

Math 33B HW#1

Chapter 2.1

8) a) $t^2 + y^2 = C^2$
 $\frac{d}{dt}(t^2 + y^2) = 0$
 $2t + 2y \frac{dy}{dt} = 0$
 $t + y \frac{dy}{dt} = 0$
 $t + y y' = 0 \checkmark$

b) $t^2 + y^2 = C^2$
 $y^2 = C^2 - t^2$
 $y = \pm \sqrt{C^2 - t^2}$
 $y' = \mp \frac{2t}{2\sqrt{C^2 - t^2}}$
 $y' = \mp \frac{t}{\sqrt{C^2 - t^2}}$
 $y = \sqrt{C^2 - t^2}, y' = -\frac{t}{\sqrt{C^2 - t^2}}$
 $y = -\sqrt{C^2 - t^2}, y' = \frac{t}{\sqrt{C^2 - t^2}}$
 $t + \sqrt{C^2 - t^2} \left(-\frac{t}{\sqrt{C^2 - t^2}}\right) = 0$
 $t - t = 0 \checkmark$

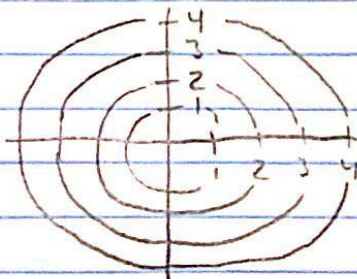
$t - \sqrt{C^2 - t^2} \left(\frac{t}{\sqrt{C^2 - t^2}}\right) = 0$
 $t - t = 0 \checkmark$

c) $\sqrt{C^2 - t^2}$ only when $t^2 \leq C^2$
 since $y' = \mp \frac{t}{\sqrt{C^2 - t^2}}$,
 $\sqrt{C^2 - t^2} \neq 0$
 $\therefore C \neq t$
 $t^2 < C^2$

$t < C, t > -C$

$-C < t < C$

d) circles with $r = C$



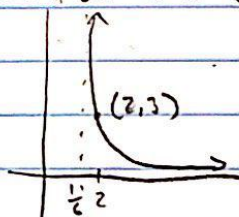
10) $y(t) = \frac{3}{6t-11}, y' = -2y^2, y(2) = 3$

$y' = -2y^2$
 $\frac{dy}{dt} = -2y^2$
 $\int \frac{dy}{y^2} = \int -2 dt$
 $-\frac{1}{y} = -2t + C$
 $\frac{1}{y} = 2t - C$
 $y = \frac{1}{2t - C}$

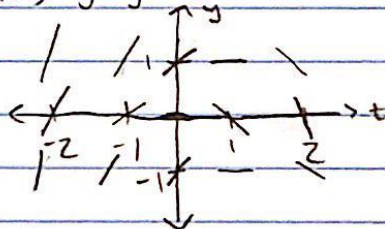
$y' = -2 \left(\frac{3}{6t-11}\right)^2 \checkmark$
 $y' = -2 \left(\frac{3}{6t-11}\right)^2 = \frac{-18}{(6t-11)^2} \checkmark$
 $6t-11 \neq 0$

$t \neq \frac{11}{6}$

\therefore interval is $(\frac{11}{6}, \infty) \rightarrow C'$ is negative



18) $y' = y^2 - t$



Chapter 2.2

12) $y' = (2xy + 2x)/(x^2 - 1)$
 $\frac{dy}{dx} = \frac{2x(y+1)}{(x^2-1)}$
 $\int \frac{dy}{y+1} = \int \frac{2x}{(x^2-1)} dx$
 $\ln|y+1| = \ln|x^2-1| + C$
 $|y+1| = e^{\ln|x^2-1| + C}$
 $|y+1| = C|x^2-1|$
 $y = C(x^2-1) + 1$

$$22) y' = (y^2 + 1)/y, y(1) = 2$$

$$\frac{dy}{dx} = \frac{(y^2 + 1)}{y}$$

$$\frac{1}{y^2 + 1} dy = dx$$

$$\frac{1}{2} \ln|y^2 + 1| = x + C$$

$$\ln|\sqrt{y^2 + 1}|$$

$$\ln\sqrt{y^2 + 1} = x + C$$

$$e^{\ln\sqrt{y^2 + 1}} = e^{x+C}$$

$$\sqrt{y^2 + 1} = Ce^x$$

$$y^2 + 1 = Ce^{2x}$$

$$y^2 = Ce^{2x} - 1$$

$$y = \pm \sqrt{Ce^{2x} - 1}$$

$$2 = \pm \sqrt{Ce^2 - 1}$$

$$Ce^2 - 1 = 4$$

$$C = \frac{5}{e^2}$$

$$y = \sqrt{5e^{2x-2} - 1}$$

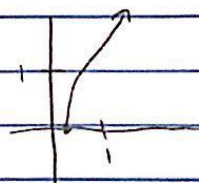
$$5e^{2x-2} - 1 > 0$$

$$e^{2x-2} > 1/5$$

$$2x - 2 > -\ln 5$$

$$x > \frac{2 - \ln 5}{2}$$

$$\frac{2 - \ln 5}{2} > x > \infty$$



$$36) x = at + by + c$$

$$x' = a + b(y')$$

$$= a + f(at + by + c)$$

$$= a + f(x) \checkmark$$

$$f(x) = x^2$$

$$x = y + t$$

$$x = a + by'$$

$$x' = a + f(x)$$

$$x' = 1 + x^2$$

$$\frac{dx}{1+x^2} = dt$$

$$\arctan x = t + C$$

$$x = \tan(t + C)$$

$$y = x - t$$

$$y = \tan(t + C) - t$$

Chapter 2.4

$$6) tx' = 4x + t^4$$

$$x' = \frac{4}{t}x + t^3 \quad t \neq 0$$

$$a(t) = \frac{4}{t}$$

$$IF = e^{\int -\frac{4}{t} dt}$$

$$= e^{-4 \ln t}$$

$$= t^{-4}$$

$$x' - \frac{4}{t}x = t^3$$

$$\int t^{-4} x' - 4t^{-5} x = \int t^{-1}$$

$$\int (t^{-4} x)' = \ln|t| + C$$

$$t^{-4} x = \ln|t| + C$$

$$x = t^4 \ln|t| + Ct^4$$

$$8) (1+x^3)y' = 3x^2y + x^2 + x^5$$

$$y' = (3x^2y + x^2 + x^5) / (1+x^3)$$

$$= x^2(3y + 1 + x^3) / (1+x^3)$$

$$y' = \frac{3x^2}{1+x^3}y + x^2$$

$$a(x) = \frac{3x^2}{1+x^3}$$

$$IF: e^{\int \frac{3x^2}{1+x^3} dx}$$

$$= e^{-\ln|1+x^3|}$$

$$= \frac{1}{1+x^3}$$

$$\frac{y}{1+x^3} - \frac{3x^2}{(1+x^3)^2}y = x^2$$

$$\left(\frac{y}{1+x^3}\right)' = \frac{x^2}{1+x^3} dx$$

$$\frac{y}{1+x^3} = \frac{1}{3} \ln|1+x^3| + C$$

$$y = \frac{1}{3}(1+x^3) \ln|1+x^3| + C(1+x^3)$$

$$12) x' - \left(\frac{n}{t}\right)x = e^t t^n$$

$$x' = \left(\frac{n}{t}\right)x + e^t t^n$$

$$a(t) = \left(\frac{n}{t}\right)$$

$$IF = e^{\int -n/t dt}$$

$$IF = e^{-n \ln t}$$

$$IF = t^{-n}$$

$$t^{-n} x' - \frac{n}{t^{n+1}} x = e^t t^n t^{-n}$$

$$(t^{-n} x)' = e^t$$

$$t^{-n} x = e^t + C$$

$$x = t^n e^t + C t^n$$

$$14) y' = y + 2x e^{2x}, y(0) = 3$$

$$y' - y = 0$$

$$y' = y$$

$$\ln y = x$$

$$y_h = e^x$$

$$y = u(x) y_h$$

$$(u(x) y_h)' = u(x) y_h + 2x e^{2x}$$

$$u'(x) y_h + u(x) y_h' = u(x) y_h + 2x e^{2x}$$

$$u'(x) y_h + u(x) (y_h' - y_h) = 2x e^{2x}$$

$$u'(x) y_h = 2x e^{2x}$$

$$u'(x) = \frac{2x e^{2x}}{y_h} = \frac{2x e^{2x}}{e^x}$$

$$u'(x) = 2x e^x$$

$$u(x) = \int 2x e^x$$

$$u = 2x, du = e^x$$

$$du = 2, u = e^x$$

$$= 2x e^x - \int 2e^x$$

$$u - u = 2x e^x - 2e^x + C$$

$$u \cdot y_h = 2x e^{2x} - 2e^{2x} + C e^x = y$$

$$3 = -2 + C$$

$$C = 5$$

$$y = 2x e^{2x} - 2e^{2x} + 5e^x$$

$$16) (1+t^2) y' + 4ty = (1+t^2)^{-2} y(1) = 0$$

$$(1+t^2) y_h' + 4ty_h = 0$$

$$y_h' = \frac{-4ty_h}{(1+t^2)}$$

$$\ln y_h = -2 \ln |1+t^2|$$

$$y_h = \frac{1}{(1+t^2)^2}$$

$$u(x) y_h = y$$

$$(1+t^2) y' = (1+t^2)^{-2} - 4ty$$

$$y' = (1+t^2)^{-3} - \frac{4ty}{(1+t^2)}$$

$$y' = \frac{-4t}{(1+t^2)} y + (1+t^2)^{-3}$$

$$(u(x) y_h)' = \frac{-4t}{(1+t^2)} u(x) y_h + (1+t^2)^{-3}$$

$$u'(x) y_h + u(x) y_h' + a u(x) y_h' = (1+t^2)^{-3}$$

$$u'(x) y_h = (1+t^2)^{-3}$$

$$u'(x) = \frac{1}{1+t^2}$$

$$u(x) = \arctan(t) + C$$

$$y = u(x) y_h$$

$$y = (\arctan(t) + C) \frac{1}{(1+t^2)^2}$$

$$y = \frac{\arctan(t)}{(1+t^2)^2} + C (1+t^2)^{-2}$$

$$0 = \frac{\arctan(1)}{4} + C \left(\frac{1}{4}\right)$$

$$-\frac{\pi}{16} = C/4$$

$$C = -\frac{\pi}{4}$$

$$y = \frac{\arctan(t)}{(1+t^2)^2} - \frac{\pi}{4} (1+t^2)^{-2}$$

$$20) y' = \cos x - y \sec x, y(0) = 1$$

$$y' = -y \sec x + \cos x$$

$$a(x) = -\sec x$$

$$IF = e^{\int \sec x dx}$$

$$IF = e^{\ln(\sec x + \tan x)}$$

$$= \sec x + \tan x$$

$$y' + \sec x y = \cos x$$

$$y'(\sec x + \tan x) + y(\sec^2 x + \tan x) = 1 + \sin x$$

$$(\sec x + \tan x) y' = 1 + \sin x$$

$$y \sec x + y \tan x = x - \cos x + C$$

$$1 + 0 = -1 + C$$

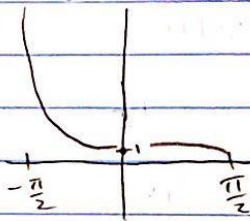
$$C = 2$$

$$y = \frac{x - \cos x + 2}{(\sec x + \tan x)}$$

$$\cos x \neq 0$$

$$x \neq \frac{\pi}{2}, -\frac{\pi}{2}$$

$$-\frac{\pi}{2} < x < \frac{\pi}{2}$$



$$24) x' = a(t)x + f(t)x^n$$

$$z = x^{1-n}$$

$$z' = (1-n)a(t)z + (1-n)f(t)$$

$$y' + y = y^2$$

$$z = y^{-1}$$

$$\frac{dz}{dx} = \frac{dz}{dy} \frac{dy}{dx}$$

$$= -y^{-2}(y^2 - y)$$

$$= -1 + \frac{1}{y} = 1 + z$$

$$IF = e^{-x}$$

$$z' = -1 + z$$

$$e^{-x}(z' - z) = -e^{-x}$$

$$(ze^{-x})' = -e^{-x}$$

$$ze^{-x} = e^{-x} + C$$

$$z = 1 + Ce^x$$

$$y = \frac{1}{1 + Ce^x}$$

$$26) p' = ap - bp^2$$

$$z = p^{-1}$$

$$\frac{dz}{dx} = \frac{dz}{dp} \frac{dp}{dx}$$

$$= -p^{-2}(ap - bp^2)$$

$$z' = -\frac{a}{p} + b = -az + b$$

$$IF = e^{\int a dx} = e^{ax}$$

$$e^{ax}(z' + az) = be^{ax}$$

$$(ze^{ax})' = be^{ax}$$

$$ze^{ax} = \frac{b}{a}e^{ax} + C$$

$$z = \frac{b}{a} + Ce^{-ax}$$

$$p = \frac{1}{\frac{b}{a} + Ce^{-ax}}$$

$$40) x' - \frac{2}{t^2}x = \frac{1}{t^2}, x(1) = 0$$

$$x' - \frac{2}{t^2}x = 0$$

$$x' = \frac{2}{t^2}x$$

$$\ln x = -\frac{2}{t}$$

$$x = e^{-2/t}$$

$$x = u(t)x_h$$

$$(u(t)x_h)' = \frac{2}{t^2}u(t)x_h + \frac{1}{t^2}$$

$$u'(t)x_h = \frac{1}{t^2}$$

$$u'(t) = \frac{e^{2/t}}{t^2} = e^{2/t}t^{-2}$$

$$u(t) = -\frac{1}{2}e^{2/t} + C$$

$$x = -\frac{1}{2}e^{2/t} \cdot e^{-2/t} + Ce^{-2/t}$$

$$x = -\frac{1}{2} + Ce^{-2/t}$$

$$0 = -\frac{1}{2} + Ce^{-2}$$

$$C = \frac{e^2}{2}$$

$$x = -\frac{1}{2} + \frac{e^2}{2}e^{-2/t}$$

$$42) a) T' + kT = 0$$

$$\frac{T'}{T} = -k$$

$$\ln T = -kt$$

$$T_h = (e^{-kt})$$

$$b) T' = 0$$

$$0 = -k(T_p - A)$$

$$T_p = A$$

$$c) T = T_h + T_p$$

$$T = (Ce^{-kt} + A)$$

$$d) T' + kT = -kA + H$$

$$T' + kT = 0$$

$$T_h = Ce^{-kt}$$

$$T' = 0$$

$$0 = -k(T_p - A) + H$$

$$T_p - A = \frac{H}{k}$$

$$T_p = \frac{H}{k} + A$$

$$T = Ce^{-kt} + \frac{H}{k} + A$$