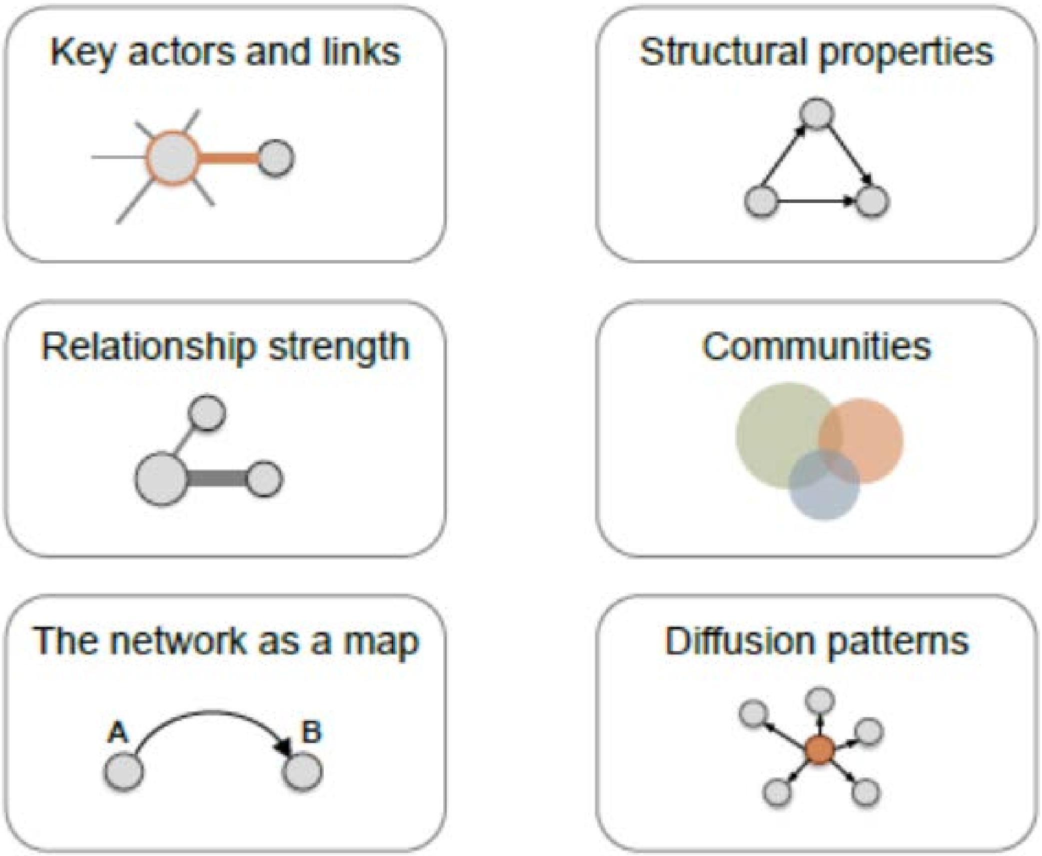
Graph Visualization

Ng Yen Kaow

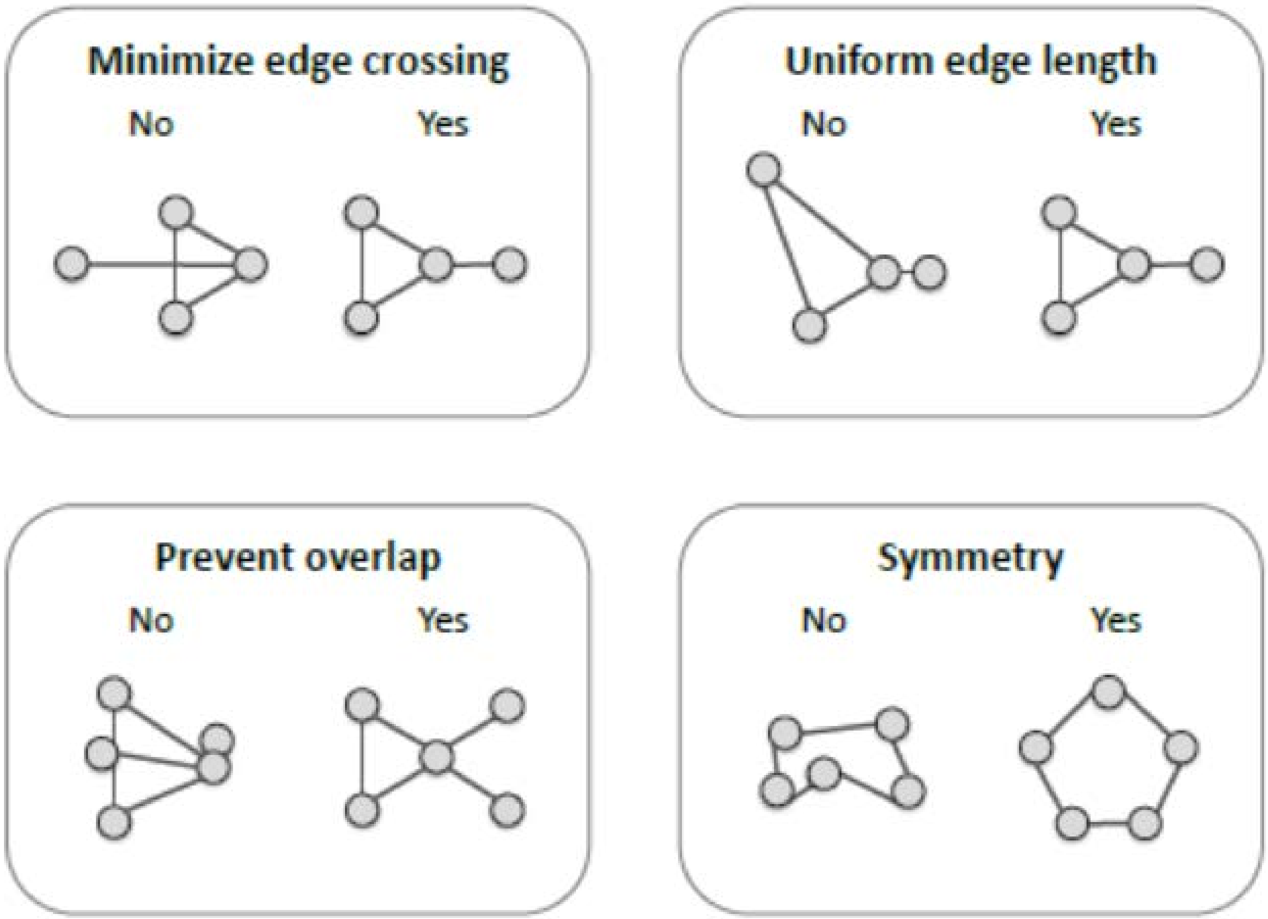
Network visualization

- From "Network Visualization with R" (kateto.net/polnet2015)
  - The goals of visualization are



Network visualization

- From “Network Visualization with R” (kateto.net/polnet2015)
  - The aesthetic issues of visualization

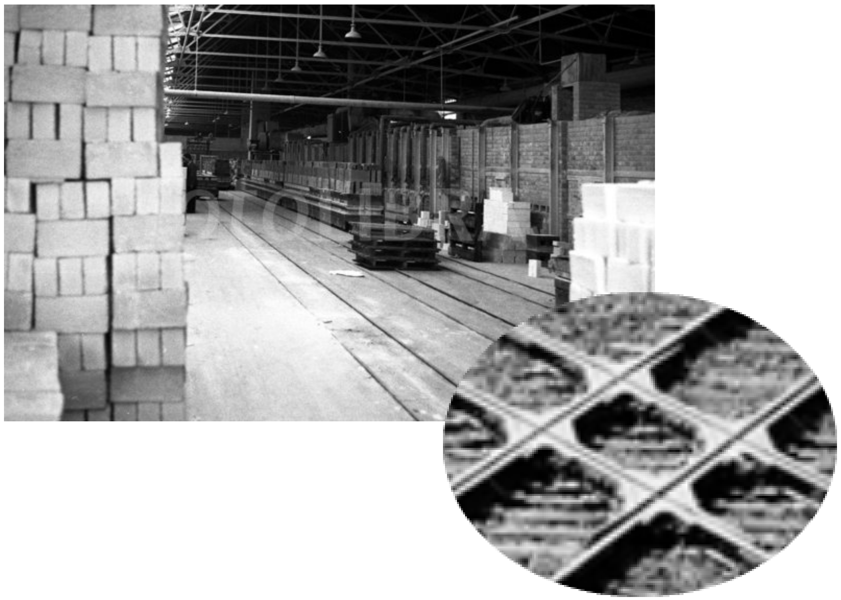


Aesthetics

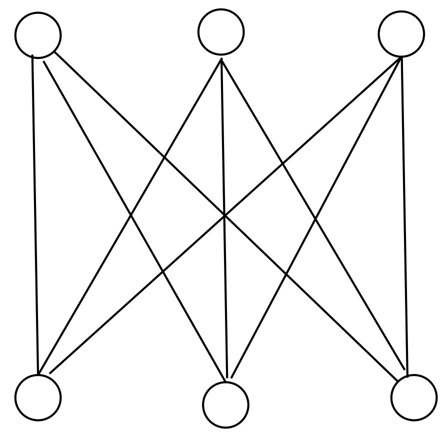
- Graph drawing is not simple
  - Turán's brick factory problem
    - Crossing number
  - Clustered planarity
  - There are conferences dedicated to these problems
    - International Symposium on Graph Drawing & Network Visualization

Turán's brick factory problem

- Origin: Hungarian mathematician Pál Turán



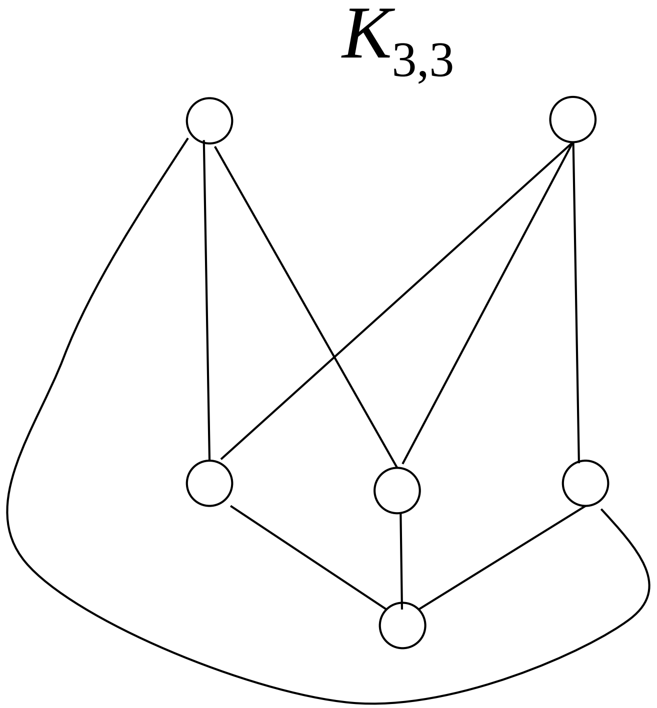
$K_{3,3}$



- How many crossings can there be in a complete bipartite graph?

Turán's brick factory problem

- For $K_{3,3}$, 1

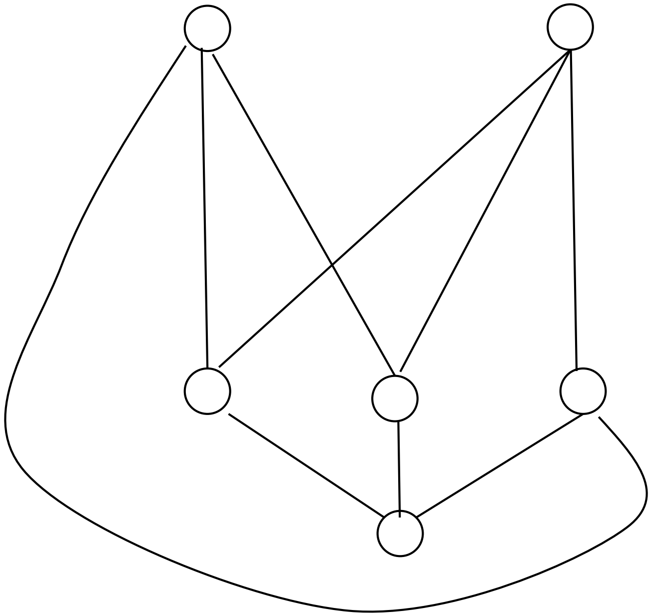


Turán's brick factory problem

- Zarankiewicz’s conjecture
  - For a complete bipartite graph $K_{m,n}$, at most $\left\lfloor \frac{n}{2} \right\rfloor \left\lfloor \frac{n-1}{2} \right\rfloor \left\lfloor \frac{m}{2} \right\rfloor \left\lfloor \frac{m-1}{2} \right\rfloor$

$$K_{3,3} : \left\lfloor \frac{3}{2} \right\rfloor \left\lfloor \frac{2}{2} \right\rfloor \left\lfloor \frac{3}{2} \right\rfloor \left\lfloor \frac{2}{2} \right\rfloor = 1$$

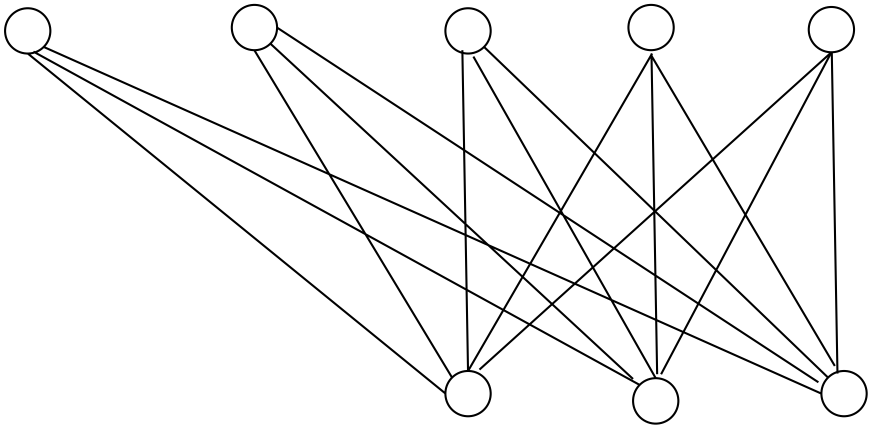
$K_{3,3}$



Turán's brick factory problem

- What about $K_{5,3}$

$$K_{5,3}:\left\lfloor\frac{5}{2}\right\rfloor\left\lfloor\frac{4}{2}\right\rfloor\left\lfloor\frac{3}{2}\right\rfloor\left\lfloor\frac{2}{2}\right\rfloor=4$$



- See if you can get 4 crossing edges

Turán's brick factory problem

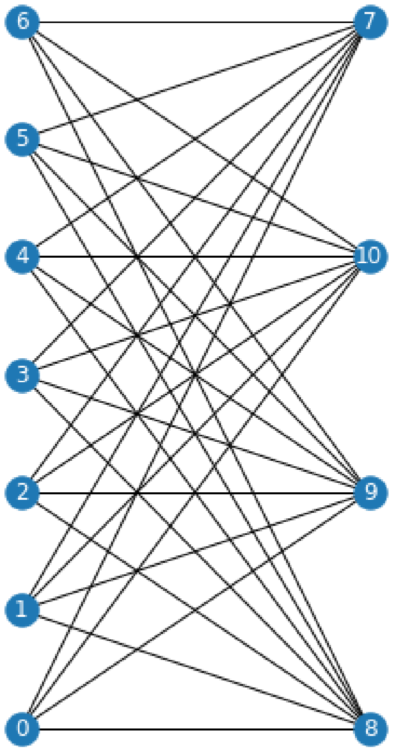
- Try $K_{7,4}$

$$K_{7,4}:\left\lfloor\frac{7}{2}\right\rfloor\left\lfloor\frac{6}{2}\right\rfloor\left\lfloor\frac{4}{2}\right\rfloor\left\lfloor\frac{3}{2}\right\rfloor=18$$

Graph layout

- R allows many graph layouts
- One of these is the bipartite layout
  - Show $K_{7,4}$ using the bipartite layout

```
import networkx as nx
import matplotlib.pyplot as plt

G = nx.complete_bipartite_graph(7, 4)

partition = nx.get_node_attributes(G, 'bipartite')

nodes = [i for i in partition.keys() if partition[i]==0]

nx.draw(G, pos=nx.bipartite_layout(G, nodes))

plt.show()
```



Other NetworkX layouts

`bipartite_layout`	Position nodes in two straight lines
`circular_layout`	Position nodes on a circle
`kamada_kawai_layout`	Position nodes using Kamada-Kawai path-length cost-function
`planar_layout`	Position nodes without edge intersections
`random_layout`	Position nodes uniformly at random in the unit square
`shell_layout`	Position nodes in concentric circles
`spring_layout`	Position nodes using Fruchterman-Reingold force-directed algorithm
`spectral_layout`	Position nodes using the eigenvectors of the graph Laplacian
`spiral_layout`	Position nodes in a spiral layout
`multipartite_layout`	Position nodes in layers of straight lines

Layered/hierarchical graph drawing

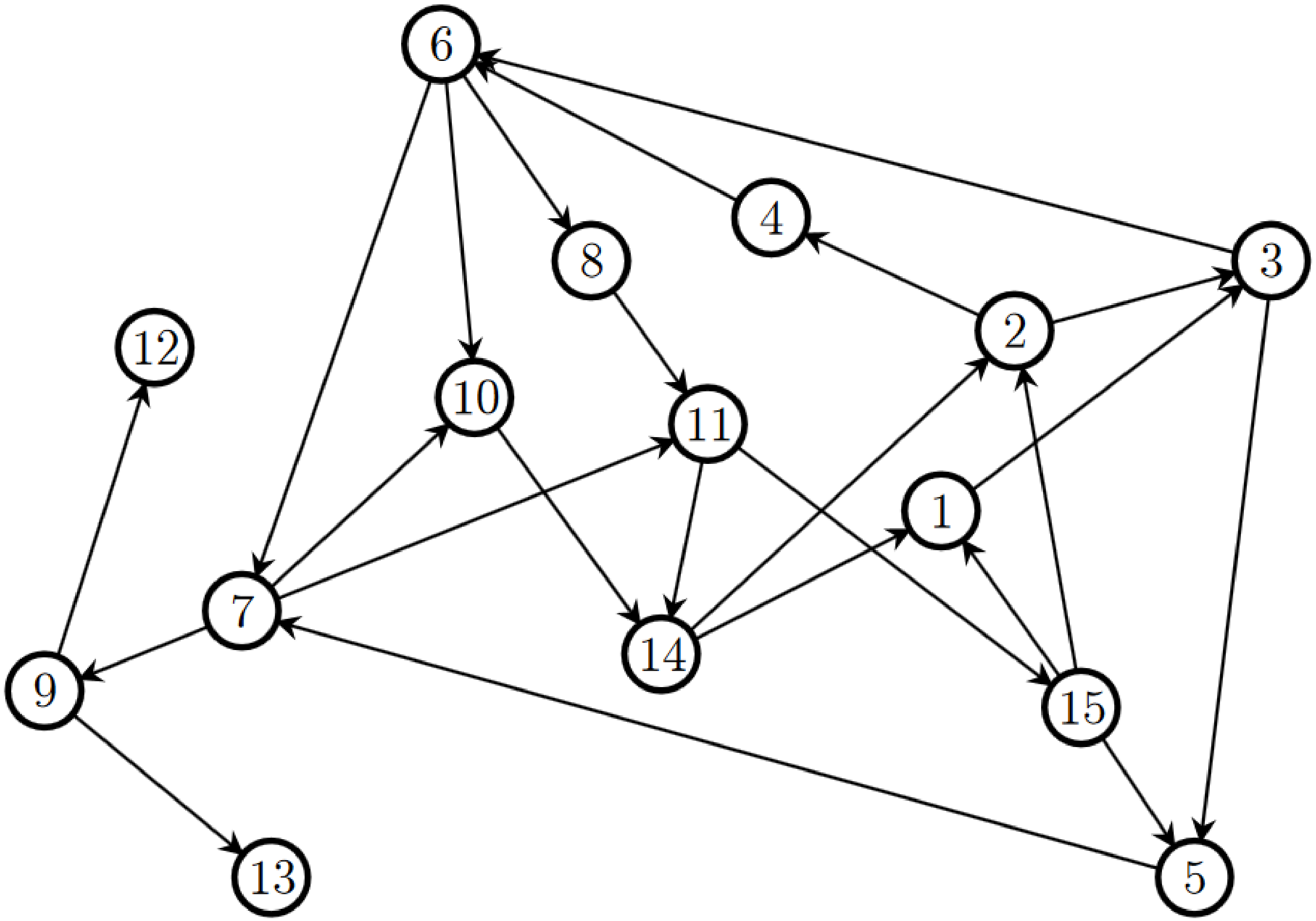
- A directed graph can be represented as a "hierarchy"
  - A cycle-free digraph (roughly)
  - Process flow diagram
  - Function call diagram
- An example is the Sugiyama layout

Sugiyama layout

- Follows the following principles as much as possible
  - Edges should point in a uniform direction
  - Short, straight edges
  - Uniformly distributed nodes
  - Minimize edge crossing

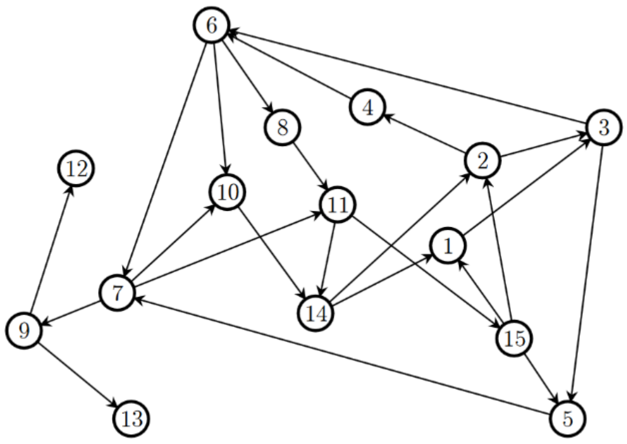
Sugiyama layout

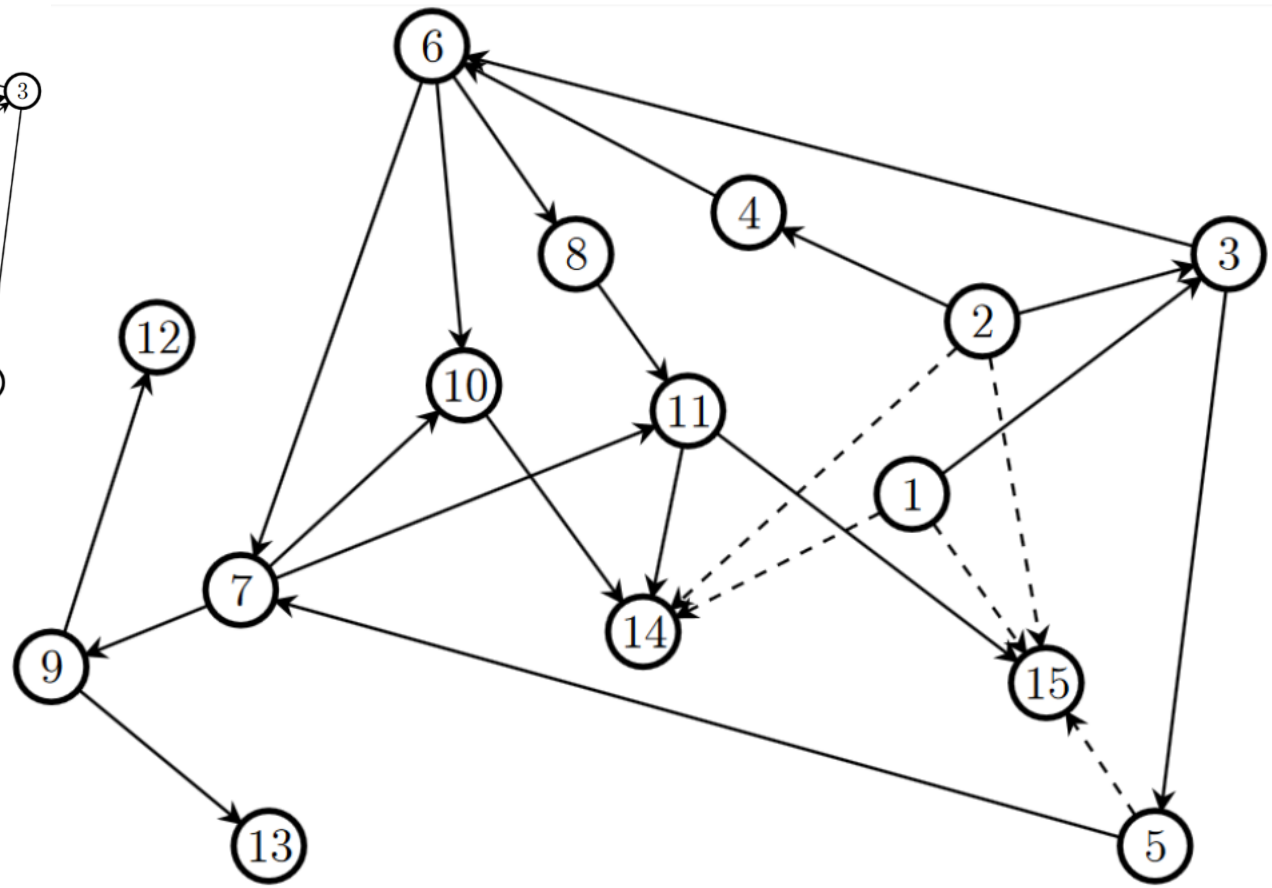
- Example: Convert the following graph to Sugiyama layout



Sugiyama layout

1. Remove cycles

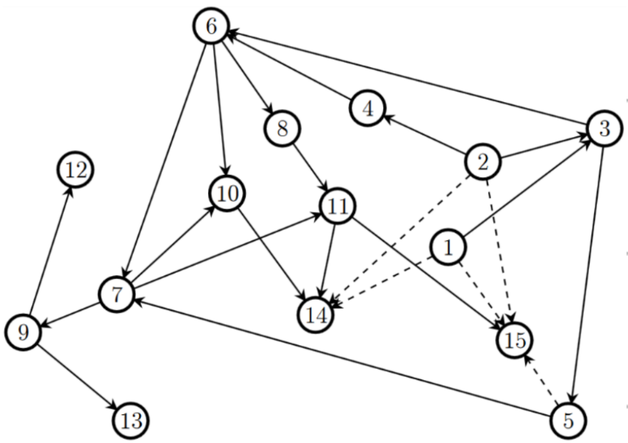


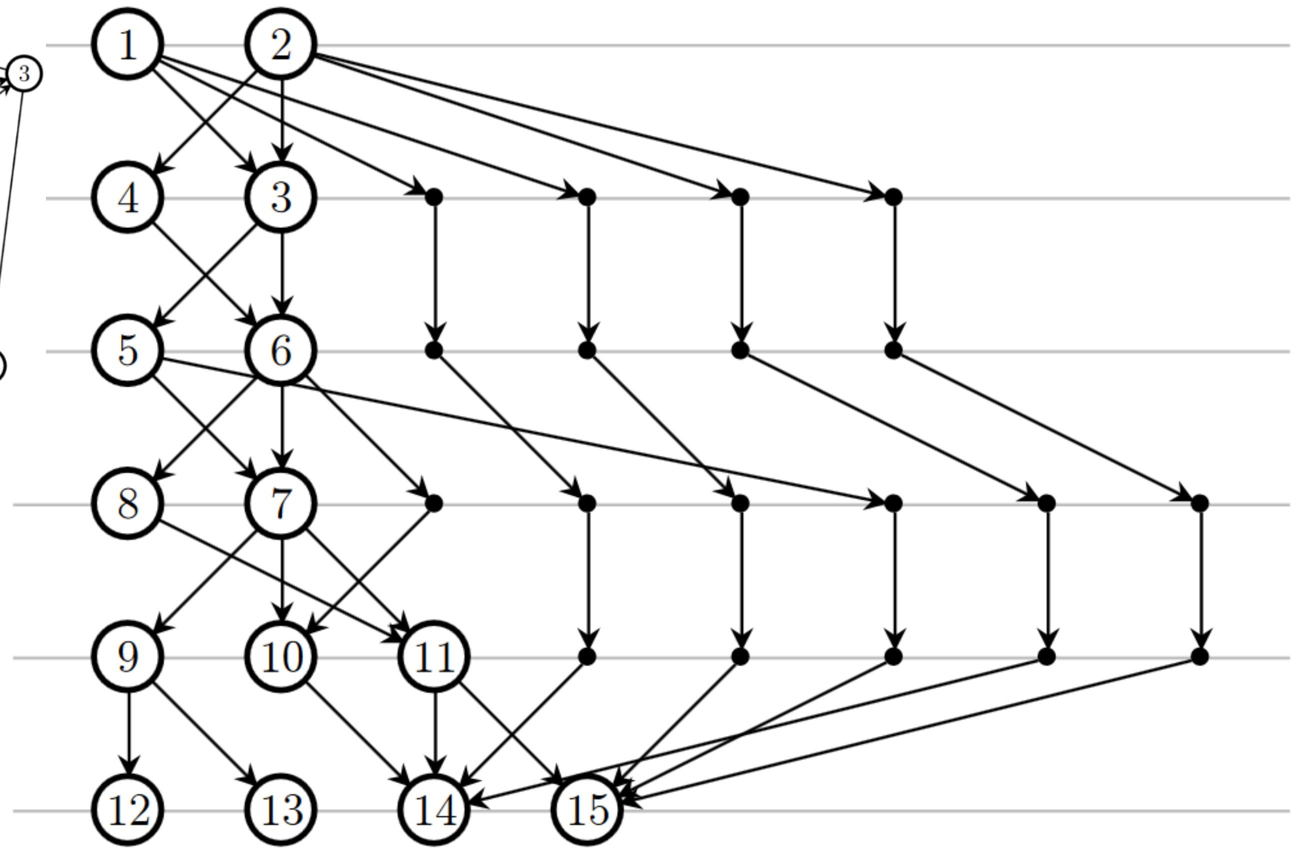


From “Handbook of Graph Drawing and Visualization”, Roberto Tamassia

Sugiyama layout

2. Arrange nodes in levels





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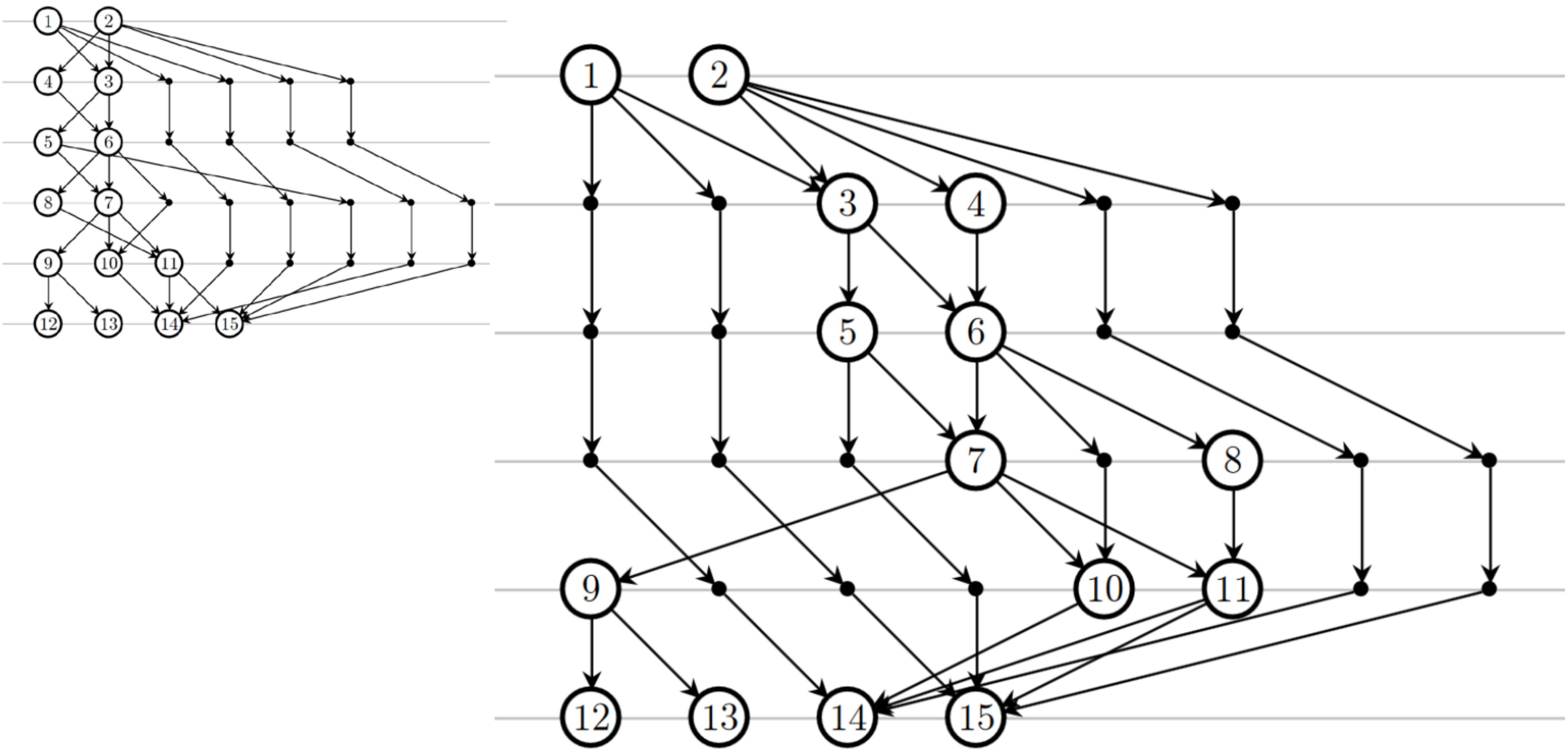
Sugiyama layout

3. Reduce crossing edges



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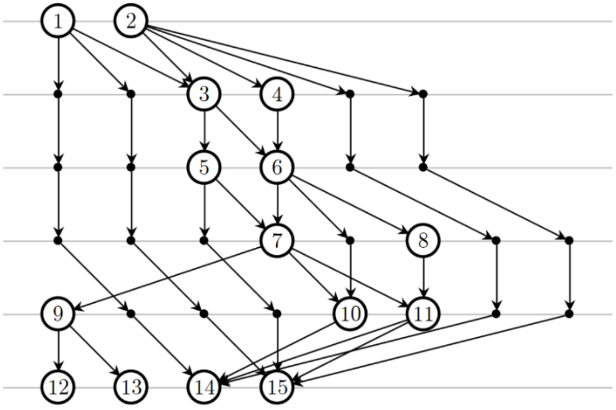
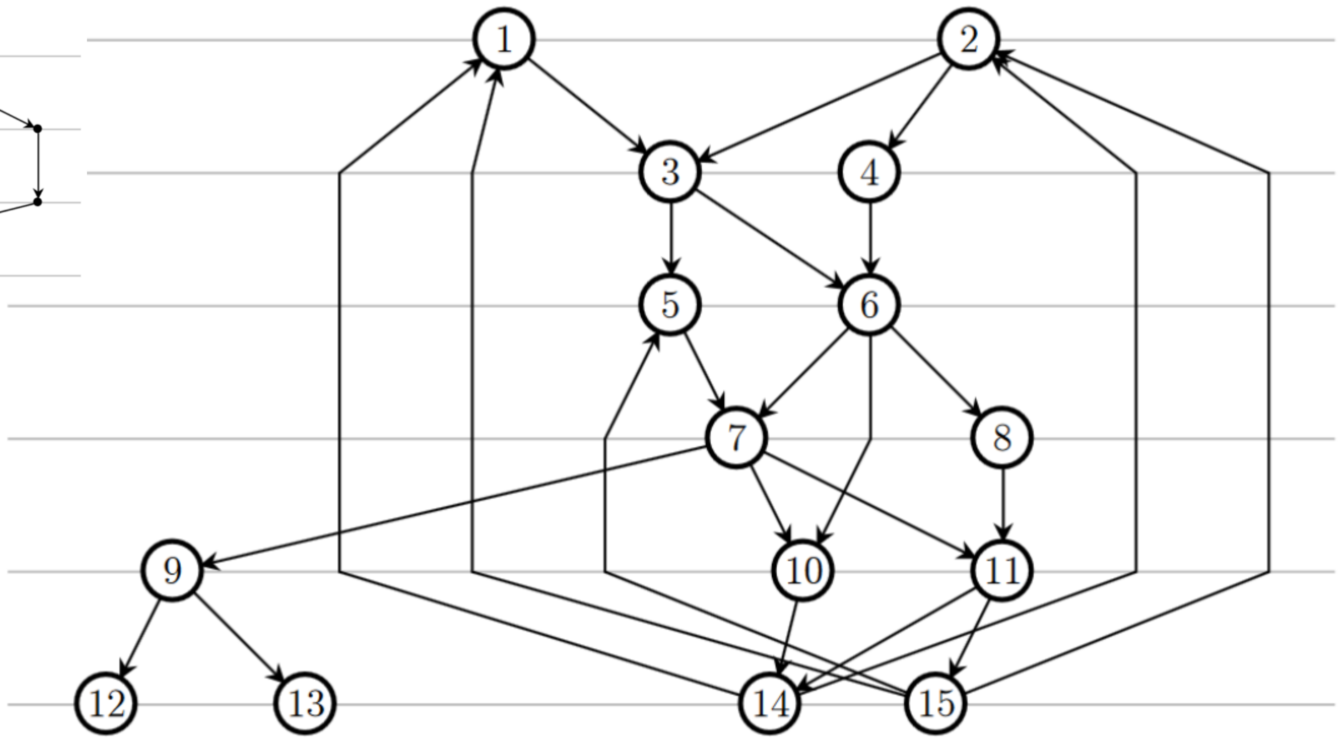
Sugiyama layout

4. Straighten edges





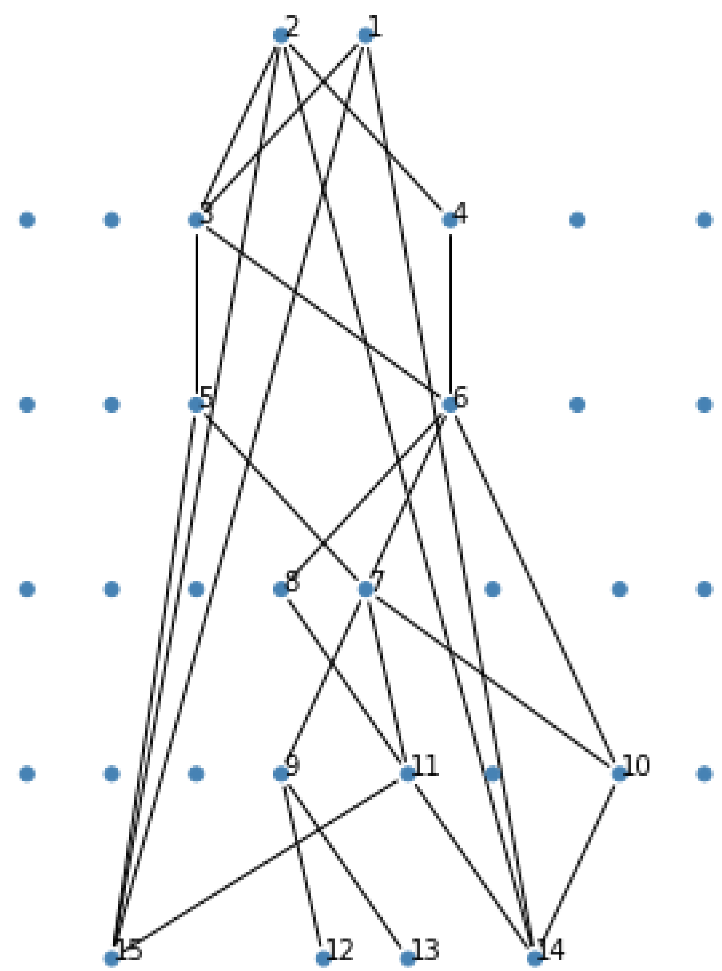
From “Handbook of Graph Drawing and Visualization”, Roberto Tamassia

Sugiyama layout

- How to do it in Python (igraph)

Regrettably this layout is not available in NetworkX

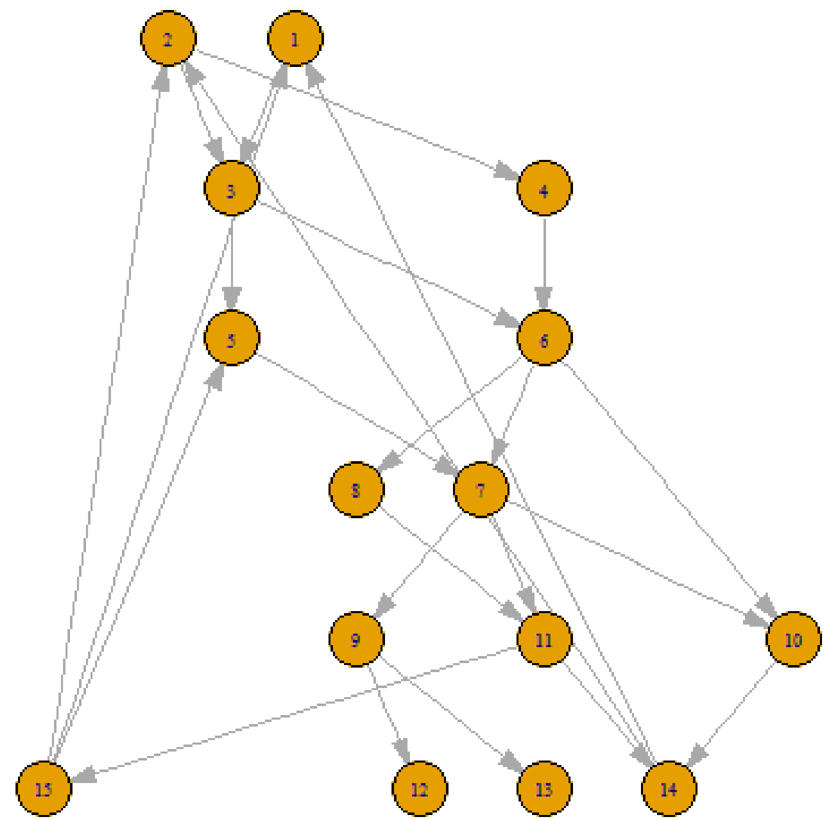
```
import igraph as ig

edges = [[x, y] for (x, y) in
np.genfromtxt('sugiyama.csv',
delimiter=',', dtype=np.integer)]

G = ig.Graph(edges=edges)

layers = [0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 4,
5, 5, 5, 5]

layout = G.layout_sugiyama(layers=layers)

_, ax = plt.subplots()

ig.plot(G, layout=layout,
vertex_label=list(range(1,16)), target=ax)

ax.invert_yaxis()
```



Sugiyama layout

- How to do it in R (igraph)

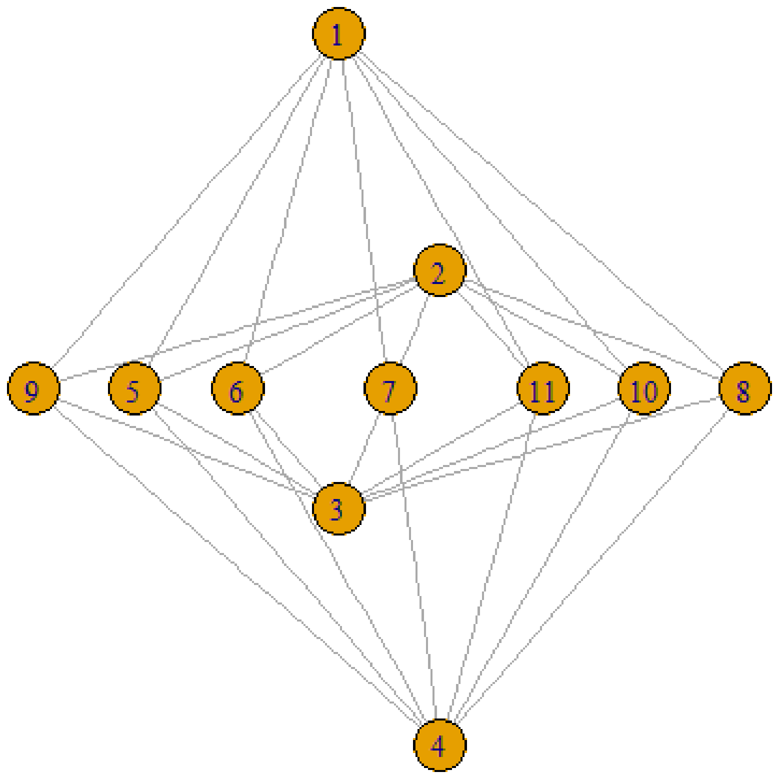
```
# Construct graph
G <-
graph_from_data_frame(sugiyama)

# Define layers
layers <- list(c(1, 2), c(4, 3),
c(5,6), c(8,7), c(9,10,11),
c(12,13,14,15))

# Perform layout
sugilayout <-
layout_with_sugiyama(G,
layers=apply(sapply(layers,
function(x) V(G)$name %in% x), 1,
which))

# Show Sugiyama plot
plot(G, layout=sugilayout$layout,
vertex.label.cex=0.9,
edge.arrow.size=0.1)
```



Same as the Python output

$K_{7,4}$ revisited

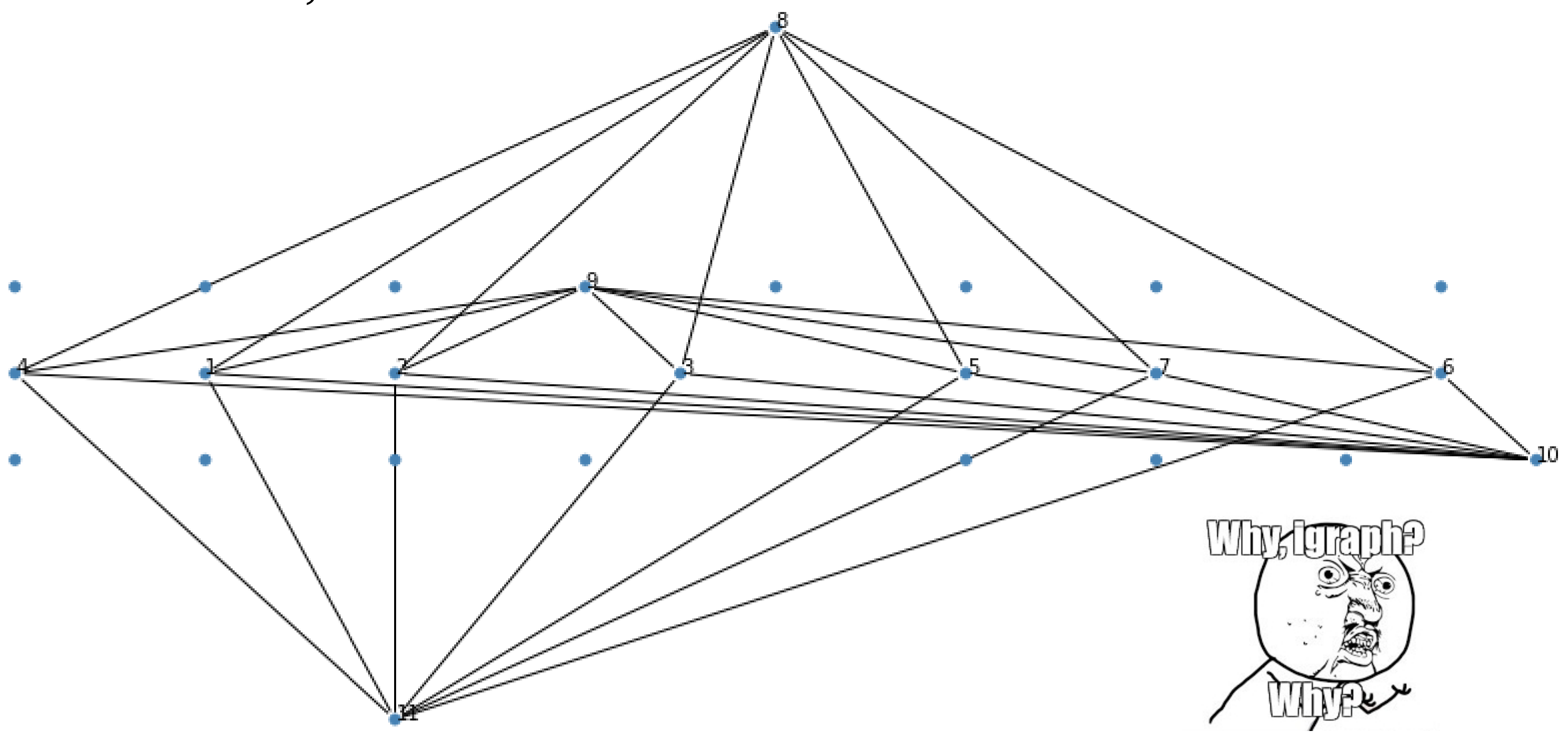
- Show $K_{7,4}$ using Sugiyama layout (in R)




```
G <- graph_from_data_frame(K74, directed=FALSE)
layers <- list(c(1), c(), c(2), c(5,6,7,8,9,10,11), c(3), c(), c(4))
sugilayout <-  layout_with_sugiyama(G, layers=apply(sapply(layers, function(x) V(G)$name
%in% x), 1, which))
plot(G, layout=sugilayout$layout, vertex.label.cex=0.9)
```

$K_{7,4}$ revisited

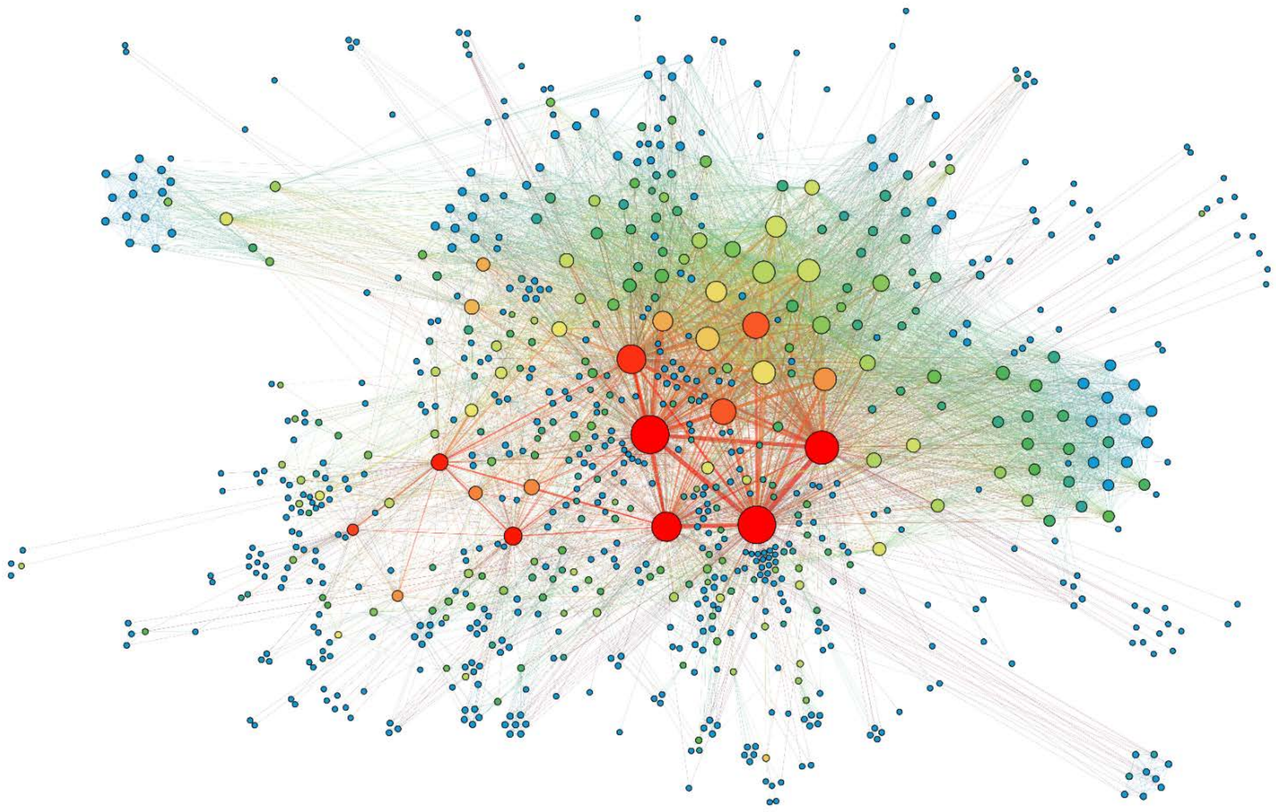
- Show $K_{7,4}$ using Sugiyama layout (Python)




```
G = nx.complete_bipartite_graph(7, 4)
edges = list(G.edges)
G = ig.Graph(edges=edges)
layers = [4, 4, 4, 4, 4, 4, 4, 0, 3, 5, 8]
layout = G.layout_sugiyama(layers=layers)
_, ax = plt.subplots()
ig.plot(G, layout=layout, vertex_label=list(range(1,12)), target=ax)
```

Force-directed graph layout

- Commonly available, e.g `nx.spring_layout`



- PalsGraph adds community information in applying the forces

Force-directed graph layout

- PalsGraph implementation
  - Each pair of vertexes i and j repel each other with a force F_r/d_{ij}
  - Each pair of vertexes i and j that are connected by an edge attracts each other with a force F_a/d_{ij}
  - (The centers of) two communities i and j repel each other with a force F_{rc}/d_{ij}
  - (The centers of) each community c attracts every vertex v in community with force $F_{ac}d_{cv}$
  - The center of the graph attracts vertices

Software for Visualization

Ng Yen Kaow

graphviz

- A set of command-line tools

`dot`	Plots hierarchical layouts of directed graphs
`neato`	Plots spring model layouts
`twopi`	Plots radial layouts
… and a few more	

- Grandfather of graph-drawing tools
- File format (“dot”) widely supported
- Used by other software
  - Source codes visualization – doxygen
    - https://www.doxygen.nl/
  - Python modules, e.g. graph-tool
- As a Python module
  - https://github.com/pygraphviz/pygraphviz

Python modules

- Most popular modules are
  - NetworkX (https://networkx.org/)
  - igraph (http://igraph.org/)
    - For R or Python or C
    - Actually, mostly made for R until recently
  - graph-tool (https://graph-tool.skewed.de/)
    - For Python only
  - Networkit (https://networkit.github.io/)
    - For Python only
- In general, graph-tool is the fastest and NetworkX is the slowest (pure Python)
  - However, NetworkX is likely the most complete, well-documented, and easy to use

Gephi

- GUI tool (but very powerful)
- https://gephi.org/
- File format (gexf) widely supported
- Export graphviz file from Python

```
def export_edge_list(dismat, labels=None, filename="edges.csv", delim=",", header=True):
    f = open(savedir + filename, 'w')
    if header:
        f.write("Source,Target\n")
    loc = np.where(dismat > 0)
    for (i, j) in [(i, j) for (i, j) in zip(loc[0], loc[1]) if i < j]:
        if labels == None:
            f.write(str(i) + delim + str(j) + "\n")
        else:
            f.write("\"" + labels[i] + "\"" + delim + "\"" + labels[j] + "\"\n")
    f.close()
```

- Allows vertices to be colored by their communities
  - See “`gephi_communities.pdf`”