EXERCISES 1.2

GOAL *Use Gauss–Jordan elimination to solve linear systems. Do simple problems using paper and pencil, and use technology to solve more complicated problems.*

In Exercises 1 through 12, find all solutions of the equations with paper and pencil using Gauss–Jordan elimination. Show all your work.

1. $\left|\begin{array}{l} x + y - 2z = 5 \\ 2x + 3y + 4z = 2 \end{array}\right|$

2. $\left|\begin{array}{l} 3x + 4y - z = 8 \\ 6x + 8y - 2z = 3 \end{array}\right|$

3. $x + 2y + 3z = 4$

4. $\left|\begin{array}{l} x + y = 1 \\ 2x - y = 5 \\ 3x + 4y = 2 \end{array}\right|$

5. $\left|\begin{array}{rl} x_3 + x_4 &= 0 \\ x_2 + x_3 &= 0 \\ x_1 + x_2 &= 0 \\ x_1 + x_4 &= 0 \end{array}\right|$

6. $\left|\begin{array}{rl} x_1 - 7x_2 + x_5 &= 3 \\ x_3 - 2x_5 &= 2 \\ x_4 + x_5 &= 1 \end{array}\right|$

7. $\left|\begin{array}{rl} x_1 + 2x_2 \quad 2x_4 + 3x_5 &= 0 \\ x_3 + 3x_4 + 2x_5 &= 0 \\ x_3 + 4x_4 - x_5 &= 0 \\ x_5 &= 0 \end{array}\right|$

8. Solve this system for the variables x_1, x_2, x_3, x_4, and x_5.

$$\left|\begin{array}{rl} x_2 + 2x_4 + 3x_5 &= 0 \\ 4x_4 + 8x_5 &= 0 \end{array}\right|$$

9. $\left|\begin{array}{rl} x_4 + 2x_5 - x_6 &= 2 \\ x_1 + 2x_2 \quad + x_5 - x_6 &= 0 \\ x_1 + 2x_2 + 2x_3 \quad - x_5 + x_6 &= 2 \end{array}\right|$

10. $\left|\begin{array}{rl} 4x_1 + 3x_2 + 2x_3 - x_4 &= 4 \\ 5x_1 + 4x_2 + 3x_3 - x_4 &= 4 \\ -2x_1 - 2x_2 - x_3 + 2x_4 &= -3 \\ 11x_1 + 6x_2 + 4x_3 + x_4 &= 11 \end{array}\right|$

11. $\left|\begin{array}{rl} x_1 + 2x_3 + 4x_4 &= -8 \\ x_2 - 3x_3 - x_4 &= 6 \\ 3x_1 + 4x_2 - 6x_3 + 8x_4 &= 0 \\ - x_2 + 3x_3 + 4x_4 &= -12 \end{array}\right|$

25. Suppose you apply Gauss–Jordan elimination to a matrix. Explain how you can be sure that the resulting matrix is in reduced row-echelon form.

26. Suppose matrix A is transformed into matrix B by means of an elementary row operation. Is there an elementary row operation that transforms B into A? Explain.

27. Suppose matrix A is transformed into matrix B by a sequence of elementary row operations. Is there a sequence of elementary row operations that transforms B into A? Explain your answer. See Exercise 26.

28. Consider an $n \times m$ matrix A. Can you transform $\operatorname{rref}(A)$ into A by a sequence of elementary row operations? See Exercise 27.

29. Is there a sequence of elementary row operations that transforms

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \text{into} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}?$$

Explain.

EXERCISES 1.3

GOAL *Use the reduced row-echelon form of the augmented matrix to find the number of solutions of a linear system. Apply the definition of the rank of a matrix. Compute the product $A\vec{x}$ in terms of the rows or the columns of A. Represent a linear system in vector or matrix form.*

1. The reduced row-echelon forms of the augmented matrices of three systems are given here. How many solutions does each system have?

a. $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$ **b.** $\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 6 \end{array}\right]$

c. $\left[\begin{array}{ccc|c} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

Find the rank of the matrices in Exercises 2 through 4.

2. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ **3.** $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ **4.** $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$