Price-management under uncertainty

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InFoMM
Industrially Focused Mathematical Modelling

How do you decide the price of your products?

Retail managers must decide the prices of hundreds of products based on company policy, revenue and profit maximisation. We will

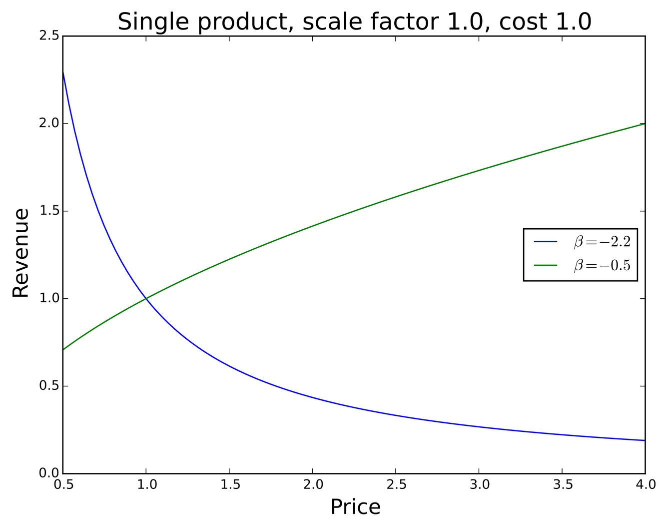
- model the demand dependence on price changes to aide in the decision making process,
- argue for a multiobjective or "soft" view of constraints,
- define a concise framework for decision making under uncertainty.

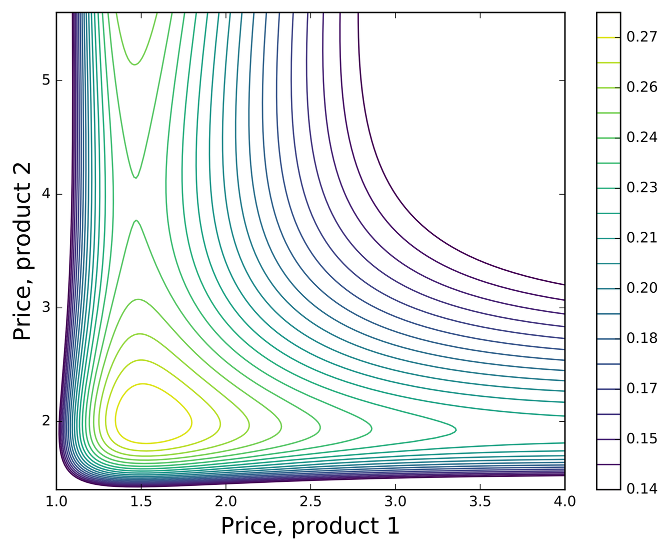
Given a product unit price, P, and product demand per unit time, $V(P)$, the *elasticity* β represents how demand changes with price [3]. If we assume constant elasticity, then

$$V(P) = \alpha P^{\beta}, \qquad \text{where } \beta = \frac{\mathrm{d}V/V}{\mathrm{d}P/P}. \tag{1}$$

The scale factor α can be used to incorporate time-dependent factors like seasonality. The price of one product can affect demand for other products, which is modelled similarly with *cross-elasticities*. Revenue and profit are central to businesses, and are defined per item as

$$\text{Revenue}(P) = PV(P), \tag{2}$$
$$\text{Profit}(P) = (P - C)V(P), \qquad \text{given } \textit{marginal cost } \text{C} \tag{3}$$





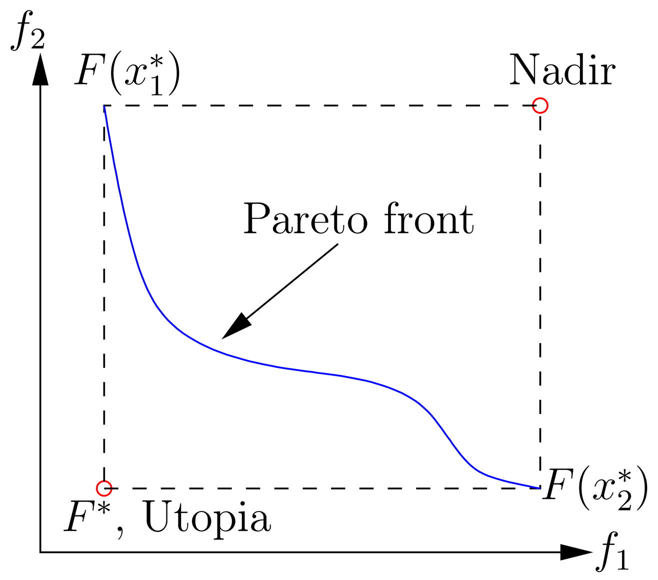
Total profit for two products

Multiobjective optimisation

Typical business decisions aim to optimise an objective $f_1(x)$, given a target $f_2(x)$. Maximising revenue without sacrificing too much profit can be expressed in a general optimisation form. Given prices $S \subset \mathbb{R}^n$, define $f_1(x) = -\sum_i \text{Revenue}_i(x)$, $f_2(x) = Pr_{\min} - \sum_i \text{Profit}_i(x)$, and solve

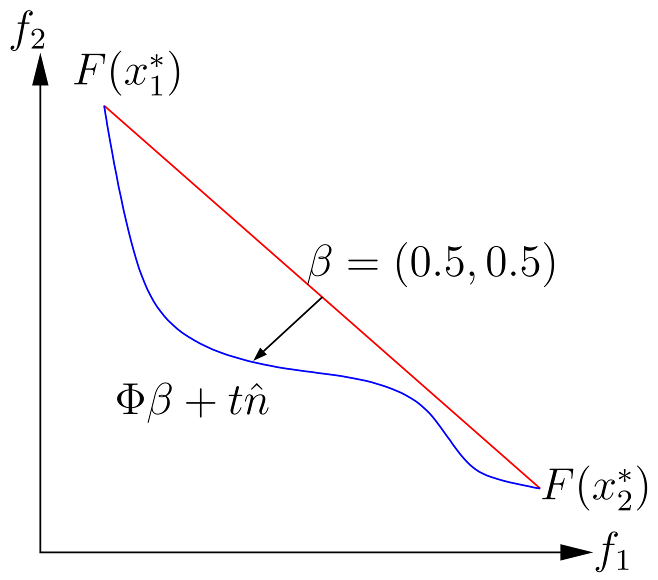
$$\min_{x \in S} \{ f_1(x) \mid f_2(x) \leq 0 \}. \tag{4}$$

Constraints should be viewed as objectives, to emphasise that tradeoffs are being made. Given objectives f_1, f_2, a *Pareto optimal* point $x^* \in S$ is one such that there are no feasible x that can improve one objective without sacrificing the other. The set of all such points is called the *Pareto front*. Active constraints force your position on the front.



Weighted Sum, 15 points. Overlaps

NBI method, 15 points. Uniform spacing



NBI subproblem

Single-objective reformulations of the problem can be used to trace out the Pareto front.

- *Weighted Sum* associates a relative importance weighting λ with one objective. It has issues including a nonlinear decision response in λ, the same decision is made with different weights, and it cannot find concave regions of the front.
- The *Normal Boundary Intersection* method [1] generates a uniformly spaced set of Pareto points by moving in a direction $\hat{n}$ away from the convex hull of individual minima, generated by the matrix Φ with columns $F(x_i^*) - F^*$.

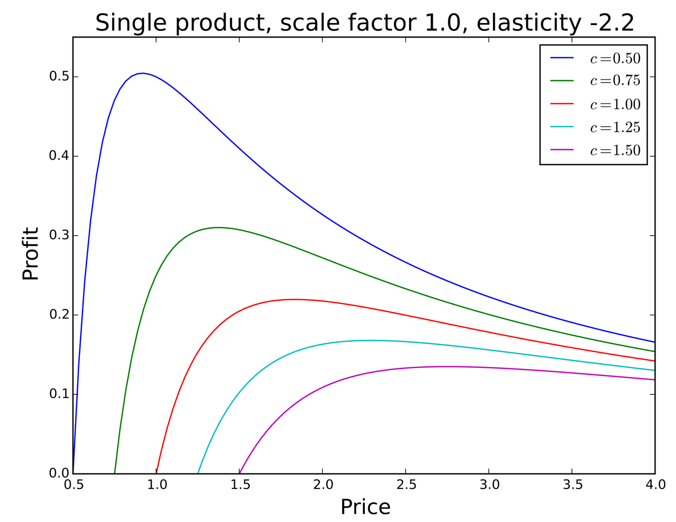
$$\min_{x \in S} \{ \lambda f_1(x) + (1 - \lambda) f_2(x) \} \qquad \max_{x \in S, t \in \mathbb{R}} \{ t \mid \Phi\beta + t\hat{n} = F(x) - F^* \} \tag{5}$$

Decisions that depend on future events

With incomplete information about our system, we model the uncertainty with random variables. This raises questions like:

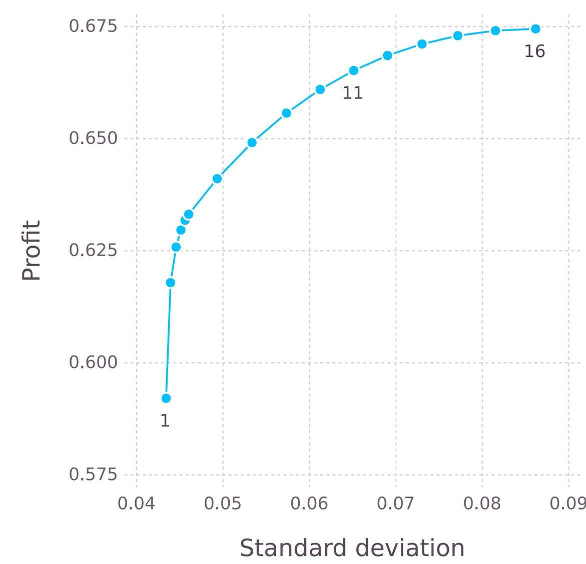
- How do we make a preference ordering between choices? The objective has several outcomes for each decision.
- What does feasibility mean? Violations of constraints are only known after we make our decision.

A motivating example in retail is the uncertainty of marginal cost for products. Calculations dependent on cost can appear in both the objective and the constraints. We make the problem deterministic with *risk measures*, functionals that represents the decision maker's attitude to risk. With the viewpoint of constraint violations as tradeoffs, this allows us to define feasibility in terms of the risk measure.

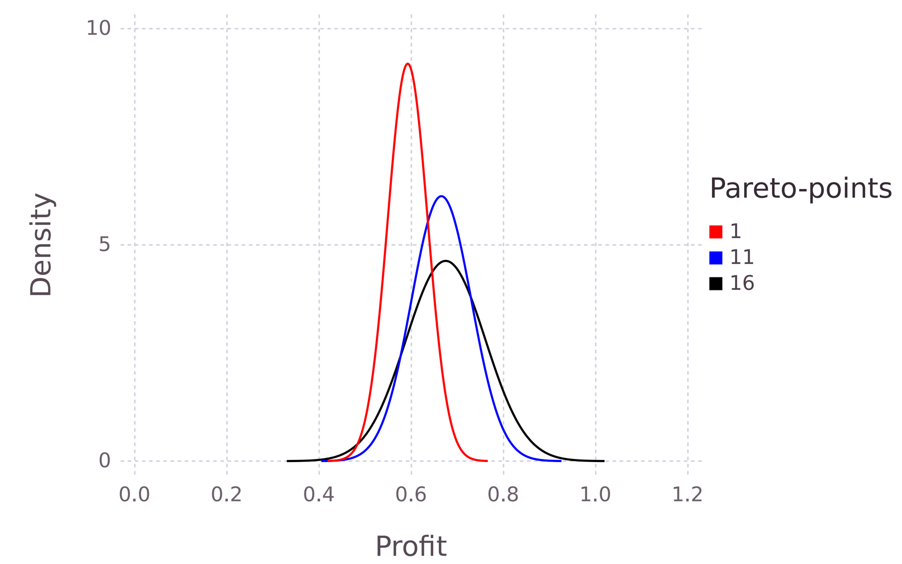


Profit depends on uncertain cost

We can for example balance optimising expected value and minimising the standard deviation. This ensures a decision is sufficiently stable, and that we are within a safe distance from violating constraints. The balance between expected value and stability fits naturally in the multiobjective framework. The following plots show results from maximising profit of two products:



Pareto front with Weighted Sum



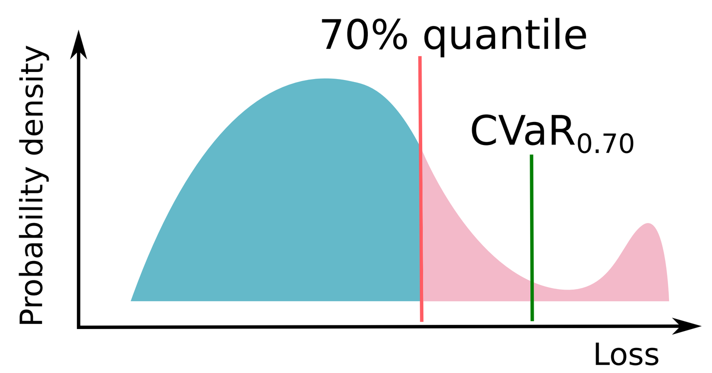
Distributions of profit, assuming costs are Gaussian

Standard deviation penalises better-than-average outcomes, and does not take into account how much we violate constraints. *Conditional Value at Risk* (CVaR_α) solves these issues by calculating the expected loss of the worst $1 - \alpha$ proportion of outcomes [2].

$$\text{CVaR}_\alpha(X) = \mathbb{E}[X \mid X \geq F_X^{-1}(\alpha)], \qquad F_X \text{ is the c.d.f. of X.} \tag{6}$$

CVaR is continuous in α and cover risk attitudes from neutral to worst-case scenarios.

$$\lim_{\alpha \to 0} \text{CVaR}_\alpha(X) = \mathbb{E}[X] \qquad \text{risk-neutral} \tag{7}$$
$$\lim_{\alpha \to 1} \text{CVaR}_\alpha(X) = \sup X \qquad \text{worst-case} \tag{8}$$



Conditional Value at Risk takes tail risk into account

Further work

We want to model marginal costs as stochastic processes, and run multi-week price optimisations based on real product data. This will let us investigate how much decisions change with different risk measures.

References

[1] I. Das and J. E. Dennis. Normal-boundary intersection: A new method for generating the pareto surface in nonlinear multicriteria optimization problems. *SIAM Journal on Optimization*, 8(3):631–657, 1998.

[2] R. T. Rockafellar. Coherent approaches to risk in optimization under uncertainty. *Tutorials in operations research*, 3:38–61, 2007.

[3] G. J. van Ryzin. Models of demand. In Ö. Özer and R. Phillips, editors, *The Oxford handbook of pricing management*. Oxford University Press, 2012.

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