Verifying correctness of Stainless programs using Coq

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Overview

The Translation Function

Definition (Translation function)

Let t be a function that assigns a (correctly typed) Coq program to every (correctly typed) Stainless program. Though t is not designed to be invertible, for simplicity let us introduce the t^{-1} notation where $t^{-1}(t(p)) = p$. Also for simplicity, let us denote $t(p)$ by $[p]$.

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def forall[T](l : List[T], p: T -> Boolean)-> forall0
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```
def forall[T](l : List[T], p: T -> Boolean)-> forall0
```

```
def forall[T](o: Option[T], p: T -> Boolean)-> forall1
```

From now on, unique names are assumed.

Translating simple types

Most constructs have exact Coq representation

- $[\text{BigInt}] = [\text{Int}] = Z$
- $[\text{Boolean}] = \mathit{bool}$ (not Prop)
- $[(a, b, \ldots)] = ([a], [b], \ldots)$
- $[f\,(p_1, p_2, \ldots)] = (f\ [p_1]\ [p_2] \ldots)$
- $[\lambda x.\ e] = (\text{fun } x \implies [e] \ldots)$

Dependent if-then-else

In the expression `if (p) then tb else fb`, we would like to propagate the boolean value of p to the branches. There are some solutions, but for more flexibility with the expressions, we defined our own.

```
Definition ifthenelse b A (e1: true = b → A) (e2: false = b
    → A): A :=
match b as B return (B = b → A) with
| true ⇒ fun H ⇒ e1 H
| false ⇒ fun H ⇒ e2 H
end eq_refl.
```

Used for:

- if then else
- non-exhaustive matches
- boolean and (&&)

Translating equality

Equality is usually translated using the coq equality.

In Coq, `a = b` is of type `Prop`.

```
Definition propInBool (P: Prop): bool :=
if (classicT P)
then true
else false.
```

Decidable type system: every Prop is True or False

```
Axiom classicT: forall P: Prop, P + ¬P.
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Exceptions: BigInt, Int, Boolean and Sets

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Contains sets with every common operation and `set_solver` tactic to solve obligations

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- generates obligations to "plug in" holes in the context
- also allows incomplete missing parameters, for which, it will generate obligations
- defines an Obligation Tactic to solve the generated obligations

The Program Library

For example, we can have the following definition:

```
Definition add (n : nat)(m: nat) (p: n >= m) : { x : nat | x
    > 2 } :=
Nat.add n 1.
```

The refined type will generate an obligation we have to prove:

```
∀ n m : nat, n ≥ m → (λ x : nat, x > 2) (n + 1)%nat
```

The Program Library

Using add, we can give an example for obligations generated to plug in holes in types:

```
Definition mul2(n: nat): nat := add n n _.
```

It will generate require us to give an expression of the type of the missing part, specifically in this case

```
∀ n: nat, n ≥ n
```

Structure

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Transforming Abstract Data Types

List Example

Best explained through an example:

```
sealed abstract class List[T] {}

case class Nil[T]() extends List[T] {}

case class Cons[T](h: T, t: List[T]) extends List[T] {}
```

Type definitions

Inductive type definitions for ADTSorts. The sematntics behind this is that an ADTSort is one of its constructors.

```
Inductive List (T: Type) :=
| Cons: T → ((List T) → (List T))
| Nil: List T.
```

Recognizing Types

We can use pattern matching to check for concrete subtype

```
Definition isCons (T: Type) (src: List T) : bool :=
match src with
| Cons _ _ _ ⇒ true
| _ ⇒ false
end.
```

Using the recognizers, we can define a type for the subtypes as a refined type.

```
Definition Cons_type (T: Type) : Type :=
{src: List T | (isCons T src = true)}.
```

Accessing Fields

Now that we have the subtypes, we can have accessors to their fields

```
Definition h (T: Type) (src: Cons_type T) : T :=
match src with
| Cons_construct _ f0 f1 ⇒ f0
| _ ⇒ let contradiction: False := _ in match contradiction with
    end
end.
```

Methods

A general method is built up from

- a function name f
- type parameters $T_1 \dots T_k$ and arguments p_1 of type U_1, p_2 of type U_2 ... , p_n of type U_n and a return type U_r
- a precondition pre of type `A => Boolean`, where $A \preceq \{U_1 x \dots x U_n\}$
- a body b of type $\{U_1 x \dots x U_n\} \Longrightarrow U_r$
- a postcondition *post* of type U_r`=> Boolean`

If there are more pre- and postcondition, they can be combined into one pre- and postcondition using conjunction.

Transforming Methods

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Preconditions can be expressed in Coq by taking an argument that states that the precondition holds, in oder means, taking an argument with the type pre $=$ true.

Postconditions can be expressed by a dependent return type. In case of a postcondition λres. post(res) the return type changes to $\{\text{res} : U_r \mid \text{post}(\text{res})\}$.

An example

```
def f[T1 ... Tk](p1: U1, ... pn: Un): Ur = {
require(pre(A))
b
} ensuring {res => post(res)}
```

Is translated to

```
Definition f (T_1: Type) ... (T_k: Type)(p_1: [U_1]) ... (p_n: [U_n])
    (prec: [pre] = true) :
{res: [U_r] | [post] res} :=
[b].
```

Transforming Recursive Methods

Translating Recursive Methods

Program library has Program Fixpoint to write recursive functions.

Makes it extremely hard to rewrite.

We used *CoqEquations* instead.

```
Equations negb (b : bool) : bool :=
  negb true := false ;
  negb false := true.
```

Persevering benefits of Program

We still want to handle pre- and postconditions using Program.

We can translate preconditions into dependent types:

```
Definition prt (T_1: Type) ... (T_k: Type)(p_1: [U_1]) ... (p_n: [U_n]) :
Type := [pre] = true
```

And postconditions just the same:

```
Definition rt (T_1: Type) ... (T_k: Type)(p_1: [U_1]) ... (p_n: [U_n])
    (prec: prt T_1 ... T_k p_1 ... p_n) : Type :=
{res: [U_r] | [post] res}
```

Persevering benefits of Program

The previous example in the recursive case would be translated to:

```
Equations f (T_1: Type) ... (T_k: Type)
          (p_1: [U_1]) ... (p_n: [U_n])
          (prec: prt T_1 ... T_k p_1 ... p_n) :
          rt T_1 ... T_k p_1 ... p_n prec :=
              f T_1 ... T_k p_1 ... p_n prec by rec
                  ignore_termination lt :=
              f T_1 ... T_k p_1 ... p_n prec :=
                  [b].
```

The `ignore_termination` is the decreasing measure in the recursion. We will have an obligation proving it to be decreasing.

Later, if we want to rewrite with the content of the function, we can rely on the fact that it will be expressed by `f_equation_1`.

Function Application

- If the method had preconditions, we pass _, so that Program will generate obligations for us.
- If there is a postcondition, the result has to be projected

```
proj1_sig (f T1 ... Tj p1 ... pn _)
```

Relation Between Proofs and Correctness

Validity - Stainless

Definition (Stainless-validity)

A Stainless program p is valid if

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$$[\text{error}] = \text{contradiction } \_$$

Contradictions can be expressed as the obligation to generate false.:

```
Definition contradiction (T: Type)(p: False): T := match p with
end.
```

Applying `contradiction` on an unknown variable will result in an obligation to derive `False` from the context.

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Theorem

For every (correctly typed) Stainless program p and its translation [p], *if* [p] *is proved to be valid by Coq, than p is also valid.*

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Theorem

For every (correctly typed) Stainless program p and its translation $[p]$*, if* $[p]$ *is proved to be valid by Coq, than p is also valid.*

Instead of proving this we will just sketch some notions why is it true.

Proof(ish)

Assumption

For every every Scala term `b: Boolean` *we assume that if* $b \rightarrow^* c$ *where* $c \in \{true, false\}$ *then* $[b] \rightarrow^*_{coq} [c]$.

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Let us assume that p does not contain any free variables.

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Fact

In Coq, if $\{\} \vdash t : T$*, then* $t \rightarrow^*_{coq} v$*, where v is a value of T*

Fact

There is no value of type `False`*.*

Automated Verification

Automatic Verification

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Define obligation tactic

Automatic Verification

How to solve obligations automatically?

Define obligation tactic

1. Fast tactics
2. Basic Tactics
3. Slow Tactics
4. Set Tactics
5. Case Analysis
6. Rewrite

Fast tactics

Some Coq tactics are fast to fail:

- cbn
- intros
- intuition
- discriminate
- ...

Fast Rewrites

Integer operations:

```
forall x y : Z, (x <=? y)= false ↔ y < x
```

Boolean operations:

```
forall a b : bool, eqb a b = true ↔ a = b
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Rewrite lemmas with Props and bools:

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forall P, propInBool P = true ↔ P
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Rewrite lemmas with Props and bools:

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```

Own rewrite lemmas about if-then-else:

```
forall b: bool, (if b then true else false)= b.
```

Fast Rewrites

Also some rewrite lemmas with dependent if-then-else:

```
forall T b e1 e2 value,
  ifthenelse b T e1 e2 = value ↔ (
  (exists H1: true = b, e1 H1 = value) ∨
  (exists H2: false = b, e2 H2 = value)
  ).
```

or

```
forall b (e1: true = b → bool),
  ifthenelse b bool e1 (fun _ ⇒ false) = true ↔
  exists H: true = b, e1 H = true.
```

Admitting termination obligations

Some obligations are related to termination, that we ignore currently.

We will have `(ignore_termination < ignore_termination)%nat` in the goal

Specific tactic to recognize it, and admit it.

```
match goal with
(...)
| |- (S ?T <= ?T)%nat ⇒
   unify T ignore_termination; apply False_ind; exact unsupported
(...)
end.
```

Basic Tactics

- Rewriting with boolean constants and value
- Destruction of exists, refinement, etc...
- Prop $\leftrightarrow$ bool rewrites

Slow Tactics

Coq's "not-so-fast" tactics:

- omega
- ring
- eauto (currently not included)

Set Tactics

Solve sets using `set_solver` of stdpp.

Also includes some basic set rewrites before:

- $\emptyset \cup s = s$
- $s \cup \emptyset = s$
- $s_1 = s_2 \implies s_3 \cup s_1 = s_3 \cup s_2$
- $s_1 = s_2 \leftrightarrow s_1 \subseteq s_2 \wedge s_2 \subseteq s_1$
- $x \in s_1 \vee x \in s_2 \leftrightarrow x \in (s_1 \cup s_2)$
- ...

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(forall H1: true = b, e1 H1 = value)→
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ifthenelse b T e1 e2 = value.
```

- perform case-analysis

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Right after fast tactics, whenever we see an **appropriate** equation we rewrite with it

Appropriate?

We only rewrite with the body of recursive functions, if their body will probably not be the subject of further refinement.

```
Rw: size T l = match l with
 | Nil T ⇒ 0
 | Cons T x xs ⇒ 1 + size T xs
end.
```

This will just introduce more branches to deal with, and we would like to perform destructing before rewriting.

Appropriate?

However, if we know that `l` is cons:

```
l, ys : List T
y: T
H : l = Cons y ys
(...)
Rw : size T l = match (Cons y ys) with
| Nil T ⇒ 0
| Cons T x xs ⇒ 1 + size T xs
end.
```

We can simplify it to:

```
  Rw: size T l = 1 + size T ys
```

Implementation

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`Admit Obligations` for dependencies

- Saves time
- Eliminates domino effect

A method is valid if the verification condition generated for it is valid, and the verification conditions of all of its dependencies are correct too.

Architecture

VerificationChecker $\rightarrow$ CoqVerificationChecker

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- Error: the verification failed, because of some internal error, most likely an error in the generated file

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CoqIO.scala: invoke coqc and check the output

Possible output:

- Valid
- Invalid: the execution terminated without solving one or more obligations
- Timeout: the verification timed out
- Error: the verification failed, because of some internal error, most likely an error in the generated file
- Canceled: the verification was canceled

Benchmark

List library

- Recursive, but not mutually recursive methods
- Inductive ADT's
- Set operations (contains, content, intersection, ...)
- Integer operations (size, indexOf, indexWhere, ...)

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- Inductive ADT's
- Set operations (contains, content, intersection, ...)
- Integer operations (size, indexOf, indexWhere, ...)

77 methods in total

Results

Verification run with 5 minutes timeout

- Valid: 66
- Invalid: 0
- Timeout: 11
- Error: 0

Conclusions

Not so bad

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Could be better

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Future work:

- Extending the supported stainless expressions
- Enhance tactics so that they handle the whole library
- Speed up tactics, external `set-solver` tactic is really slow to fail, blocks every execution requiring a rewrite with definition
- Check termination

Questions?