X<Y < 8. The joint density is

$$f(x, y) = 1/32 \qquad 0 < x < y < 8$$

The expected value and variance of Y-X, which is the time interval between Alice and Bob checking their email are, respectively,

Select one:

- a. 13, 34
- b. 19/3, -30/3
- c. 24/3, 16/3
- d. 8/3, 32/9

Question 7

Not yet answered

Marked out of 3.00

Waiting times at a service counter in a pharmacy are exponentially distributed, with expected value 10 minutes. If 100 customers come to the service counter in a day,

The standard deviation of the average waiting time of 100 customers is	Choose...	1
The probability that the waiting time of 100 customers is less than 7 minutes is	Choose...	0
The expected average waiting time of all customers is	Choose...	10

Question **8**

Not yet answered

Marked out of 4.00

Consider the experiment of drawing a random sample of 100 bags of fertilizer to see whether a factory producing them keeps good quality control. The bags specification says that the net wait of a bag is 50 pounds. Of course, we all know that the net weight put by factories in products is the average weight. A little departure from the average here and there is ok, but if the probability that there is a lot of departure from that weight is high, quality control engineers know that that is a bad sign. Something is wrong with the production process in that case.

Let X denote the actual net weight of a bag of fertilizer. The average (expected) weight of a bag of fertilizer is 50 pounds and the variance is 1.

The $E(\bar{X})$ is	Choose...	50
The probability that the average weight is between 49.75 and 50.25 is approximately equal to	Choose...	0.9876
50.25 pounds is how many standard deviations above the expected value for $\bar{X}$?	Choose...	2.5
The $\sigma_{\bar{X}}$ equals	Choose...	0.1

Question **9**

Not yet answered

Marked out of 5.00

In one of the multiple choice questions we asked:

Let

$$X_1, X_2, \ldots . X_n$$

be a set of i.i.d random variables, where

$$f(x_i) = \frac{1}{\theta}, \quad 0 < x < \theta$$

Let

$$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$$

And let's define

$$Q = \sigma_{\bar{X}}^2 + [E(\bar{X}) - \theta]^2$$

The Q equals?

Show the work you did to obtain the answer to this question. Show all steps of the calculation in detail.

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Question **10**

Not yet answered

Marked out of 5.00

In one of the multiple choice questions you are asked:

Consider an 8-hour work day. Alice and Bob each check their email just once per day. Let X and Y denote the time, respectively, until Alice and Bob each check their emails during the day. Assume that Alice always checks her email first, and that X and Y are uniform on the region where 0 < X<Y < 8. The joint density is

$$f(x, y) = 1/32 \qquad 0 < x < y < 8$$

The expected value and variance of Y-X, which is the time interval between Alice and Bob checking their email are, respectively?

Show work that led to the answer you gave in the multiple choice question.
Work must be detailed and show everything you had to compute to justify the final answer.
You may attach a pdf file but the file must contain only the answer to this question or it will not be graded.

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Question **11**

Not yet answered

Marked out of 2.00

Chapter 9, end of chapter.

Let X be a continuous random variable with the following density function:

$$f(x) = \frac{1}{2}x, \qquad 0 \leq x \leq 2$$

Let $X_1, X_2, \ldots, X_{100}$ be 100 independent and identically distributed random variables each with the that density function.

$E\left(\frac{1}{4}\sum_{i=1}^{100} X_i\right)$ Choose... 100/3

$Var\left(\frac{1}{4}\sum_{i=1}^{100} X_i\right)$ Choose... 200/144

Q4

$x_1, x_2, \ldots, x_n \rightarrow IID$

$f(x_i) \sim \text{Poisson}(\lambda),\ i=1,\ldots,n \rightarrow E(x_i) = Var(x_i) = \lambda$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{S_n}{n}$$

$$Q = \sigma^2_{\bar{x}} + [E(\bar{x}) - \lambda]^2$$

By CLT:

$$E(\bar{x}_n) = \frac{E(S_n)}{n} = \frac{n\mu}{n} = \mu$$

$$Q = \sigma^2_{\bar{x}} + [\mu - \lambda]^2$$

By the Poisson distribution:

$$\mu = \lambda$$

$$Q = \sigma^2_{\bar{x}} + [\lambda - \lambda]^2$$

$$Q = \sigma^2_{\bar{x}} = Var(\bar{x})$$

By CLT:

$$Var(\bar{x}) = \frac{Var(S_n)}{n^2} = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

By the Poisson distribution:

$$\sigma^2 = \lambda$$

$$\boxed{Q = \frac{\lambda}{n}}$$

Q7

$X_1, X_2, \ldots, X_n \Rightarrow IID$

$f(x_i) = \frac{1}{\theta}, \quad 0 < x < \theta$

$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{S_n}{n}$

$Q = \sigma_{\bar{x}}^2 + [E(\bar{x}) - \theta]^2$

By CLT:

$E(\bar{x}) = \frac{E(S_n)}{n} = \frac{n\mu}{n} = \mu$

$Var(\bar{x}) = \frac{Var(S_n)}{n^2} = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$

Find the expectation:

$\mu = \int_x x f(x) dx = \int_0^\theta \frac{x}{\theta} dx = \frac{1}{\theta}\left[\frac{1}{2}x^2\right]_0^\theta = \frac{\theta}{2}$

Find the variance:

$\sigma^2 = E(x^2) - E(x)^2$

$E(x^2) = \int_x x^2 f(x) dx = \int_0^\theta \frac{x^2}{\theta} dx = \frac{1}{\theta}\left[\frac{1}{3}x^3\right]_0^\theta = \frac{\theta^2}{3}$

$\sigma^2 = \frac{\theta^2}{3} - \left(\frac{\theta}{2}\right)^2 = \frac{\theta^2}{3} - \frac{\theta^2}{4} = \frac{\theta^2}{12}$

Plug in:

$Q = \frac{\sigma^2}{n} + [\mu - \theta]^2 = \frac{\theta^2}{12n} + \left[\frac{\theta}{2} - \theta\right]^2 = \frac{\theta^2}{12n} + \frac{\theta^2}{4} = \frac{\theta^2 + 3n\theta^2}{12n} = \boxed{\frac{(3n+1)\theta^2}{12n}}$

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Q10

$f(x,y) = \frac{1}{32}$, $0 < x < y < 8$

↳ Find $E(Y-X)$ and $Var(Y-X)$

By LOTUS:

$E(Y-X) = E(Y) - E(X)$ → Linearization of expectation

$Var(Y-X) = Var(X) + Var(Y) - 2Cov(X,Y)$

Calculate marginal distributions:

$f(x) = \int_y f(x,y)\,dy = \int_x^8 \frac{1}{32}\,dy = \left[\frac{1}{32}y\right]_x^8 = \frac{8}{32} - \frac{x}{32} = \frac{8-x}{32}$

$f(y) = \int_x f(x,y)\,dx = \int_0^y \frac{1}{32}\,dx = \left[\frac{1}{32}x\right]_0^y = \frac{1}{32}y$

Calculate expectations:

$E(X) = \int_x x f(x)\,dx = \int_0^8 x\,\frac{8-x}{32}\,dx = \frac{1}{32}\int_0^8 8x - x^2 = \frac{1}{32}\left[-\frac{(x-12)x^2}{3}\right]_0^8$

$= \frac{1}{32}\left[\frac{256}{3}\right] = \frac{8}{3}$

$E(Y) = \int_y y f(y)\,dy = \int_0^8 \frac{1}{32}y^2\,dy = \frac{1}{32}\left[\frac{y^3}{3}\right]_0^8 = \frac{1}{32}\left[\frac{512}{3}\right] = \frac{16}{3}$

$E(Y-X) = \frac{16}{3} - \frac{8}{3} = \boxed{\frac{8}{3}}$

$E(XY) = \int_{x_1}^{x_2}\int_{y_1}^{y_2} f(x,y)(xy)\,dy\,dx = \int_0^8\int_x^8 xy\left(\frac{1}{32}\right)dy\,dx$

$= \frac{1}{32}\int_0^8 \left[\frac{xy^2}{2}\right]_x^8 dx = \frac{-1}{32}\int_0^8 \frac{x^3 - 64x}{2}\,dx = -\frac{1}{64}\left[\frac{x^4}{4} - 32x^2\right]_0^8 = 16$

Calculate covariance:

$Cov(X,Y) = E(XY) - E(X)E(Y) = 16 - \left(\frac{8}{3}\right)\left(\frac{16}{3}\right) = \frac{16}{9}$

Calculate variances:

$Var(X) = E(X^2) - E(X)^2$

$E(X^2) = \int_x x^2 f(x)\,dx = \int_0^8 x^2\left(\frac{8-x}{32}\right)dx = -\frac{1}{32}\left[\frac{3x^4 - 32x^3}{12}\right]_0^8 = \frac{32}{3}$

$Var(X) = \frac{32}{3} - \left(\frac{8}{3}\right)^2 = \frac{32}{9}$

$E(Y^2) = \int_y y^2 f(y)\,dx = \int_0^8 y^2\left(\frac{1}{32}y\right)dy = \frac{1}{32}\left[\frac{1}{4}y^4\right]_0^8 = 32$

$Var(Y) = 32 - \left(\frac{16}{3}\right)^2 = \frac{32}{9}$

$Var(Y-X) = \frac{32}{9} + \cancel{\frac{32}{9}} - \cancel{2\left(\frac{16}{9}\right)} = \boxed{\frac{32}{9}}$